Clusters in hot and dense stellar matter

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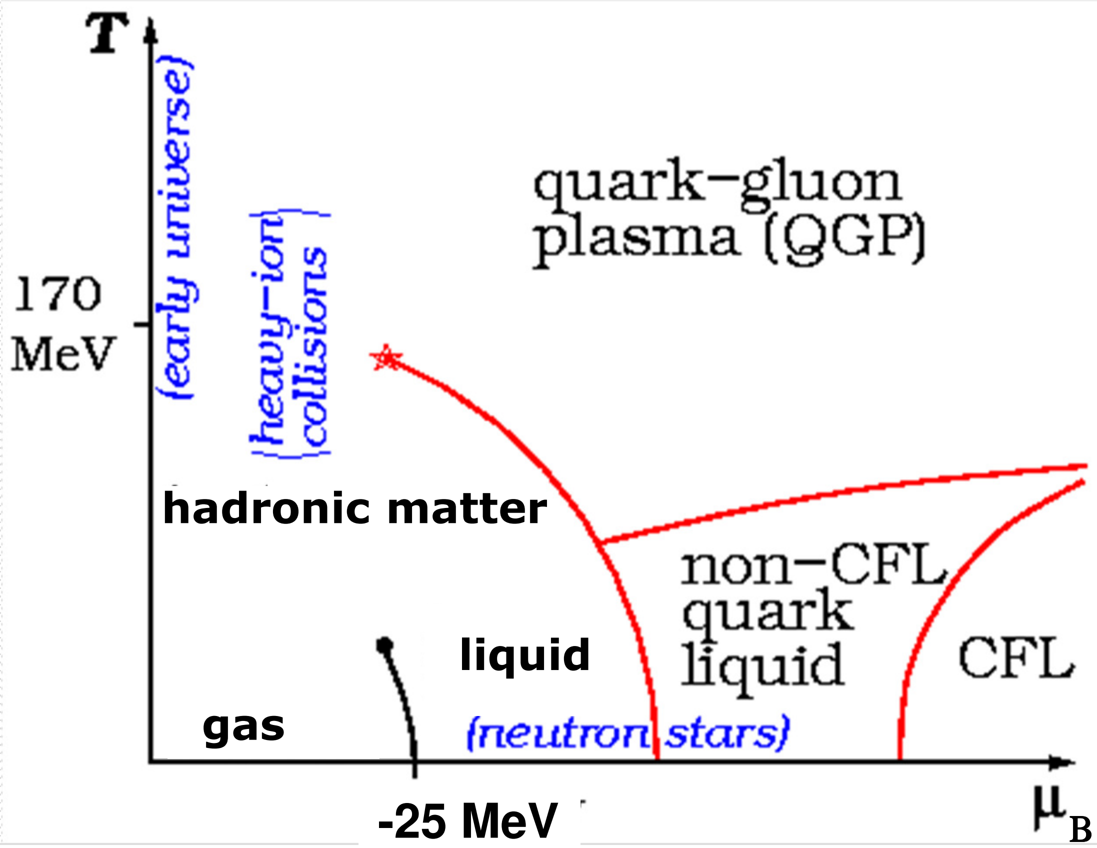
The QCD phase diagram of dense baryonic matter



This Phase Diagram is not directly relevant for astro applications:

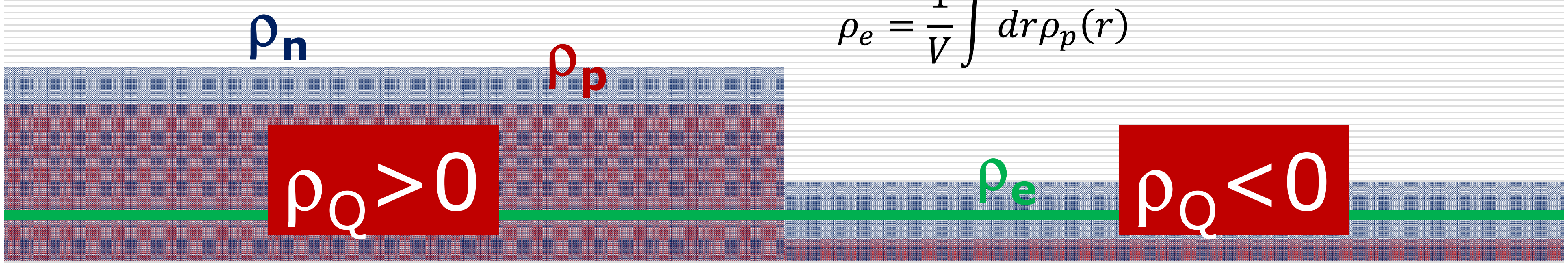
Lepton-baryon coupling leads to the emergence of clusters

