Unité mixte de recherche
CNRS-IN2P3
Université Paris-Sud

91406 Orsay Cedex
http://ipnweb.in2p3.fr

TRAPPED (LIGHT) NUCLEAR STRUCTURE: "HARMONIC EFT"

U. van Kolck
Institut de Physique Nucléaire d'Orsay
and University of Arizona

with
B. Barrett (*Arizona*)
M. Birse (*Manchester*)
J. Rotureau (*Arizona*)
I. Stetcu (*Washington*)
C.J. Yang (*Arizona*)

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Outline

- Why
- (Pionless) Effective Field Theory
- Life in the Harmonic Box
- Trapped Fermions
- Liberated Nucleons
- Conclusion & Outlook

Game Plan

Q

$M_{QCD} \sim m_N, m_\rho, 4\pi f_\pi, \ldots$

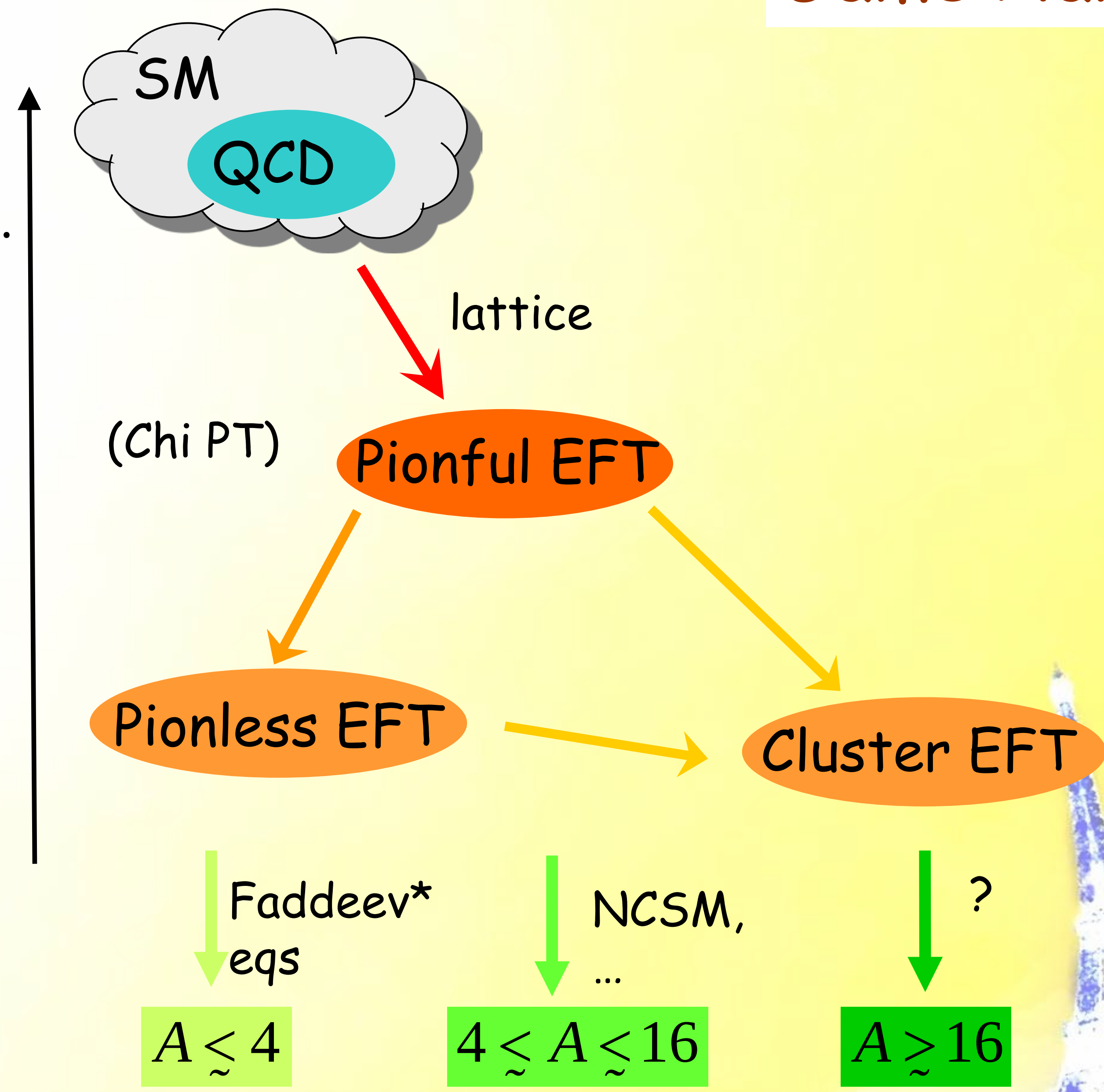
$\sim 1\,\text{GeV}$

$M_{nuc} \sim f_\pi, 1/r_{NN}, m_\pi, \ldots$

$\sim 100\,\text{MeV}$

$\aleph \sim 1/a_{NN}$

$\sim 30\,\text{MeV}$



Facts of Life

- there is *always** an underlying theory

  all interactions among low-energy d.o.f.s allowed by symmetries

- there is *always** a "model space"

  renormalization-group invariance to tame arbitrary UV cutoff

$$Q \sim m \ll M \begin{cases} T = T^{(\infty)}(Q) \sim N(M) \displaystyle\sum_{\nu=\nu_{\min}}^{\infty} \sum_i \tilde{c}_{\nu,i}(\Lambda) \left[\frac{Q}{M}\right]^{\nu} F_{\nu,i}\left(\frac{Q}{m};\frac{\Lambda}{m}\right) \\ \dfrac{\partial T}{\partial \Lambda} = 0 \end{cases}$$

normalization — parameters — non-analytic, from loops

"power counting"

truncate … $T = T^{(\bar{\nu})}\left\{1 + \mathcal{O}\left(\frac{Q}{M}, \frac{Q}{\Lambda}\right)\right\}$ ➡ want… $\Lambda \gtrsim M$

there are *always** such errors

* except, *maybe*, at the Planck scale

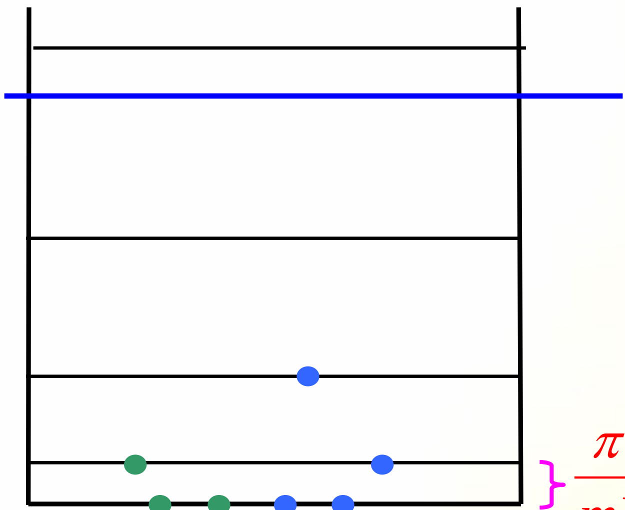
$A \gtrsim 4$

As A grows, given computational power limits number of accessible one-nucleon states

➡ IR cutoff in addition to UV cutoff

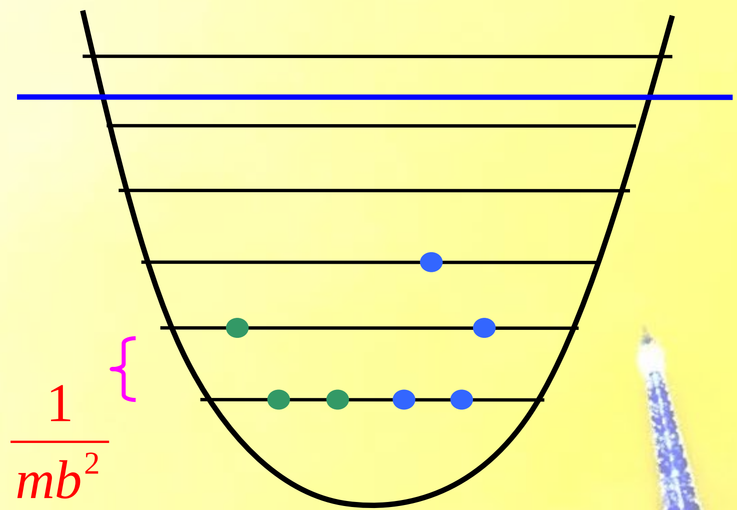
λ momentum Λ

Lattice Box



Harmonic-Oscillator Box

"No-Core Shell Model"



energy

$$\frac{N^2\pi^2}{mL^2} \qquad \frac{\Lambda^2}{2m} \qquad \frac{N_{\max}}{mb^2}$$

$$\frac{\pi^2}{mL^2} \qquad \frac{\lambda^2}{2m} \qquad \frac{1}{mb^2}$$

$L = Na$ $\qquad$ $b = \sqrt{2/m\omega}$

nuclear matter Müller *et al* '99

few nucleons Lee *et al* '05

…

finite nuclei Stetcu *et al* '06

…

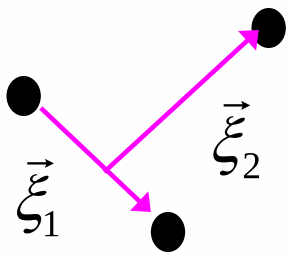
few atoms Stetcu *et al* '07

…

single particle $\phi_{nl(s)j}(\vec{r}) = N_{nl} b^{-3/2} \left(\frac{r}{b}\right)^{l} \exp\left(-r^2/2b^2\right) L_n^{(l+1/2)}\left(r^2/b^2\right) \left[Y_l(\hat{r}) \otimes \chi_s\right]_j$

generalized Laguerre polynomial

$A \leq 4$: internal (Jacobi) coordinates



$$\phi_{\{n\}}\left(\vec{\xi}_1, \vec{\xi}_2\right) = \mathcal{A}\left[\phi_{nlj}\left(\vec{\xi}_1\right)\phi_{n'l'j'}\left(\vec{\xi}_2\right)\right]_{JI}$$

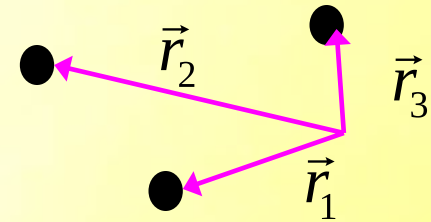
code a la

Navratil, Kamuntavicius + Barrett '00

reduced dimensions, but difficult antisymmetrization

HO Basis

$A \geq 3$: Slater-determinant



$$\phi_{\{n\}}\left(\vec{r}_1, \vec{r}_2, \vec{r}_3\right) = \begin{vmatrix} \phi_{n_1l_1j_1}(\vec{r}_1) & \phi_{n_2l_2j_2}(\vec{r}_1) & \phi_{n_3l_3j_3}(\vec{r}_1) \\ \phi_{n_1l_1j_1}(\vec{r}_2) & \phi_{n_2l_2j_2}(\vec{r}_2) & \phi_{n_3l_3j_3}(\vec{r}_2) \\ \phi_{n_1l_1j_1}(\vec{r}_3) & \phi_{n_2l_2j_2}(\vec{r}_3) & \phi_{n_3l_3j_3}(\vec{r}_3) \end{vmatrix}$$

