Solid & Superfluid Matter in Neutron Stars

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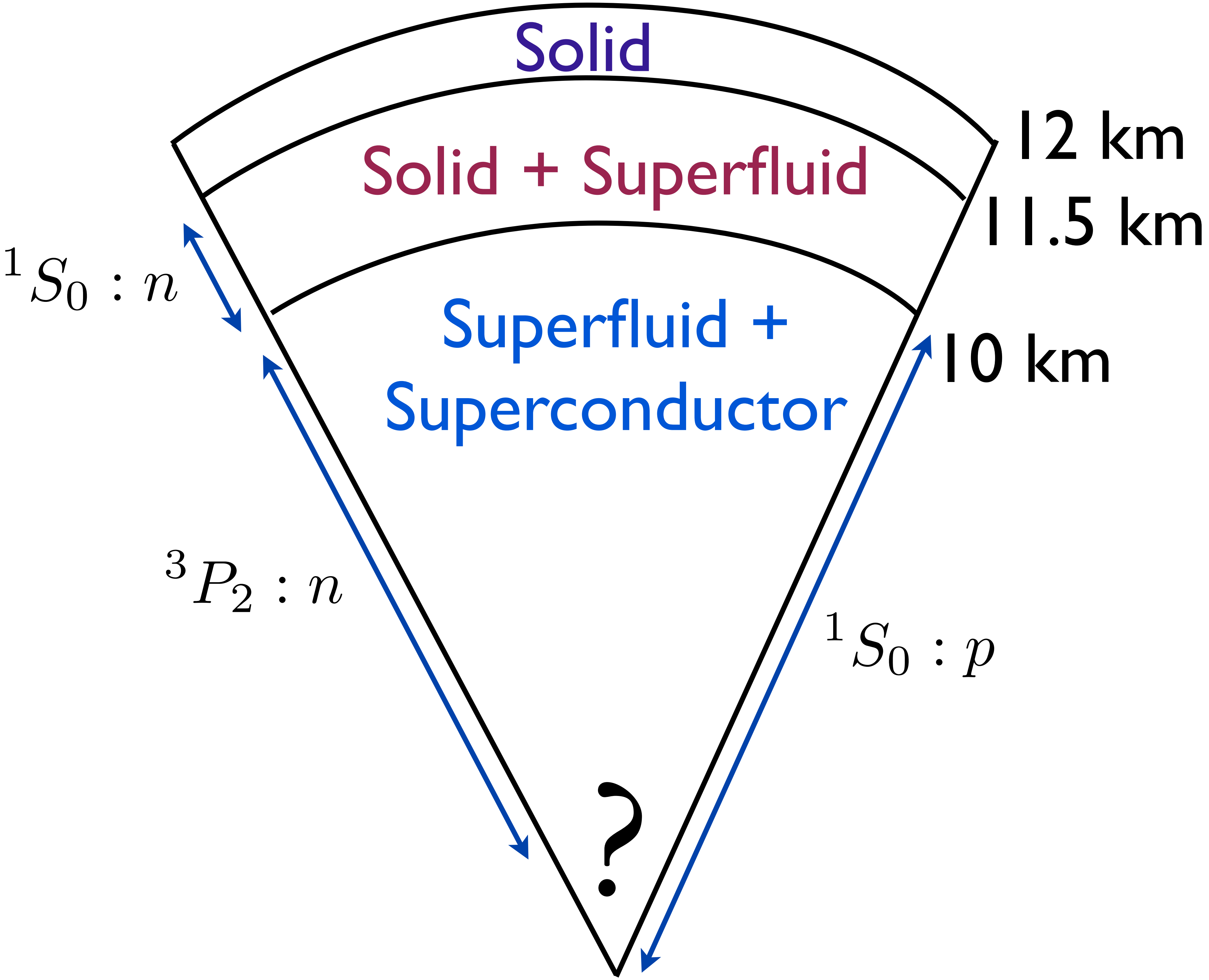
A Vanilla Neutron Star



Solid

Solid + Superfluid

Superfluid +
Superconductor

12 km

11.5 km

10 km

$^1S_0 : n$

$^3P_2 : n$

$^1S_0 : p$

?

The nucleon degree of freedom may be frozen everywhere in a cold neutron star !

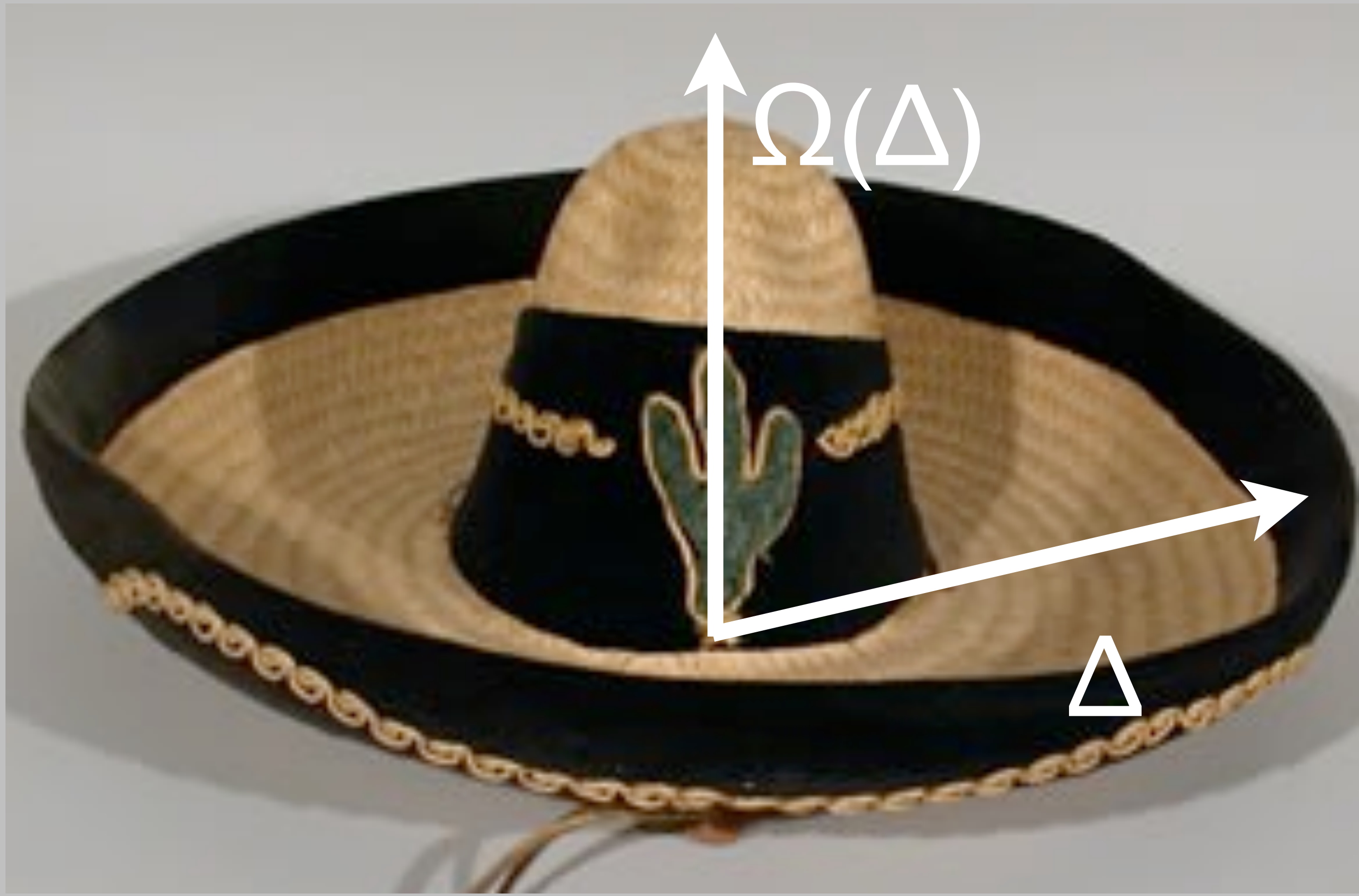
Neutron Superfluid

Attractive interactions destabilize the Fermi surface:

$$H = \sum_{k,s=\uparrow,\downarrow} (\frac{k^2}{2m} - \mu)\ a^\dagger_{k,s} a_{k,s} + g \sum_{k,p,q,s=\uparrow,\downarrow} a^\dagger_{k+q,s} a^\dagger_{p-q,s} a_{k,s} a_{p,s}$$

$$\Delta = g\langle a_{k,\uparrow} a_{p,\downarrow}\rangle \quad \Delta^* = g\langle a^\dagger_{k,\uparrow} a^\dagger_{p,\downarrow}\rangle$$

Cooper pairs leads to superfluidity



Energy gap for fermions:

$$E(p) = \sqrt{(\frac{p^2}{2M} - \mu)^2 + \Delta^2}$$

New collective mode:
Superfluid Phonon

$$\omega(k) = v_s\ k$$

Transport properties dominated by

- Outer crust: Electrons and lattice phonons.
- Inner crust: Electrons, lattice phonons and superfluid phonons.
- Core: Electrons, superfluid phonons, and angulons (Goldstone bosons associated with breaking rotational symmetry).

This is good news. Describing low energy properties of dense Fermi liquids is hard ! Low energy theory of phonons easier.

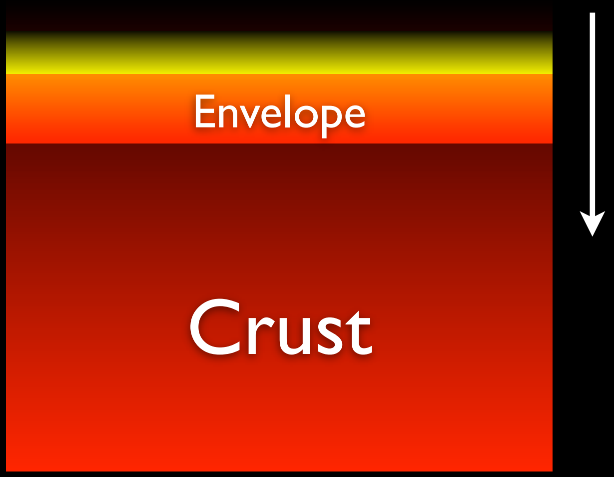
New Phenomena in Neutron Stars

- a window into the thermal and mechanical properties of the crust.

- Crustal heating and subsequent thermal relaxation in accreting neutron stars and magnetars.
- Surface temperature anisotropy in magnetars.
- Possible excitation of shear modes in the solid crusts of magnetars during giant flares.

Transiently Accreting NSs

SXRTs: High accretion followed by periods of quiescence



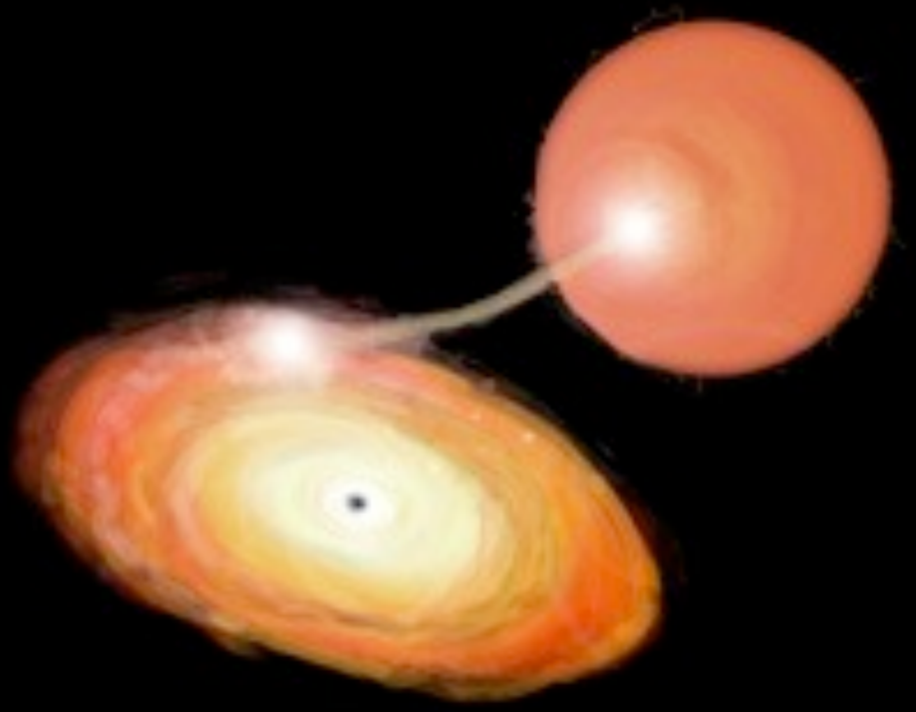
Envelope

Crust

Nuclear reactions release: ~ 1.5 MeV / nucleon

Deep crustal heating.

Brown, Bildsten Rutledge (1998)
Sato (1974), Haensel & Zdunik (1990)



Warms up old neutron stars

Transiently Accreting NSs

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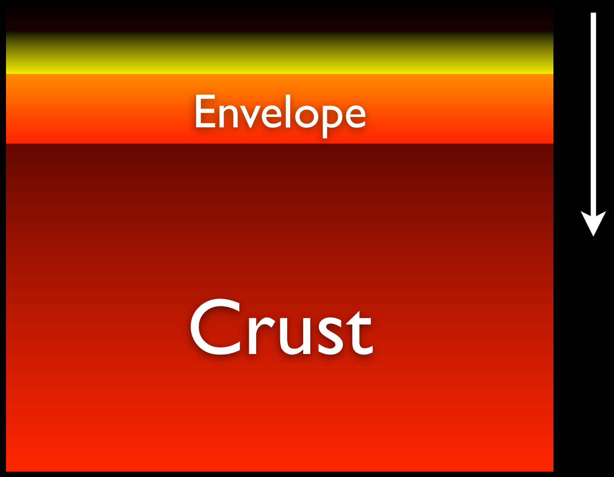

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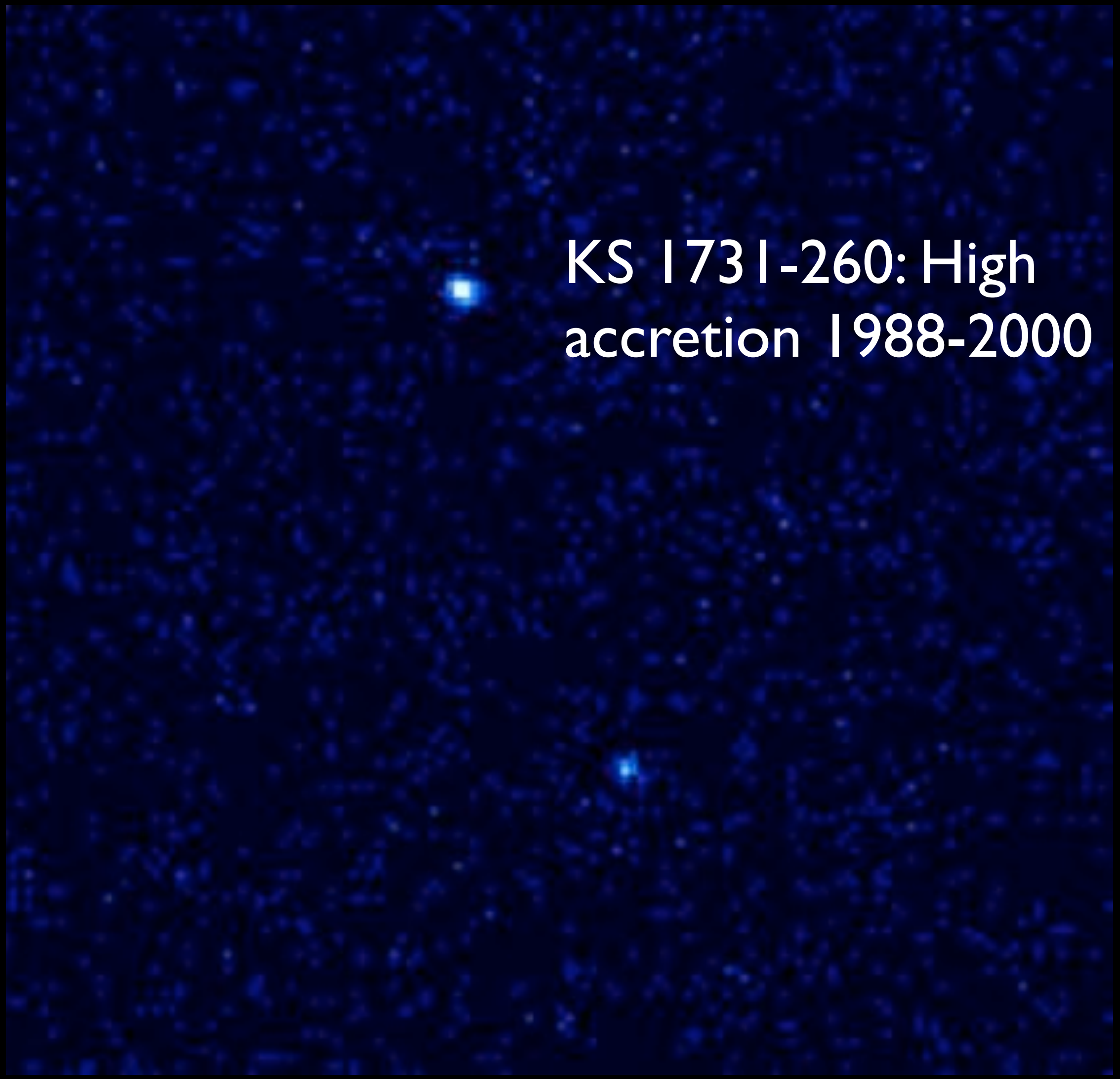
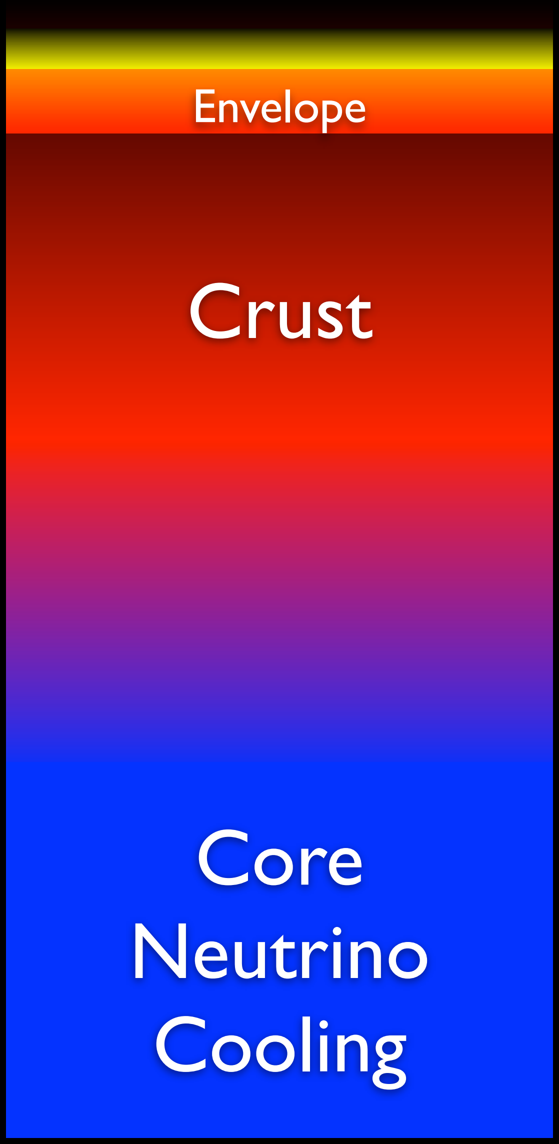


KS 1731-260: High accretion 1988-2000

Image credit: NASA/CXC/Wijnands et al.

