2000

Ultracold atomic gases:
artificial magnetic fields & "pion" condensation

Gordon Baym
University of Illinois

Facets of Strong Interaction Physics
18 January 2012
Hirschegg 40 + Bengt 60 = 100

Darmstadt 1999

GB, BF & G. Grinstein, Fluctuations and long-range order in finite-temperature pion condensates. Nucl. Phys. B. 210, 193-209 (1982).

G.B, J.-P. Blaizot & BF, Quark deconfinement transition of hot nuclear matter in the soliton bag model. Proc. Hirschegg, 1982.

GB,BF, J.-P. Blaizot, M. Soyeur & W. Czyz. Hydrodynamics of ultra-relativistic heavy ion collisions. Nuclear Physics A407, 541-570 (1983); Lecture Notes - Physics 198, Recent Progress in Many-Body Theories (1984), pp. 60-67.

BF, GB, & J.-P. Blaizot, Dynamical quark-hadron phase transitions in heavy ion collisions. Phys. Lett. 132B, 291-294 (1983).

GB, BF, & P. V. Ruuskanen, Stability of hydrodynamic flow in ultra-relativistic nucleus-nucleus collisions. Proc. Hirschegg 1987.

G.B, W. Florkowski, BF & P. V. Ruuskanen, Convective stability of hot matter in ultrarelativistic heavy-ion collisions, Nucl. Phys. A540, 659 (1992).

Quark Deconfinement Transition of Hot Nuclear Matter in the Soliton Bag Model

Hirschegg 1982

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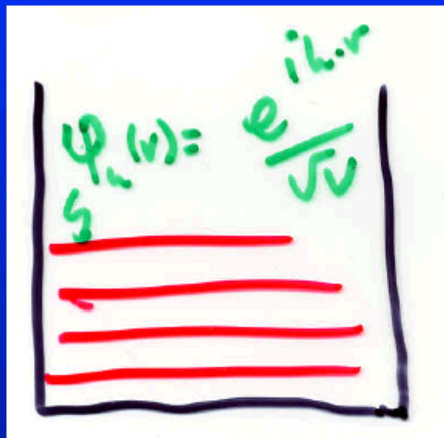
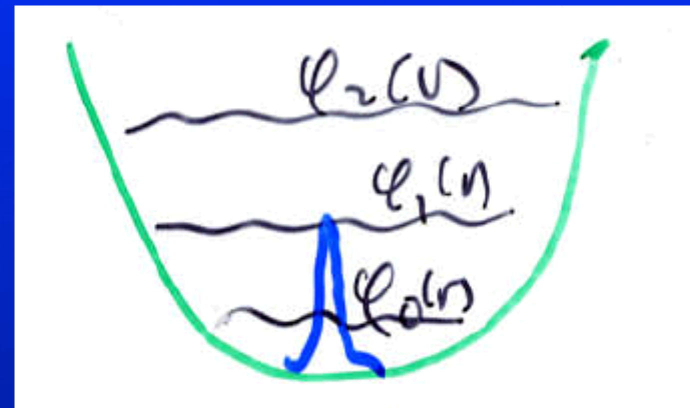
At high baryon density or temperature, one expects nuclear matter to undergo a phase transition to deconfined quark matter. Theories of the phase transition at finite baryon density have been based primarily on comparison of the free energies of the quark and hadronic phases, derived from quite inequivalent starting points,[1] rather than from a unified model. In this paper we present an attempt to derive the phase transition within a model field theory of quarks and "scalar gluons," that used by Friedberg and Lee[2,3] to construct "soliton models" for hadrons.

"In hindsight it is a pity that we never wrote these things up. At least I did not understand at the time that this work one day would become relevant."

Cold atom physics in a nutshell



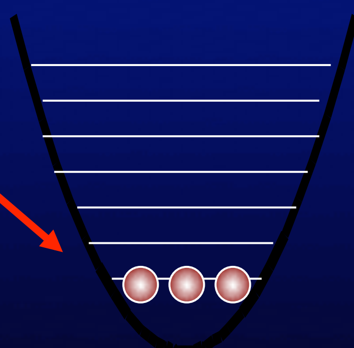
Box

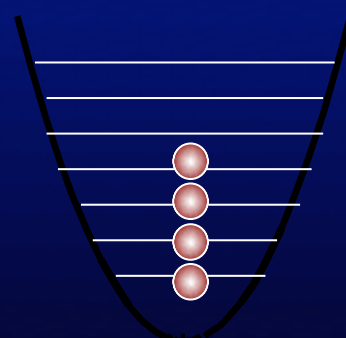


Potential well (trap)

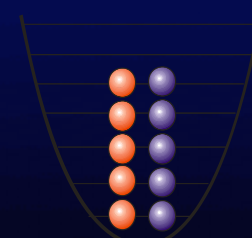
Statistics:

Bose condensate:
macroscopic occupation of single mode (generally lowest)





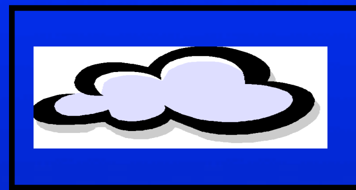
Degenerate Fermi gas



=> BCS pairing

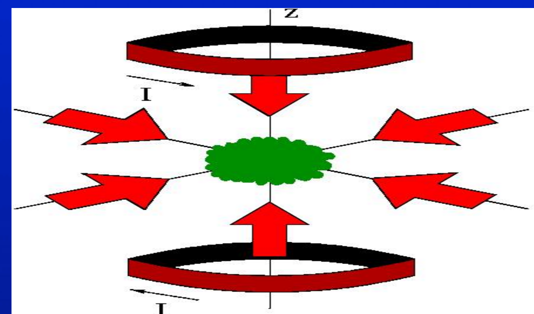
Trapped atomic experiments

Warm atomic vapor



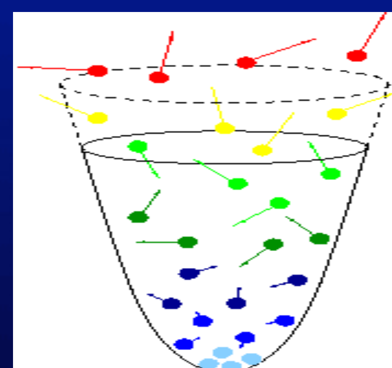
T=300K, n~ 3X10^6/cm^3

Magneto-optical trap



Laser cool to T ~ 50μK
n~10^{11}/cm^3

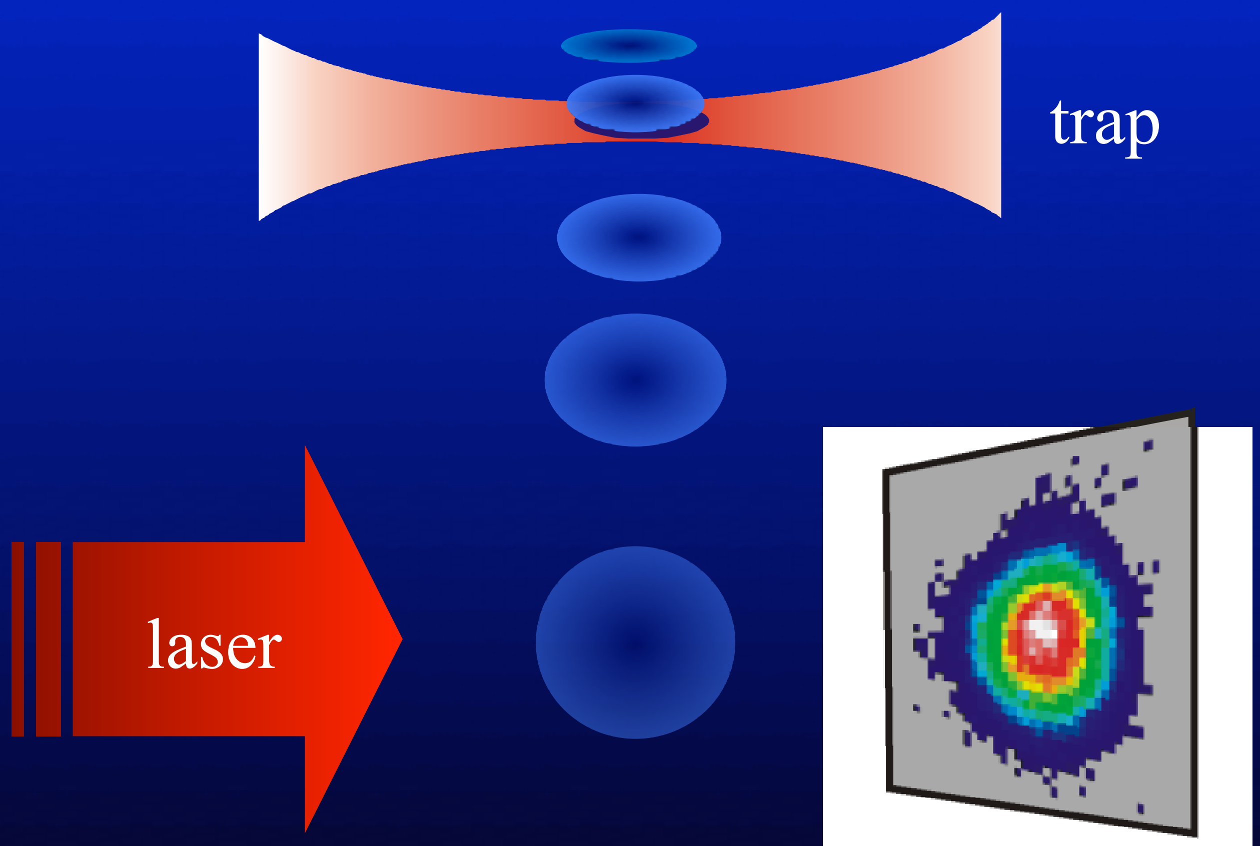
Evaporatively cool in magnetic (or optical) trap



Bosons condense,
Fermions BCS-pair
T~ 1-10^3 nK
n~ 10^{14-15}/cm^3
N ~ 10^5-10^8

Experiment, and then measure :

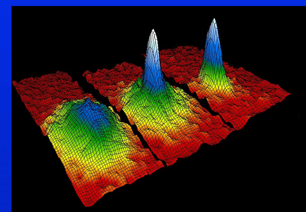
To probe system, release from trap, let expand and then image with laser:



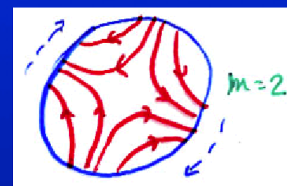
Early days of ultracold trapped atomic gases

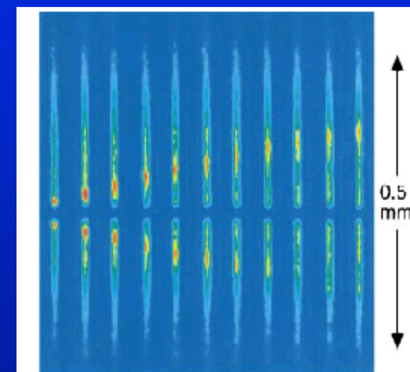
1995 = first Bose condensation of ^{87}Rb, ^{23}Na and ^{7}Li

*Structure of condensate.

