Baryon Number Fluctuations in Energy Scan Program at RHIC

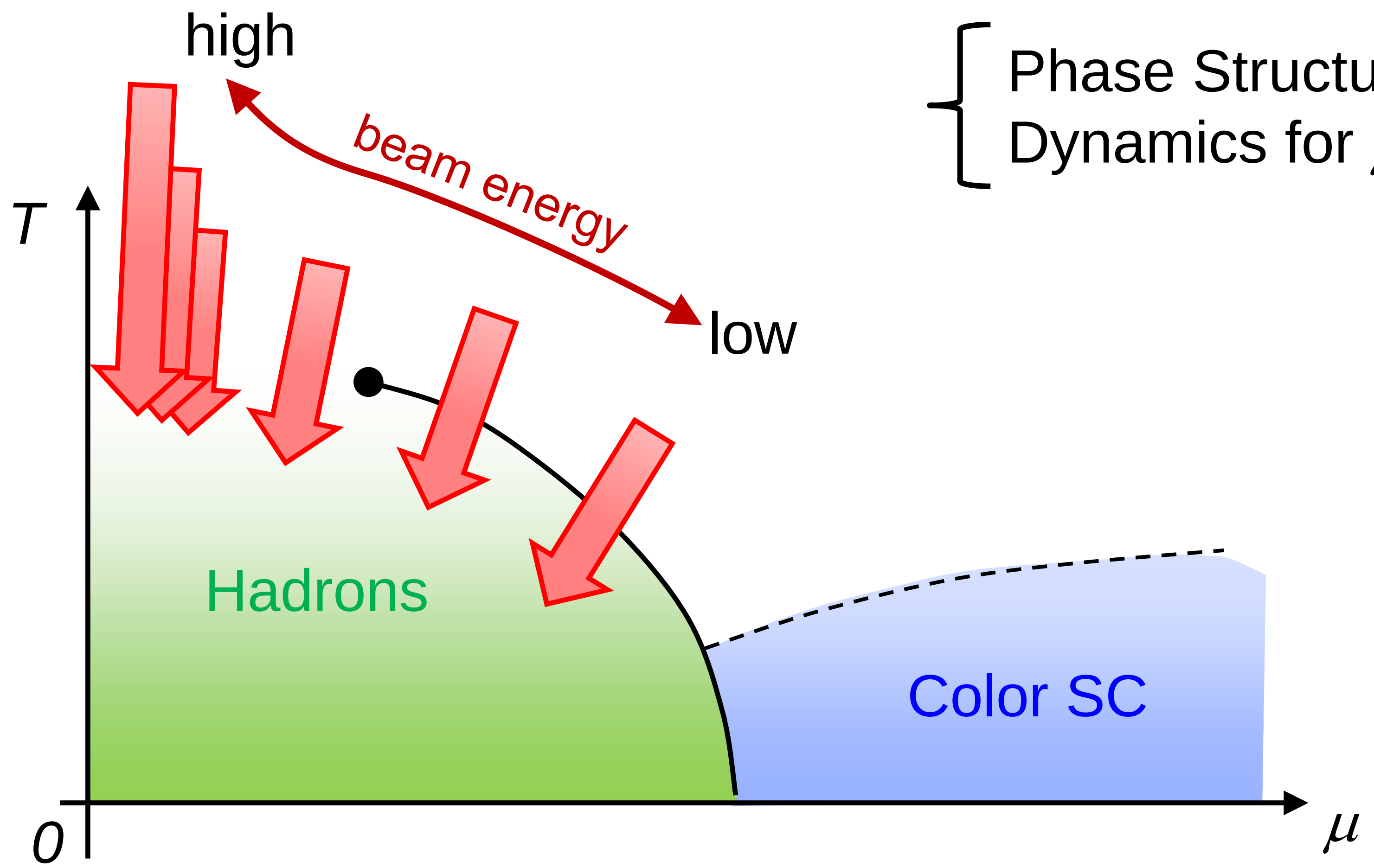
Masakiyo Kitazawa

(Osaka U.)

MK, Asakawa, arXiv:1107.2755 (to appear in PRC)

HIRSCHEGG2012, 18, Jan, 2012, Hirschegg

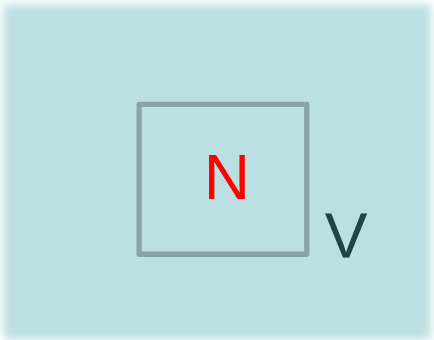
Energy Scan Program @ RHIC

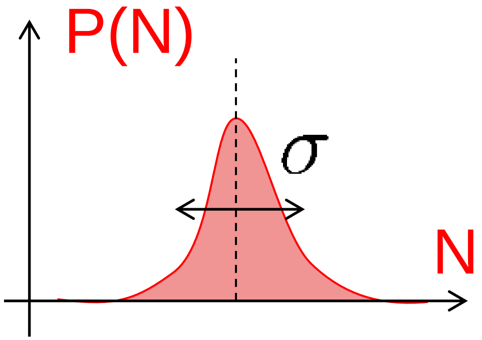


Phase Structure
Dynamics for μ~0

Fluctuations

Observables in equilibrium is fluctuating.





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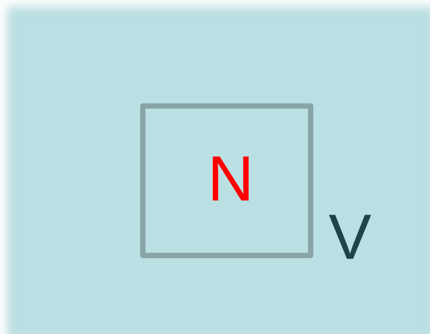
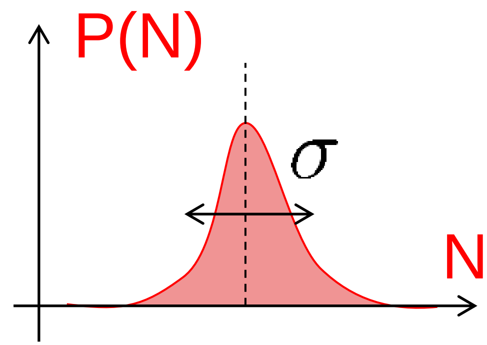




- Variance: $\langle \delta N^2 \rangle = V\chi_2 = \sigma^2$

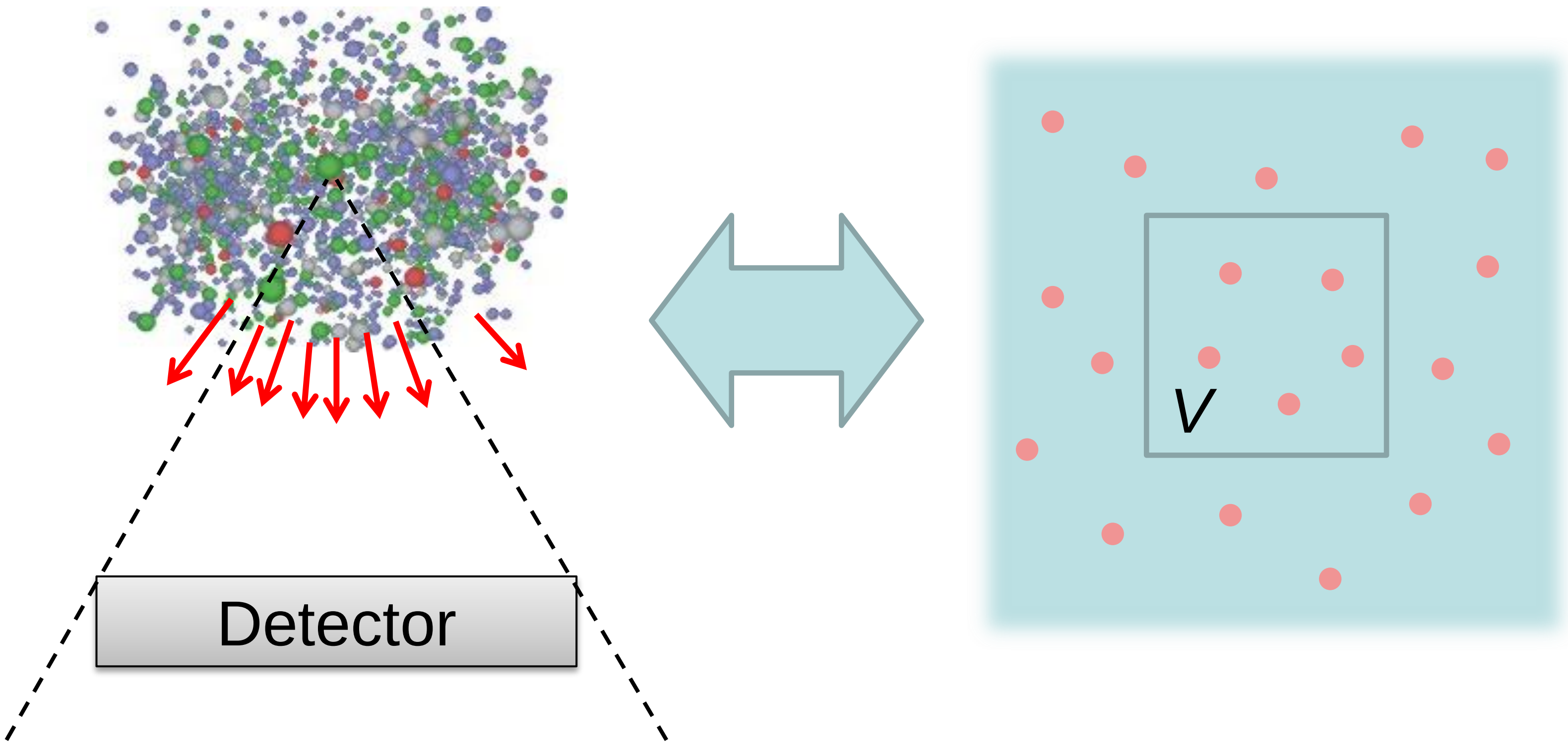
- Skewness: $S = \dfrac{\langle \delta N^3 \rangle}{\sigma^3}$

- Kurtosis: $\kappa = \dfrac{\langle \delta N^4 \rangle - 3\langle \delta N^2 \rangle^2}{\chi_2 \sigma^2}$

$$\delta N = N - \langle N \rangle$$

Event-by-Event Analysis @ HIC

Fluctuations can be measured by e-by-e analysis in experiments.



