FRG approach to QCD phase diagram with isospin chemical potential

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QCD phase diagram (2-flavor)

- Three-dimensional phase diagram
  Temperature [T]
  Quark chemical potential [μ]
  Isospin chemical potential [μ_f]

$$\mu_{\rm u} = \mu + \mu_f$$
$$\mu_{\rm d} = \mu - \mu_f$$

- Isospin chemical potential is associated by the charge of subgroup of flavor rotation (τ_3).

- charged pion condensation occur for large μ_f.

Sign problem

$$D = U^{-1} D^{\dagger} U \qquad U = \begin{pmatrix} 0 & \gamma_5 \\ \gamma_5 & 0 \end{pmatrix}$$

$$\mathrm{det[D]} = \mathrm{det[D^{\dagger}]}$$

- Quark determinant is real (μ=0,$\mu_f \neq$0).

M. Alford, A. Kapustin and F. Wilczek, Phys. Rev. D59, 054502 (1999).

- Lattice Monte Carlo simulation is available.
- We can check the reliability of effective models.

Property at T = 0

- Silver blaze
  Critical isospin chemical potential corresponds to 1/ 2 of lowest meson mass (pion).

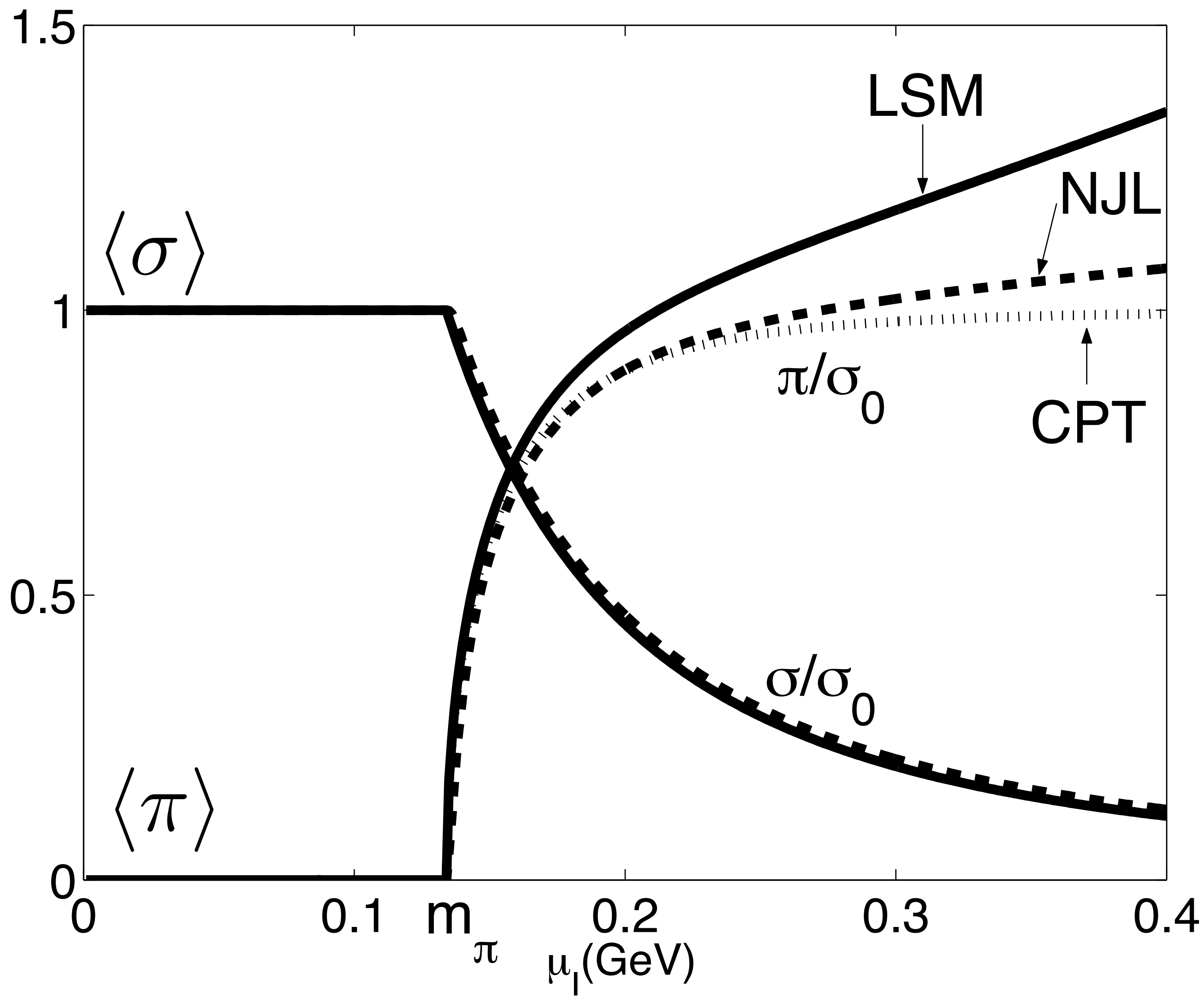
"Silver blaze" Arthur Conan Doyle

T. D. Cohen, Phys. Rev. Lett. 91, 222001 (2003); arXiv:hep-ph/0405043.

$$\mu_{cf} = \frac{1}{2} M_\pi$$

- Any effective model must realize this property.

