

# NUCLEAR CHIRAL THERMODYNAMICS and PHASES of QCD

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- ★ **Prelude: QCD Phase Diagram** (Concepts, Models, Problems)
- ★ **Main Theme: Nuclear Chiral Thermodynamics**
  - QCD interface with nuclear physics:  
**Chiral Effective Field Theory**
  - **Nuclear Equation of State** and QCD phase diagram
  - Density and temperature dependence of the  
**Chiral (Quark) Condensate**
- ★ **Outlook:** New constraints from **Neutron Stars**



Part I: Prelude

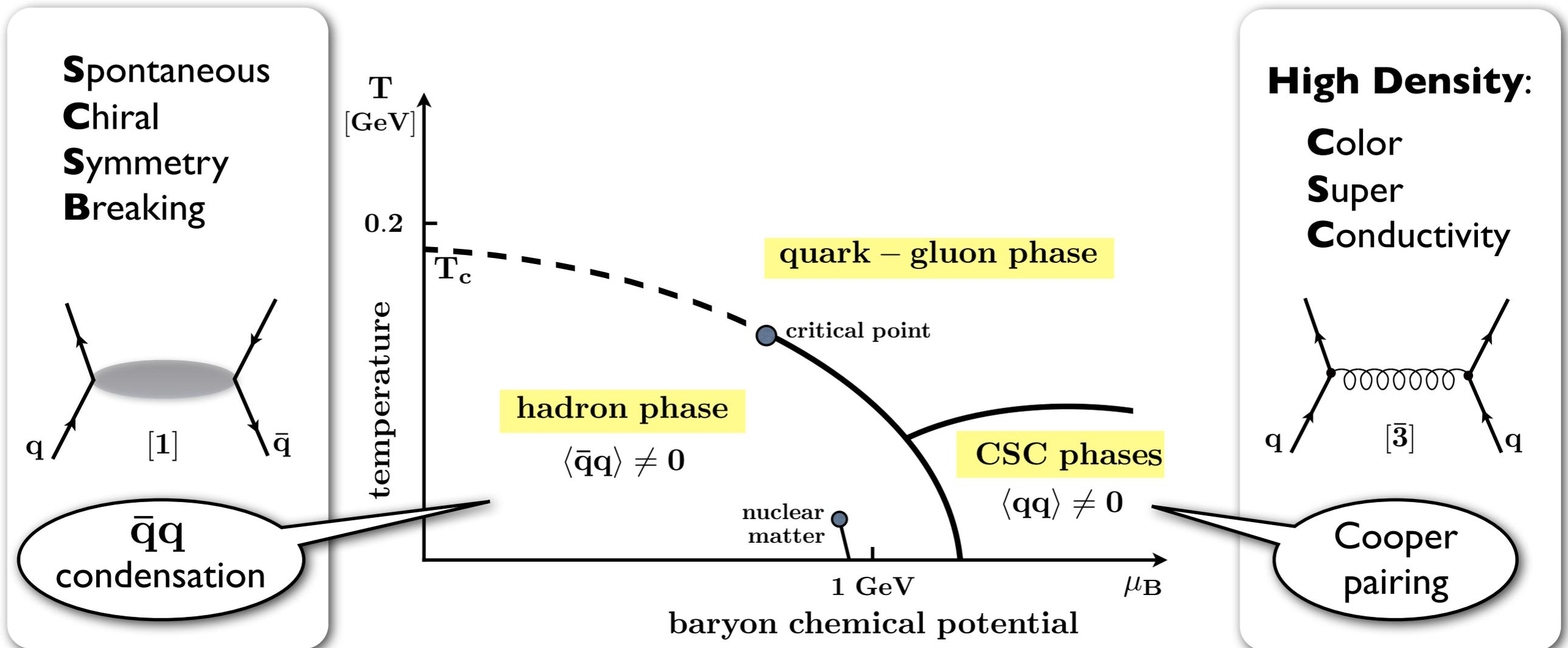
# **QCD PHASE DIAGRAM**

Visions & Facts

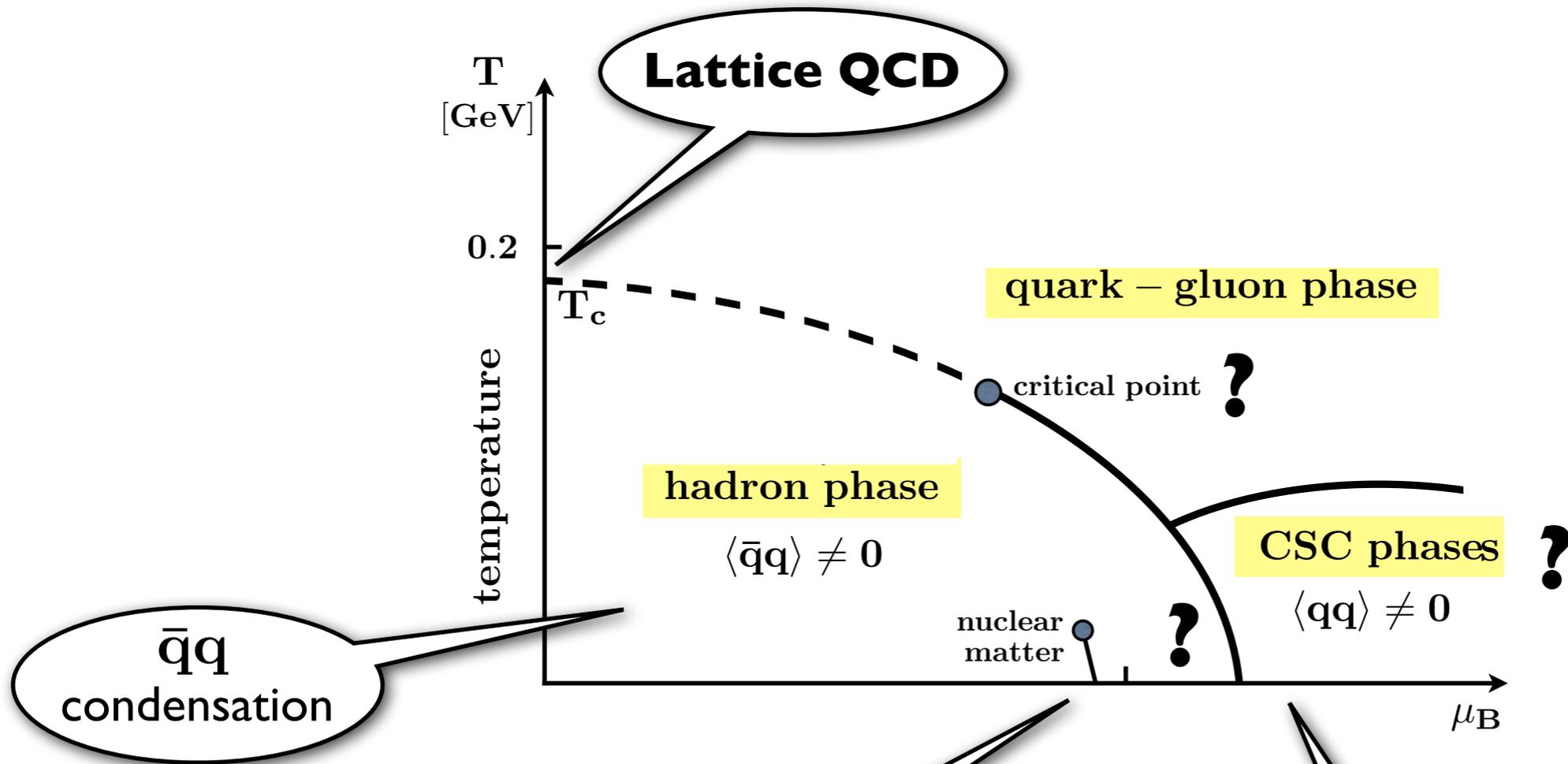


# QCD PHASE DIAGRAM

(theorists' vision)



# QCD PHASE DIAGRAM



Constraints  
from  
**Nuclear Physics**

New constraints  
from  
**Neutron Stars**



# MODELING the QCD PHASE DIAGRAM



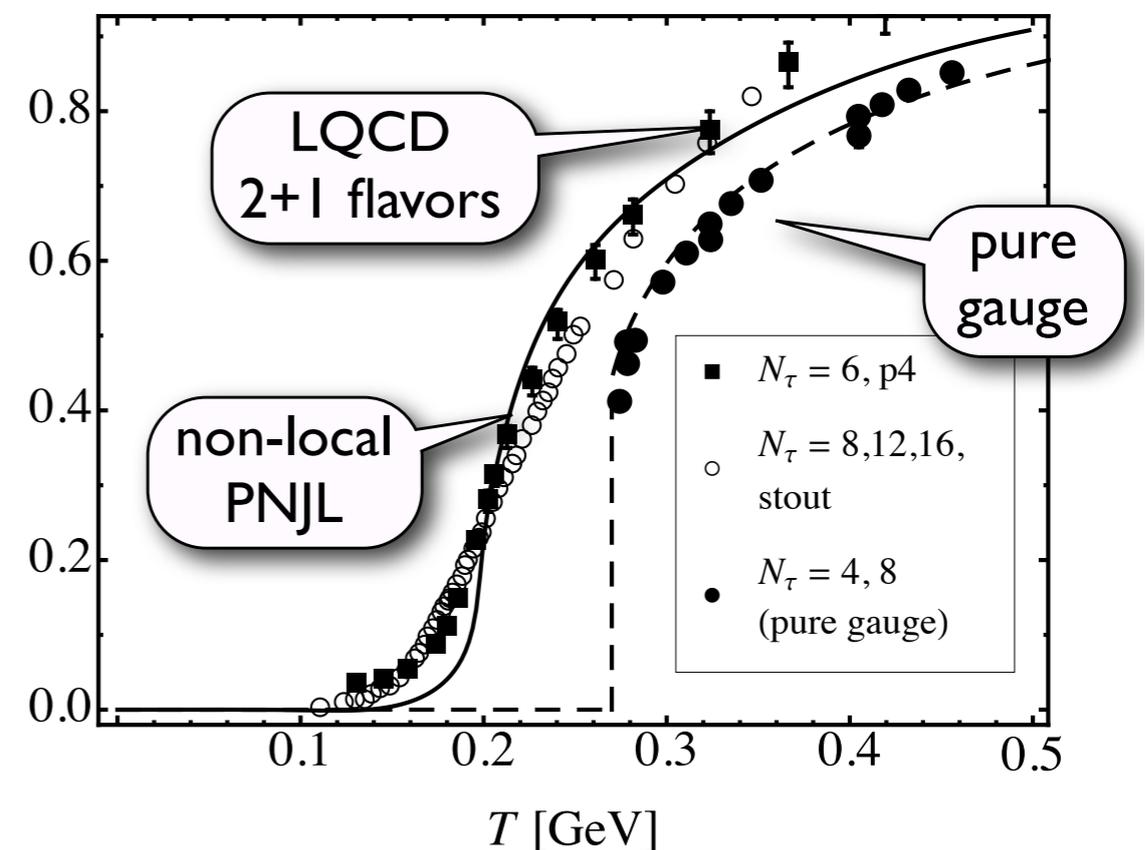
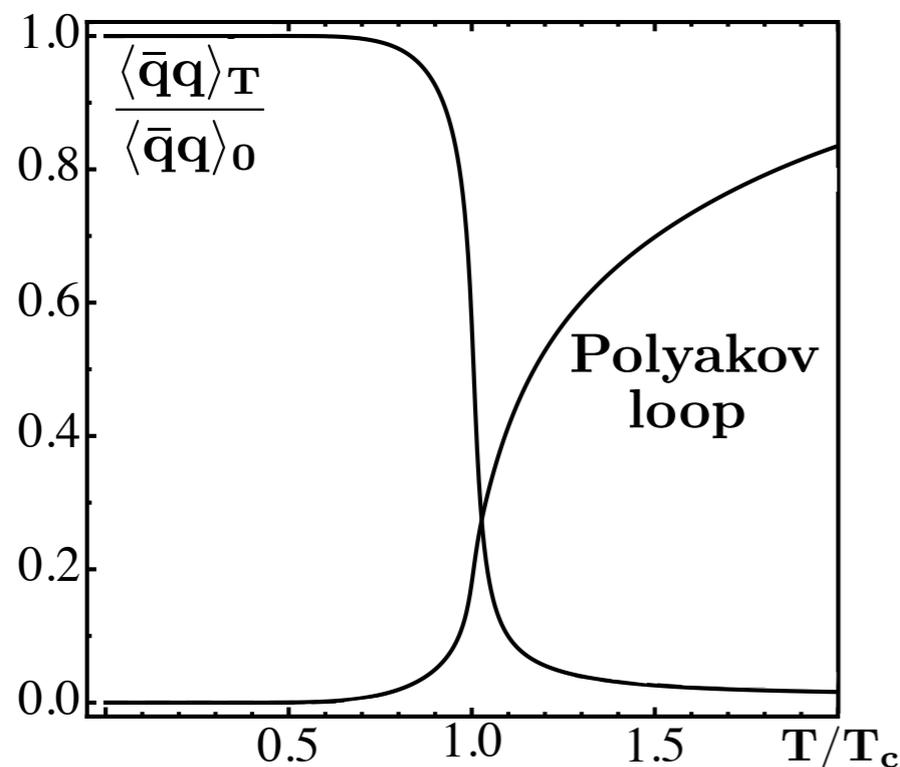
Guiding principle:  
QCD **symmetries** and symmetry **breaking** patterns

Spontaneously broken  
**chiral** symmetry  
 $SU(N_f)_R \times SU(N_f)_L$

non-local  
**PNJL**  
model

Centre  $Z(3)$  of  
 $SU(3)_c$  gauge group

● **chiral** and **deconfinement crossover** transitions (3 flavor PNJL model)



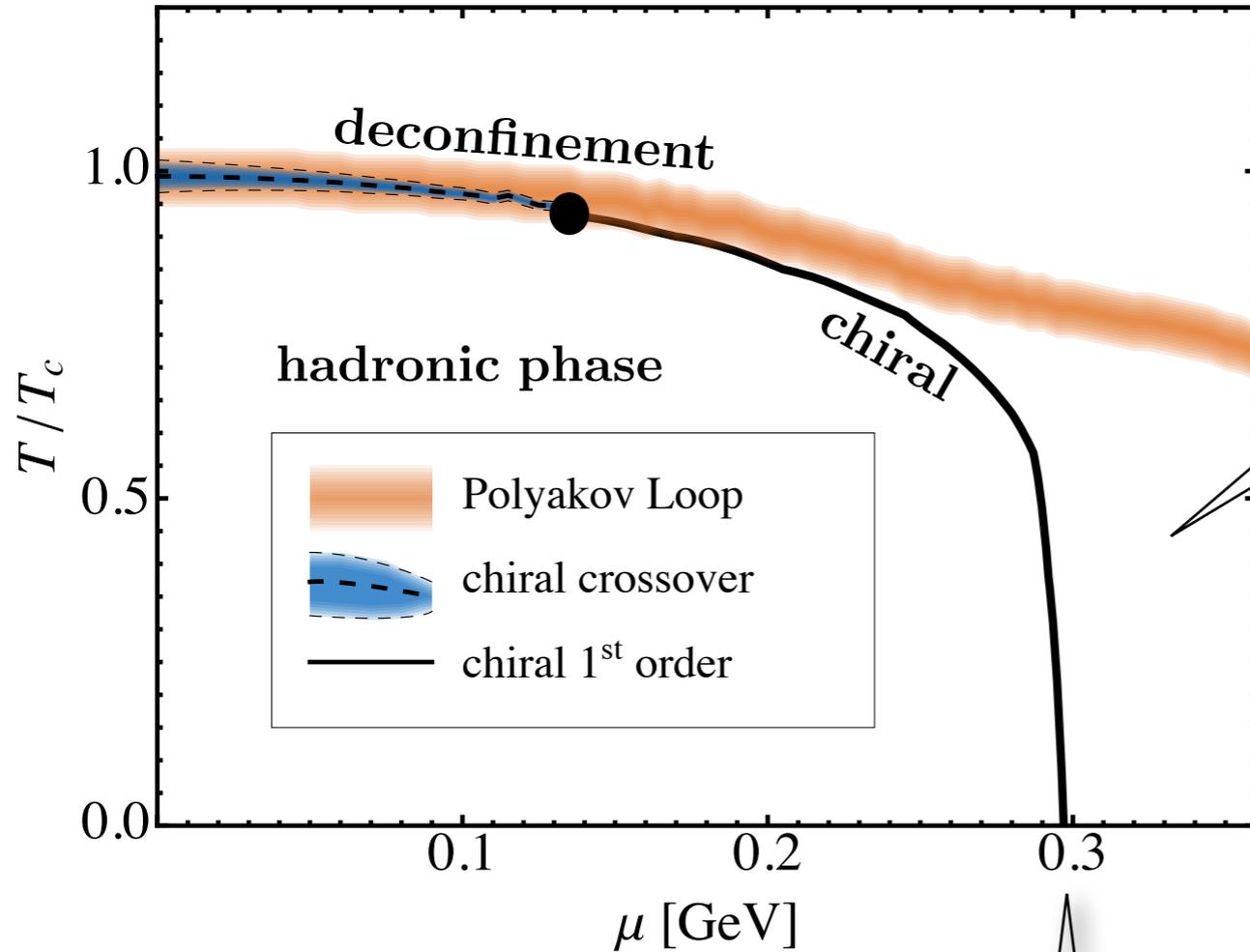
T. Hell, S. Rössner, M. Cristoforetti, W.W.: Phys. Rev. D81 (2010) 074034