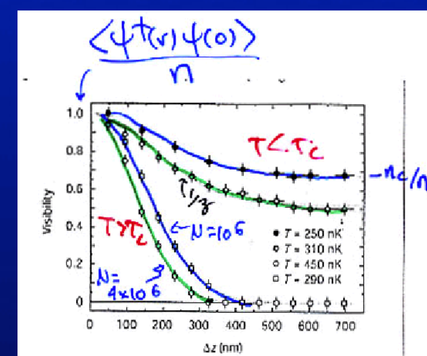


*Elementary modes: breathing, quadrupole, short wave sound, … .

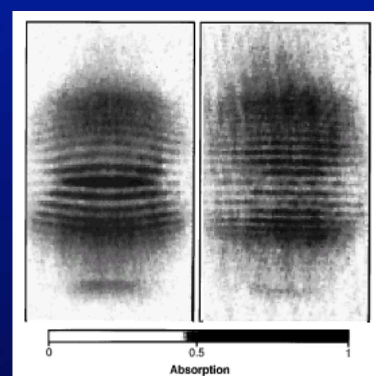




*1, 2 and 3 body correlations => evidence for BEC rather than simply condensation in space.



*Interference of condensates.



Primarily described in terms of mean field theory – Gross-Pitaevskii eq.

$$i\hbar\partial\psi(r,t)/\partial t = [-\hbar^2\nabla^2/2m + V(r) + g|\psi(r,t)|^2]\psi(r,t)$$

Turn of the century ultracold atomic systems
Strongly correlated systems

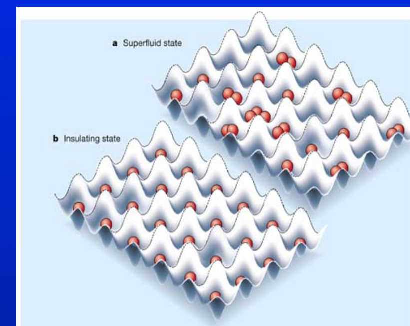
* Rapidly rotating bosons: how do many-particle Bose systems carry extreme amounts of angular momentum?

• Trapping and cooling clouds of fermionic atoms
  Degenerate Fermi gases and molecular states
  BCS pairing => new superfluid
  Crossover from BEC of molecules to BCS paired state

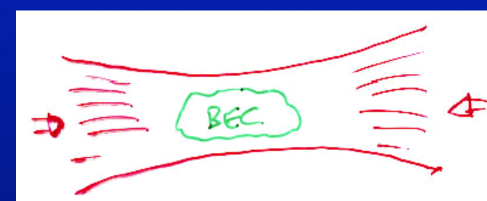
* Physics in the strong interaction limit:
  scale-free regime where $r_0 \ll n^{-1/3} \ll a$
    r_0 = range of interatomic potential ~ few Å
    n = particle density
    a = s-wave scattering length
  Realize through atomic Feshbach resonances

Newer directions in ultracold atomic systems
Novel systems

* Physics in optical lattices: Mott transition from
  superfluid to insulating states;
  low dimensional systems; 2D superfluids



* Spinor gases: trapped by laser fields.
  Physics of spin degrees of freedom
  Fragmented condensates



• Mixtures of bosons and fermions , ^{40}K+^{87}Rb

• Gases with large magnetic dipole moments, $^{160\text{-}164}Dy$

• Ultracold molecules: coherent mixtures of atoms and molecules,
  e.g., ^{87}Rb atoms and $^{87}Rb_2$ molecules;
  heteronuclear molecules: ^{6}Li+^{23}Na, ^{40}K+^{87}Rb

* Artificial magnetic fields

Artificial magnetic fields

How does one fool neutral atoms into behaving like charged particles in a magnetic field?

1) rotation

2) external lasers

3) can simulate non-Abelian gauge fields

Rotation acts as magnetic field

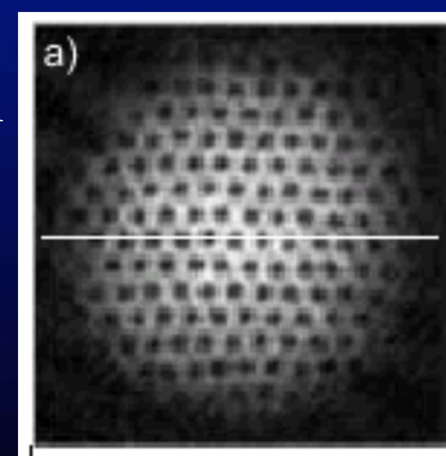
Rotate (Bose) atomic cloud in harmonic trap, **V(r) = m ω^2 r^2/2**

Energy in rotating frame:

Like a magnetic field

$$E' = \int d^3r \left[\frac{1}{2m} \left| \left(-i\nabla - m\boldsymbol{\Omega} \times \boldsymbol{r} \right) \psi \right|^2 + \left(V(\boldsymbol{r}) - \frac{1}{2} m \Omega^2 r^2 \right) |\psi|^2 + \frac{1}{2} g |\psi|^4 \right]$$

As rotation rate Ω approaches trap frequency, system flattens out,becomes effectively 2D. Centrifugal potential balances transverse trapping potential.