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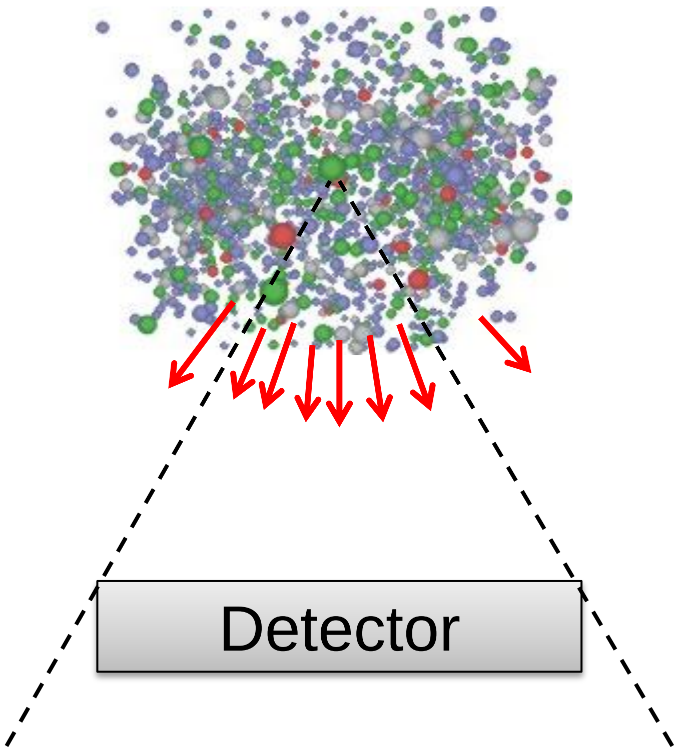
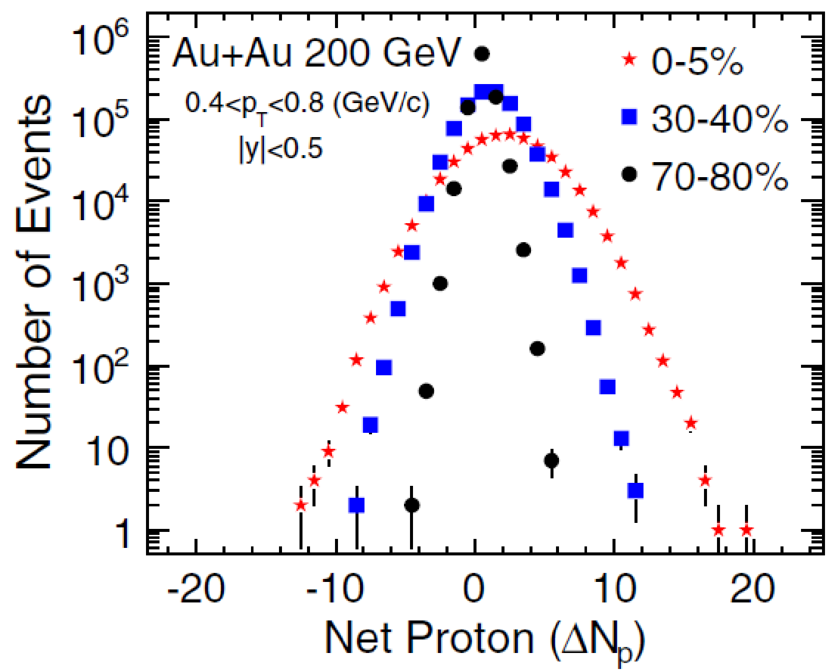


STAR, PRL**105** (2010)



Fluctuations

- Fluctuations reflect properties of matter.
  - Enhancement near the critical point

    Stephanov,Rajagopal,Shuryak('98); Hatta,Stephanov('02); Stephanov('09);…
  - Ratios between cumulants of conserved charges

    Asakawa,Heintz,Muller('00); Jeon, Koch('00); Ejiri,Karsch,Redlich('06)
  - Signs of higher order cumulants

    Asakawa,Ejiri,MK('09); Friman,et al.('11); Stephanov('11)

Fluctuations

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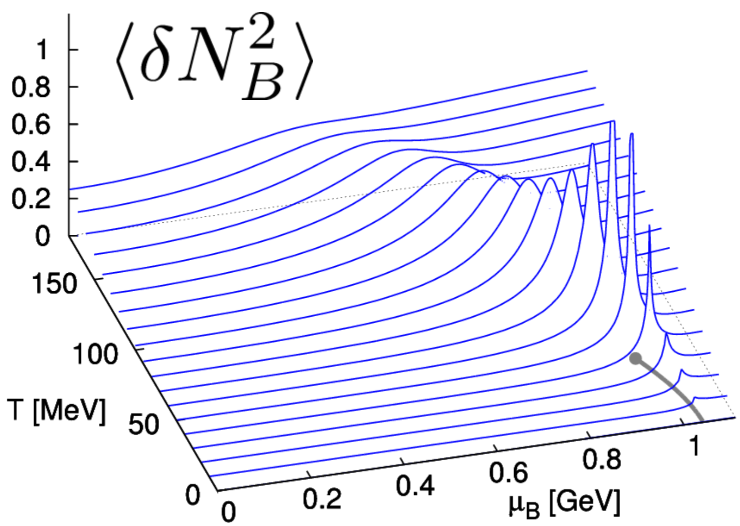
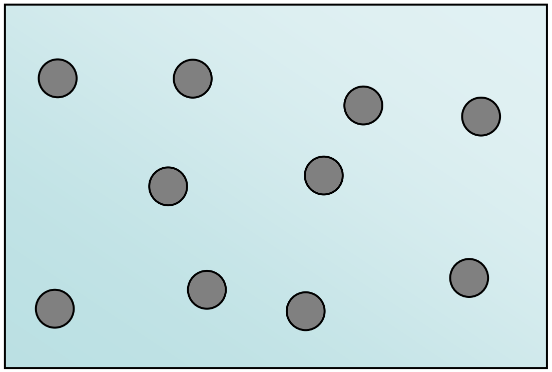
- 2nd order phase transition at the CP.

- divergences in fluctuations of
  - p_T distribution
  - freezeout T
  - baryon number, proton, chage, …



Ratios of Cumulants

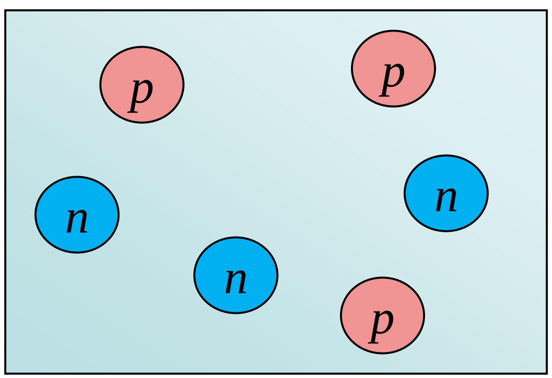
Asakawa, Heinz, Muller, ’00
Jeon, Koch, ’00
Ejiri, Karsch, Redlich, ’06



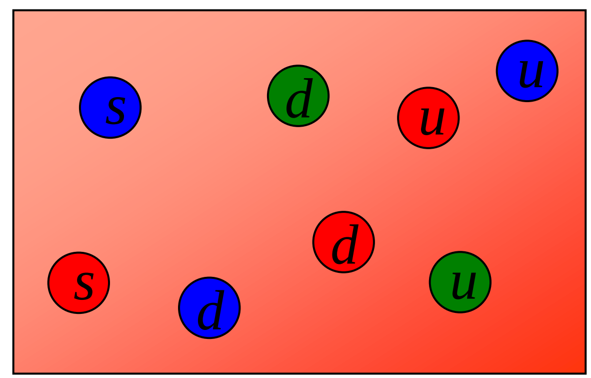
Boltzmann gas $(T, \mu \ll M)$
(=Poisson distribution)

$$\langle N \rangle = \langle \delta N^2 \rangle = \langle \delta N^3 \rangle = \langle \delta N^4 \rangle_c = \cdots$$

Hadrons: $N_B = N$



Quark–gluon: $N_B = N/3$



$$\frac{\langle \delta N_B^n \rangle_c}{\langle \delta N_B^m \rangle_c} = 1$$

$$\frac{\langle \delta N_B^n \rangle_c}{\langle \delta N_B^m \rangle_c} = \frac{1}{3^{n-m}}$$