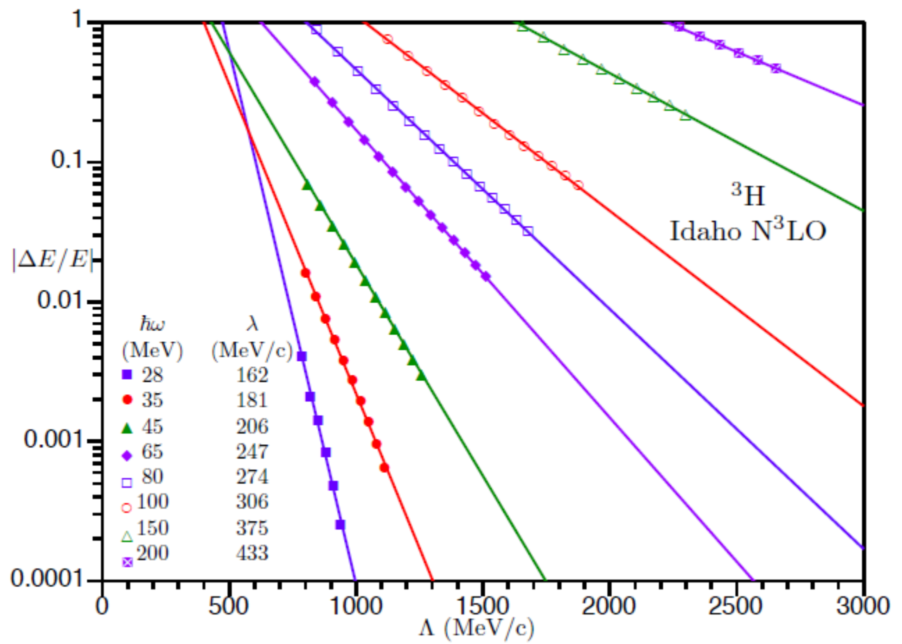
code: REDSTICK Ormand '05

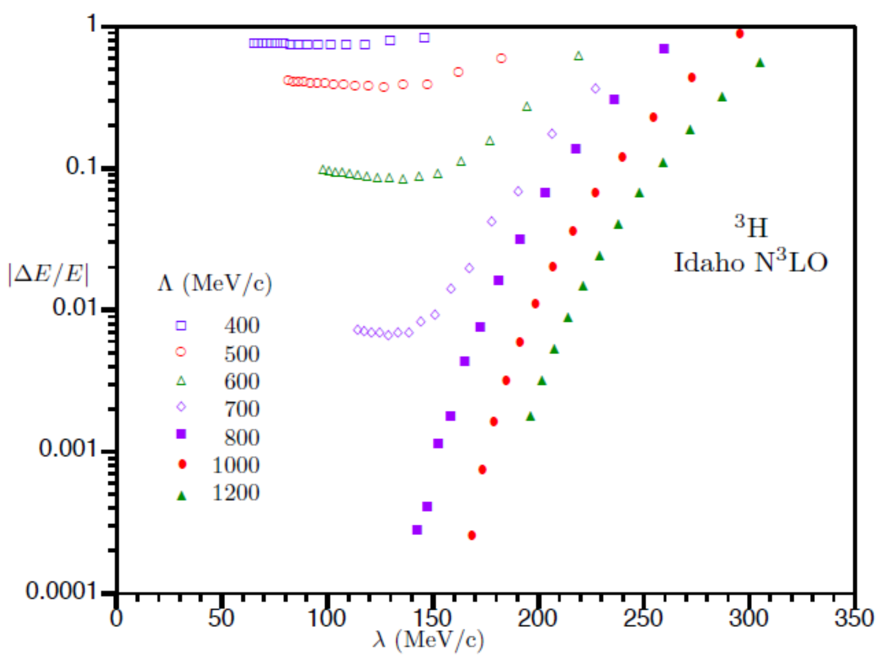
maximum number of excitations

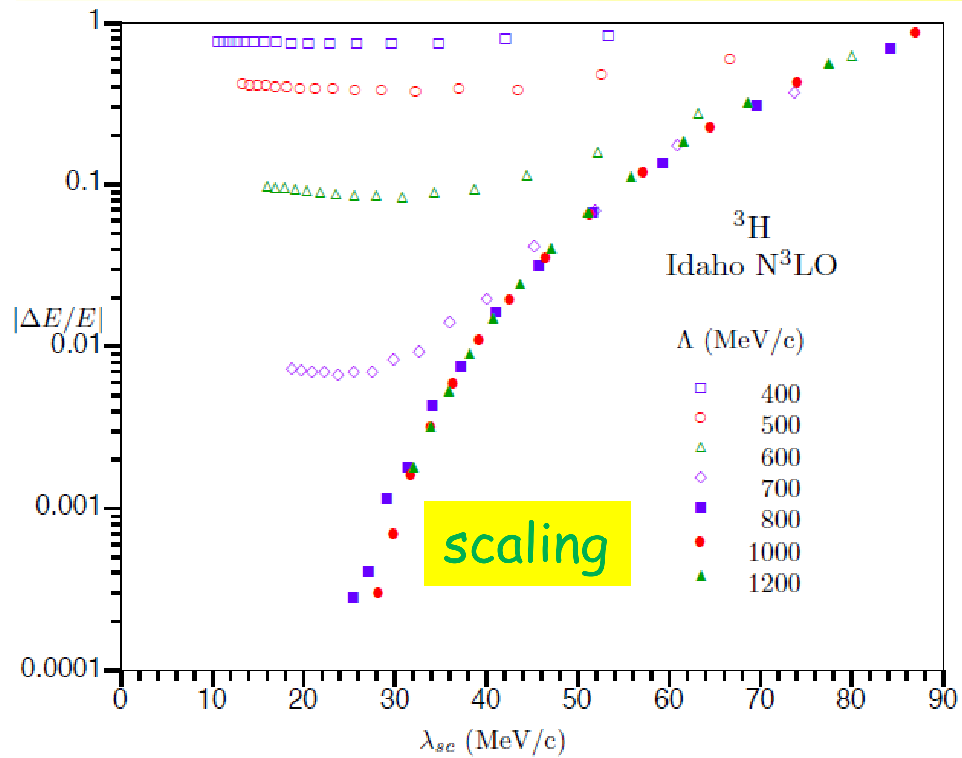
$$\psi_A(\vec{r}) = \sum_{\{n\}}^{N_{max}} A_{\{n\}} \phi_{\{n\}}(\vec{r})$$

Extrapolations in a HO basis

Coon, Avetian, Kruse, Maris, Vary + v.K., '12







$$= \frac{\lambda^2}{\Lambda}$$

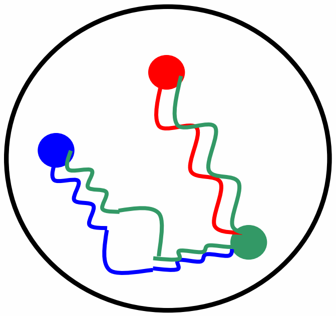
see also

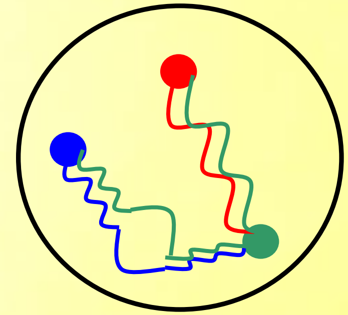
Furnstahl, Hagen + Papenbrock '12

Any EFT will do; for simplicity, start with pionless.

$r_{NN} \sim 1\ \text{fm}$

deuteron





$a_{NN} \sim 1/\aleph \cong 4.5\ \text{fm}$

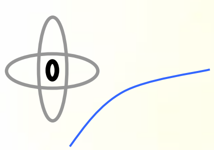
QCD: $SU(3)$ gauge theory of quarks

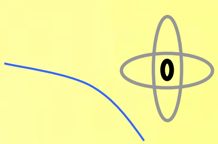
cf.

QED: $U(1)$ gauge theory of electrons and nuclei

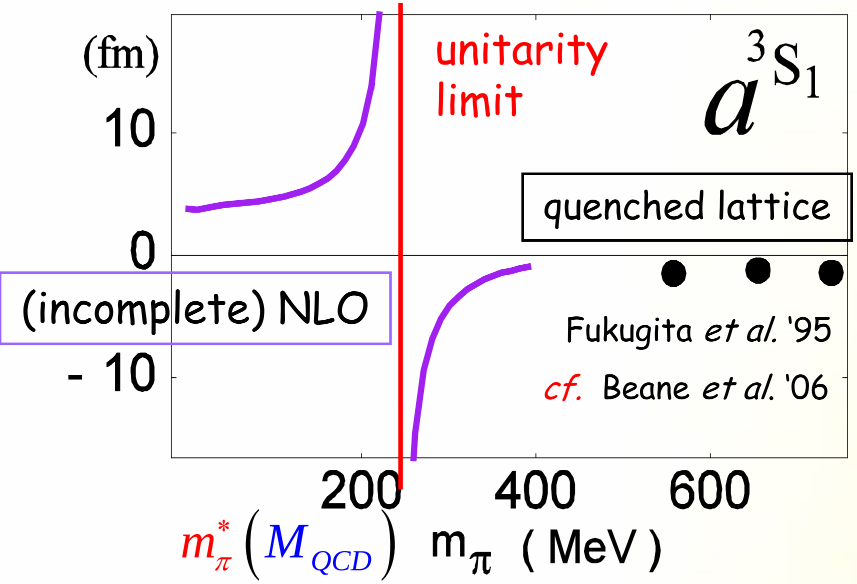
$r_{\text{He4He4}} \sim 5\ \mathring{\text{A}}$