Source: WIKIPEDIA

electron-proton coupling

- *The e.m. interaction couples the baryon and the lepton sector*
- *This is true even at the mean-field level because of electroneutrality* $f(\rho_B,\rho_Q,\rho_L) = f_B(\rho_B,\rho_Q) + f_L(\rho_L,\rho_Q)$; ρ_Q*=0*
- *Consequence: quenching of LG phase transition*

$$\rho_e = \frac{1}{V}\int dr\,\rho_p(r)$$



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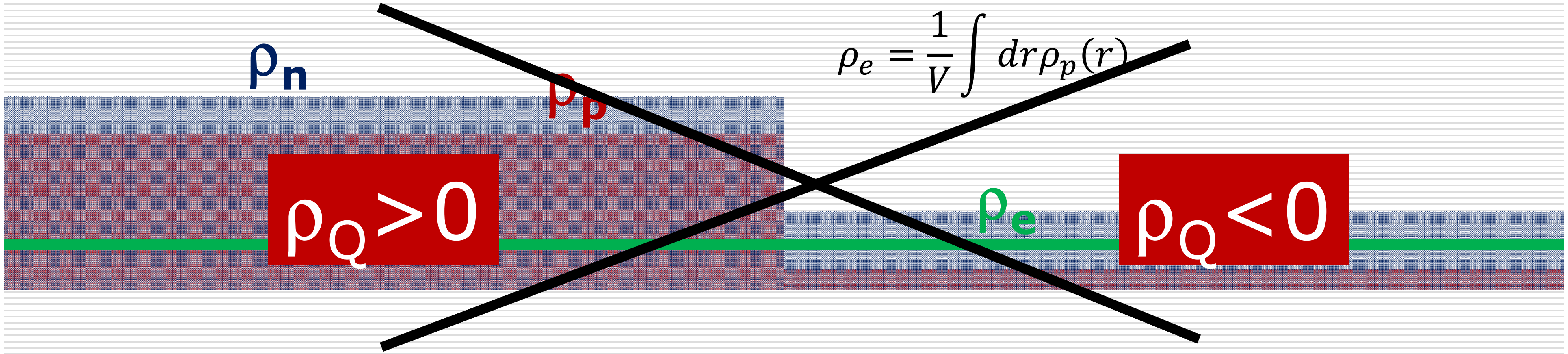

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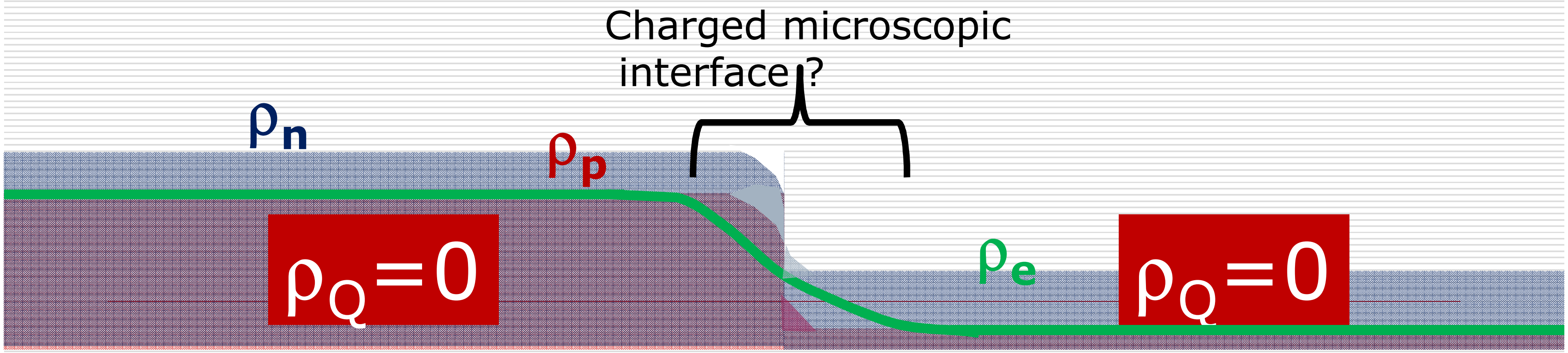
- *This is true for incompressible e: could the PT be recovered accounting for e polarisability?*



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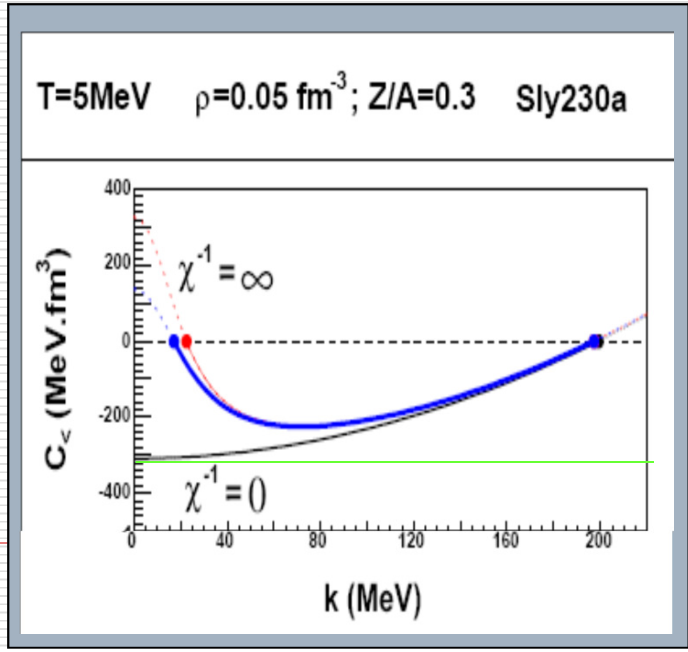
- *Consequence: quenching of LG phase transition*
- *This is true for incompressible e: could the PT be recovered accounting for e polarisability?*
- *Linear response theory:*

$$\delta\rho_q(k,r) = A_q e^{ik\cdot r} + cc \quad q = n,p,e$$

$$\delta f = \delta H - T\delta s \quad C_{pq}(k) = \frac{\partial^2 \delta f}{\partial\delta\rho_p \partial\delta\rho_q}$$