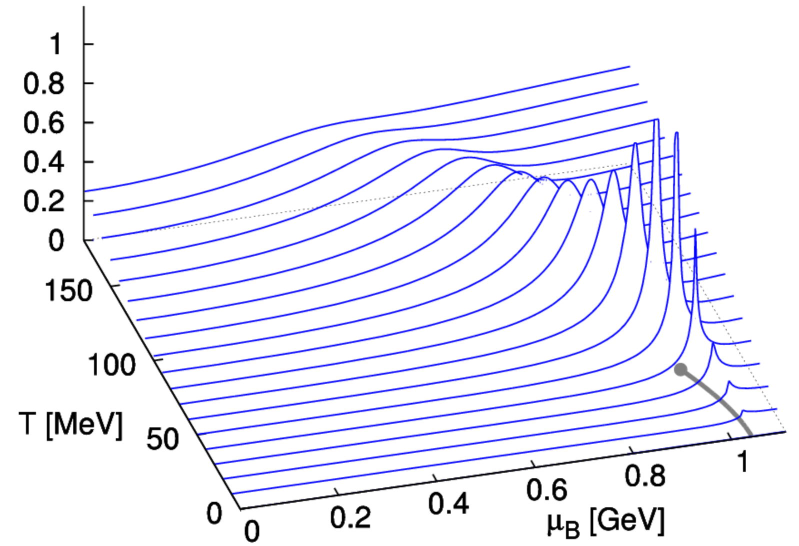
Take a Derivative of χ_B

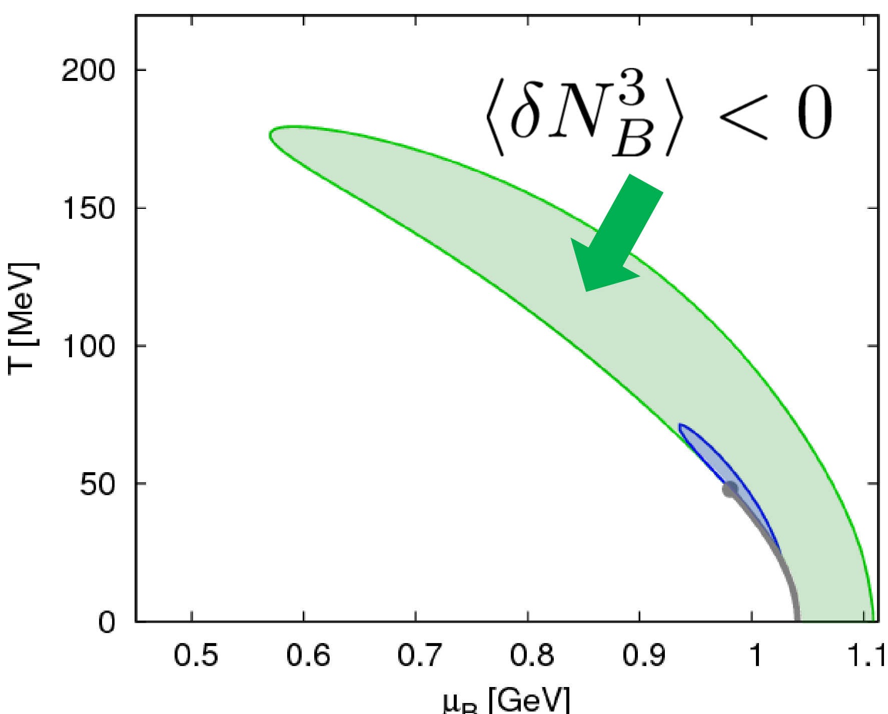
Asakawa, Ejiri, MK, 2009

- χ_B has an edge along the phase boundary

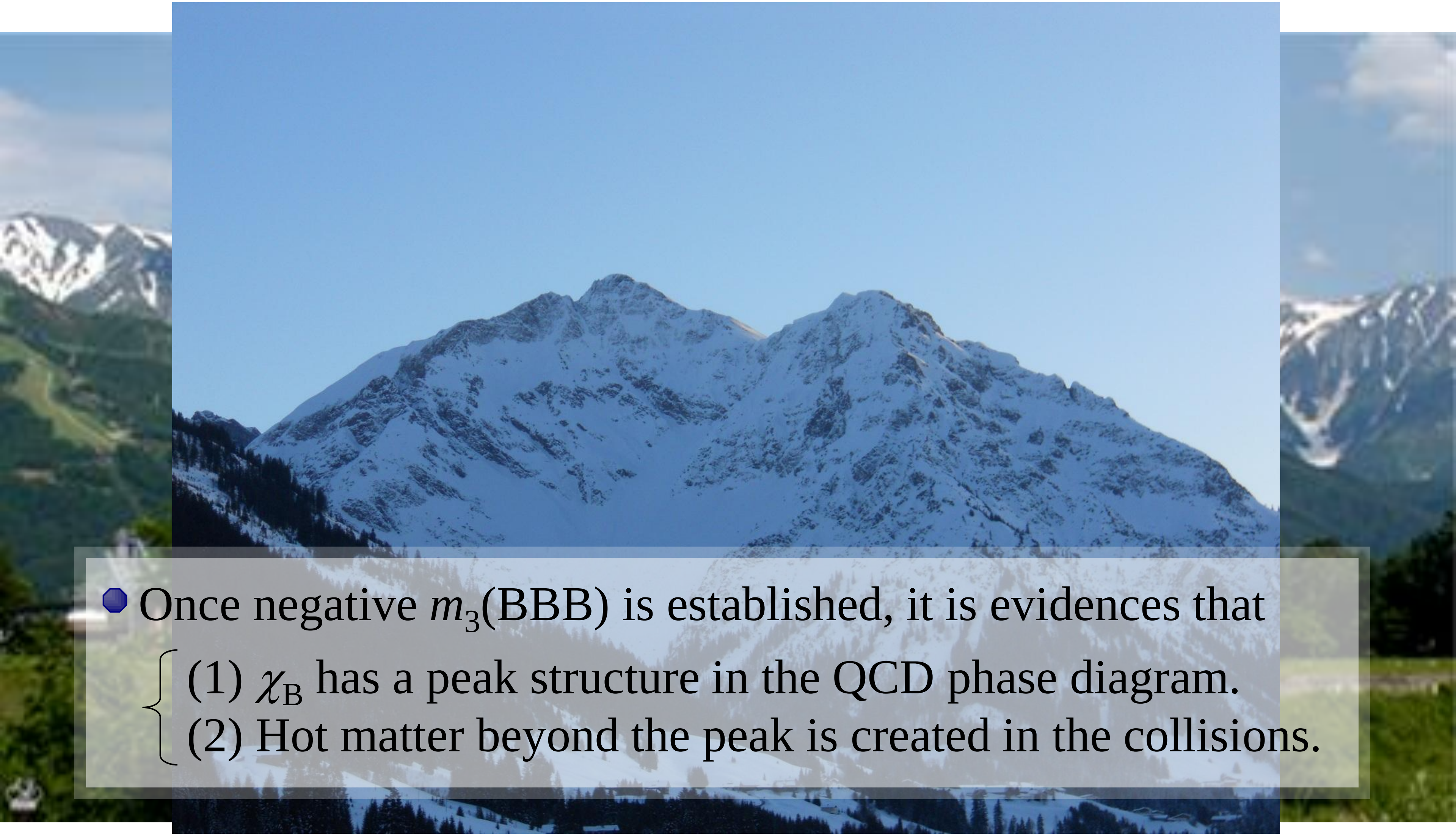
- $$\begin{cases} \chi_B = -\dfrac{1}{V}\dfrac{\partial^2\Omega}{\partial\mu_B^2} = \dfrac{\left\langle(\delta N_B)^2\right\rangle}{VT} \\ \dfrac{\partial\chi_B}{\partial\mu_B} = -\dfrac{1}{V}\dfrac{\partial^3\Omega}{\partial\mu_B^3} = \dfrac{\left\langle(\delta N_B)^3\right\rangle}{VT^2} \end{cases}$$

$\dfrac{\partial\chi_B}{\partial\mu_B}$ changes the sign at QCD phase boundary!





Impact of Negative Third Moments

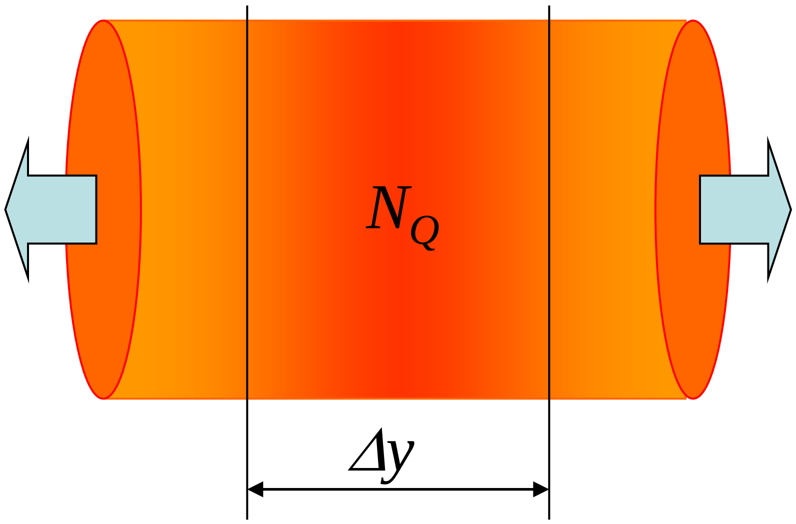


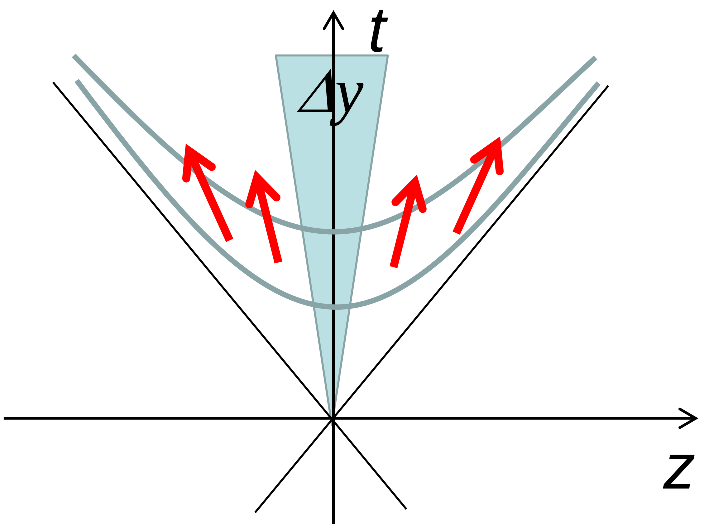
- Once negative m_3(BBB) is established, it is evidences that
  (1) χ_B has a peak structure in the QCD phase diagram.
  (2) Hot matter beyond the peak is created in the collisions.

- •**No** dependence on any specific models.
  •Just the sign! **No** normalization (such as by N_{ch}).

Fluctuations of Conserved Charges

- When is experimentally measured *D* formed?



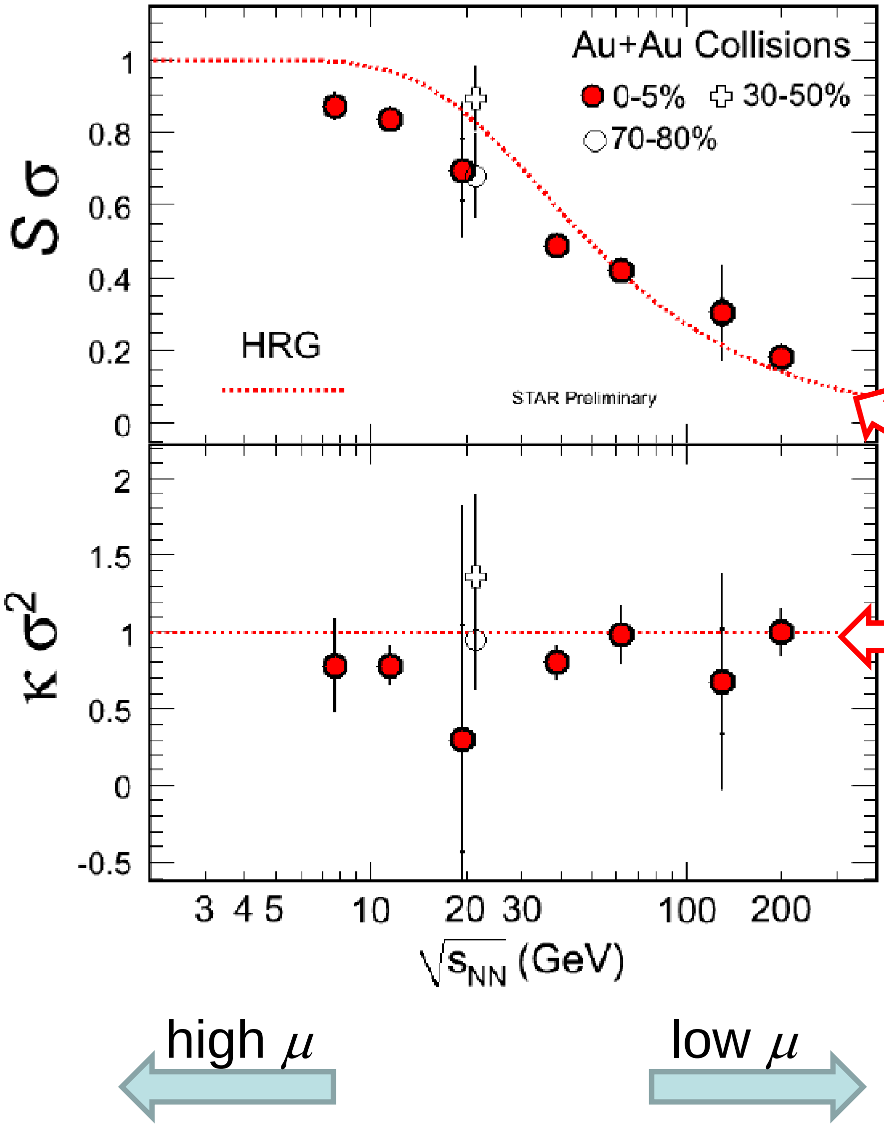


- **Conserved charges** can remember fluctuations at early stage, if diffusions are sufficiently slow.

Asakawa,Heintz,Muller ('00); Jeon,Koch ('00); Shuryak, Stephanov ('02)

Proton # Fluctuations @ STAR

2011 (Quark Matter)



$$S\sigma = \frac{\langle (\delta N_p^{(\mathrm{net})})^3 \rangle}{\langle (\delta N_p^{(\mathrm{net})})^2 \rangle},$$

$$\kappa\sigma^2 = \frac{\langle (\delta N_p^{(\mathrm{net})})^4 \rangle_c}{\langle (\delta N_p^{(\mathrm{net})})^2 \rangle}$$

