

Nuclear mass tables from energy density functionals

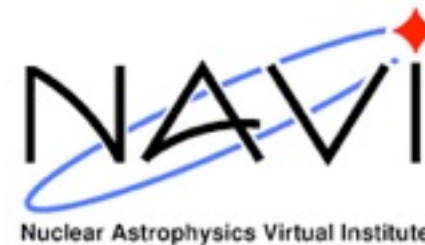
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für Bildung
und Forschung



TECHNISCHE
UNIVERSITÄT
DARMSTADT



Nuclear Astrophysics Virtual Institute



Helmholtz International Center

Outline



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1. Introduction

2. Convergence and numerical noise

3. Beyond mean field effects

4. Summary and outlook

Motivation

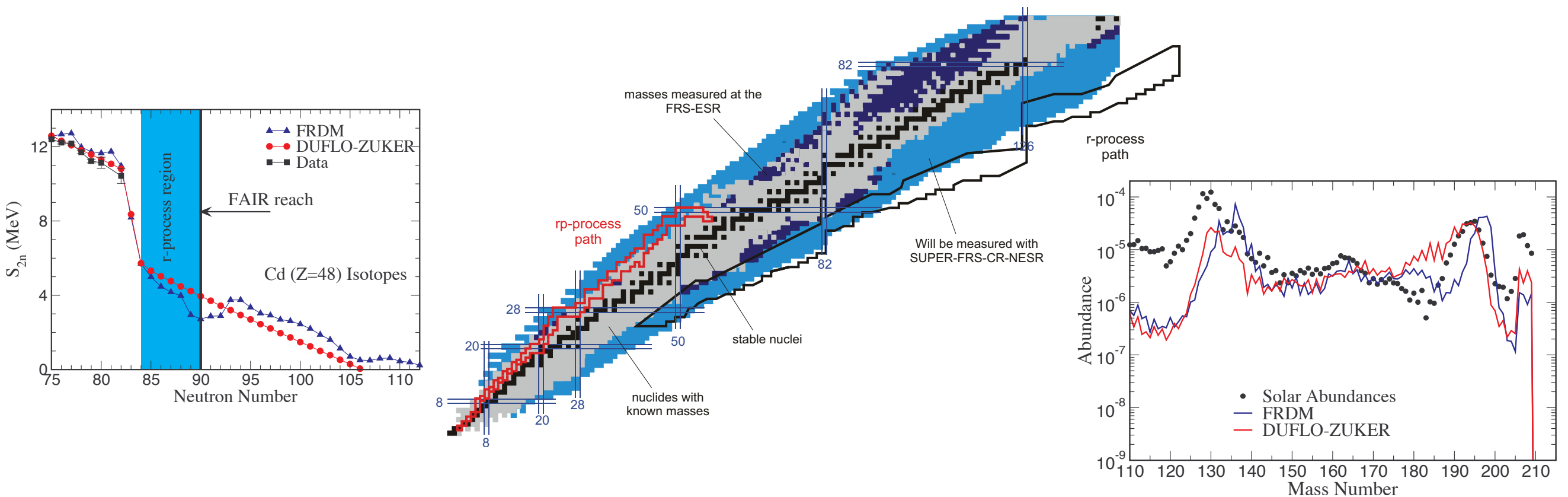
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2. Convergence and numerical noise

3. Beyond mean field effects

4. Summary and outlook

- Nuclear masses are one of the most relevant input for nucleosynthesis calculations, in particular for the r-process.
- Masses (separation energies) affect significantly (n,γ) capture rates, (γ,n) photodissociation reactions and Q -values for β -decay.
- Only few nuclei are/will be experimentally explored in the relevant region for r-process nucleosynthesis \Rightarrow we require theoretical predictions.



Courtesy: J. Mendoza-Temis

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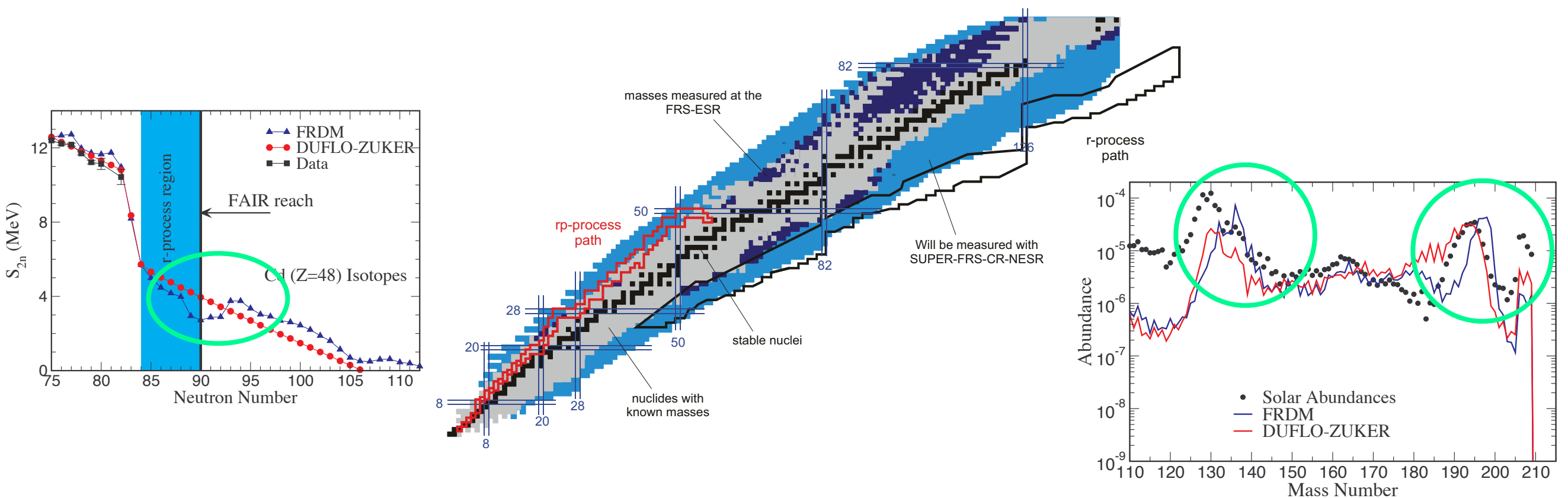
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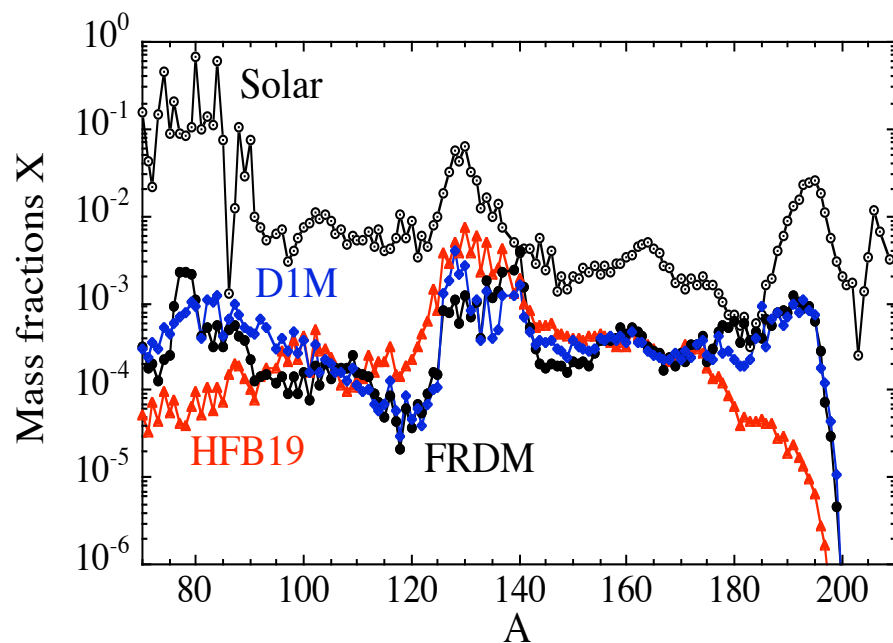


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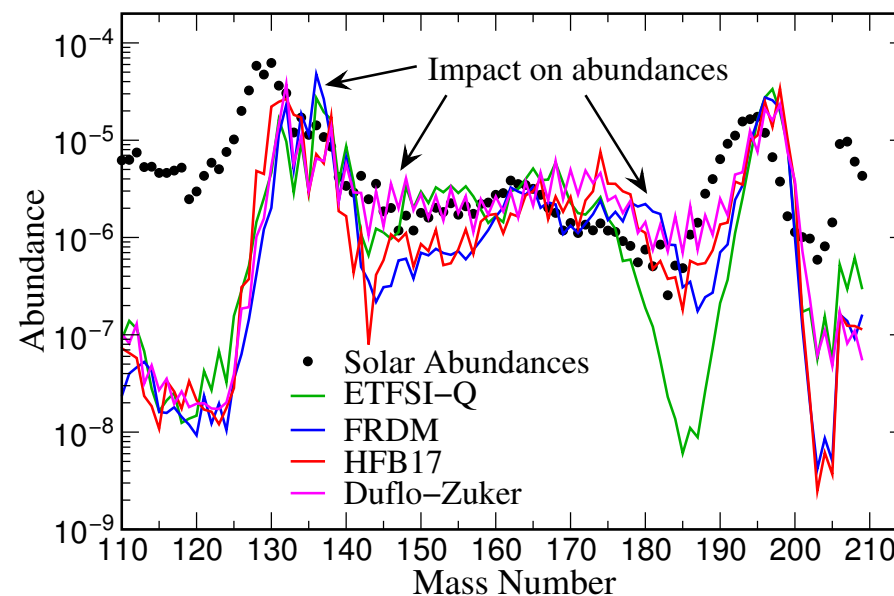
Motivation

- Impact of the nuclear mass model on r-process nucleosynthesis calculations:

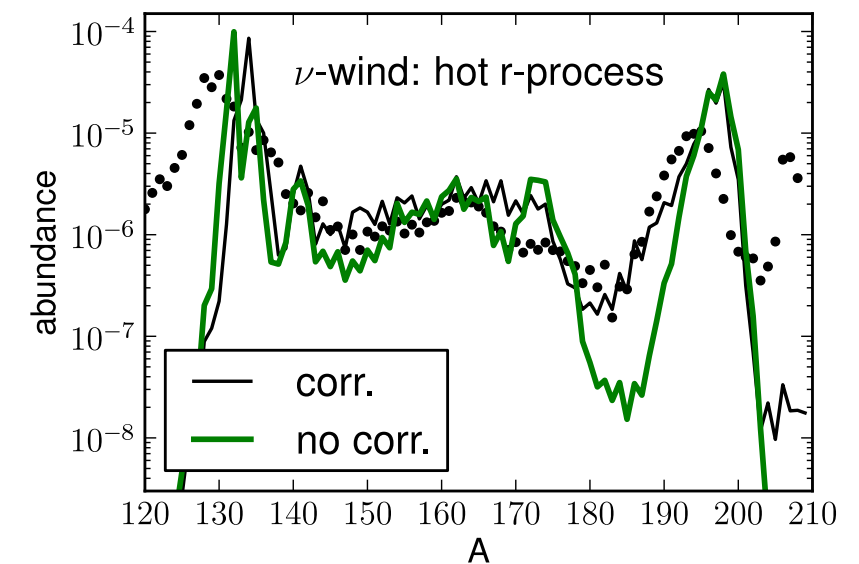
Final abundances depend on the mass model used (for the same astrophysical conditions)



Goriely et al.



Arcones and Martínez-Pinedo, PRC 83, 045809 (2011)



Arcones and Bertsch, PRL 108, 151101(2012)

Mass models

- Experimental masses where available: ~ 2149 (Audi *et al.* 2003).
- Theoretical global nuclear mass models widely used in nucleosynthesis calculations:
 - ➔ Finite Range Droplet Model (FRDM). (Möller *et al.* 1995)
 - ➔ Extended Thomas-Fermi plus Strutinsky Integral (ETFSI). (Aboussir *et al.* 1995)
 - ➔ Duflo-Zuker (DZ) functional based on Shell Model. (Duflo and Zuker 1995)
 - ➔ Self-consistent mean field models based on Hartree-Fock-Bogoliubov approximations:
 - ▶ Skyrme HFB-* (Goriely *et al.* 2009)
 - ▶ Gogny DIM (Goriely *et al.* 2009)
 - ▶ UNEDF (J. Erler *et al.* 2012)

Typical r.m.s. deviation from the experimental data ~ 0.6 MeV

Microscopic mass models

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2. Convergence and numerical noise

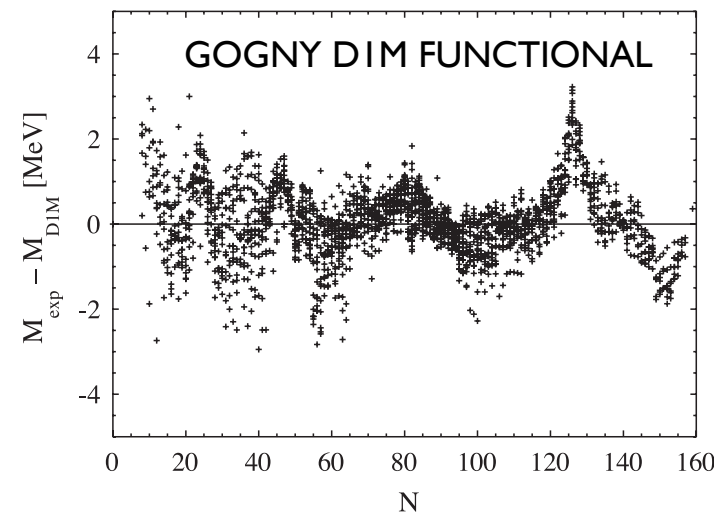
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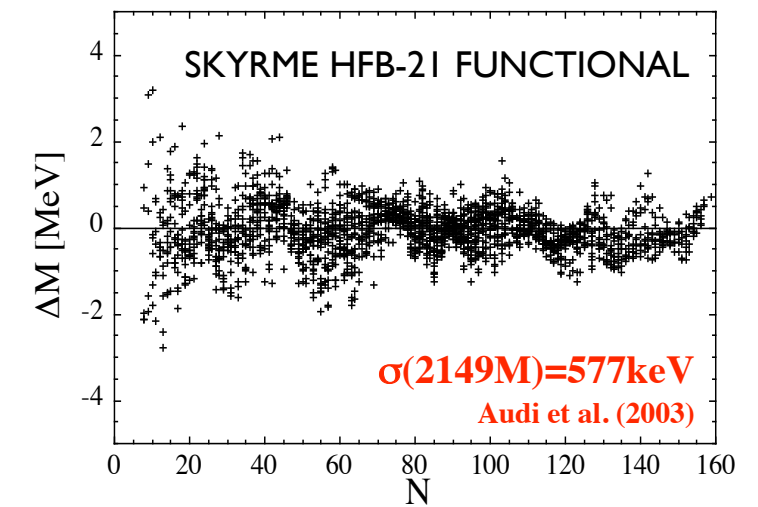
- Self-consistent mean field approximations provide a very good description of known data.

- There are still some problems in transitional regions and local uncertainties:

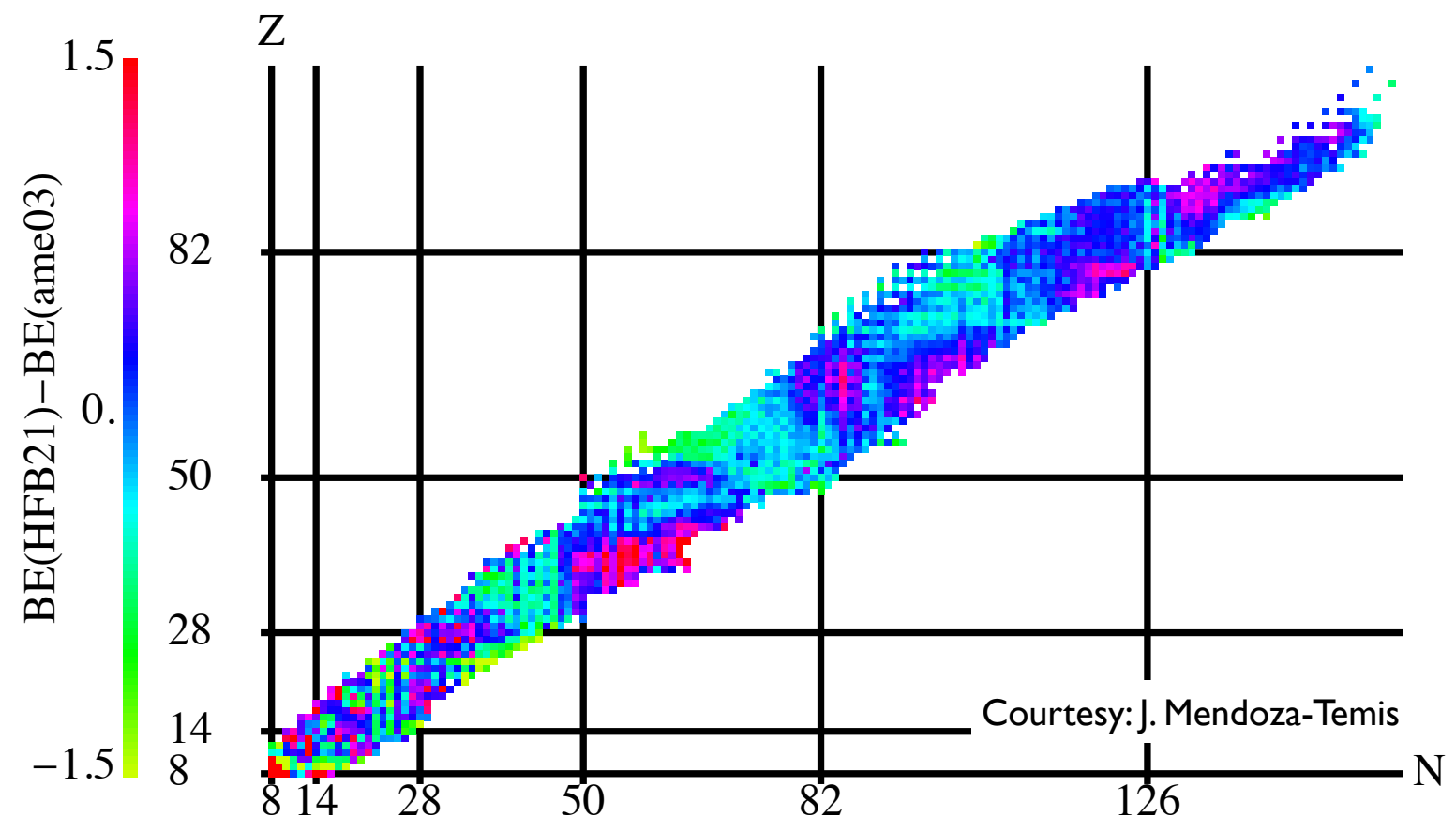
- Numerical noise.
- Some physics missing: Restoration of broken symmetries and configuration mixing.
- Nuclei with odd number of protons/neutrons are not treated in equal footing as the even-even ones



Goriely et al., PRL 102, 242501 (2009)



Goriely et al., PRL 102, 152503 (2009)



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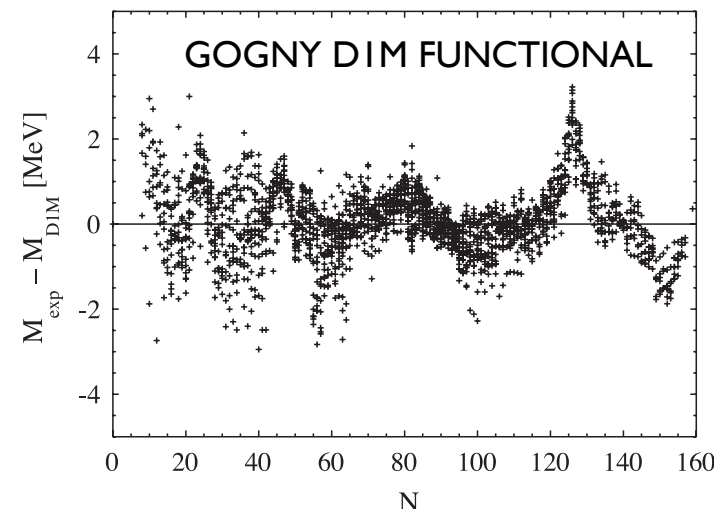
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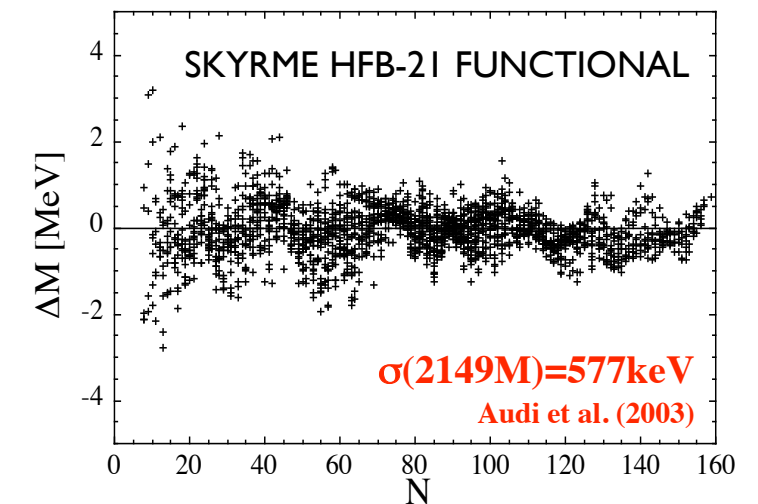
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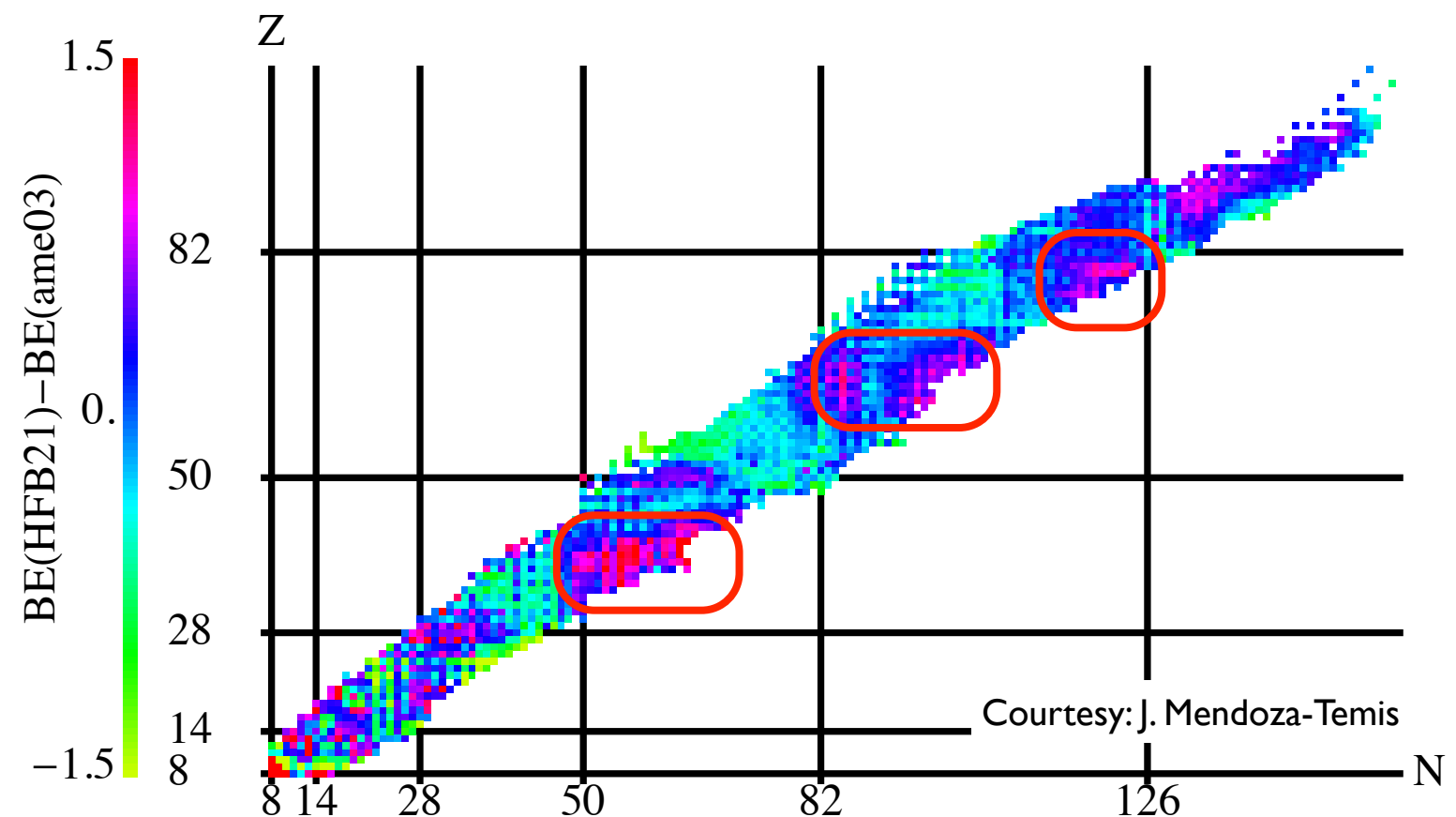
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Self-consistent (beyond) mean field description

- **Effective nucleon-nucleon interaction:**

Gogny force (DIS-DIM) that is able to describe properly many phenomena along the whole nuclear chart.

$$\begin{aligned} V(1, 2) = & \sum_{i=1}^2 e^{-(\vec{r}_1 - \vec{r}_2)^2 / \mu_i^2} (W_i + B_i P^\sigma - H_i P^\tau - M_i P^\sigma P^\tau) \\ & + iW_0(\sigma_1 + \sigma_2) \vec{k} \times \delta(\vec{r}_1 - \vec{r}_2) \vec{k} + t_3(1 + x_0 P^\sigma) \delta(\vec{r}_1 - \vec{r}_2) \rho^\alpha((\vec{r}_1 + \vec{r}_2)/2) \\ & + V_{\text{Coulomb}}(\vec{r}_1, \vec{r}_2) \end{aligned}$$

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spin-orbit term

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- **Methods of solving the many-body problem: Variational approaches**

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- **Methods of solving the many-body problem: Variational approaches**

➔ Parameters of the effective interaction are fitted to reproduce experimental data solving the many-body problem at certain level of approximation (mean field normally).