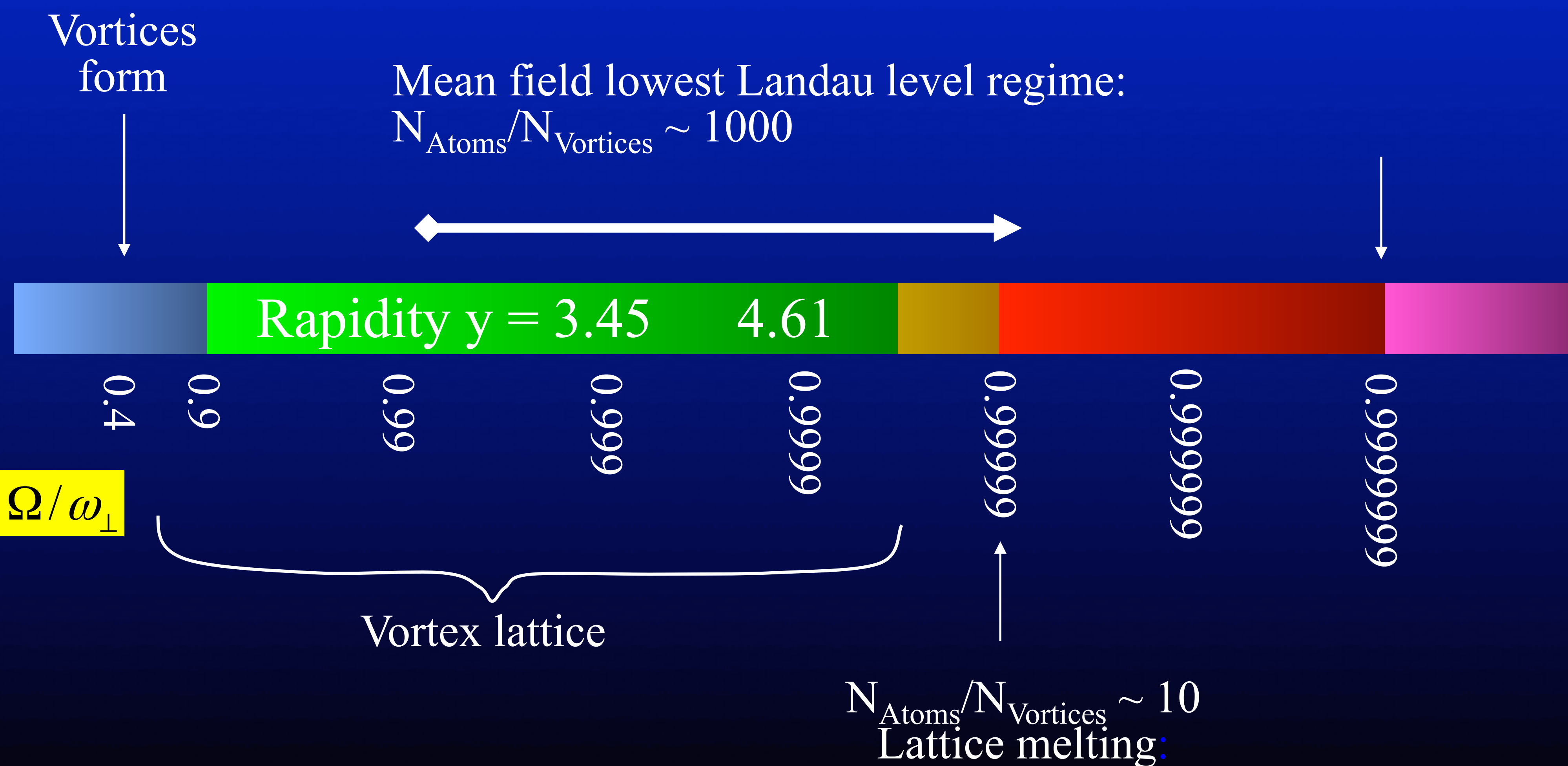


Vortices

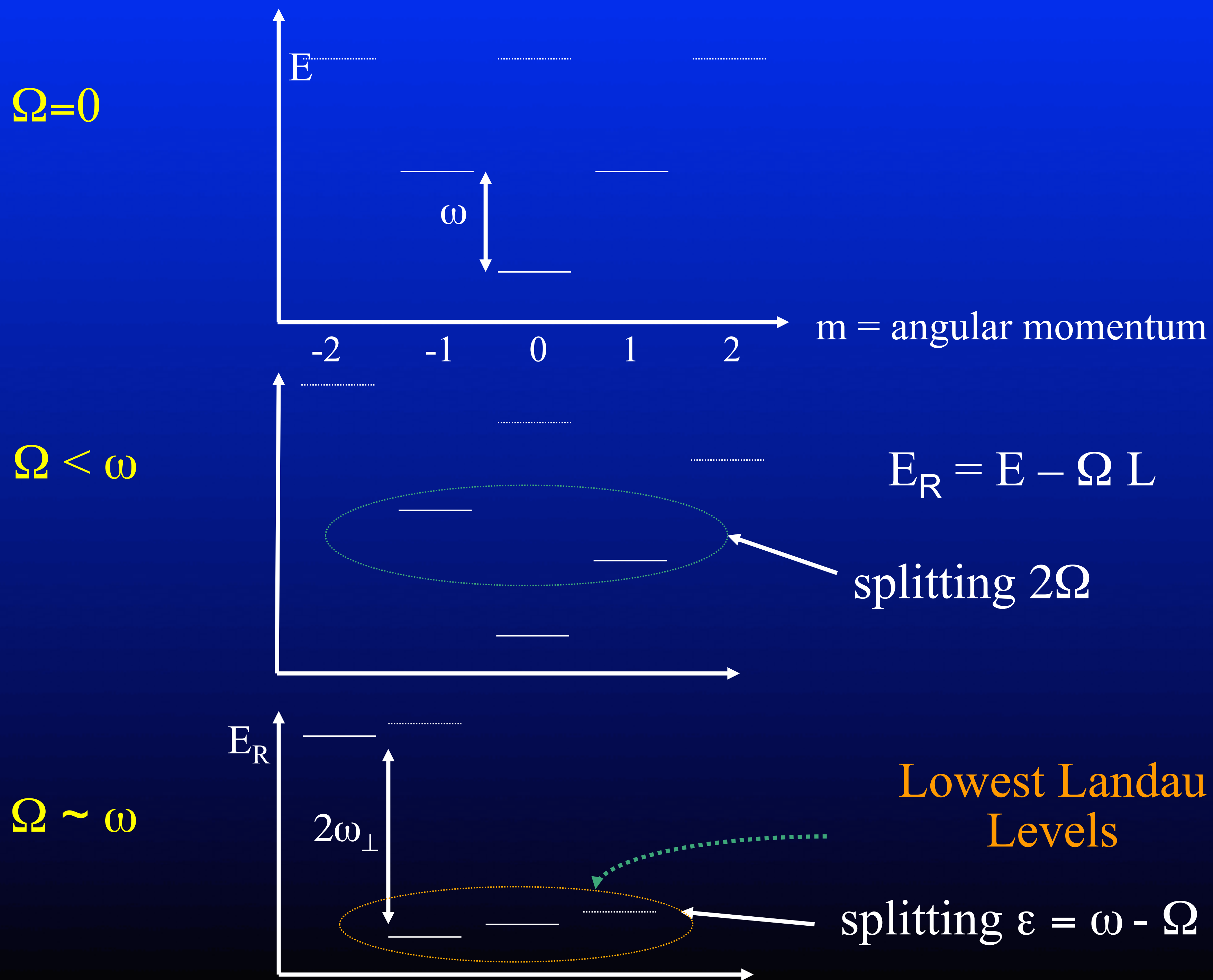
States in harmonic trap, $V(r) = m\,\omega^2 r^2/2$

Lowest Landau level regime as rotational rate Ω approaches ω
(interaction energy $gn \ll \omega \sim \Omega$): still mean field condensate

Strongly correlated states at higher $\Omega \simeq \omega$



Levels of 2d harmonic oscillator in rotating frame



Mean field lowest Landau level condensate

Ho, PRL87 (2001)

Order parameter, composed of lowest Landau levels in Coriolis force,

$$\phi_m(r) \sim z^m\, e^{-r^2/2d^2}, \quad z = x+iy,$$

becomes Laughlin-looking wave function

$$\Psi = \Sigma_m c_m (z^m e^{-r^2/2d^2})$$

d = oscillator length = $(1/m\omega)^{1/2}$

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becomes Laughlin-looking wave function

$$\Psi = (\Sigma_m c_m z^m) e^{-r^2/2d^2} = \Pi_i (z - z_i) e^{-r^2/2d^2}$$

z_i are vortex positions in lattice (in complex notation);

This state, in soft regime ($gn \ll \Omega$), is direct continuation of vortex lattice in stiff regime ($gn \gg \Omega$).

Eventually the vortex lattice melts ($N_{vortices} \sim N/10$)

Numerical simulations indicate subsequent formation of highly correlated incompressible (fractional) quantum Hall state ($N_{vortices} \sim 2N$) .

Highly degenerate LL levels allow formation of states with low occupation per level ("totally fragmented cond.").

e.g., for angular momentum/particle = N

$$\boxed{\Psi(r_1,r_2,\ldots r_N) \sim \Pi_{ij}(z_i-z_j)^2\, e^{-\Sigma_k r_k{}^2/2d^2}}$$

The problem is that one cannot rotate condensate fast enough in experiment to reach this regime. Reducing the number of particles doesn't work since one loses signal!

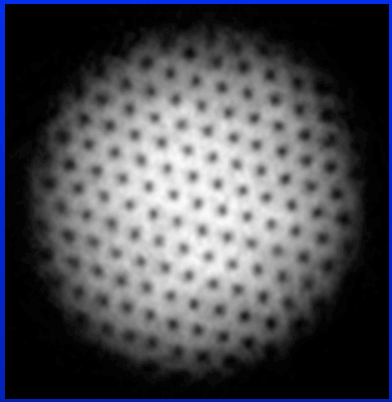
Atoms coupled to gauge fields

Abelian gauge field

$$\mathcal{H} = \frac{1}{2m}\left(\vec{p} - \vec{A}\right)^2$$

$$\vec{B} = \nabla \times \vec{A}$$

- Landau levels
- Quantized vortices
- Lattice melting (bosons)
- Quantum Hall (fermions)



Non-Abelian gauge field

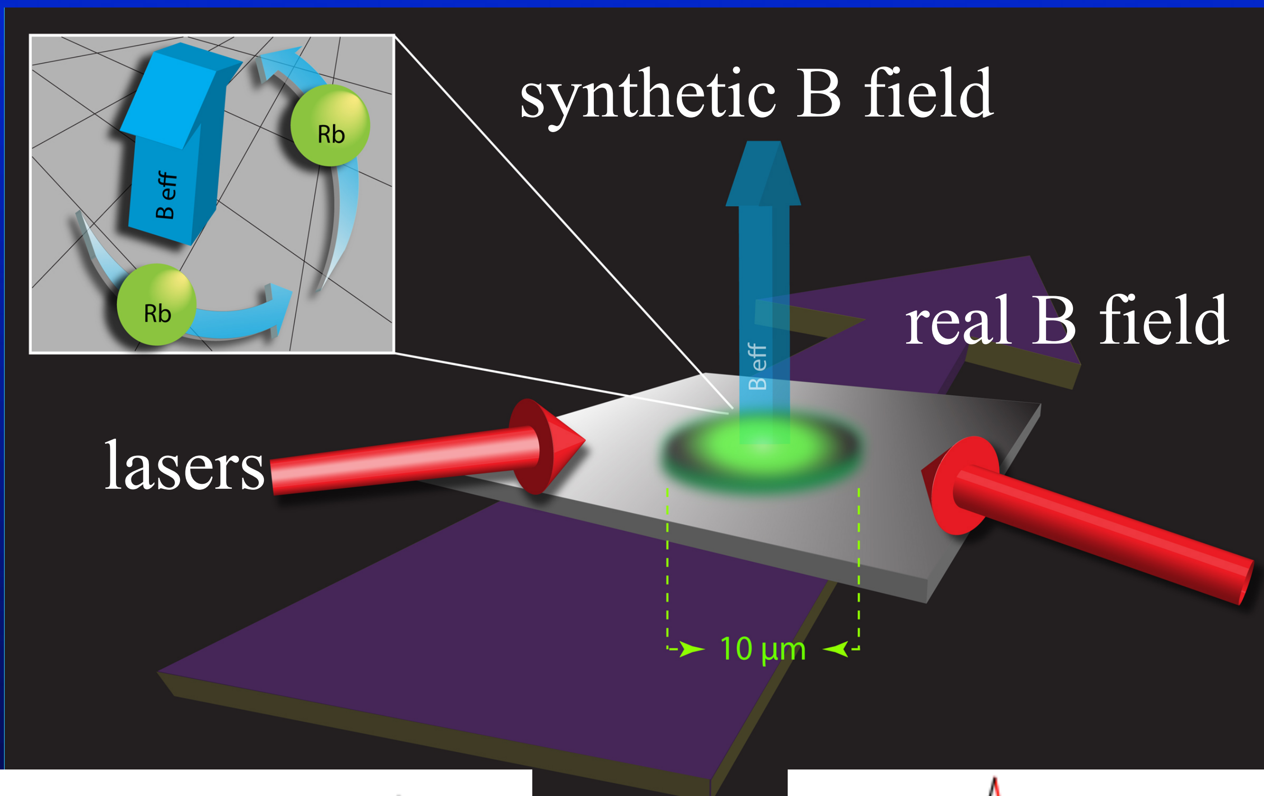
$$\mathcal{H}_{n\times n} = \frac{1}{2m}\left(\vec{p}\, I_{n\times n} - \vec{A}_{n\times n}\right)^2$$

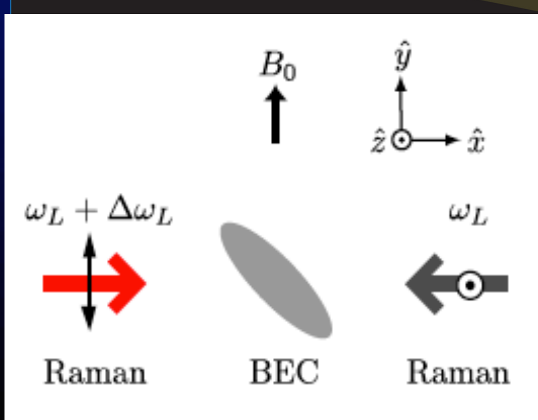
- Rashba spin-orbit coupling equivalent to non-Abelian gauge field
- Eventual connections to QCD

