New Applications of Renormalization Group Methods in Nuclear Physics

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Facets of Strong-Interaction Physics

Hirschegg, January 18, 2012
Nuclear equation of state and astrophysical applications

Light and neutron-rich nuclei

Correlations in nuclear systems
Nuclear equation of state and astrophysical applications

Unified microscopic description.

Light and neutron-rich nuclei

Correlations in nuclear systems
Chiral EFT for nuclear forces, leading order 3N forces

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<th>NN</th>
<th>3N</th>
<th>4N</th>
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<td>LO</td>
<td>$\mathcal{O}\left(\frac{Q^0}{\Lambda^0}\right)$</td>
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<td>NLO</td>
<td>$\mathcal{O}\left(\frac{Q^2}{\Lambda^2}\right)$</td>
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<td>N$^2$LO</td>
<td>$\mathcal{O}\left(\frac{Q^3}{\Lambda^3}\right)$</td>
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long (2$\pi$)  intermediate ($\pi$)  short-range

$c_1, c_3, c_4$ terms  $c_D$ term  $c_E$ term

large uncertainties in coupling constants at present:

\[
c_1 = -0.9^{+0.2}_{-0.5}, \quad c_3 = -4.7^{+1.5}_{-1.0}, \quad c_4 = 3.5^{+0.5}_{-0.2}
\]

lead to theoretical uncertainties in many-body observables
Changing the resolution: The (Similarity) Renormalization Group

- elimination of coupling between low- and high momentum components, calculations much easier
- observables unaffected by resolution change (for exact calculations)
- residual resolution dependences can be used as tool to test calculations

Not the full story:
RG transformation also changes three-body (and higher-body) interactions.
Equation of state: Many-body perturbation theory

central quantity of interest: energy per particle \( E/N \)

\[
H(\lambda) = T + V_{NN}(\lambda) + V_{3N}(\lambda) + \ldots
\]

- “hard” interactions require non-perturbative summation of diagrams
- with low-resolution interactions much more perturbative
- inclusion of 3N interaction contributions crucial
- use chiral interactions as initial input for RG evolution
• significantly reduced cutoff dependence at 2nd order perturbation theory
• small resolution dependence indicates converged calculation
• energy sensitive to uncertainties in 3N interaction
• variation due to 3N input uncertainty much larger than resolution dependence
• significantly reduced cutoff dependence at 2nd order perturbation theory
• small resolution dependence indicates converged calculation
• energy sensitive to uncertainties in 3N interaction
• variation due to 3N input uncertainty much larger than resolution dependence
• good agreement with other approaches (different NN interactions)
Constraints on the nuclear equation of state (EOS)

**A two-solar-mass neutron star measured using Shapiro delay**

P. B. Demorest\(^1\), T. Pennucci\(^2\), S. M. Ransom\(^1\), M. S. E. Roberts\(^3\) & J. W. T. Hessels\(^4,5\)

\[
M_{\text{max}} = 1.65M_\odot \rightarrow 1.97 \pm 0.04M_\odot
\]

Calculation of neutron star properties requires EOS up to high densities.

**Strategy:**

Use observations to constrain the high-density part of the nuclear EOS.
Neutron star radius constraints

incorporation of beta-equilibrium: neutron matter $\rightarrow$ neutron star matter

parametrize piecewise high-density extensions of EOS:

- use polytropic ansatz $p \sim \rho^\Gamma$
- range of parameters $\Gamma_1, \rho_{12}, \Gamma_2, \rho_{23}, \Gamma_3$ limited by physics!

see also KH, Lattimer, Pethick, Schwenk, PRL 105, 161102 (2010)
Constraints on the nuclear equation of state

use the constraints:

recent NS observation

$M_{\text{max}} > 1.97 M_{\odot}$

causality

$v_s(\rho) = \sqrt{dP/d\varepsilon} < c$

significant reduction of possible equations of state
Constraints on the nuclear equation of state

use the constraints:

NS mass

$M_{\text{max}} > 2.4 \, M_{\odot}$

causality

$v_s(\rho) = \sqrt{dP/d\varepsilon} < c$

increased $M_{\text{max}}$ systematically leads to stronger constraints
• low-density part of EOS sets scale for allowed high-density extensions
• radius constraint for typical $1.4 M_\odot$ neutron star: $9.8 - 13.4$ km
Equation of state of symmetric nuclear matter, nuclear saturation

“Very soft potentials must be excluded because they do not give saturation; they give too much binding and too high density. In particular, a substantial tensor force is required.”