T. Hell, K. Kashiwa, W.W.: Phys. Rev. D83 (2011) 114008



# PHASE DIAGRAM

## Non-local 3-flavor PNJL model calculation



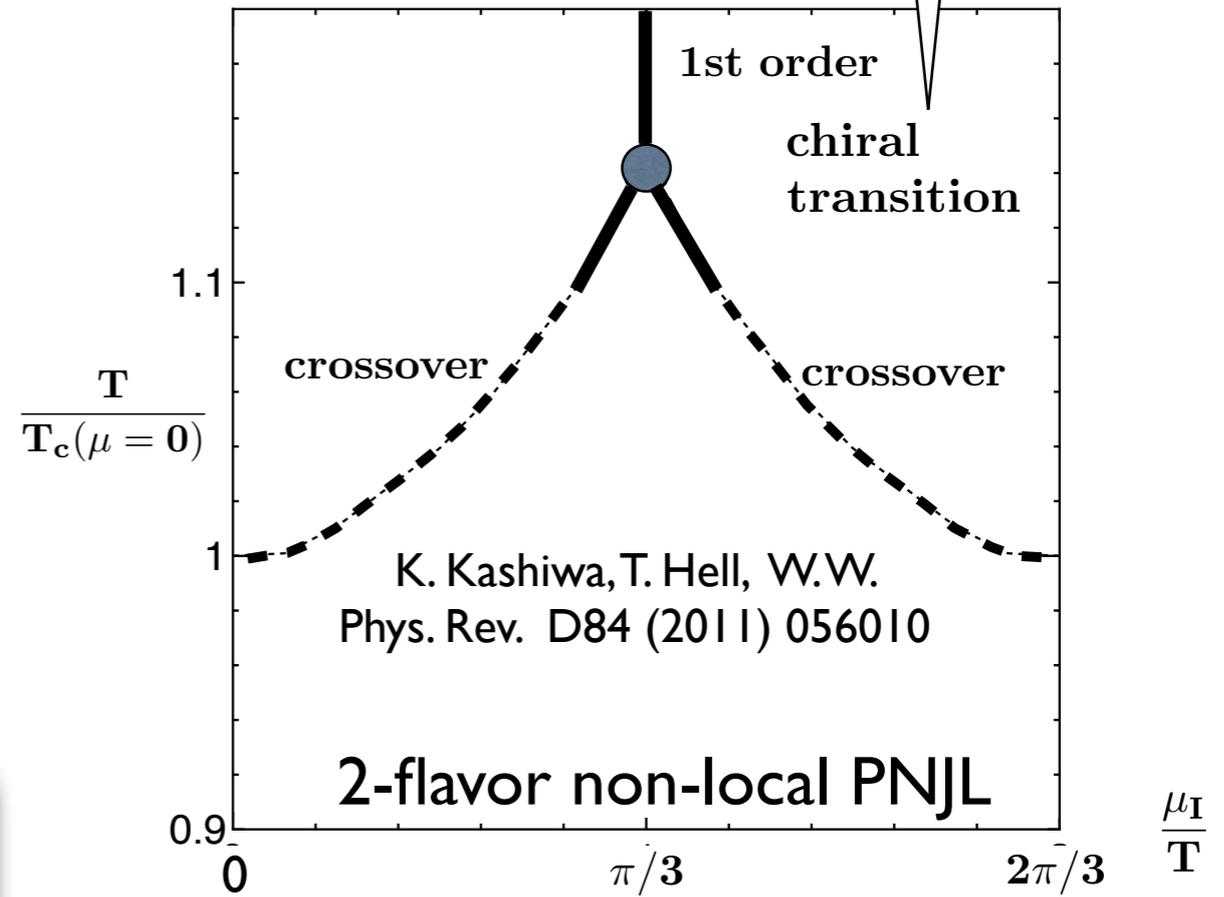
T. Hell, K. Kashiwa, W.W.:  
Phys. Rev. D83 (2011) 114008

quarkyonic ?  
L. McLerran, R. Pisarski

accessible by  
Lattice QCD  
P. de Forcrand, O. Philipsen

chiral 1st order transition line ?

● Does the **first-order line** really extend down to **low** temperatures ?



imaginary chemical potential

K. Kashiwa, T. Hell, W.W.  
Phys. Rev. D84 (2011) 056010

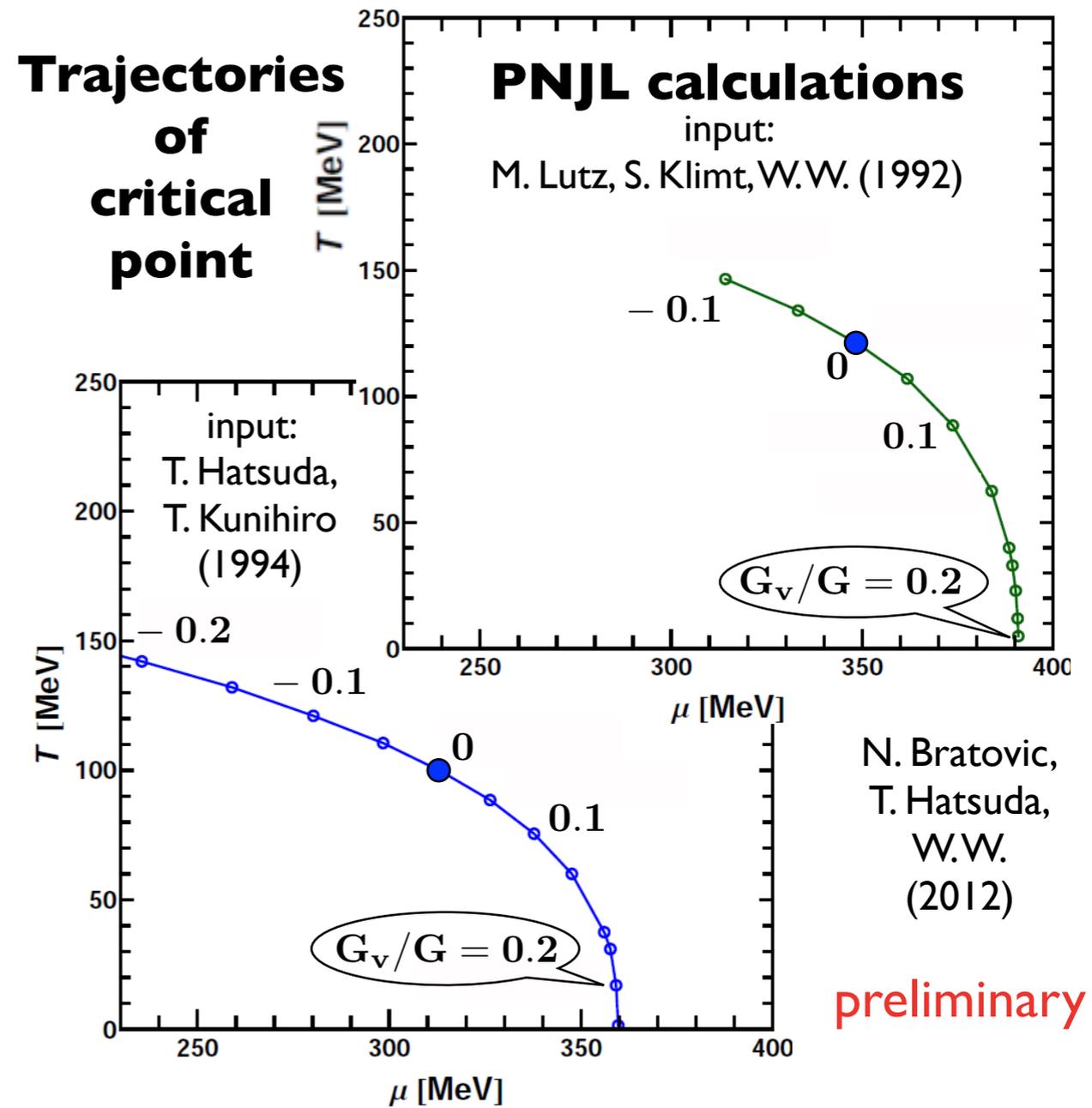
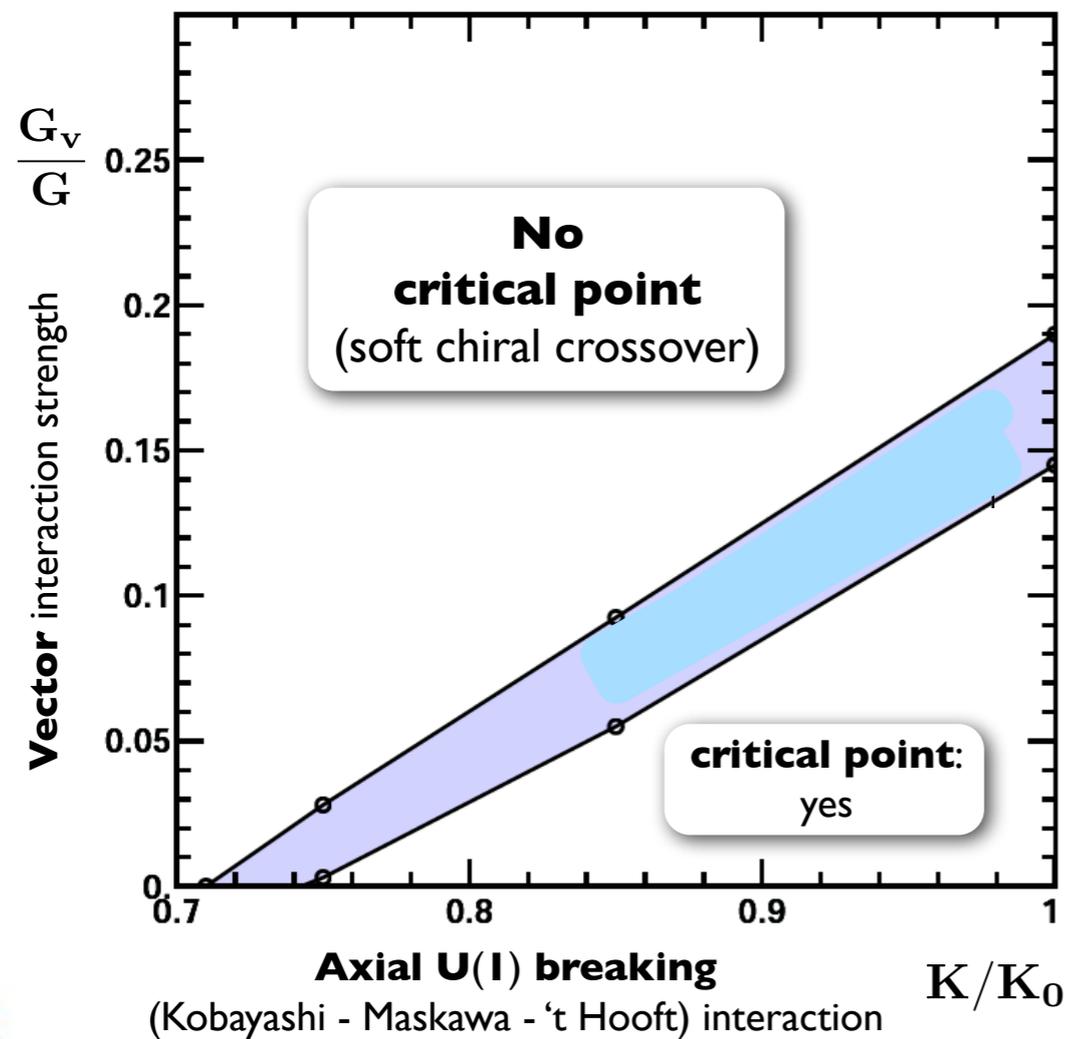


# PHASE DIAGRAM (contd.)

- PNJL analysis of Lattice QCD phase diagram at **imaginary** chemical potential suggests significant isoscalar **vector** term in the effective quark-quark interaction

$$\delta\mathcal{L}_V = -G_V (\bar{\psi}\gamma_\mu\psi)(\bar{\psi}\gamma^\mu\psi)$$

- Existence and location of critical point: extremely sensitive to
  - Strength of vector interaction
  - Axial U(1) breaking interaction