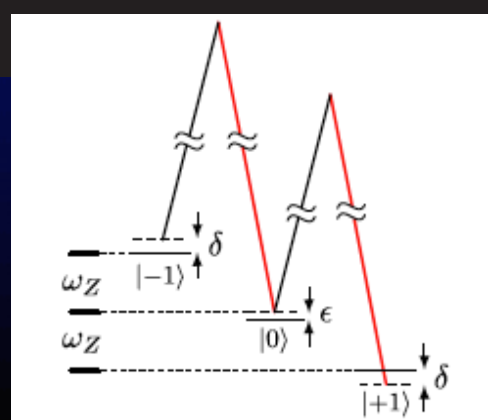
Artificial magnetic fields for neutral atoms, I

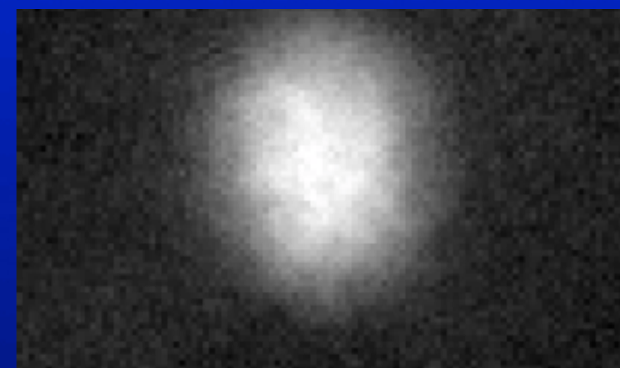
Y.-J. Lin, R. L. Compton, A. R. Perry, W.D. Phillips, J.V. Porto, & I. B. Spielman, PRL 102, 130401 (2009)

^{87}Rb BEC in optical trap: F=1 ground state, T ~ 100nK, N ~ 2.5x10^5

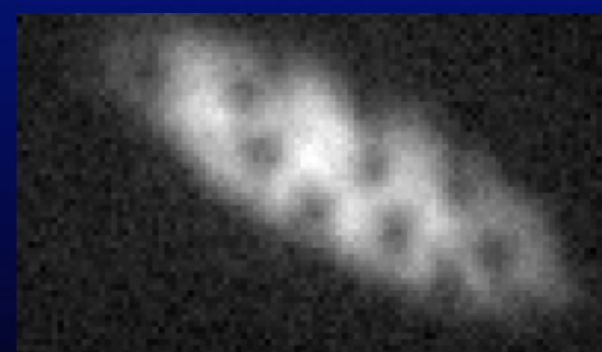








No “magnetic field”

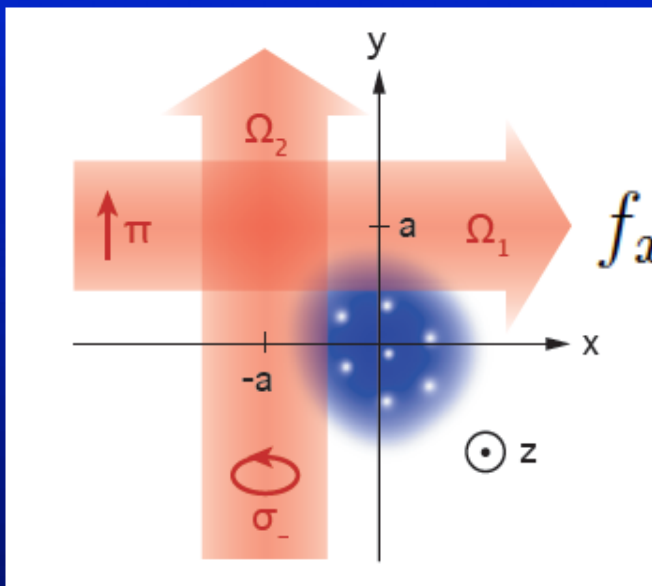


Abrikosov vortices in “magnetic field”

Artificial magnetic fields for neutral atoms, II

K. J. Günter, M. Cheneau, T. Yefsah, S. P. Rath, & J. Dalibard, PRA 79, 011604(R) (2009)

$$f_y = e^{iky-(x+a)^2/w^2}$$



$$f_x = e^{ikx-(y-a)^2/w^2}$$

Schematic setup of lasers

Two level system

Effective Hamiltonian

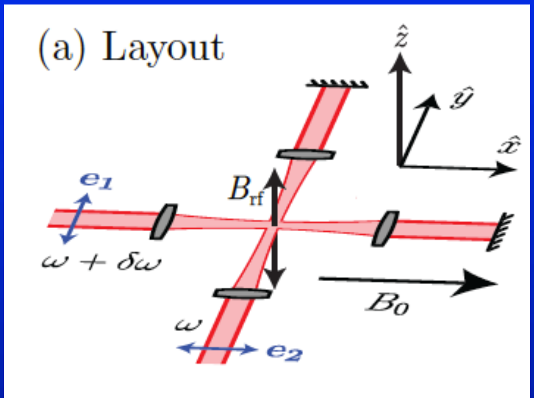
$$H = \epsilon \begin{pmatrix} |f_x|^2 & f_x f_y^* \\ f_y f_x^* & |f_y|^2 \end{pmatrix}$$

Artificial B field in z direction: $|B| = 2\hbar k a / w^2$

Induced scalar potential $\left(\hbar^2 k^2 - |B|^2 (x+y)^2\right) / 4m$

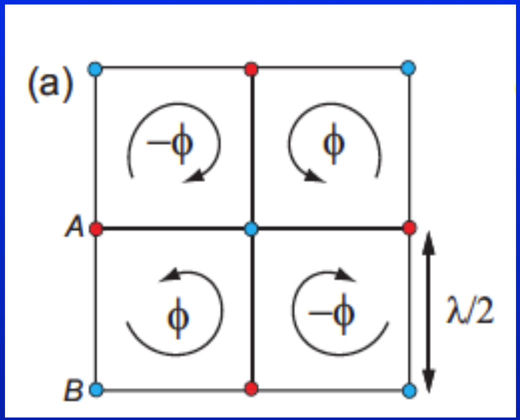
Staggered magnetic fields in optical lattices

I. Spielman, C. Morais Smith, A. Hemmerich et al.

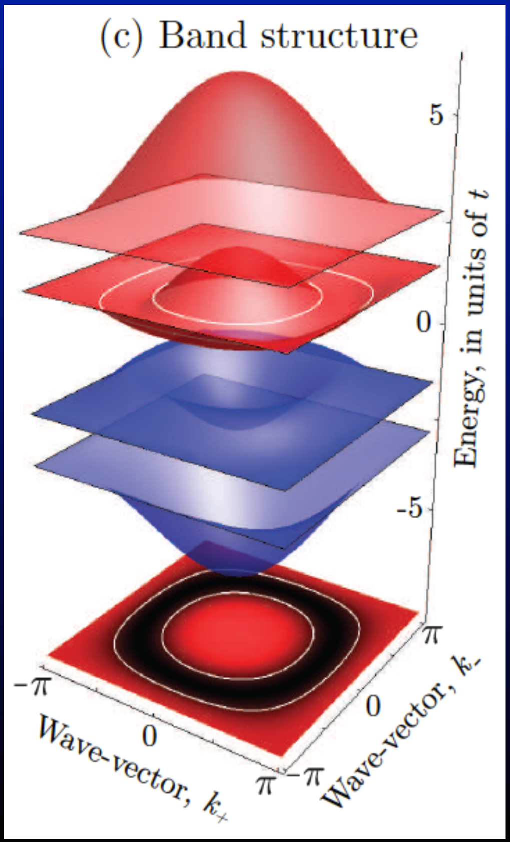


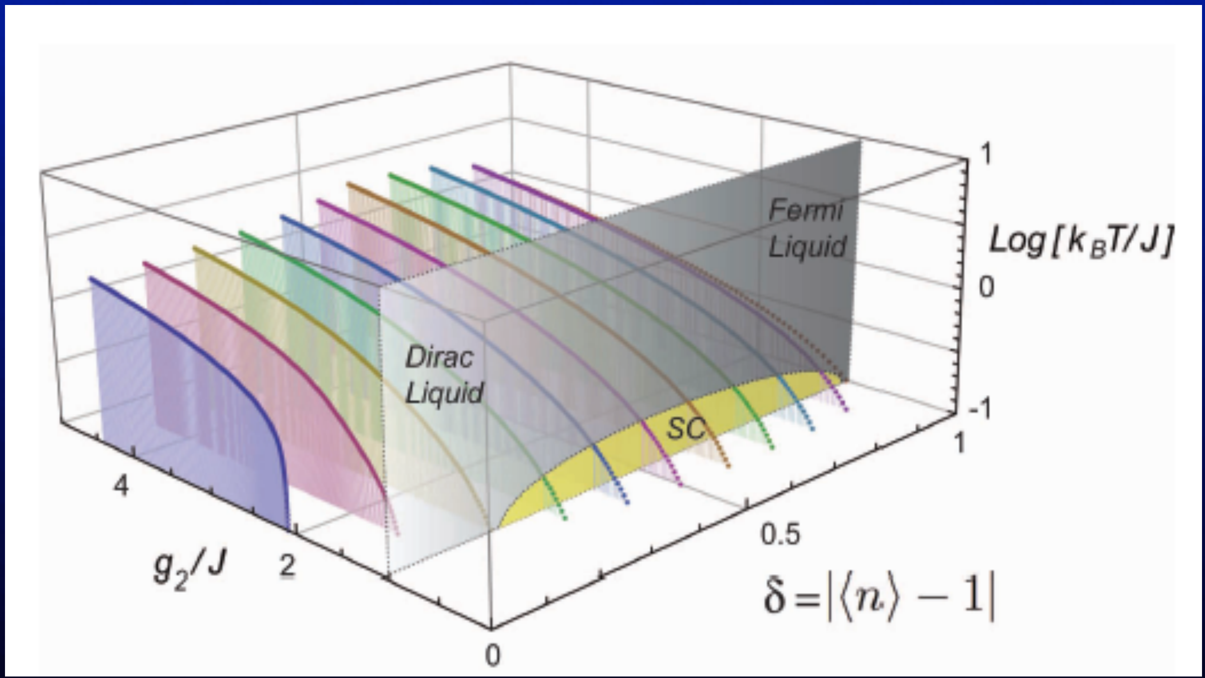
Staggered Zeeman fields

2 bichromatic lasers
=>staggered
"magnetic fields" in
optical lattice



Staggered gauge fields





Unusual states: analogs of high Tc superconductors

Cold atom simulations of QCD?