Results of another models



L. He, M. Jin, and P. Zhuang, Phys. Rev. D 71 (2005) 116001.

- Pion condensation occur.
- Another models satisfy SB relation.

The aim

- Describing QCD phase diagram on T and μ_f plane by using functional renormalization group equation on Quark Meson (QM) model
- Reproducing Silver blaze property

How to find mass

conventional way

$$\left.\frac{\partial U(\sigma)}{\partial \sigma}\right|_{\sigma=\sigma_{\min}} = 0$$

$$m_\sigma^2 = 2U' + 4\sigma^2 U''$$

$$m_\pi^2 = 2U'$$

Our way

$$\left.\frac{\partial U(\sigma)}{\partial \sigma}\right|_{\sigma=\sigma_{\min}} = 0$$

$$\Gamma_\sigma^{(0,2)}(p_0 = m_\sigma, \sigma_{\min}) = 0$$

$$\Gamma_\pi^{(0,2)}(p_0 = m_{\pi_0}, \sigma_{\min}) = 0$$

- Masses are determined by zero of 2 point function instead of curvature of potential.
- We need to calculate 2 point function.

Functional Renormalization Group (FRG)

$$k\partial_k \Gamma_k[\varphi] = \frac{1}{2}\mathrm{Tr}\left[\frac{k\partial_k R_{kB}}{R_{kB} + \Gamma_k^{(0,2)}[\varphi]}\right] - \mathrm{Tr}\left[\frac{k\partial_k R_{kF}}{R_{kF} + \Gamma_k^{(2,0)}[\varphi]}\right]$$

C. Wetterich, Phys. Lett. B301, 90 (1993)

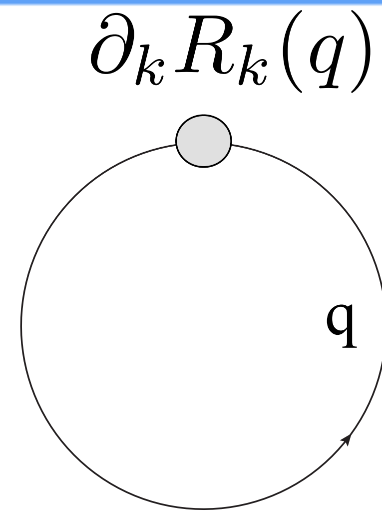
- Γ_k is effective action at scale k.