Crust Cooling

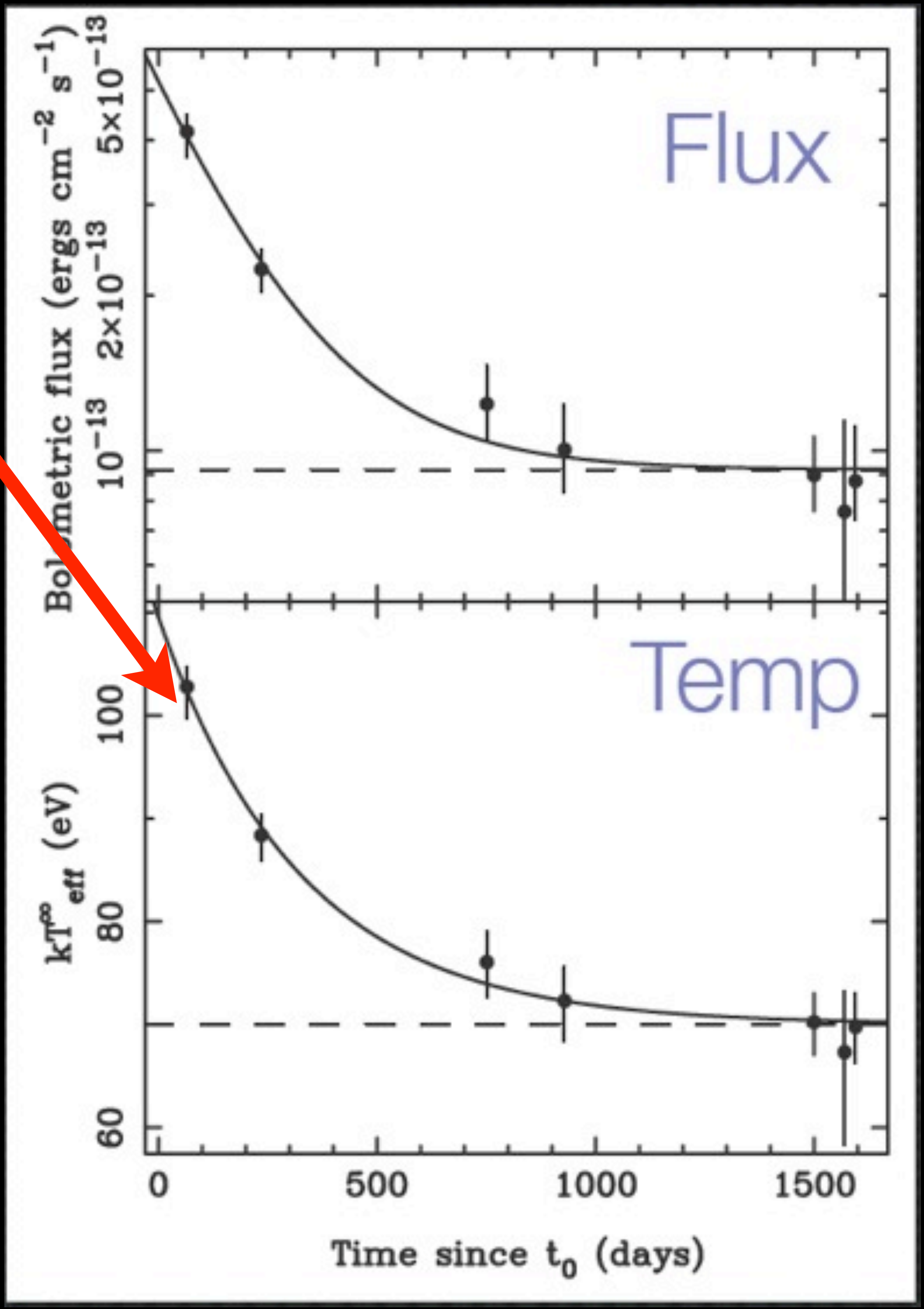
Watching NSs immediately after accretion ceases !



Crust Relaxation:

1. Initial temperature profile.
2. Thermal conductivity.
3. Heat capacity.

Shternin & Yakovlev (2007)
Cumming & Brown (2009)



Cackett, et al. (2006)

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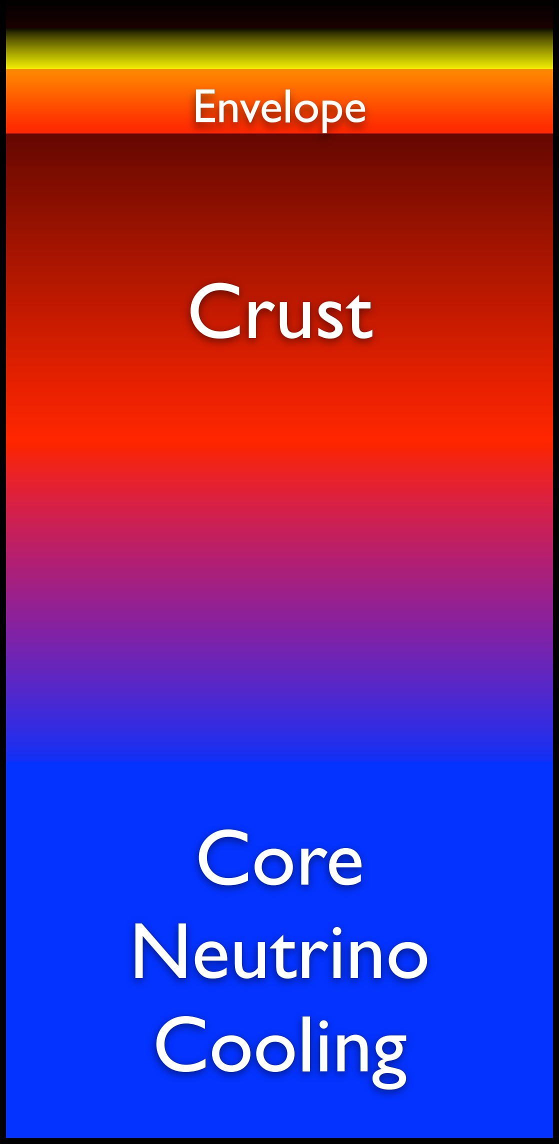

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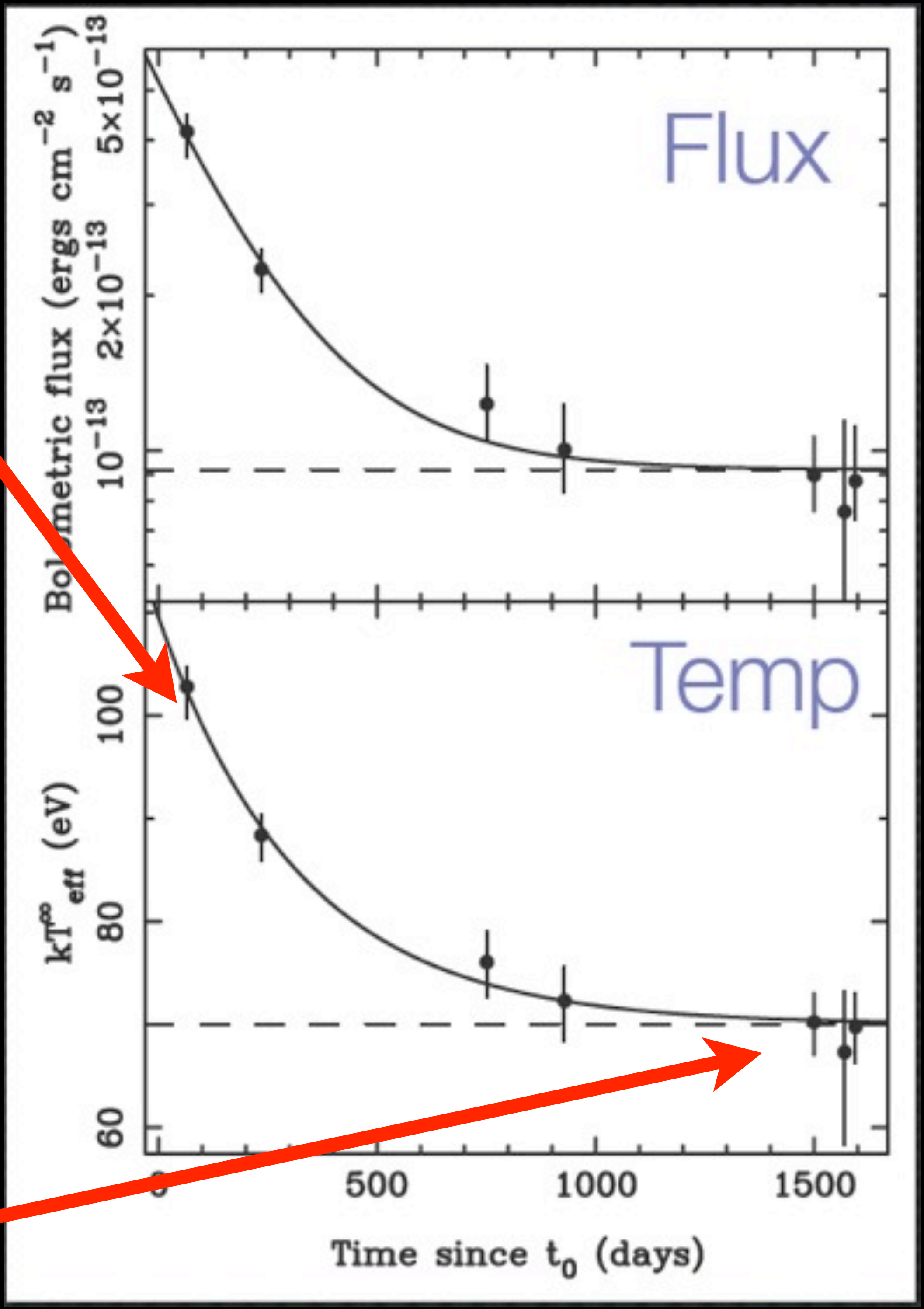
During quiescence we see the "Core Temperature"



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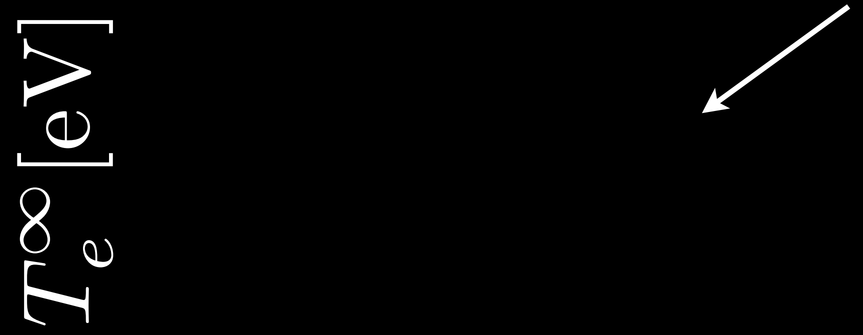
Thermal Relaxation

- Crust relaxes during quiescence.

Shternin & Yakovlev (2007), Brown & Cumming (2009)