Self-consistent mean field

Hartree-Fock-Bogoliubov (HFB)

Variational space: $\{|\Phi(\vec{q})\rangle\}$ set of **product-type** wave functions which fulfill:

- Quasiparticle vacua:

$$\alpha_k(\vec{q})|\Phi(\vec{q})\rangle = 0$$

- Most general linear combination of the arbitrary single particle basis:

$$\alpha_k^\dagger(\vec{q}) = \sum_l U_{lk}(\vec{q})c_l^\dagger + V_{lk}(\vec{q})c_l$$

- Fermionic operators:

$$\{\alpha_k^\dagger(\vec{q}), \alpha_{k'}(\vec{q})\} = \delta_{kk'}; \{\alpha_k^\dagger(\vec{q}), \alpha_{k'}^\dagger(\vec{q})\} = \{\alpha_k(\vec{q}), \alpha_{k'}(\vec{q})\} = 0$$

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$|\text{HFB}(\vec{q})\rangle$ **Product Type**

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**I. finite basis!!
convergence?**

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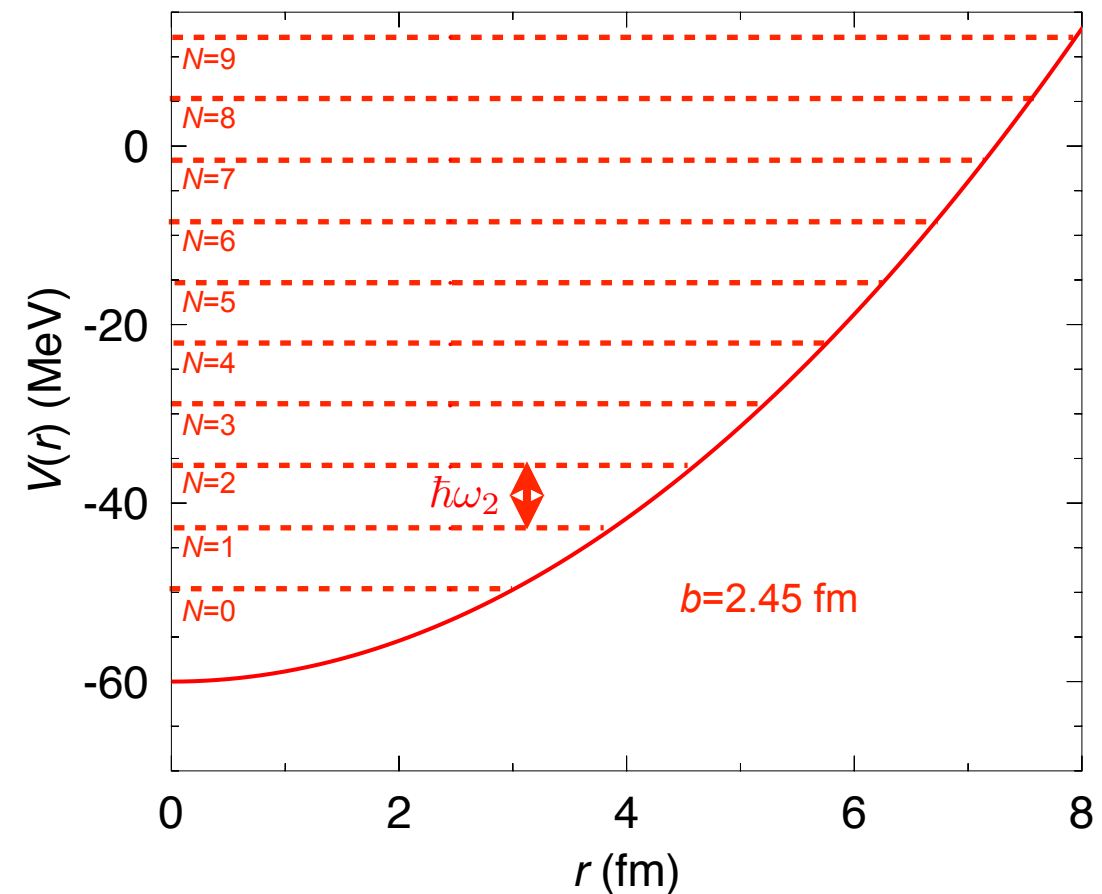
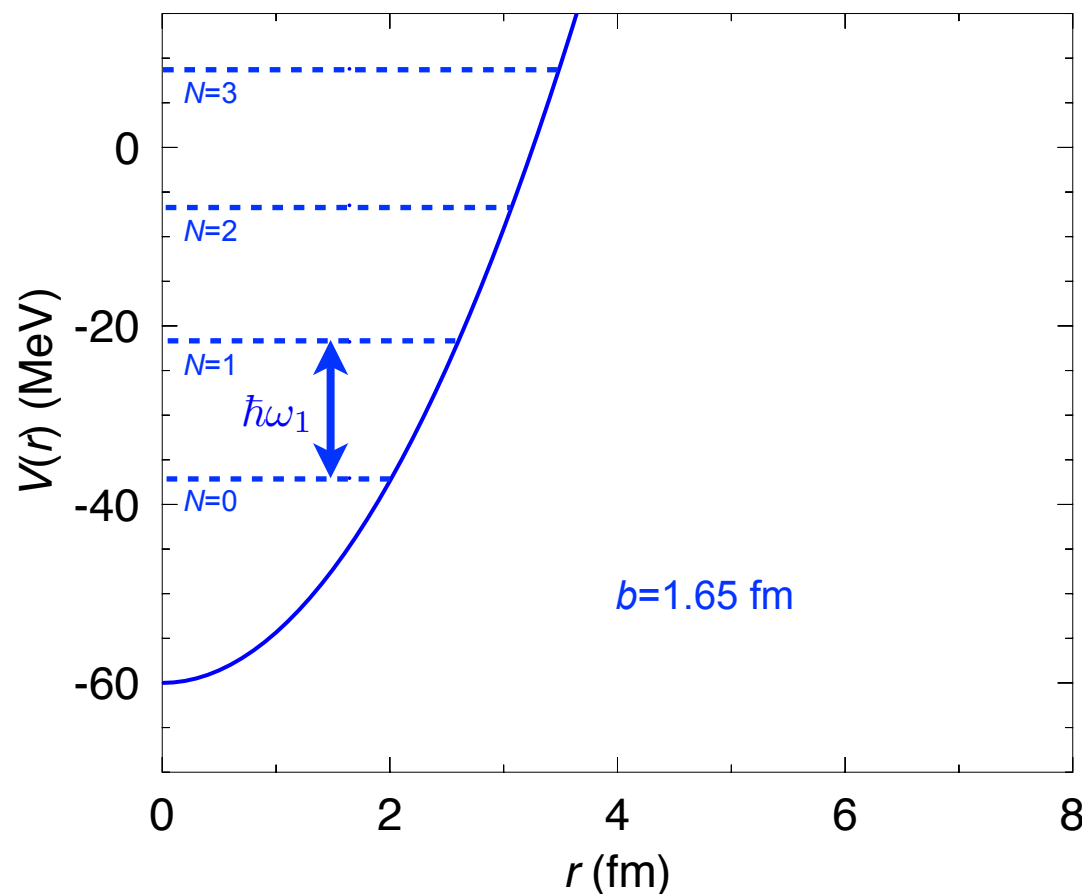
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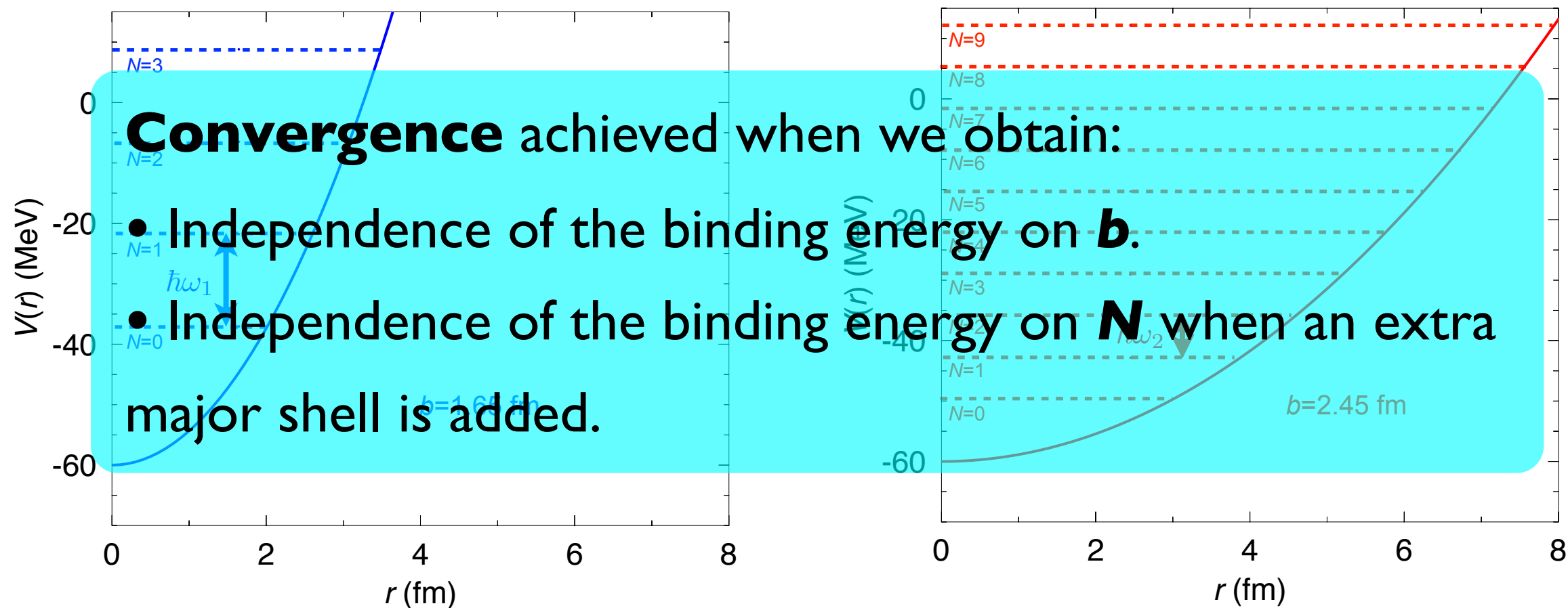


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Convergence

Examples in *ab-initio* calculations

P. Maris et al., Phys. Rev C 79 (2009)

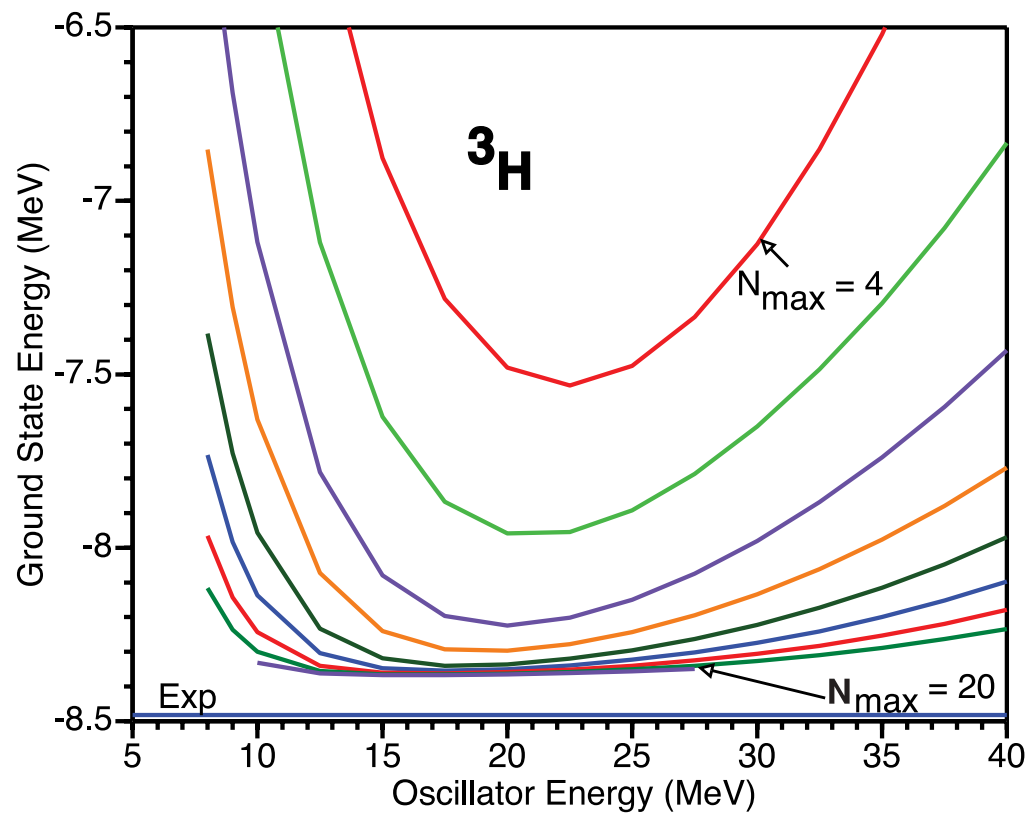


FIG. 10. (Color online) Calculated ground-state energy of ${}^3\text{H}$ as a function of the oscillator energy, $\hbar\Omega$, for selected values of N_{max} . The curve closest to experiment corresponds to the value $N_{\text{max}} = 20$ and successively higher curves are obtained with N_{max} decreased by two units for each curve.

R. Roth, Phys. Rev C 79 (2009)

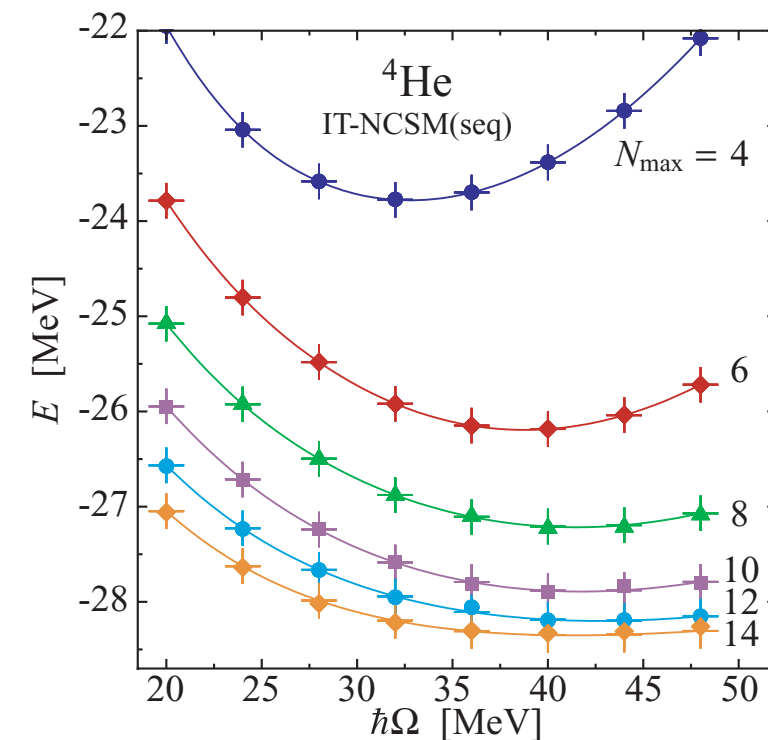


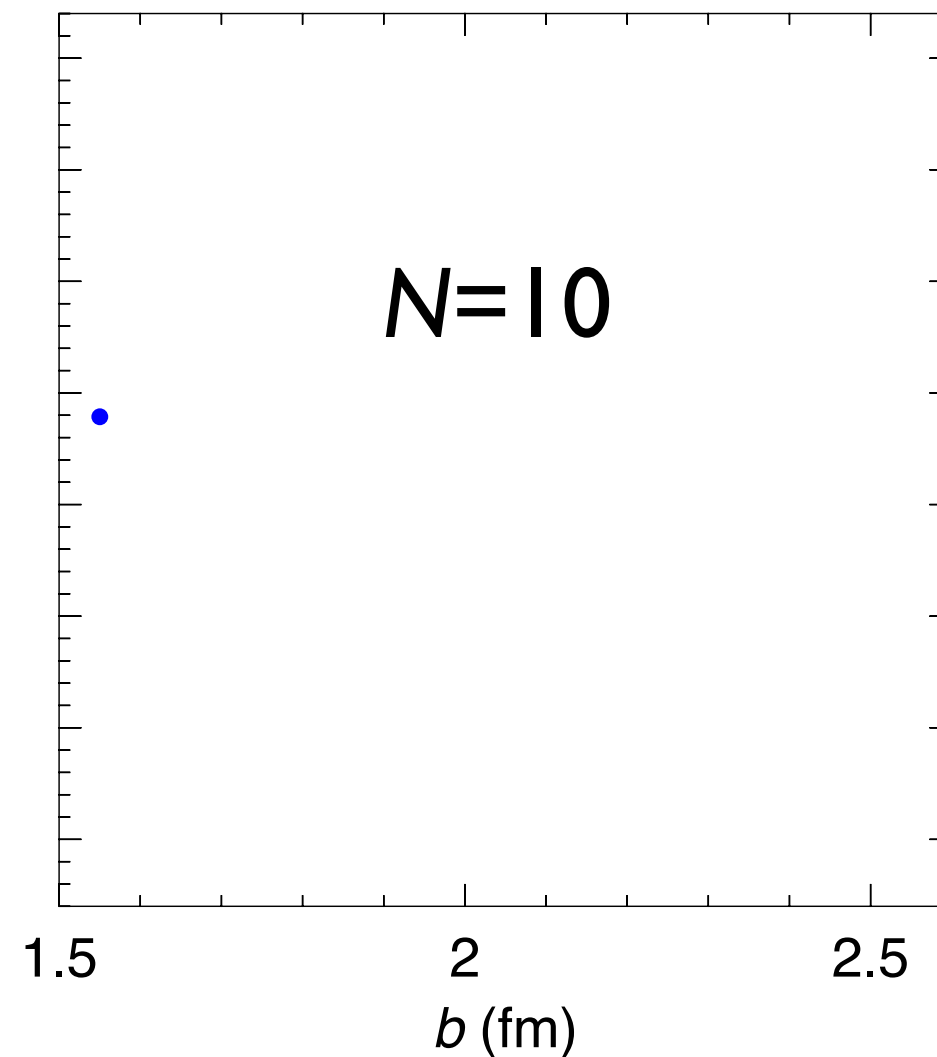
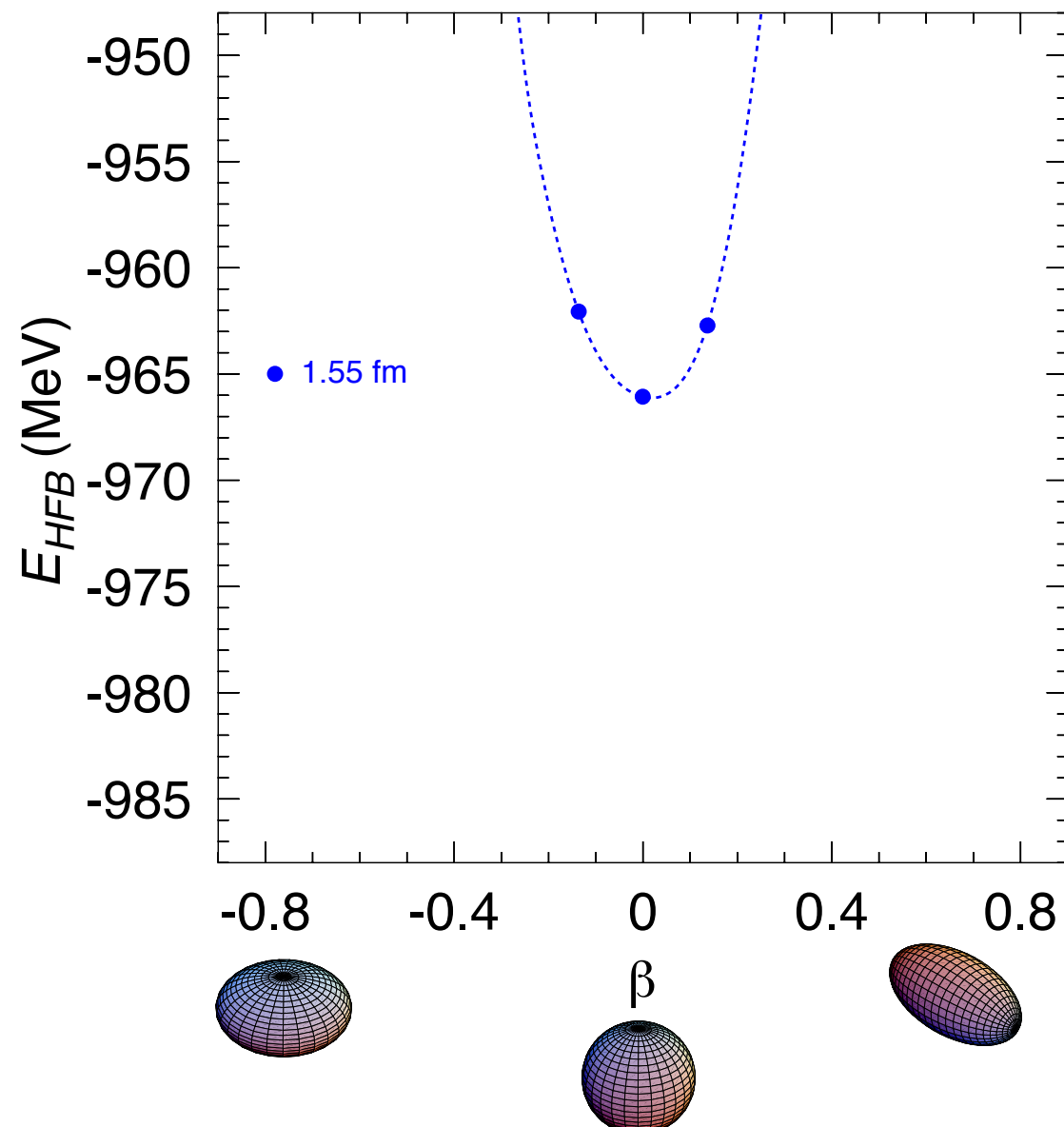
FIG. 7. (Color online) Ground-state energies of ${}^4\text{He}$ obtained for the V_{UCOM} interaction as a function of the oscillator frequency $\hbar\Omega$ for different $N_{\text{max}}\hbar\Omega$ model spaces. Results of IT-NCSM(seq) calculations (solid symbols) are compared with full NCSM calculations (crosses).

Convergence

Effects of deformation on the convergence

Example:

β^-	
Cd116	
0+	
7.49	
Ag115	

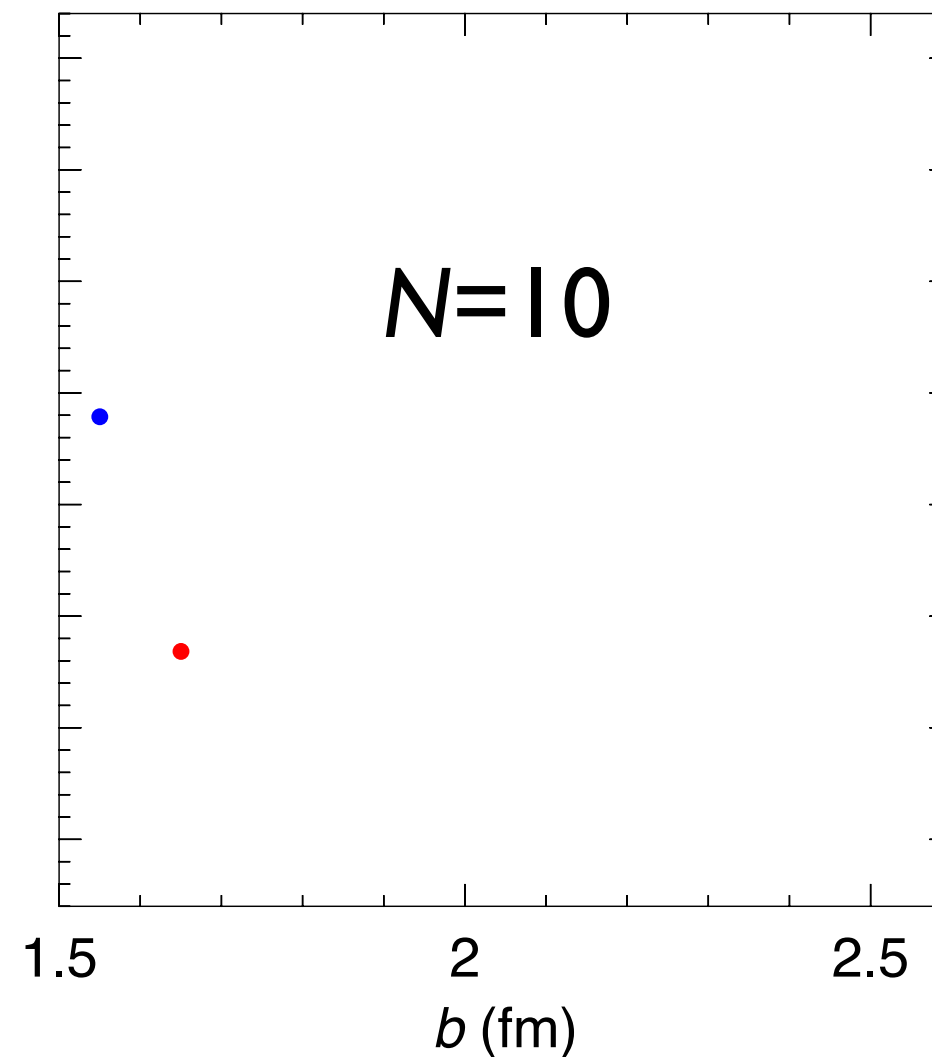
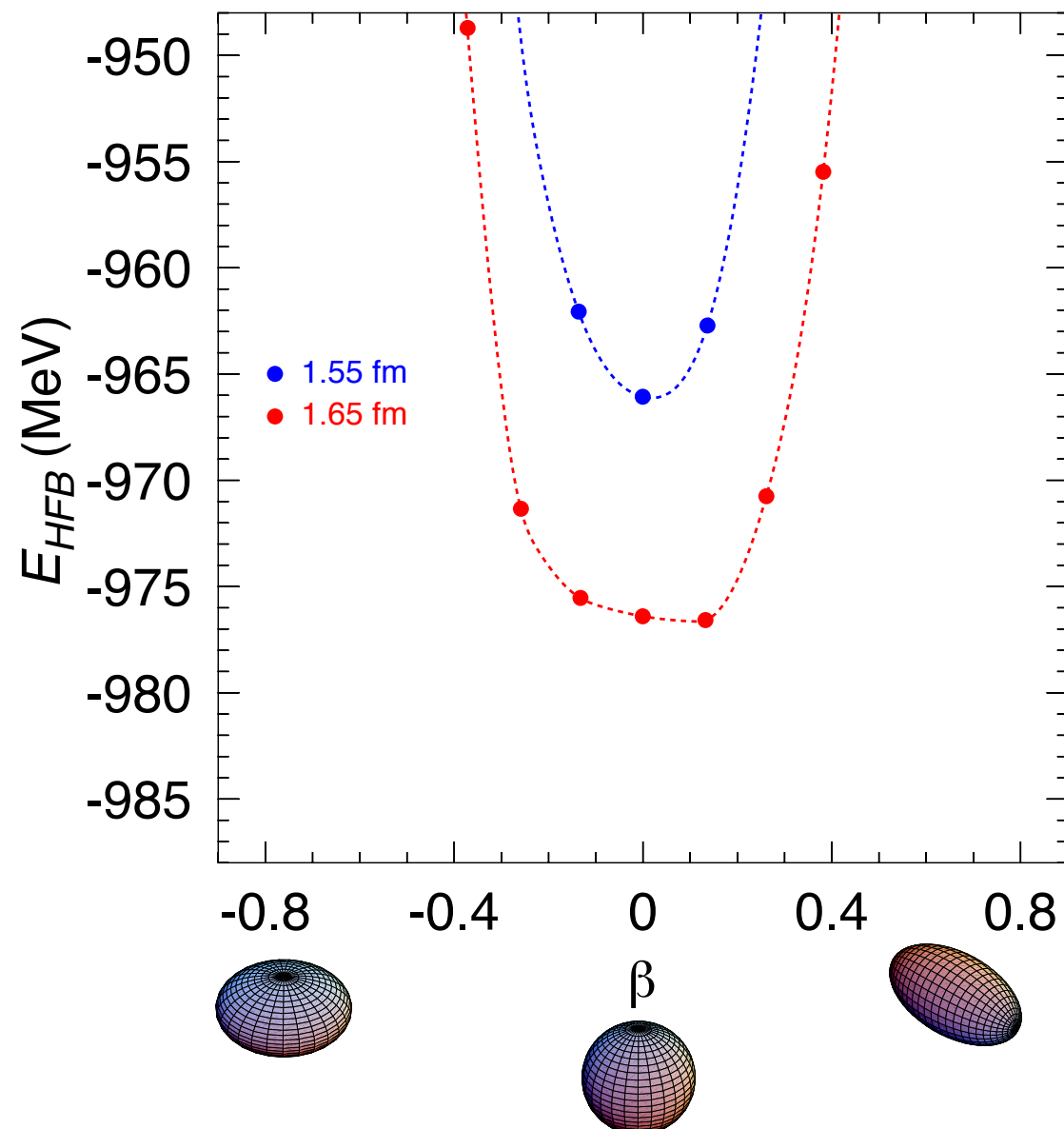