$\Gamma_{k=\Lambda} = S$ ⟶ $\Gamma_{k=0} = \Gamma$

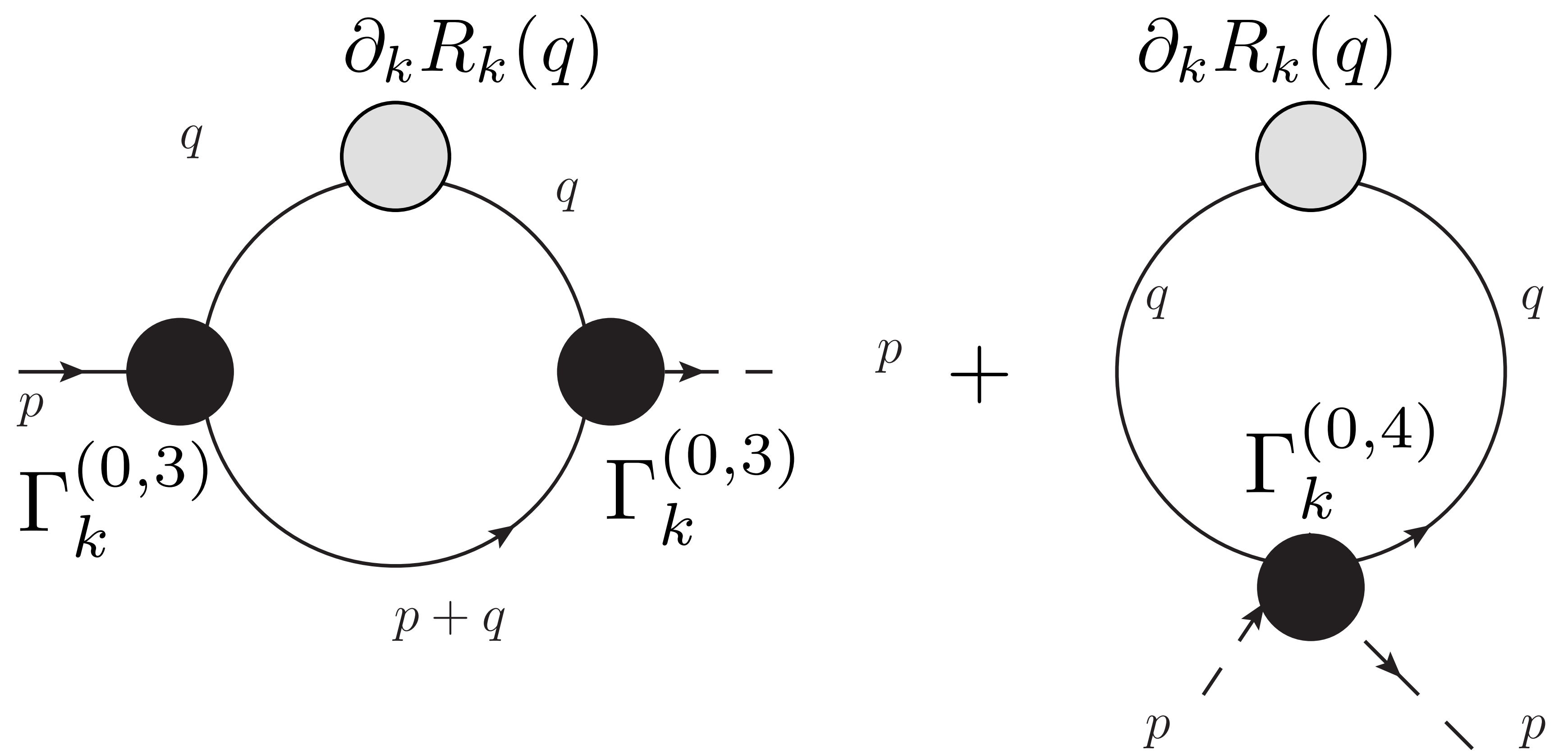
classical → quantum

Diagramatic representation

$$\frac{\partial \Gamma_k[\sigma]}{\partial k} \sim$$



$$\frac{\partial \Gamma_k^{(0,2)}[p,\sigma]}{\partial k} \sim$$



- Equations never close.
- We need some truncation.