HRG:
Hadron Resonance Gas
||
Free gas composed of all hadrons & resonances

Proton # Fluctuations @ STAR

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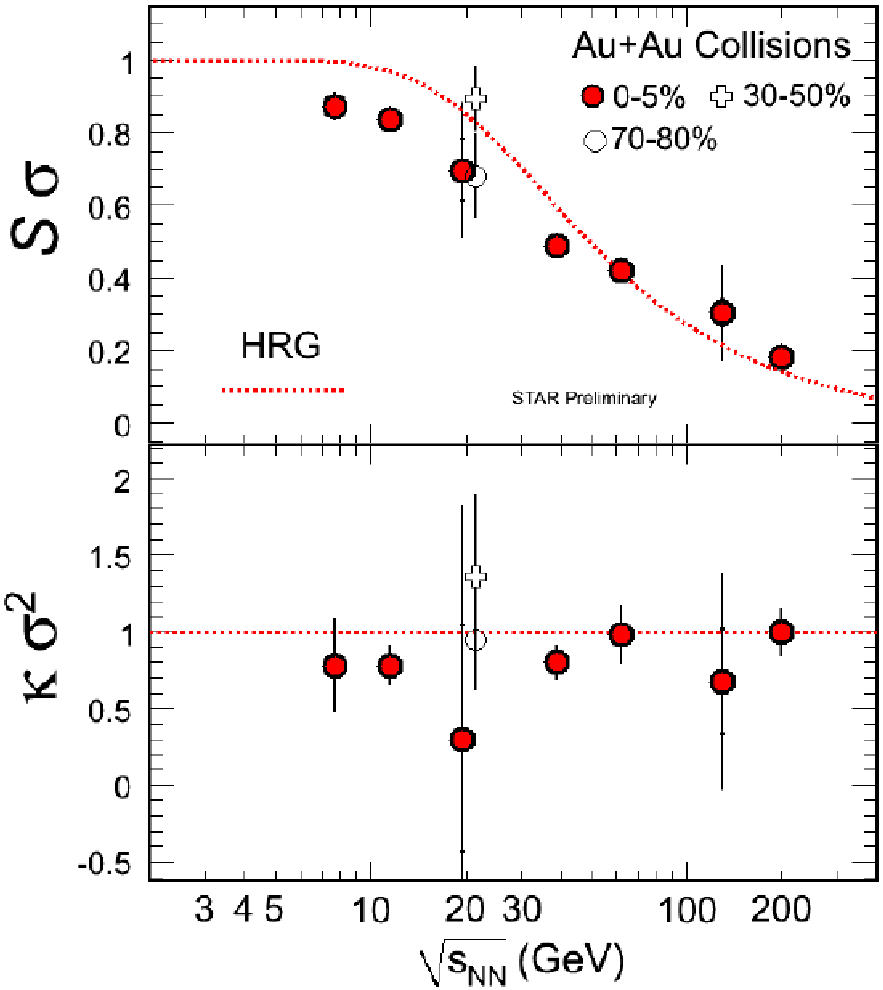


high μ ← → low μ

$$S\sigma = \frac{\langle(\delta N_p^{(\mathrm{net})})^3\rangle}{\langle(\delta N_p^{(\mathrm{net})})^2\rangle},$$

$$\kappa\sigma^2 = \frac{\langle(\delta N_p^{(\mathrm{net})})^4\rangle_c}{\langle(\delta N_p^{(\mathrm{net})})^2\rangle}$$

- No negative *S*
- No suppression of $\kappa\sigma^2$

Do fireballs forget all information in the early stage?

$$\boxed{\langle \delta N_p^n \rangle} \qquad \boxed{\langle \delta N_{\mathrm{B}}^n \rangle}$$

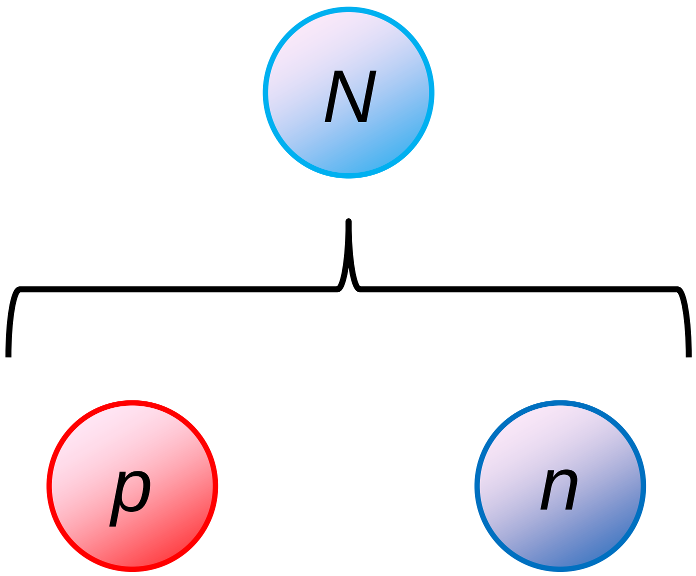
- In equilibrated free nucleon gas,

$$\langle \delta N_{\mathrm{B}}^n \rangle_c = 2 \langle \delta N_p^n \rangle_c$$

- If the medium is not equilibrated,

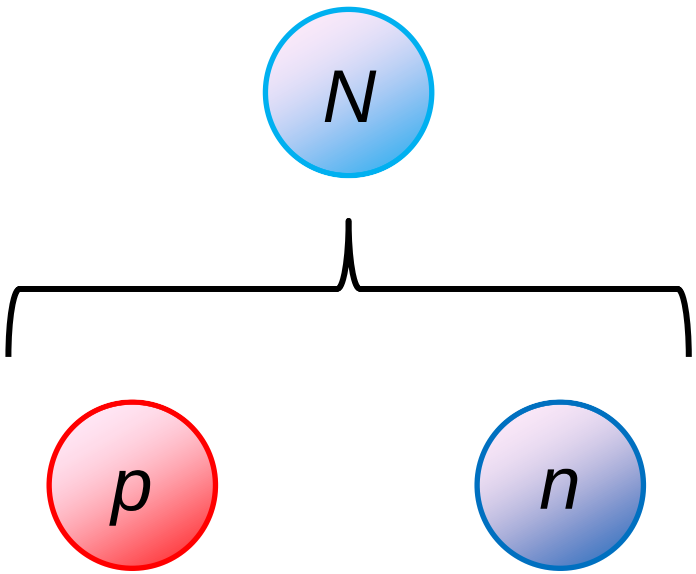
$$\langle \delta N_{\mathrm{B}}^n \rangle_c \neq 2 \langle \delta N_p^n \rangle_c$$

Nucleon Isospin as Two Sides of a Coin

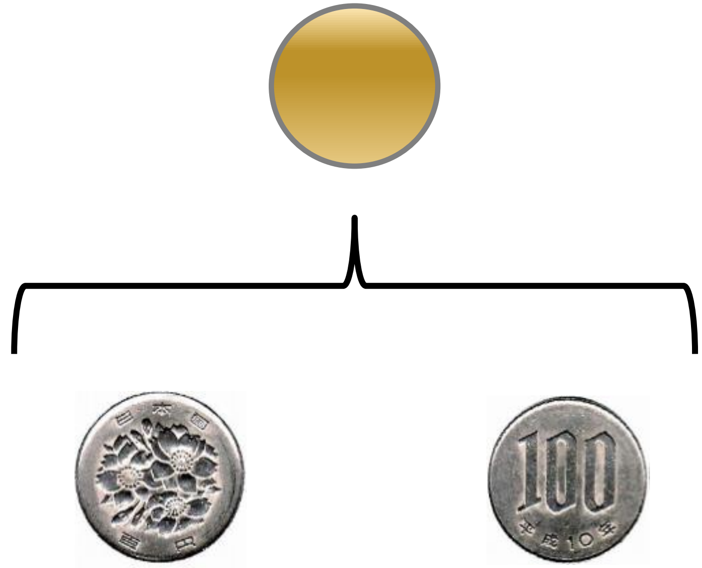


Nucleons have
two isospin states.

Nucleon Isospin as Two Sides of a Coin



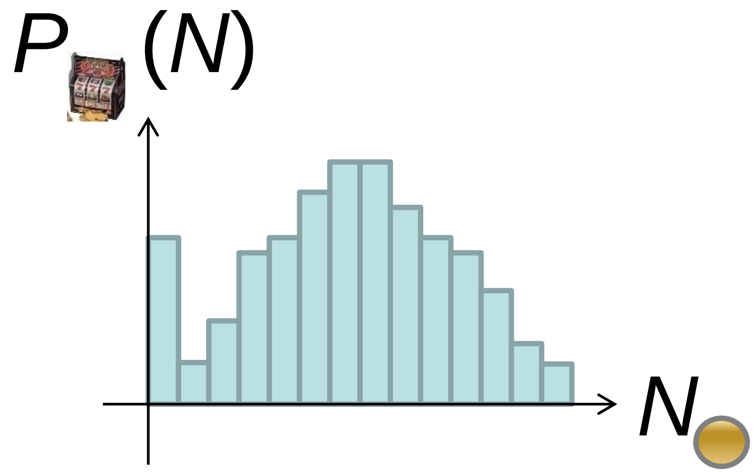
a coin

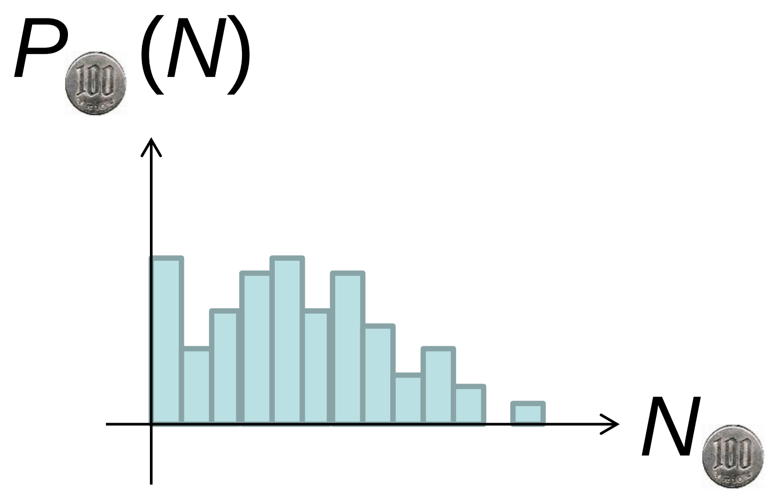


Nucleons have
two isospin states.

Coins have two sides.

Slot Machine Analogy

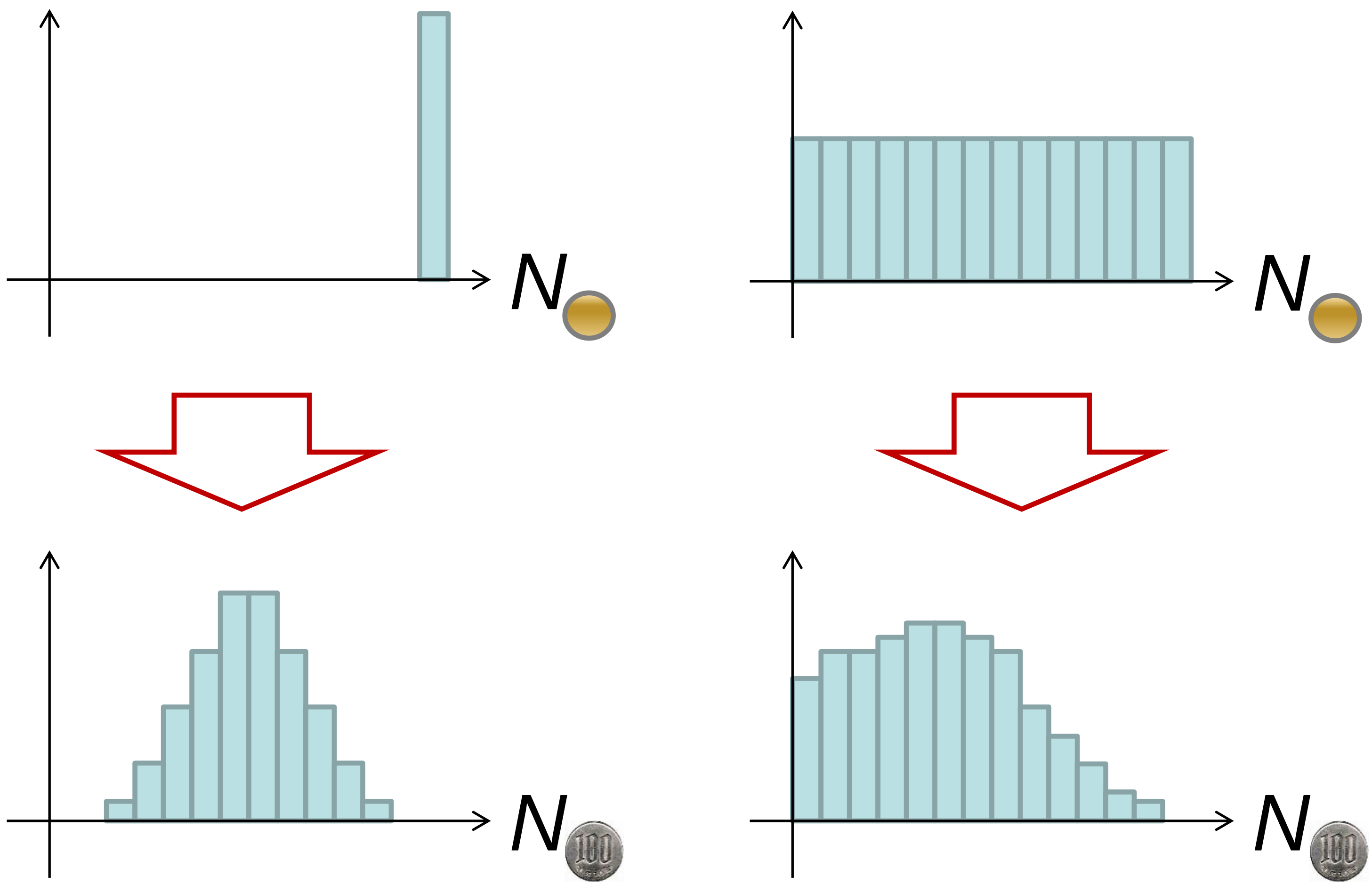




Extreme Examples

Fixed # of coins

Constant probabilities



Baryon Number Fluctuations are Better

- Since it is a conserved charge
  - Expectation values are well defined
  - possible slow diffusion in hadronic stage