$C_<(k) < 0$ unstable



C.Ducoin et al NPA 789 (2007) 403,
PRC 75 (2007) 065805.

electron-proton coupling

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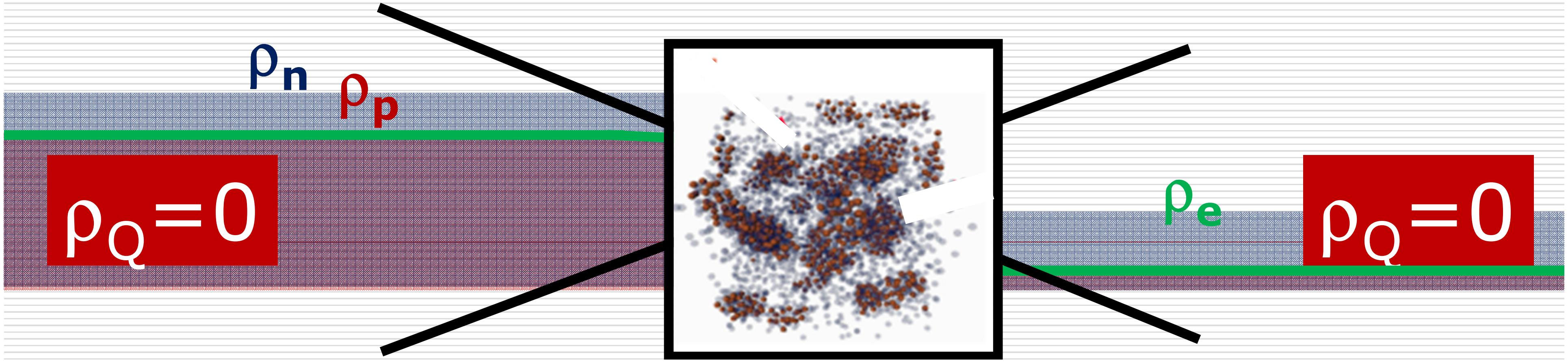
- *Consequence: quenching of LG phase transition*
- ***Cannot** be recovered accounting for e polarisability*
- *Continuous transition through a cluster phase*
- *EoS at* $\rho<\rho_0$ *without nuclei is not correct !*



A.Raduta,F.G.,PRC 82:065801 (2010)
PRC 85:025803 (2012)

The extended NSE model

- Mixture of nucleons, clusters of all sizes, γ, e^-, e^+, ν

$$Z = Z_{lep}(\beta,\mu_e) Z_\gamma(\beta) Z_n(\beta,\mu_n,\mu_p) Z_N$$

- Nucleons treated in the Skyrme-HF approximation

$$Z_n = \exp\left(\beta(V - V_N)\left(\frac{\hbar^2\tau_n}{3m_n^*} + \frac{\hbar^2\tau_p}{3m_p^*} + \langle \hat{h}_{sp}\rangle - \langle \hat{h}_{mf}\rangle\right)\right) \cdot \exp\beta(\mu_n\rho_n + \mu_p\rho_p)$$

- Nuclei form a statistical ensemble of excited clusters interacting via Coulomb and excluded volume

$$Z_N(\beta, A, \tilde{\mu}) = \sum_{\{n_a\}} \prod_{a=2}^{\infty} \frac{\omega_{a\tilde{\mu}}^{n_a}}{n_a!} = \frac{1}{A}\sum_{a=2}^{A} a\,\omega_{a\tilde{\mu}} Z_N(\beta, A-a, \tilde{\mu})$$

$$\omega_{a\tilde{\mu}} = (V - V_{N+n}) \sum_{i=-a}^{a} g_{ai}(\beta)\left(\frac{m_{ai}}{2\pi\beta}\right)^{3/2} e^{-\beta\left(e_{ai}(\rho,\rho_p) - \tilde{\mu} i\right)}$$

- Thermodynamic consistency between the different components

$$\mu_i^{nucleons} = \mu_i^{clus} \qquad i = n, p$$

$$P = P^{nucleons} + P^{clus} \quad ; \quad \rho_i = \rho_i^{nucleons} + \rho_i^{clus}$$