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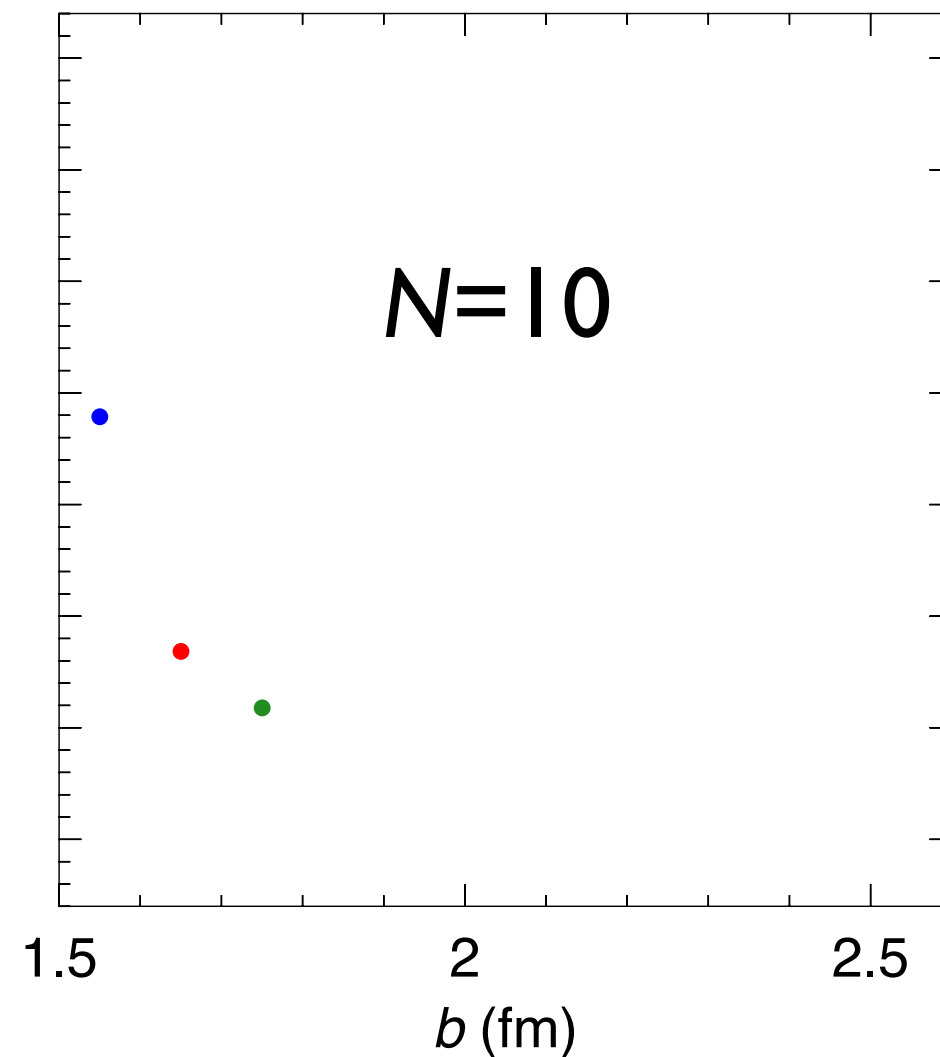
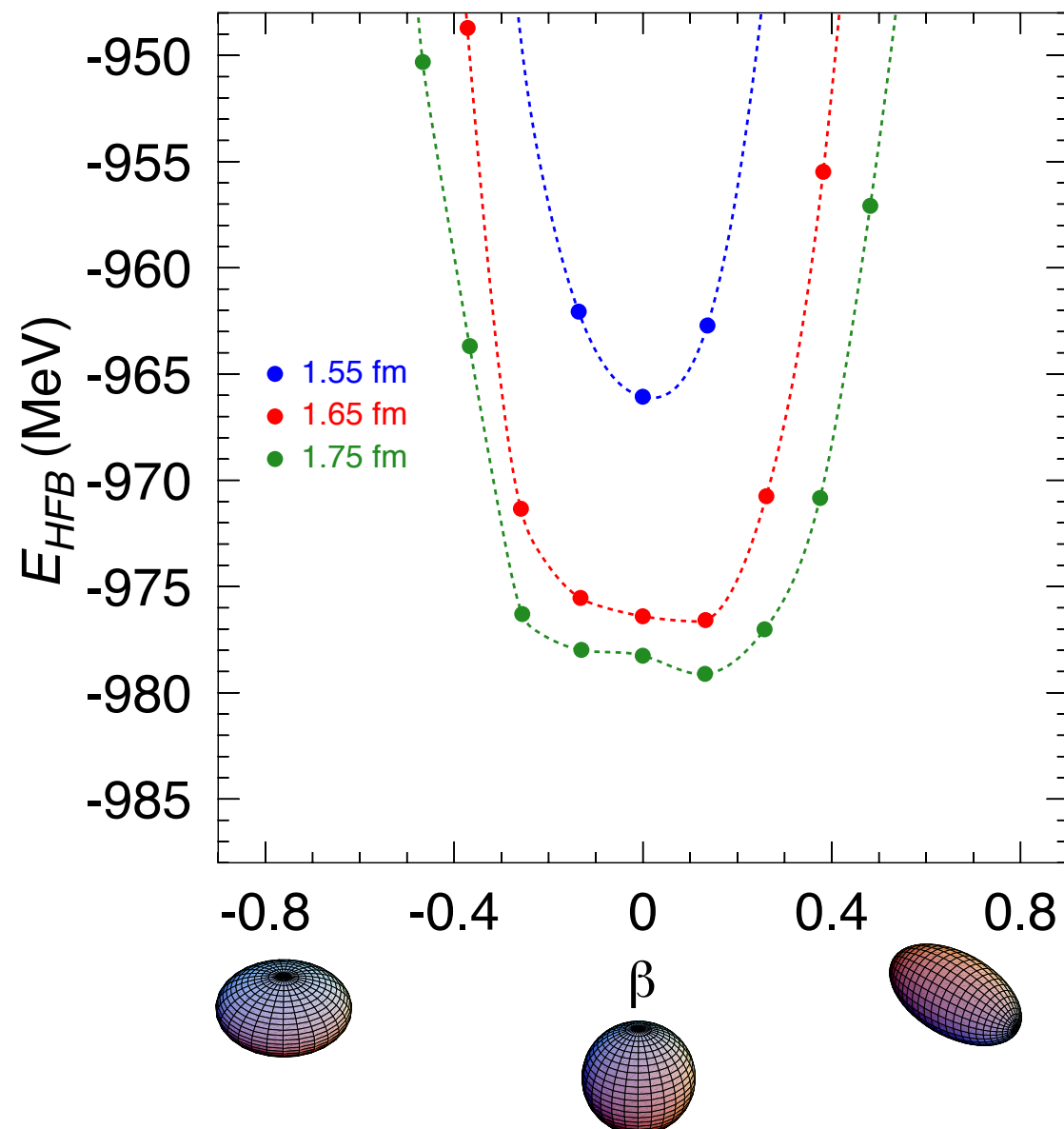


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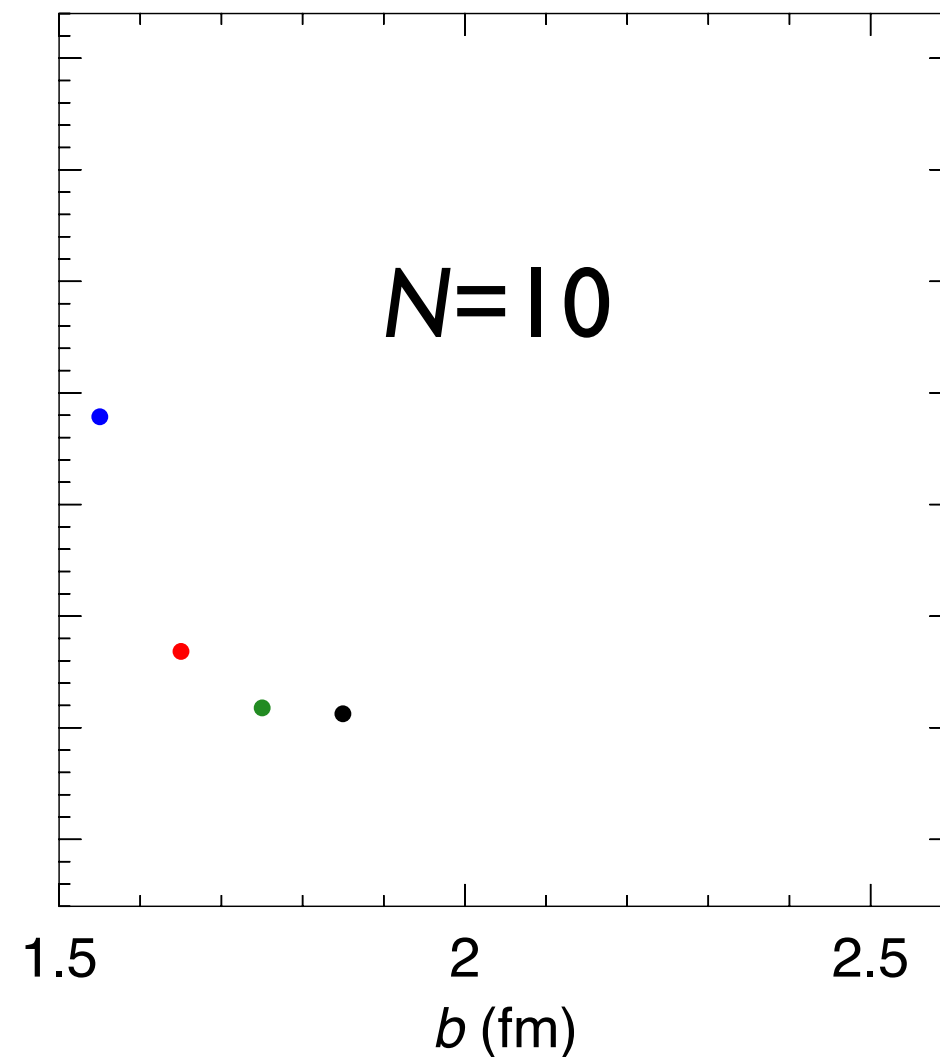
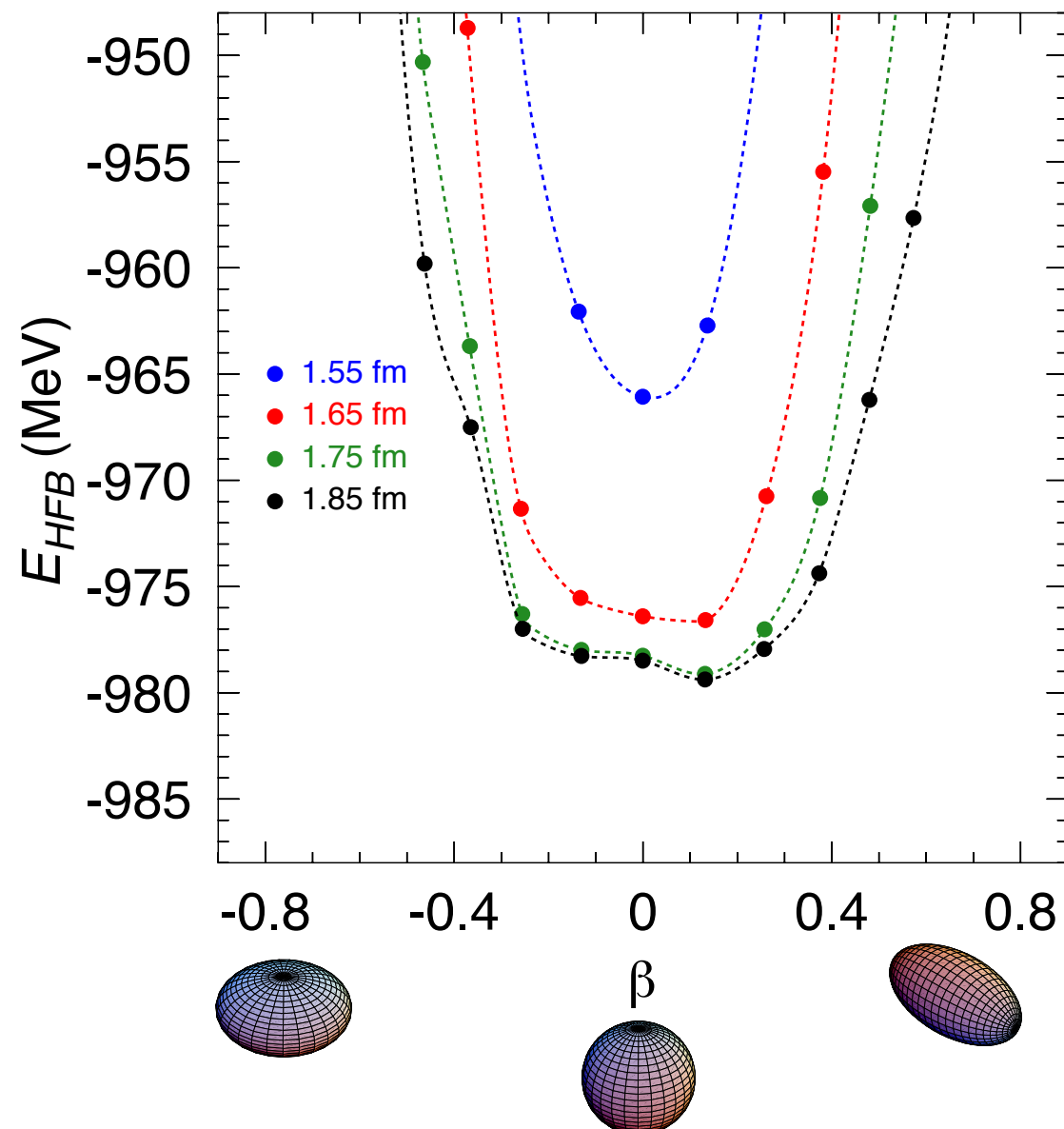


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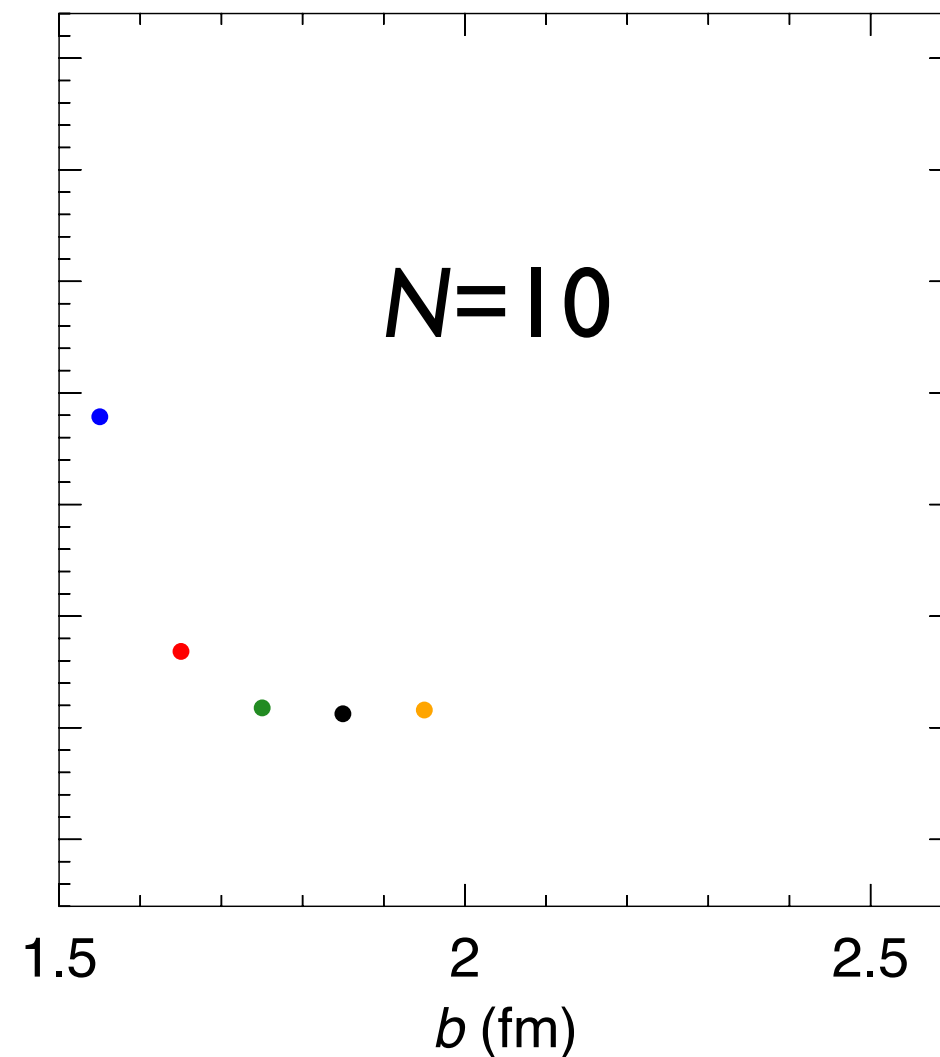
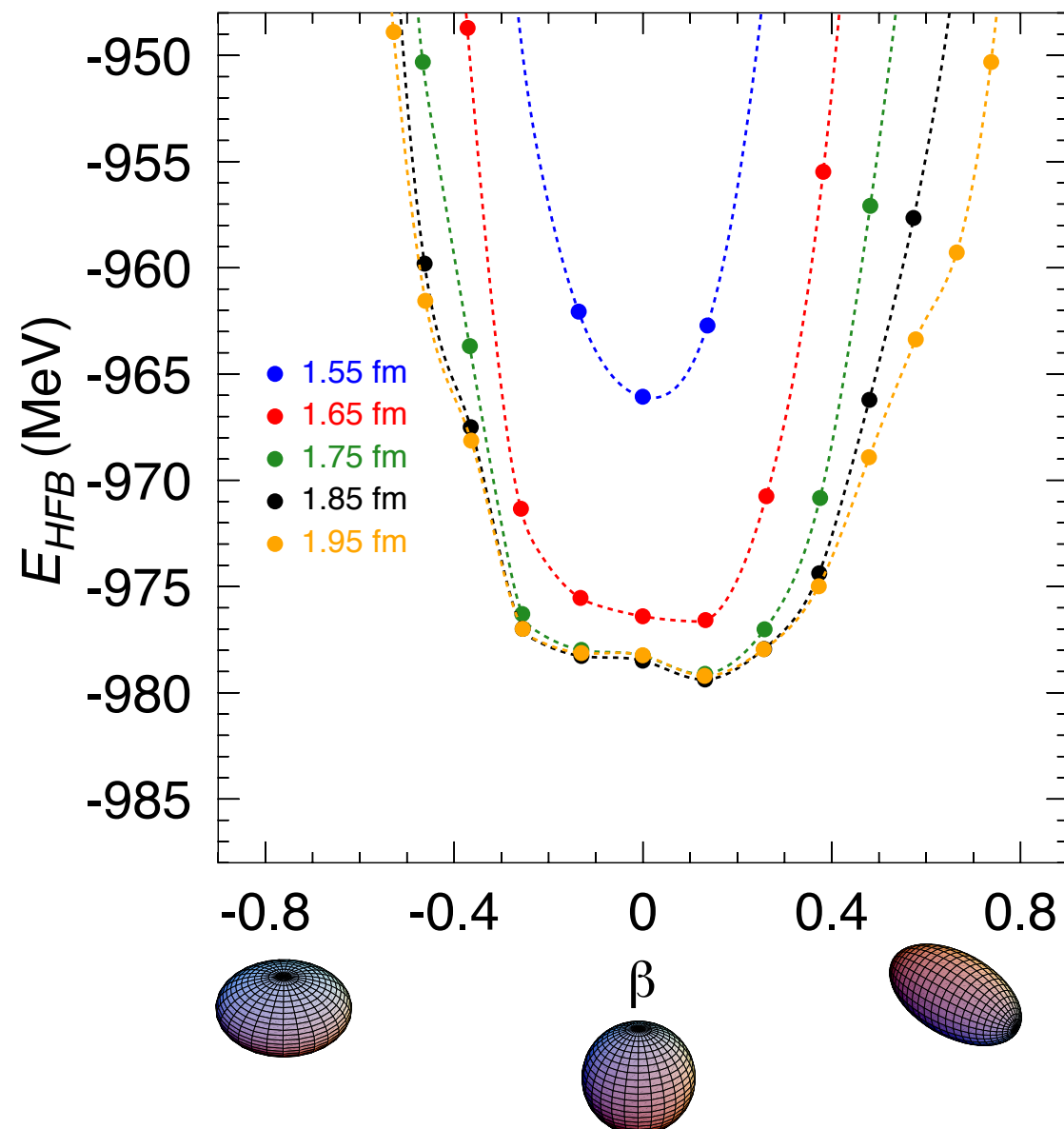


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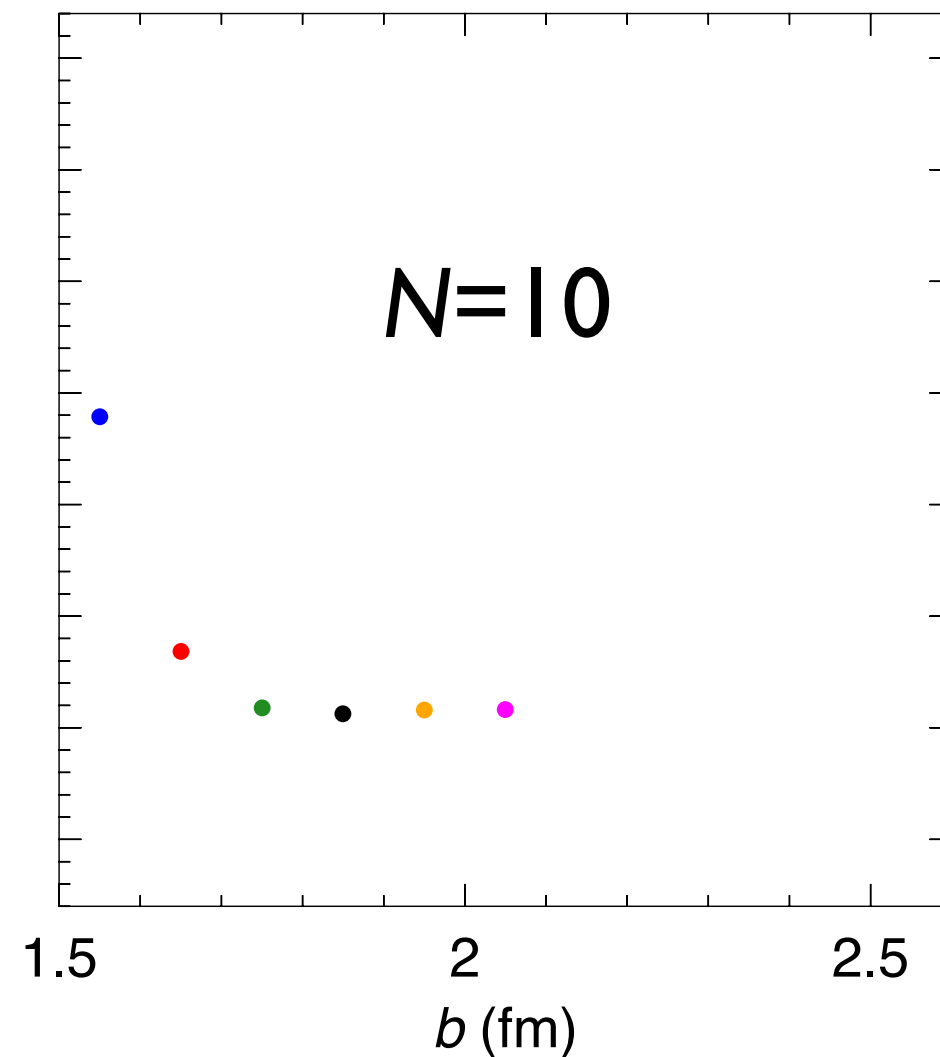
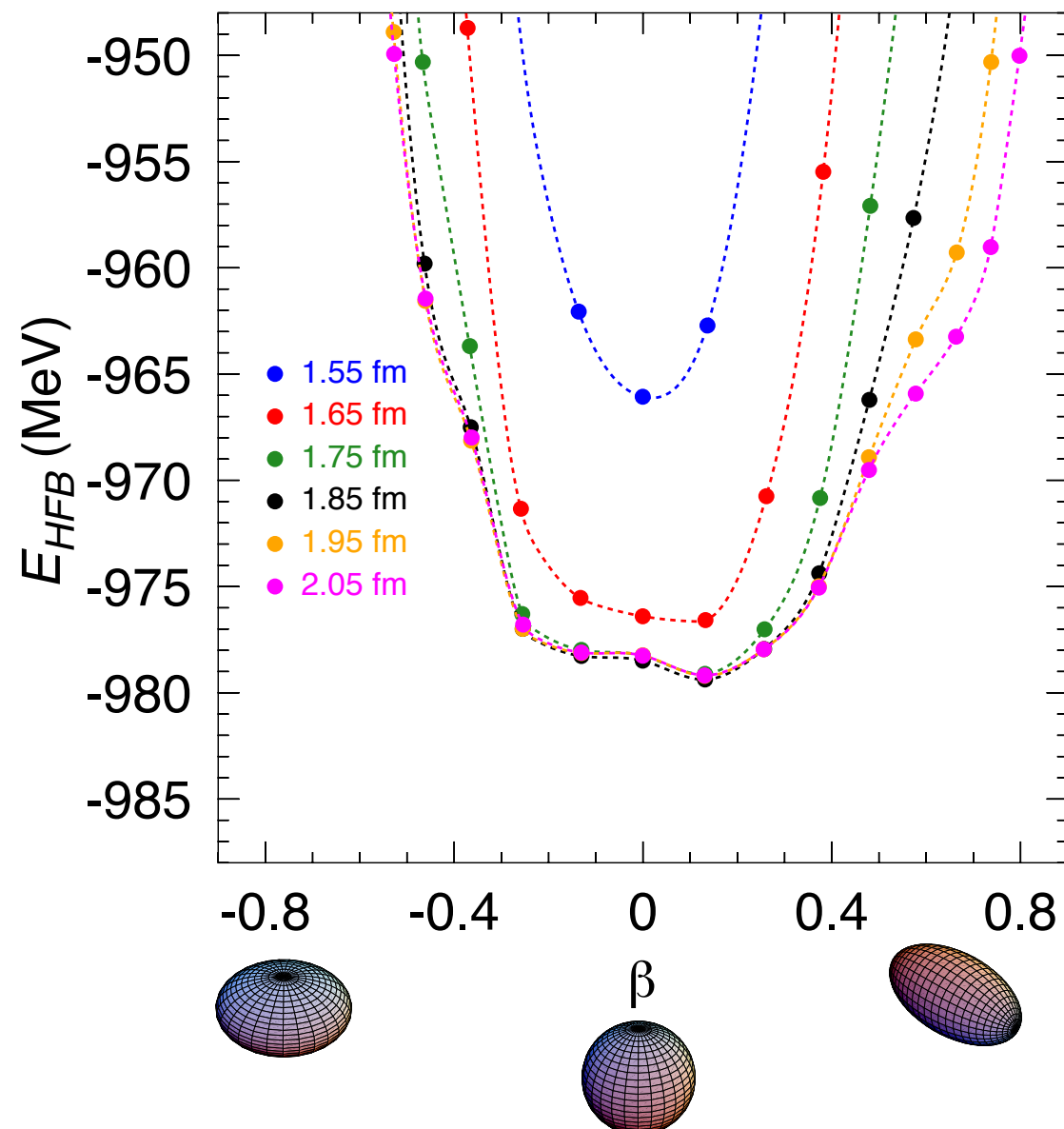


Convergence

Effects of deformation on the convergence

Example:

β^-	β^+
Cd116	
0+	
7.49	
Ag115	

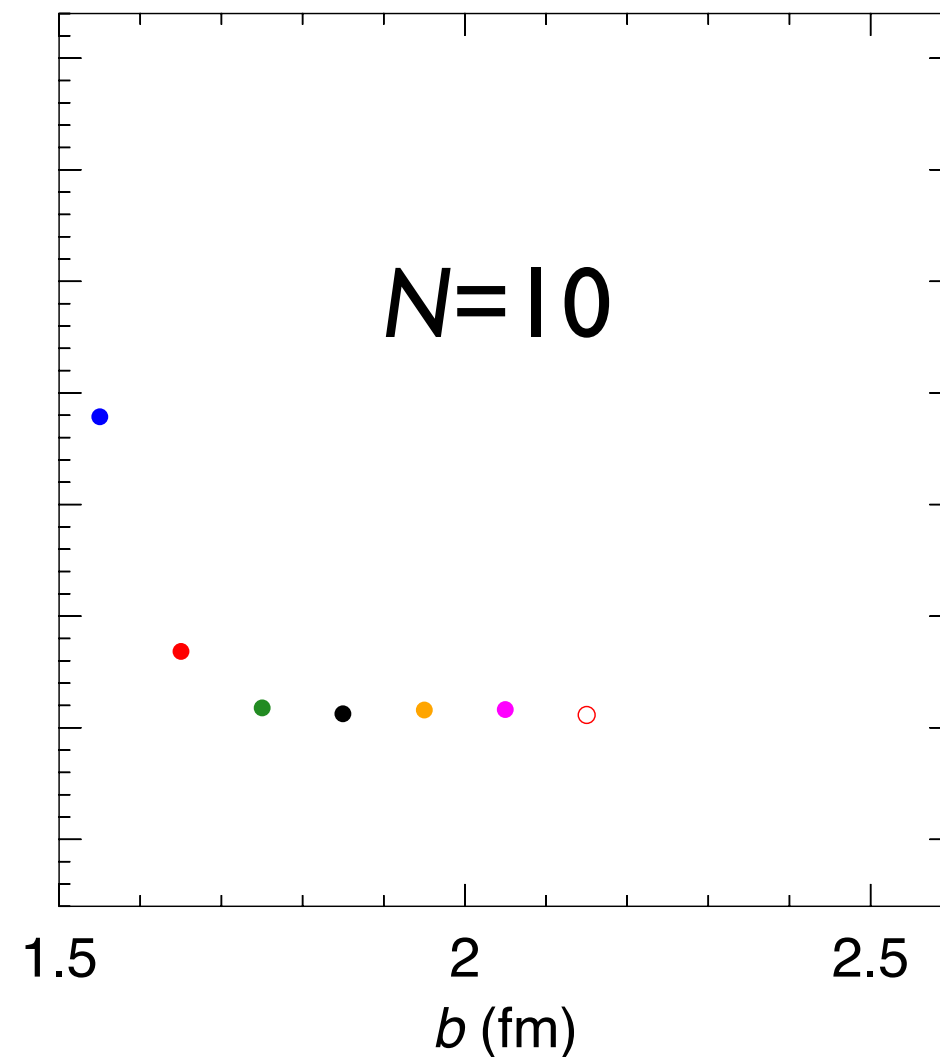
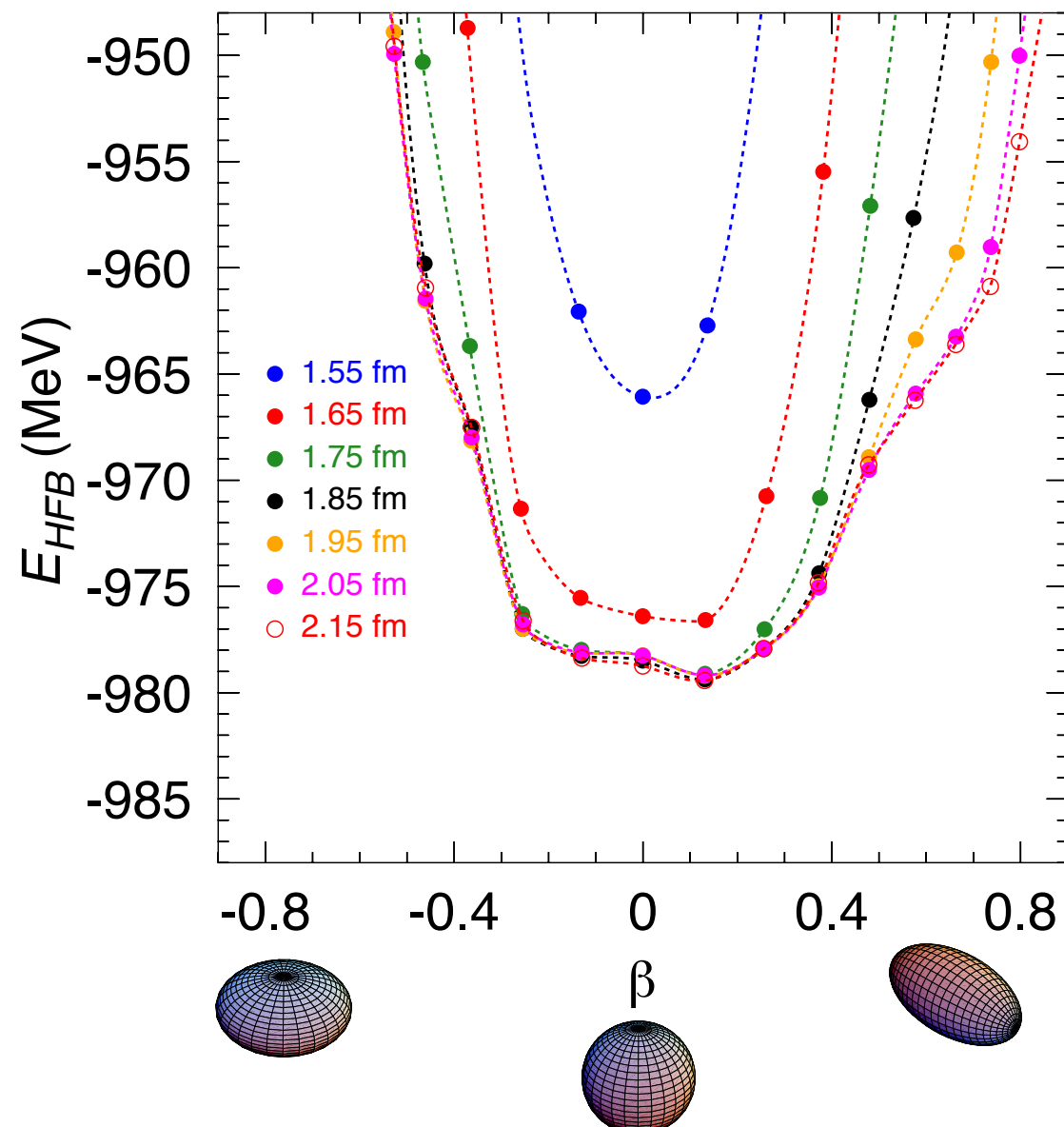


Convergence

Effects of deformation on the convergence

Example:

β^-	
Cd116	
0+	
7.49	
Ag115	

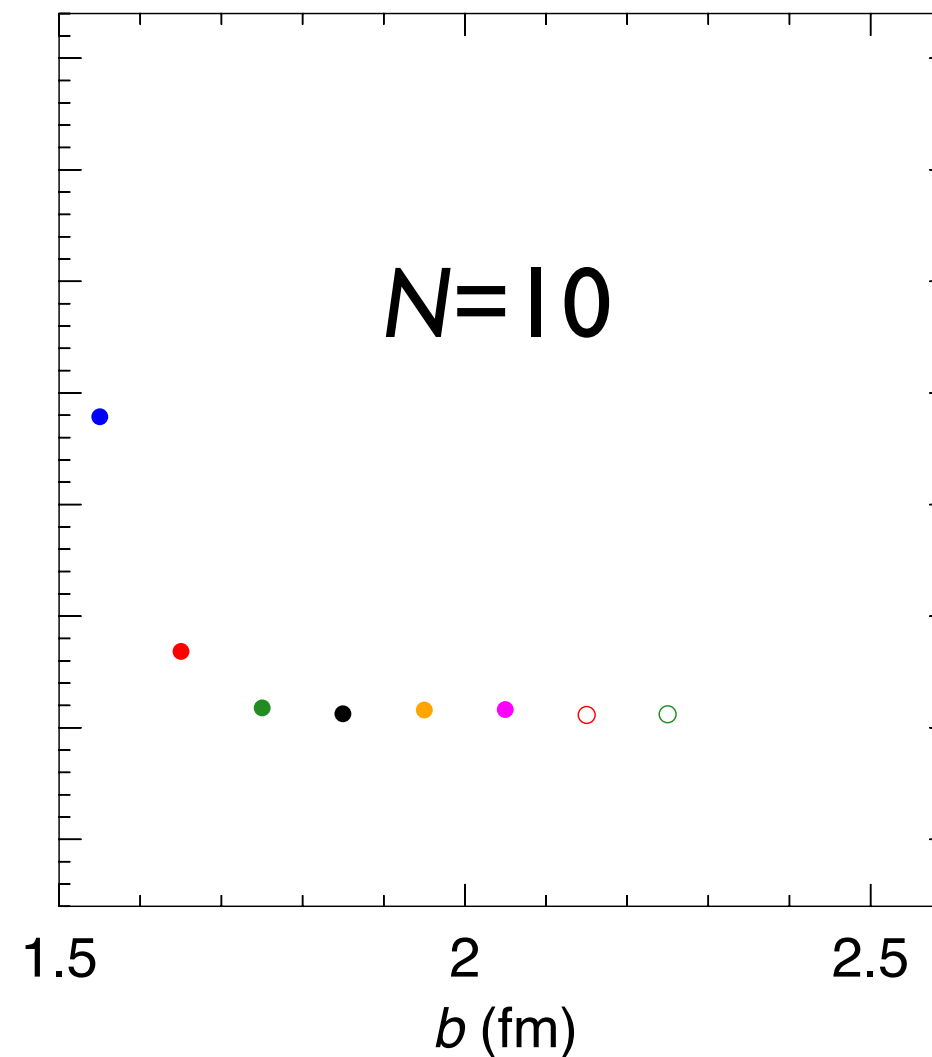
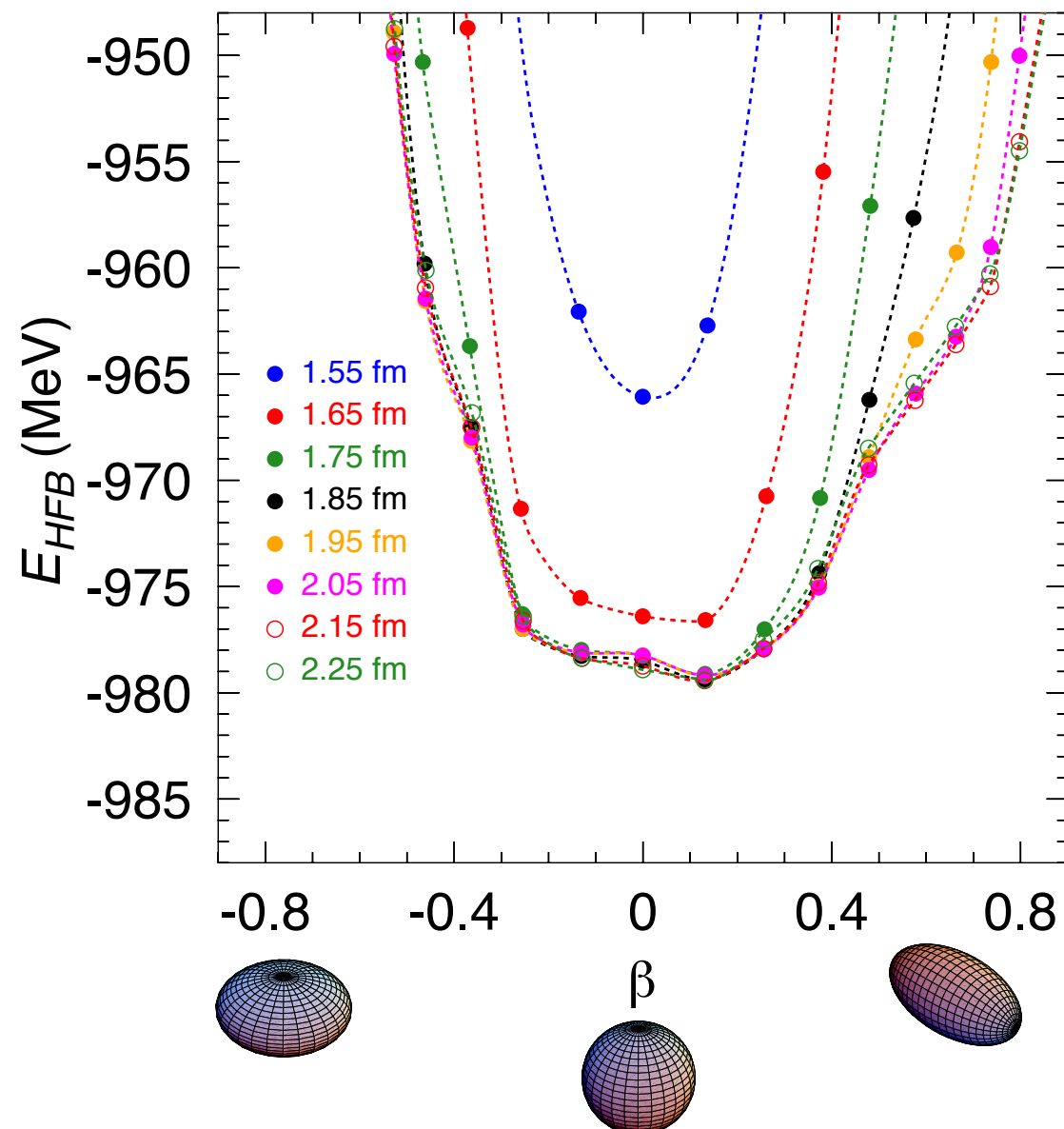


Convergence

Effects of deformation on the convergence

Example:

β^-	β
Cd116	
0+	
7.49	
β^-	β
Ag115	

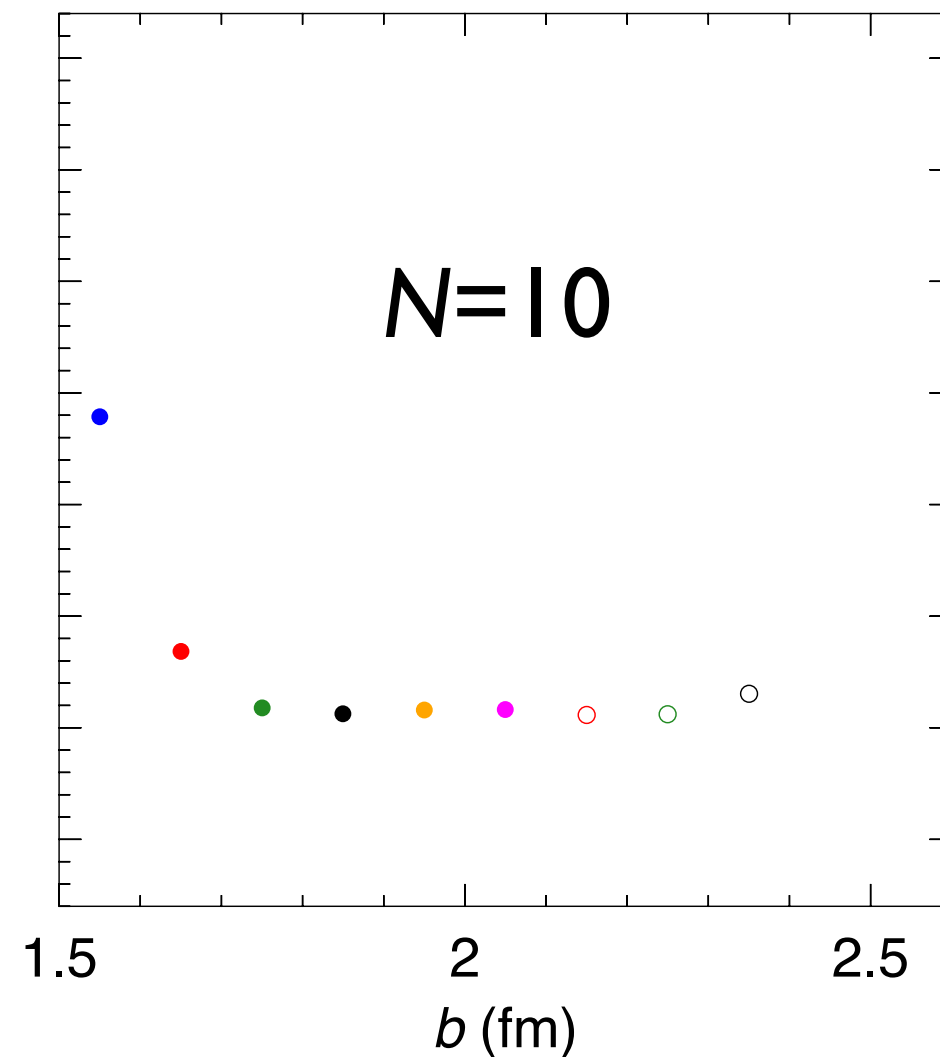
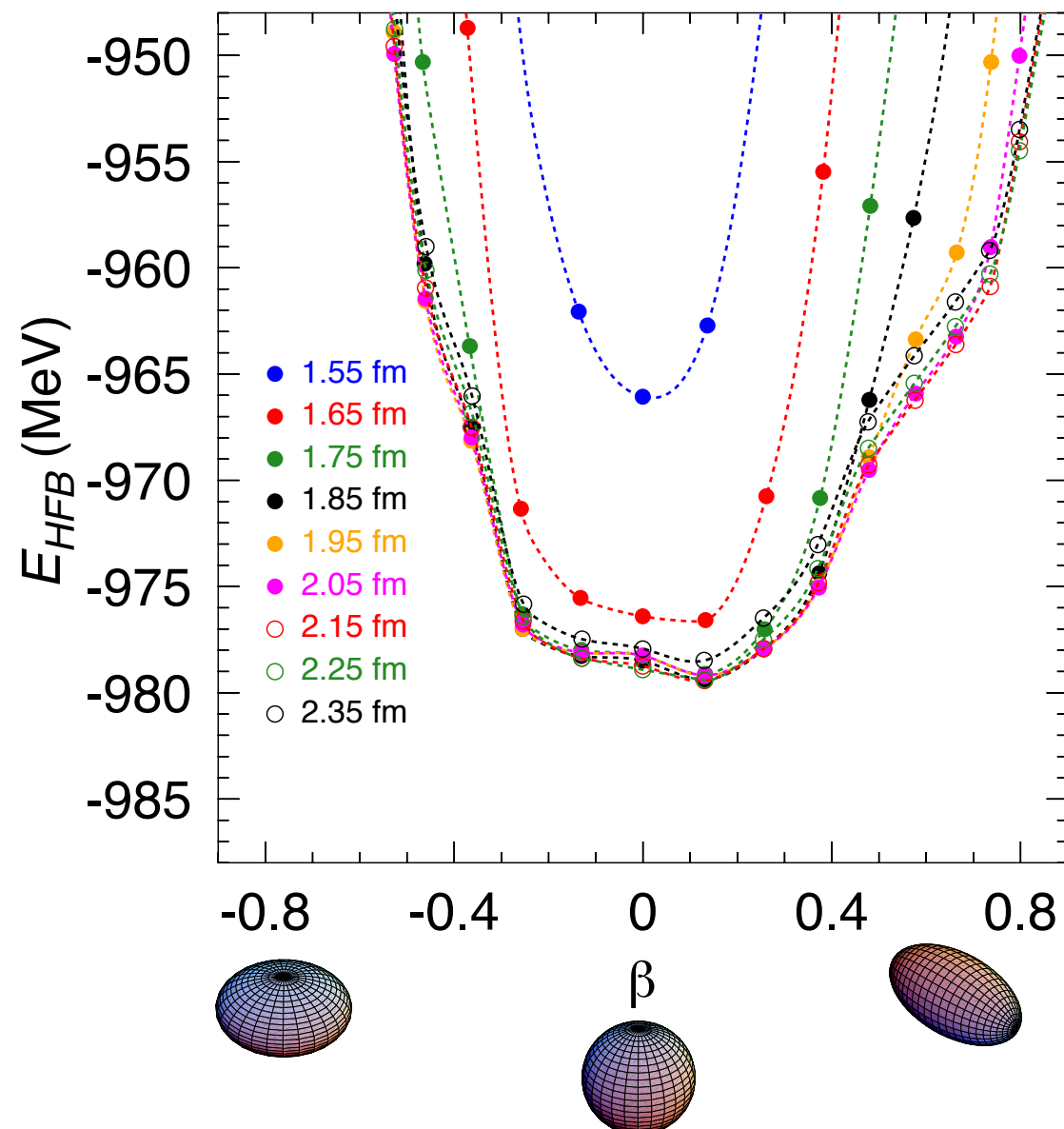


Convergence

Effects of deformation on the convergence

Example:

β^-
Cd116
0+
7.49
Ag115

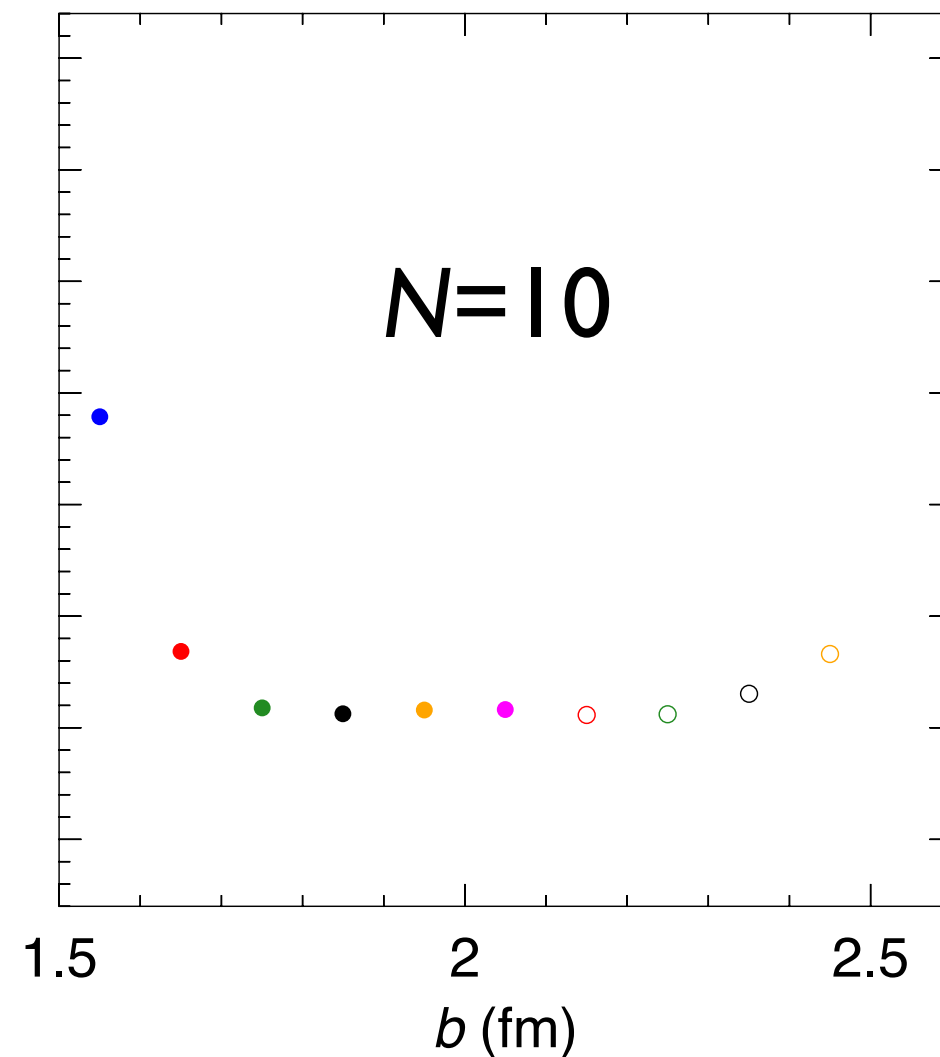
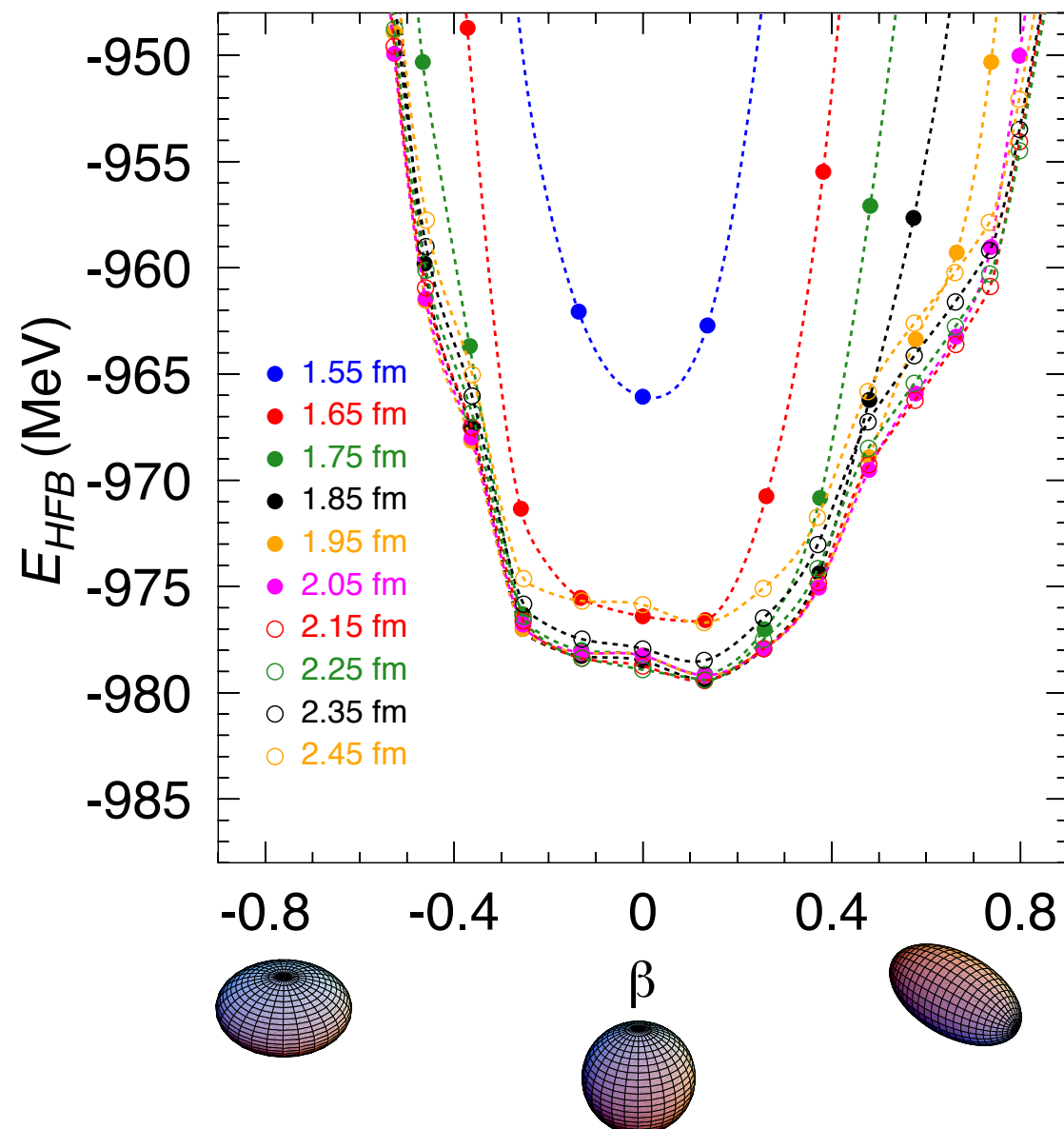