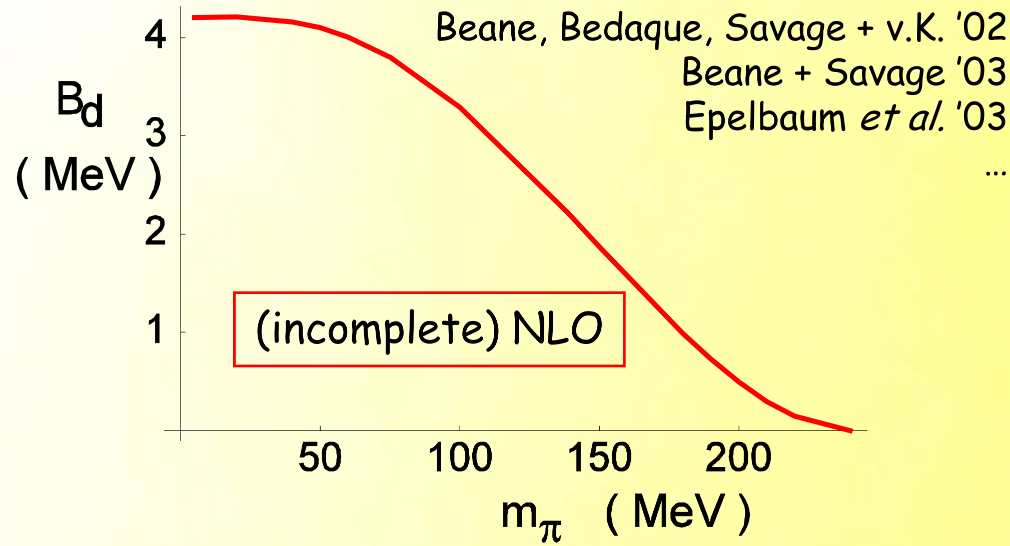
He4 dimer





$a_{\text{He4He4}} \sim 1/\aleph \cong 125\ \mathring{\text{A}}$

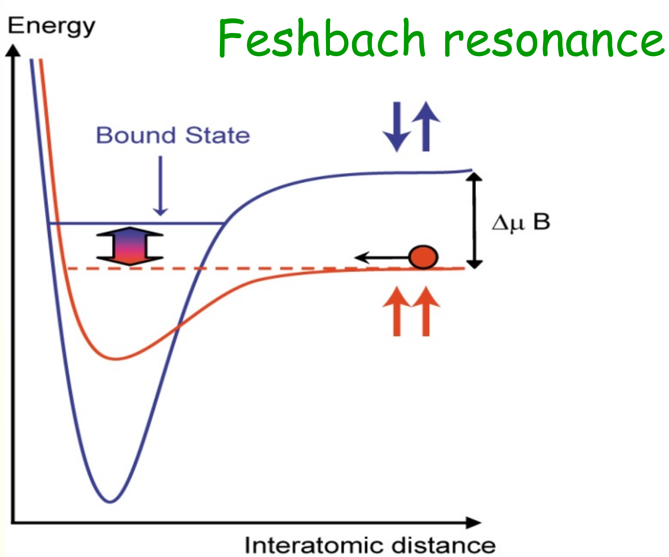


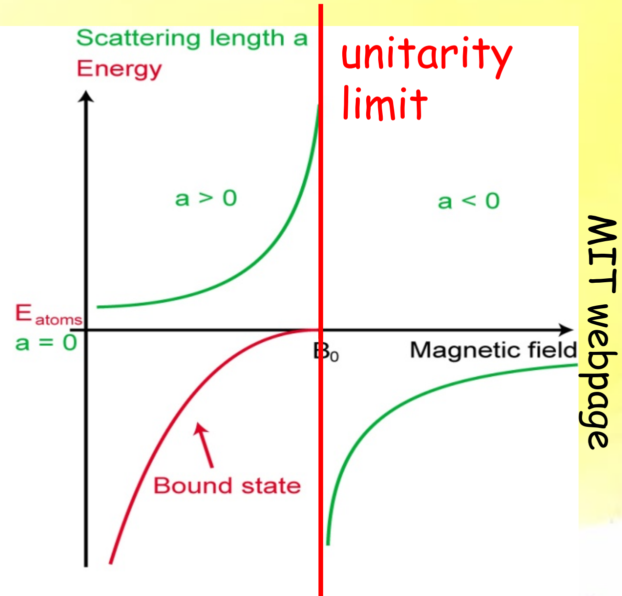


QCD near a
Feshbach resonance
in pion mass

Scale $\aleph \sim \frac{m_\pi - m_\pi^*}{m_\pi^*} M_{nuc}$ emerges

cf.
atoms as
magnetic field
varies





$Q \sim \aleph \ll M_{nuc}$

contact EFT

- degrees of freedom: nucleons
- symmetries: Lorentz, $\not{P}$, $\not{T}$
- expansion in: $\dfrac{Q}{M_{nuc}} = \begin{cases} Q/m_N & \text{non-relativistic} \\ Q/m_\pi, \cdots & \text{multipole} \end{cases}$ $\sim \dfrac{1}{3}$

Universality:
first orders apply also to atoms $M_{nuc} \to 1/l_{vdW}$ *where* $V(r) = -\dfrac{l_{vdW}^4}{2mr^6} + \ldots$

$$\mathcal{L}_{EFT} = N^+\left(i\partial_0 + \frac{\nabla^2}{2m_N}\right)N + C_0 N^+N\,N^+N + D_0 N^+N\,N^+N\,N^+N$$
$$+ N^+\frac{\nabla^4}{8m_N^3}N + C_2 N^+N\left(N^+\nabla^2 N + \text{H.c.}\right) + \ldots$$

(omitting spin, isospin)

two-body sector ~ effective-range expansion

v.K. '97 '99
Kaplan, Savage + Wise '98
Gegelia '98
...

$$V_{ij} = \underbrace{C_0 \delta^{(3)}\left(\vec{r}_i - \vec{r}_j\right)}_{\text{LO}} + \underbrace{C_2\left[\nabla^2 \delta^{(3)}\left(\vec{r}_i - \vec{r}_j\right) + \ldots\right]}_{\text{NLO}} + \underbrace{C_4\left[\nabla^4 \delta^{(3)}\left(\vec{r}_i - \vec{r}_j\right) + \ldots\right]}_{\text{NNLO}} + \ldots$$

LO: a_2

NLO: a_2, r_2

NNLO: a_2, r_2

v.K. '97
Kaplan, Savage + Wise '98
Gegelia '98

$$V_{ijk} = \underbrace{D_0 \delta^{(3)}\left(\vec{r}_i - \vec{r}_j\right)\delta^{(3)}\left(\vec{r}_j - \vec{r}_k\right)}_{\text{LO}} + \underbrace{D_2\left[\nabla^2 \delta^{(3)}\left(\vec{r}_i - \vec{r}_j\right)\delta^{(3)}\left(\vec{r}_j - \vec{r}_k\right) + \ldots\right]}_{\text{NNLO}} + \ldots$$

LO: a_3

NNLO: a_3, r_3

Bedaque, Hammer + v.K. '99
Hammer + Mehen '00
...

$$V_{ijkl} = \underbrace{E_0 \delta^{(3)}\left(\vec{r}_i - \vec{r}_j\right)\delta^{(3)}\left(\vec{r}_j - \vec{r}_k\right)\delta^{(3)}\left(\vec{r}_k - \vec{r}_l\right)}_{\text{not LO, NLO}} + \ldots$$

not LO, NLO

Platter, Hammer + Meissner '04
Griesshammer, Hoffmann + Kirschner '09

Untrapped nucleons

LO

$$H_A^{(0)} = \frac{1}{2m_N A}\sum_{[i<j]}\left(\vec{p}_i - \vec{p}_j\right)^2 + C_{0[0]}\sum_{[i<j]_0}\delta^{(3)}\left(\vec{r}_i - \vec{r}_j\right)$$

$S=0$ pairs

$$+C_{0[1]}\sum_{[i<j]_1}\delta^{(3)}\left(\vec{r}_i - \vec{r}_j\right) + D_0\sum_{[i<j<k]}\delta^{(3)}\left(\vec{r}_i - \vec{r}_j\right)\delta^{(3)}\left(\vec{r}_j - \vec{r}_k\right)$$

$S=1$ pairs $S=1/2$ triplets

EFT PC effectively justifies (modified) cluster approximation

$$H_A^{(0)}\psi_A^{(0)}\left(\vec{r}\right) = E_A^{(0)}\psi_A^{(0)}\left(\vec{r}\right)$$

Stetcu, Barrett +v.K., '07

parameters fitted to d, t, a ground-state energies
predicted 4He excited, 6Li ground energies

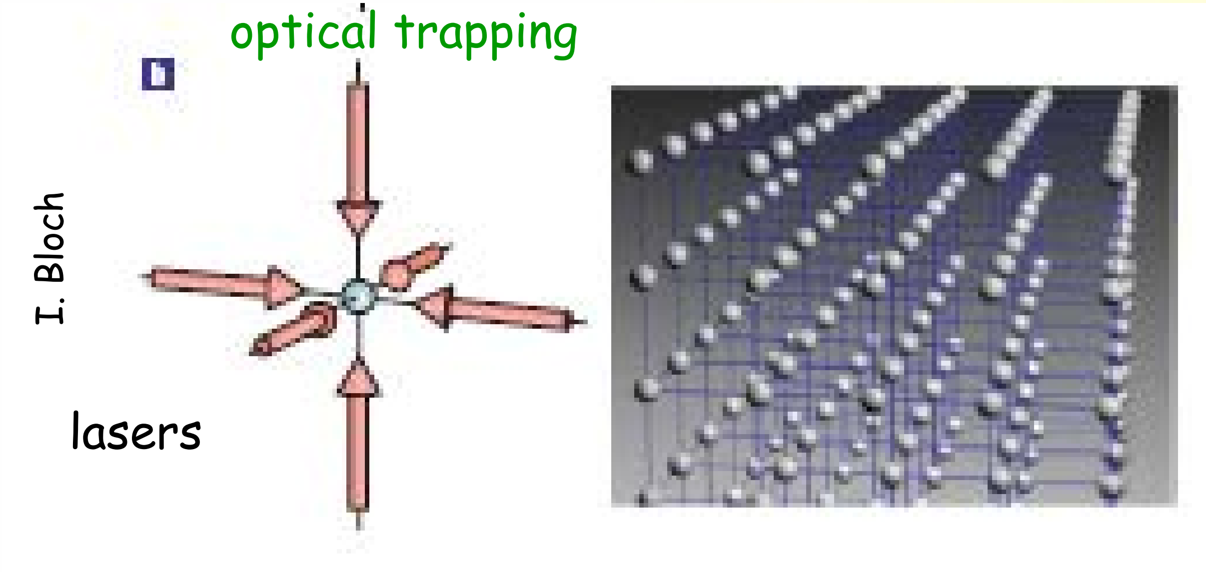
works within ~30%

but parameters proliferate: *e.g.*, at NLO two more 2-body parameters
can we fit them to scattering data?

$C_{2[0]}(\Lambda)$, $C_{2[1]}(\Lambda)$

Yes, trap them!