data from
MXB 1659

Thermal Relaxation

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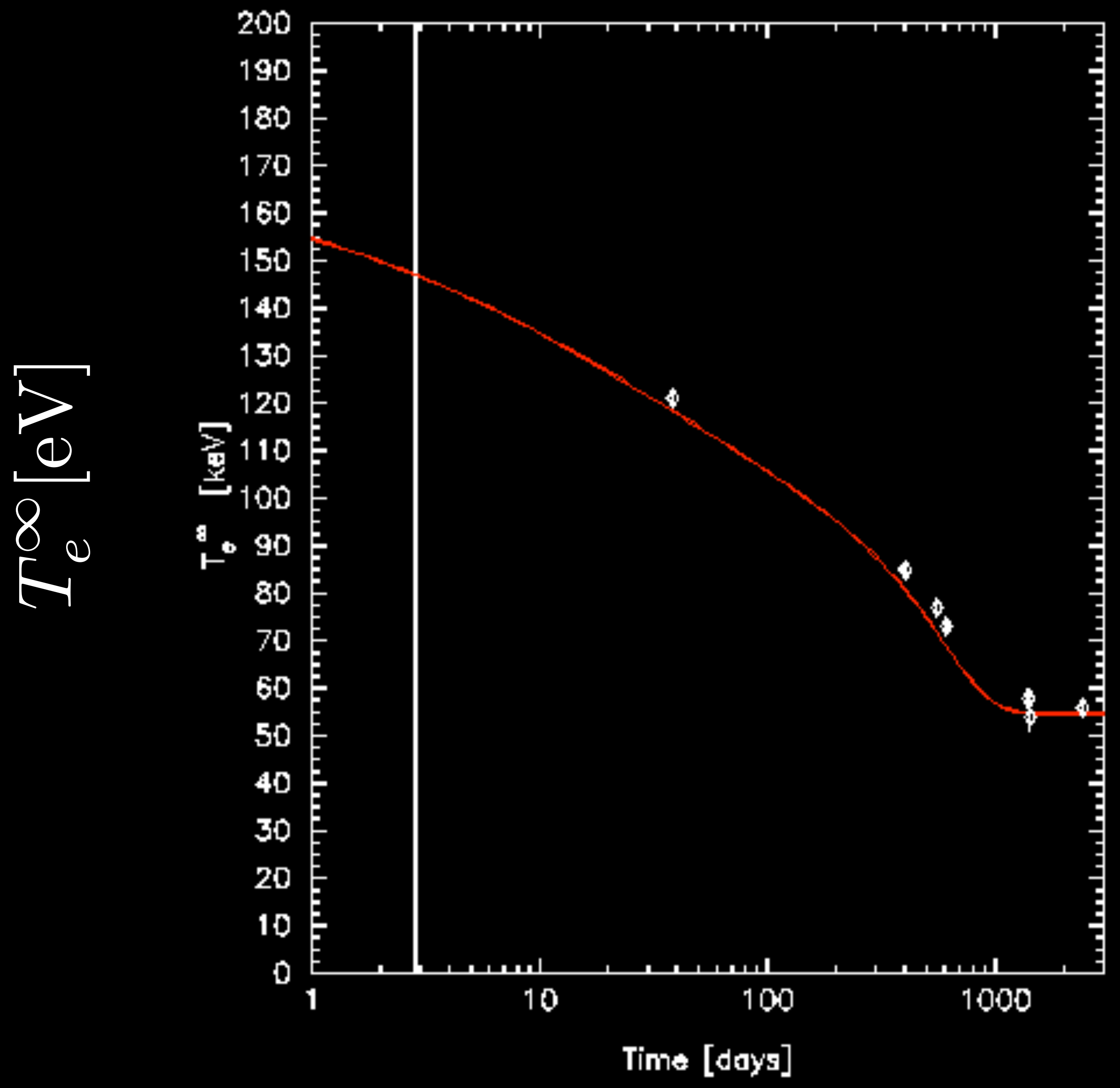
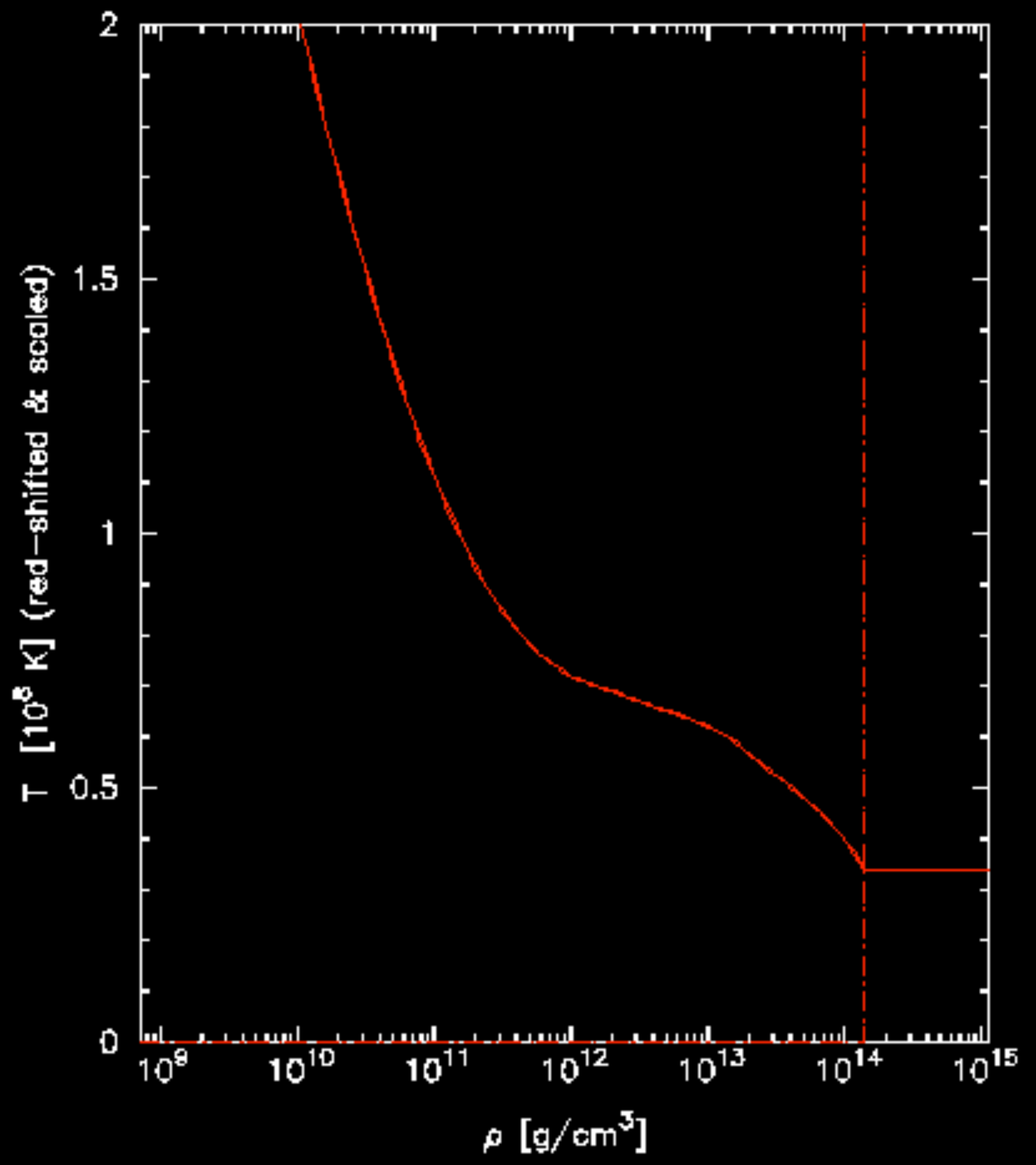
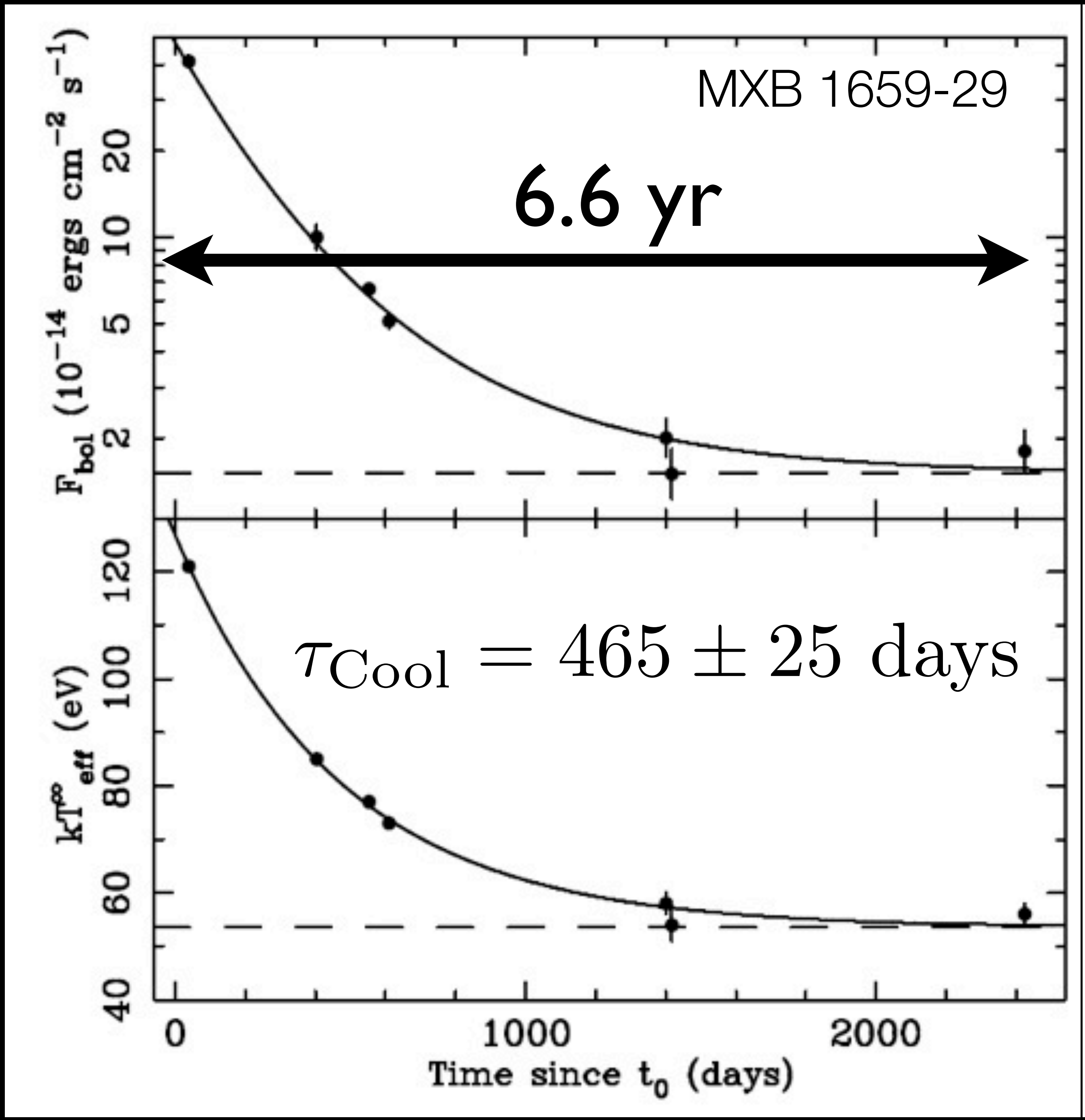




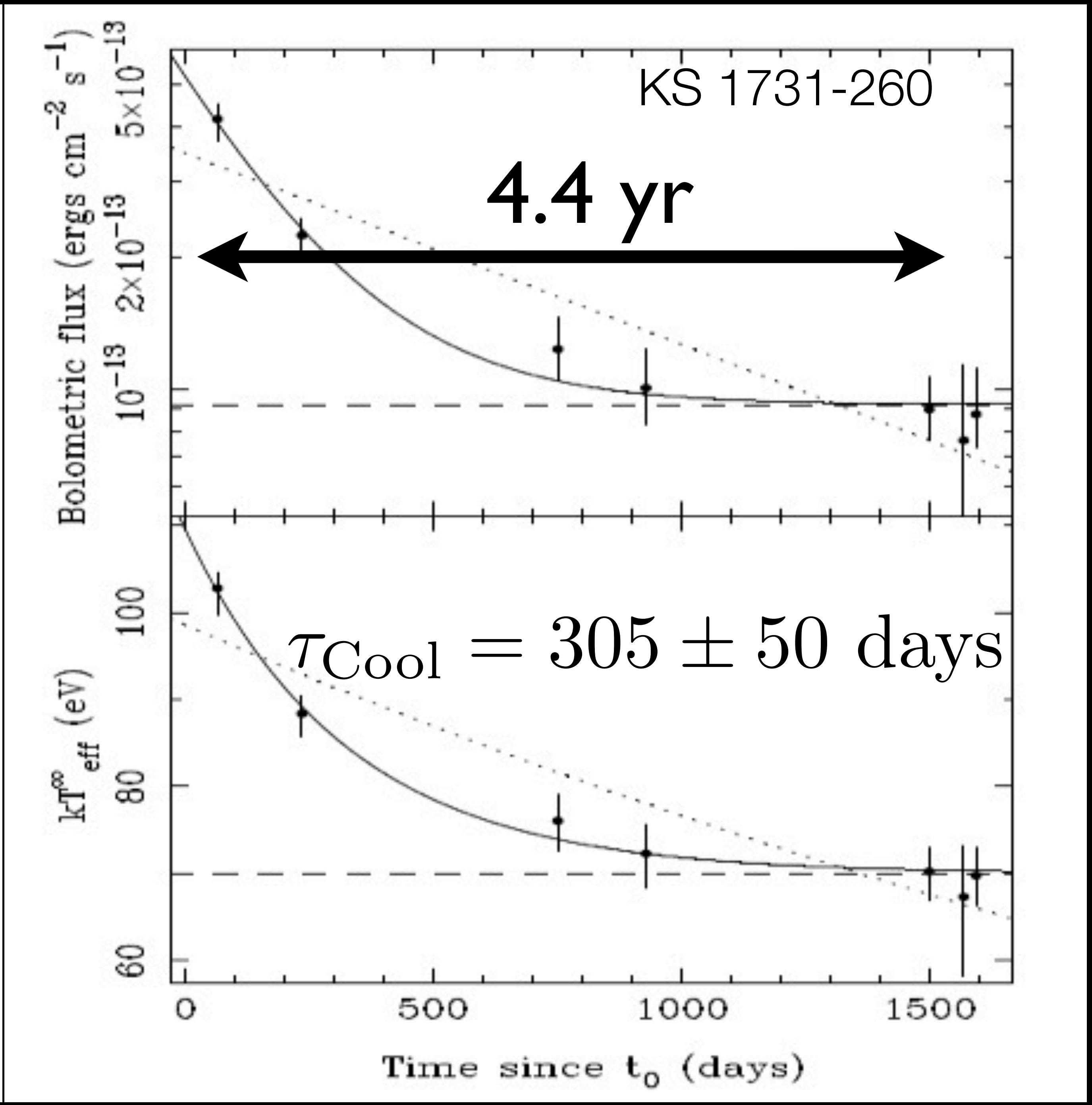
More than one source !



Cackett et al. 2006

6.6 yr

$\tau_{\mathrm{Cool}} = 465 \pm 25$ days



Cackett et al. 2008

4.4 yr

$\tau_{\mathrm{Cool}} = 305 \pm 50$ days

Connecting to Crust Microphysics

$$\tau_{\mathrm{Cool}} \simeq \frac{C_V}{\kappa}\,(\Delta R)^2$$

Connecting to Crust Microphysics

Crustal Specific Heat

$$\tau_{\mathrm{Cool}} \simeq \frac{C_V}{\kappa} \, (\Delta R)^2$$

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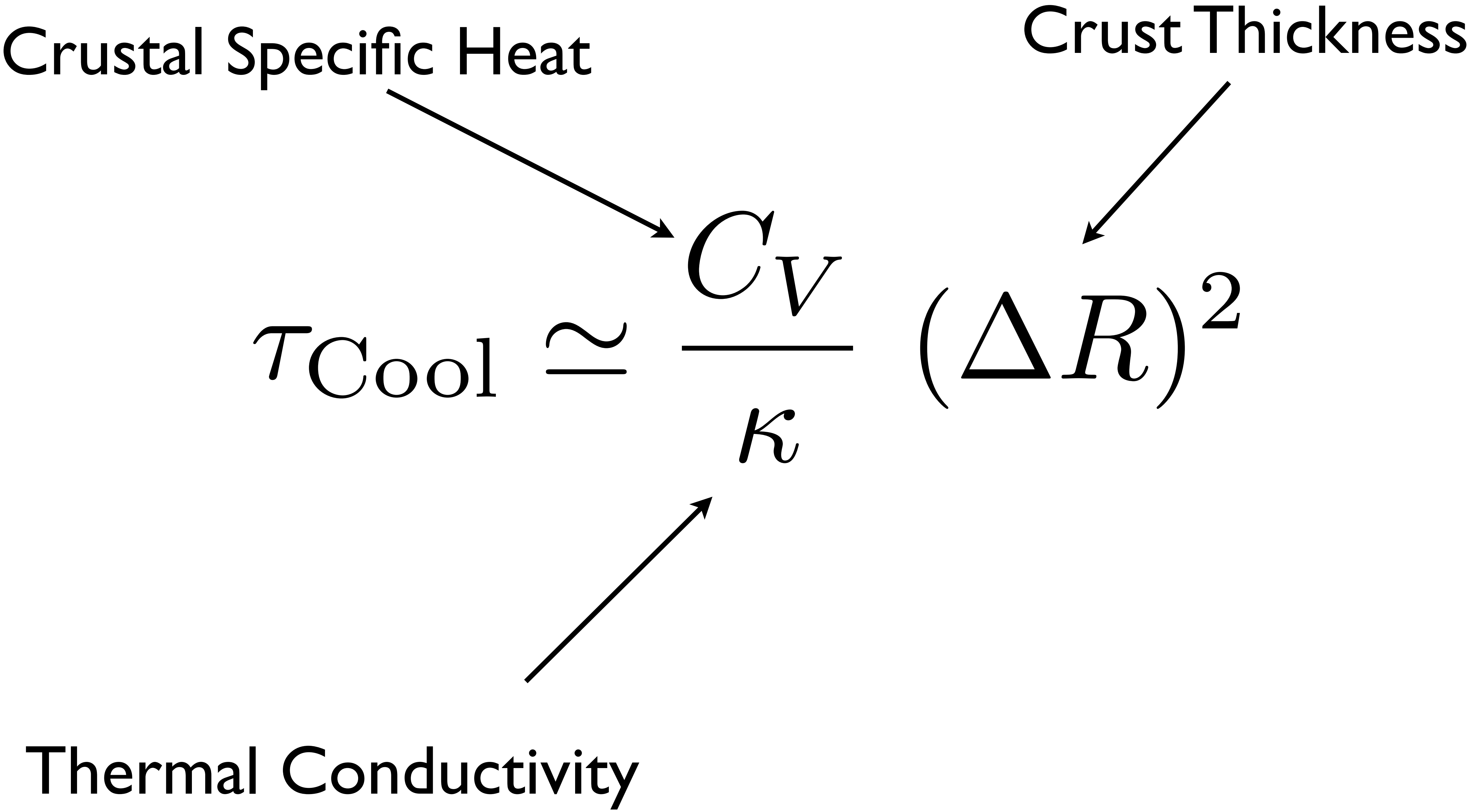
Thermal Conductivity