Asakawa,Heintz,Muller ('00); Jeon, Koch ('00)

- Clearest observable for enhancement near CP
  - Same critical exponents for baryon, charge, proton #

Hatta, Stephanov ('02)

- But singular contribution is largest in baryon #

- Ratio of cumulants behaves most drastically

Reconstructing Total Coin Number

$$P_{\,}(N_{\,}) = \sum_{\,} P_{\,}(N_{\,}) B_{1/2}(N_{\,}; N_{\,})$$

$$B_p(k;N) = p^k(1-p)^{N-k}\ {}_kC_N$$:binomial distr. func.

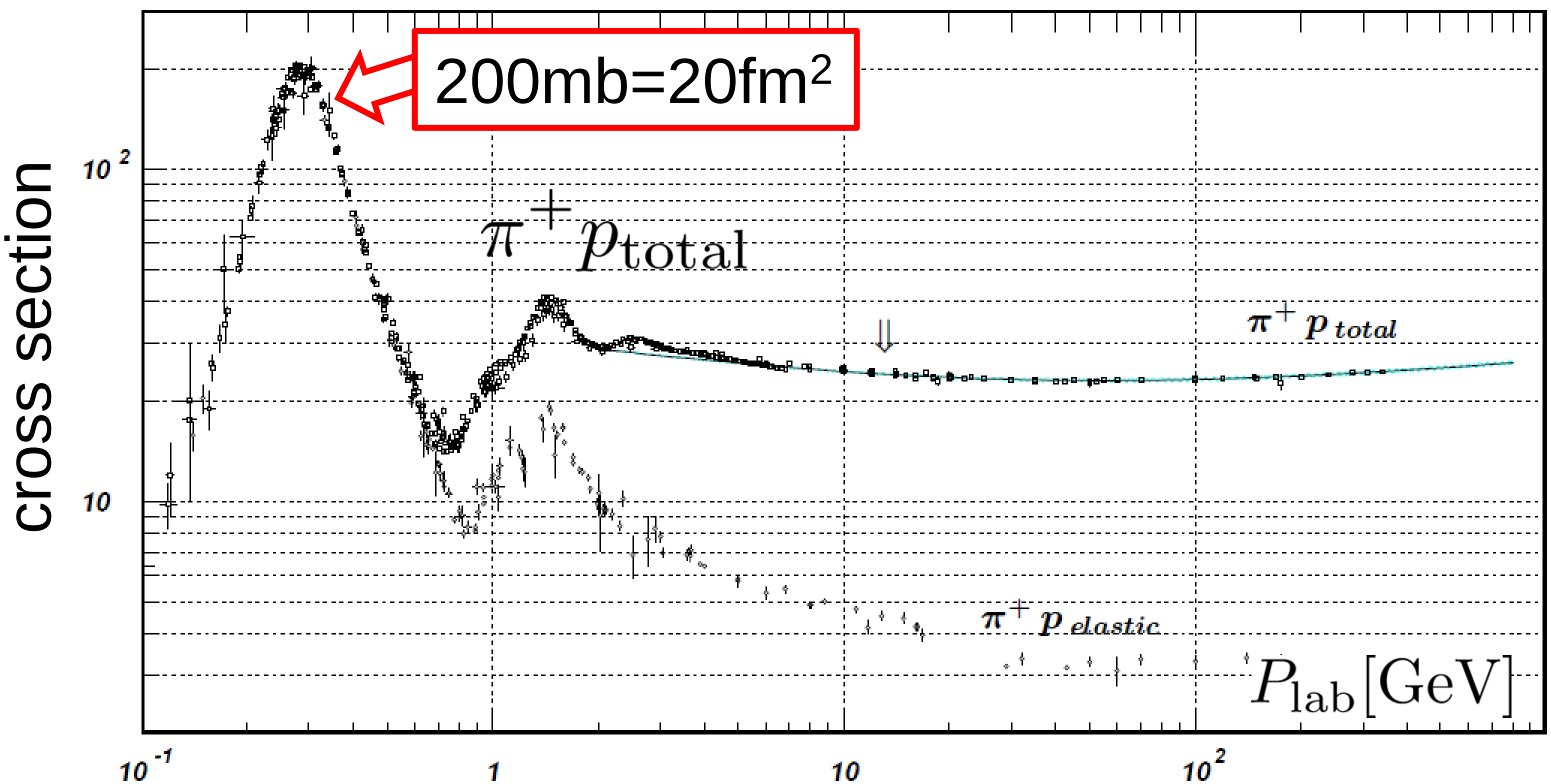
Nucleon Isospin in Hadronic Medium

- Isospin of baryons can vary after chemical freezeout via charge exchange reactions mediated by $\Delta(1232)$:

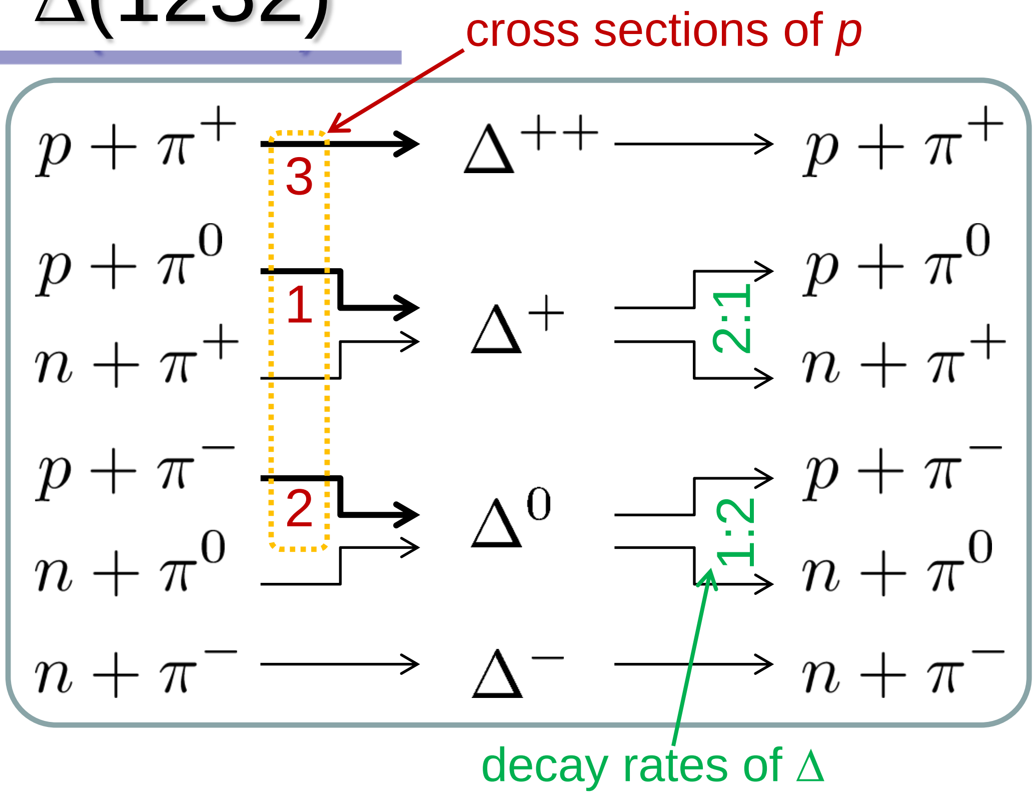
$$p, n \searrow \quad \pi \nearrow \quad \Delta(1232) \quad \nearrow p, n \quad \searrow \pi$$

$$I = 3/2$$

$$\Gamma \simeq 1.8\ [\mathrm{fm}]$$

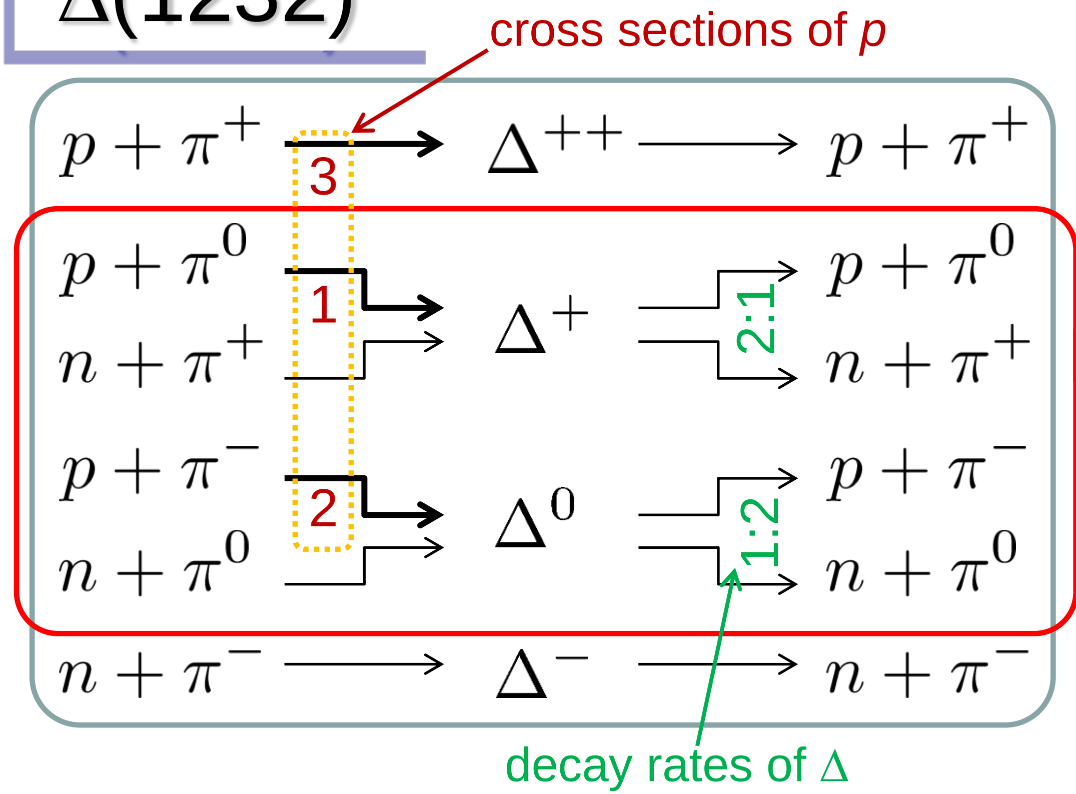


Δ(1232)



cross sections of p
$p + \pi^+$
Δ^{++}
$p + \pi^+$
3
$p + \pi^0$
1
Δ^+
2:1
$p + \pi^0$
$n + \pi^+$
$n + \pi^+$
$p + \pi^-$
2
Δ^0
1:2
$p + \pi^-$
$n + \pi^0$
$n + \pi^0$
$n + \pi^-$
Δ^-
$n + \pi^-$
decay rates of Δ