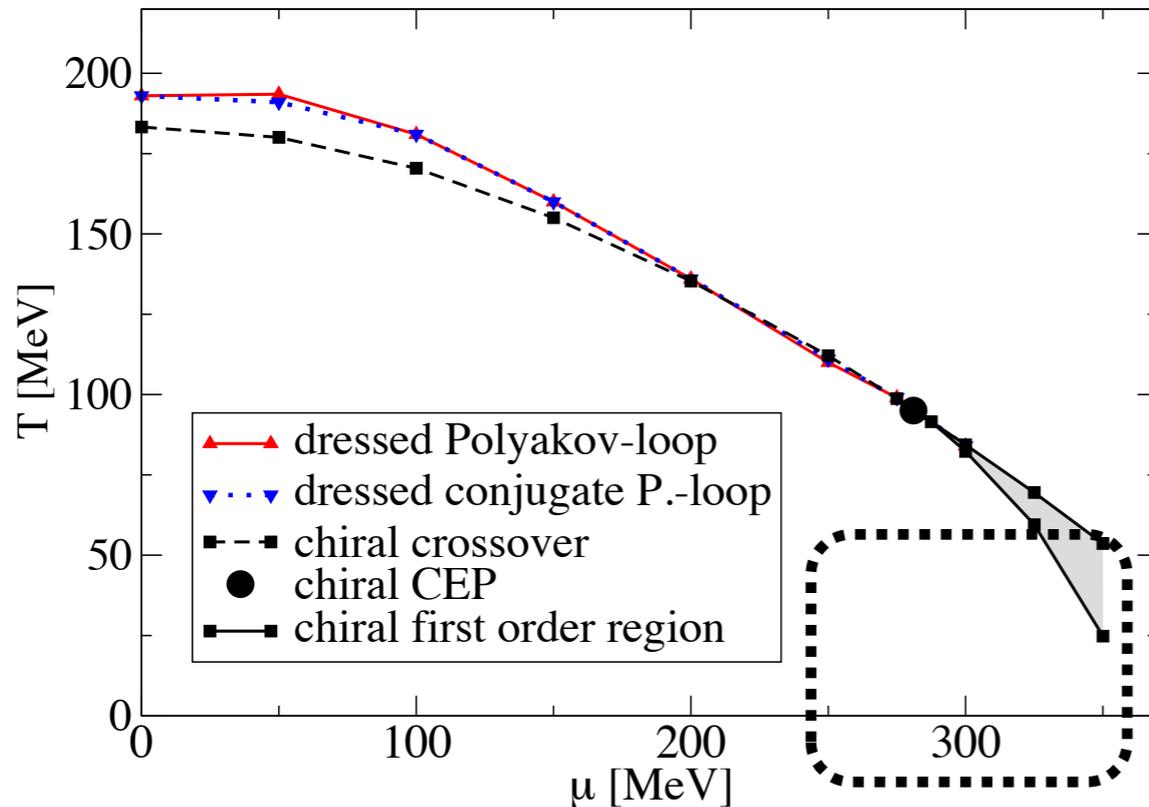
preliminary



# PHASE DIAGRAM (contd.)

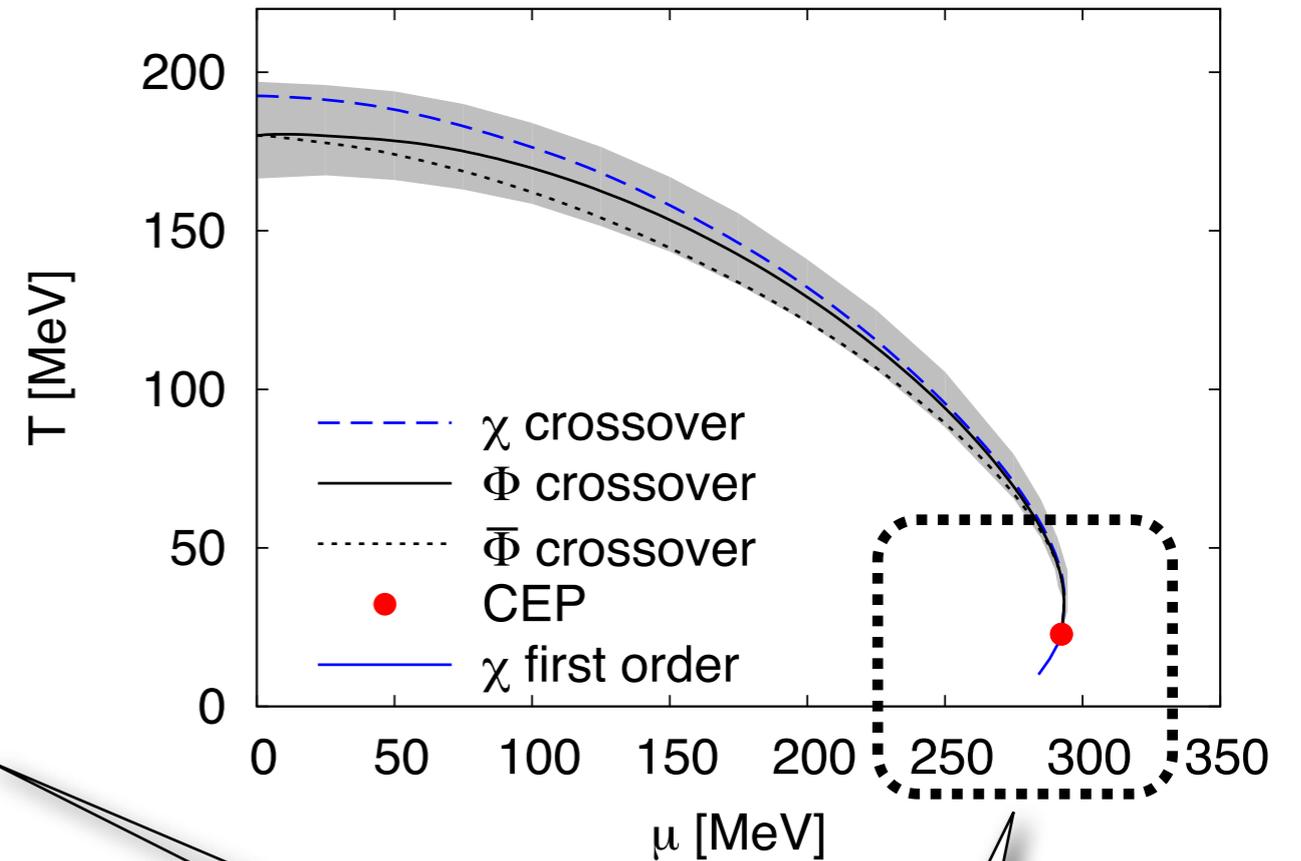
## Dyson - Schwinger QCD

C.S. Fischer, J. Luecker, J.A. Mueller: PLB 702 (2011) 438



## Polyakov - Quark-Meson model

T. K. Herbst, J. Pawłowski, B.-J. Schäfer: PRL 106 (2011) 58



baryon densities

$$\rho_B = \frac{1}{3} \left( \frac{\partial P}{\partial \mu} \right)_T$$

in the range  
 $0.1 - 0.2 \text{ fm}^{-3}$

**nuclear  
terrain !**

Quarks are **not** the **relevant active quasiparticles** at low temperatures and at baryon chemical potentials  $\mu_B \lesssim 1 \text{ GeV}$

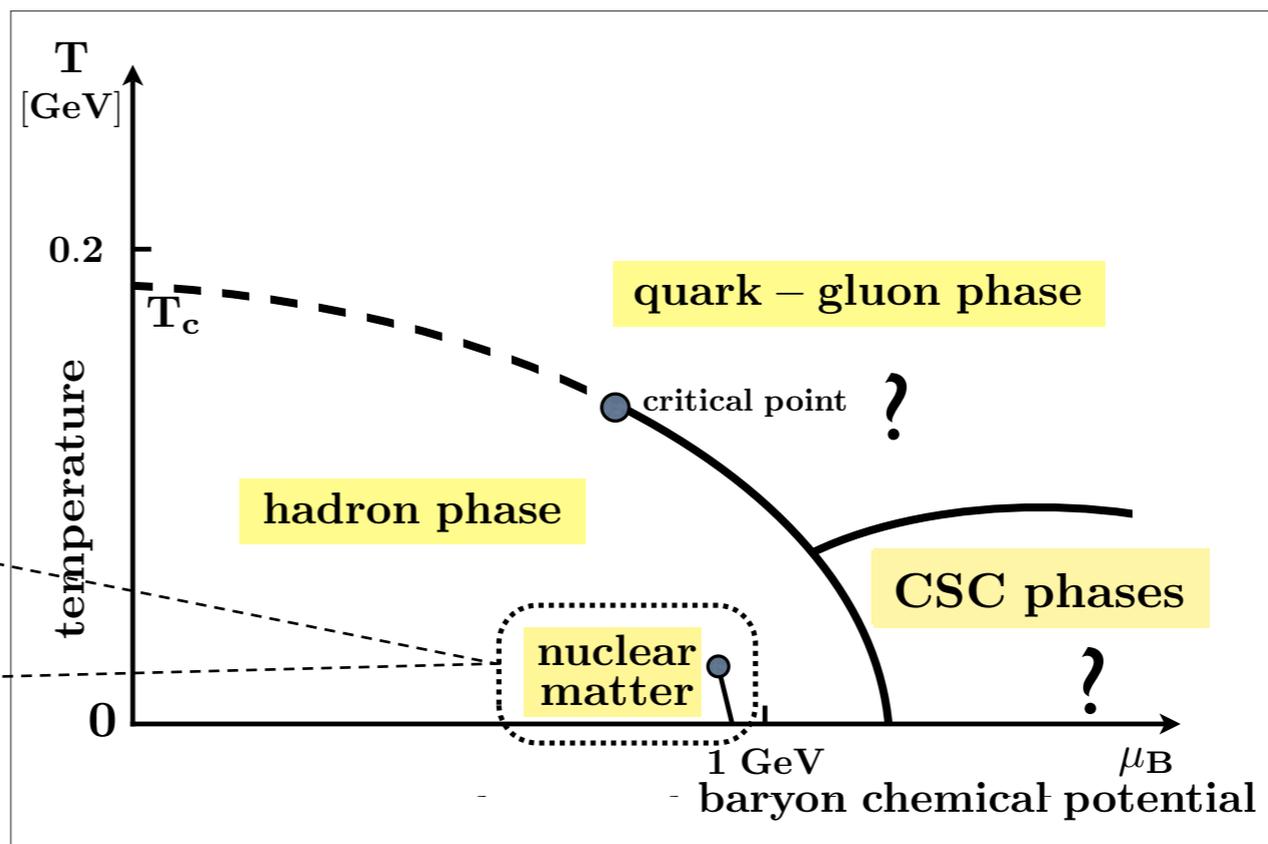
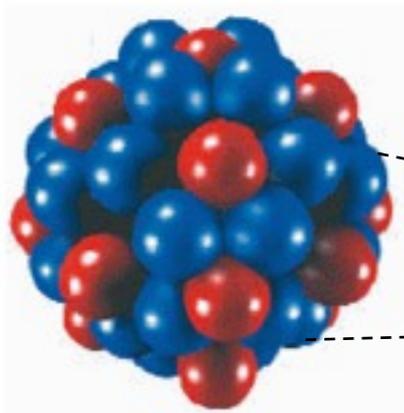
Part II:

**NUCLEAR  
CHIRAL  
THERMODYNAMICS**



# NUCLEAR MATTER and QCD PHASES

nuclei



## Scales in nuclear matter:

- momentum scale:  
**Fermi momentum**
- NN distance:
- energy per nucleon:
- compression modulus:

$$k_F \simeq 1.4 \text{ fm}^{-1} \sim 2m_\pi$$

$$d_{NN} \simeq 1.8 \text{ fm} \simeq 1.3 m_\pi^{-1}$$

$$E/A \simeq -16 \text{ MeV}$$

$$K = (260 \pm 30) \text{ MeV} \sim 2m_\pi$$



# PIONS and NUCLEI

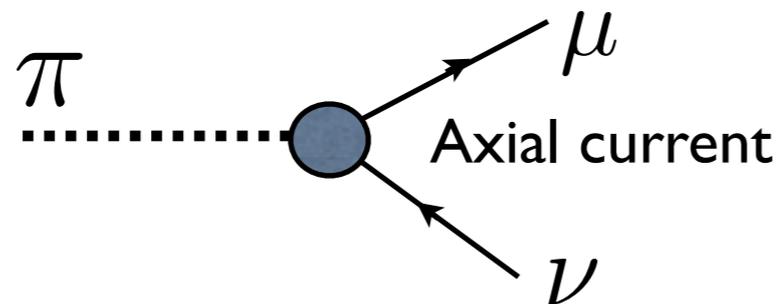
in the context of **LOW-ENERGY QCD**

- **CONFINEMENT** of quarks and gluons in hadrons
- Spontaneously broken **CHIRAL SYMMETRY**

**LOW-ENERGY / LOW-TEMPERATURE QCD:**  
**E**ffective **F**ield **T**heory of **weakly** interacting  
**Nambu-Goldstone Bosons (PIONS)**

representing QCD at (energy and momentum) scales

$$Q \ll 4\pi f_\pi \sim 1 \text{ GeV}$$



$$f_\pi = 92.4 \text{ MeV}$$

**spontaneous**  
symmetry breaking

$$m_\pi^2 f_\pi^2 = -m_q \langle \bar{\psi}\psi \rangle + \mathcal{O}(m_q^2)$$

**explicit**  
symmetry breaking



# CHIRAL EFFECTIVE FIELD THEORY

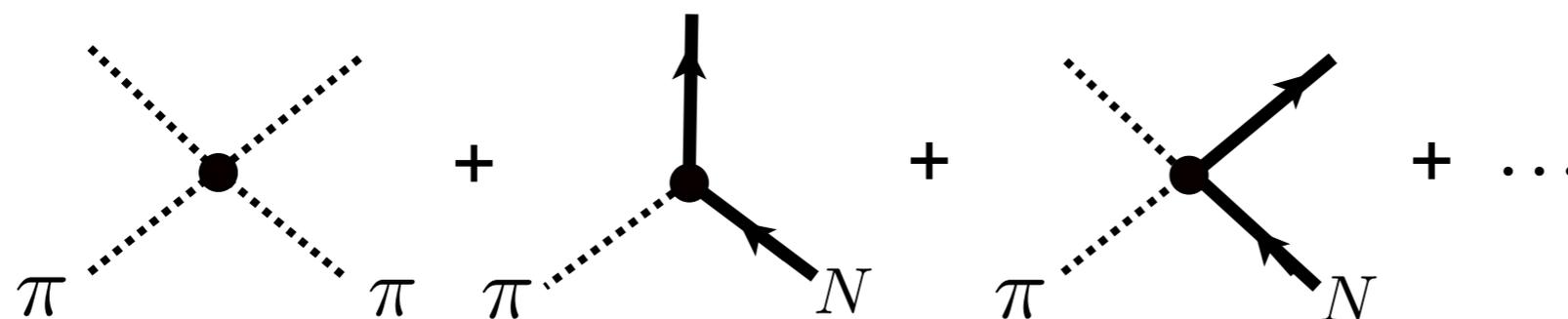
- Systematic framework at interface of QCD and Nuclear Physics

- Interacting systems of **PIONS** (light / fast) and **NUCLEONS** (heavy / slow):

$$\mathcal{L}_{eff} = \mathcal{L}_\pi(U, \partial U) + \mathcal{L}_N(\Psi_N, U, \dots)$$

$$U(x) = \exp[i\tau_a \pi_a(x) / f_\pi]$$

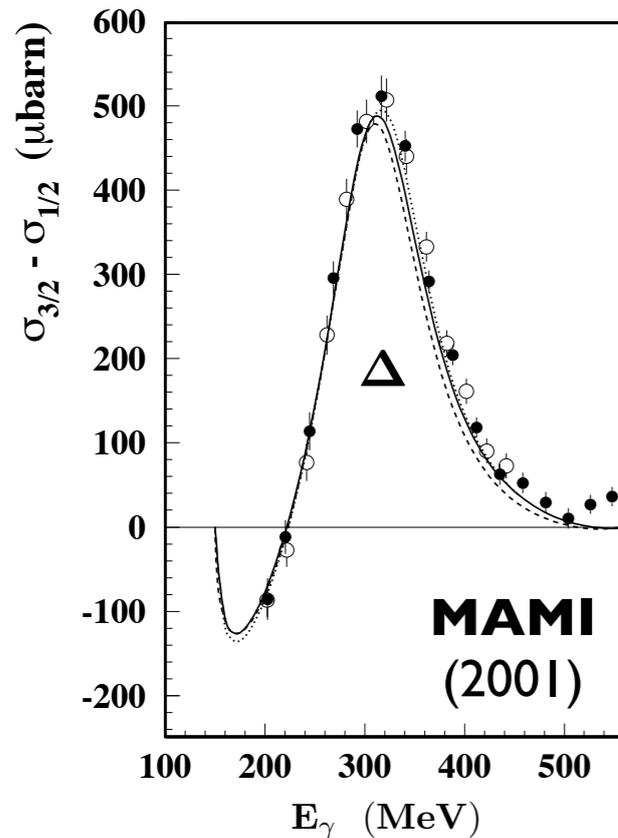
- Construction of Effective Lagrangian: **Symmetries**



**short  
distance  
dynamics:  
contact terms**

# Explicit $\Delta(1230)$ DEGREES of FREEDOM

- **Large spin-isospin polarizability** of the Nucleon

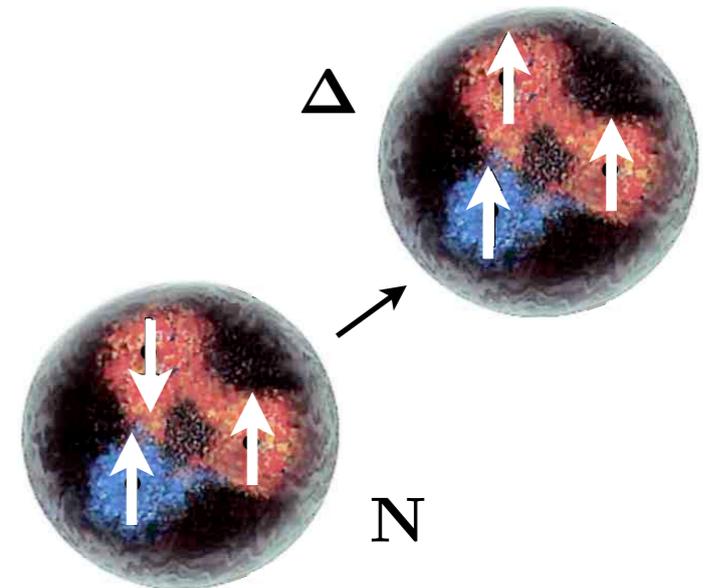


example: polarized Compton scattering

$$\beta_\Delta = \frac{g_A^2}{f_\pi^2 (M_\Delta - M_N)} \sim 5 \text{ fm}^3$$

$$M_\Delta - M_N \simeq 2 m_\pi \ll 4\pi f_\pi$$

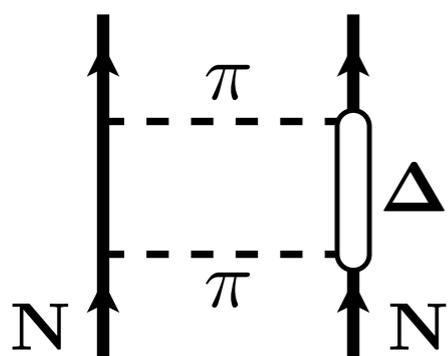
(small scale)