Trapped fermions

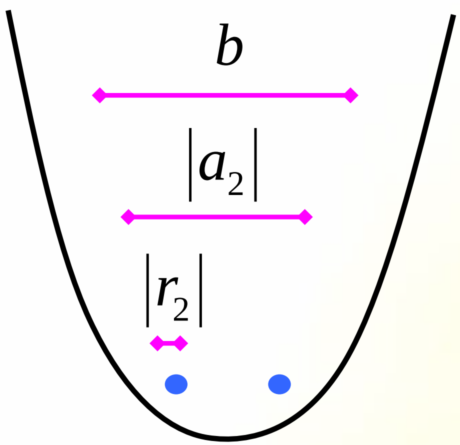


$$V(\vec{r}) \propto \alpha(\omega_L)\left|\vec{E}(\vec{r})\right|^2$$
$$\propto \sum_i \sin^2(k_L r_i)$$
$$\approx k_L^2 \vec{r}^{\,2}$$

standing waves

low-tunneling regime

(band insulator)



$$\frac{b}{|r_2|} \gg 1$$ universal behavior

$$\frac{b}{|a_2|}\begin{cases}\to \infty \\ \lesssim 1 \\ \to 0\end{cases}$$

- $\to \infty$: untrapped limit
- $\lesssim 1$: significant trap effects
- $\to 0$: only low-energy scale given by b; some semi-analytical results known

test our method

Rotureau, Stetcu, Barrett + v.K., '09

Life in the Box

$$H_A = \frac{\omega}{2}\left\{\sum_{i=1}^{A}\left[\frac{1}{2}b^2 p_i^2 + 2\frac{r_i^2}{b^2}\right] + 2\mu_2 b^2 V\left(\left\{\vec{r}_i - \vec{r}_j\right\}\right)\right\} = H_A^{(cm)} + H_A^{(rel)}$$

two-body reduced mass $\mu_2 = m/2$

S waves only in LO

LO $$H_A^{(0)}\left|\psi_A^{(0)}\right\rangle = E_A^{(0)}\left|\psi_A^{(0)}\right\rangle$$

NLO $$E_A^{(1)} = \left\langle\psi_A^{(0)}\left|V_A^{(1)}\right|\psi_A^{(0)}\right\rangle$$

NNLO $$E_A^{(2)} = \left\langle\psi_A^{(0)}\left|V_2^{(2)}\right|\psi_A^{(0)}\right\rangle + \frac{1}{2}\left\{\left\langle\psi_A^{(0)}\left|V_2^{(1)}\right|\psi_A^{(1)}\right\rangle + \left\langle\psi_A^{(1)}\left|V_2^{(1)}\right|\psi_A^{(0)}\right\rangle\right\}$$

etc.

$A=2$

Stetcu, Barrett, Vary + v.K. '08
Rotureau, Stetcu, Barrett + v.K. '10

LO $H_2^{(0)}\left|\psi_2^{(0)}\right\rangle = E_2^{(0)}\left|\psi_2^{(0)}\right\rangle$

$$\Rightarrow \frac{2\pi b}{\mu_2 C_0^{(0)}\left(N_{2\max},\omega\right)} = -\frac{2}{\pi^{1/2}}\sum_{n=0}^{N_{2\max}/2}\frac{L_n^{(1/2)}(0)}{2n+3/2-\left(E_2^{(0)}/\omega\right)}$$

input one $\frac{E_2^{(0)}}{\omega} = \frac{E_2^{(0)}}{\omega}\left(\frac{b}{a_2}\right)$ ⇨ determine $C_0^{(0)}\left(N_{2\max},\omega\right)$ ⇨ calculate other levels

e.g. lowest level

NLO $E_2^{(1)} = \left\langle\psi_2^{(0)}\right|V_2^{(1)}\left|\psi_2^{(0)}\right\rangle = \ldots$

input second level ⇨ determine $C_2^{(1)}\left(N_{2\max},\omega\right)$ ⇨ calculate other levels

NNLO $E_2^{(2)} = \left\langle\psi_2^{(0)}\right|V_2^{(2)}\left|\psi_2^{(0)}\right\rangle + \frac{1}{2}\left\{\left\langle\psi_2^{(0)}\right|V_2^{(1)}\left|\psi_2^{(1)}\right\rangle + \left\langle\psi_2^{(1)}\right|V_2^{(1)}\left|\psi_2^{(0)}\right\rangle\right\} = \ldots$

input third level ⇨ determine $C_4^{(2)}\left(N_{2\max},\omega\right)$ ⇨ calculate other levels

etc.

Where do levels come from?

$$N_{2\max} \to \infty$$

$$\psi_2(0<r\ll b) \propto \frac{1}{r}\left\{1-2\frac{\Gamma\left(\frac{3}{4}-\frac{E_2}{2\omega}\right)}{\Gamma\left(\frac{1}{4}-\frac{E_2}{2\omega}\right)}\frac{r}{b}+\mathcal{O}\left(\frac{r^2}{b^2}\right)\right\}$$

$$=\left[1-\mu a_2 r_2 E+\ldots\right]\frac{r}{a_2}$$

$$\Rightarrow \quad \frac{\Gamma\left(\frac{3}{4}-\frac{E_2}{2\omega}\right)}{\Gamma\left(\frac{1}{4}-\frac{E_2}{2\omega}\right)}=\frac{b}{2a_2}\left\{1-\frac{a_2 r_2}{b^2}\frac{E_2}{\omega}+\ldots\right\}$$

LO

NLO, NNLO

Busch *et al.* '98
Blume + Greene '02
Block + Holthaus '02
Bolda, Tiesinga + Julienne '02
...

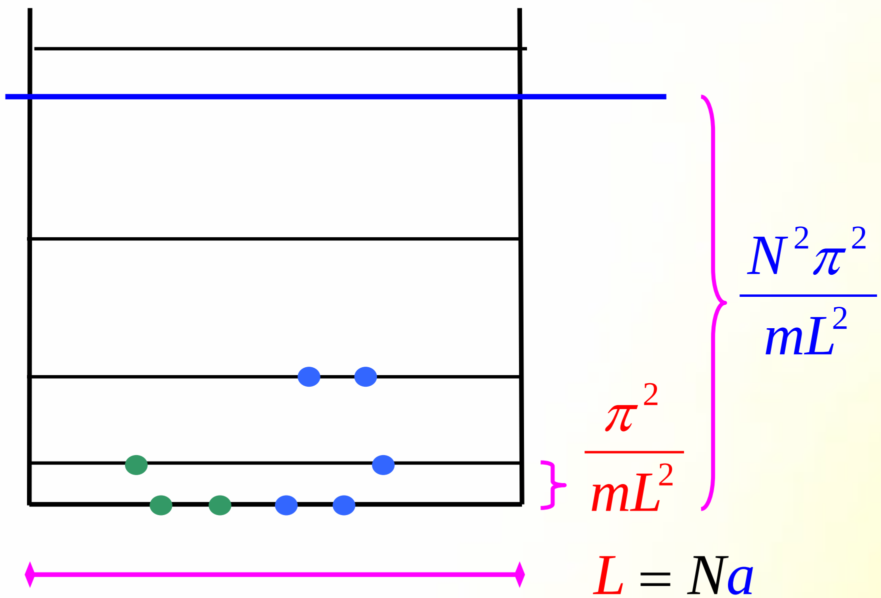
$$\frac{b}{a_2}\to\infty$$

$$\begin{cases} \dfrac{E_{2,0}}{\omega}=-\dfrac{b^2}{a_2^2}+\ldots & \text{untrapped bound state} \\ \dfrac{E_{2,n}}{\omega}=-\dfrac{1}{2}+2n+\ldots \quad (n=1,2,\ldots) & \text{scattering states} \end{cases}$$

Lattice EFT

Lattice Box

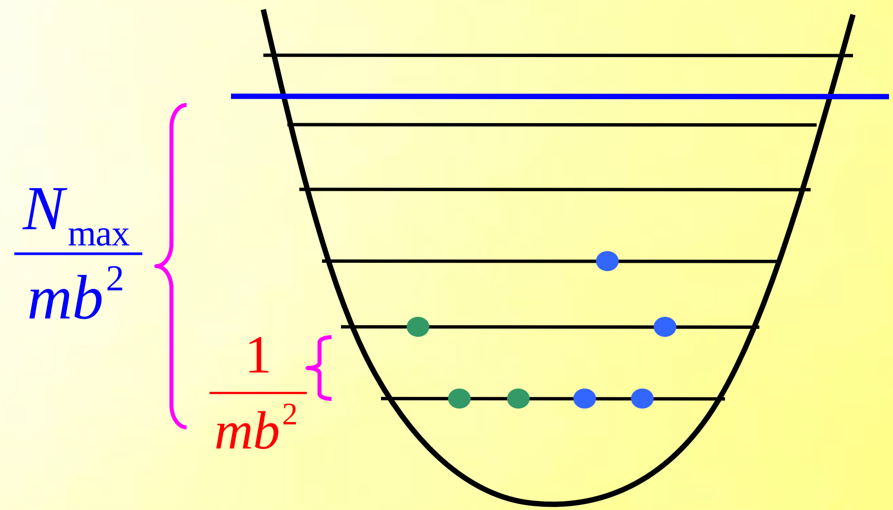
cf. Fukuda + Newton '54



$L = Na$

Harmonic EFT

Harmonic-Oscillator Box



$b = \sqrt{2/m\omega}$

Parameters fitted to *E* from

$$2N - 2\pi \sum_{\mathbf{n}}^{|\mathbf{n}|<N} \frac{1}{(2\pi\mathbf{n})^2 - mEL^2} = -\frac{\sqrt{mEL^2}}{2}\cot\delta(E)$$

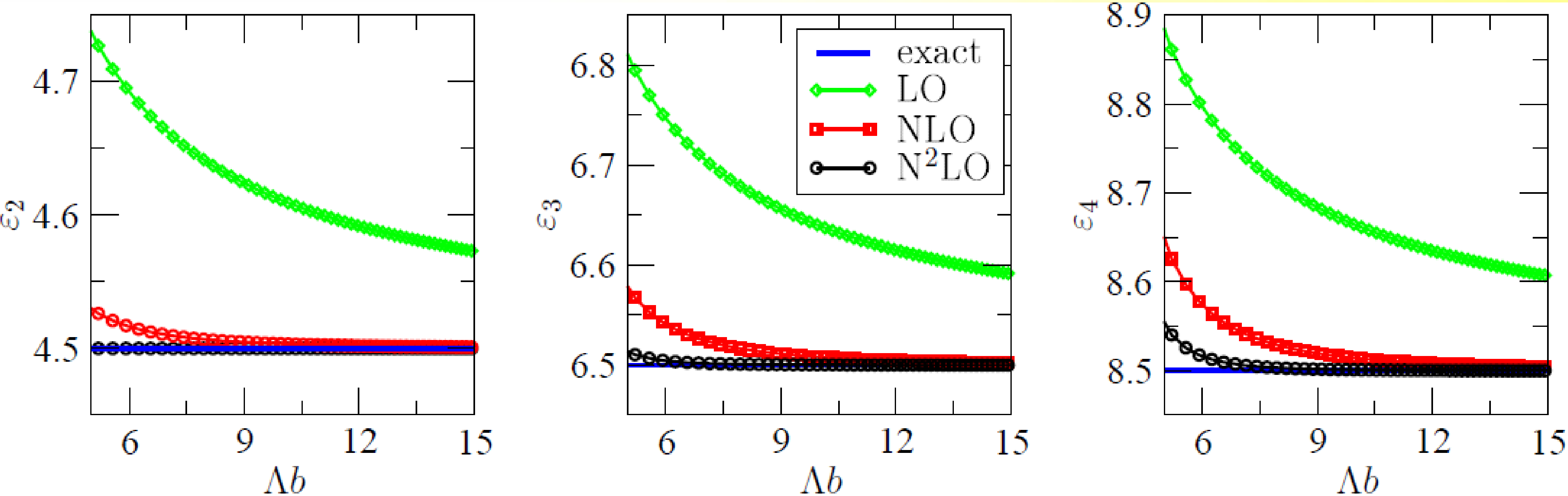
Luescher '91

$$\frac{\Gamma\left(\frac{3}{4} - \frac{mEb^2}{2}\right)}{\Gamma\left(\frac{1}{4} - \frac{mEb^2}{2}\right)} = -\frac{\sqrt{mEb^2}}{2}\cot\delta(E)$$