Hans Bethe (1971)

\[ V_{\text{low } k} \text{ NN from } N^3 \text{LO (500 MeV)} \]
\[ 3\text{NF fit to } E_3^H \text{ and } r_{4\text{He}} \text{ } \Lambda_{3\text{NF}} = 2.0 \text{ fm}^{-1} \]

\[ 3\text{rd order pp+hh} \]
\[ \Lambda = 1.8 \text{ fm}^{-1} \text{ NN only} \]
\[ \Lambda = 2.8 \text{ fm}^{-1} \text{ NN only} \]

KH, Bogner, Furnstahl, Nogga, PRC(R) 83, 031301 (2011)

empirical nuclear saturation properties

\[ n_S \sim 0.16 \text{ fm}^{-3} \]
\[ E_{\text{binding}}/N \sim -16 \text{ MeV} \]

\[ \bar{l}_S \sim 1.8 \text{ fm} \]
• nuclear saturation delicate due to cancellations of large kinetic and potential energy contributions

• 3N forces are essential! 3N interactions fitted to $^3$H and $^4$He properties
Equation of state of symmetric nuclear matter,
Nuclear saturation

- saturation point consistent with experiment, **without free parameters**
- cutoff dependence at 2nd order significantly reduced
- 3rd order contributions small
- cutoff dependence consistent with expected size of 4N force contributions
• binding energy results from cancellations of much larger kinetic and potential energy contributions
• chiral hierarchy of many-body terms preserved for considered density range
• cutoff dependence of natural size, consistent with chiral exp. parameter $\sim 1/3$
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Hierarchies of many-body contributions

- Binding energy results from cancellations of much larger kinetic and potential energy contributions
- Chiral hierarchy of many-body terms preserved for considered density range
- Cutoff dependence of natural size, consistent with chiral exp. parameter $\sim 1/3$
RG evolution of 3N interactions

- So far:
  intermediate ($c_D$) and short-range ($c_E$) 3NF couplings fitted to few-body systems at different resolution scales:

  \[ E_{3H}^3 = -8.482 \, \text{MeV} \quad \text{and} \quad r_{4\text{He}}^4 = 1.95 - 1.96 \, \text{fm} \]

  \[ \rightarrow \quad \text{coupling constants of natural size!} \]

- Ideal case: evolve 3NF consistently with NN to lower resolution using the RG

  - has been achieved in oscillator basis (Jurgenson, Roth)
  - promising results in very light nuclei
  - problems in heavier nuclei (see talk by R. Roth)
  - not suitable for infinite systems

\[ V_{\text{low } k} \text{ NN from } N^3\text{LO (500 MeV)} \]

3NF fit to $E_{3H}^3$ and $r_{4\text{He}}^4$ $A_{3NF} = 2.0 \, \text{fm}^{-1}$

\[ 3\text{rd order pp+hh} \]

\[ A = 1.8 \, \text{fm}^{-1} \quad A = 2.8 \, \text{fm}^{-1} \]

\[ A = 1.8 \, \text{fm}^{-1} \text{ NN only} \quad A = 2.8 \, \text{fm}^{-1} \text{ NN only} \]
RG evolution of 3N interactions in momentum space

Three-body Faddeev basis:

\[ |pq\alpha\rangle_i \equiv |p_i q_i; [(LS) J(ls_i) j] \mathcal{J} \mathcal{J}_z (T t_i) T T_z \rangle \]