Connecting to Crust Microphysics



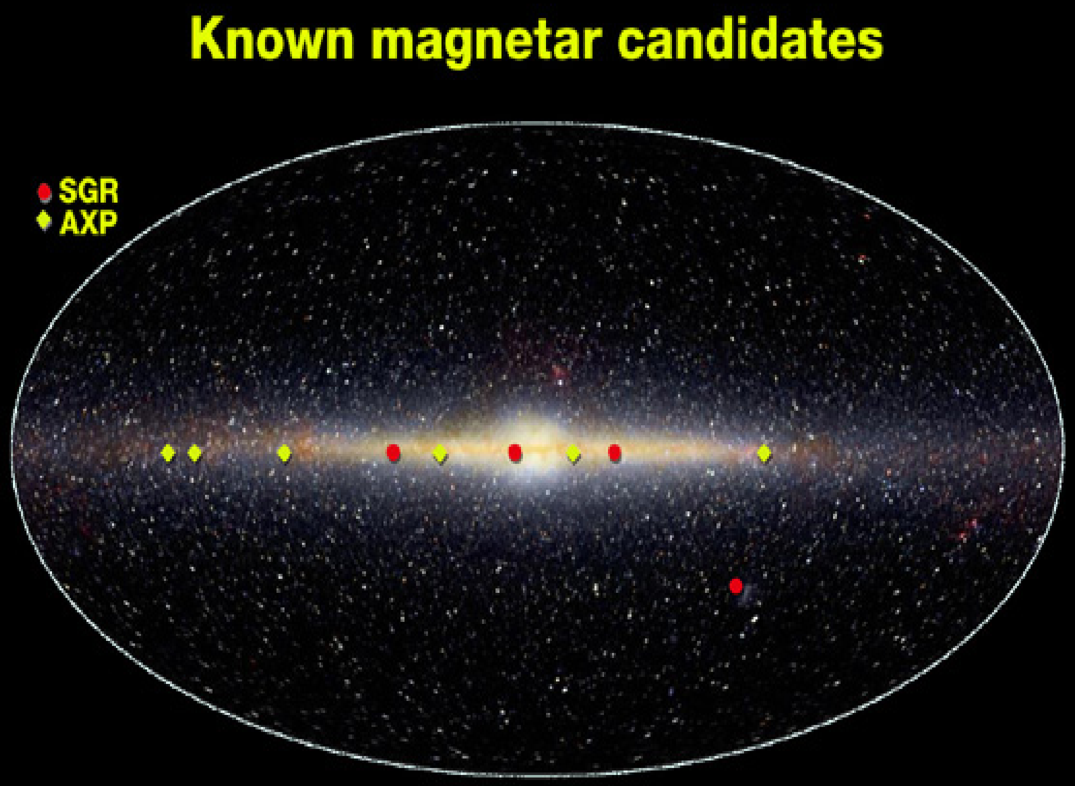
Crustal Specific Heat

Crust Thickness

$$\tau_{\rm Cool} \simeq \frac{C_V}{\kappa} (\Delta R)^2$$

Thermal Conductivity

Explosions on Magnetars: Giant Flares

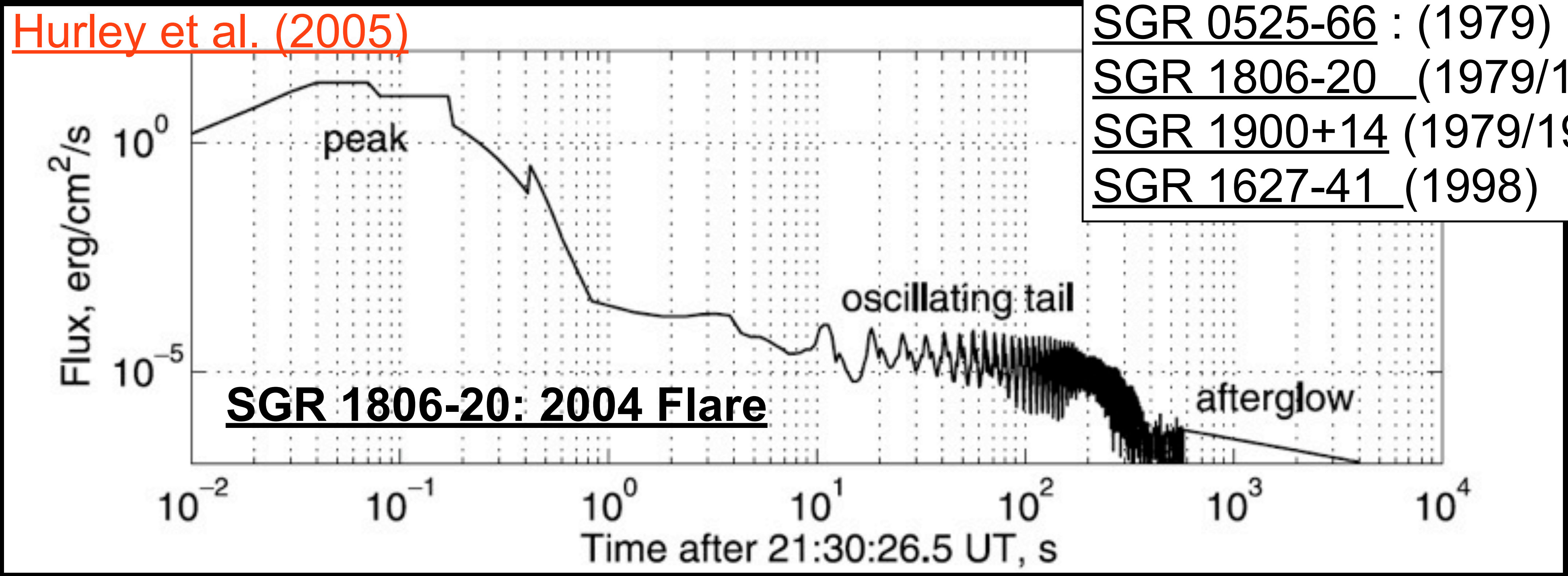


Anomalous X-Ray Pulsars (10)
Soft Gamma Repeaters (8)

Inferred to have surface fields of the order of 10^{15} Gauss.

http://www.physics.mcgill.ca/~pulsar/magnetar/main.html

SGRs exhibit powerful outburst ~ 10^{46} ergs/s

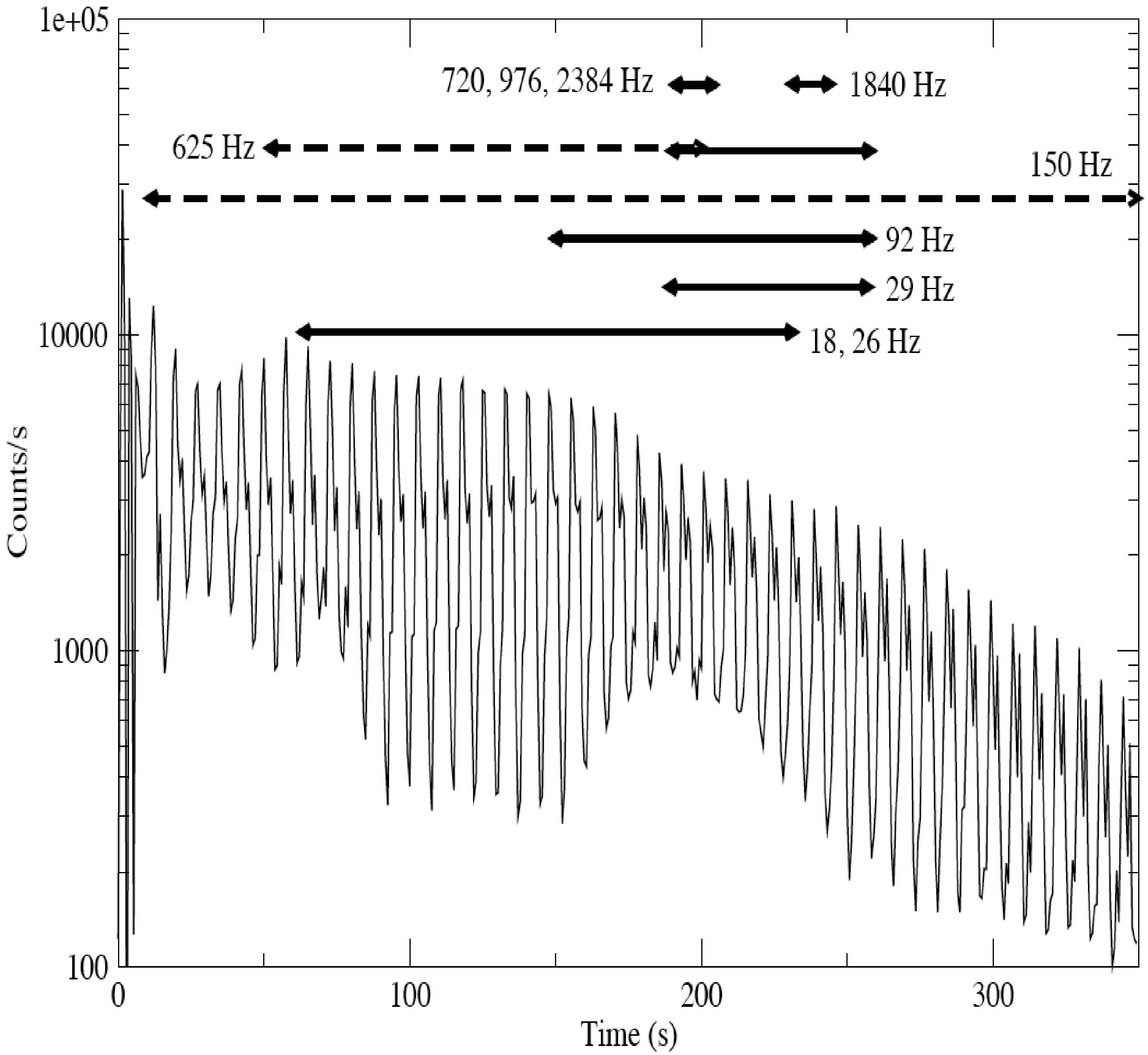


Hurley et al. (2005)

SGR 0525-66 : (1979)
SGR 1806-20 (1979/1986/2004)*
SGR 1900+14 (1979/1986/1998)
SGR 1627-41 (1998)

QPOs are likely to be shear modes in the solid crust

Duncan (1998), Strohmayer, Watts (2006)



SGR 1806
2004 Giant Flare

$$\omega_{\mathrm{n=1}} \simeq \frac{\pi \boxed{v_t}}{R} \frac{\Delta R}{R}$$

$$\omega_{\mathrm{n=0,l=2}} \simeq \frac{2 \boxed{v_t}}{R}$$

Similar frequencies observed in 2 sources.

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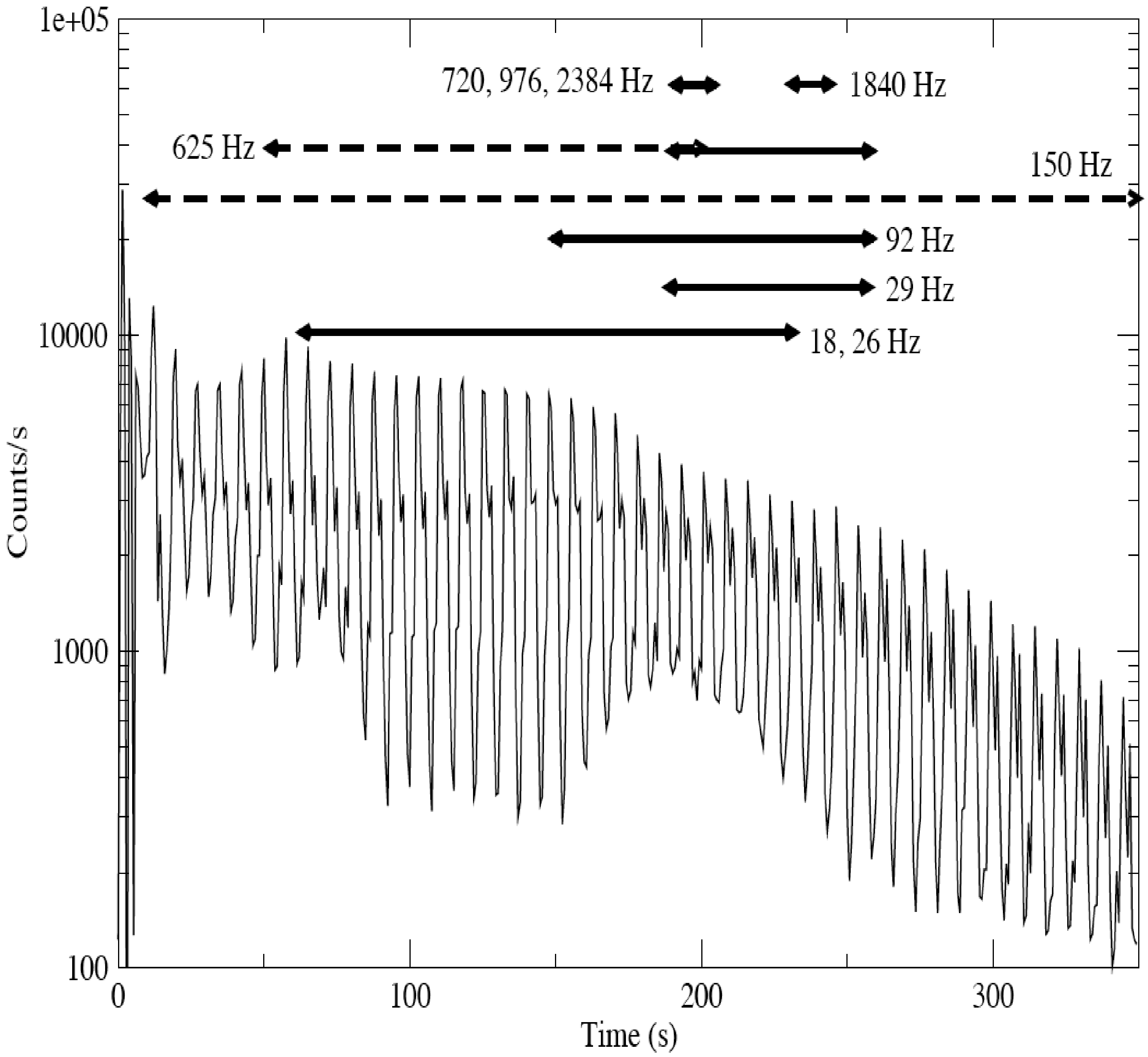

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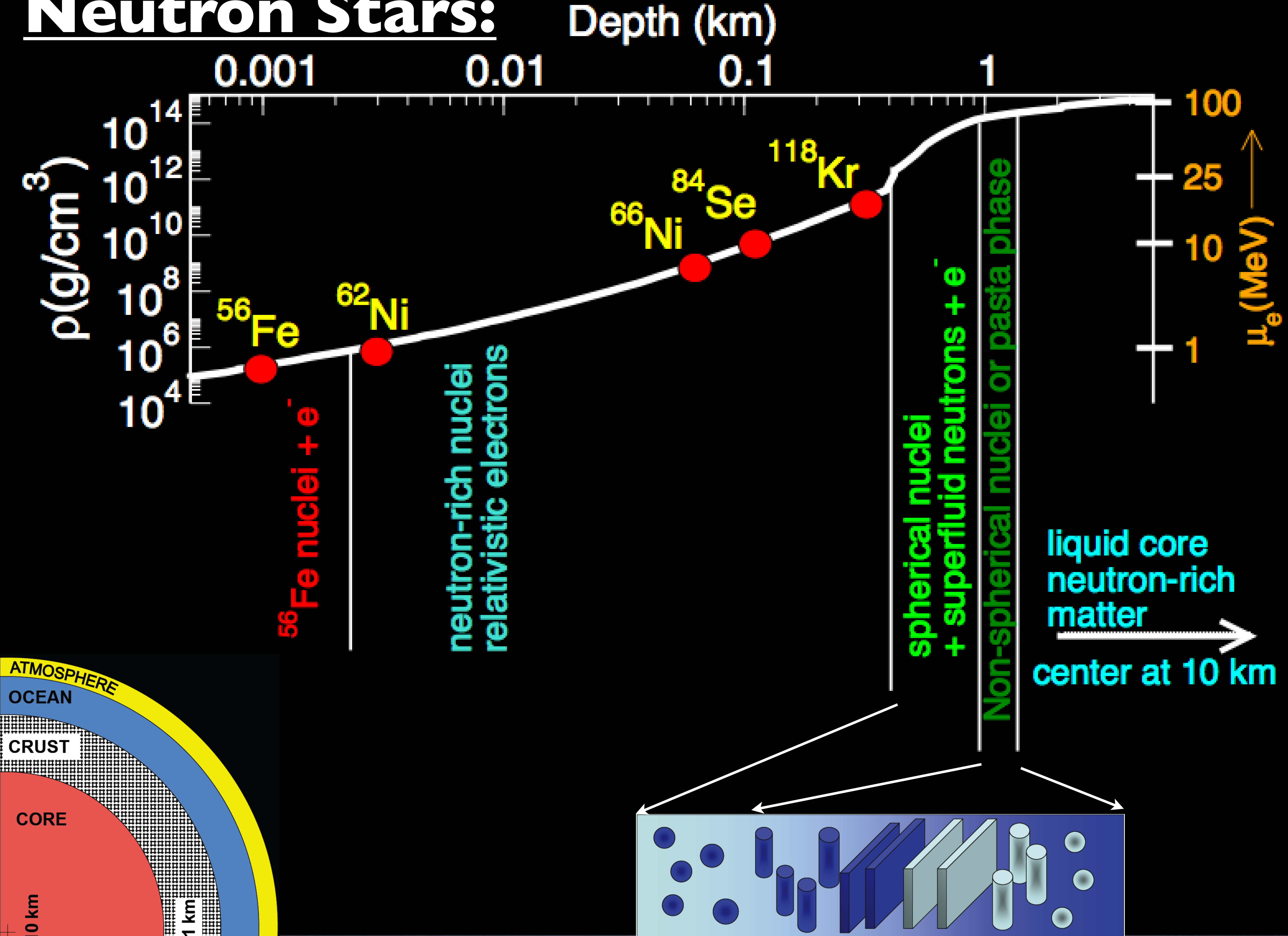
$$\omega_{\mathrm{n=1}} \simeq \frac{\pi v_t}{R} \frac{\Delta R}{R}$$

$$\omega_{\mathrm{n=0,l=2}} \simeq \frac{2 v_t}{R}$$

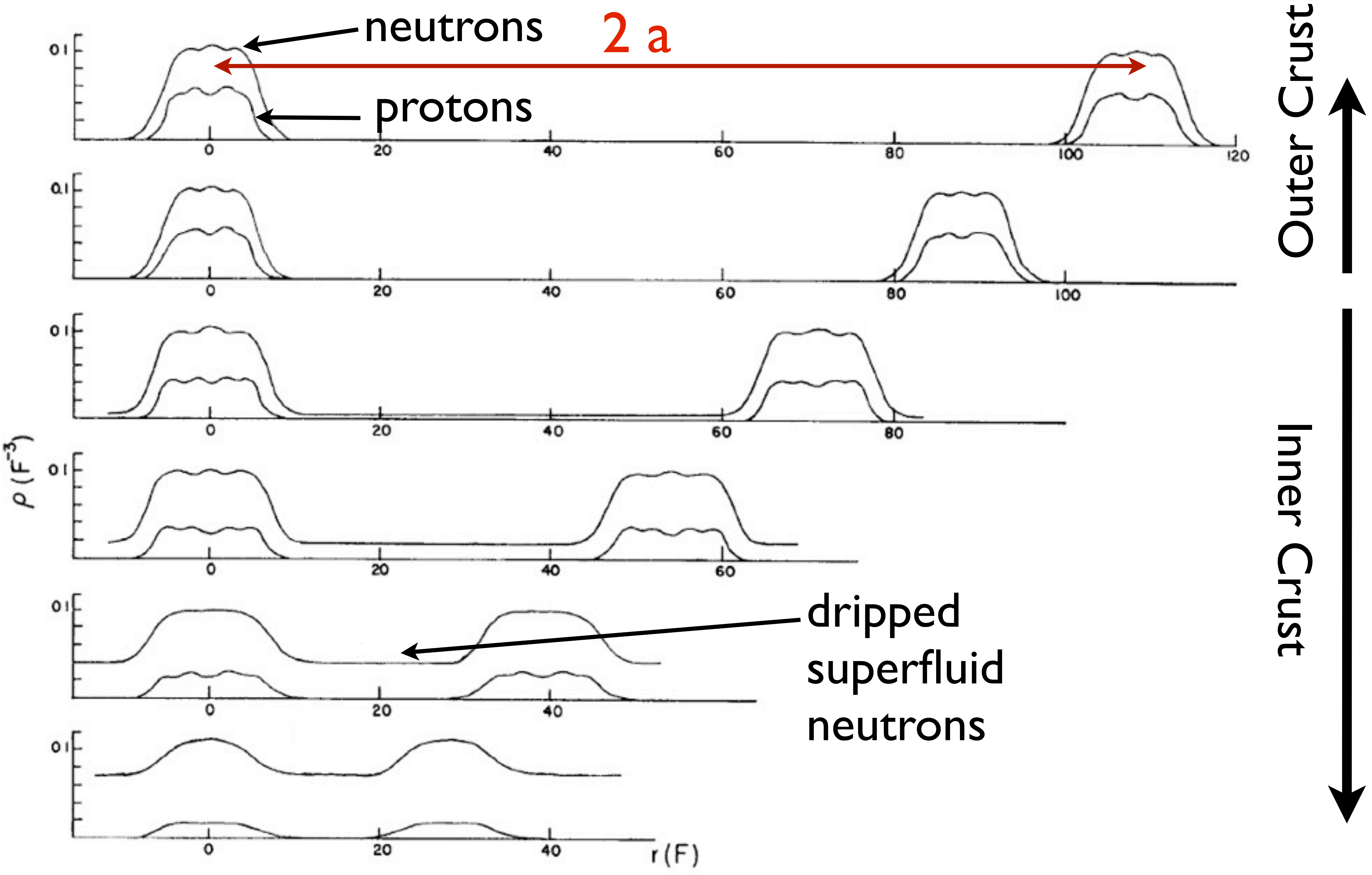
Shear mode velocity

Similar frequencies observed in 2 sources.