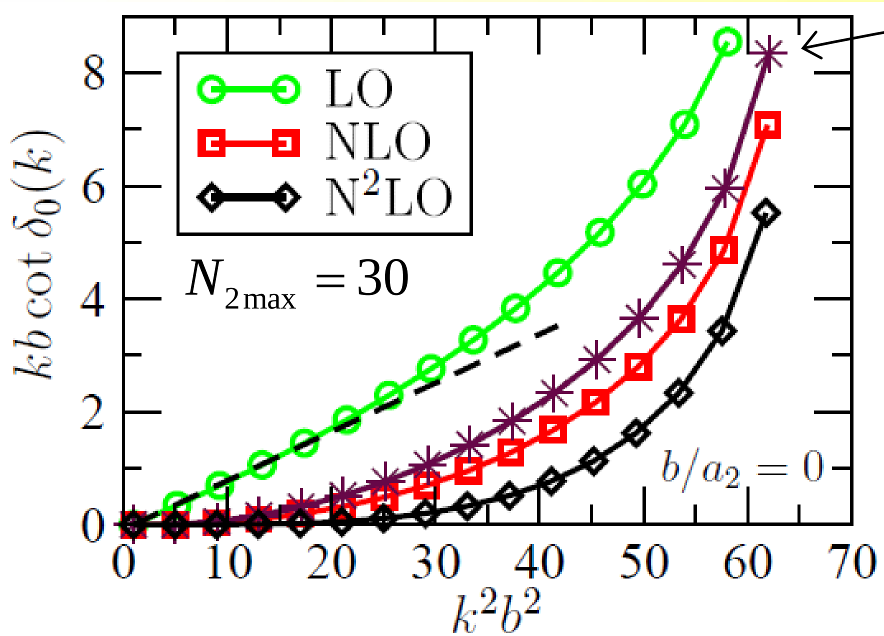
Busch *et al.* '98

at unitarity

Stetcu, Barrett, Vary + v.K. '08
Rotureau, Stetcu, Barrett + v.K. '10



$\varepsilon_n \equiv E_{2,n} / \omega$



NNLO Hamiltonian fully diagonalized: worse than NLO!

cf. Luu *et al*. '10

Rotureau, Stetcu, Barrett, Birse + v.K. '10

$A \geq 3$ include few-body forces

$$N_{A\max} \geq N_{2\max} \begin{cases} 1) \quad N_{A\max} \gg N_{2\max} & \Rightarrow E_A = E_A\left(N_{2\max}, \omega\right) \\ 2) \quad N_{2\max} \gg 1 & \end{cases}$$

$\frac{b}{a_2} \to \infty$

- lowest states: free-space bound states
  - binding energy info $B_{A,0} = -E_{A,0}, \ldots$
- other states: scattering states
  - phase-shift info, for example:

$$\frac{\Gamma\left(\frac{3}{4} - \frac{E_{A,n} - E_{A-1,0}}{2\omega}\right)}{\Gamma\left(\frac{1}{4} - \frac{E_{A,n} - E_{A-1,0}}{2\omega}\right)} = -\sqrt{\frac{E_{A,n} - E_{A-1,0}}{2\omega}} \cot \delta_{1,A-1} \left(\frac{2}{b_{1,A-1}} \sqrt{\frac{E_{A,n} - E_{A-1,0}}{2\omega}}\right)$$

$$b_{1,A-1} = \frac{1}{\sqrt{\mu_{1,A-1}\omega}}$$

S-wave phase shift for particle/lighter b.s. scattering

Trapped two-component fermions: $S = 1/2$

$$V = \sum_{[i<j]_0} \left\{ C_0 \delta^{(3)}\left(\vec{r}_i - \vec{r}_j\right) + C_2 \left[\nabla^2 \delta^{(3)}\left(\vec{r}_i - \vec{r}_j\right) + \ldots\right] + C_4 \left[\nabla^4 \delta^{(3)}\left(\vec{r}_i - \vec{r}_j\right) + \ldots\right] \right\}$$

$S = 0$ pairs

S wave only in LOs

up to NNLO

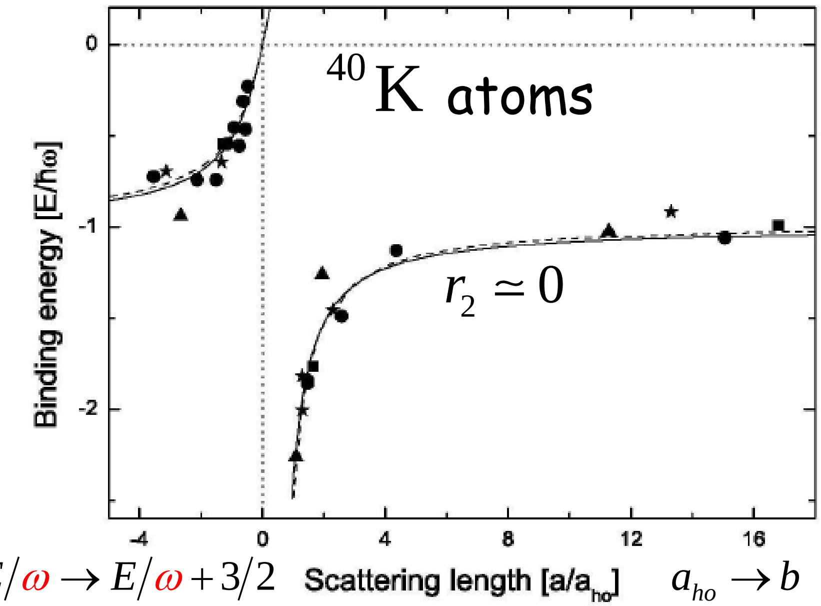
no $\begin{cases} S = 1 \text{ two-body force} \\ \text{three-body force in LOs} \end{cases}$

+ HO is physics

$A = 2$ fit to data *e.g.*

$A \geq 3$ no fit

Stoeferle et al (ETH) '06



$E/\omega \to E/\omega + 3/2$ $\quad a_{ho} \to b$

$\frac{b}{a_2} \to -\infty$ $\quad \frac{E_A}{\omega} =$ filling of HO shells

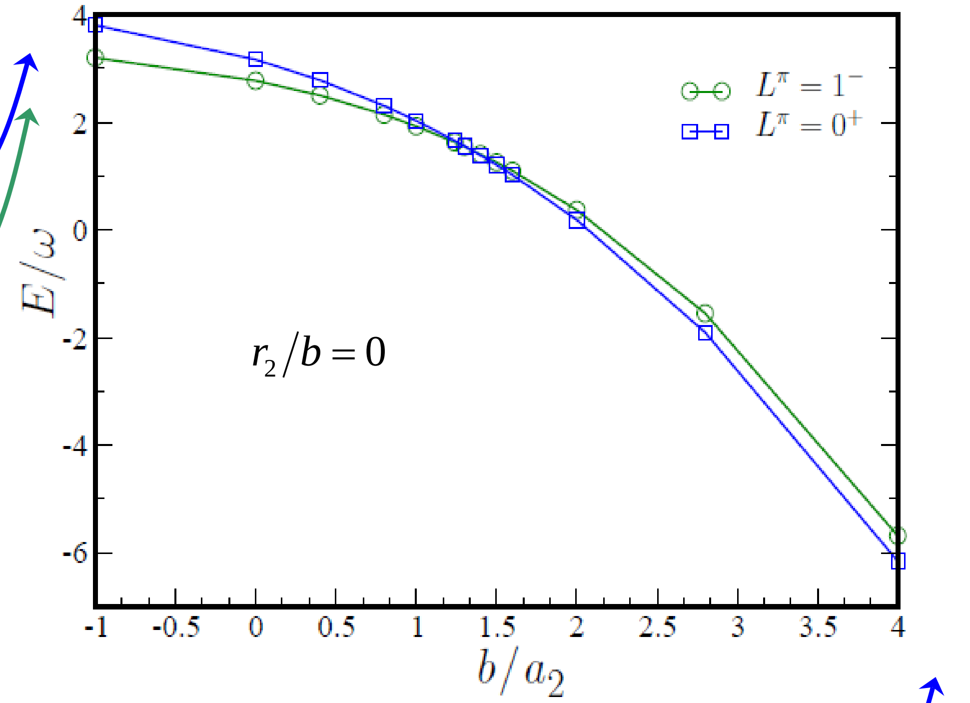
$\frac{b}{|a_2|} \to 0$ $\quad \frac{E_A}{\omega} = \varepsilon_A\left(N_{2\max}\right)$ (independent of ω since *b* only scale)

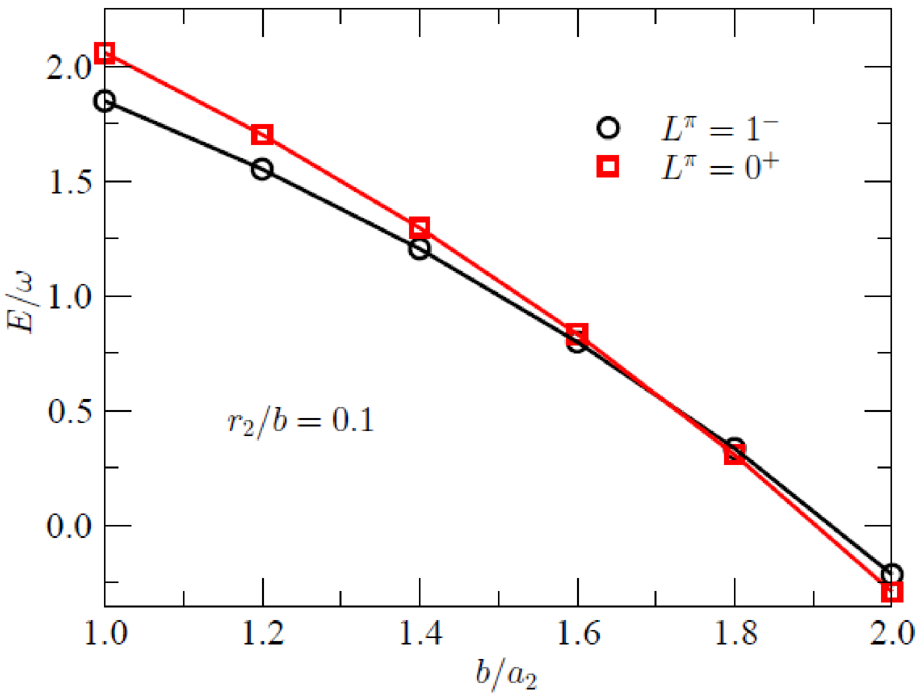
Stetcu, Barrett, Vary + v.K., '07
Kerstner + Duan '07
Rotureau, Stetcu, Barrett, Birse + v.K. '10

$$\frac{E_3}{\omega} \to \begin{cases} 5 & 1S2P \\ 4 & 2S1P \end{cases}$$

$A = 3$

inversion of g.s. parity!