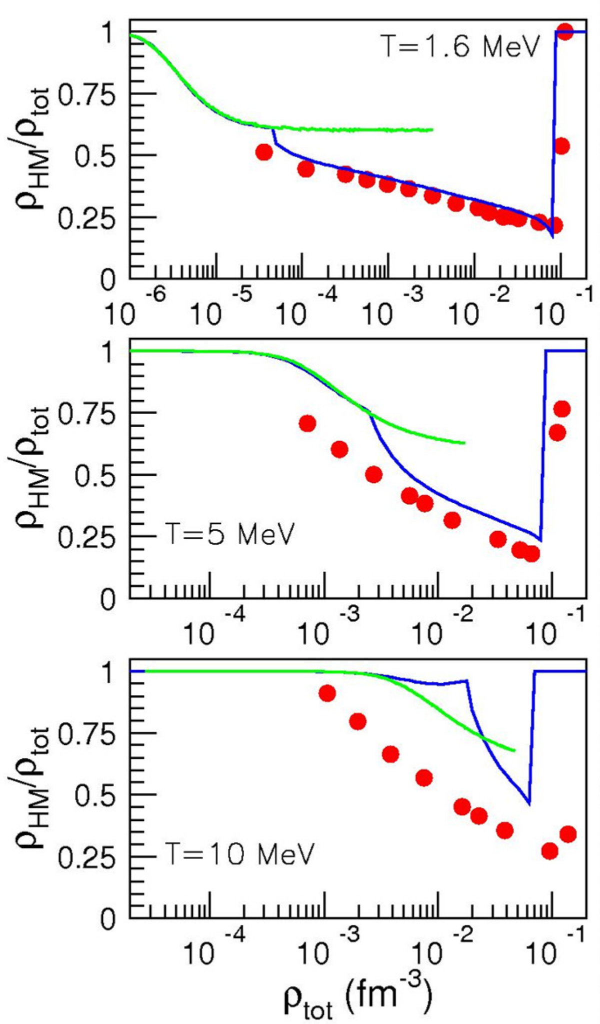
The crust-core transition

Lines: **LS** + **virial** EOS
Symbols: this work
y_p=0.2

T=1.6 MeV

T=5

T=10



- Crust-core transition naturally occurs

Matter composition: cluster contribution

Lines: **LS EOS**
Symbols: this work
$y_p=0.2$

T=1.6 MeV

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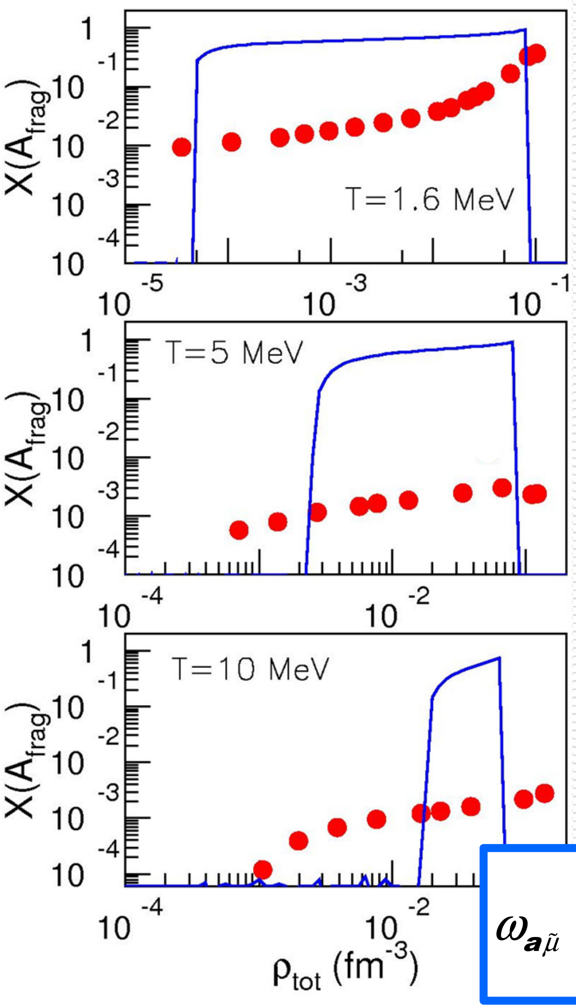
T=10



- No artificial discontinuities
- Decreasing cluster size with increasing temperature
- Clusters still important at T=10 MeV

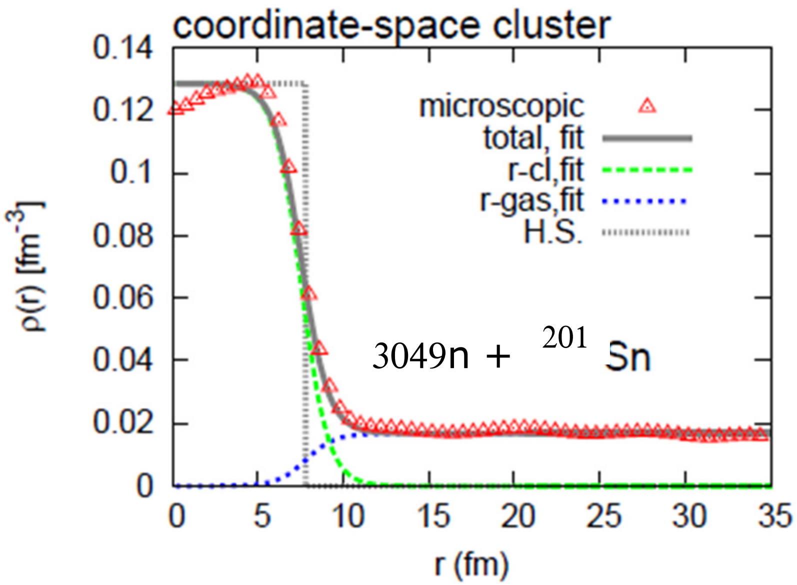
BUT

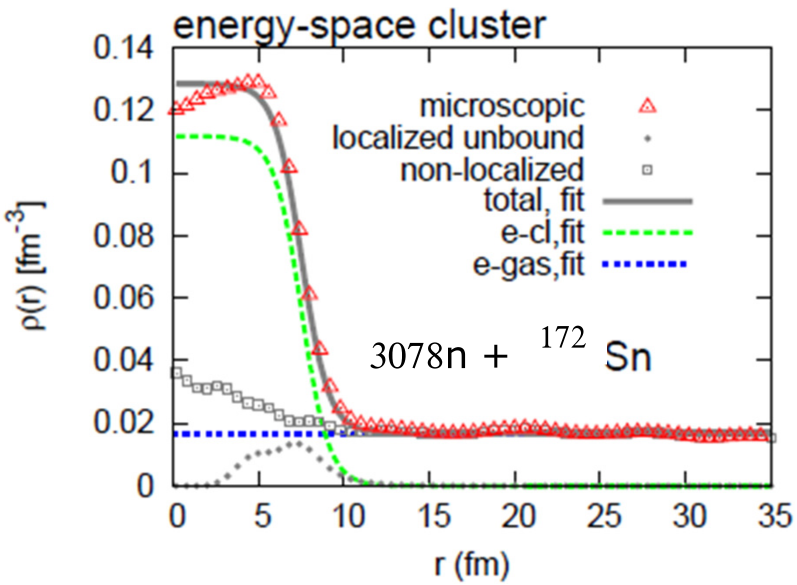
- Coulomb screening and excluded volume are the only in-medium effects of the model