Δ(1232)



$$p+\pi \rightarrow \Delta^{+,0}$$

$$\rightarrow p:n$$

$$= 5:4$$

$\Delta(1232)$

cross sections of p

$$p+\pi^+ \xrightarrow{3} \Delta^{++} \longrightarrow p+\pi^+$$

$$\left.\begin{array}{l} p+\pi^0 \xrightarrow{1} \\ n+\pi^+ \longrightarrow \end{array}\right\} \Delta^+ \xrightarrow{2:1} \left\{\begin{array}{l} p+\pi^0 \\ n+\pi^+ \end{array}\right.$$

$$\left.\begin{array}{l} p+\pi^- \xrightarrow{2} \\ n+\pi^0 \longrightarrow \end{array}\right\} \Delta^0 \xrightarrow{1:2} \left\{\begin{array}{l} p+\pi^- \\ n+\pi^0 \end{array}\right.$$

$$n+\pi^- \longrightarrow \Delta^- \longrightarrow n+\pi^-$$

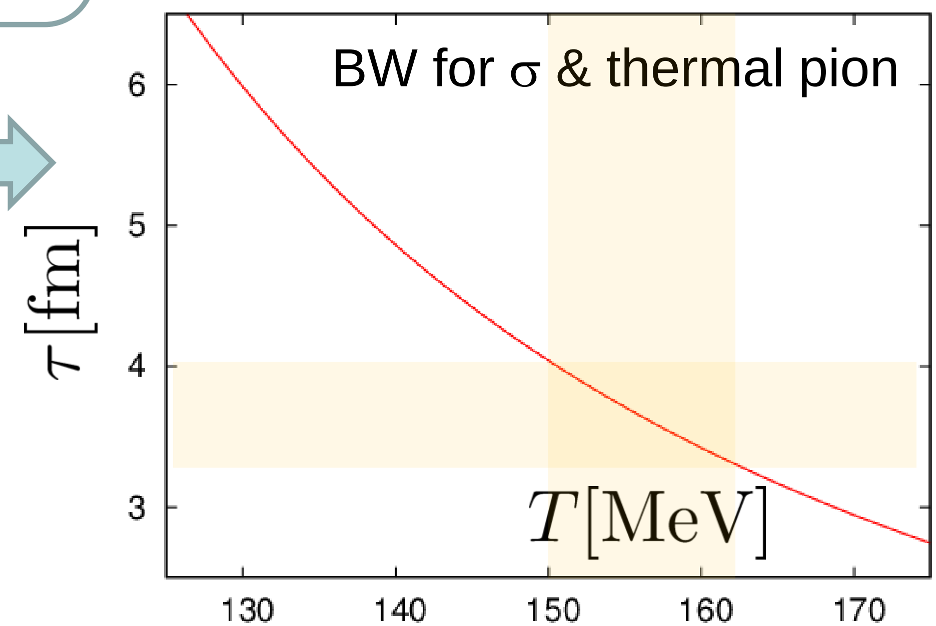
decay rates of Δ

$$p+\pi \to \Delta^{+,0}$$
$$\to p:n$$
$$= 5:4$$

Lifetime to create Δ^+ or Δ^0

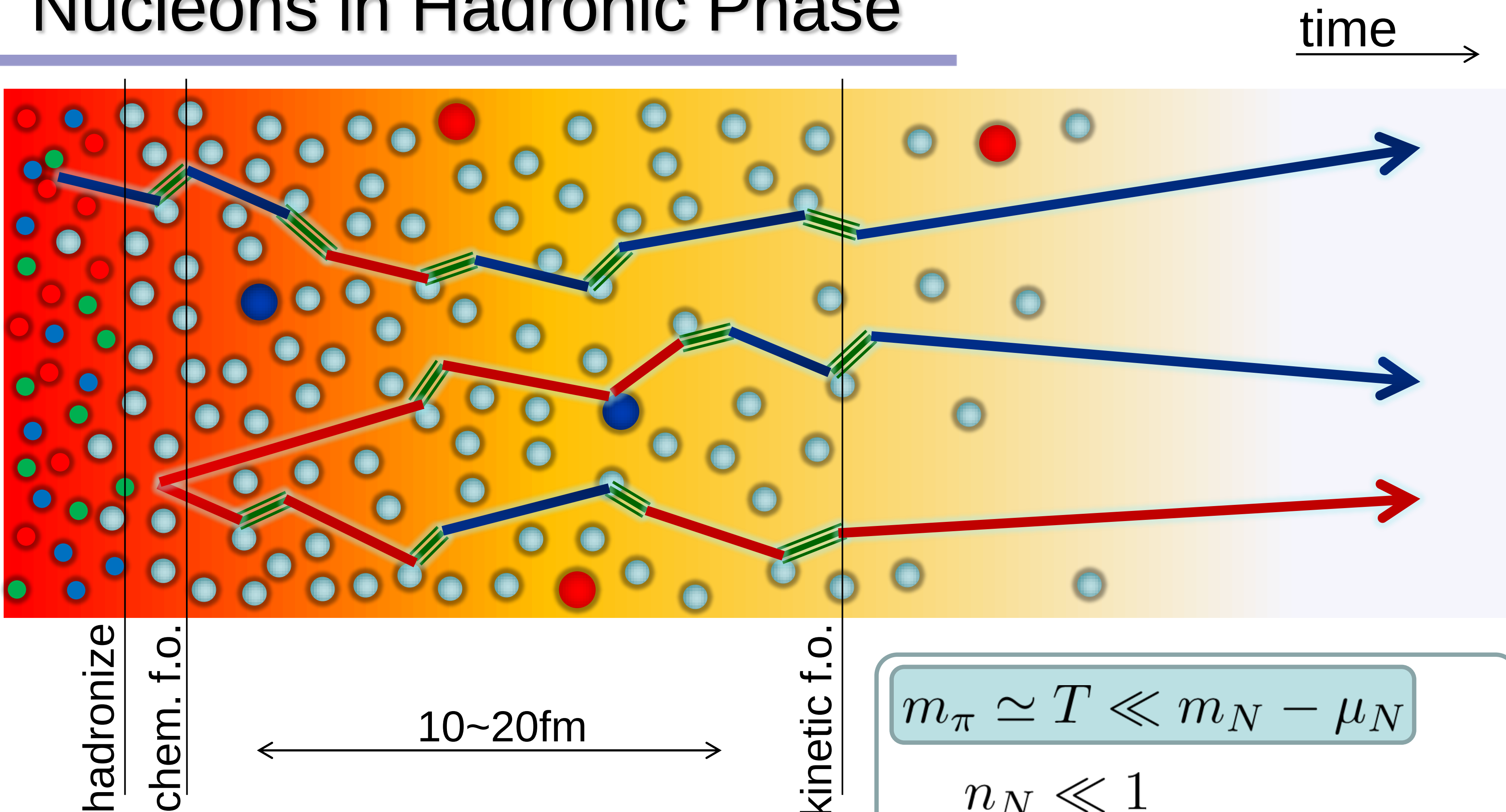
$$\tau^{-1} = \int \frac{d^3k_\pi}{(2\pi)^3}\sigma(E_{\rm cm})v_\pi n(E_\pi)$$

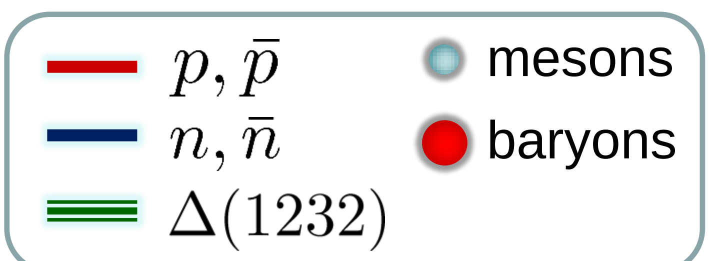
$$\tau \ll (\text{freezeout time}) \simeq 20[\text{fm}]$$



c.f.) Nonaka, Bass, 2007

Nucleons in Hadronic Phase





$$m_\pi \simeq T \ll m_N - \mu_N$$

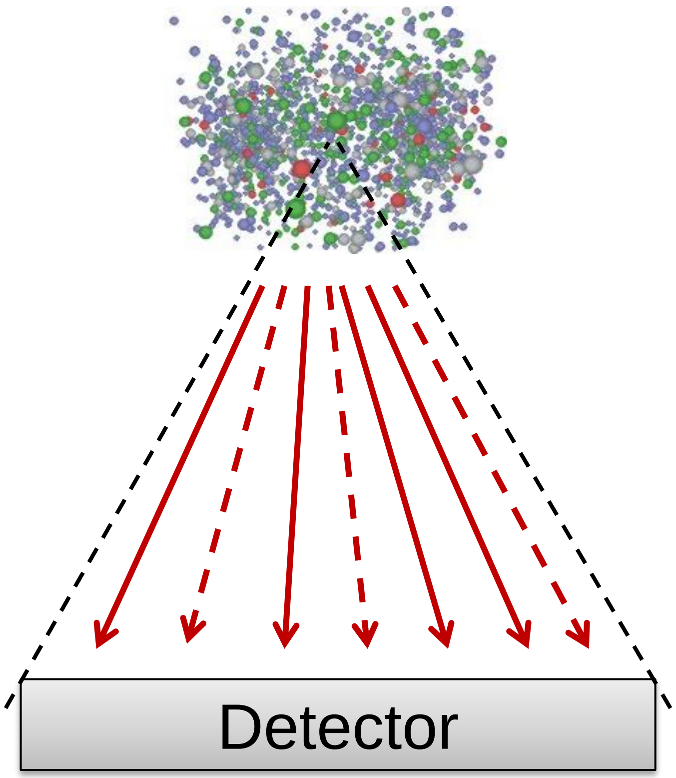
$n_N \ll 1$

- rare NN collisions
- no quantum corr.

$n_N \ll n_\pi$

- many pions

Probability Distribution $\mathcal{P}(N_p, N_n, N_{\bar{p}}, N_{\bar{n}})$



- $\left\{ \begin{array}{l} \longrightarrow N_N \text{ nucleons} \\ \dashrightarrow N_{\bar{N}} \text{ anti-nucleons} \end{array} \right.$

  $\Rightarrow F(N_N, N_{\bar{N}})$

- $N_N \left\{ \begin{array}{l} N_p \text{ protons} \\ N_n \text{ neutrons} \end{array} \right.$

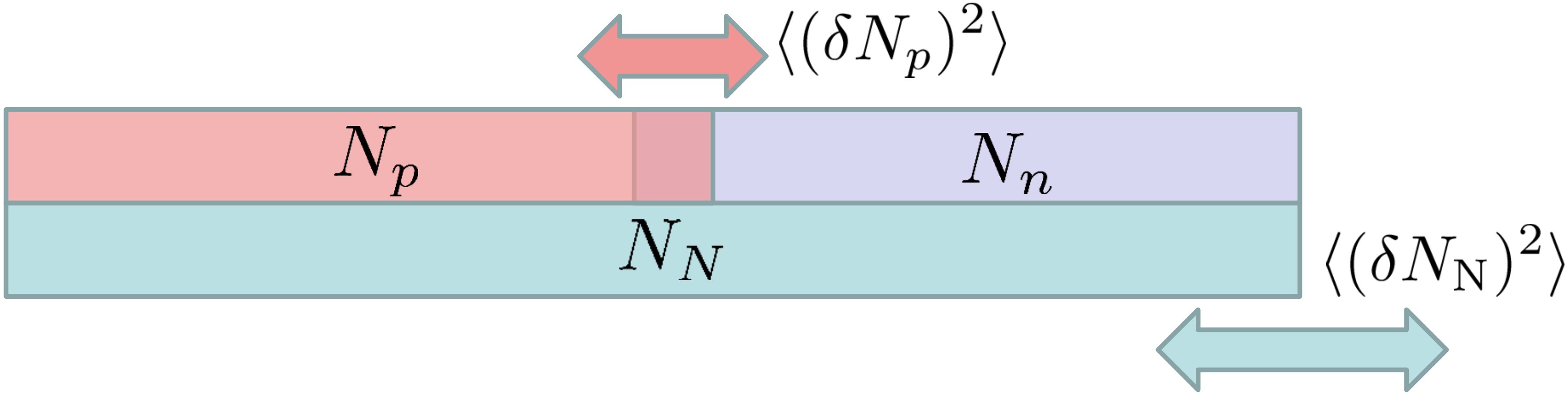
  $\Rightarrow B(N_p; N_N)$

  binomial distribution func.

$$\mathcal{P}(N_p, N_n, N_{\bar{p}}, N_{\bar{n}})$$
$$= F(N_N, N_{\bar{N}}) B(N_p; N_N) B(N_{\bar{p}}; N_{\bar{N}})$$

- for any phase space in the final state.