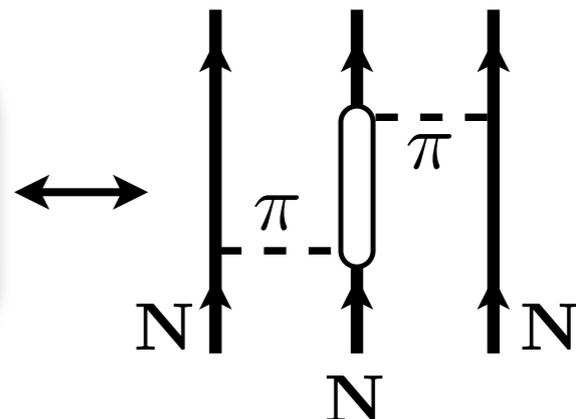
- **Pionic Van der Waals** - type intermediate range central potential

N. Kaiser, S. Gerstendörfer, W.W., NPA637 (1998) 395

N. Kaiser, S. Fritsch, W.W., NPA750 (2005) 259



$$V_c(r) = -\frac{9 g_A^2}{32 \pi^2 f_\pi^2} \beta_\Delta \frac{e^{-2m_\pi r}}{r^6} P(m_\pi r)$$



**strong 3-body interaction**

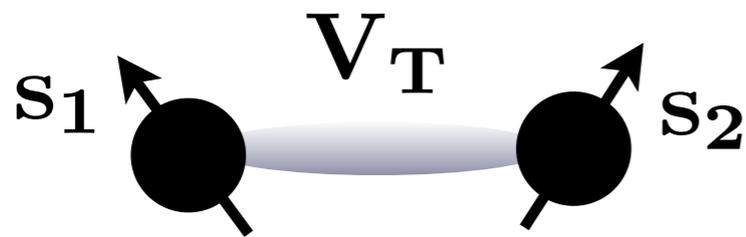
J. Fujita, H. Miyazawa (1957)

Pieper, Pandharipande, Wiringa, Carlson (2001)

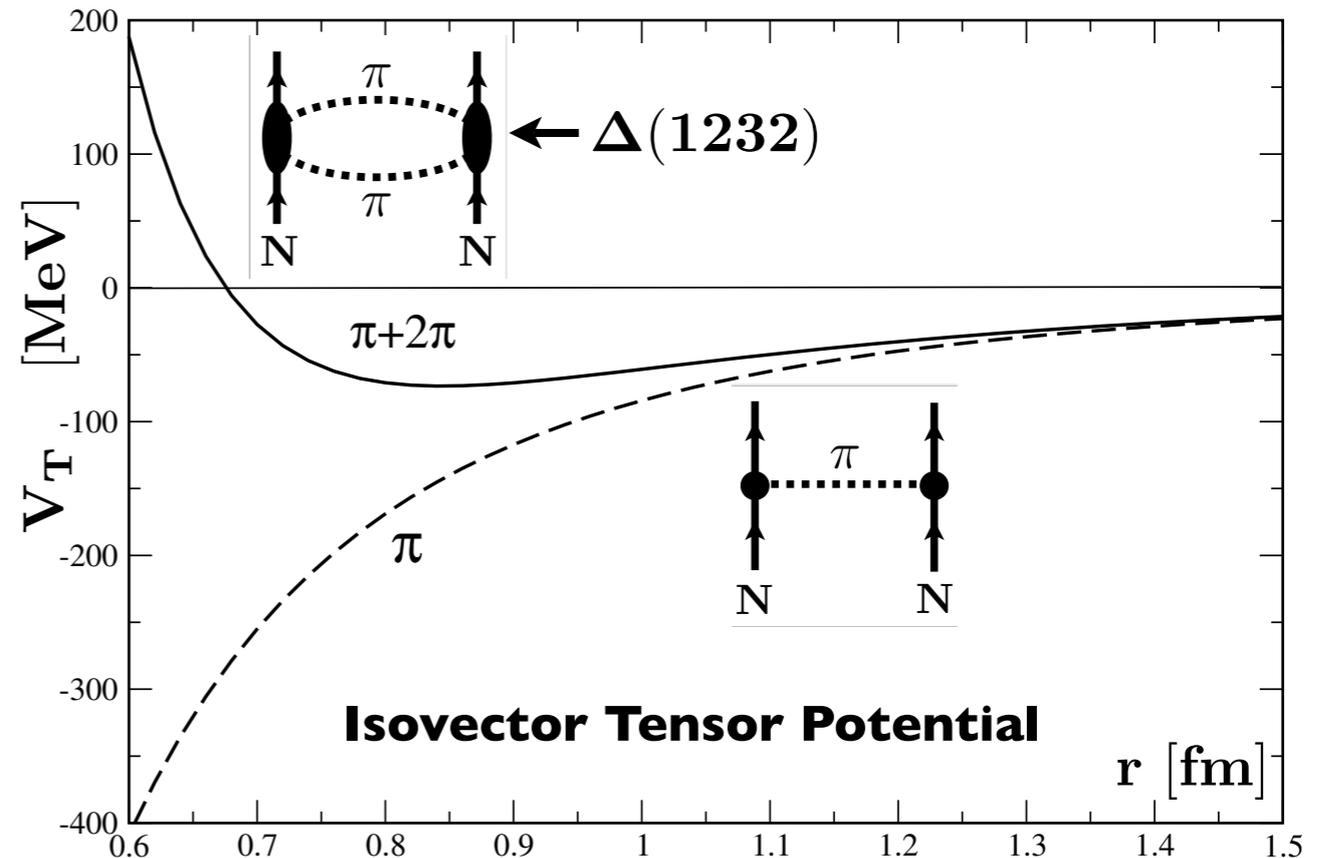


# Important pieces of the CHIRAL NUCLEON-NUCLEON INTERACTION

- **ISOVECTOR TENSOR FORCE**

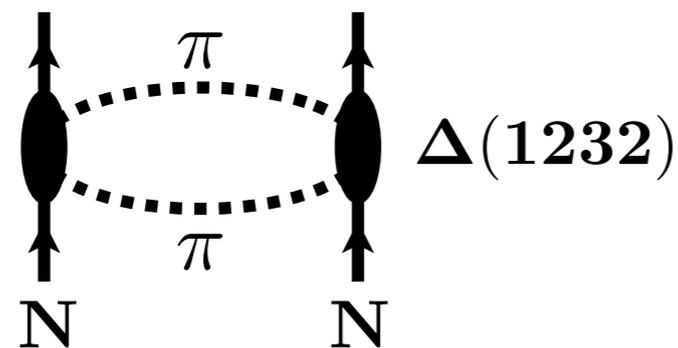


- note: **no**  $\rho$  meson



N. Kaiser, S. Gerstendörfer, W.W.: Nucl. Phys.A 637 (1998) 395

- **CENTRAL ATTRACTION** from **TWO-PION EXCHANGE**



- note: **no**  $\sigma$  boson

**Van der WAALS** - like force:

$$V_c(r) \propto -\frac{\exp[-2m_\pi r]}{r^6} P(m_\pi r)$$

... at intermediate and long distance

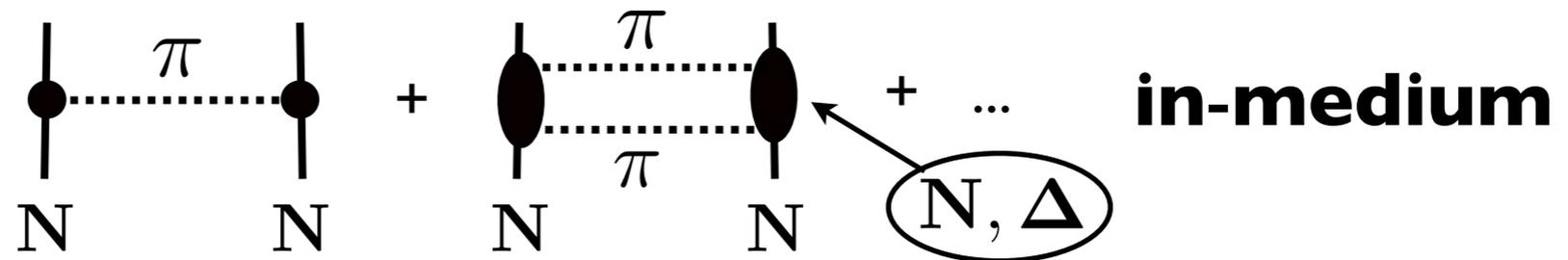
# CHIRAL DYNAMICS and the NUCLEAR MANY-BODY PROBLEM

N. Kaiser, S. Fritsch, W.W. (2002 - 2005)

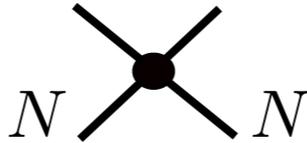
- **Small scales:**  $k_F \sim 2 m_\pi \sim M_\Delta - M_N \ll 4\pi f_\pi$
- **PIONS** (and **DELTA** isobars) as **explicit degrees of freedom**

## IN-MEDIUM CHIRAL PERTURBATION THEORY

pion exchange processes in presence of filled **Fermi sea**



2nd order **TENSOR** force + nucleon's **SPIN-ISOSPIN** polarizability

short-distance dynamics:  **contact interactions** (incl. **resummations**)



# IN-MEDIUM CHIRAL PERTURBATION THEORY

- **Loop expansion of (In-Medium) Chiral Perturbation Theory**



Systematic expansion of **ENERGY DENSITY**  $\mathcal{E}(k_F)$  in **powers of Fermi momentum** [modulo functions  $f_n(k_F/m_\pi)$ ]  
 (works for  $k_F \ll 4\pi f_\pi \sim 1 \text{ GeV}$ )

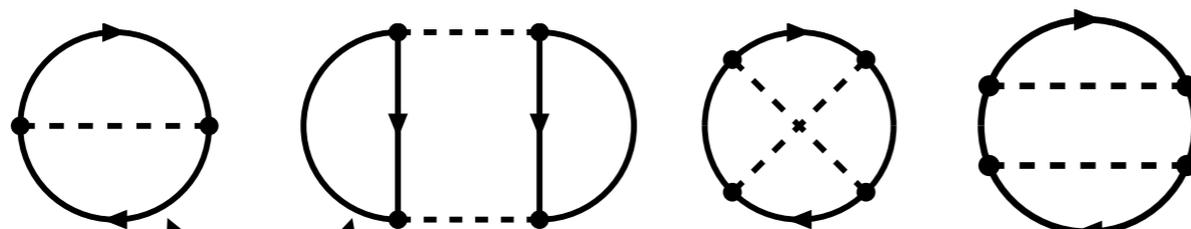
- **Finite nuclei  $\leftrightarrow$  energy density functional**

J.W. Holt, N. Kaiser, W.W.: Eur. Phys. J. A 47 (2011) 128

many quantitatively successful applications throughout the nuclear chart

e.g. P. Finelli et al.: Nucl. Phys. A 770 (2007) 1

- Nuclear **thermodynamics**: compute **free energy density**



(3-loop order)

N. Kaiser, S. Fritsch, W.W.  
(2002-2004)

**in-medium**  
nucleon propagators  
incl. Pauli blocking



# NUCLEAR MATTER

- **In-medium ChPT**  
3-loop ( $\pi, \mathbb{N}, \Delta$ )

- **Input** parameters:  
two contact terms

- basically:  
analytic calculation

- **Output:**

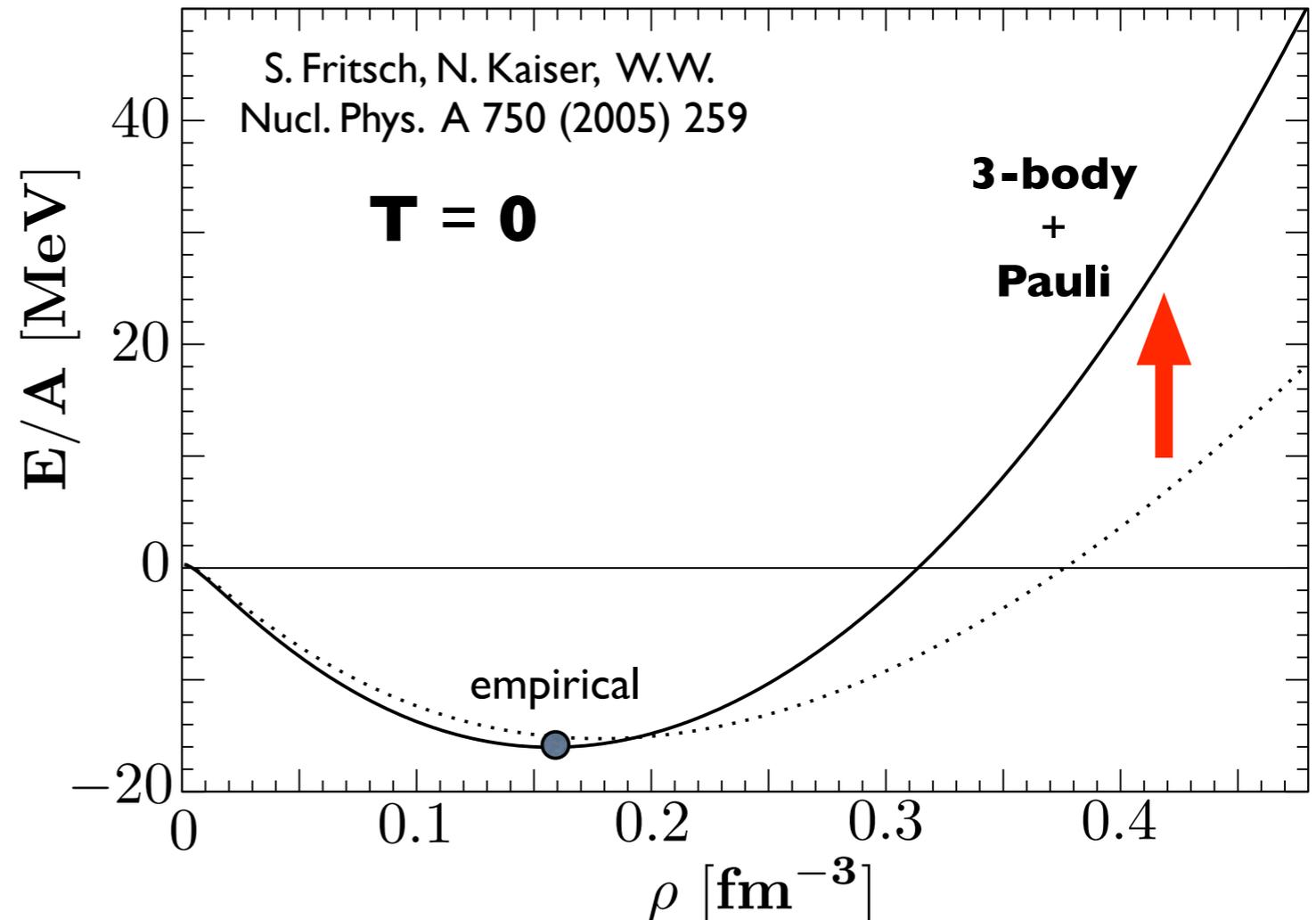
- ▶ Binding & saturation

$$E_0/A = -16 \text{ MeV}, \quad \rho_0 = 0.16 \text{ fm}^{-3}, \quad K = 290 \text{ MeV}$$

- ▶ Realistic (complex, momentum dependent) single-particle potential  
... satisfying Hugenholtz - van Hove and Luttinger theorems (!)