$$\omega_{a\tilde{\mu}} = (V - V_{N+n}) \sum_{i=-a}^{a} g_{ai}(\beta) \left(\frac{m_{ai}}{2\pi\beta} \right)^{3/2} e^{-\beta\left(e_{ai}(\rho_p) - \tilde{\mu} i\right)}$$

Clusters in the medium

- In medium effects depend on the definition of what is a cluster

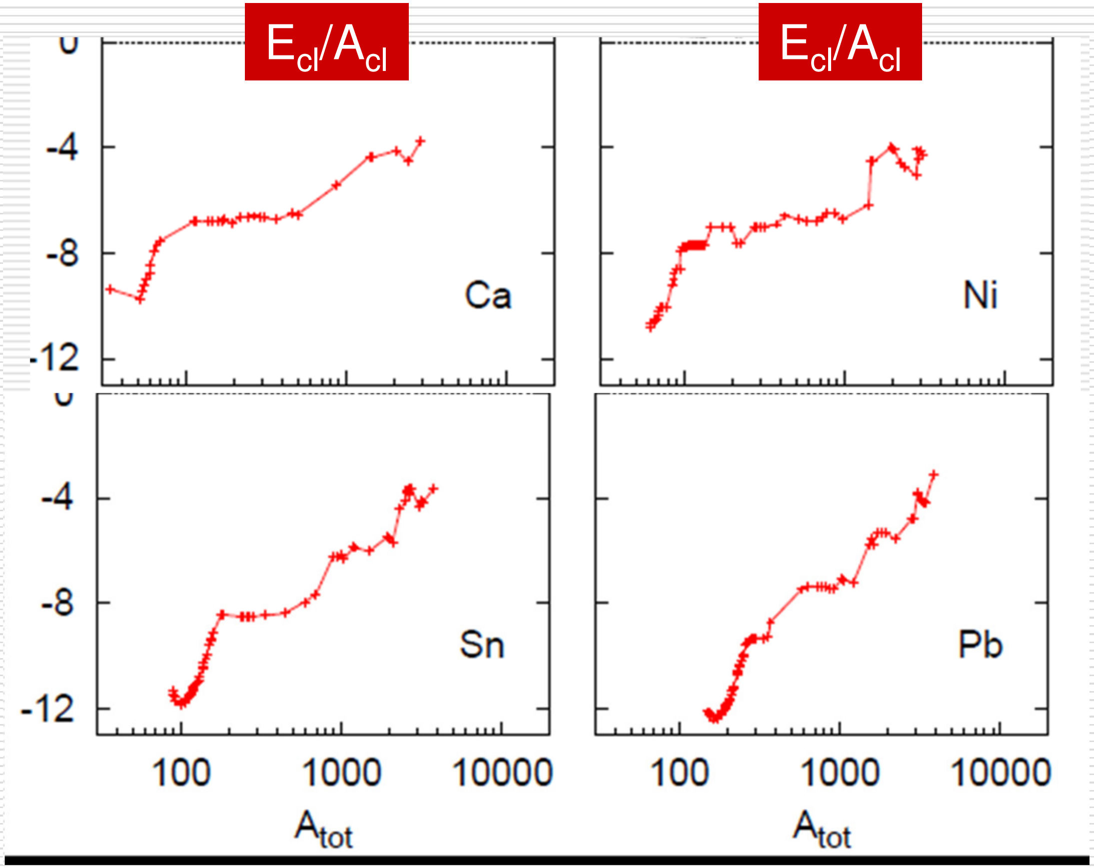




Cluster: density fluctuation
=>excluded volume applies
=>bulk energy is ~unaffected
=>surface energy is not though.

Cluster: localized wave functions
=>excluded volume does not apply
=>binding energy shift
=>cutoff on the excited states

Hints from microscopic calculations



T=0 HF in the Wigner-Seitz cell: Sly4 coordinate space clusters

Reduced binding in the medium

$$E_{cl}\,(A_{cl},\,\delta_{cl},\,\rho_{gas})$$

To disentangle the different effects: model the density profile and use the LDA

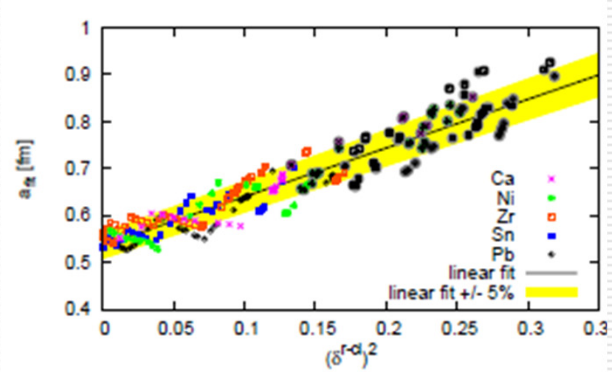
$$E_{LDA} = \int d^3r\varepsilon_{HF}(\rho(\vec{r}))$$