Nucleon & Proton Number Fluctuations



$$\left\{\begin{aligned}\langle(\delta N_p^{(\rm net)})^2\rangle &= \frac{1}{4}\langle(\delta N_{\rm N}^{(\rm net)})^2\rangle + \frac{1}{4}\langle N_{\rm N}^{(\rm tot)}\rangle \\ \langle(\delta N_{\rm N}^{(\rm net)})^2\rangle &= 4\langle(\delta N_p^{(\rm net)})^2\rangle - 2\langle N_p^{(\rm tot)}\rangle\end{aligned}\right.$$

- for isospin symmetric medium
- effect of isospin density <10%
- Similar formulas up to any order!

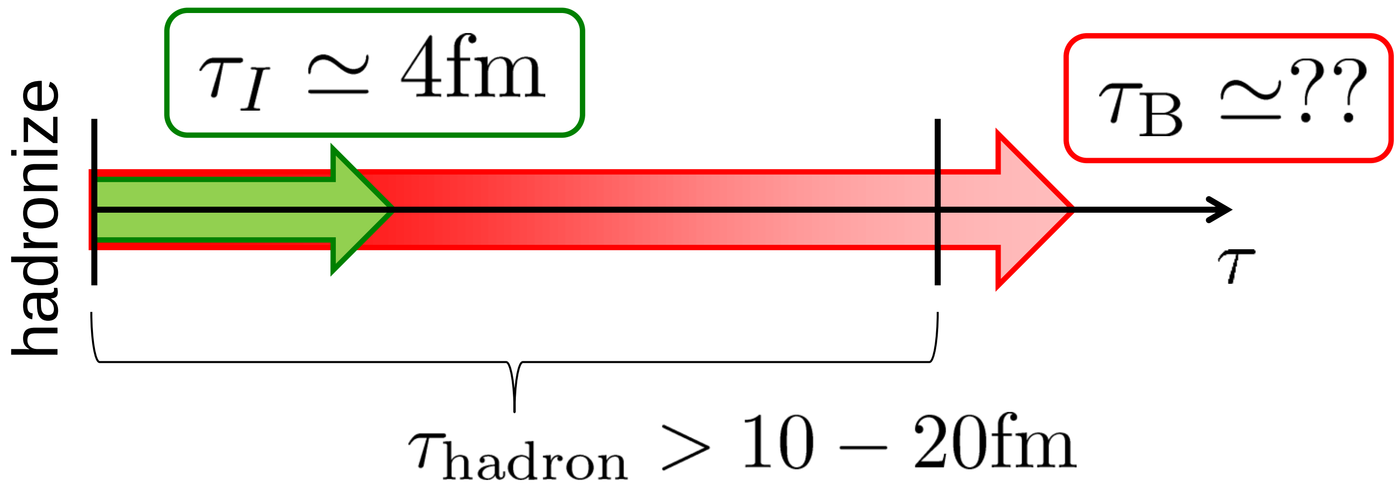
For free gas

$$\langle(\delta N_p^{(\rm net)})^2\rangle = \frac{1}{2}\langle(\delta N_{\rm N}^{(\rm net)})^2\rangle$$

Time Scales

- Time scales of fireballs:

$$\begin{cases} \tau_I & \text{: time scale to realize isospin binomiality} \\ \tau_{\rm B} & \text{: time scale of baryon number diffusion} \\ \tau_{\rm hadron} & \text{: life-time of hadronic medium in HIC} \end{cases}$$



Effect of Isospin Distribution

(1) $N_B^{(\mathrm{net})} = N_B - N_{\bar{B}}$ deviates from the equilibrium value.

(2) Boltzmann (Poisson) distribution for $N_B, N_{\bar{B}}$.

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(1) $N_B^{(\mathrm{net})} = N_B - N_{\bar{B}}$ deviates from the equilibrium value.

(2) Boltzmann (Poisson) distribution for $N_B, N_{\bar{B}}$.

$$
\left\{
\begin{aligned}
2\langle(\delta N_p^{(\mathrm{net})})^2\rangle &= \frac{1}{2}\langle(\delta N_{\mathrm{B}}^{(\mathrm{net})})^2\rangle + \frac{1}{2}\langle(\delta N_{\mathrm{B}}^{(\mathrm{net})})^2\rangle_{\mathrm{free}} \\
2\langle(\delta N_p^{(\mathrm{net})})^3\rangle &= \frac{1}{4}\langle(\delta N_{\mathrm{B}}^{(\mathrm{net})})^3\rangle + \frac{3}{4}\langle(\delta N_{\mathrm{B}}^{(\mathrm{net})})^3\rangle_{\mathrm{free}} \\
2\langle(\delta N_p^{(\mathrm{net})})^4\rangle_c &= \frac{1}{8}\langle(\delta N_{\mathrm{B}}^{(\mathrm{net})})^4\rangle_c + \cdots
\end{aligned}
\right.
$$

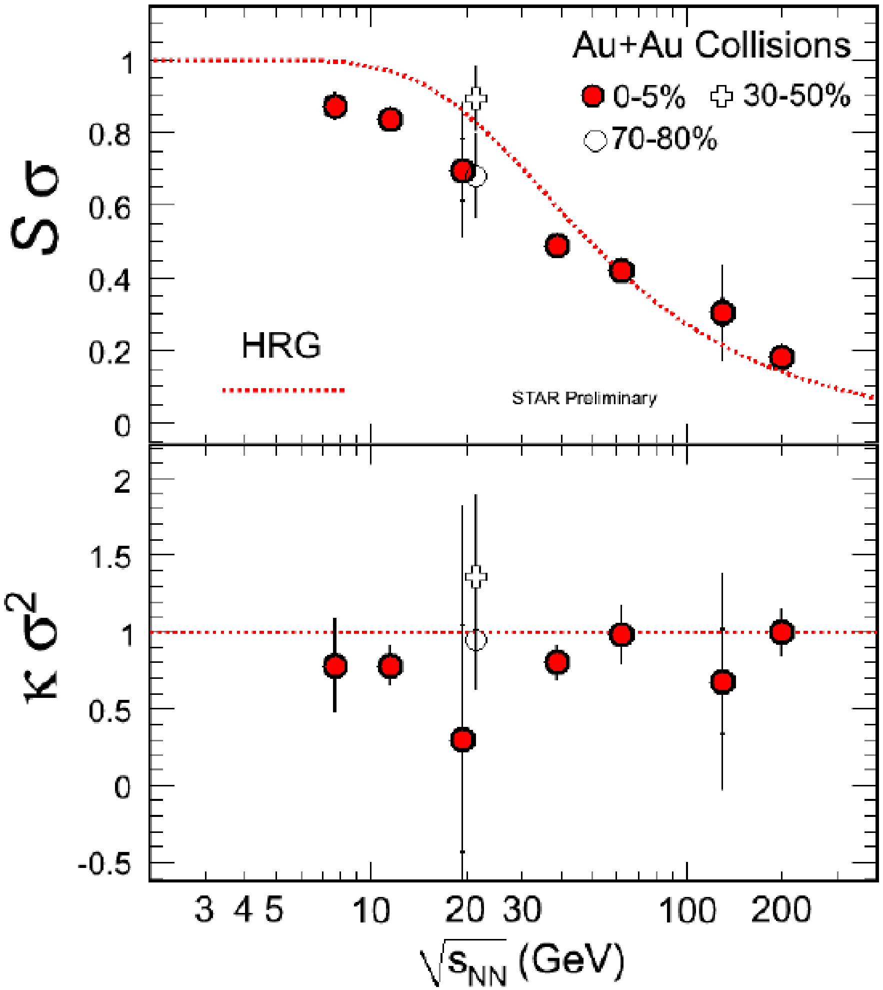
genuine info. noise

For free gas

$$2\langle(\delta N_p^{(\mathrm{net})})^n\rangle_c = \langle(\delta N_{\mathrm{N}}^{(\mathrm{net})})^n\rangle_c$$

Proton # Fluctuations @ STAR

2011 (Quark Matter)



high μ

low μ

$$S\sigma = \frac{\langle(\delta N_p^{(\mathrm{net})})^3\rangle}{\langle(\delta N_p^{(\mathrm{net})})^2\rangle},$$

$$\kappa\sigma^2 = \frac{\langle(\delta N_p^{(\mathrm{net})})^4\rangle_c}{\langle(\delta N_p^{(\mathrm{net})})^2\rangle}$$

- No negative S
- No suppression of $\kappa\sigma^2$

Do fireballs forget all information in the early stage?

No. Not necessarily!

Strange Baryons

Decay Rates:

$\boxed{\Lambda}$ $m_\Lambda \simeq 1116[\mathrm{MeV}]$

$$\Rightarrow \; p : n \simeq 1.6 : 1$$

$\boxed{\Sigma}$ $m_\Sigma \simeq 1190[\mathrm{MeV}]$

$$\Rightarrow \; p : n \simeq 1 : 1.8$$

Decay modes:

$$\Lambda \rightarrow \begin{cases} p + \pi^- & 64\% \\ n + \pi^0 & 36\% \end{cases}$$

$$\Sigma^+ \rightarrow \begin{cases} p + \pi^0 & 52\% \\ n + \pi^+ & 48\% \end{cases}$$

$$\Sigma^0 \rightarrow \Lambda \rightarrow \begin{cases} p + \pi^- & 64\% \\ n + \pi^0 & 36\% \end{cases}$$

$$\Sigma^- \longrightarrow n + \pi^-$$

Regarding these ratios even, protons from these decays is incorporated into the binomial distribution. Then, $N_N \rightarrow N_B$

Summary

- **Baryon and proton number fluctuations are different**.
  To see non-thermal effects in heavy ion collisions, baryon number's is better.

- Formulas to reveal baryon # cumulants in experiments.

- Experimental analysis of baryon # fluctuations may verify
  - signals of QCD phase transition
  - speed of baryon number diffusion in the hadronic stage.