- ▶ Asymmetry energy  $A(k_F^0) = 34 \text{ MeV}$

- ▶ Quasiparticle interaction and Landau parameters



J.W. Holt, N. Kaiser, W.W.  
Nucl. Phys. A 870 (2011) 1,  
arXiv:1111.1924 [nucl-th]  
(NPA (2012), in print)

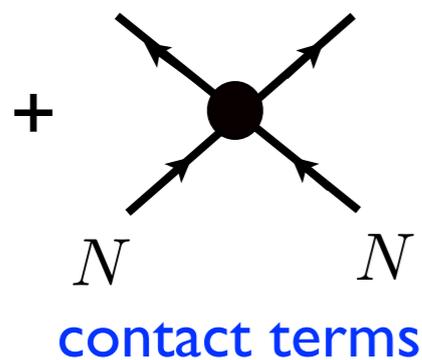
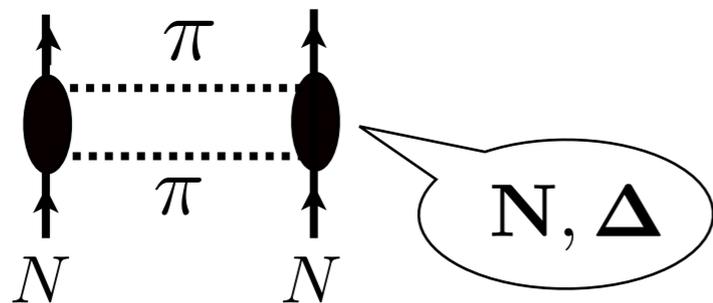


# NUCLEAR THERMODYNAMICS

## NUCLEAR CHIRAL (PION) DYNAMICS

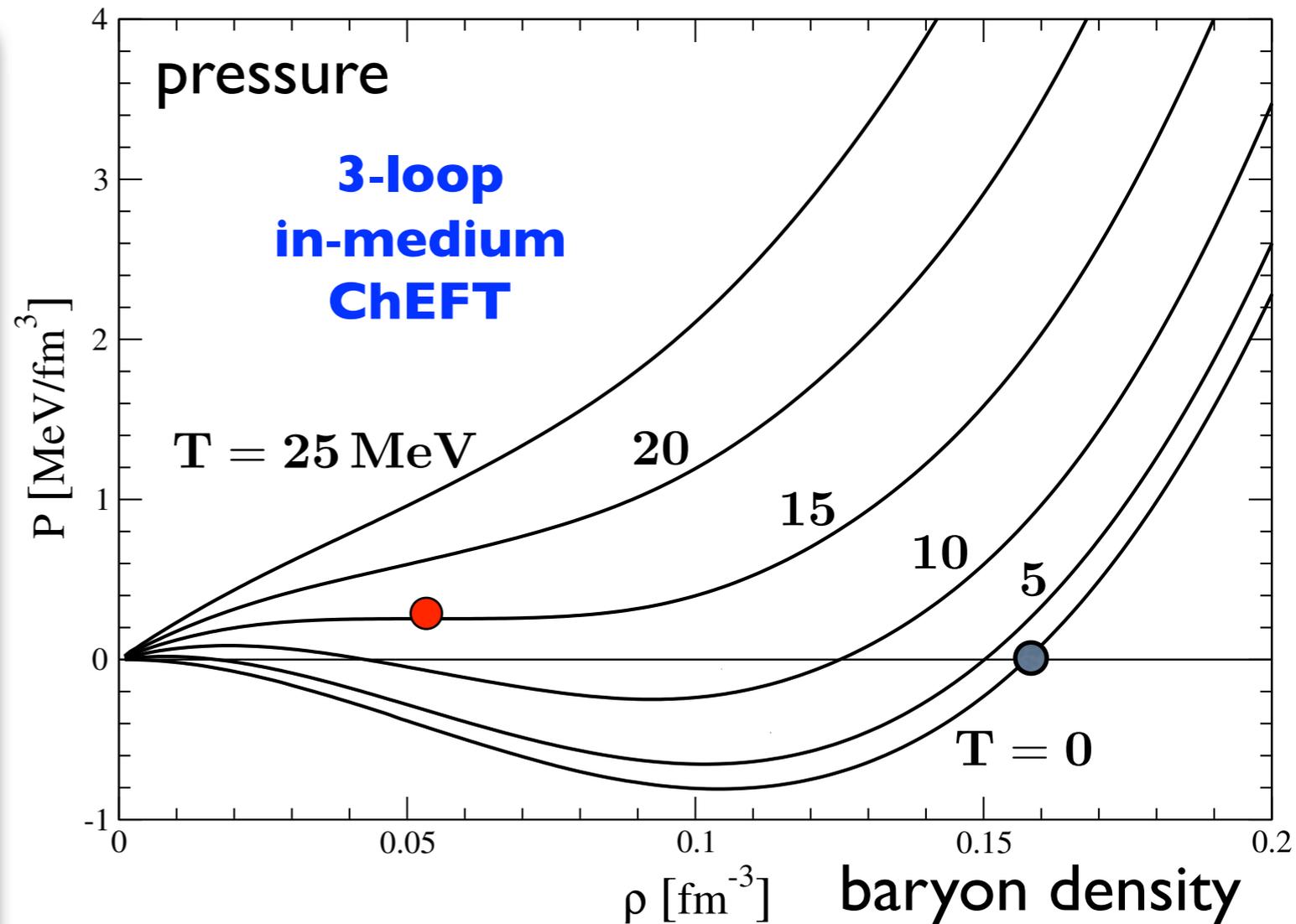
### BINDING & SATURATION:

Van der Waals + Pauli



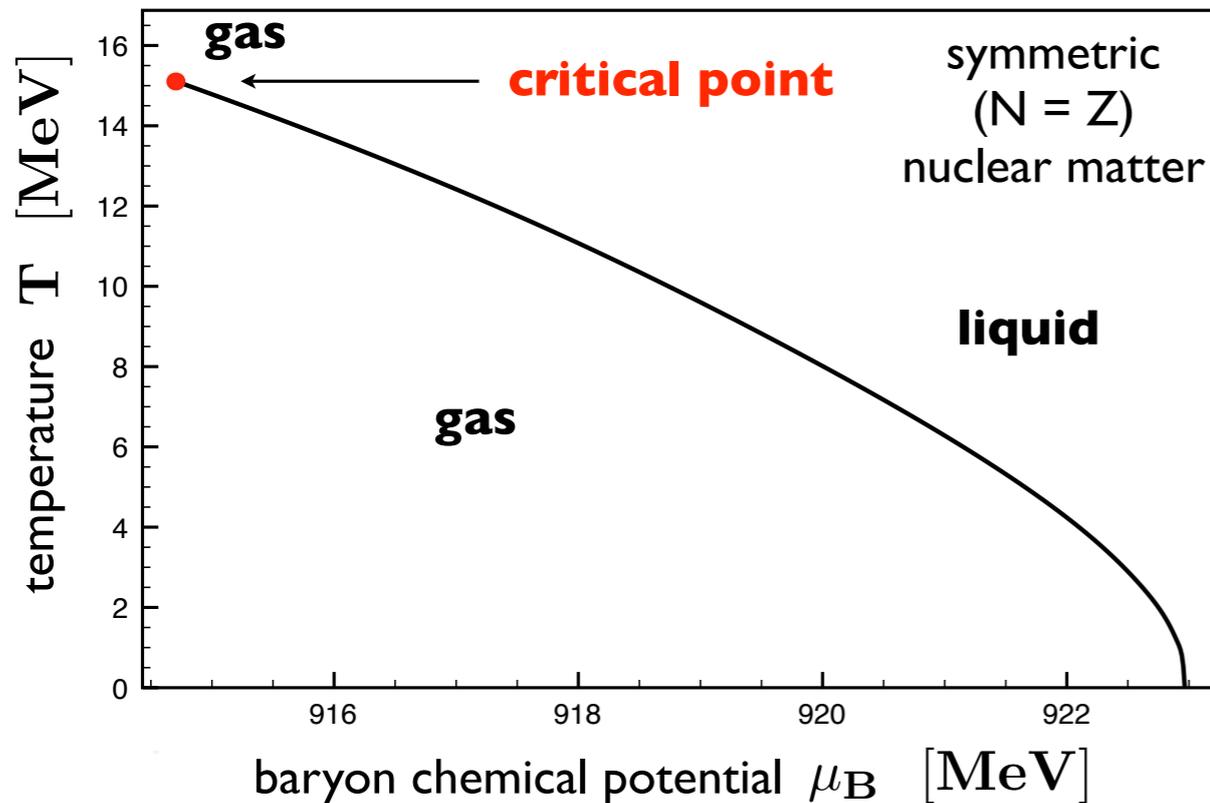
+ 3-body forces

## nuclear matter: equation of state



Liquid - Gas Transition at  
Critical Temperature  $T_c = 15 \text{ MeV}$   
(empirical:  $T_c = 16 - 18 \text{ MeV}$ )

# PHASE DIAGRAM of NUCLEAR MATTER

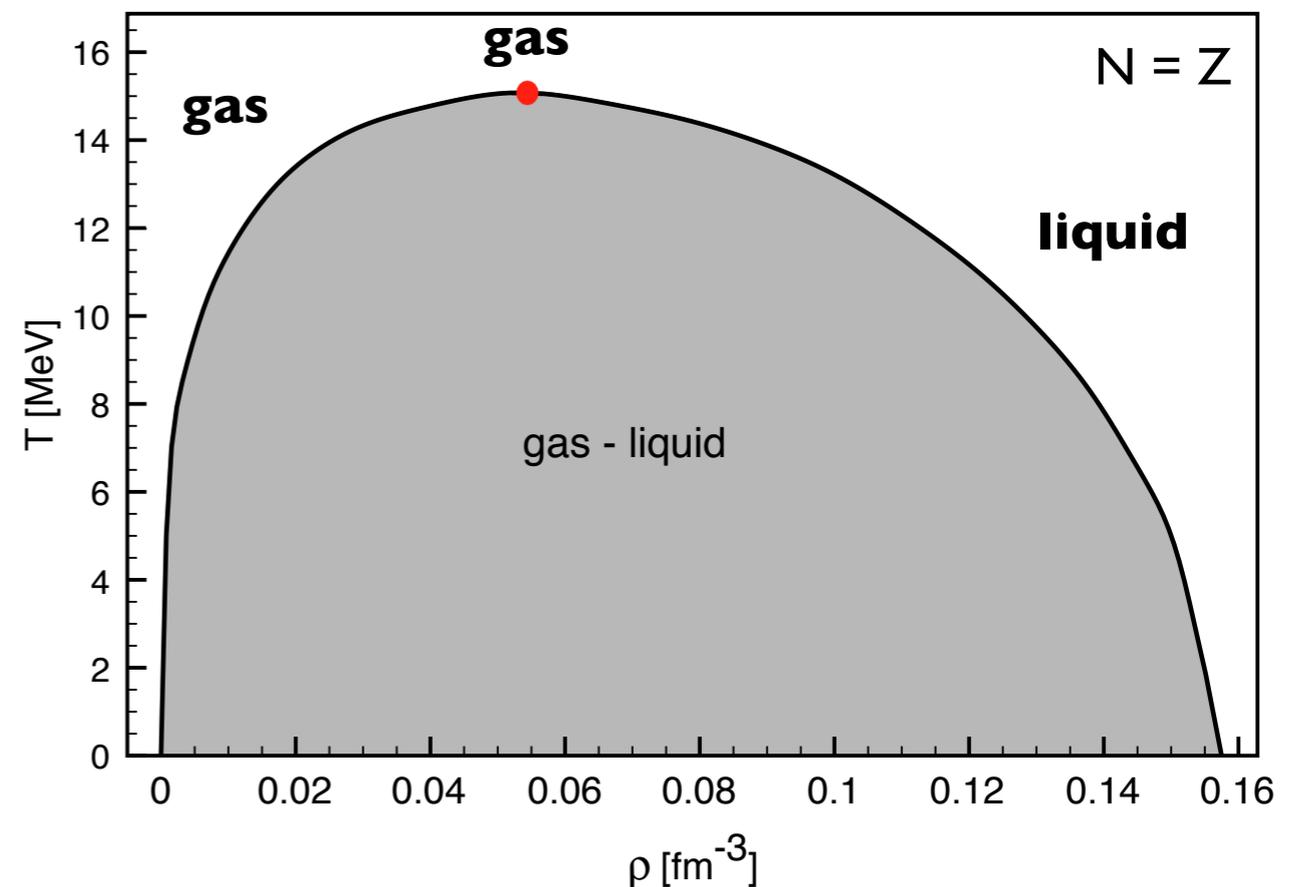


- Pion-nucleon dynamics incl. delta isobars
- Short-distance NN contact terms
- Three-body forces

- In-medium **chiral effective field theory** (3-loop calculation of free energy density)

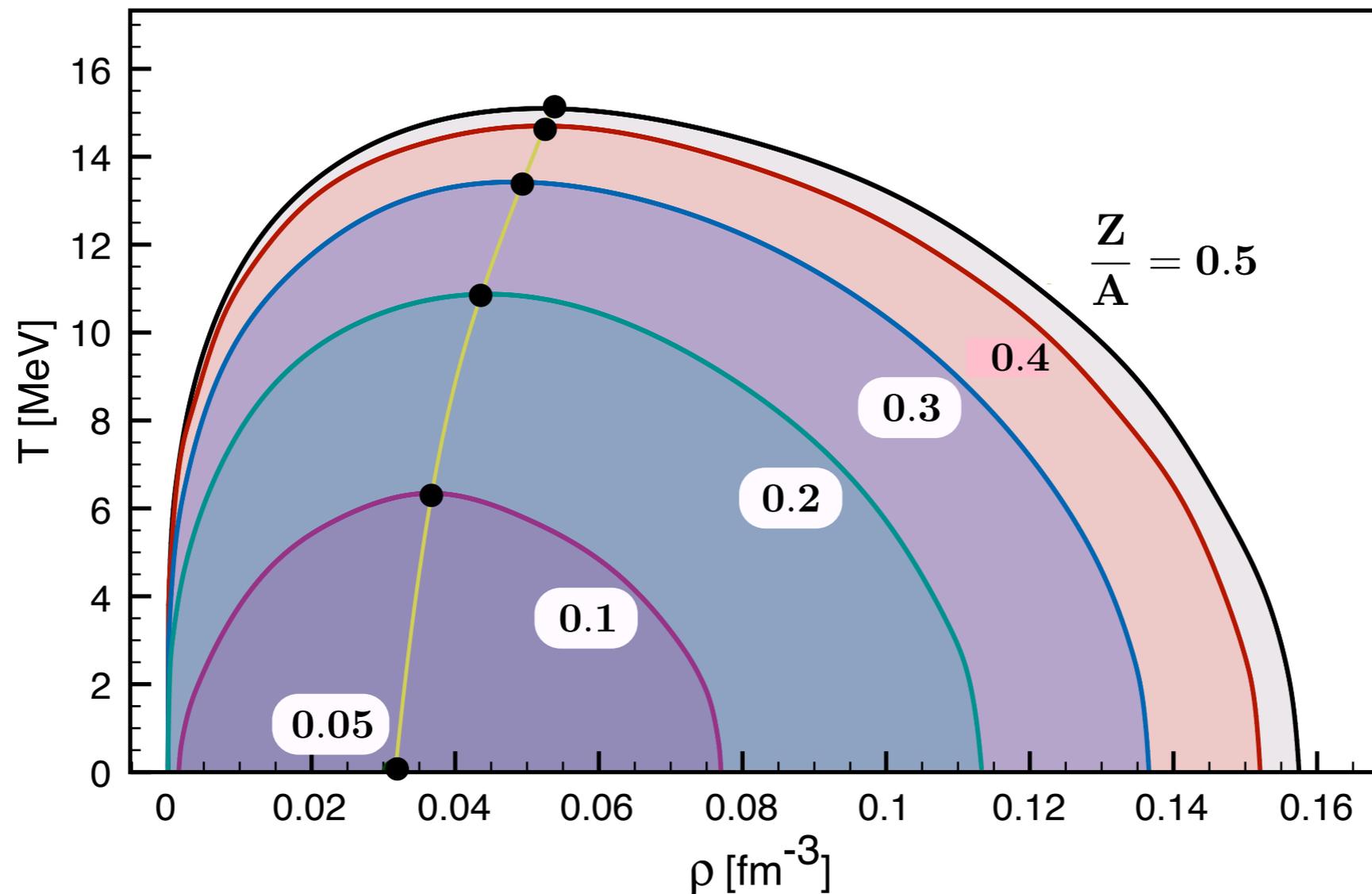
S. Fritsch, N. Kaiser, W.W.: NPA 750 (2005) 259

S. Fiorilla, N. Kaiser, W.W.  
arXiv:1111.3688 [nucl-th], Nucl. Phys. A (2012), in print



# PHASE DIAGRAM of NUCLEAR MATTER

- Trajectory of **CRITICAL POINT** for **asymmetric matter** as function of proton fraction  $Z/A$