$$\frac{E_3}{\omega} \approx -\frac{b^2}{a_2^2}$$

$(\text{atom+dimer})_{S\ \text{wave}}$

NNLO

cliparts.com

Rotureau, Stetcu, Barrett, Birse + v.K. '10 cf. von Stecher, Blume + Greene '08

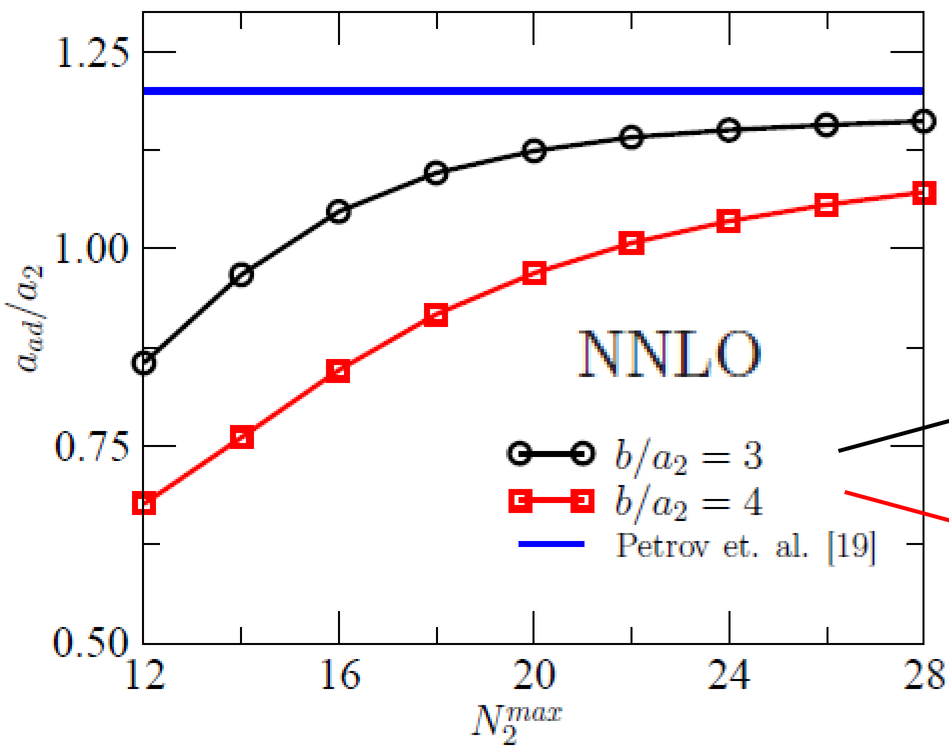
$$\frac{\Gamma\left(\frac{3}{4}-\frac{E_{3,n}-E_{2,0}}{2\omega}\right)}{\Gamma\left(\frac{1}{4}-\frac{E_{3,n}-E_{2,0}}{2\omega}\right)}=\frac{b_{1,2}}{2a_{ad}}-\frac{r_{ad}}{b_{1,2}}\frac{E_{3,n}-E_{2,0}}{2\omega}+\ldots$$

3-body energy above dimer g.s.

$$b_{1,2}=\frac{1}{\sqrt{\mu_{1,2}\omega}}$$

use two levels, eliminate r_{ad} :

$A=3$



better precision at smaller cutoffs

better dimer inside trap

Liberated nucleons

add $\begin{cases} S=1 \text{ two-body force} \\ \text{three-body force in LOs} \end{cases}$

+ HO is <u>not</u> physics

$$V = \sum_{S=0,1} \sum_{[i<j]_S} \left\{ C_{0[S]}\, \delta^{(3)}\left(\vec{r}_i - \vec{r}_j\right) + C_{2[S]} \left[\nabla^2 \delta^{(3)}\left(\vec{r}_i - \vec{r}_j\right) + \ldots \right] \right\}$$

$$+ D_0 \sum_{[i<j<k]} \delta^{(3)}\left(\vec{r}_i - \vec{r}_j\right) \delta^{(3)}\left(\vec{r}_j - \vec{r}_k\right)$$