Local Potential approximation (LPA)

$$
\begin{aligned}
\Gamma_k[\phi] &= \bar{\psi}\left[i\partial\!\!\!/ + g(\sigma + i\gamma_5\vec{\pi}\cdot\vec{\tau}) + \mu_f\gamma_0\tau_3\right]\psi \\
&+ \frac{1}{2}\partial\sigma\partial\sigma + \frac{1}{2}\partial\pi_0\partial\pi_0 + \frac{1}{2}\vec{\partial}\pi_+\vec{\partial}\pi_+ + \frac{1}{2}\vec{\partial}\pi_-\vec{\partial}\pi_- \\
&+ (\partial_0 + 2\mu_f)(\pi_+ + i\pi_-)(\partial_0 - 2\mu_f)(\pi_+ - i\pi_-) \\
&+ U_k(X = \sigma^2 + \pi_3^2, Y = \pi_+^2 + \pi_-^2) - c\sigma
\end{aligned}
$$

- LPA leads to the flow equation for effective action.
- We neglect wave function renormalization.

BMW approximation

$q << k$

$$\Gamma^{(0,3)}_{abi}[p,-p+q,-q] \to \Gamma^{(0,3)}_{abi}[p,-p,0] = \frac{\partial \Gamma^{(0,2)}_{ab}[p]}{\partial \phi_i}$$

$$\Gamma^{(0,4)}_{abij}[p,-p,q,-q] \to \Gamma^{(0,4)}_{abij}[p,-p,0,0] = \frac{\partial^2 \Gamma^{(0,2)}_{ab}[p]}{\partial \phi_i \partial \phi_j}$$

J. P. Blaizot, R. Mendez-Galain and N. Wschebor, Phys. Rev. E 74, 051116 (2006)

- Equations are closed up to two point function.

Consistency

$$\Gamma_{\sigma}^{(0,2)}[p=0] = M_{\sigma,\mathrm{curv}}^2 \qquad \Gamma_{\pi}^{(0,2)}[p=0] = M_{\pi,\mathrm{curv}}^2$$

- BMW approximation does not have consistency with curvature mass.
- We can realize consistency under the another approximation (RPA- like).

$$\Gamma_{abi}^{(0,3)}[p,-p,0] = \frac{\partial \Gamma_{ab}^{(0,2)}[p]}{\partial \phi_i} \to \frac{\partial^3 U}{\partial \phi_a \partial \phi_b \partial \phi_i}$$

$$\Gamma_{abij}^{(0,4)}[p,-p,0,0] = \frac{\partial^2 \Gamma_{ab}^{(0,2)}[p]}{\partial \phi_i \partial \phi_j} \to \frac{\partial^4 U}{\partial \phi_a \partial \phi_b \partial \phi_j \partial \phi_k}$$