S. Fiorilla,  
N. Kaiser,  
W.W.  
(2011)

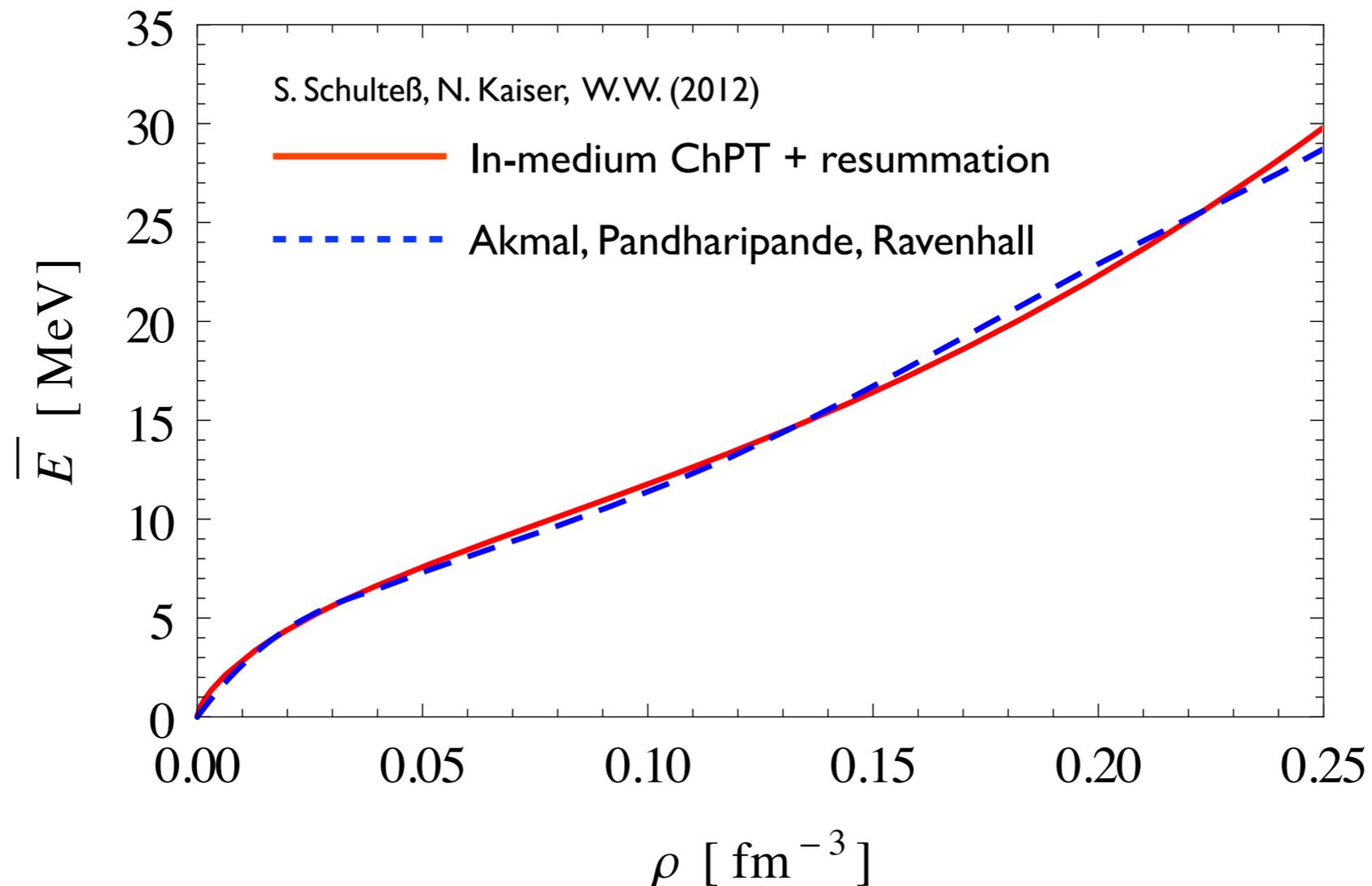
... determined almost entirely by  
**isospin** dependent (one- and two-) **pion** exchange dynamics



# NEUTRON MATTER

- In-medium chiral effective field theory (3-loop) with resummation of short distance contact terms (large nn scattering length,  $a_s = 19$  fm)

N. Kaiser, Nucl. Phys.A 860 (2011) 370



- perfect agreement with sophisticated many-body calculations



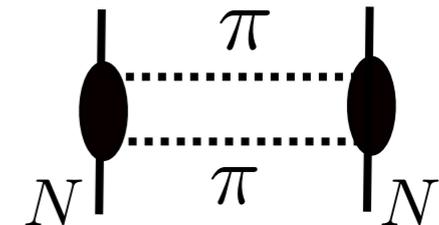
# CHIRAL CONDENSATE at finite BARYON DENSITY

- Chiral (quark) condensate  $\langle \bar{q}q \rangle$ :  $m_\pi^2 f_\pi^2 = -2 m_q \langle \bar{q}q \rangle$   
Order parameter of spontaneously broken chiral symmetry in QCD
- Hellmann - Feynman theorem:  $\langle \Psi | \bar{q}q | \Psi \rangle = \langle \Psi | \frac{\partial \mathcal{H}_{\text{QCD}}}{\partial m_q} | \Psi \rangle = \frac{\partial \mathcal{E}(m_q; \rho)}{\partial m_q}$

sigma term

$$m_q \frac{\partial M_N}{\partial m_q}$$

**in-medium  
chiral  
effective  
field theory**



$$\frac{\langle \bar{q}q \rangle_\rho}{\langle \bar{q}q \rangle_0} = 1 - \frac{\rho}{f_\pi^2} \left[ \frac{\sigma_N}{m_\pi^2} \left( 1 - \frac{3 p_F^2}{10 M_N^2} + \dots \right) + \frac{\partial}{\partial m_\pi^2} \left( \frac{E_{\text{int}}(p_F)}{A} \right) \right]$$

(free) Fermi gas  
of nucleons

nuclear interactions  
(dependence on pion mass)



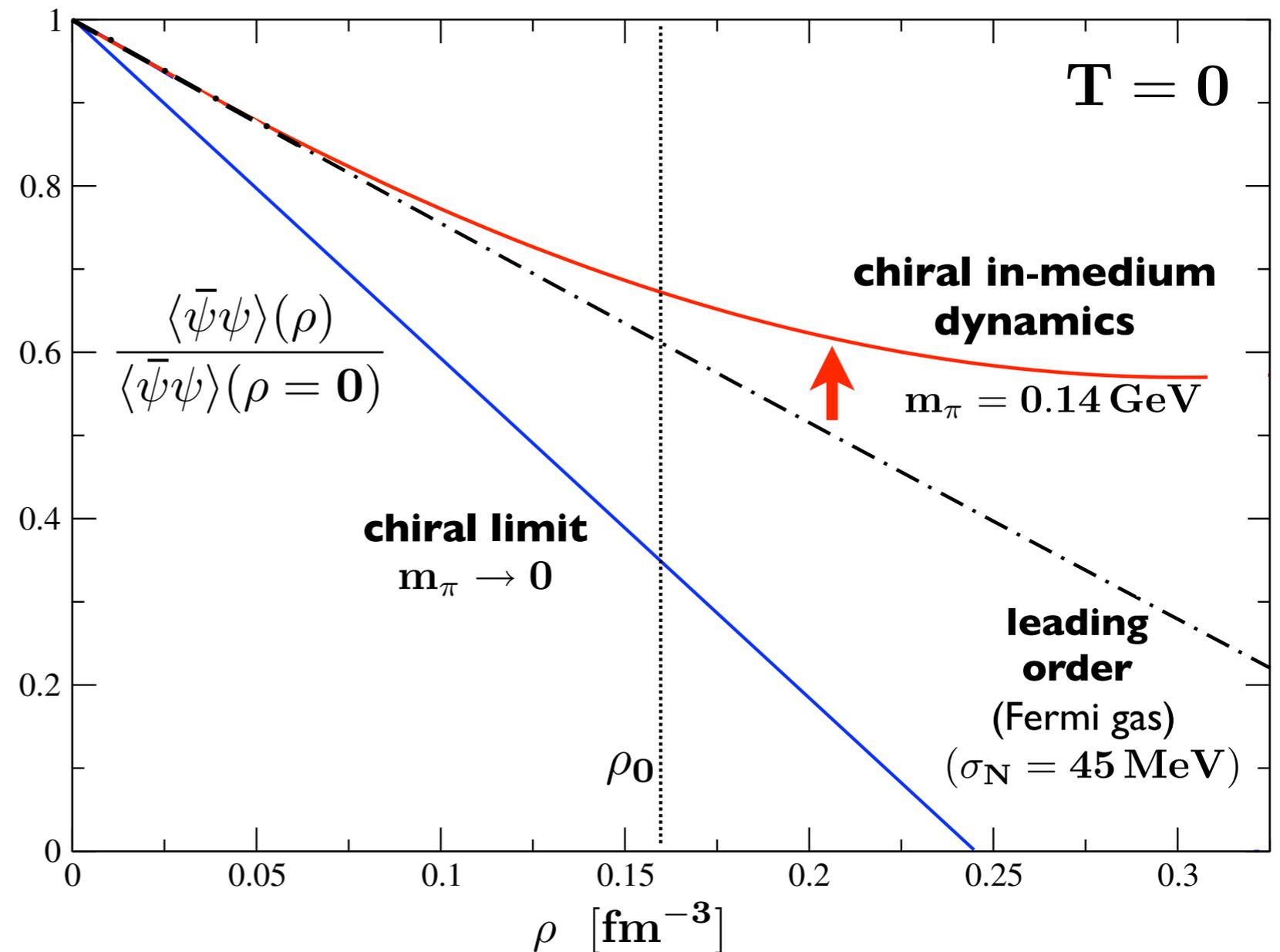
# CHIRAL CONDENSATE: DENSITY DEPENDENCE

- In-medium Chiral Effective Field Theory**

(NLO 3-loop)

constrained by **realistic nuclear equation of state**

N. Kaiser, Ph. de Homont, W.W.  
Phys. Rev. C 77 (2008) 025204



- Substantial **change of symmetry breaking scenario** between chiral limit  $m_q = 0$  and physical quark mass  $m_q \sim 5 \text{ MeV}$
- Nuclear Physics** would be **very different** in the **chiral limit** !



# CHIRAL CONDENSATE: DENSITY and TEMPERATURE DEPENDENCE

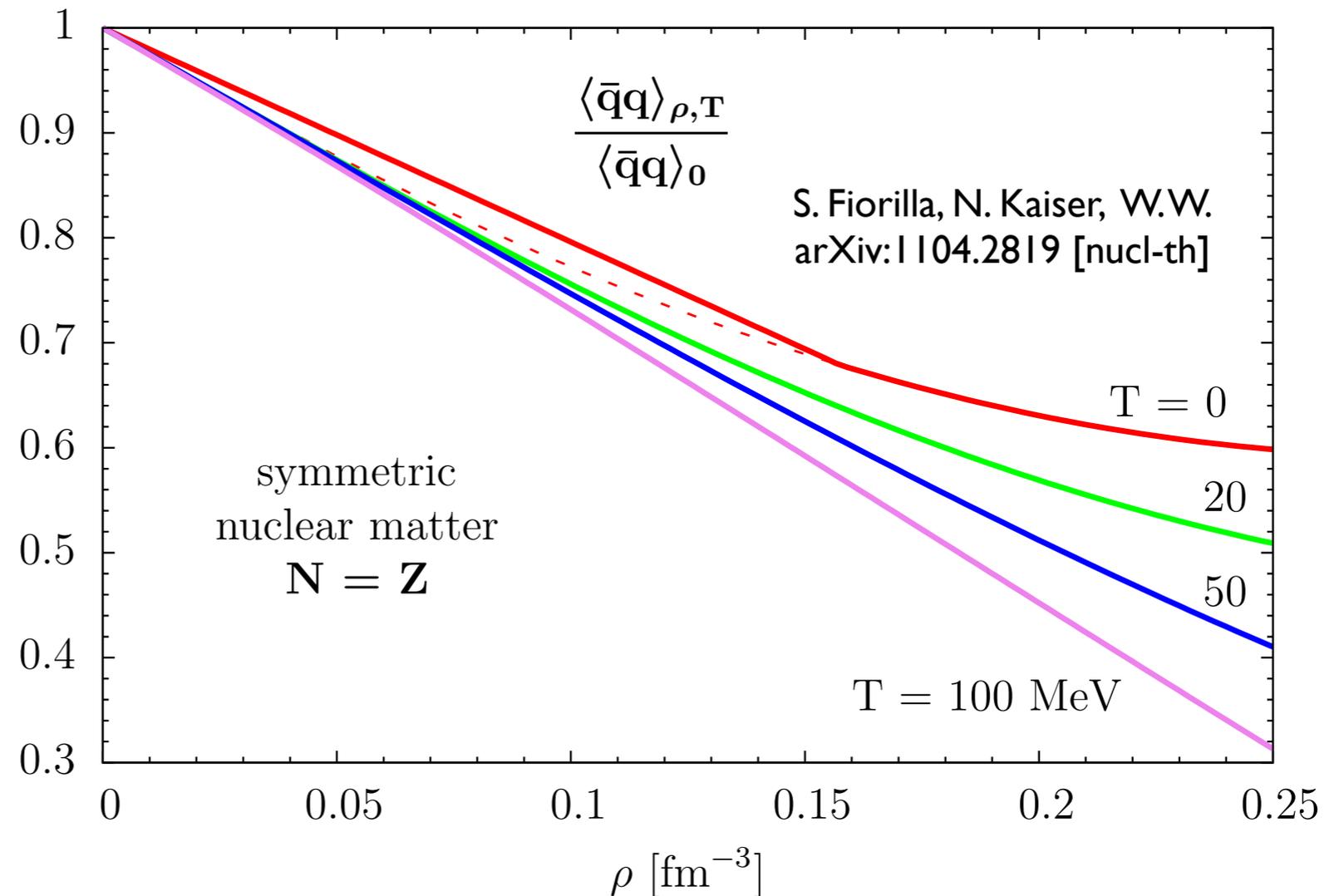
- Free energy density  $\mathcal{F}(\mathbf{m}_q; \rho, \mathbf{T})$

$$\langle \Psi | \bar{q}q | \Psi \rangle_{\rho, \mathbf{T}} = \frac{\partial \mathcal{F}(\mathbf{m}_q; \rho, \mathbf{T})}{\partial \mathbf{m}_q}$$

## In-medium Chiral Effective Field Theory

(NLO 3-loop)

constrained by  
**realistic nuclear  
equation of state**



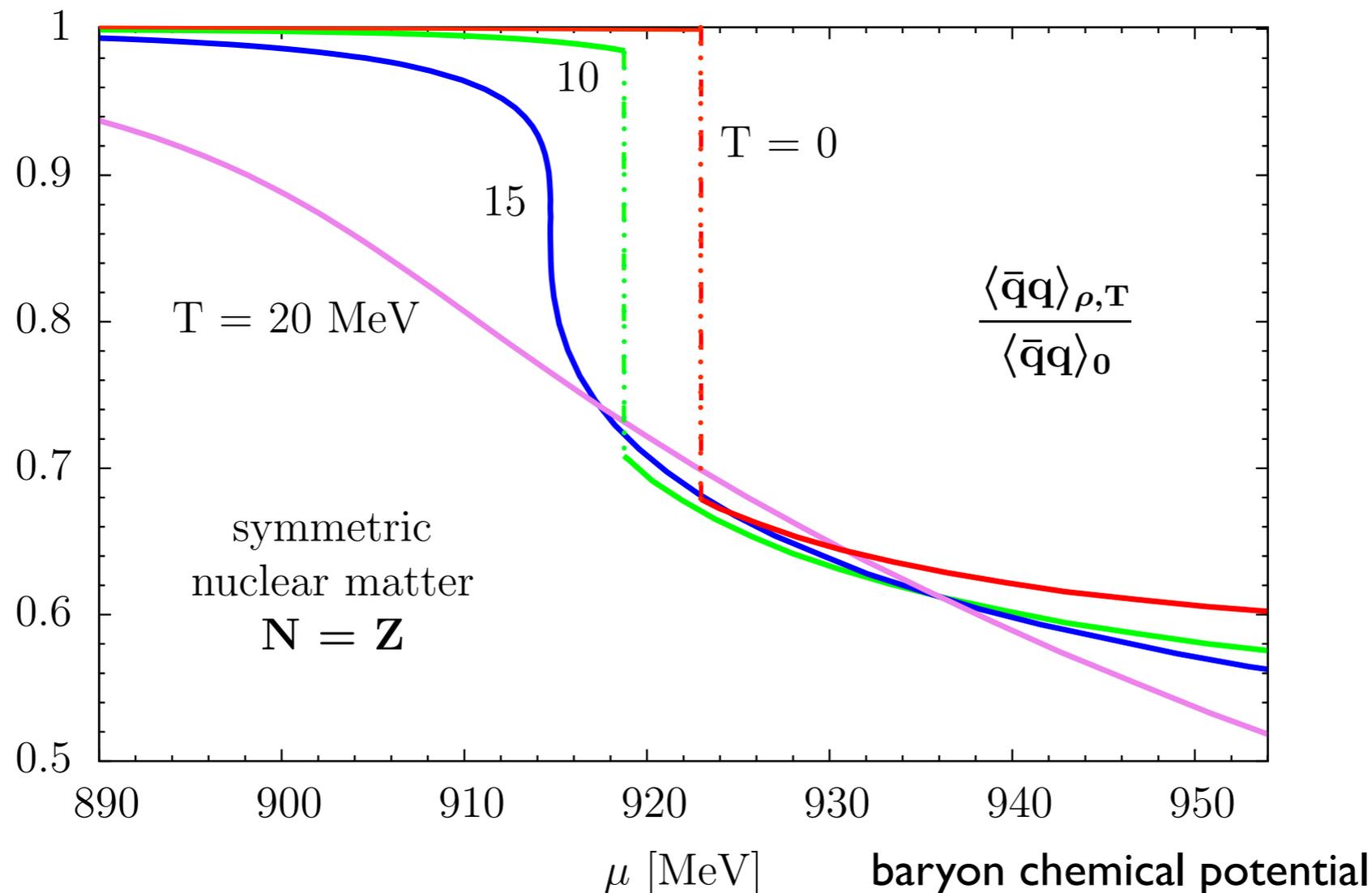
- No indication** of first order **chiral phase transition** for

$$\rho \lesssim 2 \rho_0, \quad \mathbf{T} \lesssim 100 \text{ MeV}$$



# CHIRAL CONDENSATE:

Dependence on  
TEMPERATURE and BARYON CHEMICAL POTENTIAL



- **Liquid-gas** phase transition leaves its signature also in chiral condensate
- but: **no** tendency toward **chiral first order transition** in the range

$$\mu_B \lesssim 1 \text{ GeV}$$



Outlook:

**New Constraints**  
from  
**NEUTRON STARS**



# A two-solar-mass neutron star measured using Shapiro delay

P. B. Demorest<sup>1</sup>, T. Pennucci<sup>2</sup>, S. M. Ransom<sup>1</sup>, M. S. E. Roberts<sup>3</sup> & J. W. T. Hessels<sup>4,5</sup>

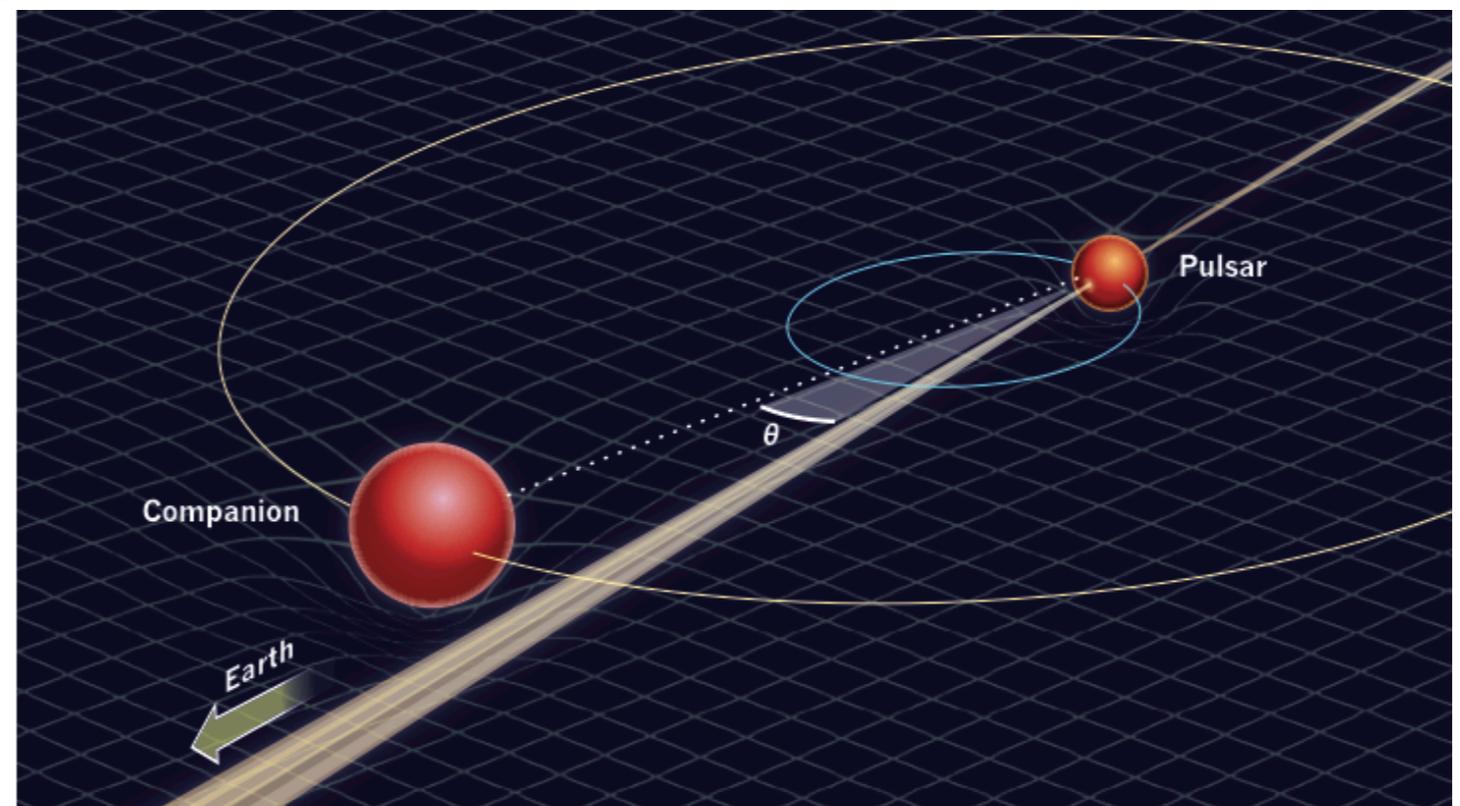
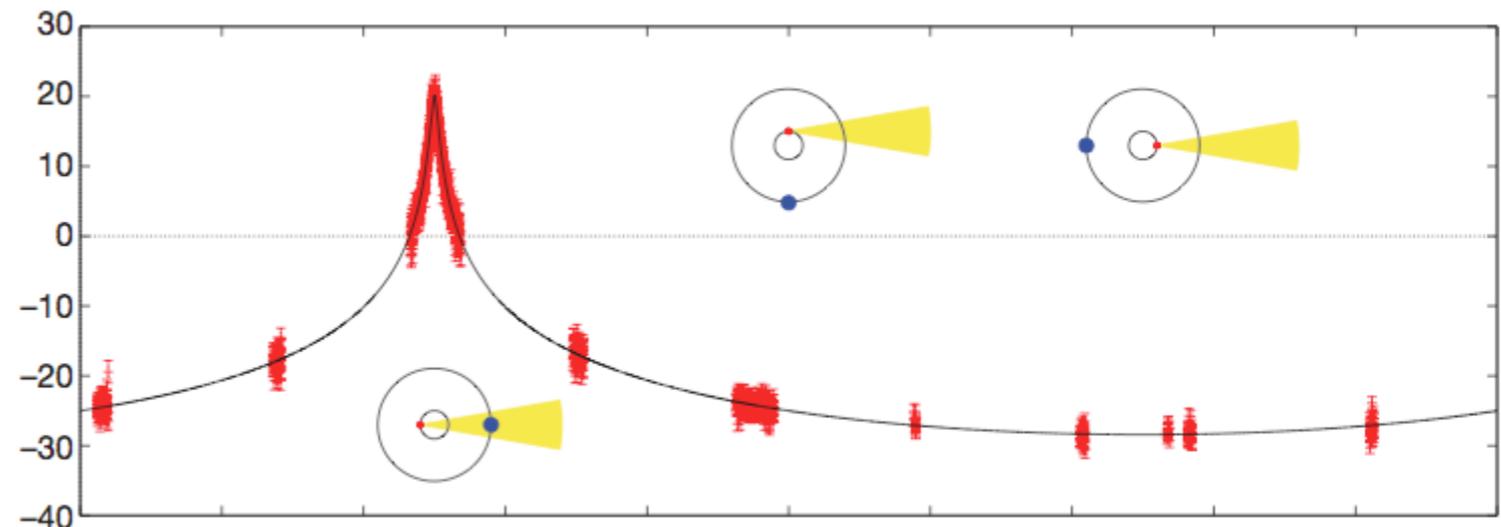
Nature, Oct. 28, 2010

direct measurement of  
neutron star mass from  
increase in travel time  
near companion

J1614-2230

most edge-on binary  
pulsar known ( $89.17^\circ$ )  
+ massive white dwarf  
companion ( $0.5 M_{\text{sun}}$ )

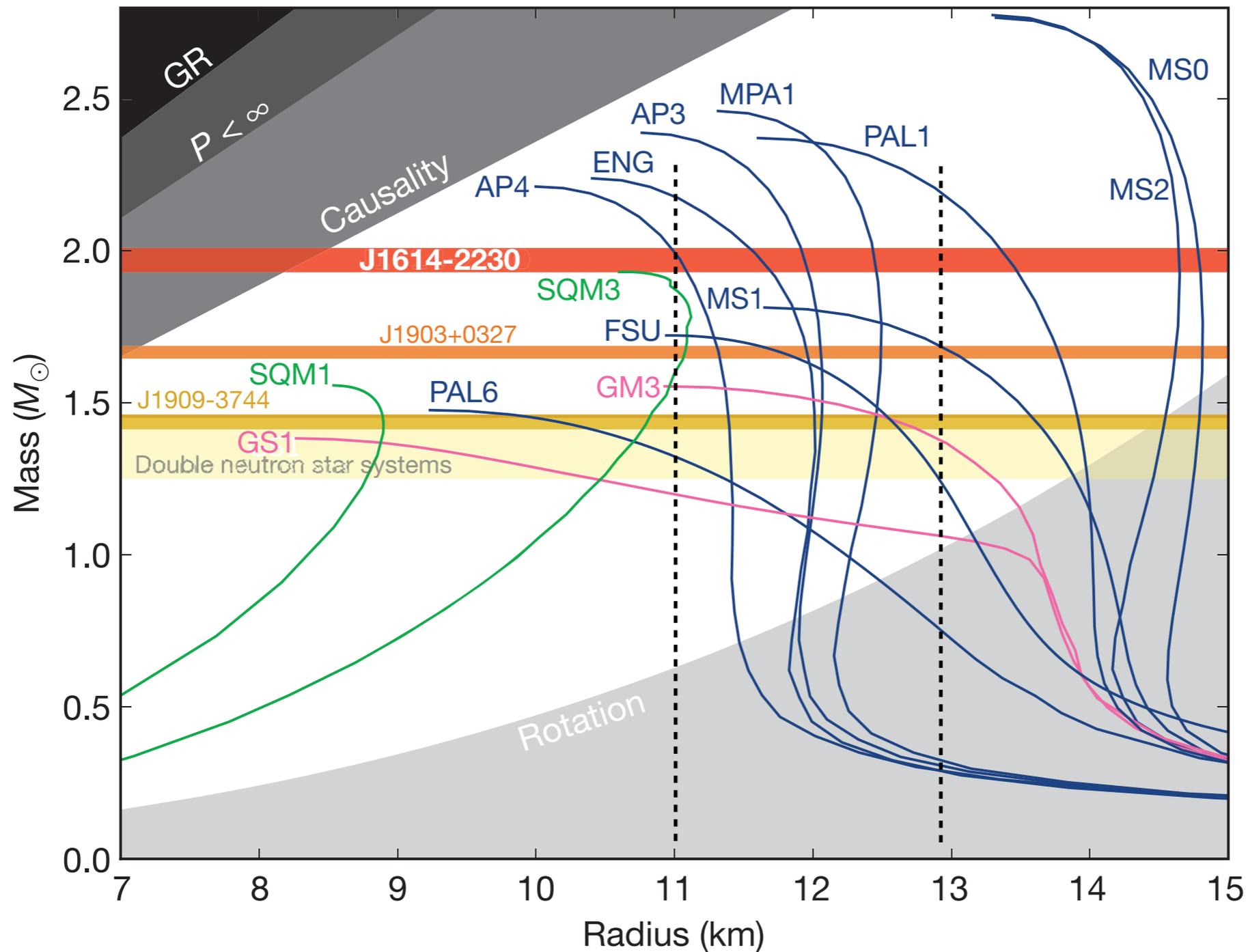
heaviest neutron star  
with  $1.97 \pm 0.04 M_{\text{sun}}$



# TWO-SOLAR-MASS NEUTRON STAR

... observed using Shapiro delay

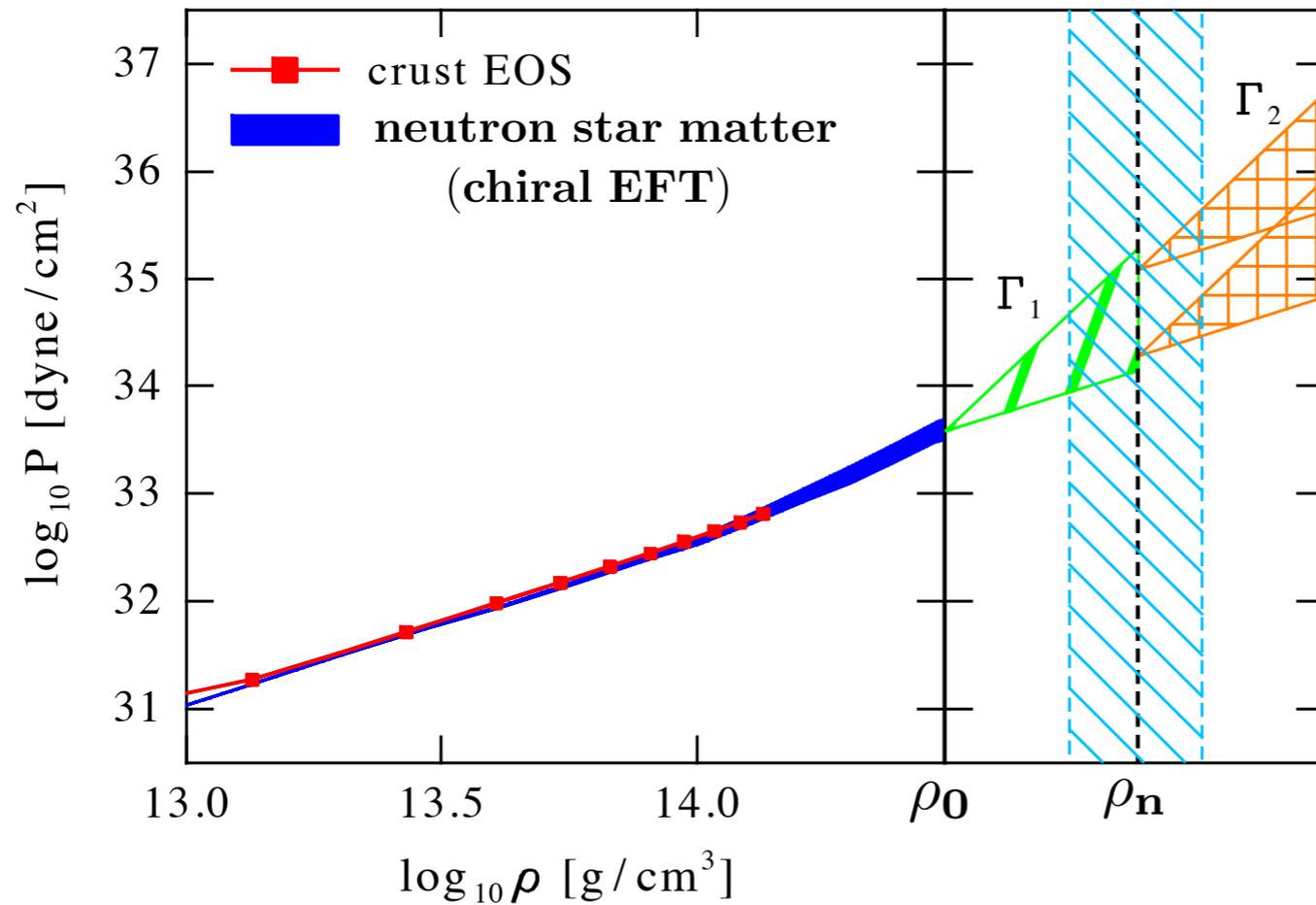
P.B. Demorest et al., Nature 467 (2010) 1081