1) Analog models, e.g., three hyperfine states ⇔ quarks of 3 colors

2) Can simulate external magnetic fields. Major challenge is to induce electromagnetic-like interactions between atoms! (Dipolar atoms)

3) Cold atoms as analog computer. ex. Hubbard model.
   Can one eventually do simulations of lattice gauge theory?
   Will one be able eventually to address fields on each link of lattice, or at least more locally?

Eventual goal:
SU(3) quantum chromodynamics with quarks.

“Confinement” of three atomic fermions on lattice; formation of “nucleons”

A, Rapp, G. Zarand, C. Honerkamp, & W. Hofstetter , PRL 98 (2007), PRB 77 (2008)

Hubbard model with 3 internal degrees of freedom

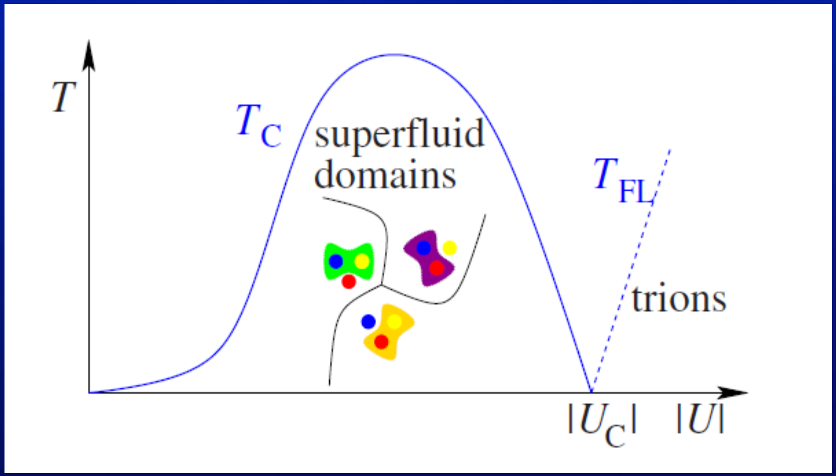
Red, Green, Blue

$$\hat{H} = -t \sum_{\langle i,j \rangle, \alpha} \hat{c}^{+}_{i\alpha} \hat{c}_{j\alpha} + \sum_{\alpha \neq \beta} \sum_{i} \frac{U_{\alpha\beta}}{2} (\hat{n}_{i\alpha} \hat{n}_{i\beta}),$$

3 lowest h.f. states of ^{6}Li

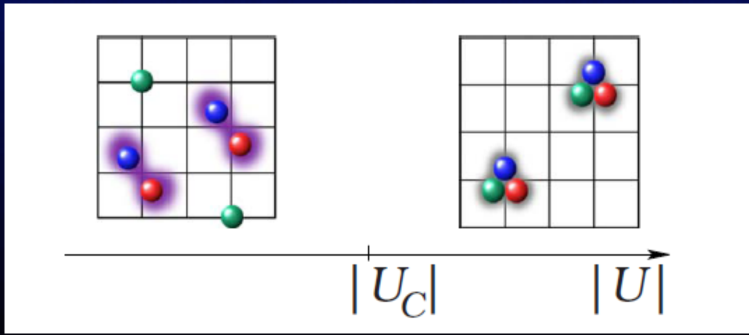
Small |U|: (U < 0)

Superfluid of two species
(“color superfluid”)



Large |U|:

Three particles bind together
(formation of trions = “baryons”)



Rashba-Dresselhaus spin-orbit coupling

Expt: Y.-J. Lin, K. Jiménez-García, & I.B. Spielman, Nature 471, 83 (2011)

Rashba-Dresselhaus Hamiltonian acting on 2-component spinors (2 hyperfine states) :

$$\mathcal{H}_0 = \frac{1}{2m}\left(\vec{p}I - \vec{A}\right)^2 = \frac{1}{2m}\begin{pmatrix} p^2 & 2\kappa(p_x - i\eta p_y) \\ 2\kappa(p_x + i\eta p_y) & p^2 \end{pmatrix} + const$$

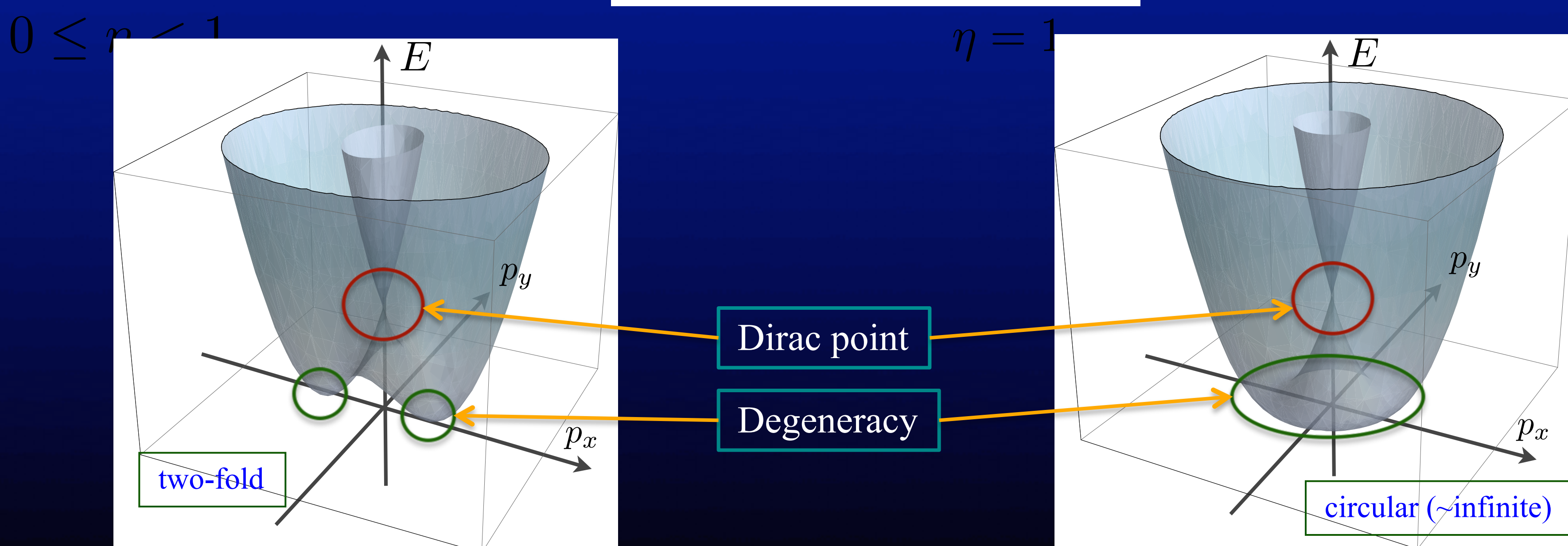
$$0 \leq \eta < 1$$

$$\vec{A} = -\kappa\left(\sigma_x, \eta\sigma_y, 0\right)$$

Single-particle dispersion:

$$\epsilon_{\pm}(\mathbf{p}) = \frac{(p_\perp \pm \kappa)^2 + p_z^2}{2m}$$

$$\eta = 1$$



Structure of BECs?

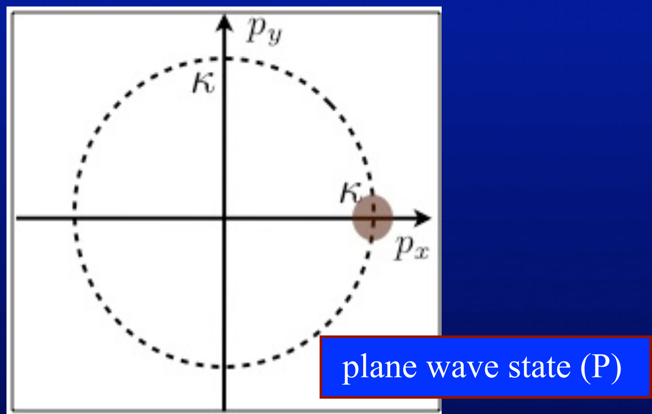
Effects of interactions: mean-field theory

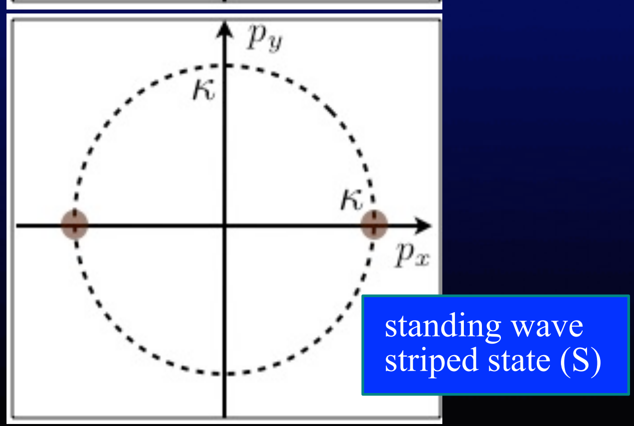
C. Wang, C. Gao, C.-M. Jian, & H. Zhai, PRL 105, 160403 (2010)

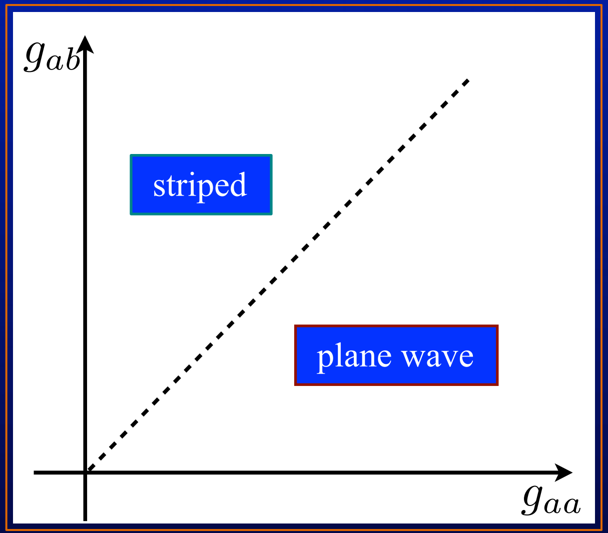
s-wave interaction

$$\mathcal{H}_{\mathrm{int}}(\mathbf{r},\mathbf{r}') = \sum_{\sigma,\sigma'=a,b} g_{\sigma\sigma'}\delta(\mathbf{r}-\mathbf{r}'), \quad \begin{cases} g_{aa} = g_{bb} > 0 \\ g_{ab} = g_{ba} > 0 \end{cases} \qquad a, b : \text{spinor indices}$$