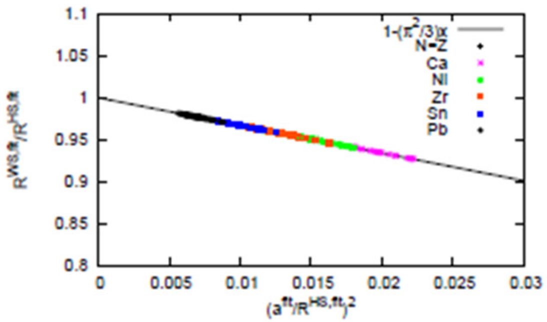
Modelling the density profile

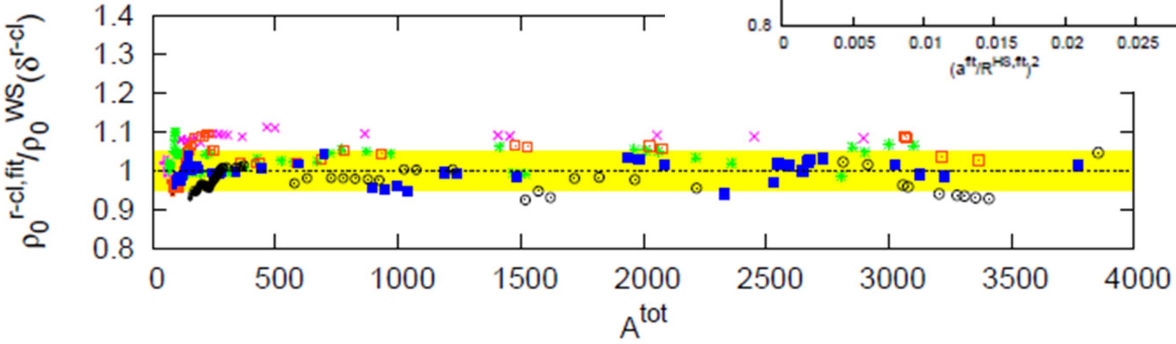
$$\rho^q_{cl}(r) = \frac{\rho^q_{eq}(\delta_{bulk})}{1 + exp\,\frac{r - R^q}{a^q}}$$

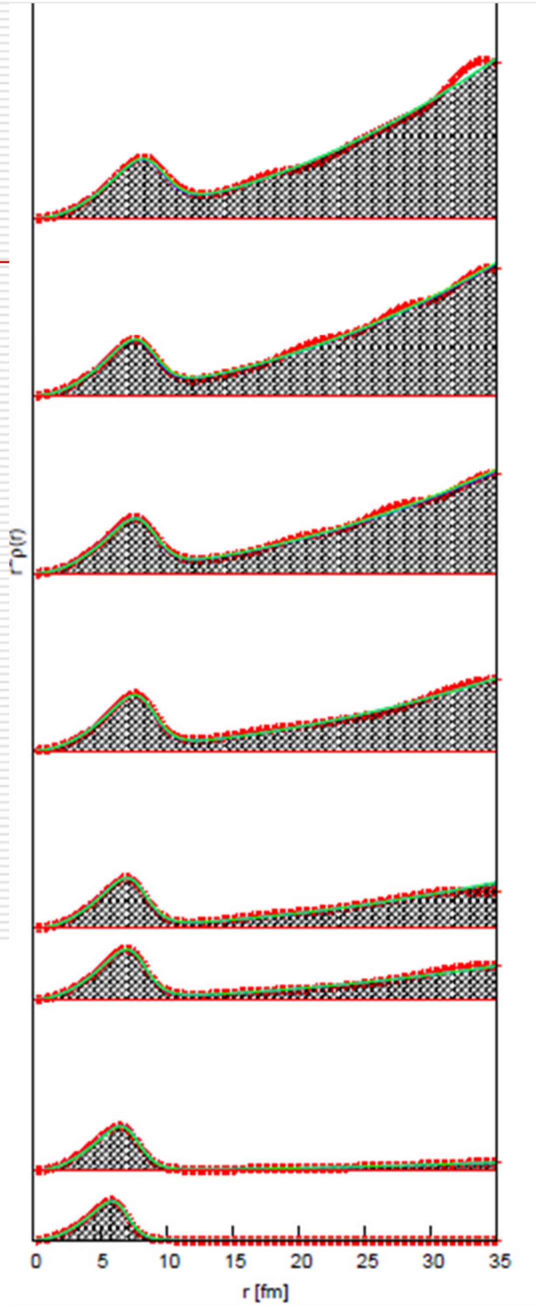
$$\rho^q_{gas}(r) = \frac{\rho^q_{gas}}{1 + exp\,\frac{R^q - r}{a^q}}$$