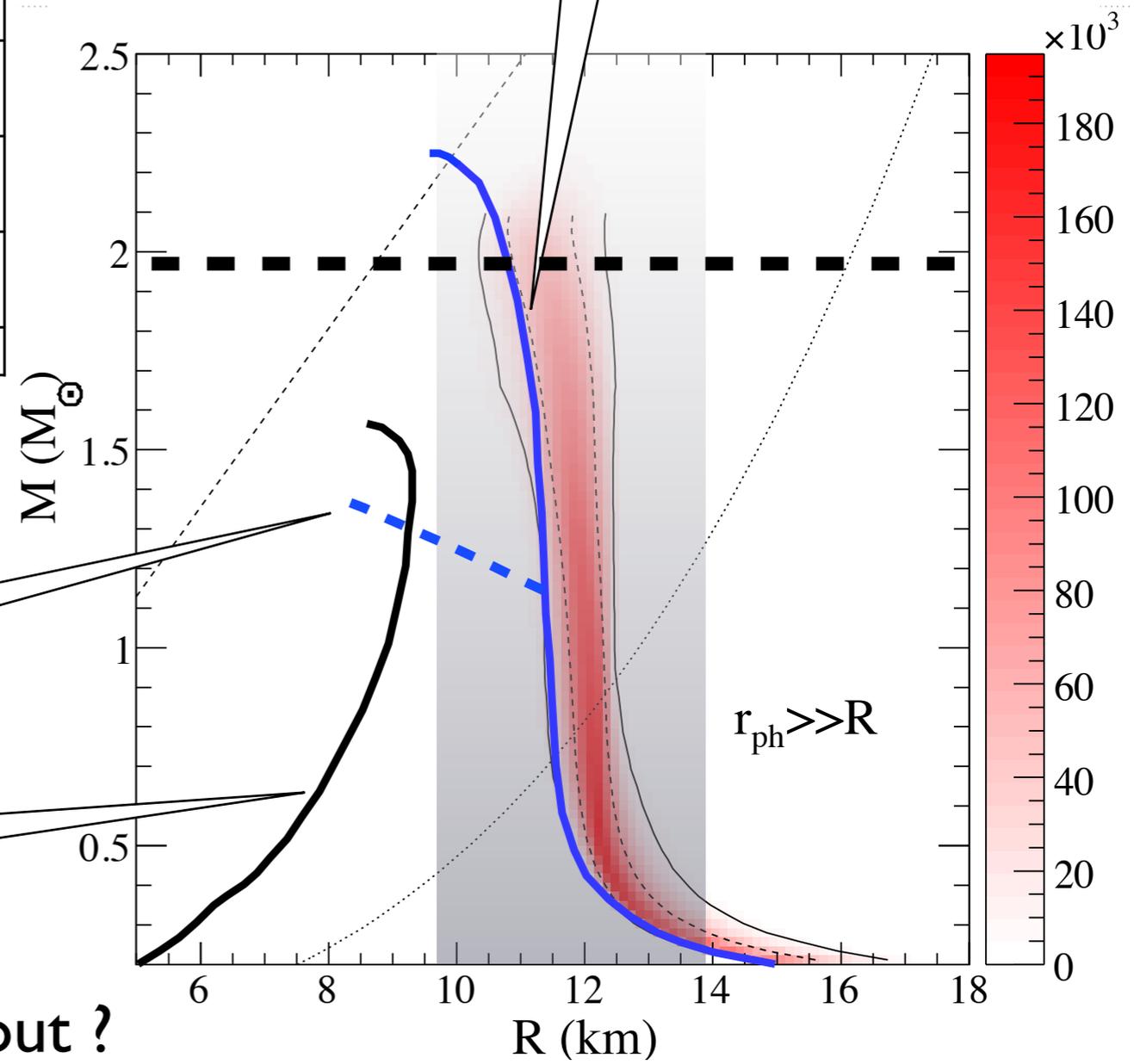
# News from NEUTRON STARS

K. Hebeler, J. Lattimer, C. Pethick, A. Schwenk  
PRL 105 (2010) 161102

A.W. Steiner, J. Lattimer, E.F. Brown  
Astroph.J. 722 (2010) 33



realistic “nuclear” EoS  
(Illinois)



● New constraints from **EFT** and **neutron star observables**

kaon condensate

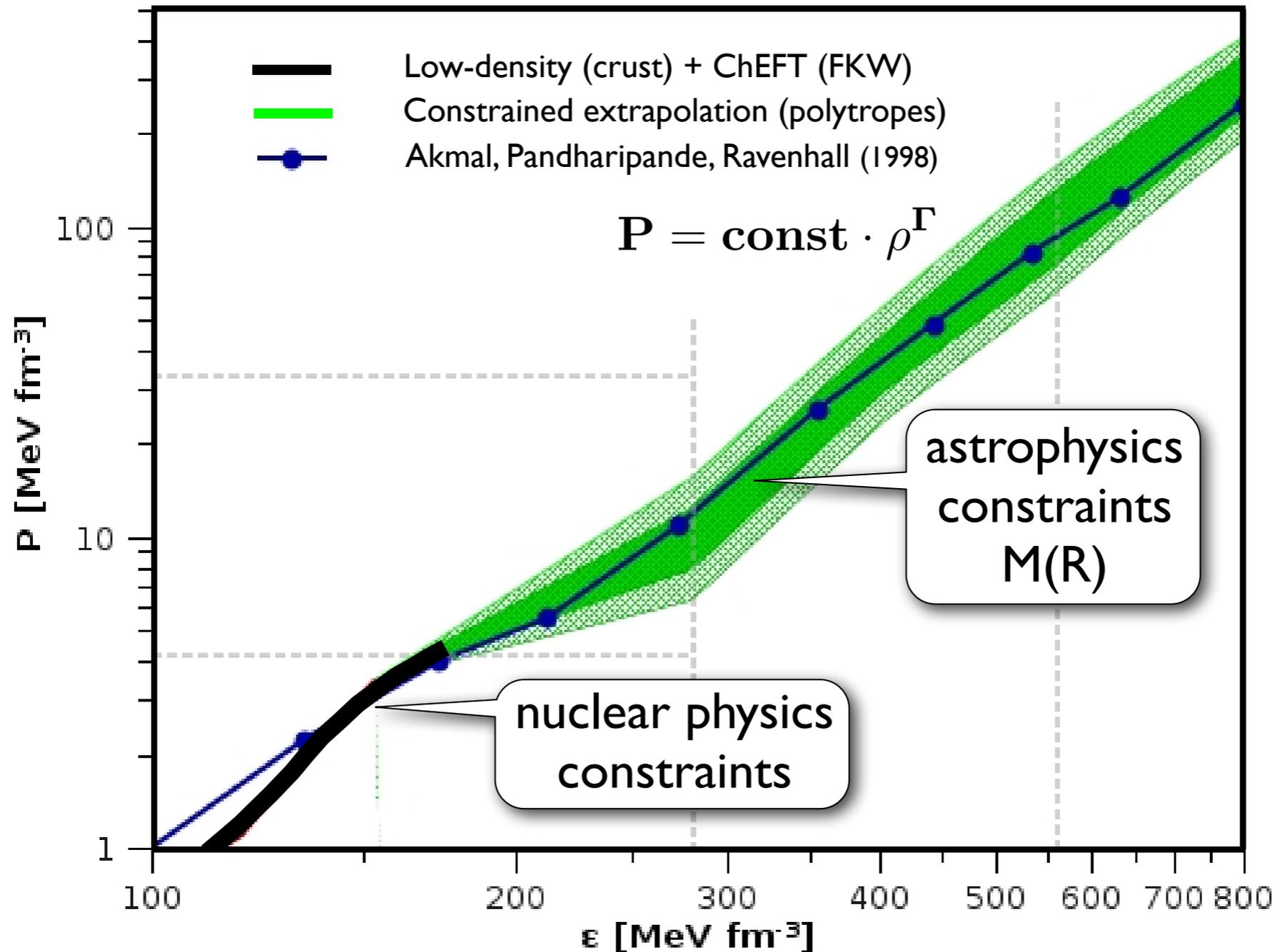
quark matter

● “**Exotic**” equations of state ruled out?



# NEUTRON STAR MATTER

## Equation of State



Bernhard Röttgers,  
W.W.  
(2011)

- Including new neutron star constraints plus **Chiral Effective Field Theory** at lower density



# SUMMARY

- Exploration of **QCD phase diagram**:  
progress concerning basic symmetry breaking patterns
  - ▶ Lattice QCD (restricted to small quark chemical potentials)
  - ▶ Models (PNJL, PQM) (but: nuclear physics constraints missing)
  - ▶ Dyson-Schwinger QCD ( -- same problem -- )
- **Nuclear thermodynamics** based on  
In-medium **Chiral Effective Field Theory**  
Fermi liquid ↔ interacting Fermi gas (1st order transition)
  - ▶ **No** indication of first order **chiral** transition in the range  
 $\rho \lesssim 2\rho_0, T \lesssim 100 \text{ MeV}$
  - ▶ Major challenge: design **QCD phase diagram** that is consistent with established hadronic and nuclear physics
- New **dense & cold matter** constraints from **neutron stars**:
  - ▶ Mass - radius relation; observation of two-solar-mass n-star
  - ▶ “Non-exotic” equation of state works best !



*The End*

*thanks to:*

Nino Bratovic  
Jeremy Holt

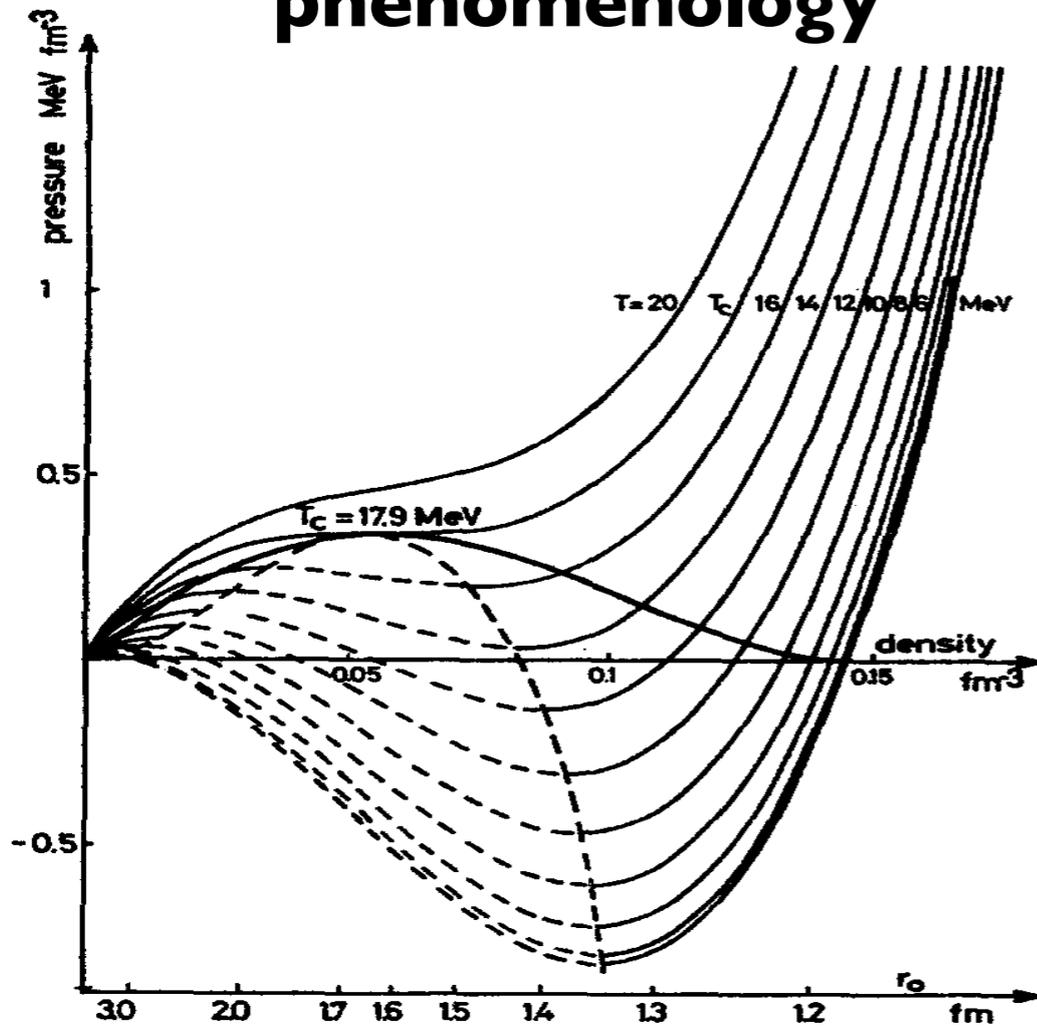
Salvatore Fiorilla  
Norbert Kaiser

Thomas Hell  
Kouji Kashiwa



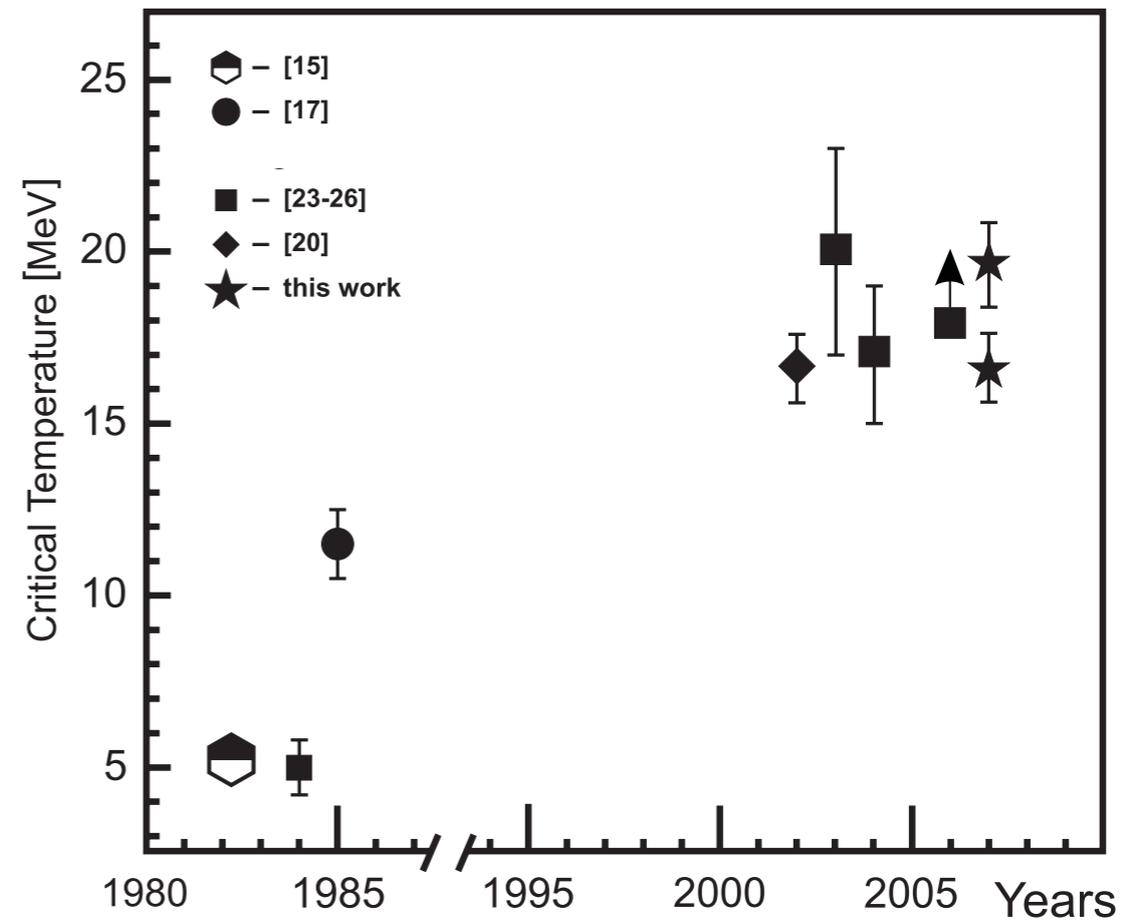
# NUCLEAR THERMODYNAMICS

## Skyrme phenomenology



G. Sauer, H. Chandra, U. Mosel  
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## Multifragmentation and fission analysis



V.A. Karnaukhov et al. :  
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