Neutron Stars:



Microscopic Structure of the Crust



Baym Pethick & Sutherland (1971) Negele & Vautherin (1973)

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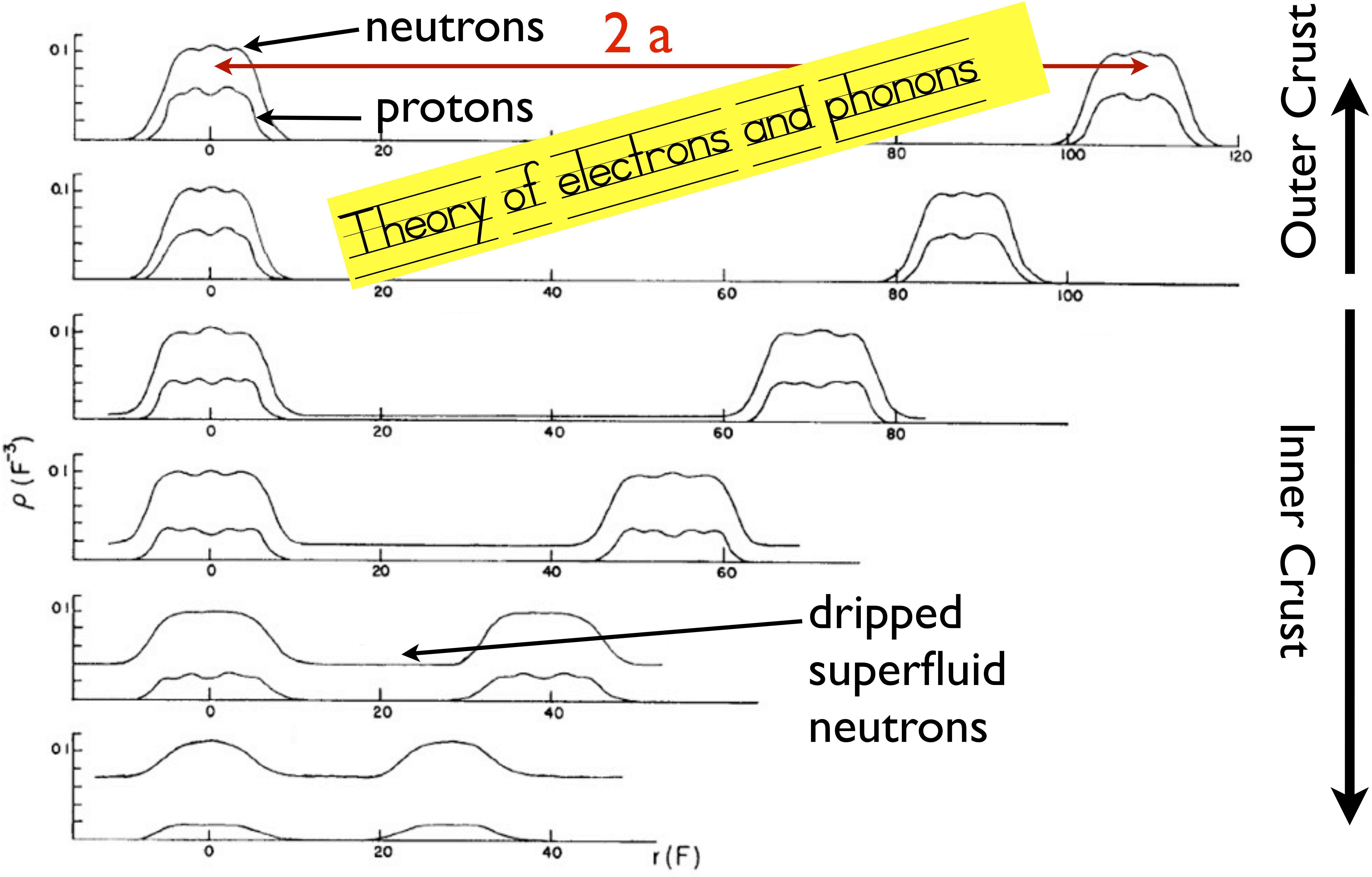


neutrons

protons

2 a

Theory of electrons and phonons

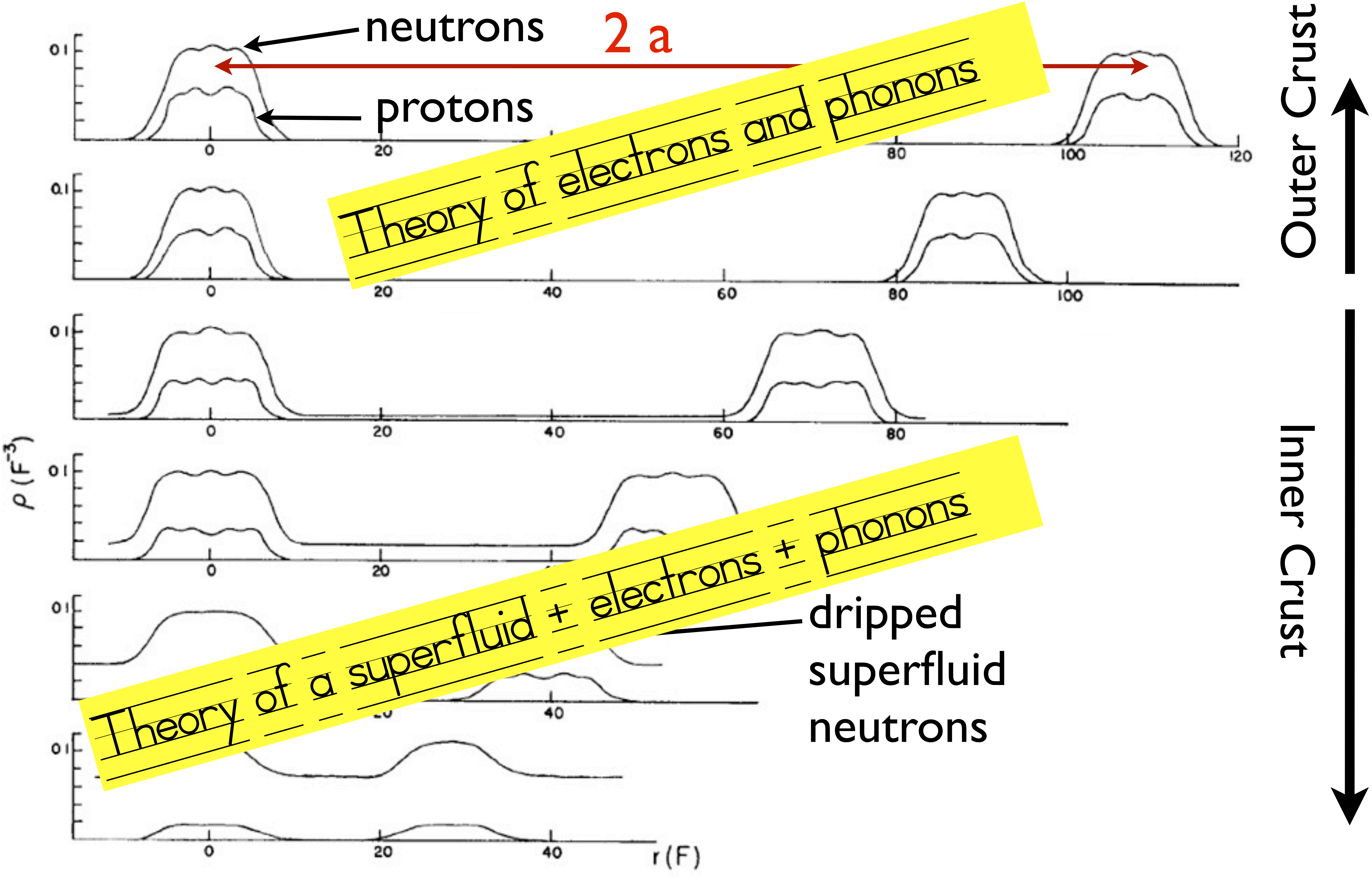
dripped superfluid neutrons

Outer Crust

Inner Crust

Baym Pethick & Sutherland (1971)   Negele & Vautherin (1973)

Microscopic Structure of the Crust



neutrons

2 a

protons

Theory of electrons and phonons

Outer Crust

Inner Crust

Theory of a superfluid + electrons + phonons

dripped superfluid neutrons

Baym Pethick & Sutherland (1971) Negele & Vautherin (1973)

Electrons are (nearly) free

- Electrons are dense,degenerate and relativistic.

$$n_e = Z\ n_I \qquad k_{\rm Fe} \approx E_{\rm Fe} \simeq 25 - 75\ {\rm MeV} \gg m_e$$

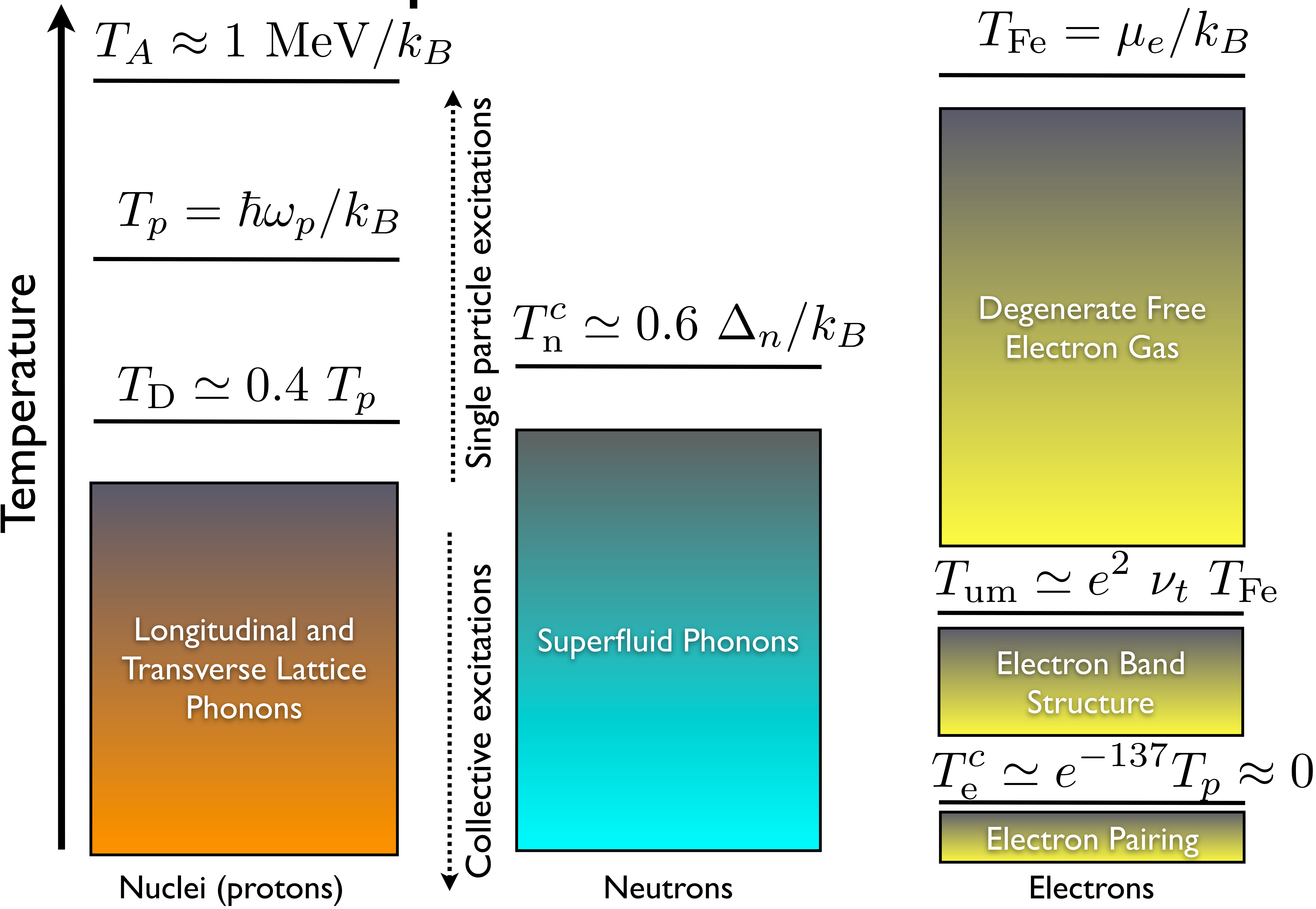
- Band gaps are small and restricted to small patches in the Fermi surface.

$$\frac{V_{\rm e-i}}{E_{\rm Fe}} \simeq \alpha_{\rm em}\ Z^{2/3} \ll 1 \qquad \frac{\delta_{\rm e}}{E_{\rm Fe}} \simeq \frac{4\alpha_{\rm em}}{3\pi} \approx 10^{-3}$$

- Pairing energy is negligible.

$$T_c \simeq \omega_p^{\rm ion}\ \exp\left(-\frac{v_{Fe}}{\alpha_{\rm em}}\right) \approx 0$$

Separation of Scales



Temperature

$T_A \approx 1 \text{ MeV}/k_B$

$T_p = \hbar\omega_p/k_B$

$T_{\rm D} \simeq 0.4\ T_p$

Longitudinal and Transverse Lattice Phonons

Nuclei (protons)

Single particle excitations

Collective excitations

$T_{\rm n}^c \simeq 0.6\ \Delta_n/k_B$

Superfluid Phonons

Neutrons

$T_{\rm Fe} = \mu_e/k_B$

Degenerate Free Electron Gas

$T_{\rm um} \simeq e^2\ \nu_t\ T_{\rm Fe}$

Electron Band Structure

$T_{\rm e}^c \simeq e^{-137} T_p \approx 0$

Electron Pairing

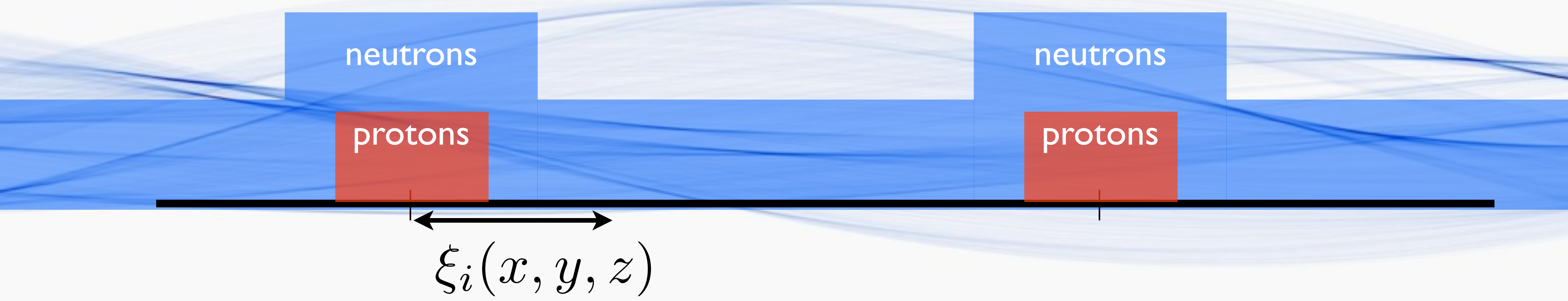
Electrons

Low Energy Theory of Phonons

Proton (clusters) move collectively on lattice sites. Displacement is a good coordinate.

Neutron superfluid: Goldstone excitation is the phase of the condensate.