With constant g's, bosonic ground state has either all particles in one state (k) or superposition of two states (k,-k)





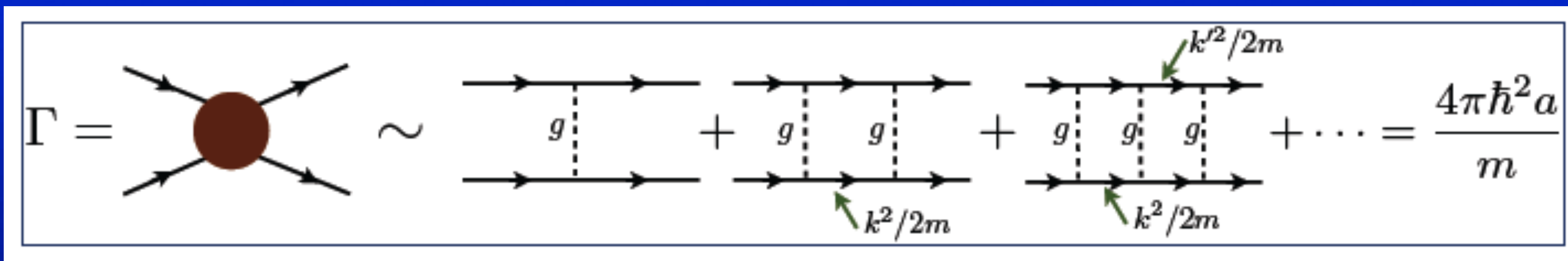


How to relate g's to observables?
Cannot replace g's by scattering lengths, a, via g=4πa/m.

Effective interaction = t-matrix

T. Ozawa and GB, PR A 84, 043622 (2011)

Effective interaction of low energy particles with s-wave interaction in free space with low energy given by t-matrix:

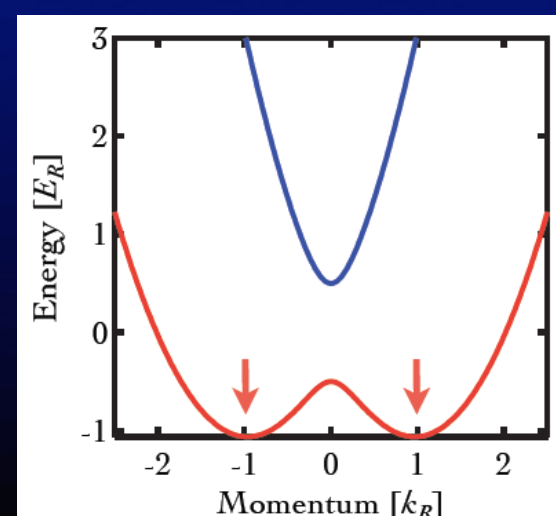


Dispersion is not quadratic in the presence of spin-orbit coupling, Possible ultraviolet divergence:

$4\pi\hbar^2 a/m.$

$$\int^{\Lambda} \frac{d^3k}{(2\pi)^3} \frac{m}{k_z^2 + (|k_\perp| - \kappa)^2} \to \frac{m}{2\pi^2}\left(\Lambda + \frac{\pi\kappa}{2}\ln\Lambda + \cdots\right)$$

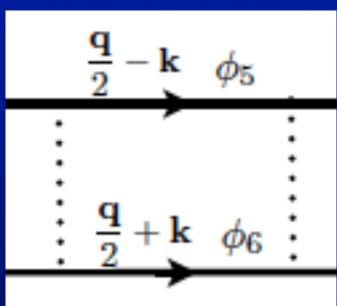
linear term renormalized as usual, but log term remains for single branch spectrum. With both lower and upper branches log goes away.



T-matrix with non-quadratic dispersion relation

$$\epsilon_\pm(\mathbf{p}) = \frac{1}{2m}\left[\left(\sqrt{p_x^2+\eta^2 p_y^2} \pm \kappa\right)^2 + (1-\eta^2)p_y^2 + p_z^2\right]$$

Non-trivial infrared behavior as relative momentum → 0:



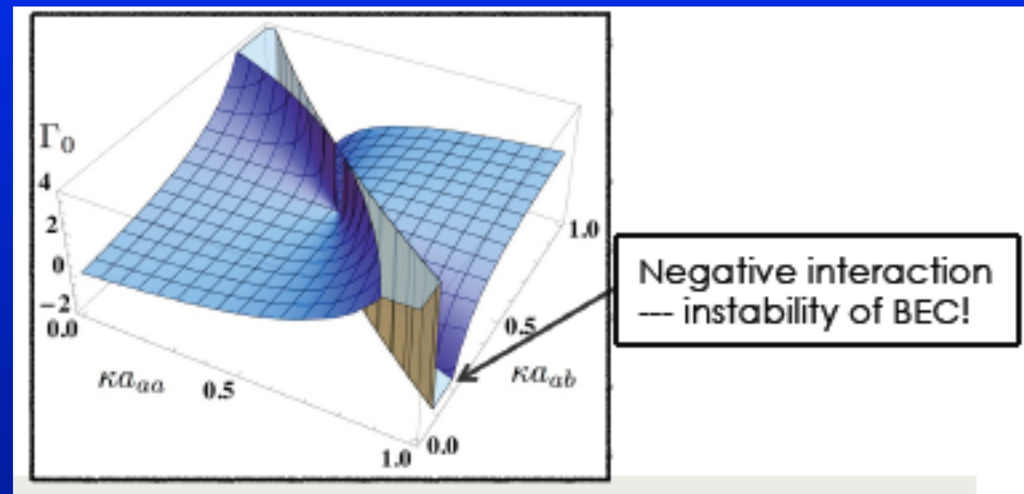
$$\sim \int_0 \frac{d^3k}{(2\pi)^3} \frac{2m}{(|\mathbf{k}_\perp + \mathbf{q}/2| - \kappa)^2 + (|-\mathbf{k}_\perp + \mathbf{q}/2| - \kappa)^2 + 2k_z^2}$$

$$\to -\frac{m\kappa}{2\pi}\ln q.$$

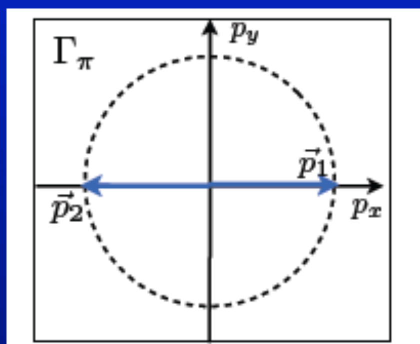
Thus for isotropic spin orbit coupling effective interaction vanishes as 1/ln q as q → 0

Interaction of condensate particles

$\Gamma_0 = t\,(\mathbf{K} + \mathbf{K} \rightarrow \mathbf{K} + \mathbf{K})$, $\quad\Gamma_\pi = t\,(-\mathbf{K} + \mathbf{K} \rightarrow -\mathbf{K} + \mathbf{K})$



a_{aa} = intra-species scatt. length
a_{ab} = inter-species scatt. length



$\Gamma_\pi = 0$ for isotropic Rashba interaction

With t-matrix as effective interaction, determine ground state phases in mean field:

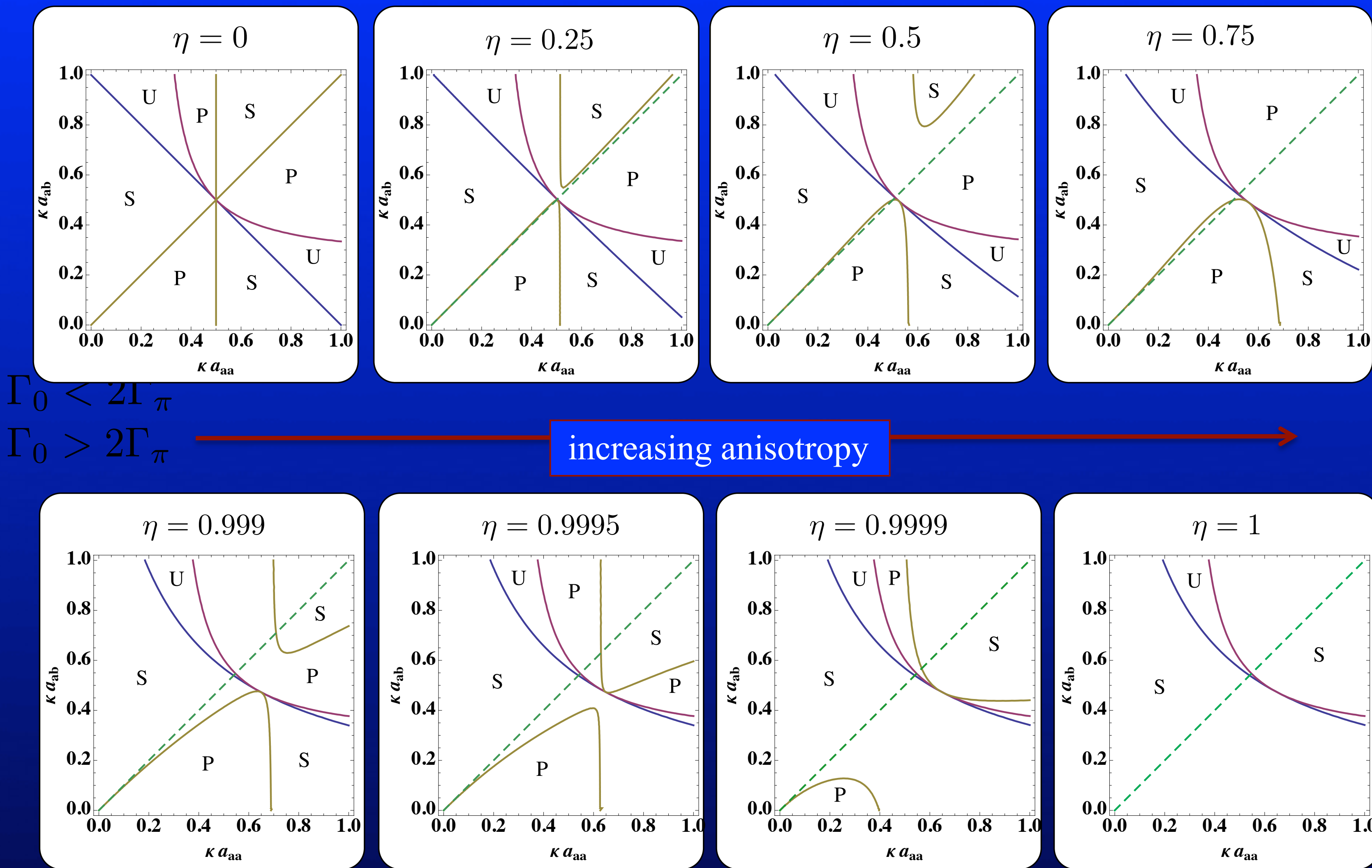
$\Gamma_0 < 0 \Rightarrow$ BEC unstable to collapse (U)

$\Gamma_0 < 2\,\Gamma_\pi \Rightarrow$ form condensate from single momentum: "plane wave" phase, P

$\Gamma_0 > 2\,\Gamma_\pi \Rightarrow$ form condensate from K and -K: "striped" phase, S

Ground state phases

(T.Ozawa & GB,PR A85, 013612 (2012))



For isotropy, no plane wave state; only striped phase or unstable.