R^q, a^q, δ_{bulk} analytic functions of N,Z, ρ^n_{gas}, ρ^p_{gas}









An analytical in-medium cluster energy

$$\rho^q(r) = \rho^q_{cl}(r) + \rho^q_{gas}(r)$$

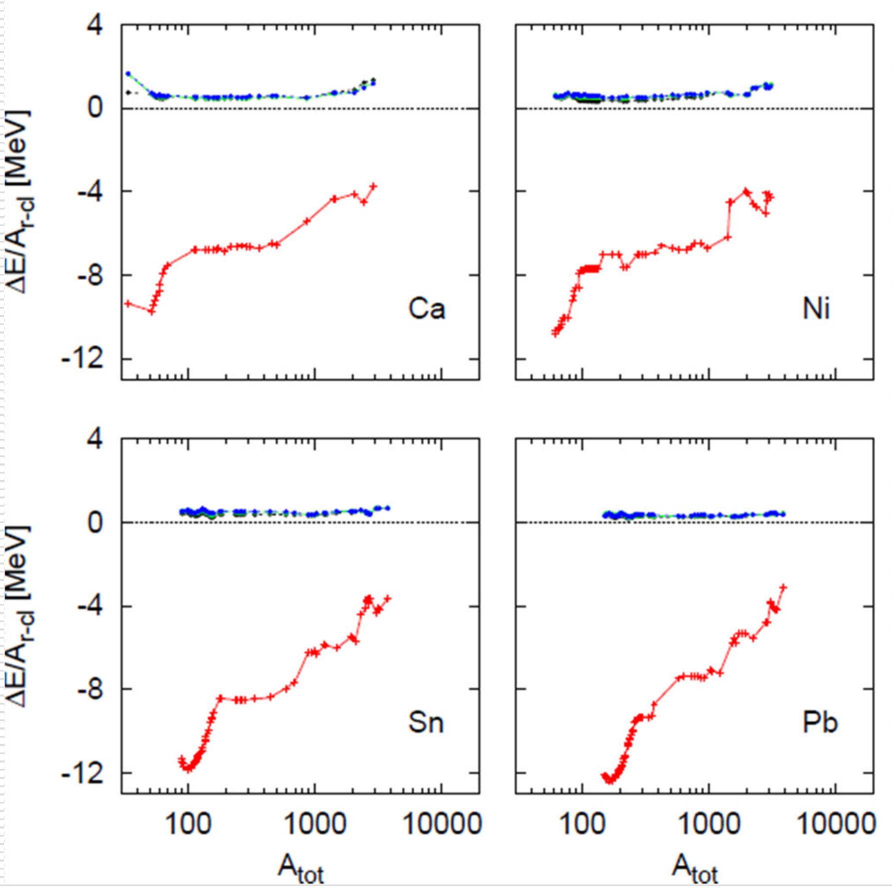
$$E^m_{cl}(A,\delta,\rho_{gas}) = \int d^3r \varepsilon_{HF}\left(\rho^n(r),\rho^p(r)\right) - \varepsilon_{HF}\left(\rho^n_{gas},\rho^p_{gas}\right)\left(V_{WS} - A/\rho_{eq}\right)$$

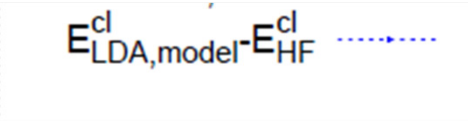
$$=\frac{\varepsilon_{HF}\left(\rho^n_{eq},\rho^p_{eq},\right)}{\rho_{eq}}A + E^m_{surf}=e^m_{bulk}\left(\rho_{gas},\delta\right)A + e^m_{surf}\left(\rho_{gas},\delta\right)A^{2/3}$$

In-medium modification of the mass formula parameters:

$$\delta e^m_{bulk}\left(\rho_{gas},\delta\right)=e^m_{bulk} - a_V$$
$$\delta e^m_{surf}\left(\rho_{gas},\delta\right)=e^m_{surf} - a_S$$

Quality of the LDA





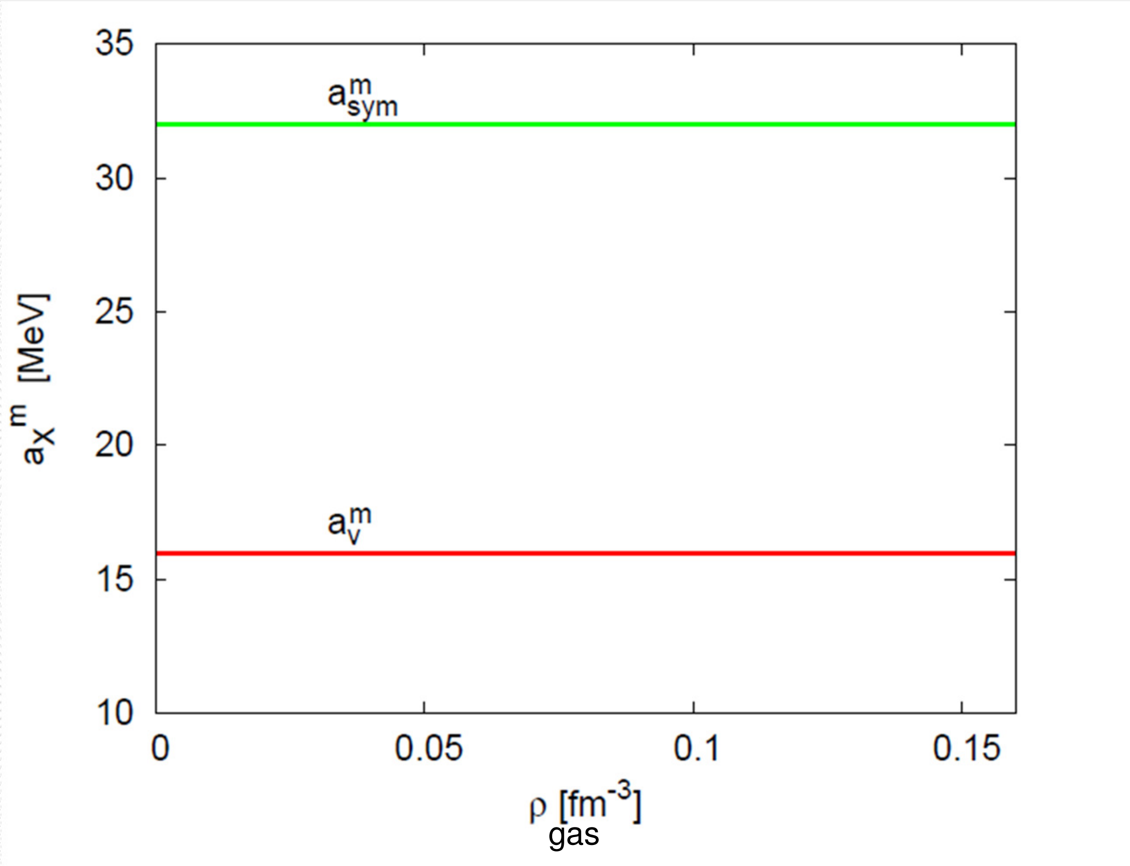
The deviation between the microscopic calculation and the LDA modelling is independent of the medium