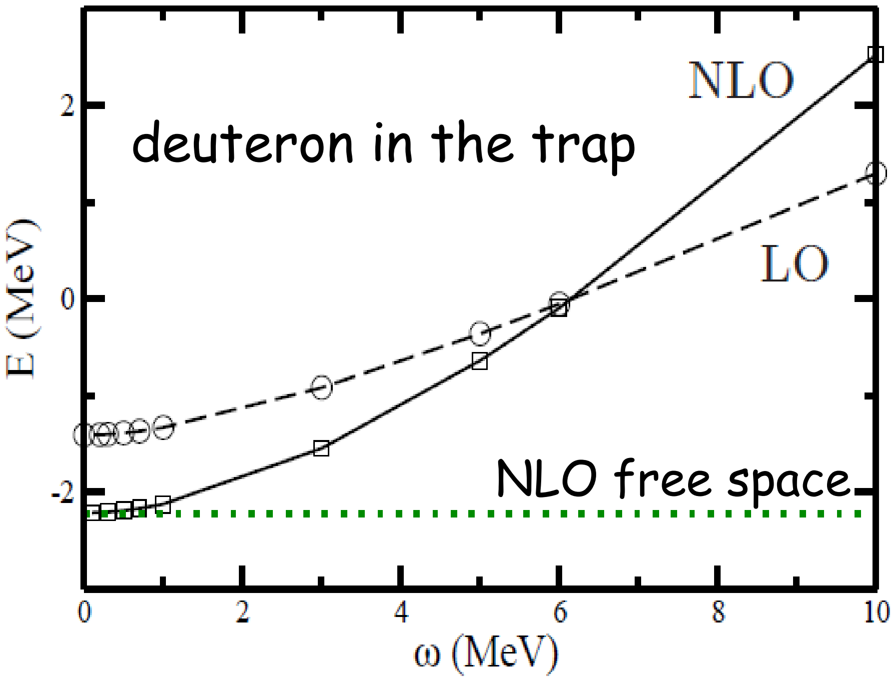
single parameter

$S = 1/2$ triplets

S wave only

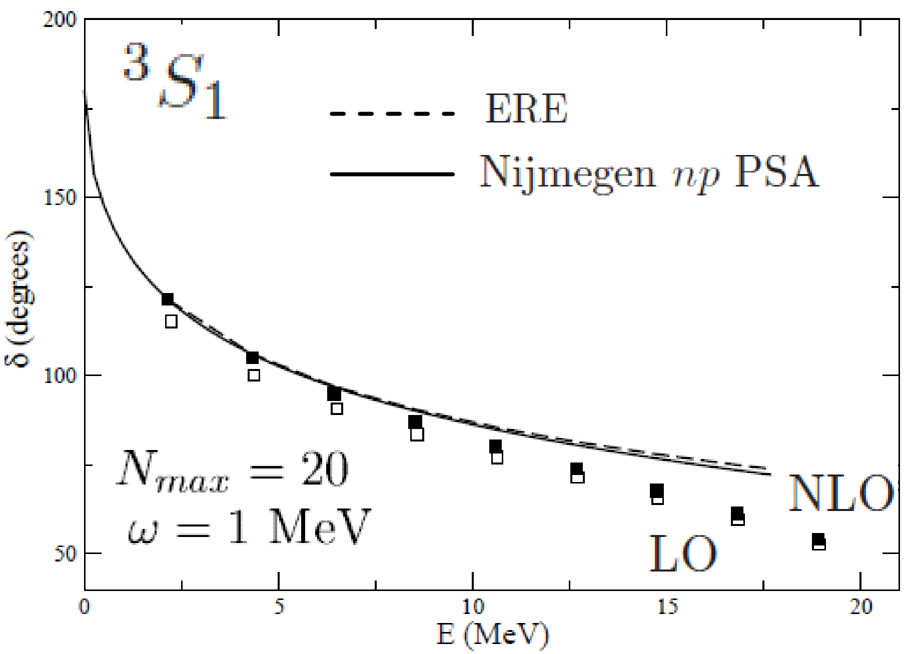
up to NLO

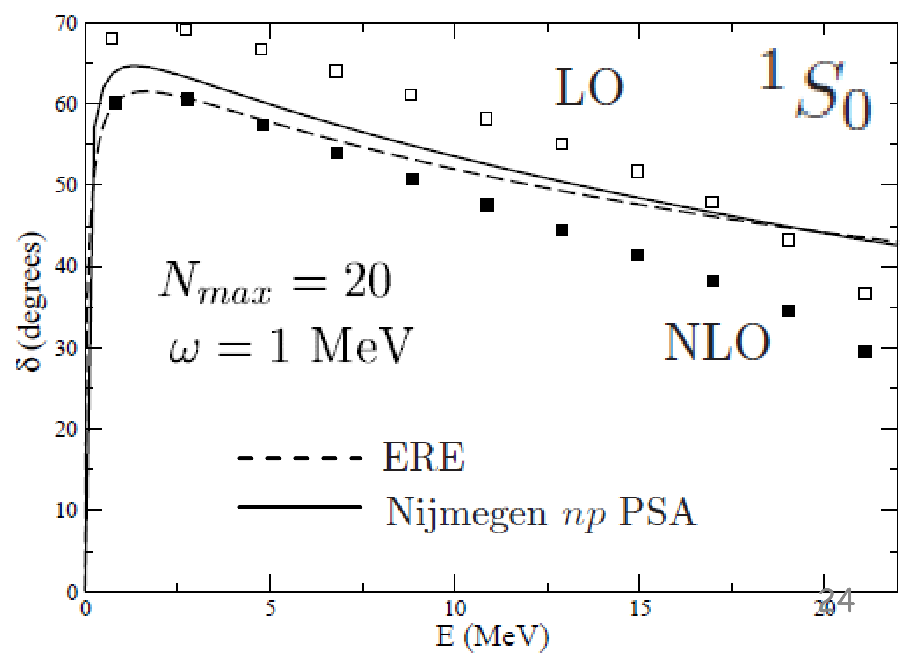
Rotureau, Stetcu, Barrett + v.K. '11



$A = 2$

NLO





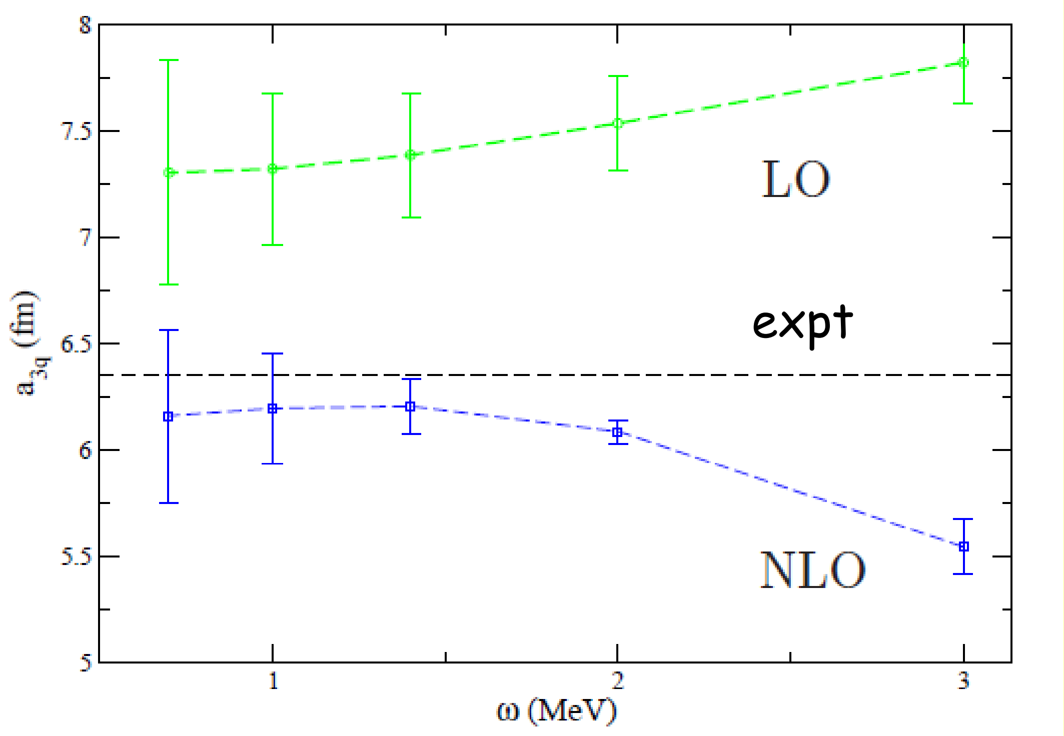
Rotureau, Stetcu, Barrett + v.K. '11

$A = 3$

$I = 1/2,\ J^{\pi} = 3/2^{+}$

$^{4}a_{3} = 6.35 \pm 0.02$ fm

Dilg *et al.* '71



cf. NNLO in free space

$^{4}a_{3} = 6.33 \pm 0.10$ fm

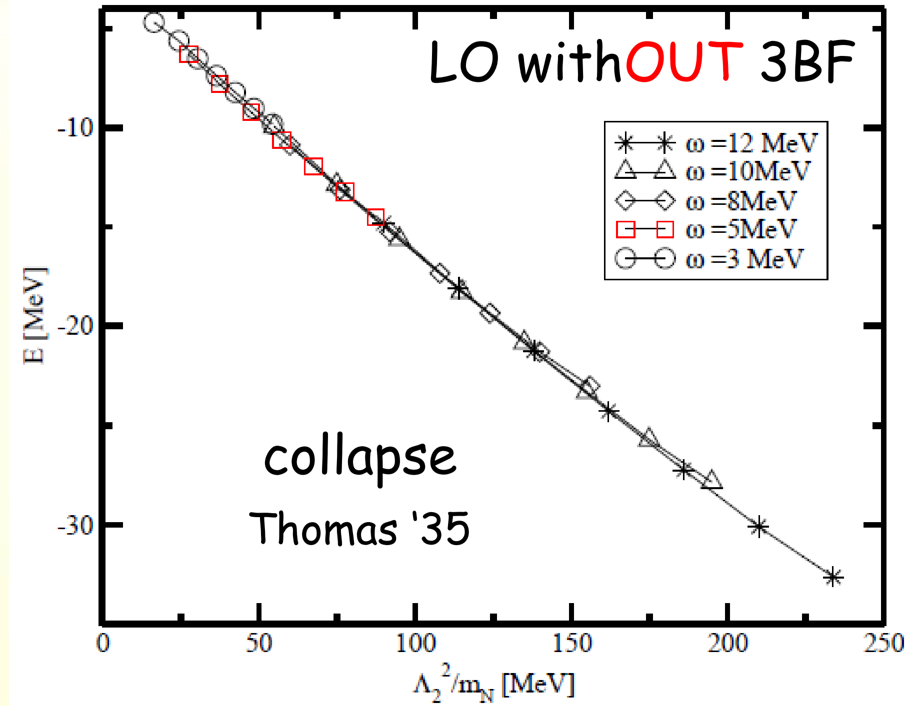
Bedaque, Hammer + v.K. '98

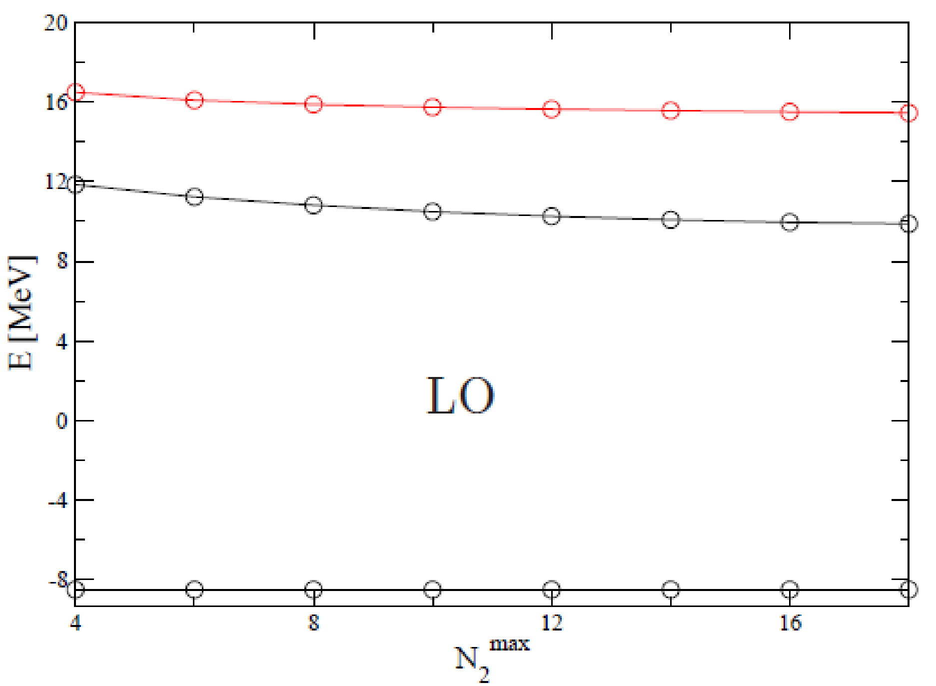
$A = 3$ $\quad I = 1/2,\ J^{\pi} = 1/2^{+}$