Much richer structure than with simple replacement of bare couplings by scattering lengths

Finite temperature?? Effects of trap??

FLUCTUATIONS AND LONG-RANGE ORDER IN FINITE-TEMPERATURE PION CONDENSATES*

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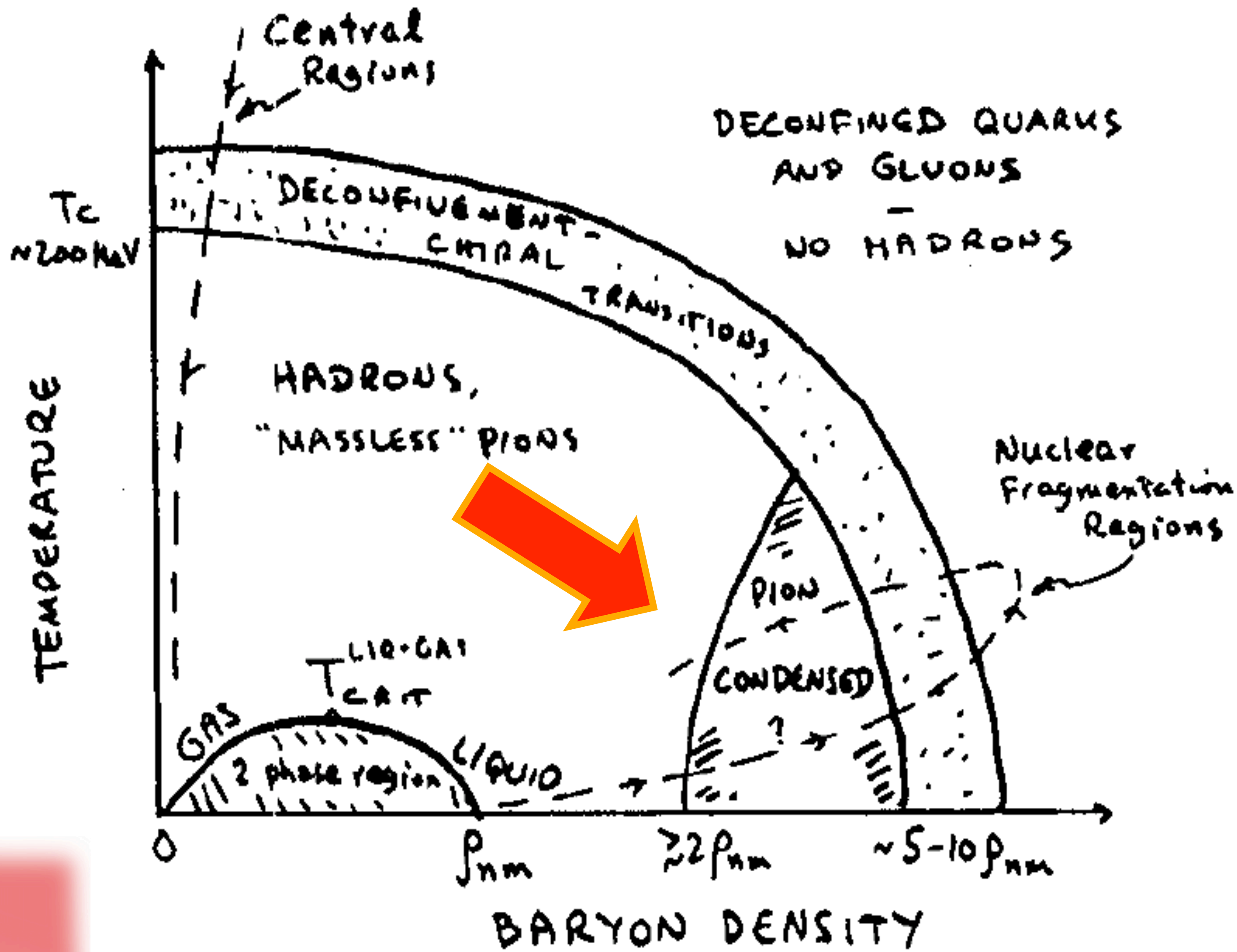
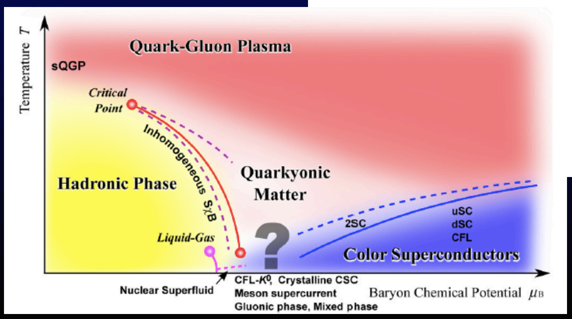
We study the stability of the neutral and charged pion-condensed phases of nuclear matter against fluctuations of the order parameter. At finite temperatures pion condensates with an order parameter varying in only one dimension are, as we show, prohibited, while such condensates are allowed at zero temperature. Condensates that vary in two and three dimensions can be stable at all temperatures. Another allowed state, which may be favored energetically, is a quasi-ordered one-dimensional condensate characterized by long-range pion field correlations decaying only algebraically in space; insufficient experimental resolution may, however, limit one's ability to distinguish such a one-dimensional structure from true one-dimensional long-range order. Finally, we calculate the normal modes and the pion propagator in a charged one-dimensional running-wave condensate, explicitly illustrating the effect of long-range Coulomb forces on the order-parameter fluctuations.

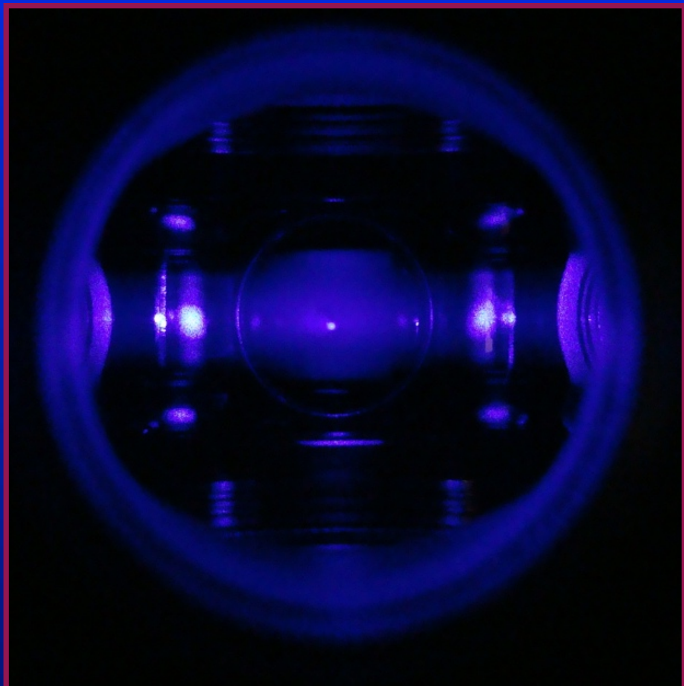
Simulating "pion condensed" phases of nuclear matter with dipolar fermionic atoms (^{161}Dy, ^{163}Dy) and molecules (RbCs)