How to calculate real time correlation

$$\partial_k \Gamma^{(0,2)}(\omega_n) = f(\omega_n, \phi)$$

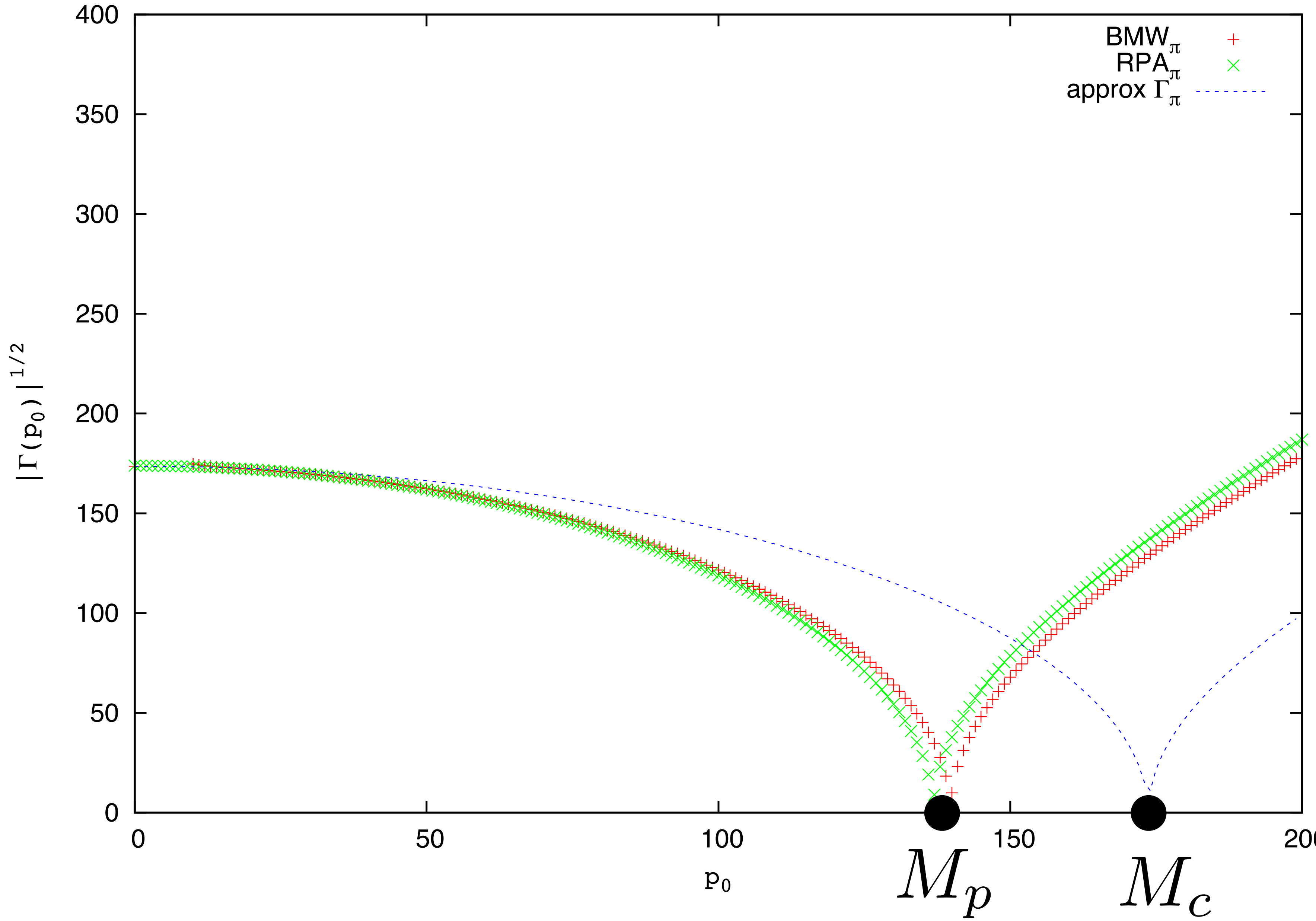
$$\downarrow$$

$$\partial_k \Gamma_R^{(0,2)}(p_0) = \mathrm{Re\ f}(\omega_\mathrm{n} \to \mathrm{ip_0} + \epsilon, \phi)$$

- After all analytic calculation along the Matsubara formalism, we make analytic continuation.
- We directory solve flow equation for real time correlation function.