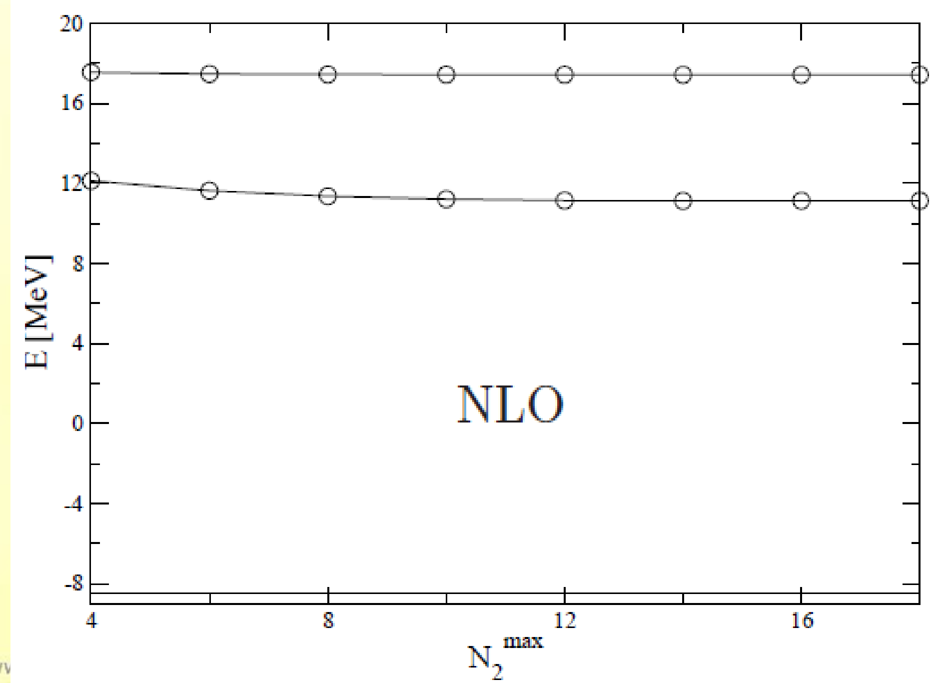
similar for bosons

Toelle, Hammer + Metsch '10

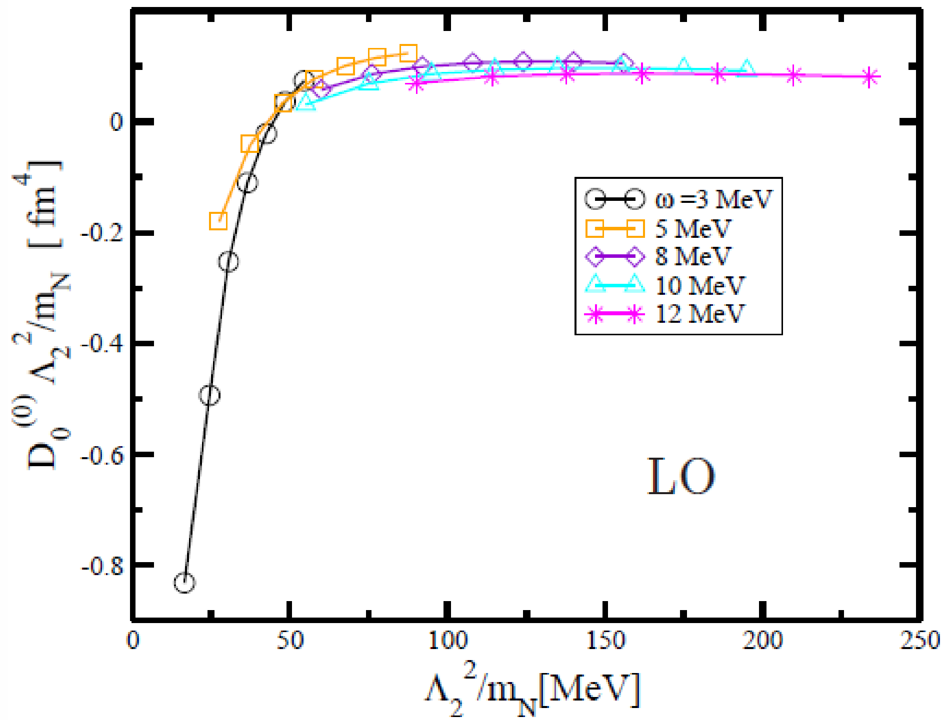
fit 3BF to triton BE







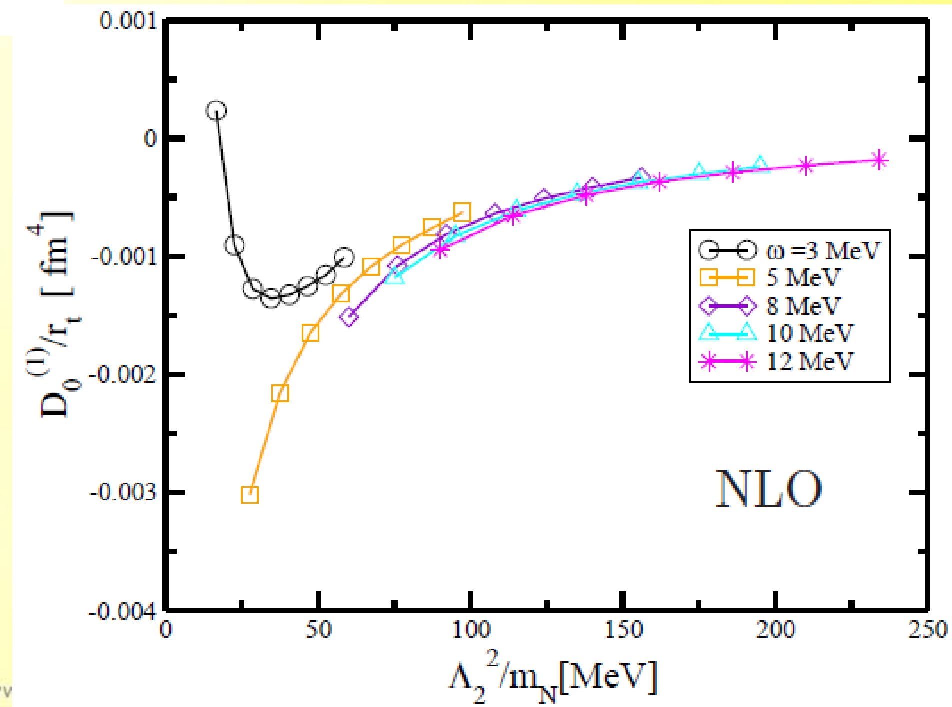
Rotureau, Stetcu, Barrett + v.K. '11



fit 3BF to triton BE

cf. limit cycle

Bedaque, Hammer + v.K. '99



What next?

- other ERE parameters in Nd scattering
- four- and more-nucleon systems
- pionful EFT

$Q \sim m_\pi \ll M_{QCD}$

Pionful EFT

- degrees of freedom: nucleons, pions, deltas (+ Roper + ?)

$$m_\Delta - m_N \sim 2m_\pi \quad \left(m_{N'} - m_N \sim 3m_\pi, \ldots\right)$$

- symmetries: Lorentz, P̸, T̸, chi̸ral

$$D_\mu = \frac{1}{1+\boldsymbol{\pi}^2/4f_\pi^2}\partial_\mu \qquad \mathcal{D}_\mu = \partial_\mu + \frac{i}{2f_\pi^2}\left(\boldsymbol{\pi}\times D_\mu\boldsymbol{\pi}\right)\cdot \mathbf{t}^{(I)}$$

spontaneous: pion decay constant $f_\pi \sim M_{QCD}/4\pi$

explicit: pion mass $m_\pi^2 \sim m_q M_{QCD}$

$$\mathcal{L}_{EFT} = \frac{1}{2}D_\mu\boldsymbol{\pi}\cdot D^\mu\boldsymbol{\pi} - \frac{m_\pi^2}{2}\frac{\boldsymbol{\pi}^2}{1+\boldsymbol{\pi}^2/4f_\pi^2} + N^+\left(i\mathcal{D}_0 + \frac{\vec{\mathcal{D}}^2}{2m_N}\right)N + \frac{g_A}{2f_\pi}N^+\vec{S}\boldsymbol{\tau}N\cdot\cdot\vec{\mathcal{D}}\boldsymbol{\pi}$$

$$+C_0\, N^+N\, N^+N + C_2'\, N^+N\left(\vec{\mathcal{D}}N^+\right)\cdot\vec{\mathcal{D}}N + \ldots$$

other spin/isospin combos, more derivatives, powers of pion mass, deltas, Ropers, *etc.*

- expansion in:

$$\frac{Q}{M_{QCD}} \sim \begin{cases} Q/m_N & \text{non-relativistic} \\ Q/m_\rho, \ldots & \text{multipole} \\ Q/4\pi f_\pi & \text{pion loop} \end{cases} \qquad \sim \frac{1}{5}$$

hidden-charm molecules $NN \to D\bar{D}^* + \bar{D}D^*$

Fleming, Kusunoki, Mehen + v.K. '07
Pavon Valderrama '11 …

New features of the force, already at LO:

- finite-range component
- singular long-range component
- coupling between channels

} tensor force

$1/r^3$ singularity

tensor operator couples l and $l+2$ for $S=1$

range $R \sim 1/m_\pi$

$$V = \frac{g_A^2}{48\pi f_\pi^2} \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \left[\left(\frac{1}{r^2} + \frac{3m_\pi}{r} + 3m_\pi^2 \right) \left(\vec{\sigma}_1 \cdot \hat{r}\, \hat{r} \cdot \vec{\sigma}_2 - \frac{\vec{\sigma}_1 \cdot \vec{\sigma}_2}{3} \right) + m_\pi^2\, \vec{\sigma}_1 \cdot \vec{\sigma}_2 \right] \frac{e^{-m_\pi r}}{r}$$

$$+ \frac{1 - \vec{\sigma}_1 \cdot \vec{\sigma}_2}{4} C_{0[0]} \delta^{(3)}(\vec{r}\,)$$

$$+ \frac{3 + \vec{\sigma}_1 \cdot \vec{\sigma}_2}{4} \left\{ C_{0[1]} \delta^{(3)}(\vec{r}\,) + C'_{2[1]} \left[\left(\frac{\partial \delta^{(3)}(\vec{r}\,)}{\partial r} + \frac{2}{r} \delta^{(3)}(\vec{r}\,) \right) \frac{\partial}{\partial r} + \delta^{(3)}(\vec{r}\,) \frac{\partial^2}{\partial r^2} \right] \right\}$$

$$+ \ldots$$

Yang, Rotureau, Barrett
+ v.K., in preparation

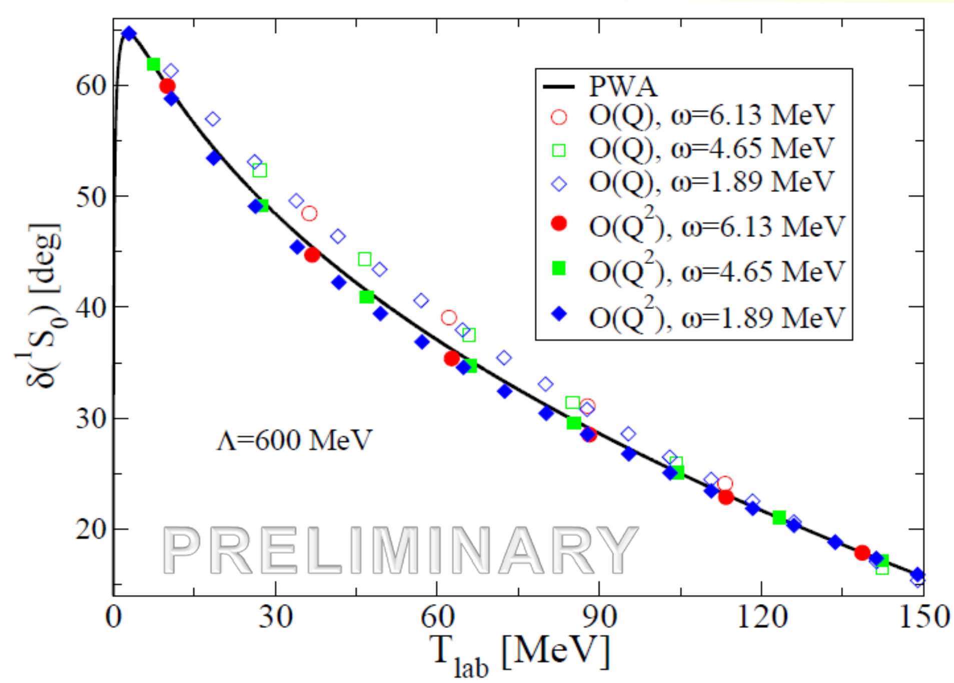
Uncoupled channels

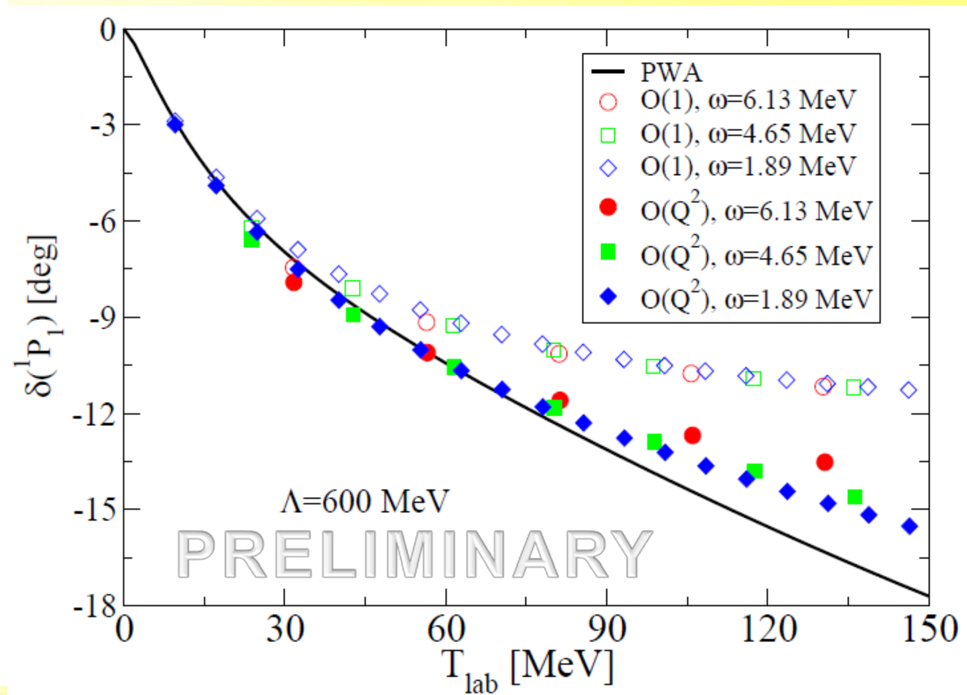
following the same steps as before, in the limit $N_{2\max} \to \infty$

$$\frac{\Gamma\left(\frac{2l+3}{4}-\frac{mEb^2}{2}\right)}{\Gamma\left(\frac{1-2l}{4}-\frac{mEb^2}{2}\right)}\left[1+\mathcal{O}\left(R^2/b^2\right)\right]=(-1)^{l+1}\left(\frac{\sqrt{mEb^2}}{2}\right)^{2l+1}\cot\delta(E)$$

cf. Luu *et al*. '10

Examples





Conclusion

- ✓ EFT can be solved in HO basis with scattering input
- ✓ Nucleons with pionless EFT in HO similar to trapped atoms near a Feshbach resonance
- ✓ Convergence improves with increasing order
- ✓ Few-body binding energies and scattering parameters can be calculated
- ✓ More extensive calculations with more nucleons and in pionful EFT are needed