Convergence

Effects of deformation on the convergence

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7.49
Ag115

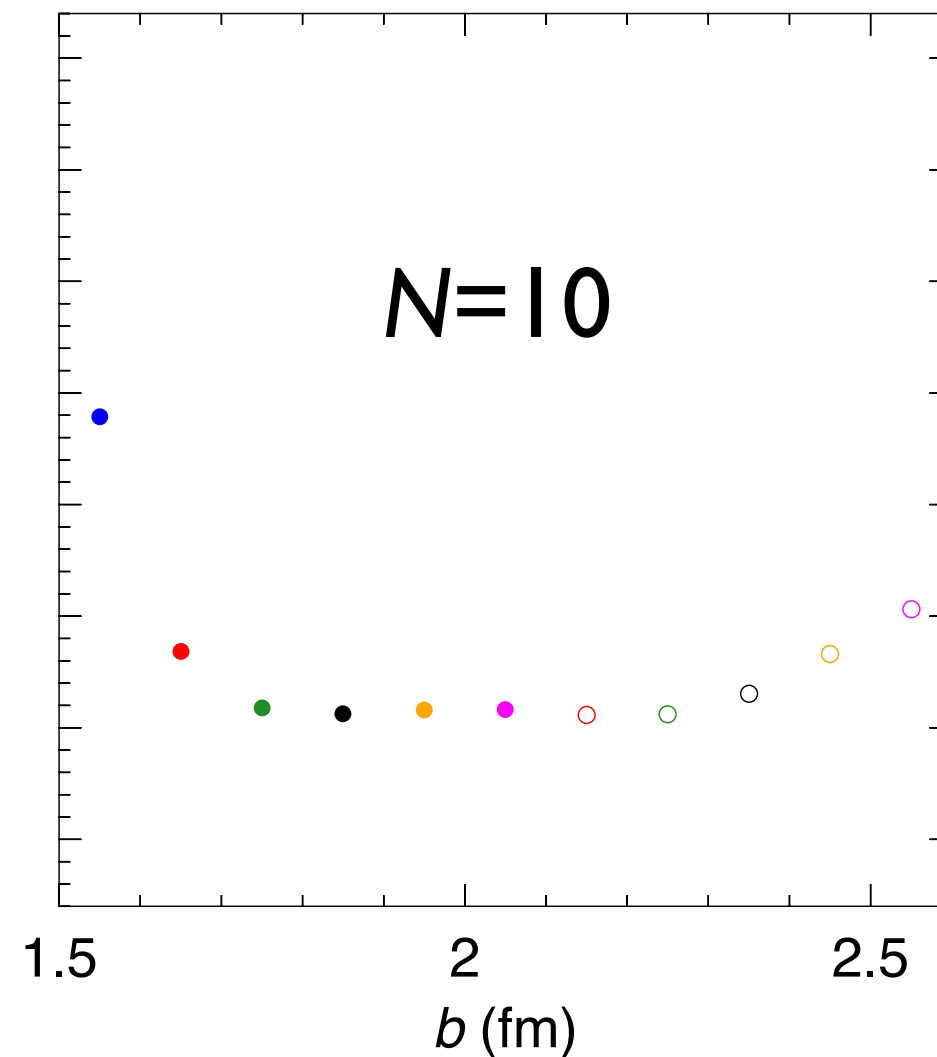
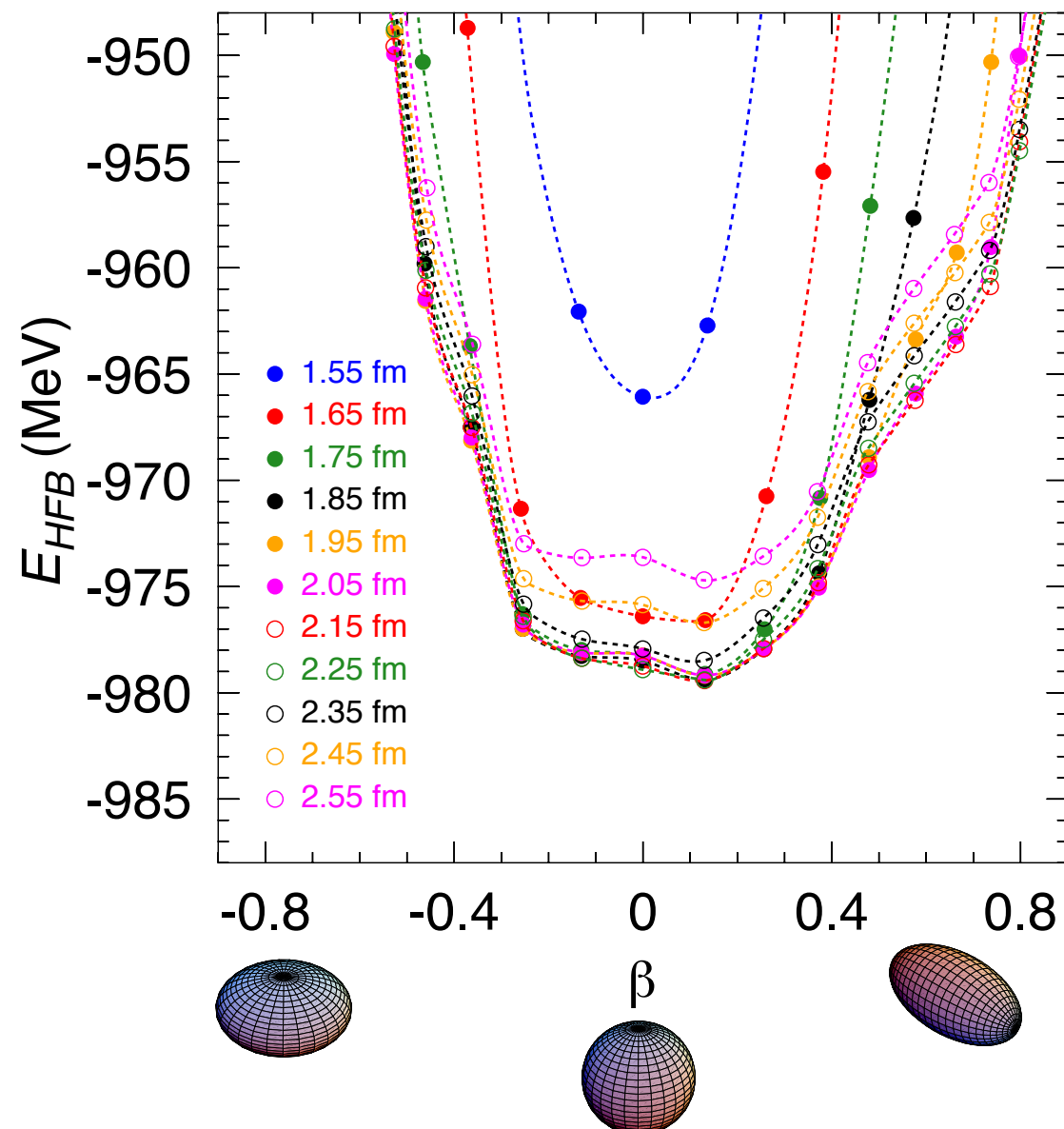


Convergence

Effects of deformation on the convergence

Example:

β^-
Cd116
0+
7.49
Ag115

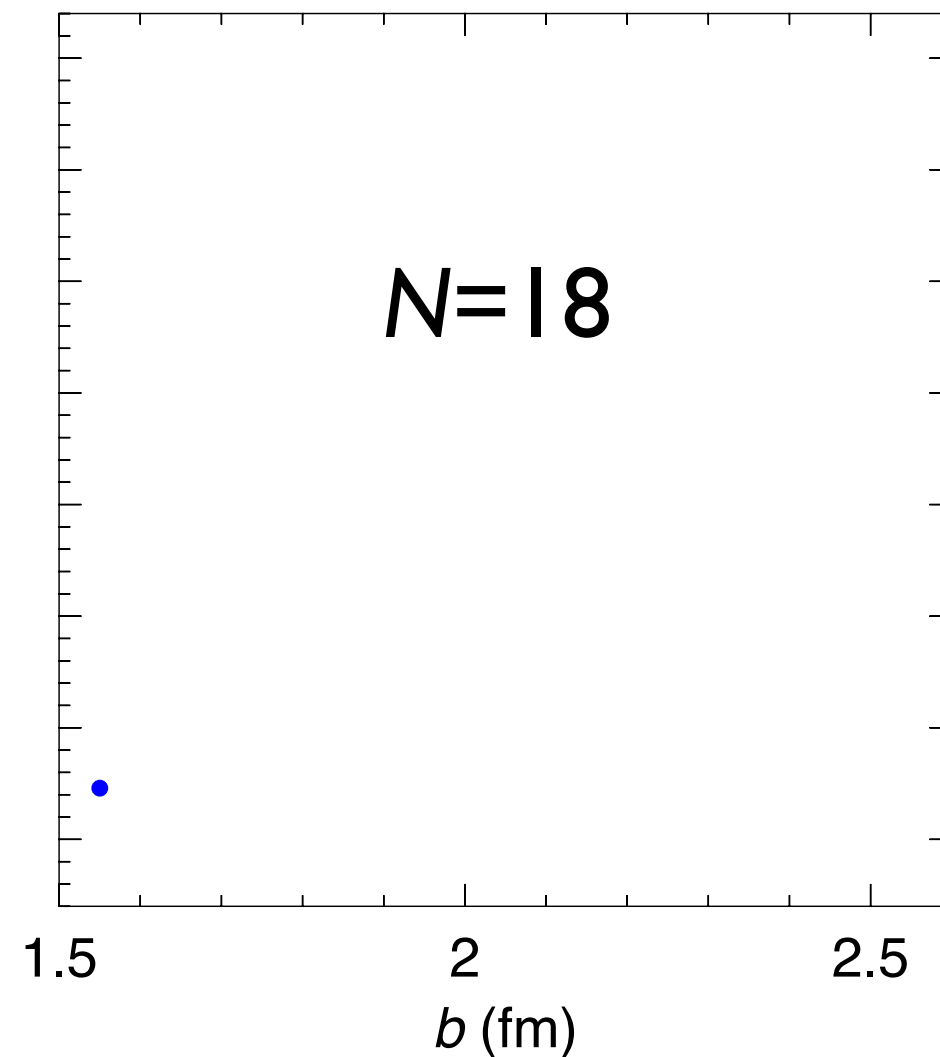
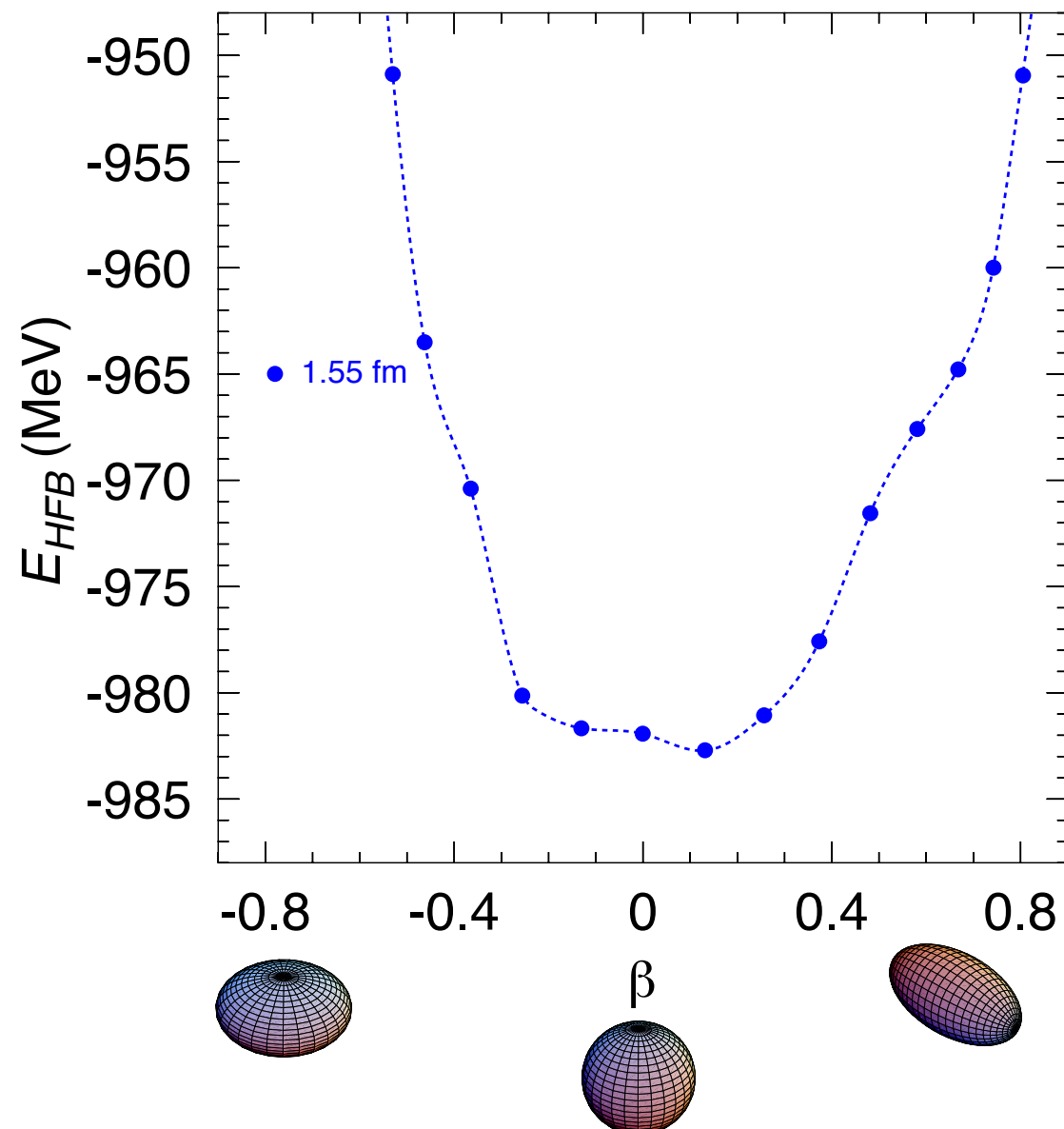


Convergence

Effects of deformation on the convergence

Example:

β^-	
Cd116	
0+	
7.49	
Ag115	

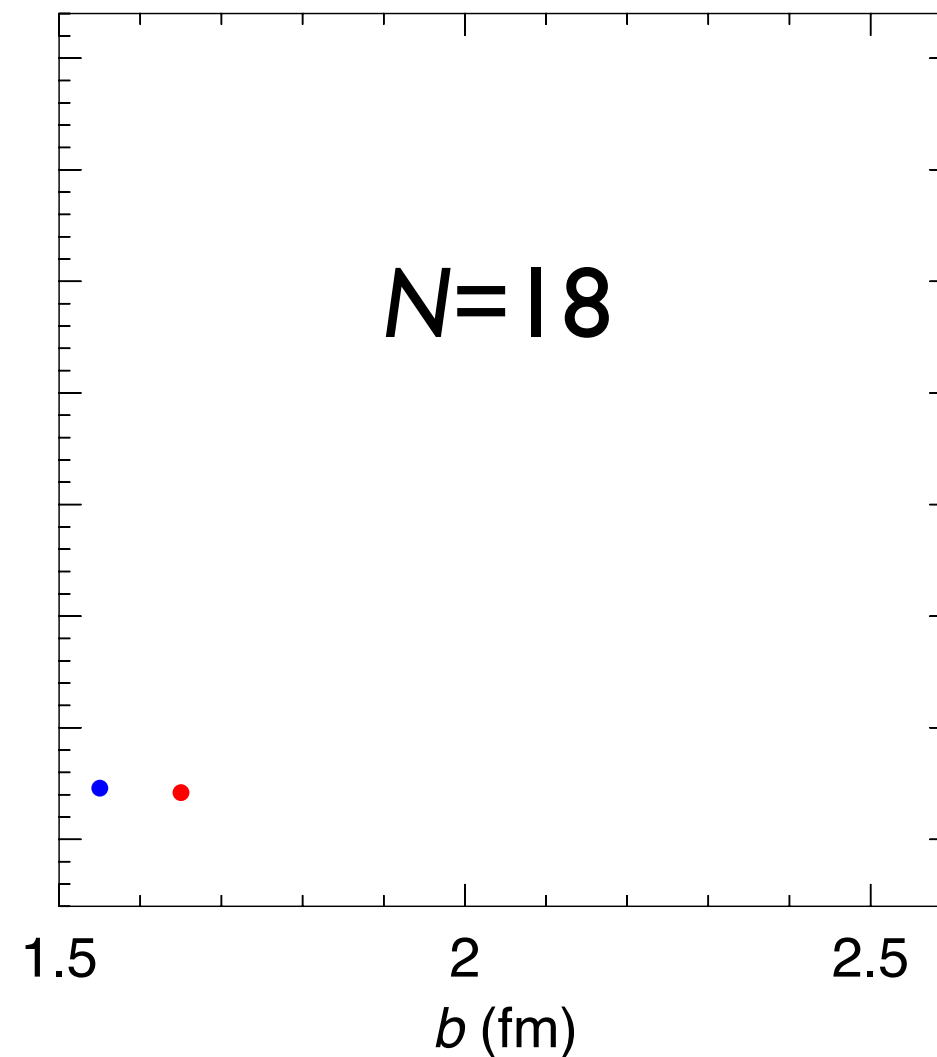
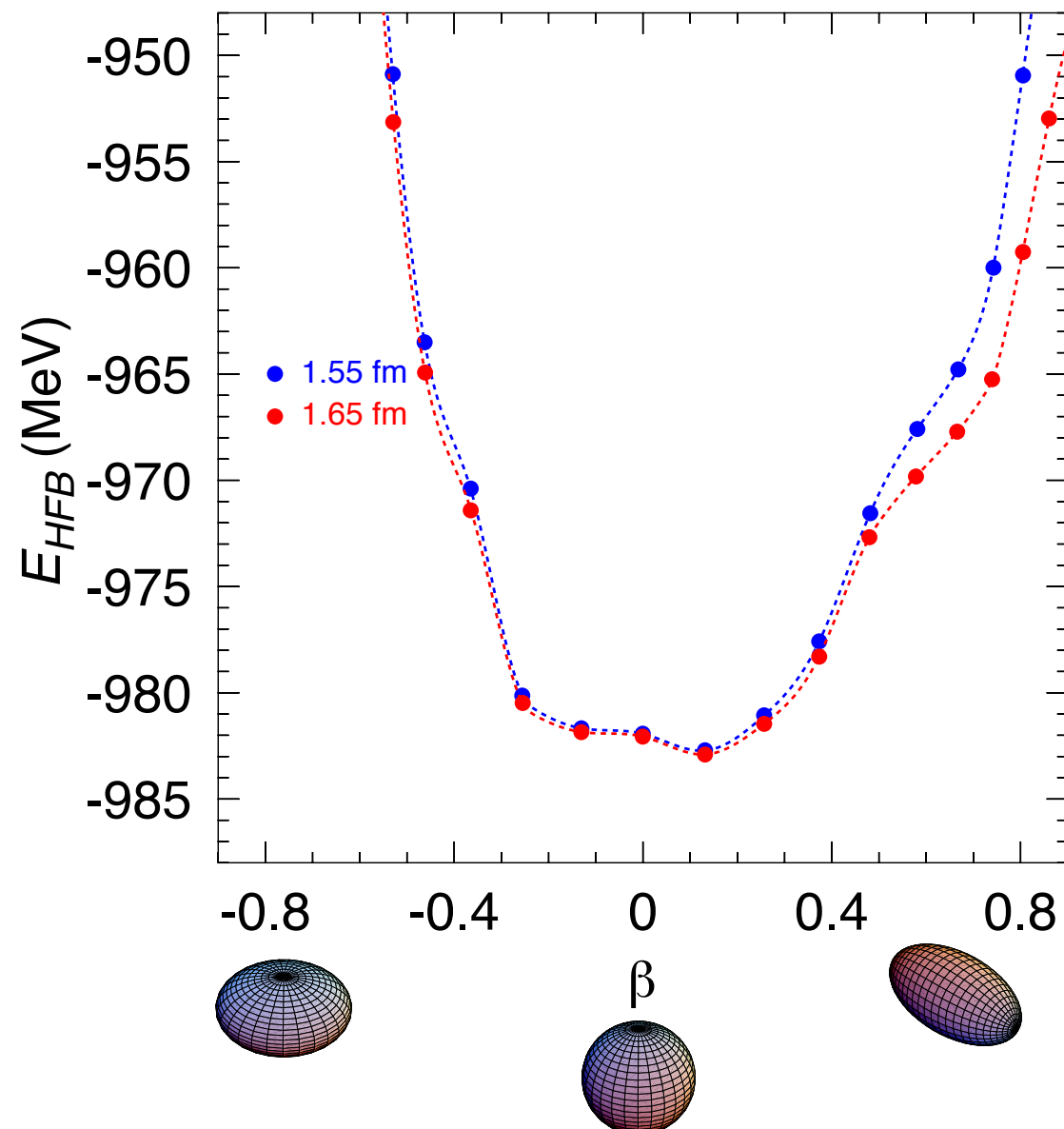


Convergence

Effects of deformation on the convergence

Example:

β^-	
Cd116	
0+	
7.49	
Ag115	

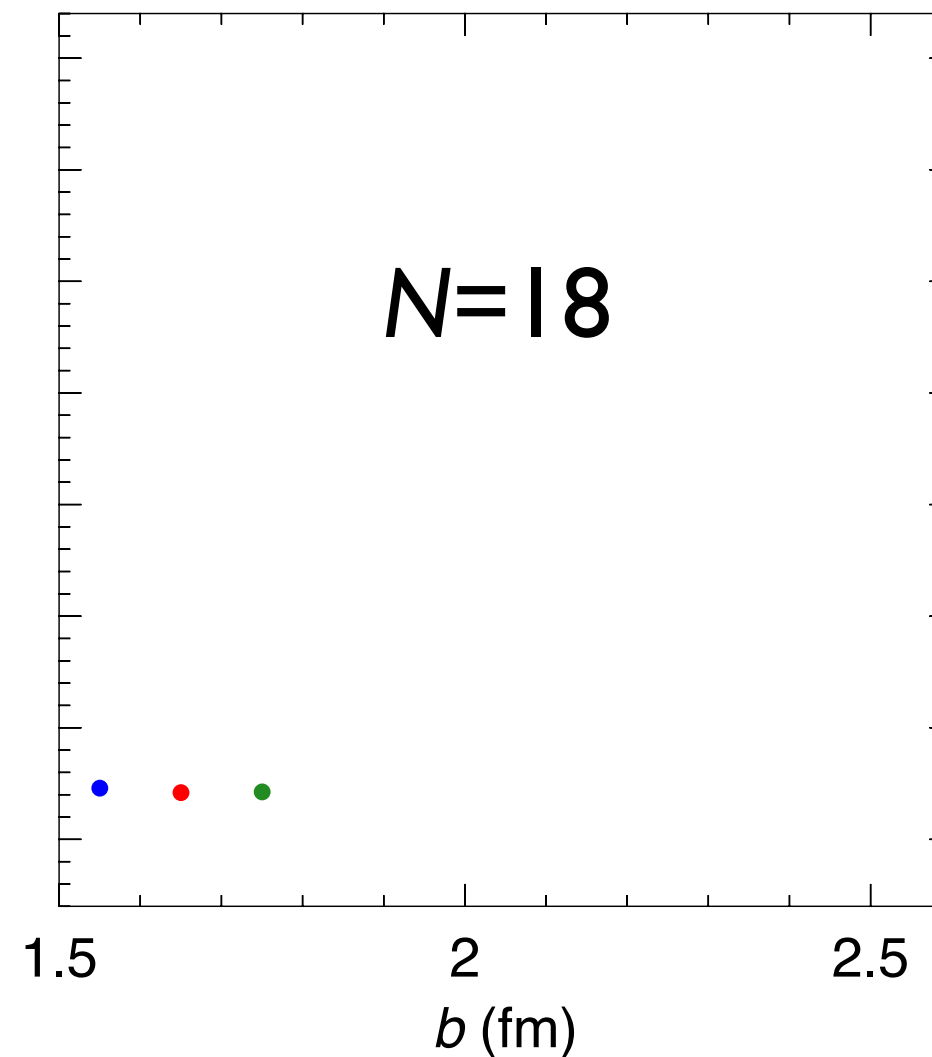
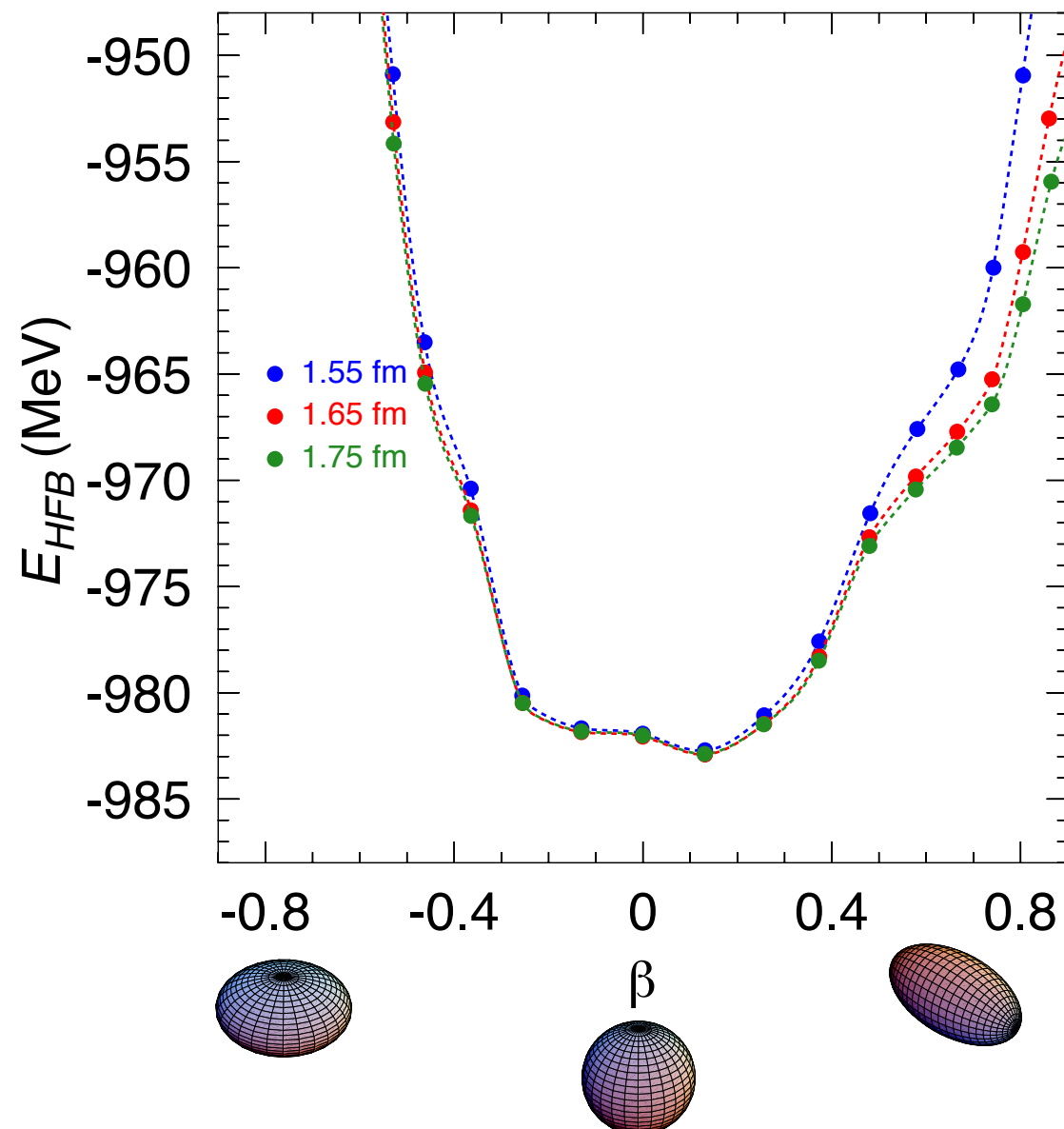


Convergence

Effects of deformation on the convergence

Example:

β^-	β^+
Cd116	
0+	
7.49	
Ag115	

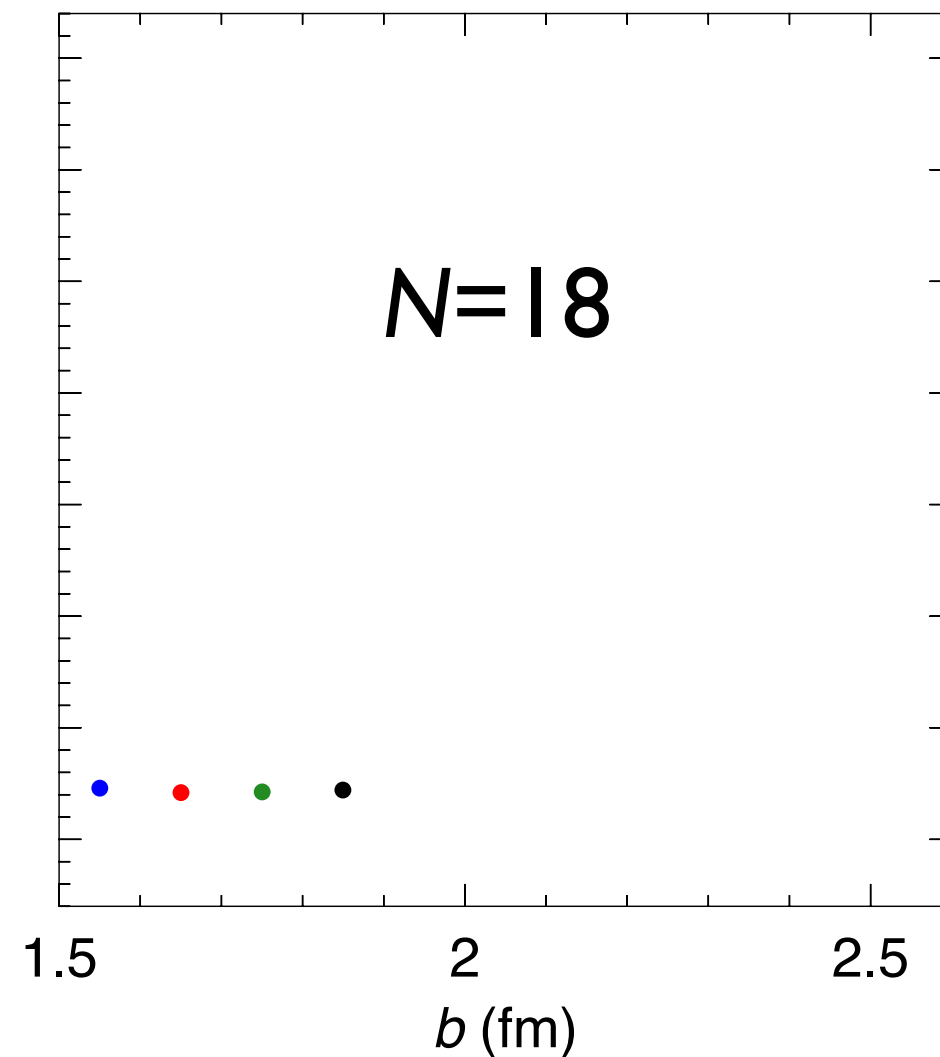
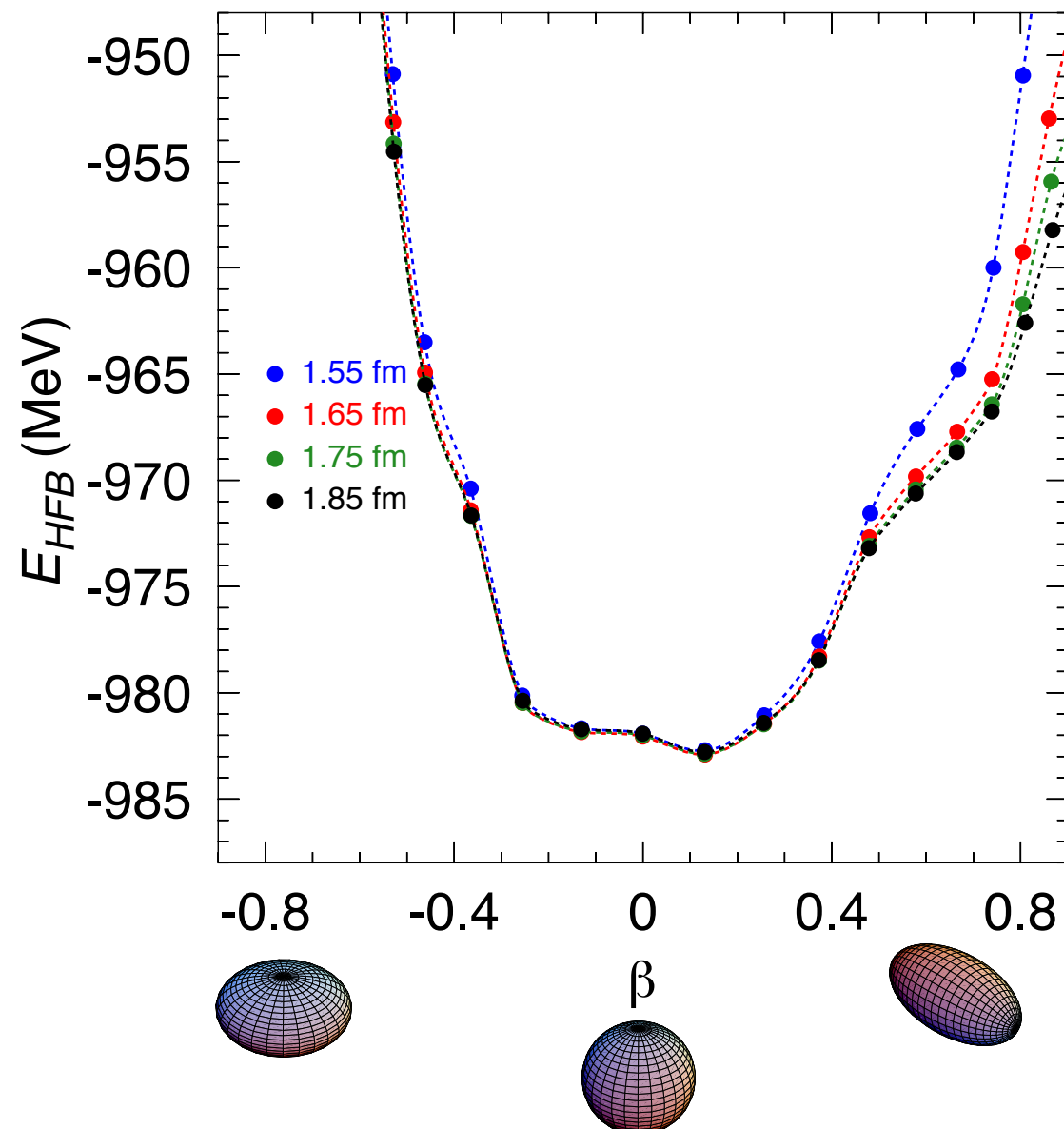


Convergence

Effects of deformation on the convergence

Example:

β^-	β^+
Cd116	
0+	
7.49	
Ag115	

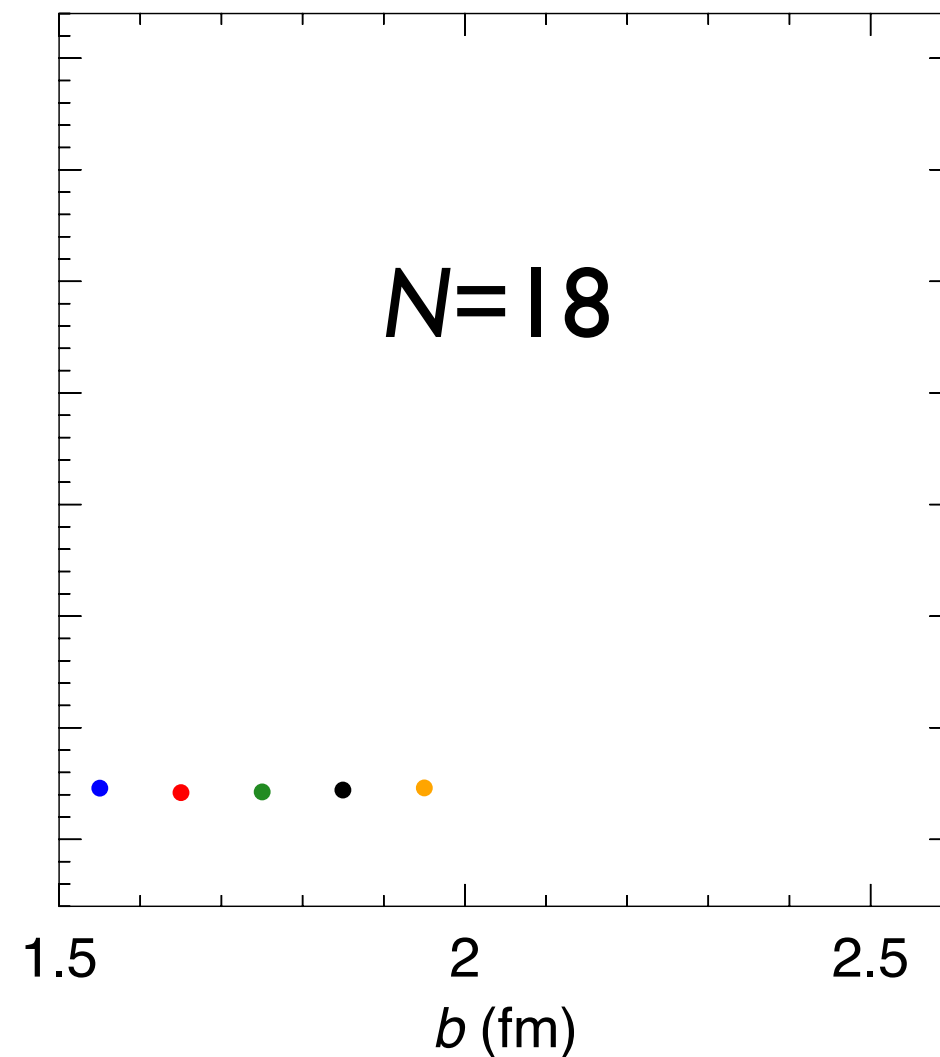
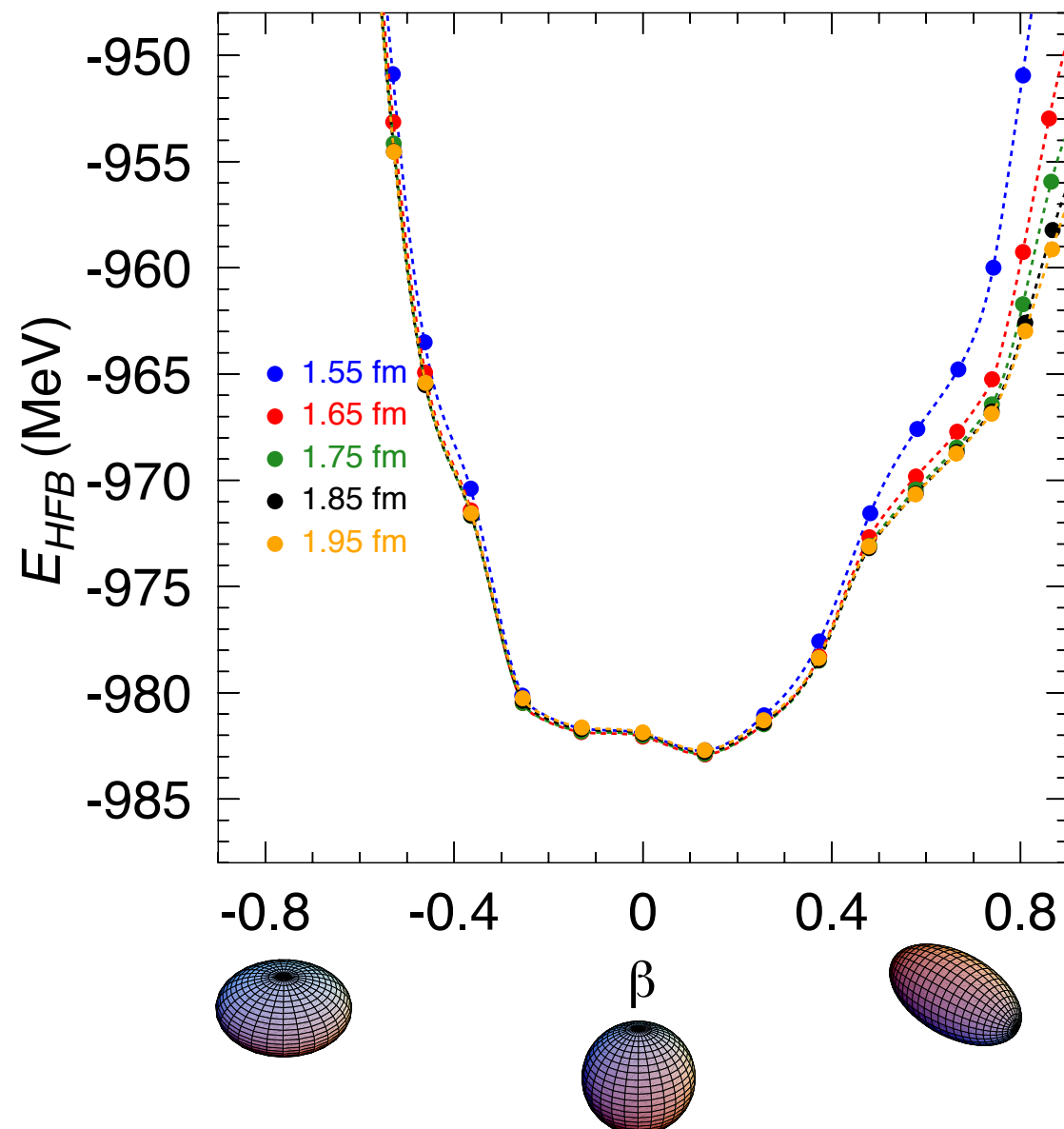


Convergence

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Example:

β^-	
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0+	
7.49	
Ag115	

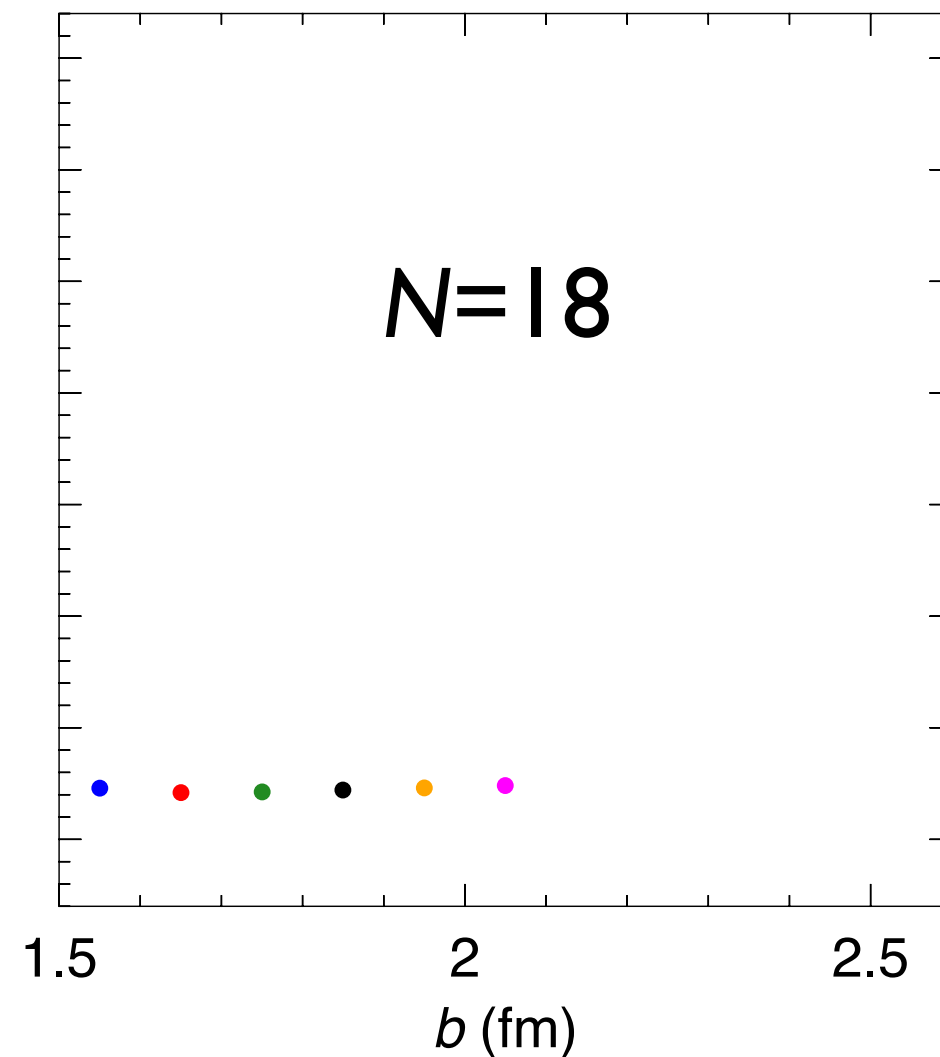
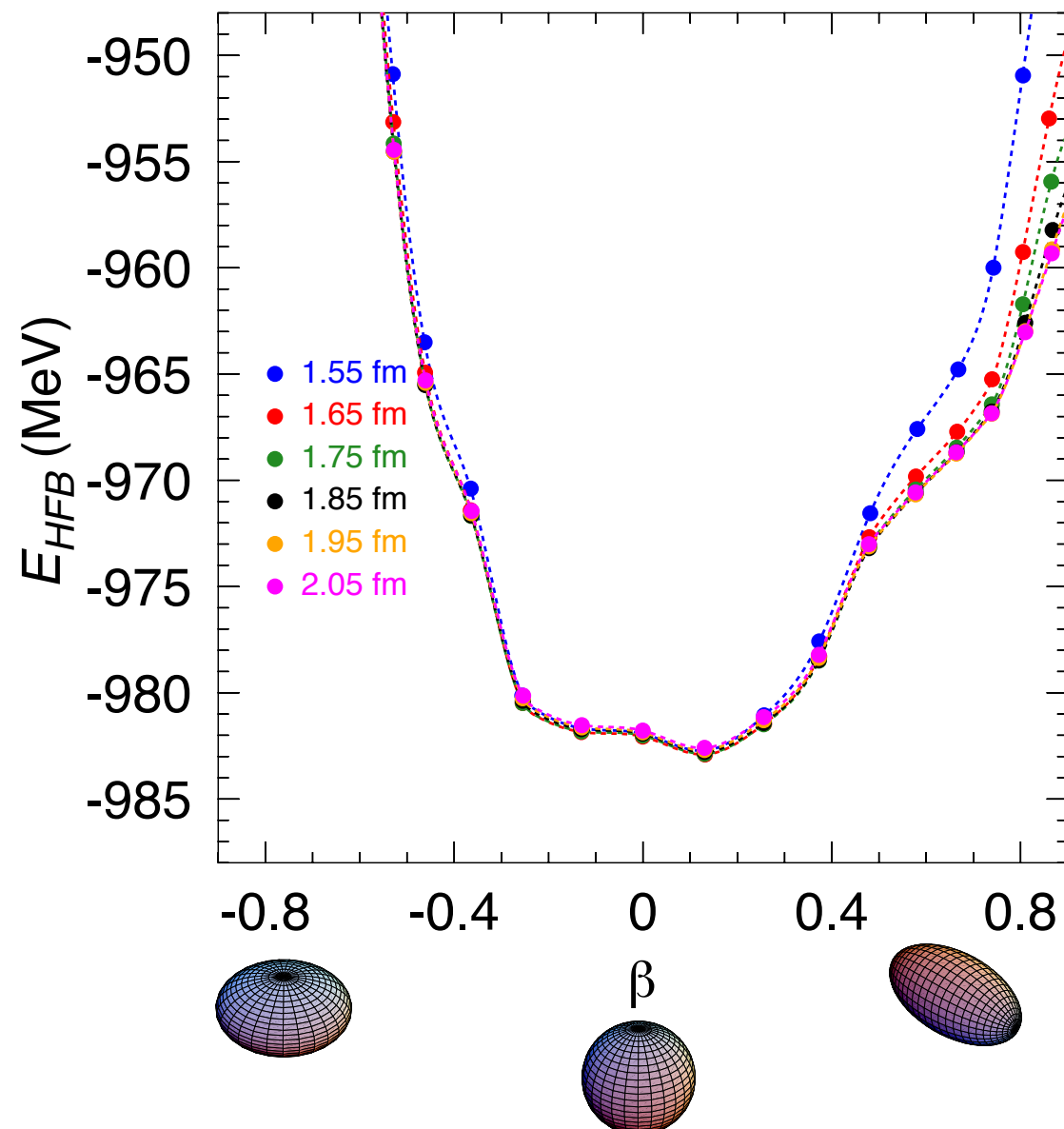


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Effects of deformation on the convergence

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β^-	
Cd116	
0+	
7.49	
Ag115	

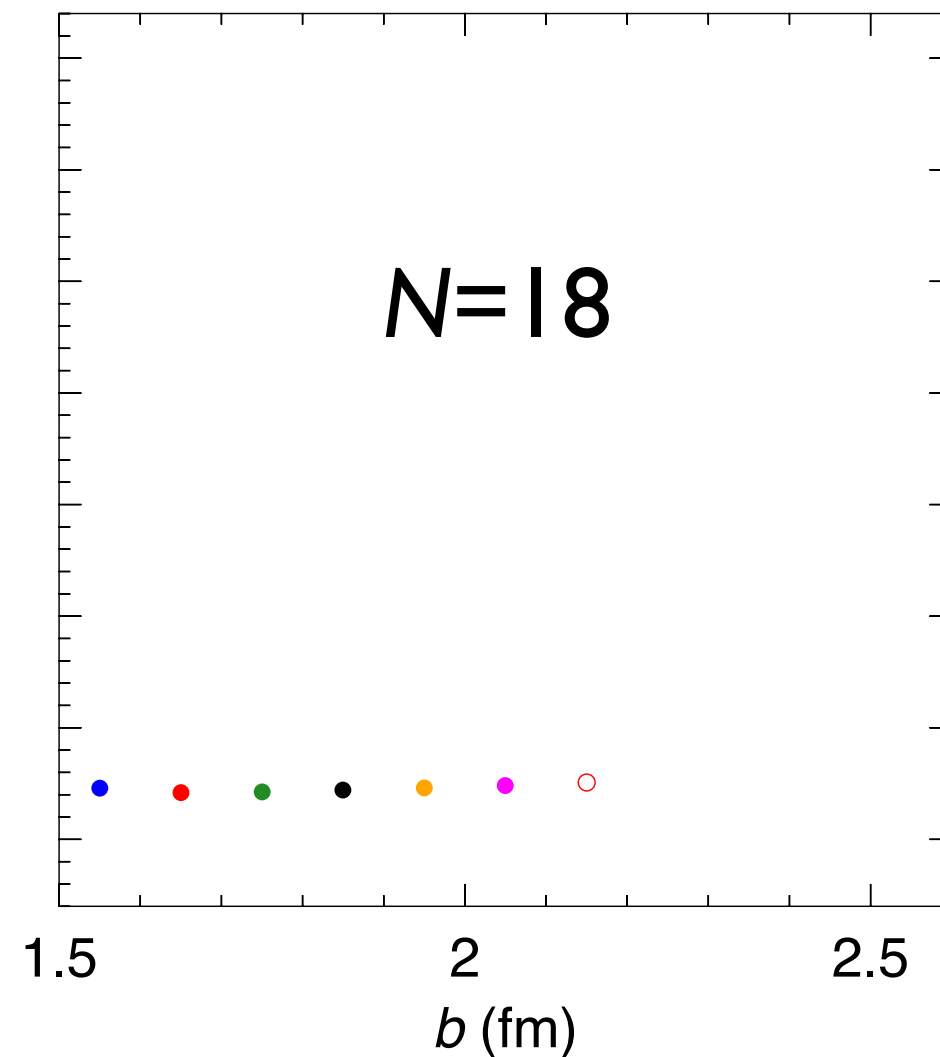
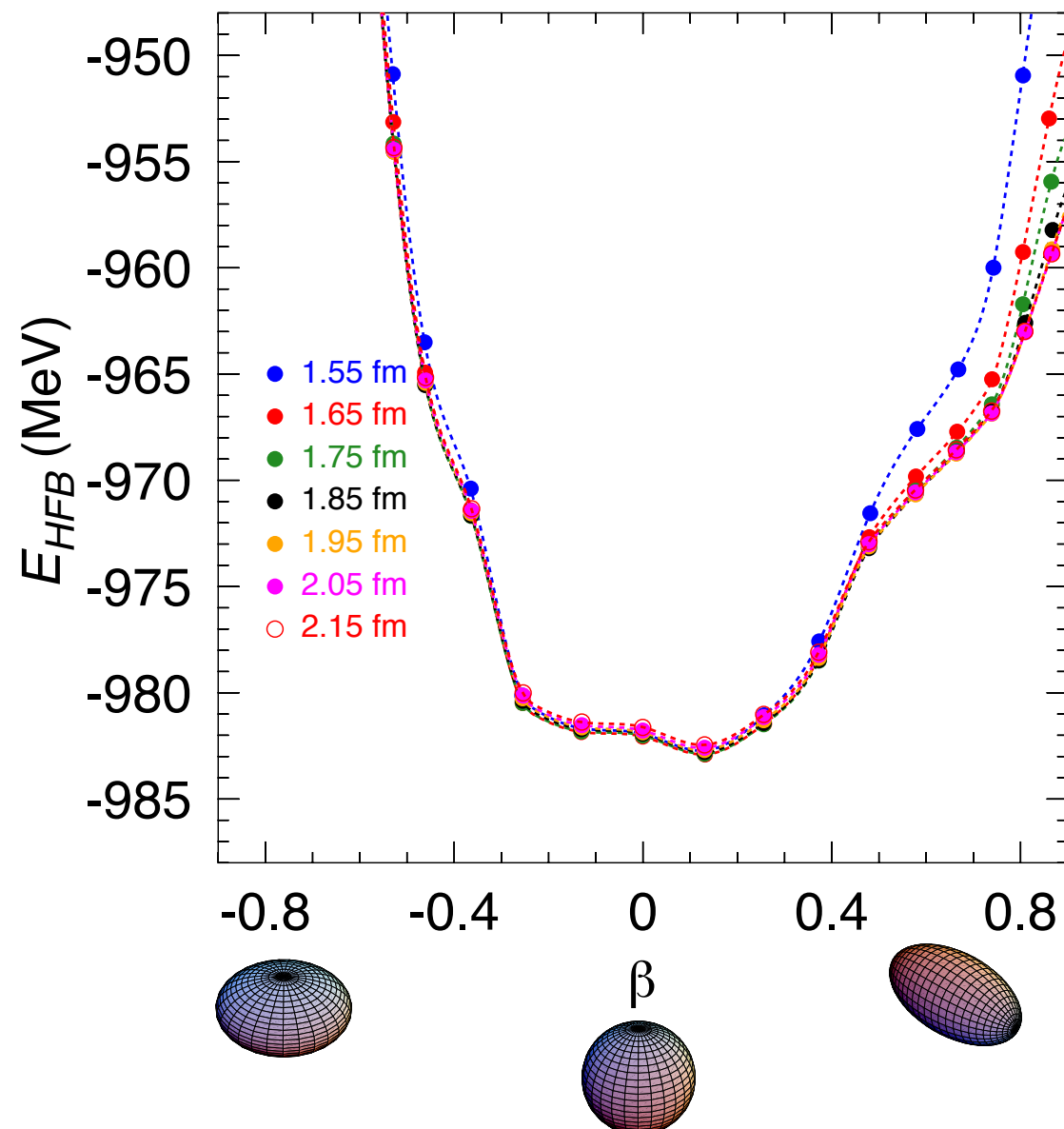


Convergence

Effects of deformation on the convergence

Example:

β^-	β
Cd116	
0+	
7.49	
β^-	β
Ag115	

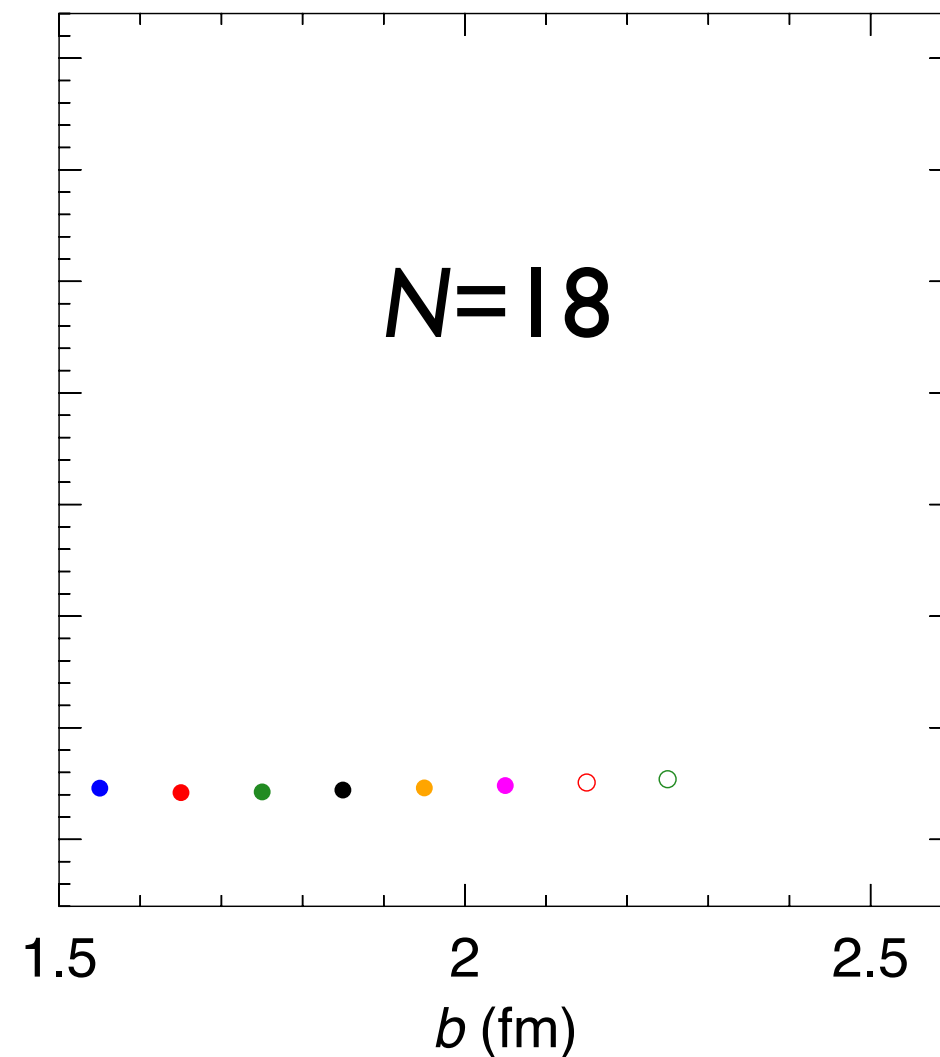
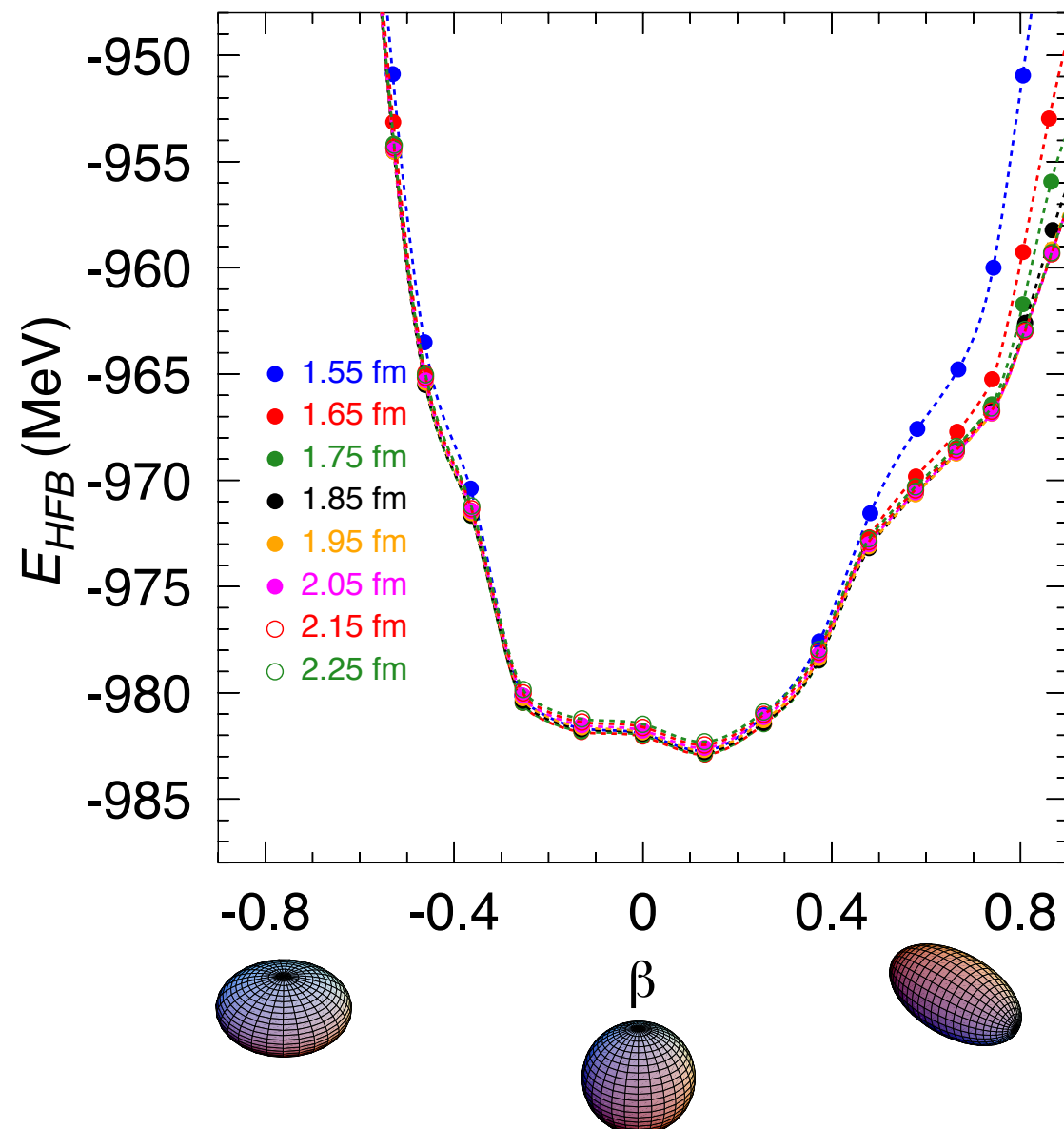


Convergence

Effects of deformation on the convergence

Example:

β^-	β
Cd116	
0+	
7.49	
β^-	β
Ag115	

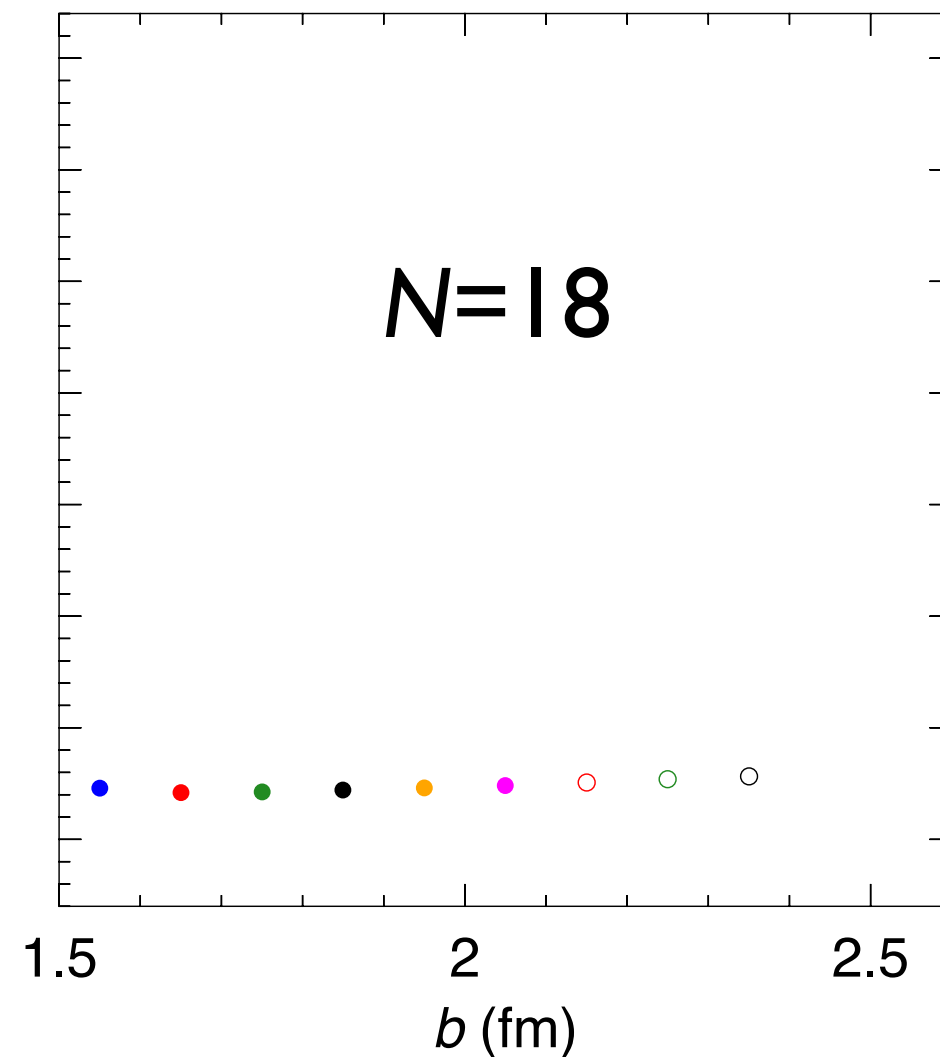
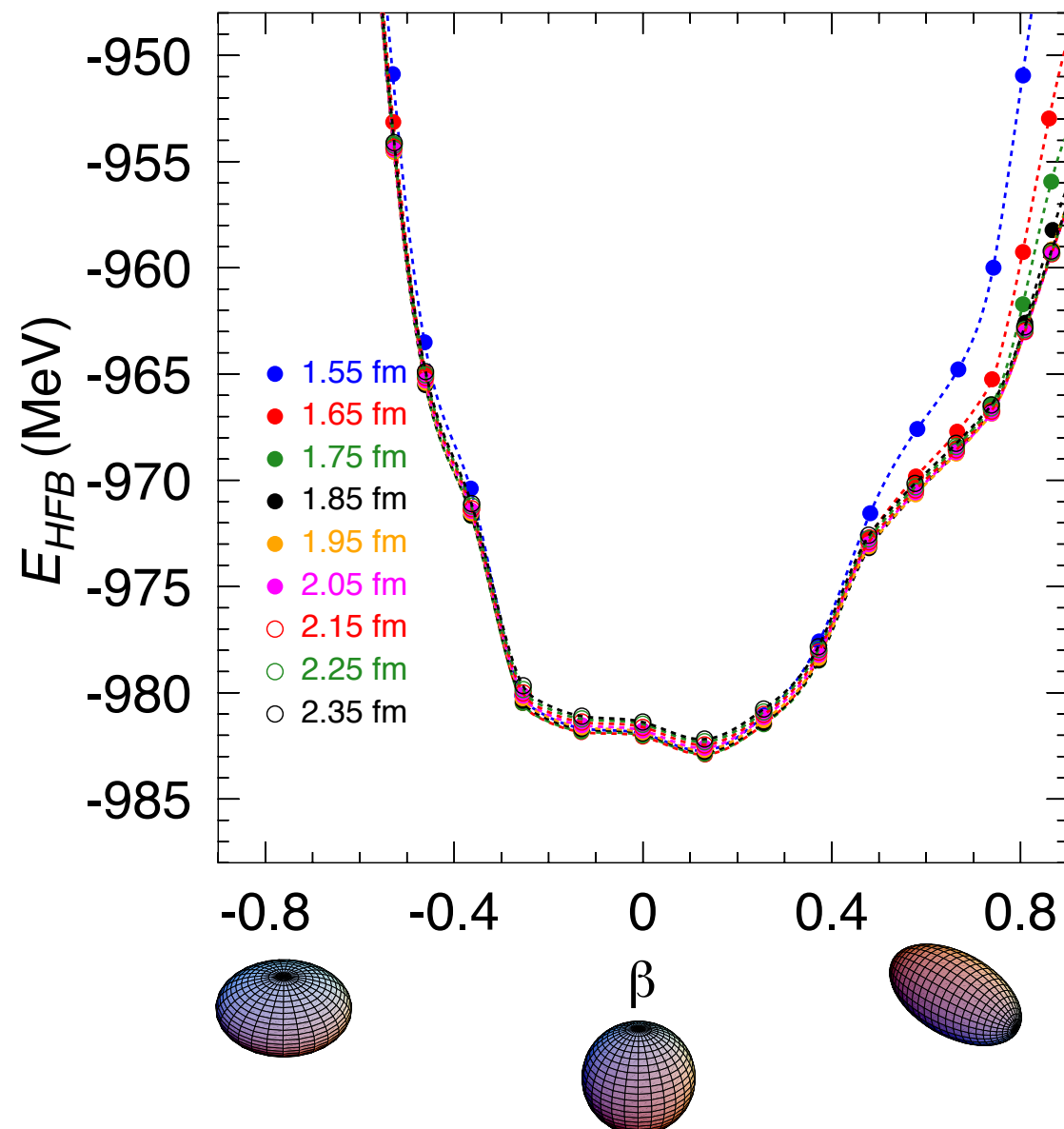


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Example:

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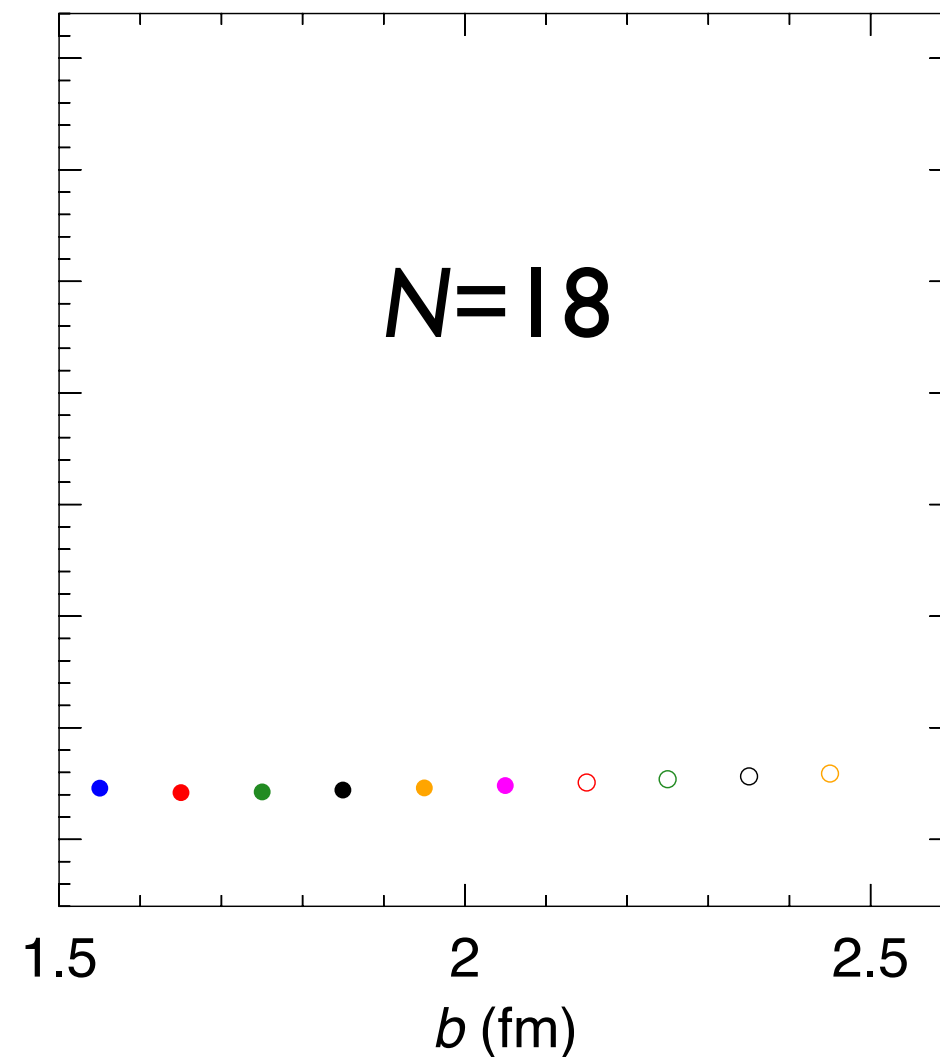
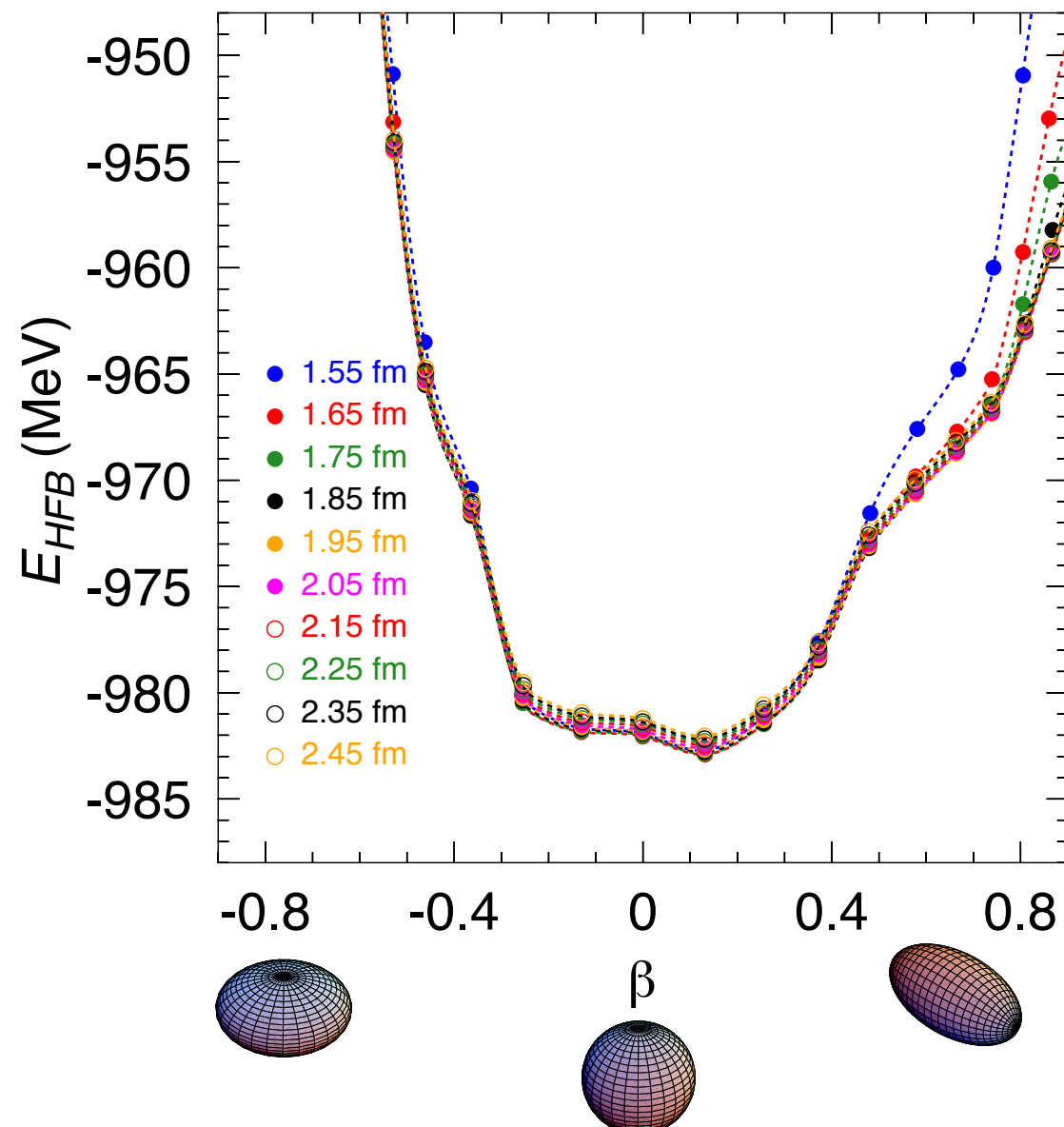


Convergence

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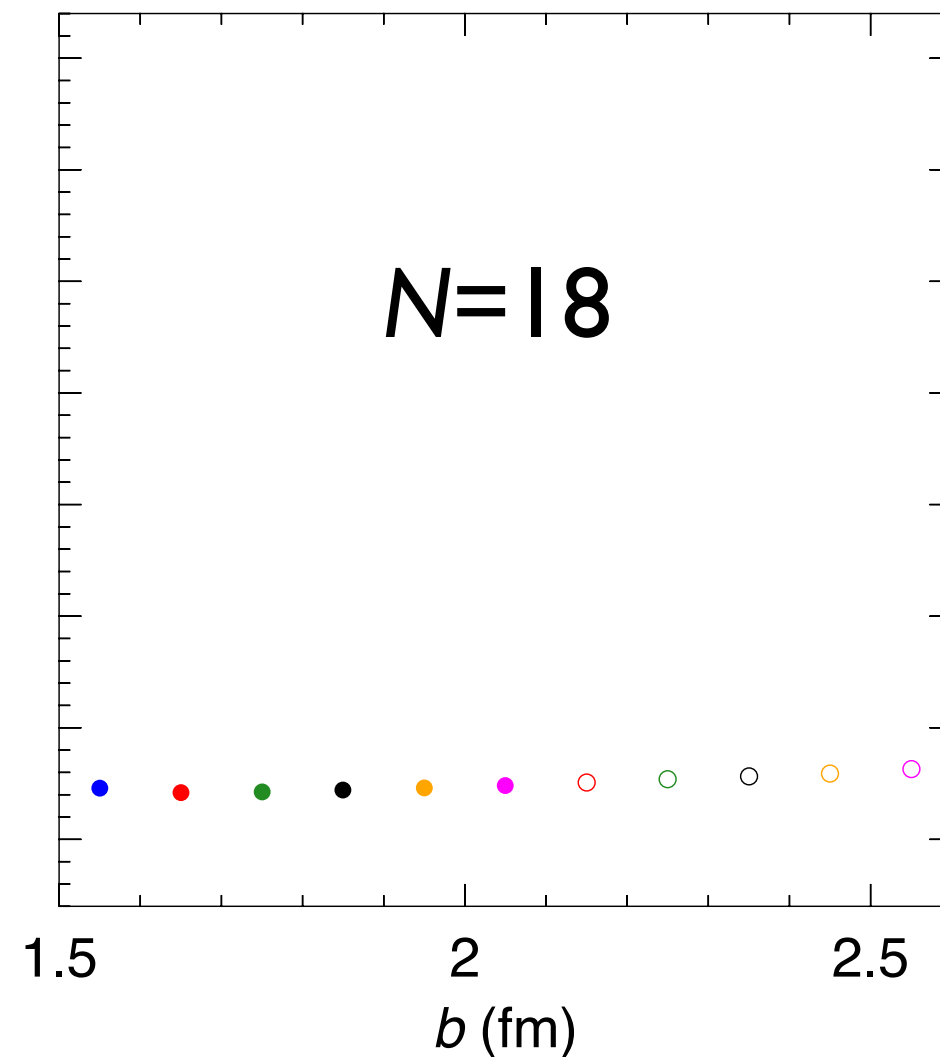
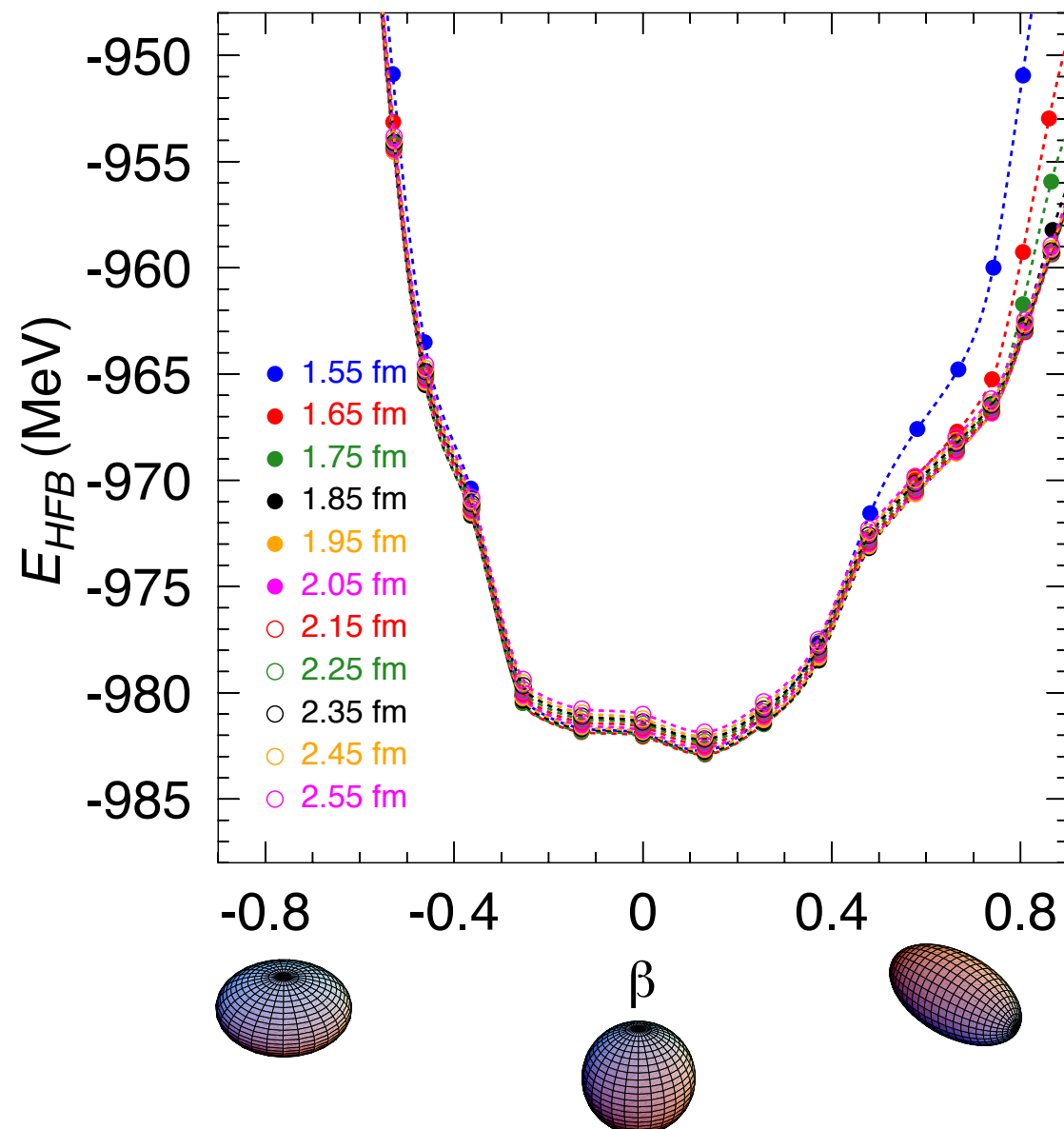


Convergence

Effects of deformation on the convergence

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Cd116	
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7.49	
Ag115	