Low Energy Theory of Phonons



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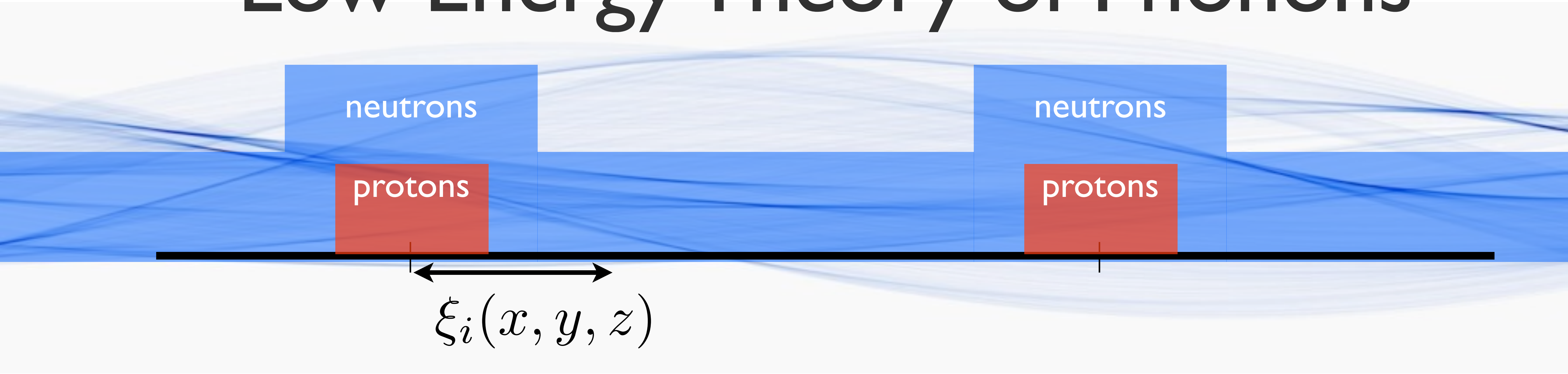

Proton (clusters) move collectively on lattice sites. Displacement is a good coordinate.

Neutron superfluid: Goldstone excitation is the phase of the condensate.

$$\langle \psi_\uparrow(r)\psi_\downarrow(r)\rangle = |\Delta| \; \exp\left(-2i \; \theta\right)$$

“coarse-grain”

Collective coordinates:

Vector Field: $\xi_i(r,t)$
Scalar Field: $\phi(r,t)$

Low Energy Effective Theory

$$\begin{aligned}\mathcal{L}_{\mathrm{EFT}}^{\mathrm{sPh}} &= \frac{1}{2}\,(\partial_o\phi)^2 + \frac{1}{2}v\,(\partial_i\phi)^2 + \frac{1}{f_s}\,\partial_o\phi\;\psi^\dagger\psi + \frac{1}{\Lambda_s^2}\,(\partial_o\phi)^3 + \cdots \\ \mathcal{L}_{\mathrm{EFT}}^{\mathrm{lPh}} &= \frac{1}{2}\,(\partial_o\xi)^2 + \frac{1}{2}c\,(\partial_i\xi_i)^2 + \frac{1}{f_l}\,\partial_i\xi^i\;\psi^\dagger\psi + \frac{1}{\Lambda_l^2}\,(\partial_i\xi^i)^3 + \cdots\end{aligned}$$

↑ kinetic terms

↑ coupling to Fermions

↑ self-coupling

$$\mathcal{L}_{\mathrm{sPh-lPh}} = g\;\partial_0\phi\;\partial_i\xi^i + \gamma\;\partial_i\phi\;\partial_0\xi^i + \frac{1}{\Lambda^2}\;\partial_0\phi\;\partial_i\xi^i\partial_i\xi^i + \cdots$$

↑ lPh-sPh mixing

Low Energy Effective Theory

$$\mathcal{L}^{\rm sPh}_{\rm EFT} = \frac{1}{2}\left(\partial_o\phi\right)^2 + \frac{1}{2}v\left(\partial_i\phi\right)^2 + \frac{1}{f_s}\,\partial_o\phi\;\psi^\dagger\psi + \frac{1}{\Lambda_s^2}\left(\partial_o\phi\right)^3 + \cdots$$

$$\mathcal{L}^{\rm lPh}_{\rm EFT} = \frac{1}{2}\left(\partial_o\xi\right)^2 + \frac{1}{2}c\left(\partial_i\xi_i\right)^2 + \frac{1}{f_l}\,\partial_i\xi^i\;\psi^\dagger\psi + \frac{1}{\Lambda_l^2}\left(\partial_i\xi^i\right)^3 + \cdots$$

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↑ lPh-sPh mixing

↑ sPh→2 lPh

Low energy constants related to ground state thermodynamics

Pethick, Chamel, Reddy (2010), Cirigliano, Reddy & Sharma (2011)

Bare modes:

superfluid phonon

$$v_\phi = \sqrt{\frac{n_n^{\mathrm{c}}}{m}\frac{\partial \mu_n}{\partial n_n}}$$

density of conduction neutrons

longitudinal lattice phonon

$$v_\ell = \sqrt{\frac{\tilde{K} + 4S/3}{\rho_{\mathrm{I}}}}$$

transverse lattice phonon

$$v_t = \sqrt{\frac{S}{(n_p + n_n^b)m}}$$

density of entrained neutrons

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$$v_\phi = \sqrt{\frac{n_n^{\mathrm{c}}}{m}\frac{\partial \mu_n}{\partial n_n}}$$

density of conduction neutrons

longitudinal lattice phonon

$$v_\ell \approx \frac{\omega_p}{q_{\mathrm{TFe}}} = \sqrt{\frac{n_p}{n_p + n_n^{\mathrm{b}}}\,\frac{n_p}{m}\,\frac{\partial \mu_e}{\partial n_e}}$$

transverse lattice phonon

$$v_t = \sqrt{\frac{S}{(n_p + n_n^b)m}}$$

density of entrained neutrons

Phonon mixing and drag

$$\mathcal{L}_{\rm sPh-lPh} = g\ \partial_0\phi\ \partial_i\xi_i + \gamma\ \partial_i\phi\ \partial_0\xi_i$$

density-density interaction:

$$g = -\ \frac{n_p\ v_\phi}{\sqrt{n_n^c(n_p + n_n^b)}}\frac{\partial n_n}{\partial n_p}$$

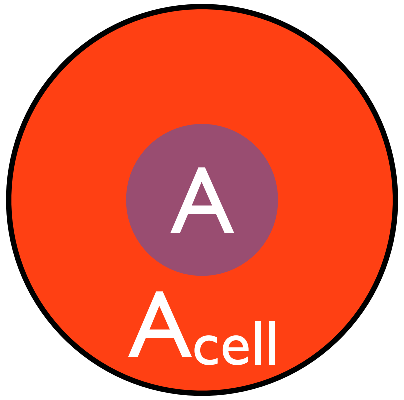
velocity-velocity interaction:

$$\gamma = \frac{n_n^b\ v_\phi}{\sqrt{n_n^c(n_p + n_n^b)}}$$

Entrainment

Chamel (2005)
Carter, Chamel & Haensel (2006)

$n_n^b \neq$ number of “bound” neutrons.



A=N+Z

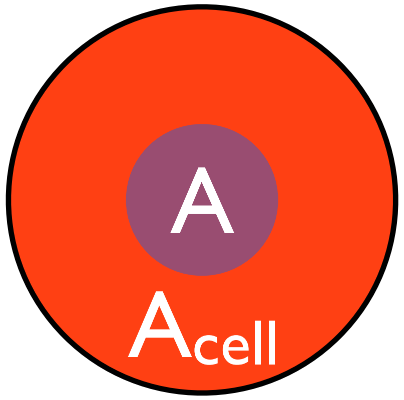
Bragg scattering off the lattice is important.

$$n_n^{\rm c} = \frac{m}{24\pi^3\hbar^2}\sum_\alpha \int_{\rm F} |\nabla_{\boldsymbol{k}}\varepsilon_{\alpha\boldsymbol{k}}|{\rm d}\mathcal{S}^{(\alpha)}$$

$$n_n^b = n_n - n_n^c$$

Entrainment

Chamel (2005)
Carter, Chamel & Haensel (2006)



A=N+Z

$n_n^b \neq$ number of "bound" neutrons.

Bragg scattering off the lattice is important.

neutron single-particle energy

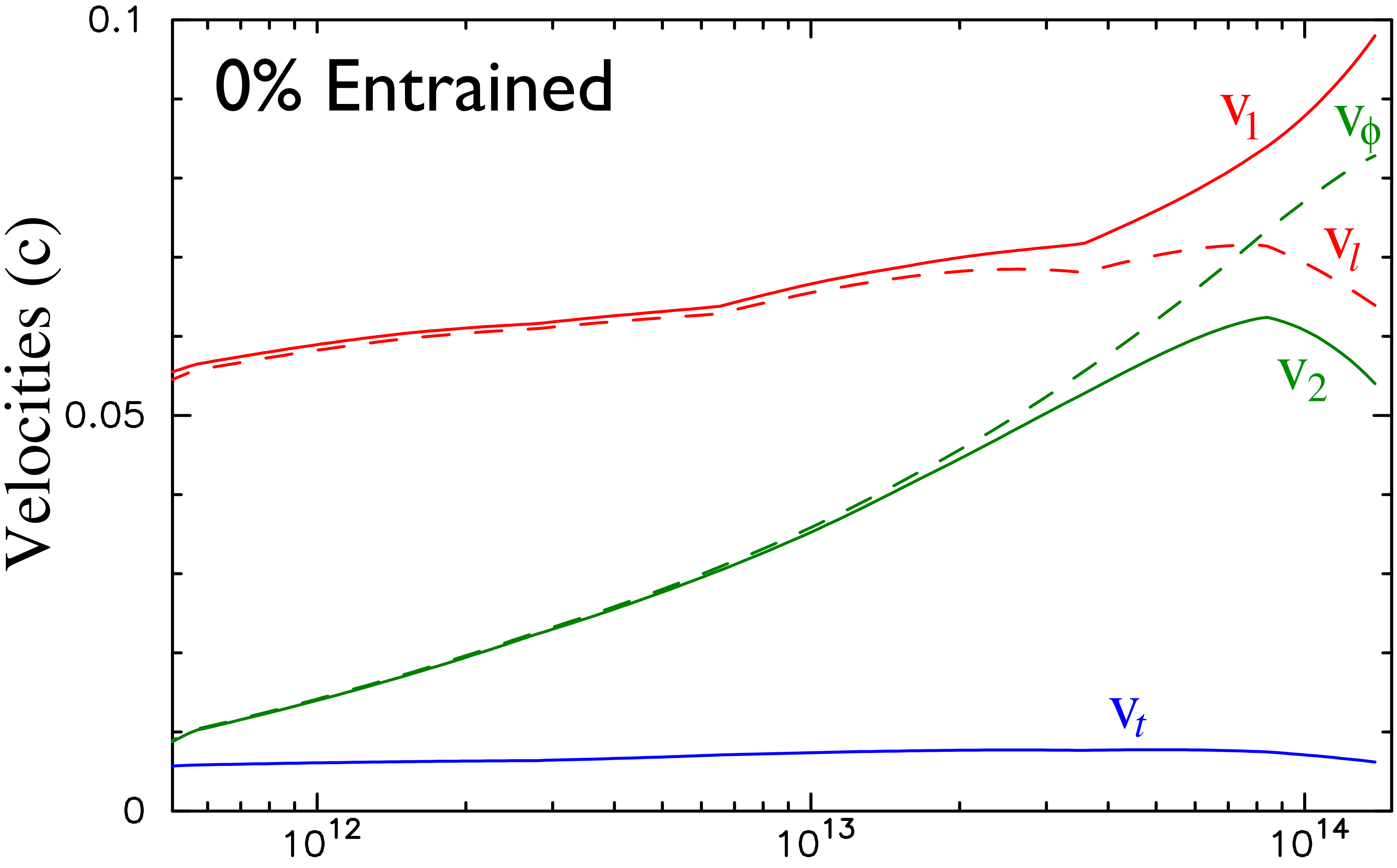
$$n_n^{\mathrm{c}} = \frac{m}{24\pi^3\hbar^2}\sum_\alpha \int_{\mathrm{F}} |\nabla_{\boldsymbol{k}}\varepsilon_{\alpha\boldsymbol{k}}|\,\mathrm{d}\mathcal{S}^{(\alpha)}$$

$$n_n^b = n_n - n_n^c$$

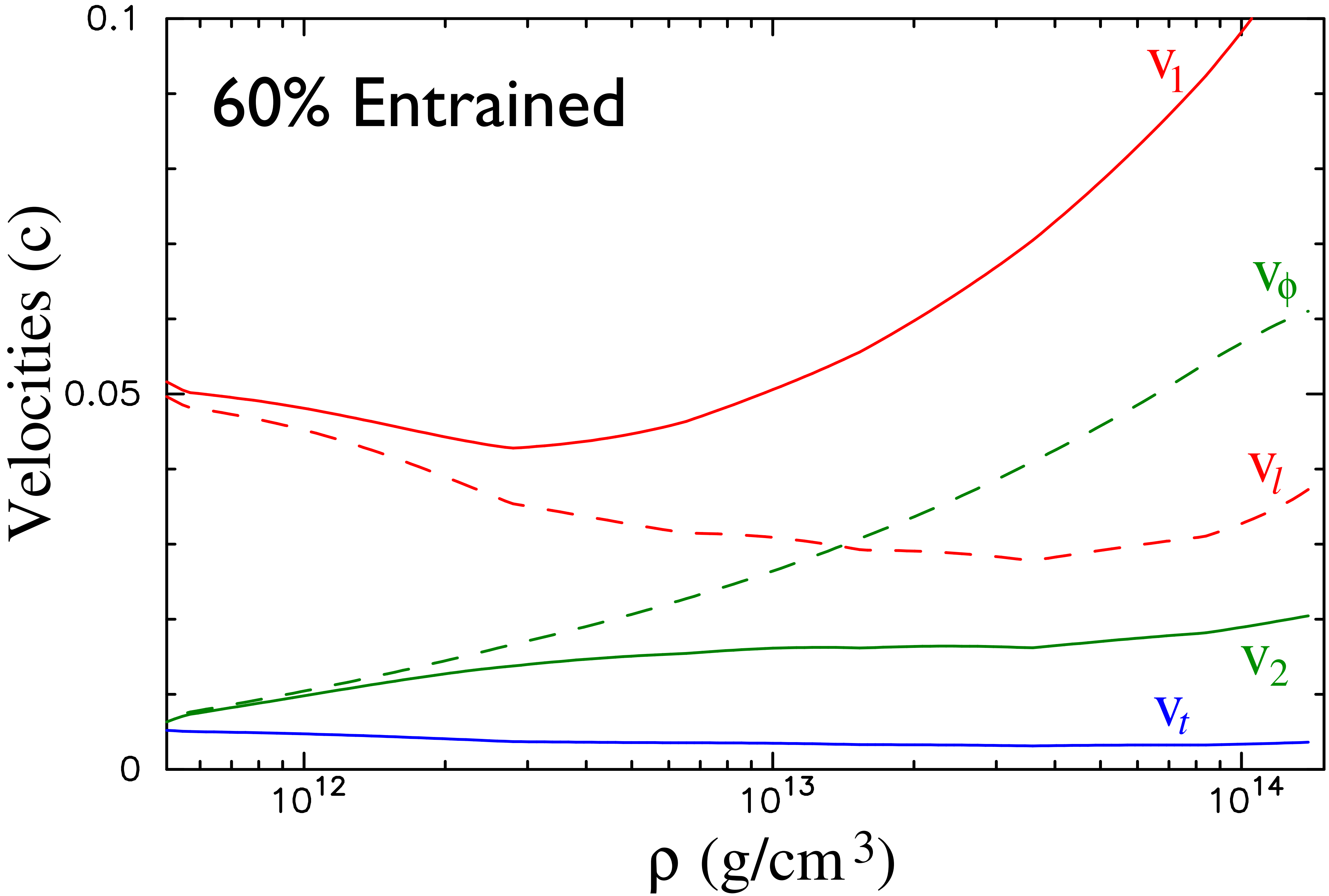
Complex interplay of nuclear and band structure effects.
The nuclear surface and disorder are likely to play a role.

Longitudinal lattice phonons and superfluid phonons are strongly coupled by entrainment.