pion 2 point function

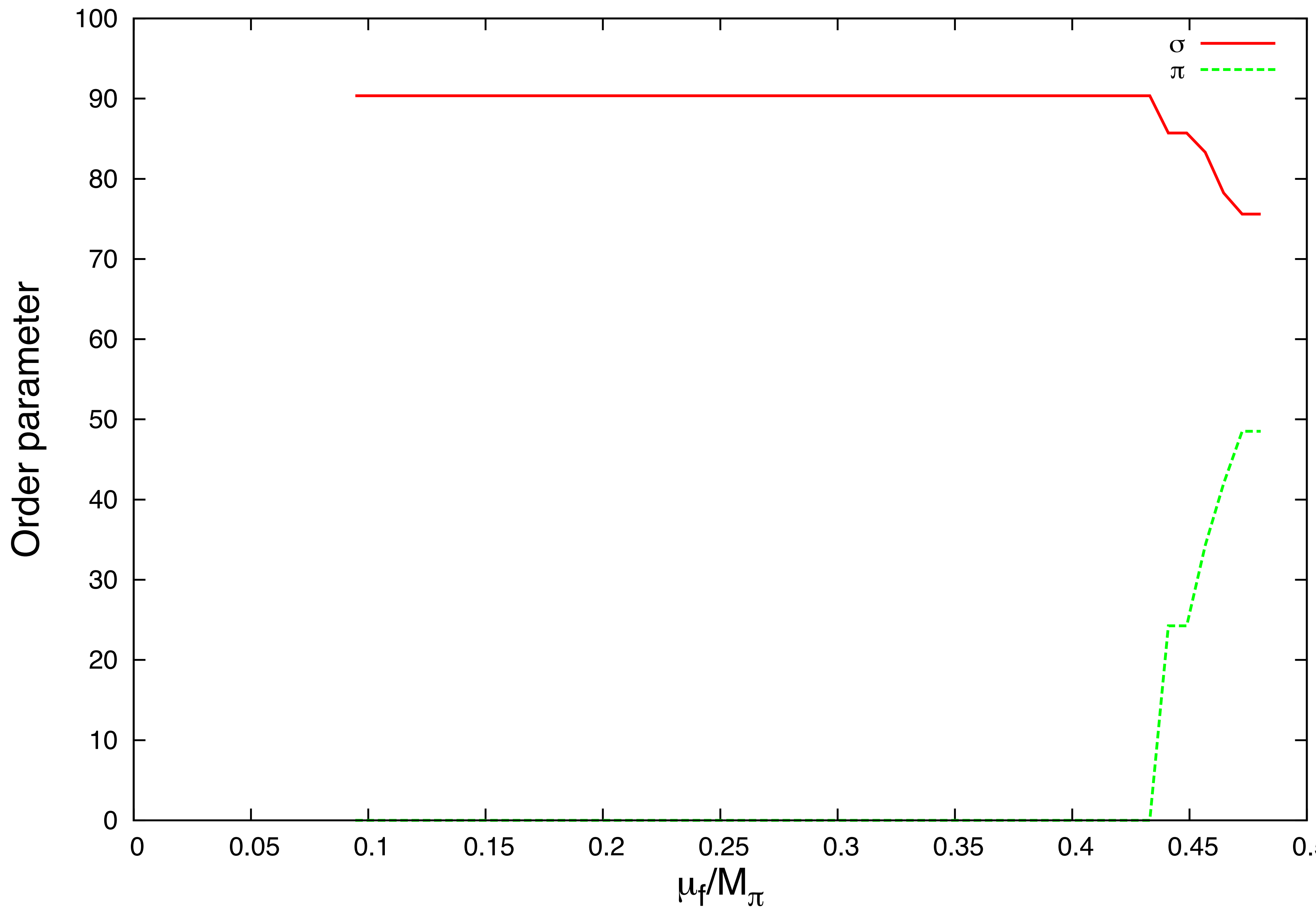
At the vacuum



$$\text{Approx } \Gamma = -p_0^2 + M_c^2$$

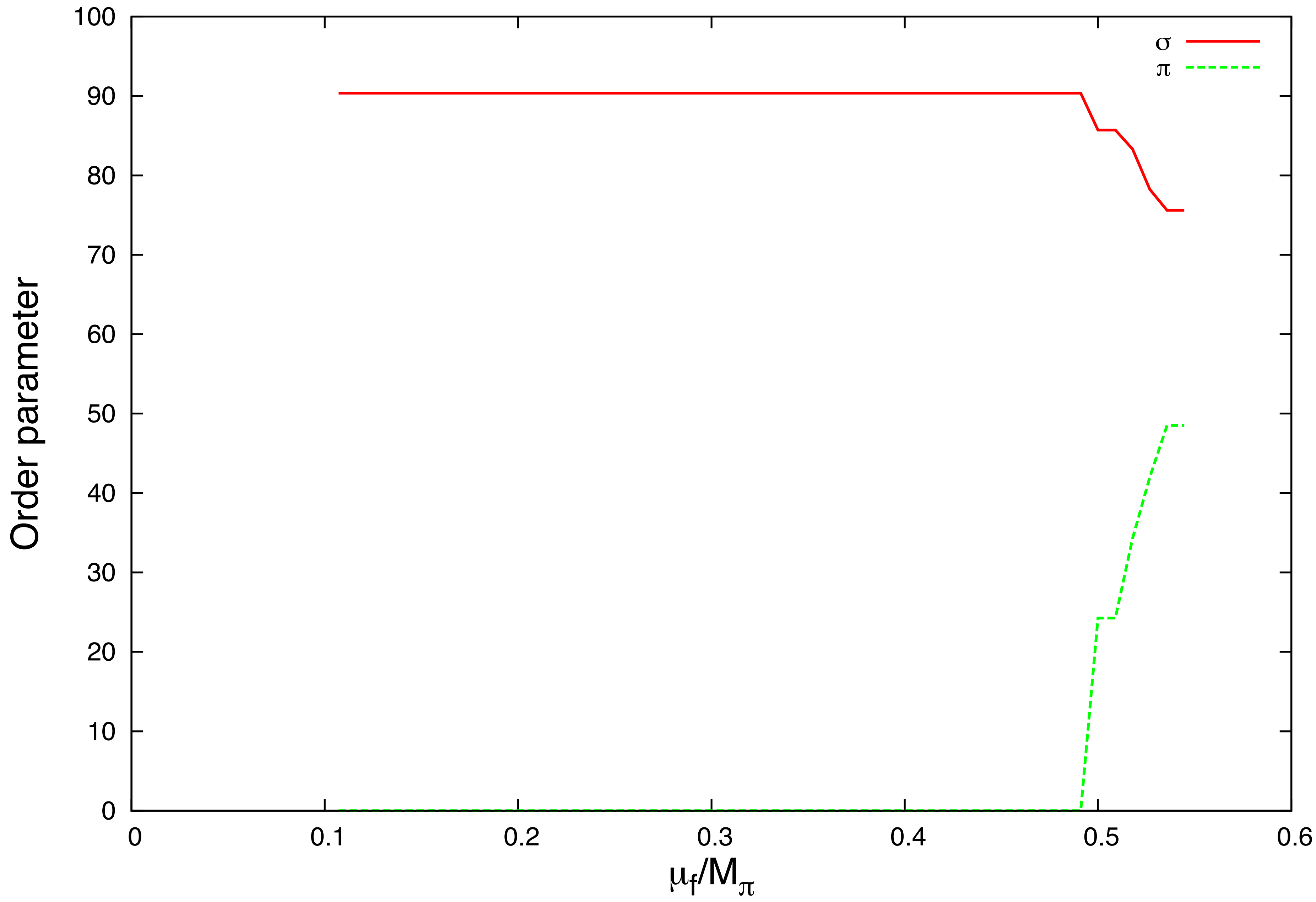
- In this parameter set, LPA and BMW have good agreement at zero external momentum.

phase transiton ($T = 0$)



- normalized by curvature mass (M_c)
- There is the difference between onset of pion condensation and curvature mass

phase transition ($T = 0$)



- normalized by pole mass (M_p)
- pole mass almost satisfy silver blaze relation

Results

- The value of curvature mass and pole mass are different at 17%.
- Numerical result support pole mass.
- We must use pole mass as meson masses instead of curvature mass.