=> $\delta e^{m}_{bulk}(\rho_{gas},\delta)$,

$\delta e^{m}_{surf}(\rho_{gas},\delta)$

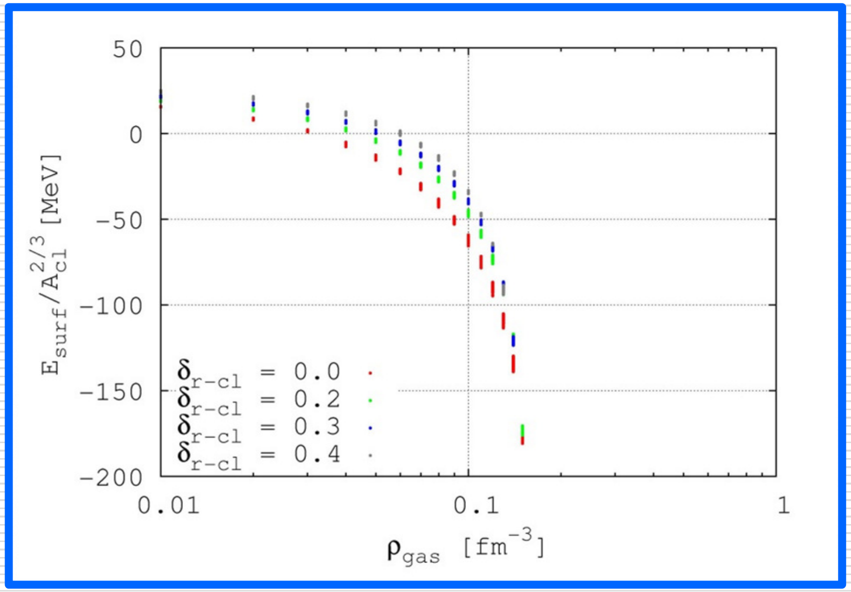
will be correct

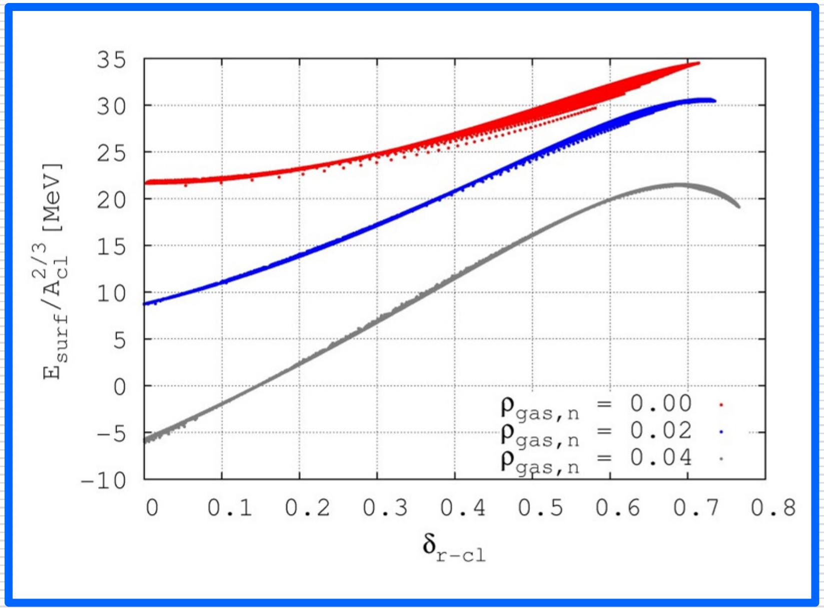
In-medium energies



- No modification of the bulk energy
  (accounted by the
  excluded volume term)

In-medium energies





Conclusions

- *Clusters d.o.f. essential to describe hot and dense stellar matter*
  - *Due to the Coulomb quenching of the LG phase transition*
  - ⇒ *Wide distribution of exotic clusters in stellar conditions*
  - ⇒ *Energetics very different from the vacuum*
  - *LDA modelization of in-medium effects with Sly4:*
    - *Decreasing surface energy with increasing ρ*
    - *Increasing surface-symmetry energy*
  - *Good reproduction of HF calculations in the WS cell*