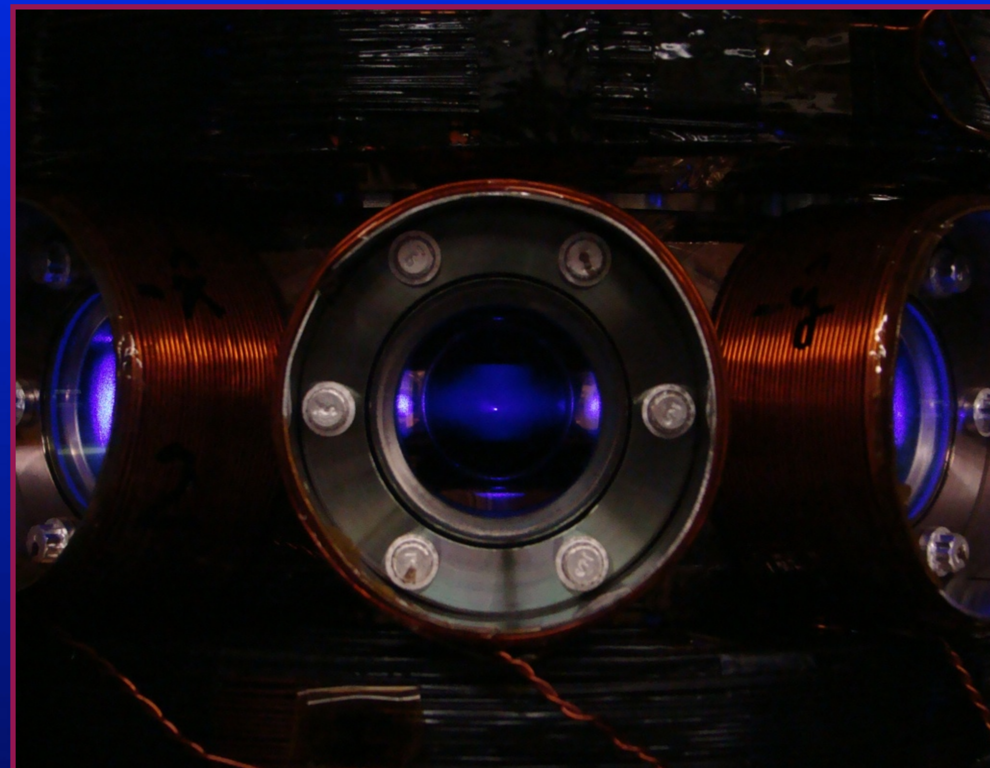




World's first Dysprosium MOT

^{164}Dy, ^{163}Dy, ^{162}Dy, ^{161}Dy, ^{160}Dy



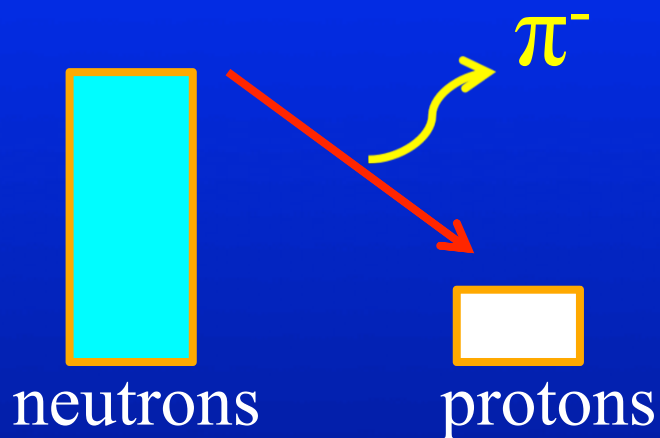


~ 2 x 10^8 atoms

density ~10^{10} - 10^{11}/cm^3

Ben Lev's lab, Urbana

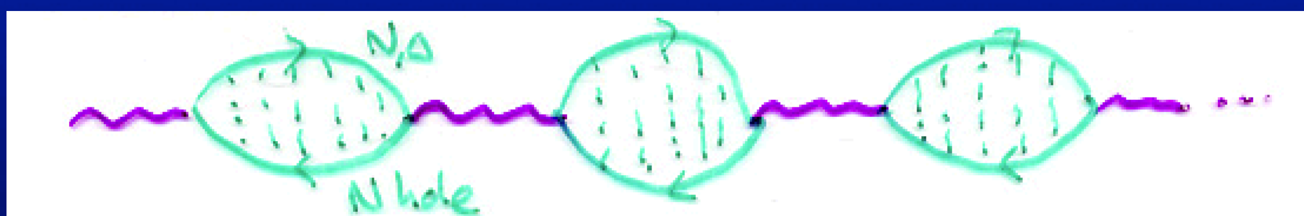
Fermi seas of neutrons and protons in neutron star matter



Neutron Fermi energy > proton Fermi energy plus pion rest mass ($m_\pi c^2$)

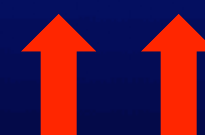
=> BEC of pi mesons – pion condensate.

Technically, have a soft collective spin-isospin instability



driven by nuclear tensor interaction, between spins σ:

$$H_{tensor} \sim \frac{e^{-m_\pi r}}{r^3}\left[3(\vec{\sigma}_1 \cdot \vec{r})(\vec{\sigma}_2 \cdot \vec{r}) - \vec{\sigma}_1 \cdot \vec{\sigma}_2\right]$$

favors 

similar, except for sign to magnetic dipolar interaction

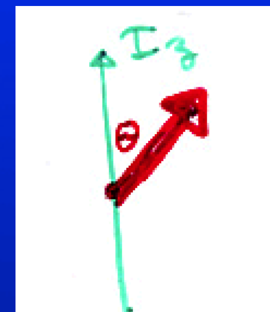
$$H_{dipole} \sim -\frac{1}{r^3}\left[3(\vec{d}_1 \cdot \vec{r})(\vec{d}_2 \cdot \vec{r}) - \vec{d}_1 \cdot \vec{d}_2\right]$$

favors 

Charged pion condensation:

Above critical density have transition to new state with nucleons rotated in isospin space:

$$|N'\rangle = \cos\theta\,|n\rangle + \sin\theta\,|p\rangle$$
$$|P'\rangle = -\sin\theta\,|n\rangle + \cos\theta\,|p\rangle$$



$$\langle \pi^- \rangle \sim e^{i k \cdot r}$$

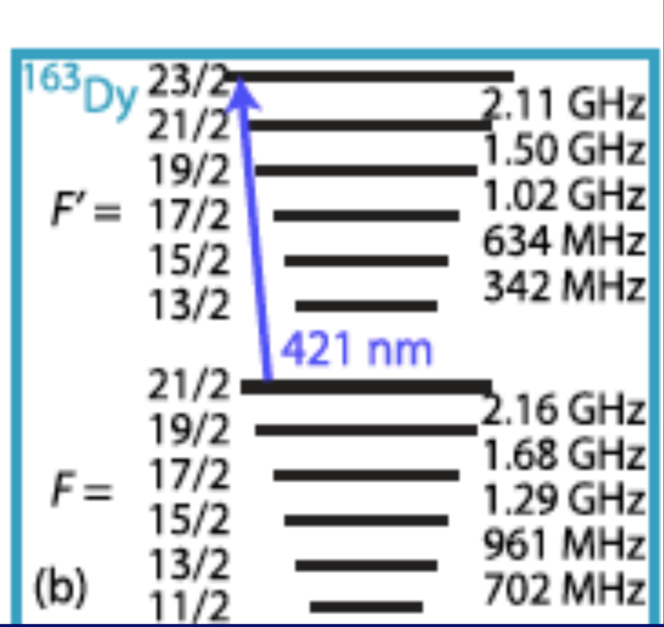
Neutral pion condensation

Form spatially varying spin-isospin wave:

$$\langle \pi^0 \rangle \sim \cos \vec{k}\cdot\vec{r}$$

"Charged pion" condensate

Atomic dysprosium has magnetic moment = $10\mu_B$.
Trapped in Urbana by Lev



Populate 11/2 state ⇔ n

At high density get spontaneous excitation of 13/2 ⇔ p

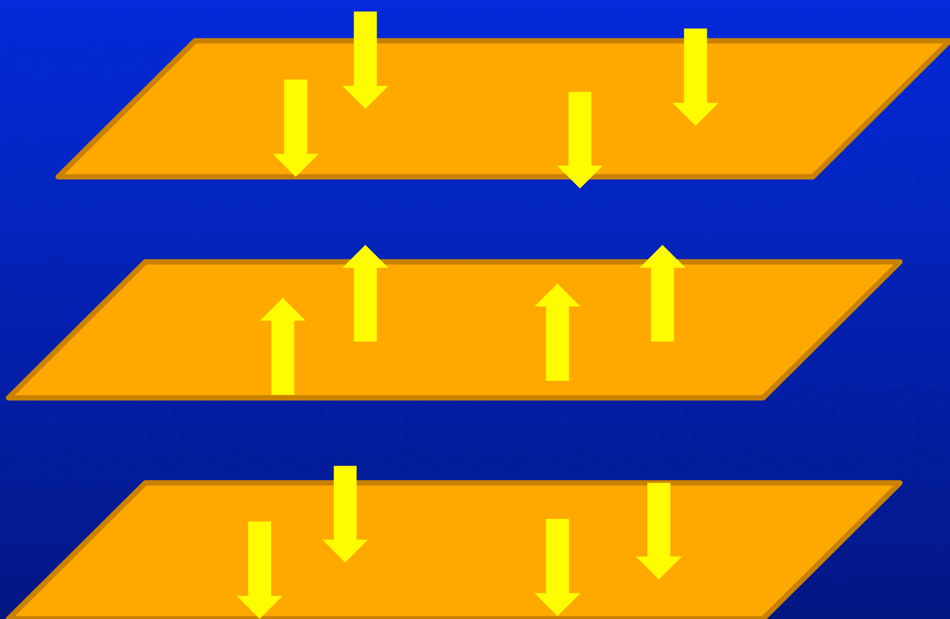
with macroscopic magnetic field ⇔ charged pion field.

^{163}Dy angular mom = 21/2

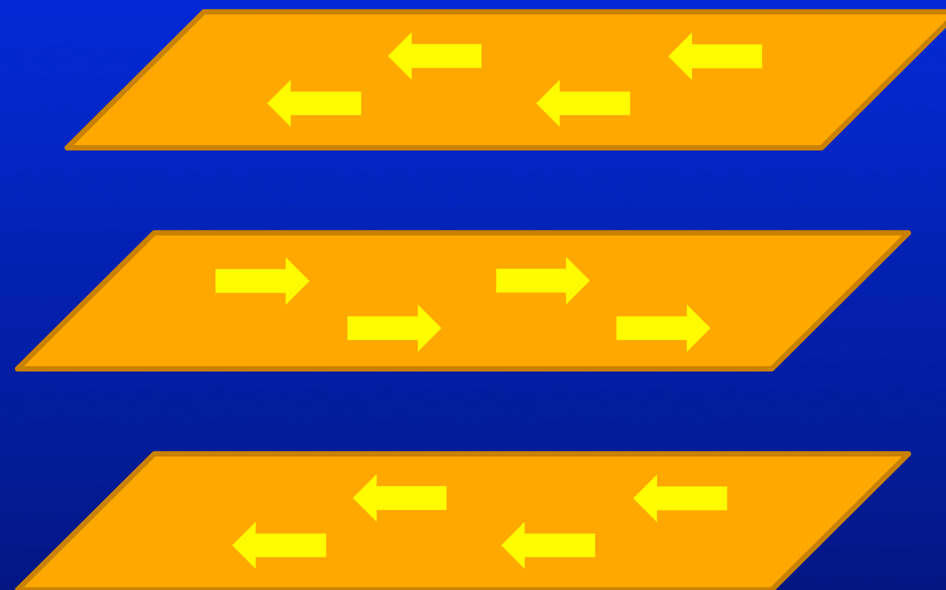
GB, T. Hatsuda, K. Maeda in progress

Realization of “neutral meson-condensate” in cold atoms

Baym, Hatsuda and Maeda (in progress)



$$(-\nabla^2 + m_\pi{}^2)\,\varphi_c(\boldsymbol{r}) = (f/m_\pi)\,\boldsymbol{\nabla}\cdot\langle\psi^\dagger\boldsymbol{\sigma}\psi\rangle$$



$$(-\nabla^2 + m_\rho{}^2)\,\boldsymbol{\rho}_c(\boldsymbol{r}) = (f_\rho/m_\rho)\,\boldsymbol{\nabla}\times\langle\psi^\dagger\boldsymbol{\sigma}\psi\rangle$$

Dipolar fermionic atom = Neutron
Photon = Massless rho-meson

to our dear friend and colleague, Bengt,
50 dodecades old!