SRG flow equations for NN and 3N interactions:

\[
\frac{dH_s}{ds} = [\eta_s, H_s] \quad \eta_s = [T_{\text{rel}}, H_s]
\]

\[
\frac{dV_{ij}}{ds} = [[[T_{ij}, V_{ij}], T_{ij} + V_{ij}] ,
\frac{dV_{123}}{ds} = [[[T_{12}, V_{12}], V_{13} + V_{23} + V_{123}]
+ [[[T_{13}, V_{13}], V_{12} + V_{23} + V_{123}]
+ [[[T_{23}, V_{23}], V_{12} + V_{13} + V_{123}]
+ [[[T_{\text{rel}}, V_{123}], H_s]]

RG evolution of 3N interactions in momentum space

Invariance of $E_{gs}^{3H}$ within 16 keV for consistent chiral interactions at $N^2$LO
RG evolution of 3N interactions in momentum space

\[ \xi^2 = p^2 + \frac{3}{4} q^2 \]

\[ \tan \theta = \frac{2p}{\sqrt{3}q} \]

same decoupling patterns like in NN interactions

Universality in 3N interactions at low resolution

To what extent are 3N interactions constrained at low resolution?

- only two low-energy constants $c_D$ and $c_E$
- 3N interactions give only subleading contributions to observables

phase-shift equivalence  common long-range physics

(approximate) universality of low-resolution NN interactions
remarkably reduced model dependence for typical momenta $\sim 1 \text{ fm}^{-1}$, matrix elements with significant phase space well constrained at low resolution

- new momentum structures induced at low resolution
- study based on $N^2\text{LO}$ chiral interactions, improved universality at $N^3\text{LO}$
Applications

• application to infinite systems

  → equation of state, systematic study of induced many-body contributions

• transformation of evolved interactions to oscillator basis

  → application to finite nuclei, complimentary to HO evolution (no core shell model, coupled cluster)

• study of alternative generators

  ▶ different decoupling patterns (e.g. \( V_{\text{low } k} \))
  ▶ improved efficiency of evolution
  ▶ suppression of many-body forces
Correlations in nuclear systems

What is this vertex?

\[ A(e,e'p) \]

- detection of knocked out pairs with large relative momenta
- excess of np pairs over pp pairs

**Short-range-correlation interpretation:**

- \( q \): low rel. momentum
- \( k' \): high rel. momentum

**Explanation in terms of low-momentum interactions?**

Correlations in nuclear systems

What is this vertex?

- detection of knocked out pairs with large relative momenta
- excess of np pairs over pp pairs

Short-range-correlation interpretation:

- $q$: low rel. momentum
- $k'$: high rel. momentum


Vertex depends on the resolution!
RG provides systematic way to calculate such processes at low resolution.
 Scaling in nuclear systems

- scaling behavior of momentum distribution function:
  \[ \rho_{NN}(q, Q = 0) \approx C_A \times \rho_{NN, \text{Deuteron}}(q, Q = 0) \] at large \( q \)

- dominance of np pairs over pp pairs

- “hard” (high resolution) interaction used, calculations hard!

- dominance explained by short-range tensor forces
Nuclear scaling at low resolution

\[ \langle \psi_\lambda | O_\lambda | \psi_\lambda \rangle \] factorizes into a low-momentum structure and a universal high momentum part if the initial operator only weakly couples low and high momenta → explains scaling!

RG transformation of pair density operator (induced many-body terms neglected):

“simple” calculation of pair density at low resolution in nuclear matter:

\[ \langle \rho(P, q) \rangle = \]
Nuclear scaling at low resolution

- pair-densities approximately resolution independent
- significant enhancement of np pairs over nn pairs due to tensor force
- reproduction of previous results using a “simple” calculation at low resolution!

High-resolution experiments can be explained by low-resolution methods! Opens door to study other electro-weak processes and higher-body correlations.
Summary

• low-resolution interactions allow simpler calculations for nuclear systems
• observables invariant under resolution changes, interpretation of results can change!
• chiral EFT provides systematic framework for constructing nuclear Hamiltonians
• 3N interactions are essential at low resolution
• nucleonic matter equation of state based on low-resolution interactions consistent with empirical constraints
• constraints for the nuclear equation of state and structure of neutron stars

Outlook

• RG evolution of three-nucleon interactions: microscopic study of light nuclei and nucleonic matter using chiral nuclear interactions at low resolution
• RG evolution of operators: nuclear scaling and correlations in nuclear systems