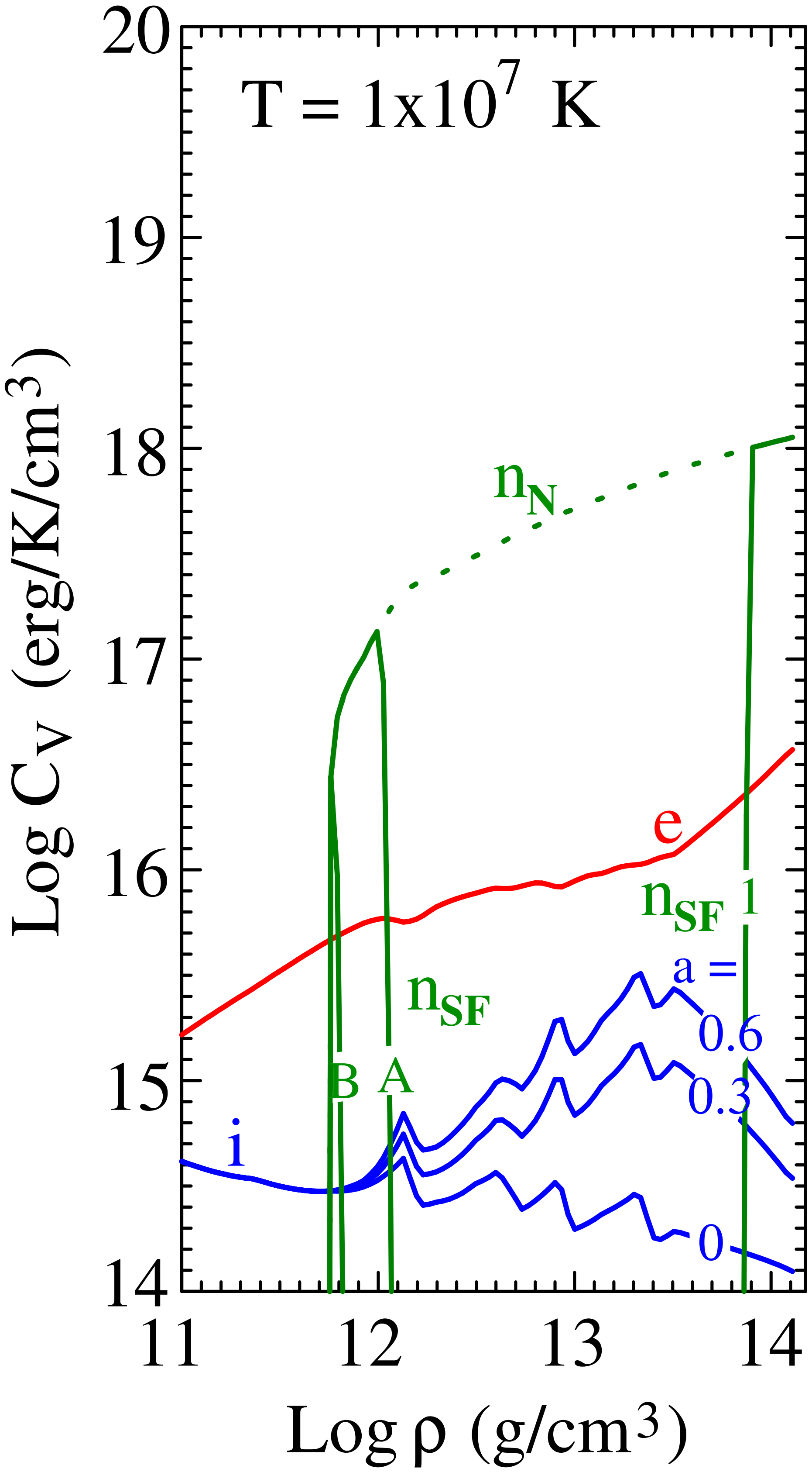
Mixed & Entrained Modes



Mixed & Entrained Modes



Crustal Specific Heat



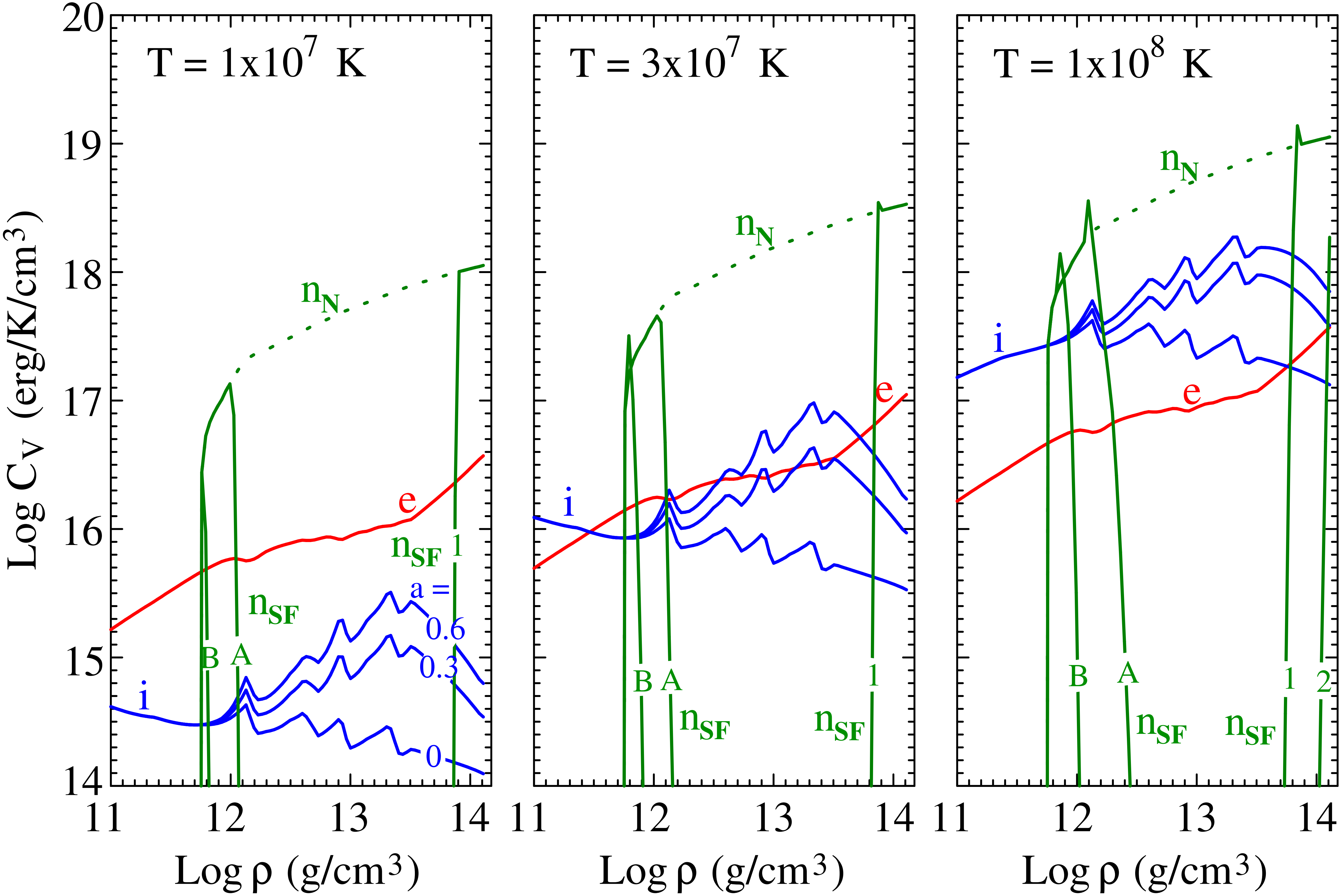
Electrons: $C_v^e = \frac{1}{3}\mu_e^2\, T$

Ions: $C_v^{\rm lph} = \frac{2\pi^2}{15}\left(\frac{T^3}{v_l^3} + \frac{2\,T^3}{v_t^3}\right)$

Neutrons:
$$\begin{cases} C_v^{\rm sph} = \frac{2\pi^2}{15}\frac{T^3}{v_\phi^3} & (T \ll T_c) \\ C_v^{\rm neutron} = \frac{1}{3}\, m_n\, k_{\rm Fn}\, T & (T > T_c) \end{cases}$$

Page & Reddy (2012)

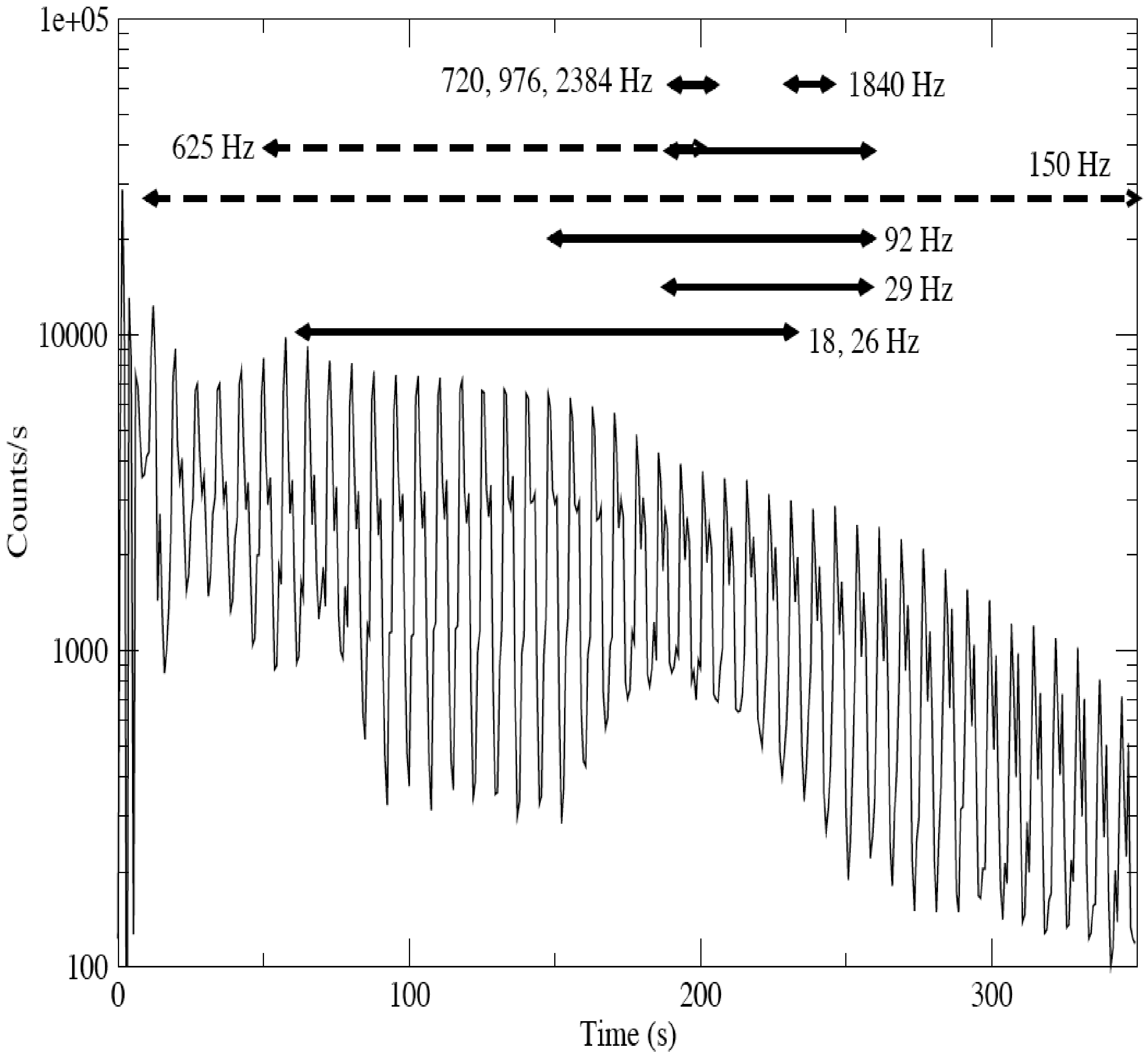
Crustal Specific Heat



Page & Reddy (2012)

These are the same velocities probed in QPOs.

2004 Giant Flare



$$\omega_{\mathrm{n=1}} \simeq \frac{\pi v_t}{R}\frac{\Delta R}{R}$$

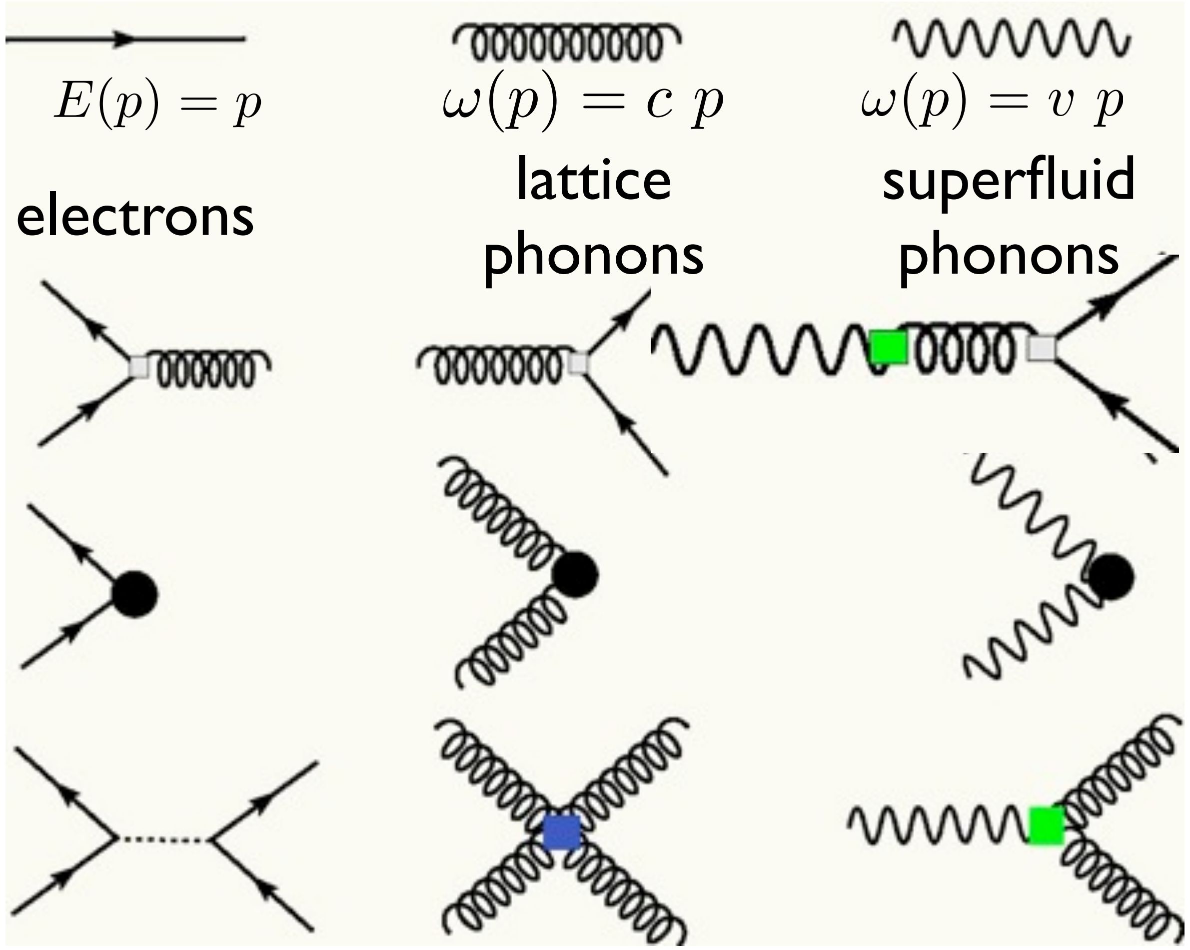
$$\omega_{\mathrm{n=0,l=2}} \simeq \frac{2 v_t}{R}$$

Can connect thermal properties to seismology !

Thermal Conduction

$$\kappa = \frac{1}{3}\, C_v \times v \times \lambda$$

- Dissipative processes:

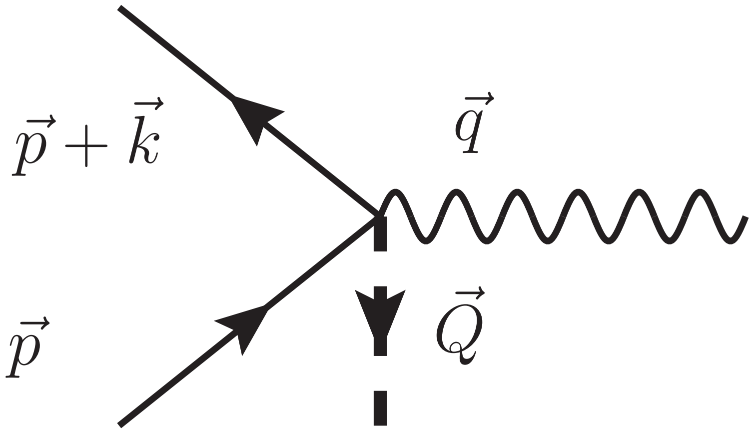


Cirigliano, Reddy & Sharma (2011)

- Umklapp is important:

$$\frac{k_{\rm Fe}}{q_{\rm D}} = \left(\frac{Z}{2}\right)^{1/3} > 1$$

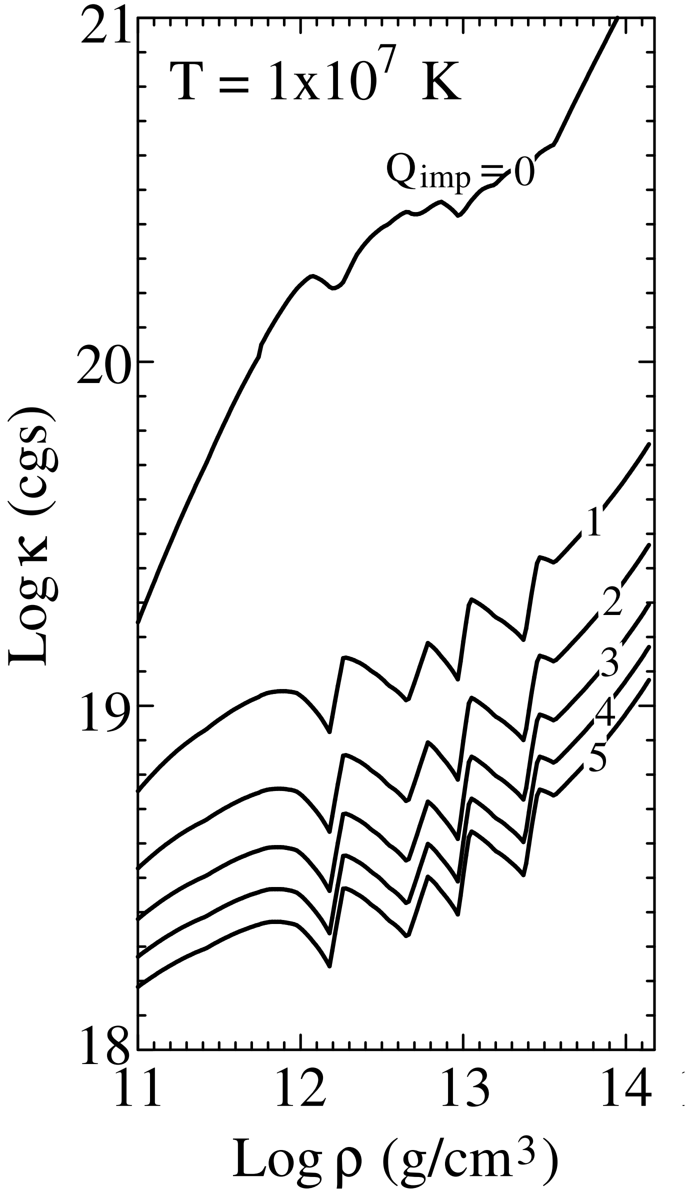
Electron Bragg scatters and emits a transverse phonon.