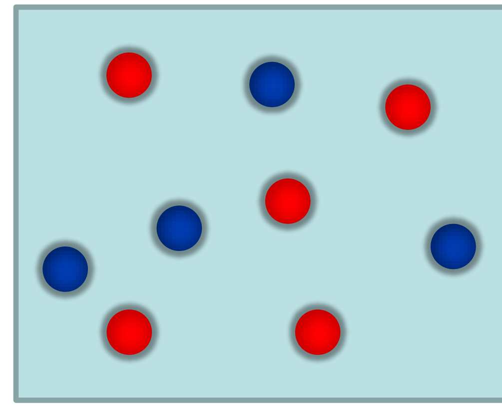
Future Work

- Distribution function itself
- Incorporating effects of efficiency and acceptance in exp.
  / correlation between isospin fluctuations of pions
- Discussion on dynamical evolution of fluctuations

Free Nucleon Gas

$T, \mu_{\mathrm{B}} \ll m_{\mathrm{N}}$ ⇨ Poisson distribution $P_\lambda(N)$

$$\mathcal{P}(N_p, N_n) = P_\lambda(N_p) P_\lambda(N_n)$$
$$= P_{2\lambda}(N_p + N_n) B_{1/2}(N_p; N_p + N_n)$$

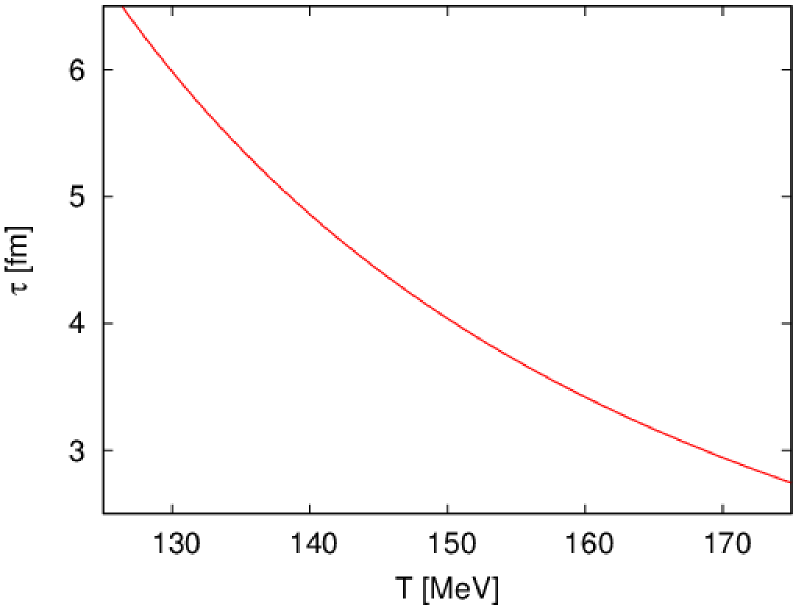


- The factrization is satisfied in free nucleon gas.

$$\mathcal{P}_{\mathrm{free}}(N_p, N_n, N_{\bar{p}}, N_{\bar{n}}) = \underbrace{P_{\bar{N}_{\mathrm{N}}}(N_{\mathrm{N}}) P_{\bar{N}_{\bar{\mathrm{N}}}}(N_{\bar{\mathrm{N}}})}_{F(N_{\mathrm{N}}, N_{\bar{\mathrm{N}}}) = P_{\bar{N}_{\mathrm{N}}}(N_{\mathrm{N}}) P_{\bar{N}_{\bar{\mathrm{N}}}}(N_{\bar{\mathrm{N}}})} B(N_p; N_{\mathrm{N}}) B(N_{\bar{p}}; N_{\bar{\mathrm{N}}})$$

Nucleon Time Scales in Fireballs

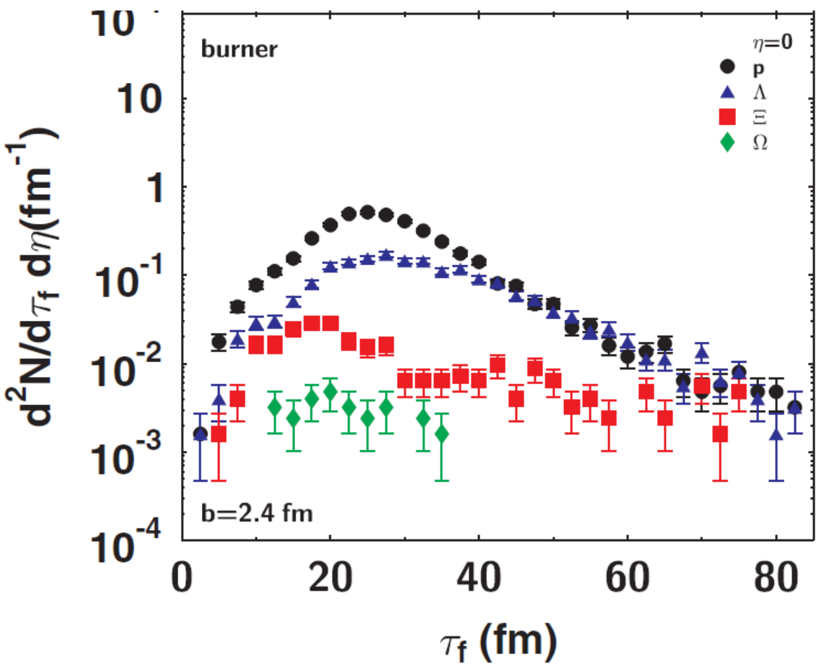
Mean time to create $\Delta^{+,0}$



$$\tau_\Delta = 3 \sim 4[\mathrm{fm}]$$

Freeze-out time

Nonaka, Bass, 2007



$$\tau_{\mathrm{f.o.}} > 20[\mathrm{fm}]$$

3rd & 4th Order Fluctuations

$$N_{\mathrm{B}} \to N_p$$

$$\langle (\delta N_p^{(\mathrm{net})})^3 \rangle = \frac{1}{8} \langle (\delta N_{\mathrm{B}}^{(\mathrm{net})})^3 \rangle + \frac{3}{8} \langle \delta N_{\mathrm{B}}^{(\mathrm{net})} \delta N_{\mathrm{B}}^{(\mathrm{tot})} \rangle,$$

$$\begin{aligned} \langle (\delta N_p^{(\mathrm{net})})^4 \rangle_c = & \frac{1}{16} \langle (\delta N_{\mathrm{B}}^{(\mathrm{net})})^4 \rangle_c + \frac{3}{8} \langle (\delta N_{\mathrm{B}}^{(\mathrm{net})})^2 \delta N_{\mathrm{B}}^{(\mathrm{tot})} \rangle \\ & + \frac{3}{16} \langle (\delta N_{\mathrm{B}}^{(\mathrm{tot})})^2 \rangle - \frac{1}{8} \langle N_{\mathrm{B}}^{(\mathrm{tot})} \rangle, \end{aligned}$$

$$N_p \to N_{\mathrm{B}}$$

$$\begin{aligned} \langle (\delta N_{\mathrm{B}}^{(\mathrm{net})})^3 \rangle = & 8 \langle (\delta N_p^{(\mathrm{net})})^3 \rangle - 12 \langle \delta N_p^{(\mathrm{net})} \delta N_p^{(\mathrm{tot})} \rangle \\ & + 6 \langle N_p^{(\mathrm{net})} \rangle, \end{aligned}$$

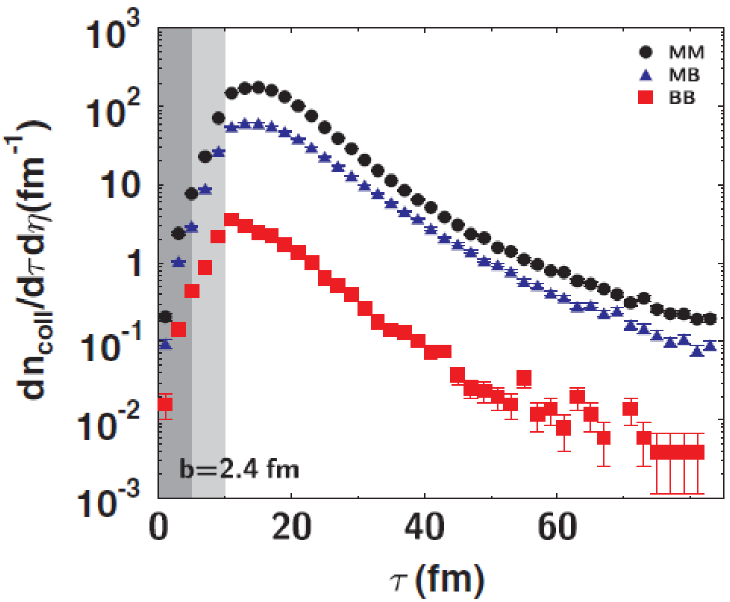
$$\begin{aligned} \langle (\delta N_{\mathrm{B}}^{(\mathrm{net})})^4 \rangle_c = & 16 \langle (\delta N_p^{(\mathrm{net})})^4 \rangle_c - 48 \langle (\delta N_p^{(\mathrm{net})})^2 \delta N_p^{(\mathrm{tot})} \rangle \\ & + 48 \langle (\delta N_p^{(\mathrm{net})})^2 \rangle + 12 \langle (\delta N_p^{(\mathrm{tot})})^2 \rangle - 26 \langle N_p^{(\mathrm{tot})} \rangle, \end{aligned}$$

Isospin Distributions

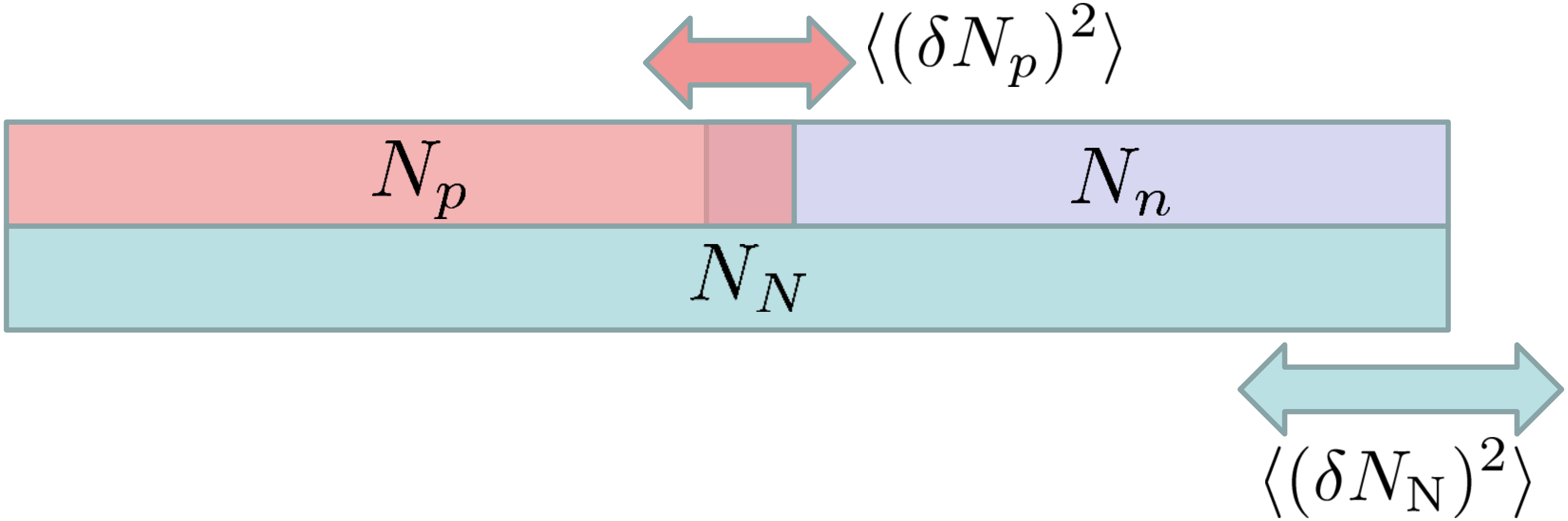
- Large pion density
  - Small nucleon density because M_N/T<<1
  - For top RHIC energy, N_π~$20N_N$
- Nucleons exclusively interact with pions
  - Rare NN collisions
  - Huge $\pi\pi$ reactions

All formations and decays of Δ take place independently

Nonaka, Bass, '07



Baryon & Proton Number Fluctuations



- In general, fluctuations of N_{N} and N_{p} are different.
- Due to the isospin fluctuations, N_{p} fluctuations tend to be close to equilibrium (Poissonian) ones.