Phase diagram (mean field)

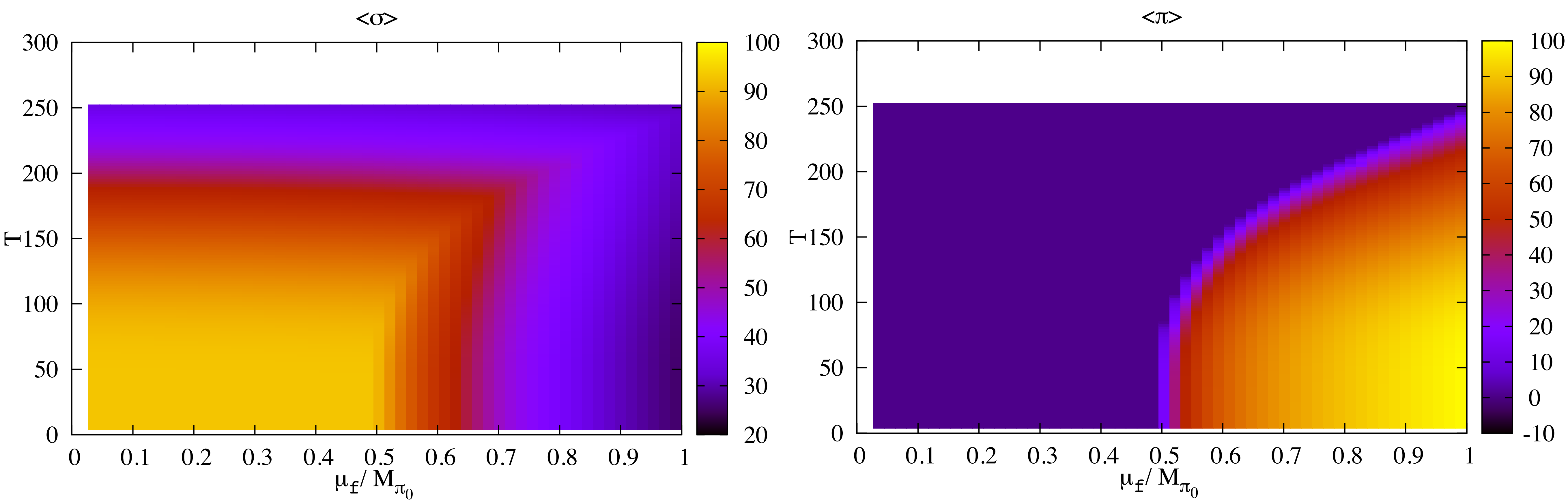
$$\frac{\partial U_K}{\partial k} = \text{~~Meson part~~} + \text{Fermion part}$$

$$U_\Lambda = a(X+Y) + b(X+Y)^2 - 2\mu_f^2 Y - c\sqrt{X}$$

$a = 0.1\Lambda^2[\text{Mev}^2]$
$b = 3.5$
$c = 0.08\Lambda^3[\text{Mev}^3]$
$\Lambda = 600[\text{Mev}]$
$g = 3.2$

- Neglect meson contribution for flow equation.
- Only quark determinant term remains (mean field).

phase diagram



- phase transition of sigma is cross over.
- For large isospin chemical potential sigma became small.

Conclusion

- We consider phase diagram with isospin chemical potential by using Functional renormalization group equation.
- We calculate real time mesonic 2 point function and redefine meson masses by pole of correlation function.

Thank you for your attention