Flowers & Itoh (1976)

Electron Conduction

$$\kappa_e = \frac{1}{9}\mu_e^2 \, T \, \lambda_e$$



Electron-phonon:

$$\begin{cases} \lambda_e^{\text{ph}} \propto v_t^3 / T^2 & T \geq T_{\text{um}} \\ \lambda_e^{\text{ph}} \propto v_l^4 / T^3 & T \ll T_{\text{um}} \end{cases}$$

$$T_{\text{um}} = (4e^3/9\pi)\, v_t \, k_{\text{Fe}}$$

Electron-impurity:

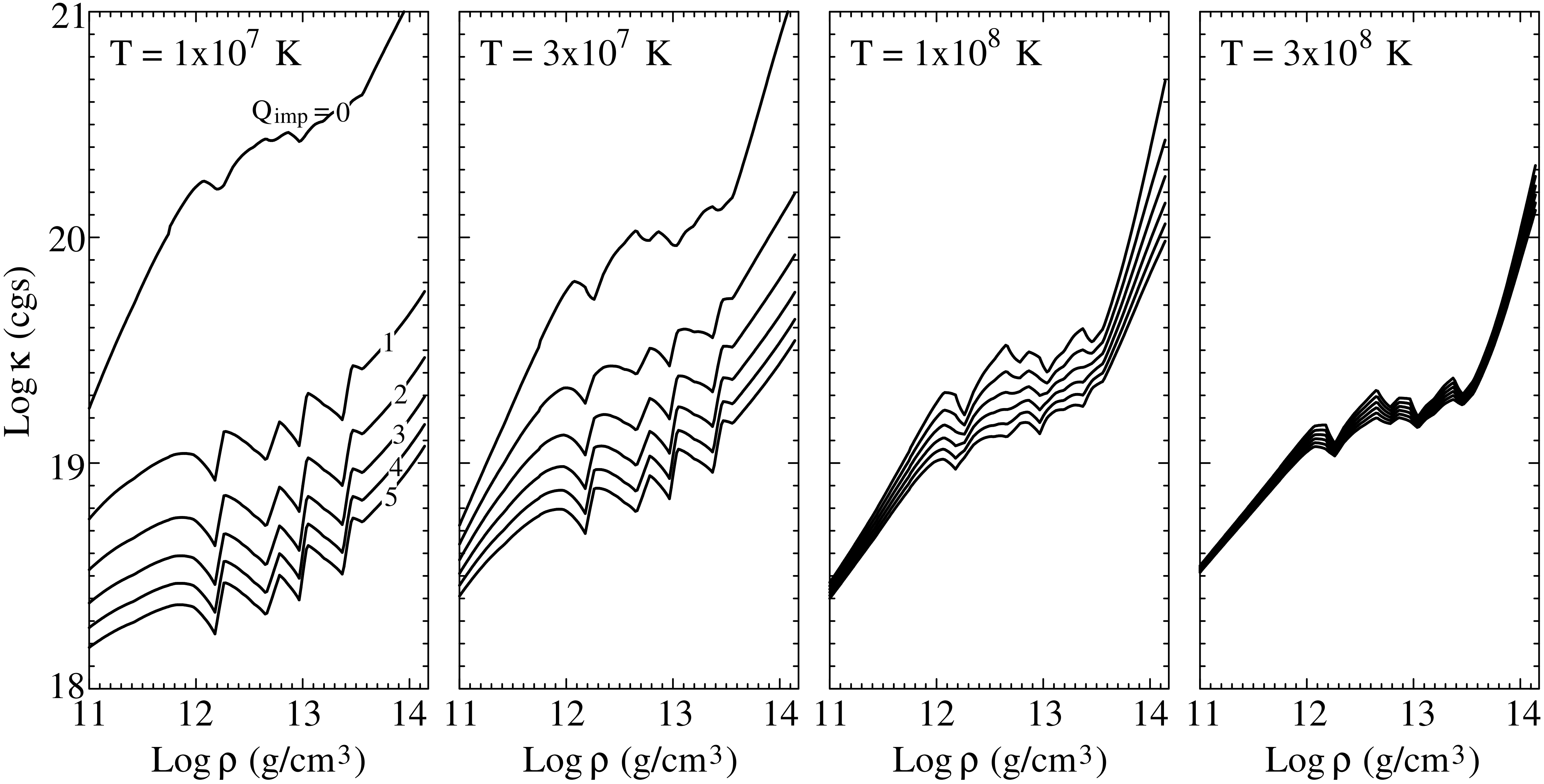
$$\lambda_e^{\text{imp}} = \frac{3\pi\langle Z\rangle}{4e^4 Q_{\text{imp}}\, k_{\text{Fe}}} \Lambda^{-1} \qquad Q_{\text{imp}} = \frac{1}{n_{\text{ion}}} \sum_i n_i \,(Z_i - \langle Z\rangle)^2$$

Impurity scattering is important at low temperature.

Flowers & Itoh (1976)

Electron Conduction

$$\kappa_e = \frac{1}{9}\mu_e^2\, T\, \lambda_e$$



Impurity scattering is important at low temperature.

Flowers & Itoh (1976)

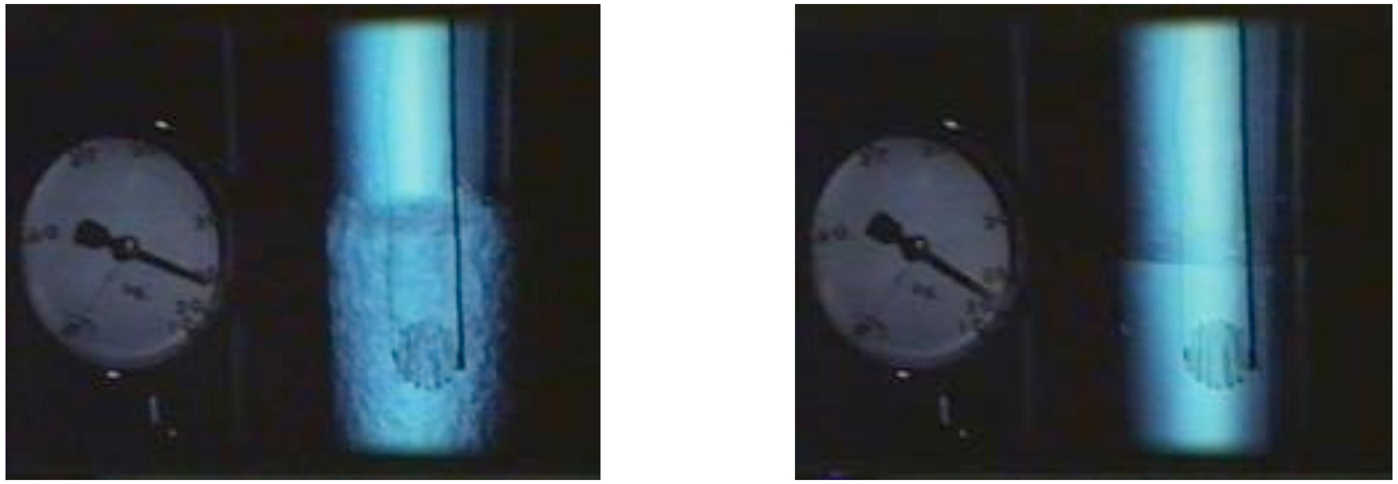
Superfluid Conduction

Its impossible to sustain a temperature gradient in bulk superfluid helium !

$$\vec{Q} = S^{(\mathrm{sPh})} T \vec{v}_n$$

$$S^{(\mathrm{sPh})} = \frac{1}{3} C_v^{(\mathrm{sPh})} = \frac{2\pi^2}{15\ c_s^3} T^3$$

Photographs: JF Allen and JMG Armitage (St Andrews University 1982).



$T>T_c$ $T<T_c$

Two fluid model: Counter-flow transports heat.
(Its the superfluid phonon fluid)

The velocity is limited only by fluid dynamics: (i) boundary shear viscosity or (ii) superfluid turbulence.

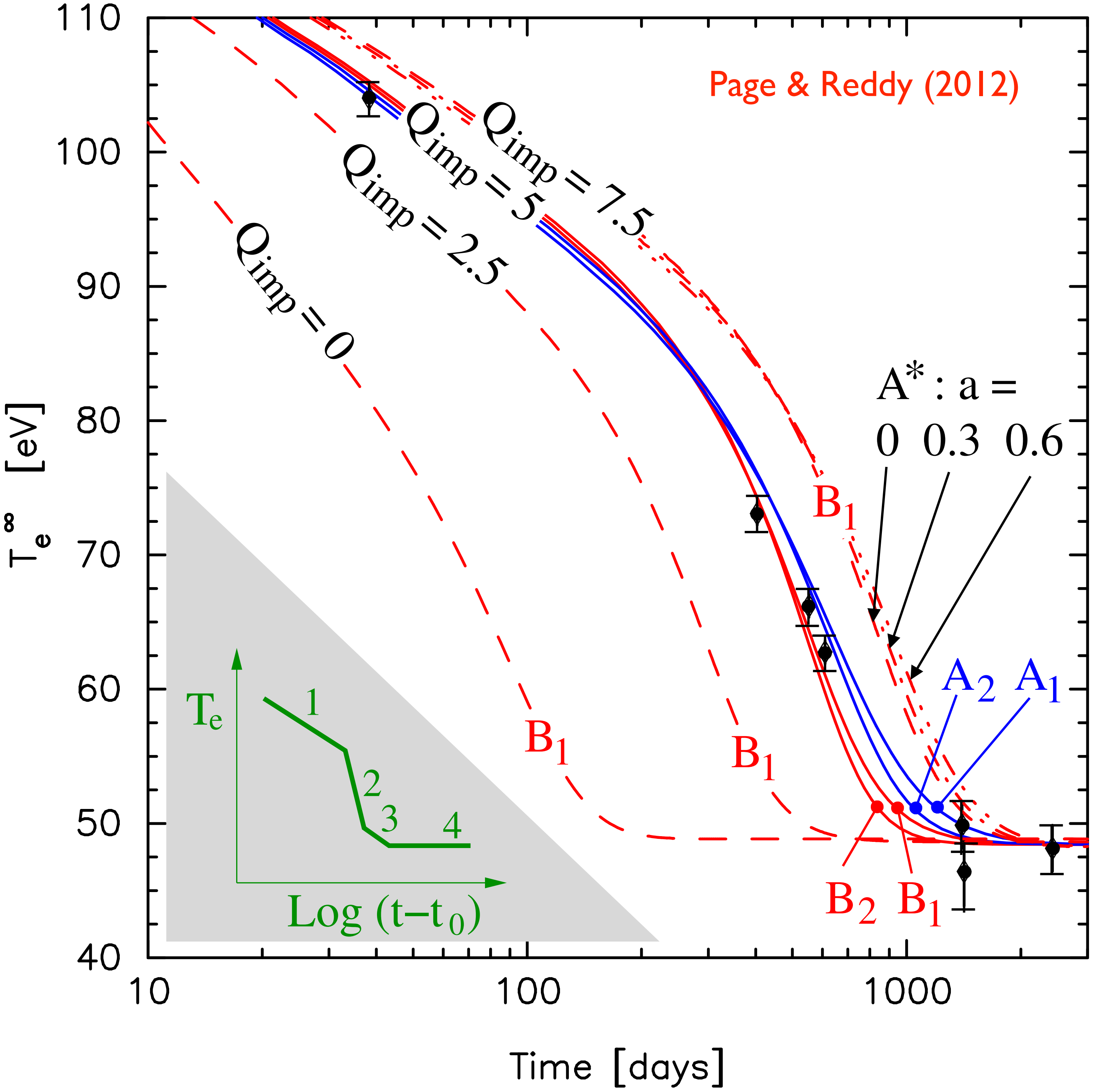
Why does this not occur in neutron stars ?
Answer: Fluid motion is damped by electrons.

Aguilera, Cirigliano, Reddy & Sharma (2009)

Unraveling thermal relaxation

- Late time signal is sensitive to inner crust thermal and transport properties.
- Impurity parameter can be fixed at earlier times. Shternin & Yakovlev (2007) Brown & Cumming (2009)
- Variations in the pairing gap (changes the fraction of normal neutrons) are discernible !

Page & Reddy (2012)



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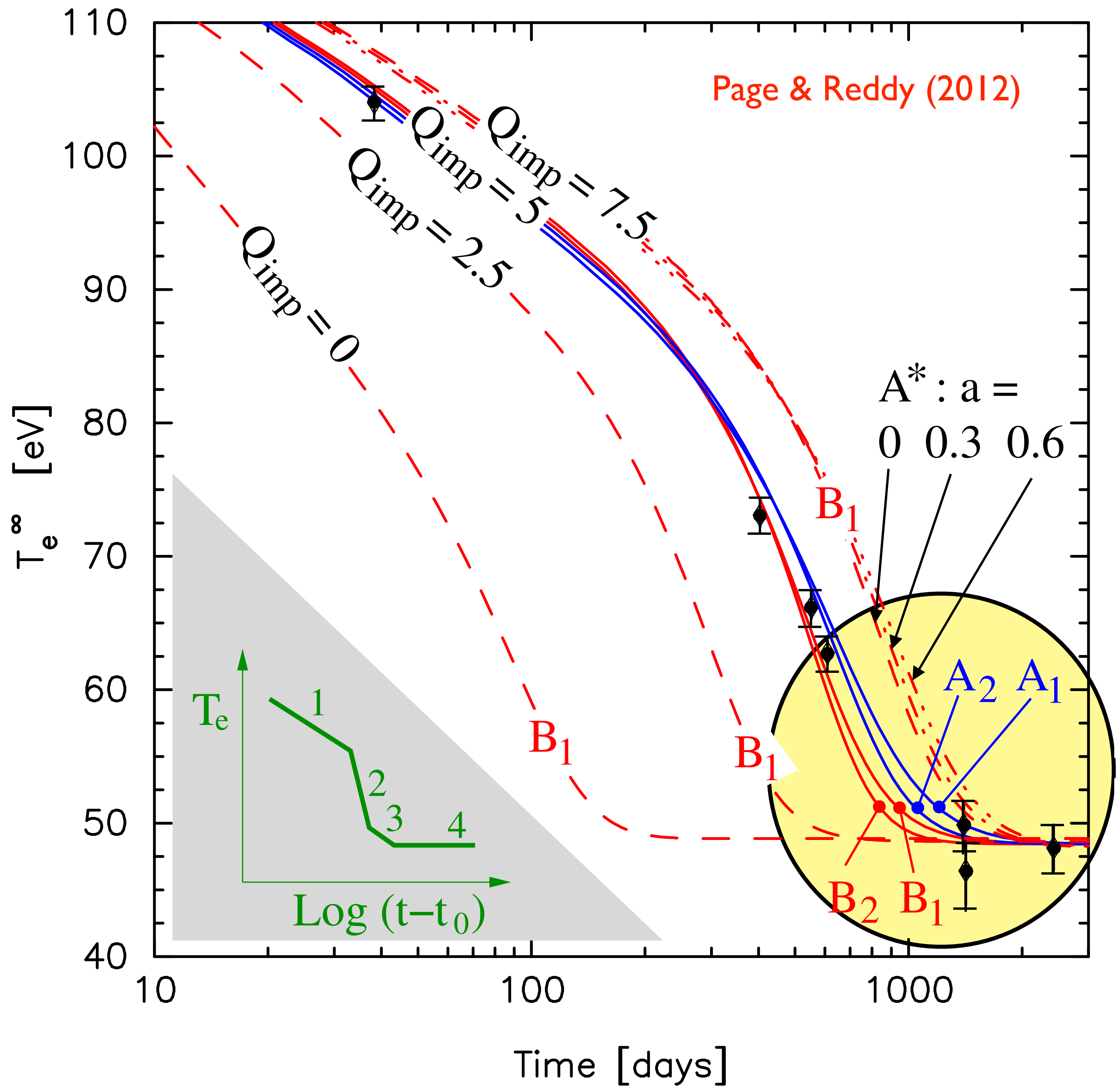

A: Low T_c - large normal fraction

B: High T_c- small normal fraction

Summary

- Thermal relaxation in neutron stars is sensitive to the low temperature properties of the crust.
- Thermal and transport properties of the inner crust (super - solid) can be calculated in terms of a few low-energy constants (LEC) of a effective theory for phonons and electrons.
- To calculate the LECs we require a microscopic description of the ground state of exotic neutron rich phases at sub-nuclear density.
- Potentially observable and likely to have implications for both thermal evolution and seismology !