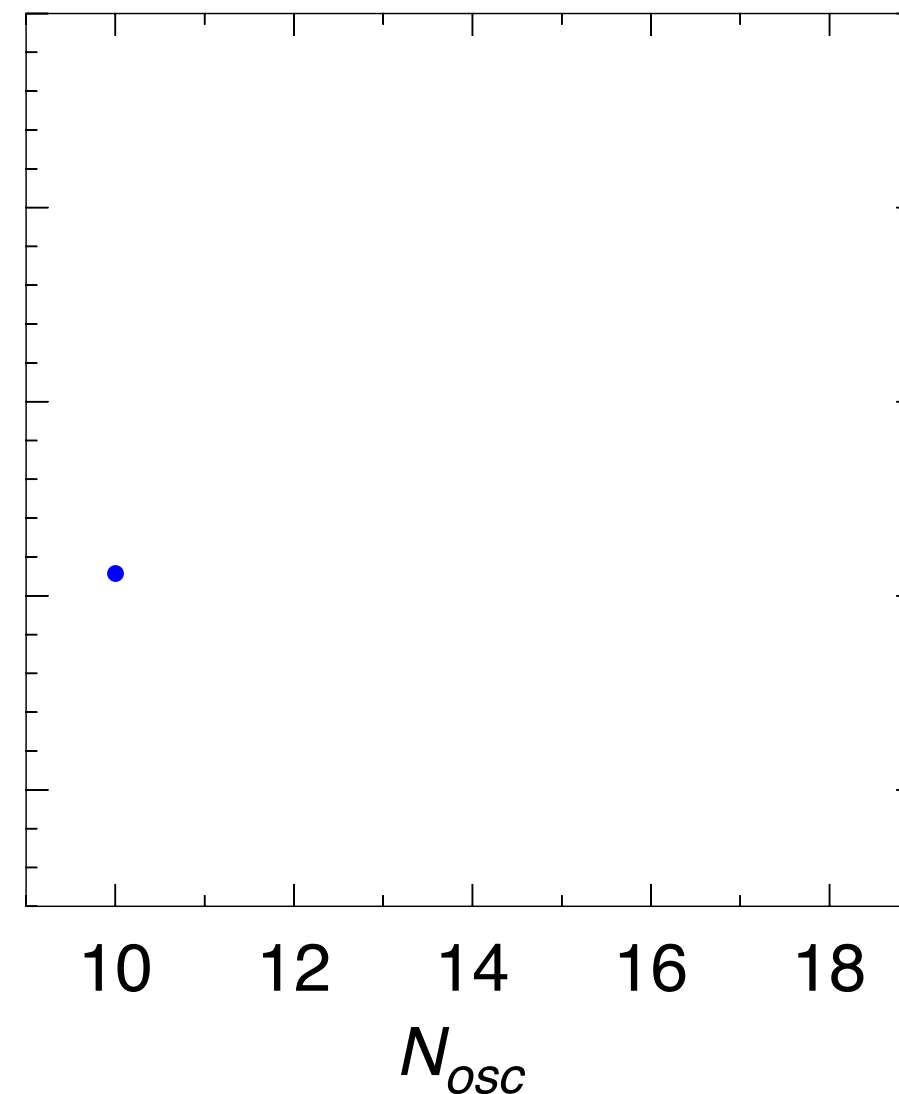
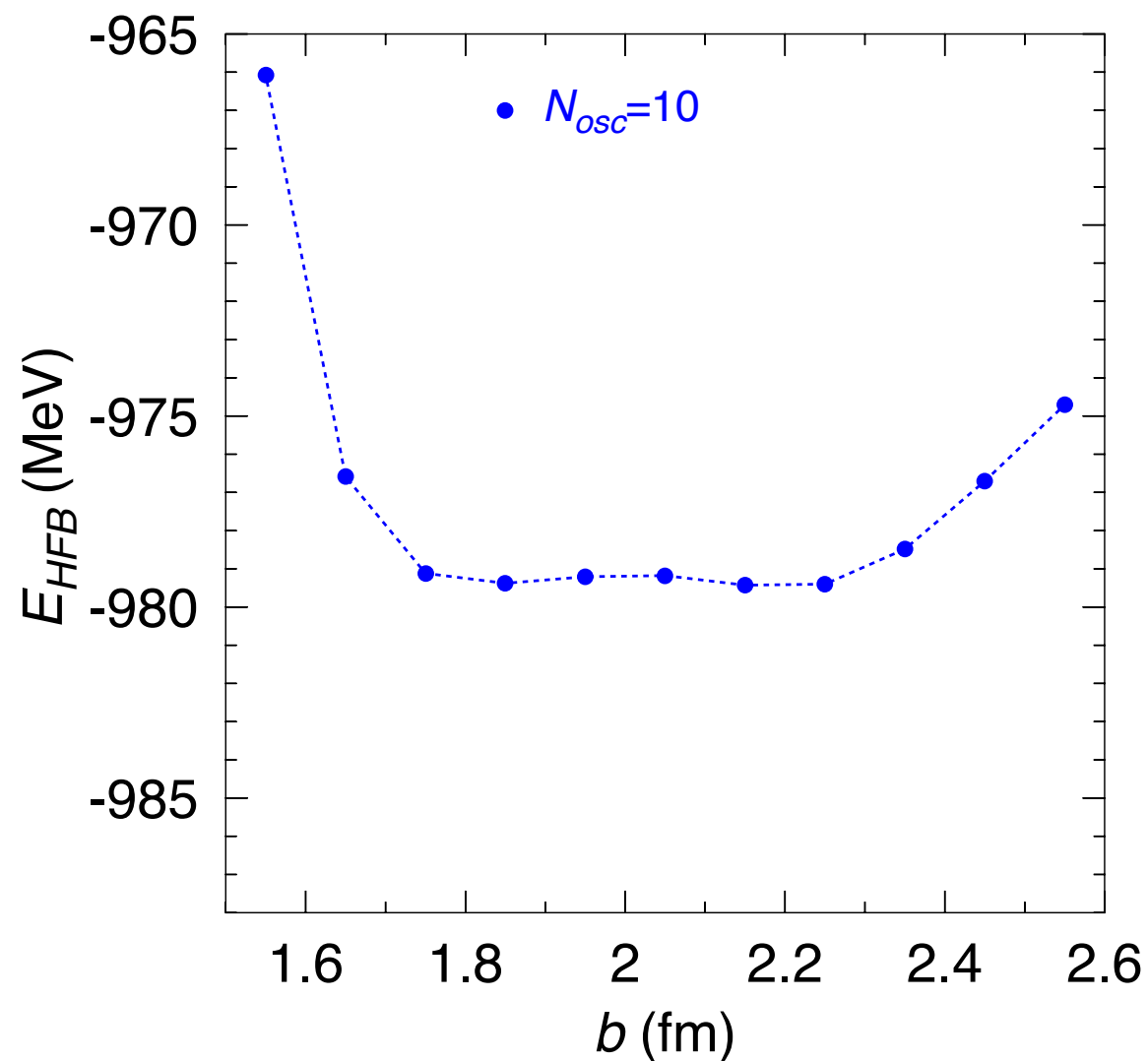


Convergence

Final convergence

Example:

β^-	
Cd116	
0+	
7.49	
Ag115	

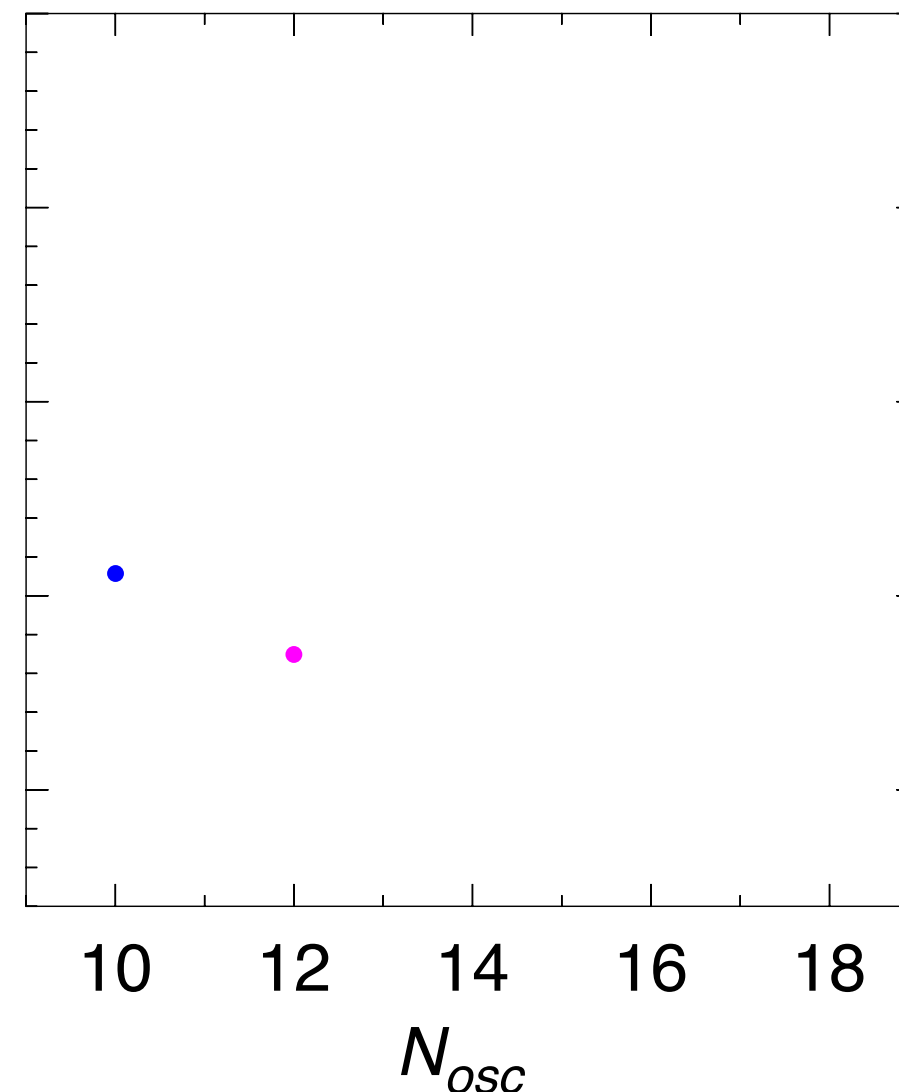
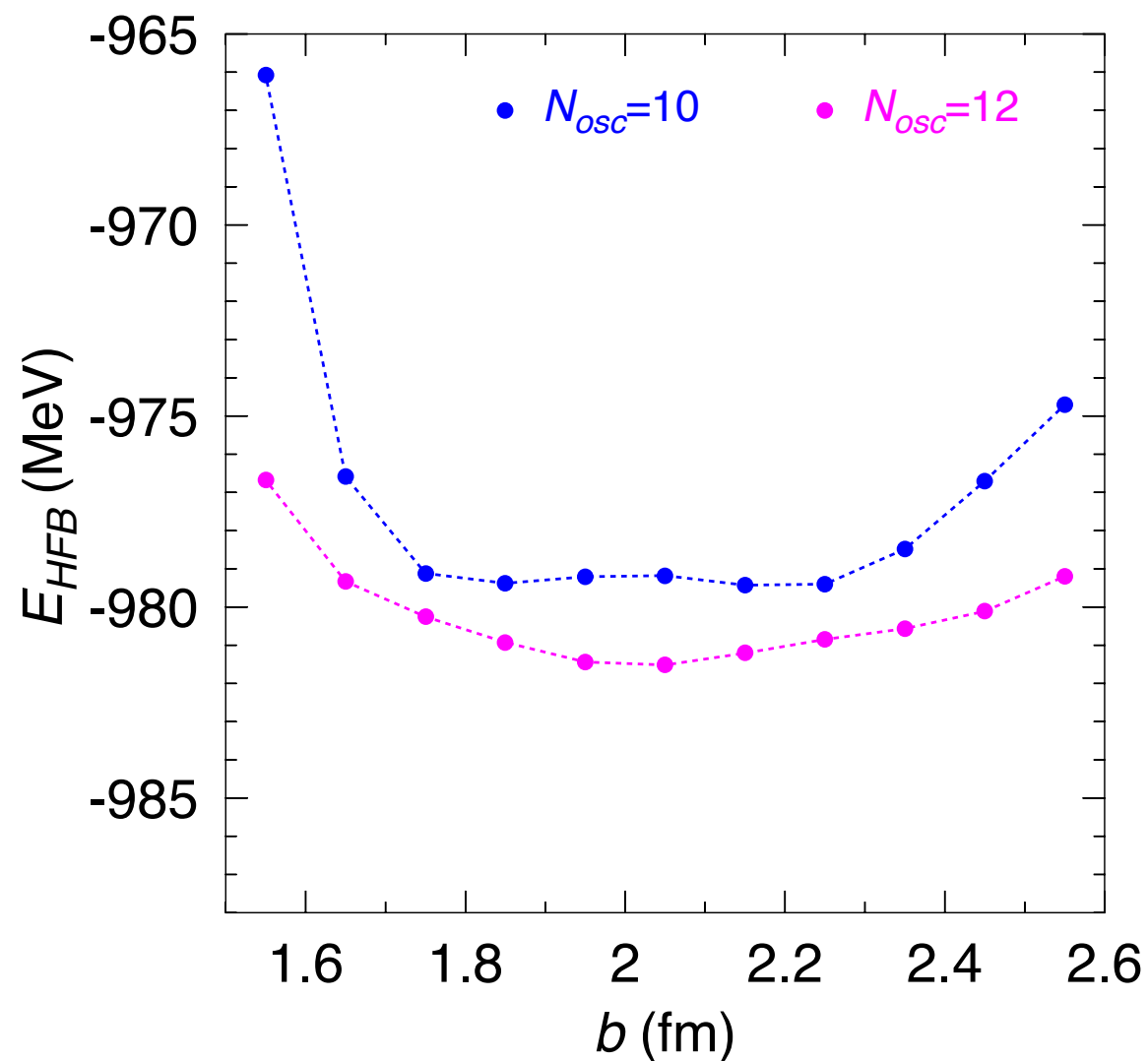


Convergence

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Example:

β^-	
Cd116	
0+	
7.49	
Ag115	

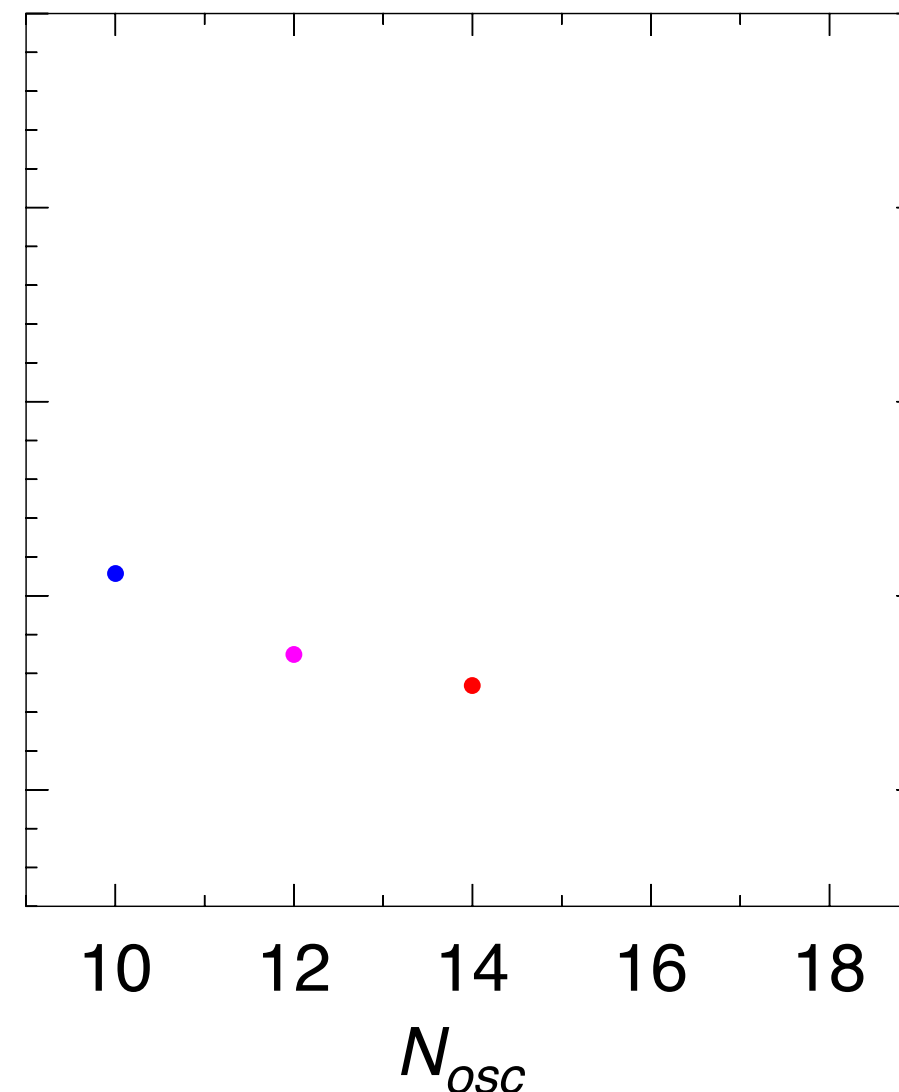
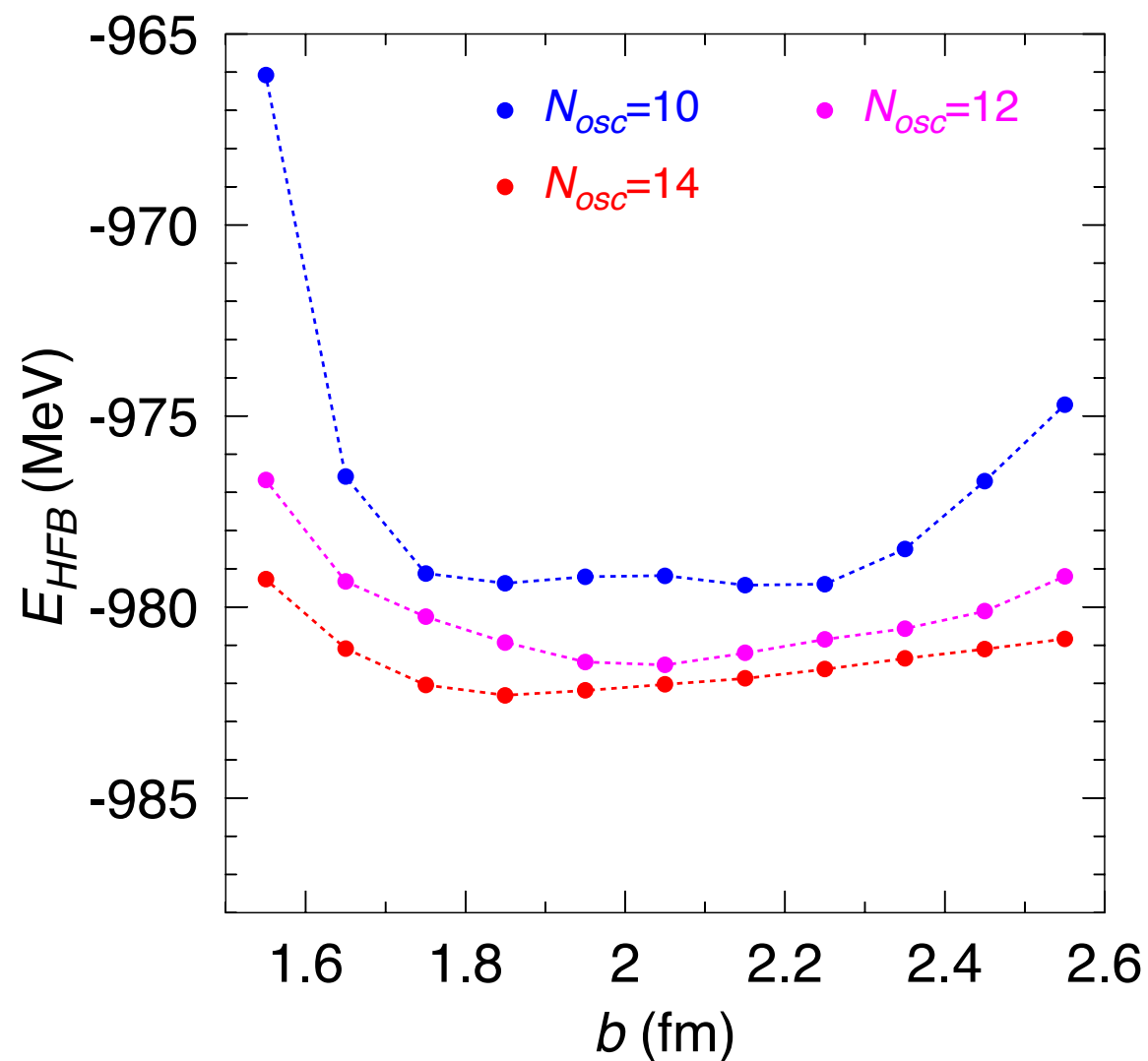


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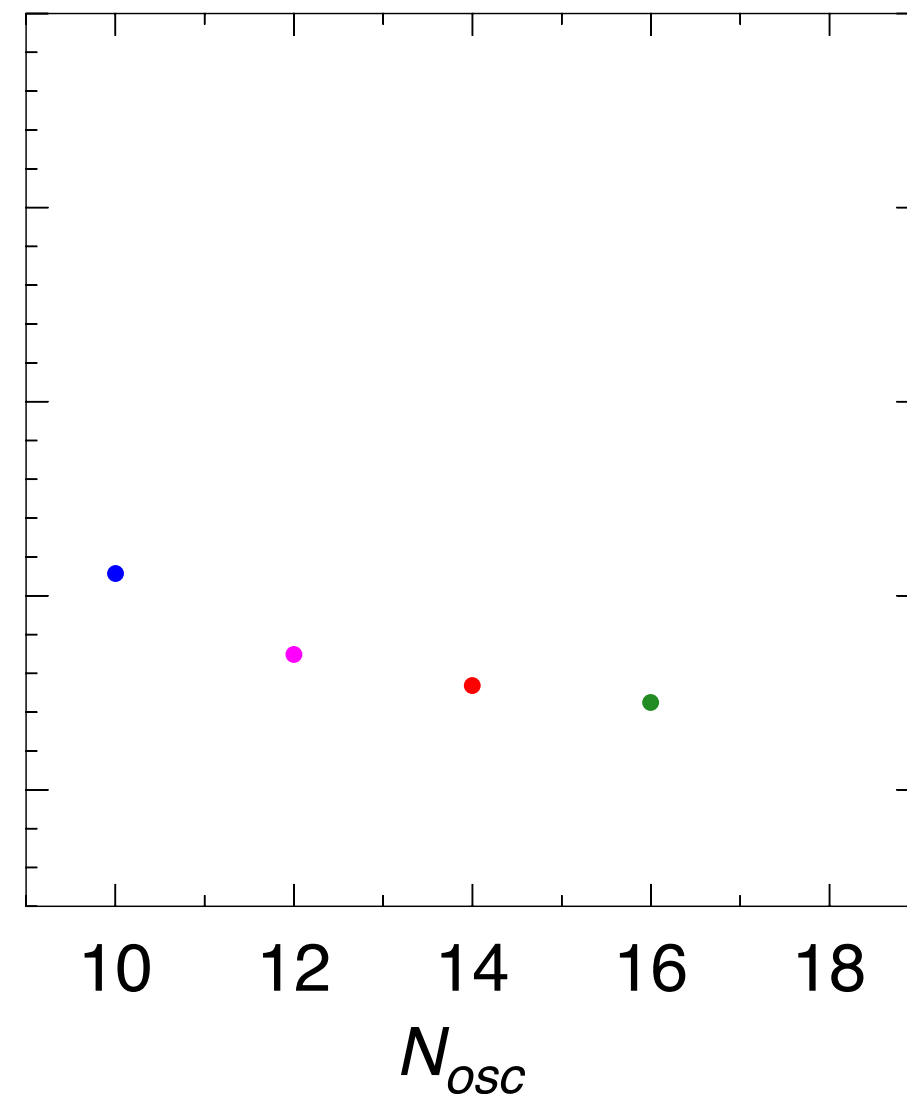
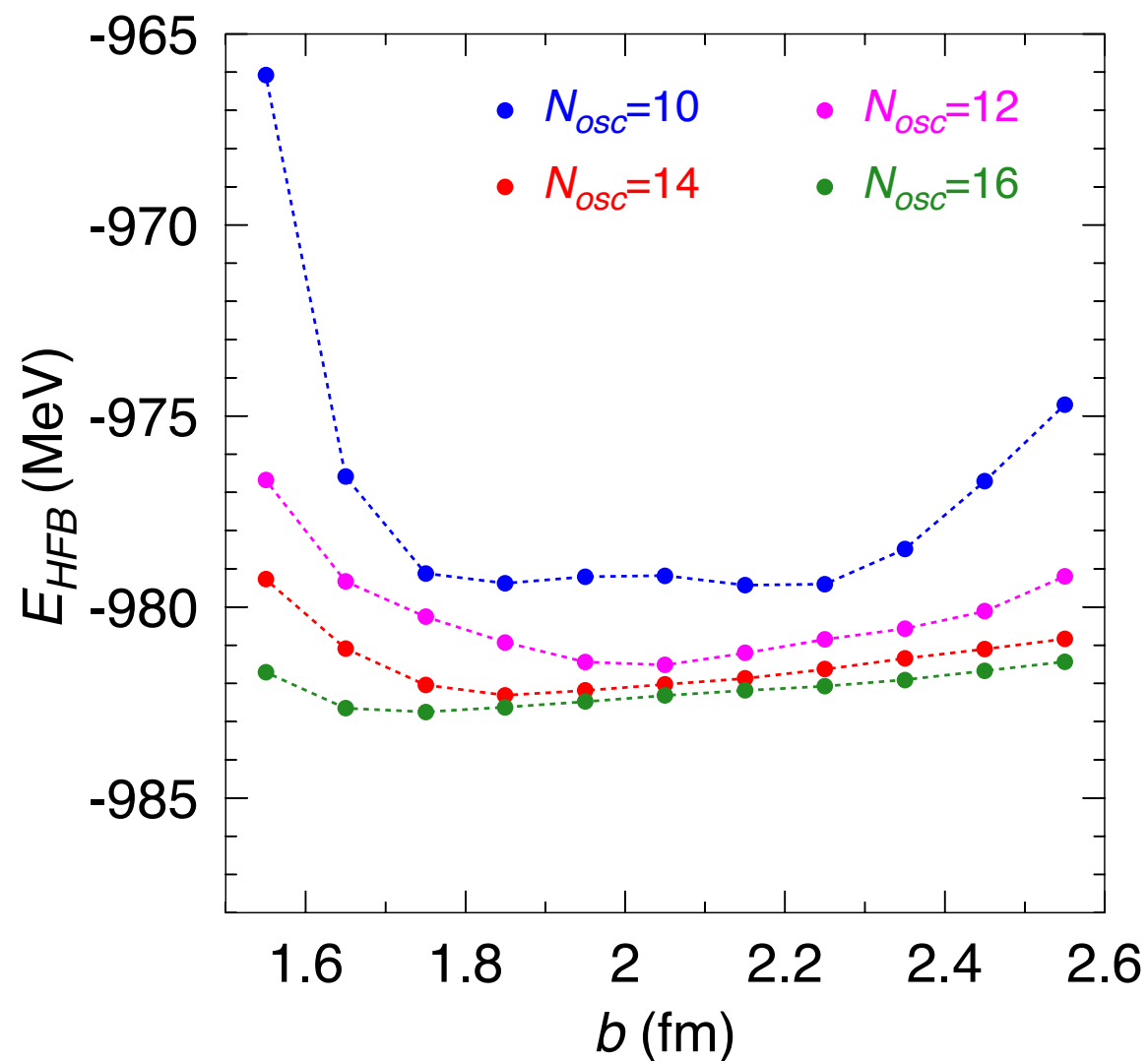


Convergence

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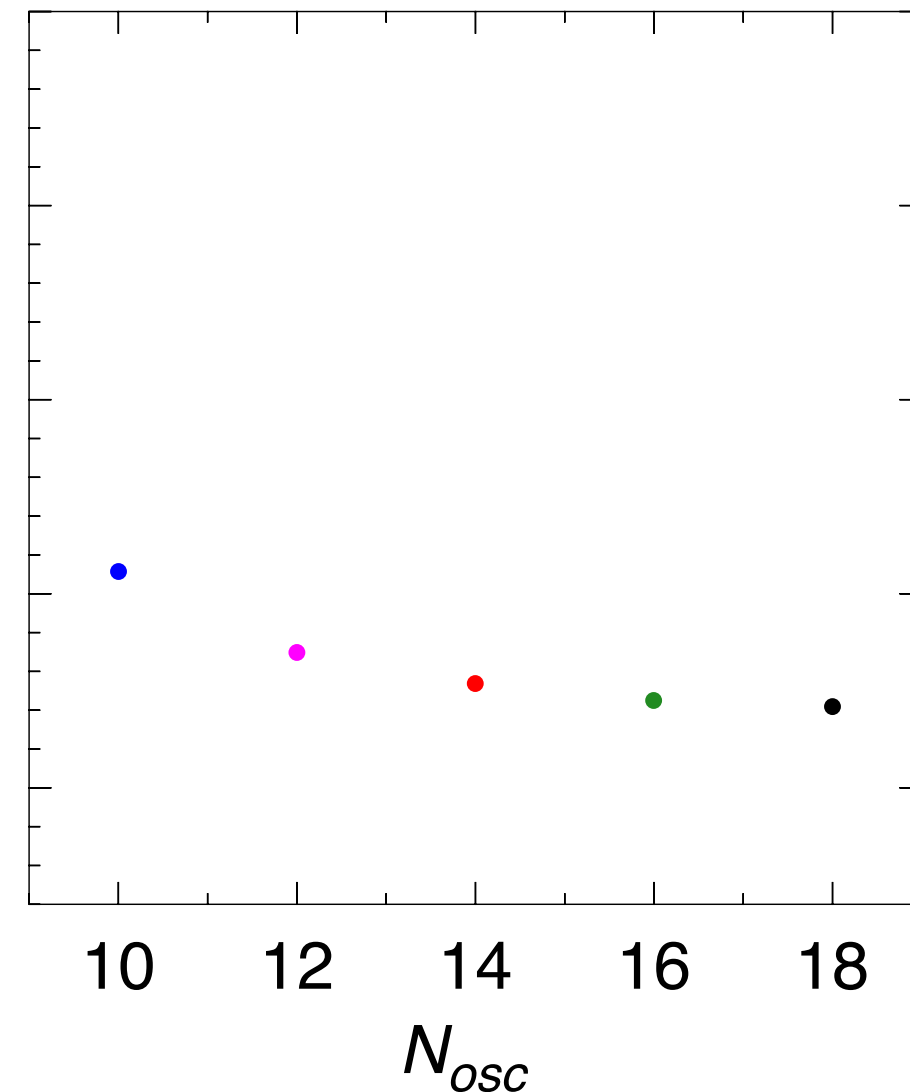
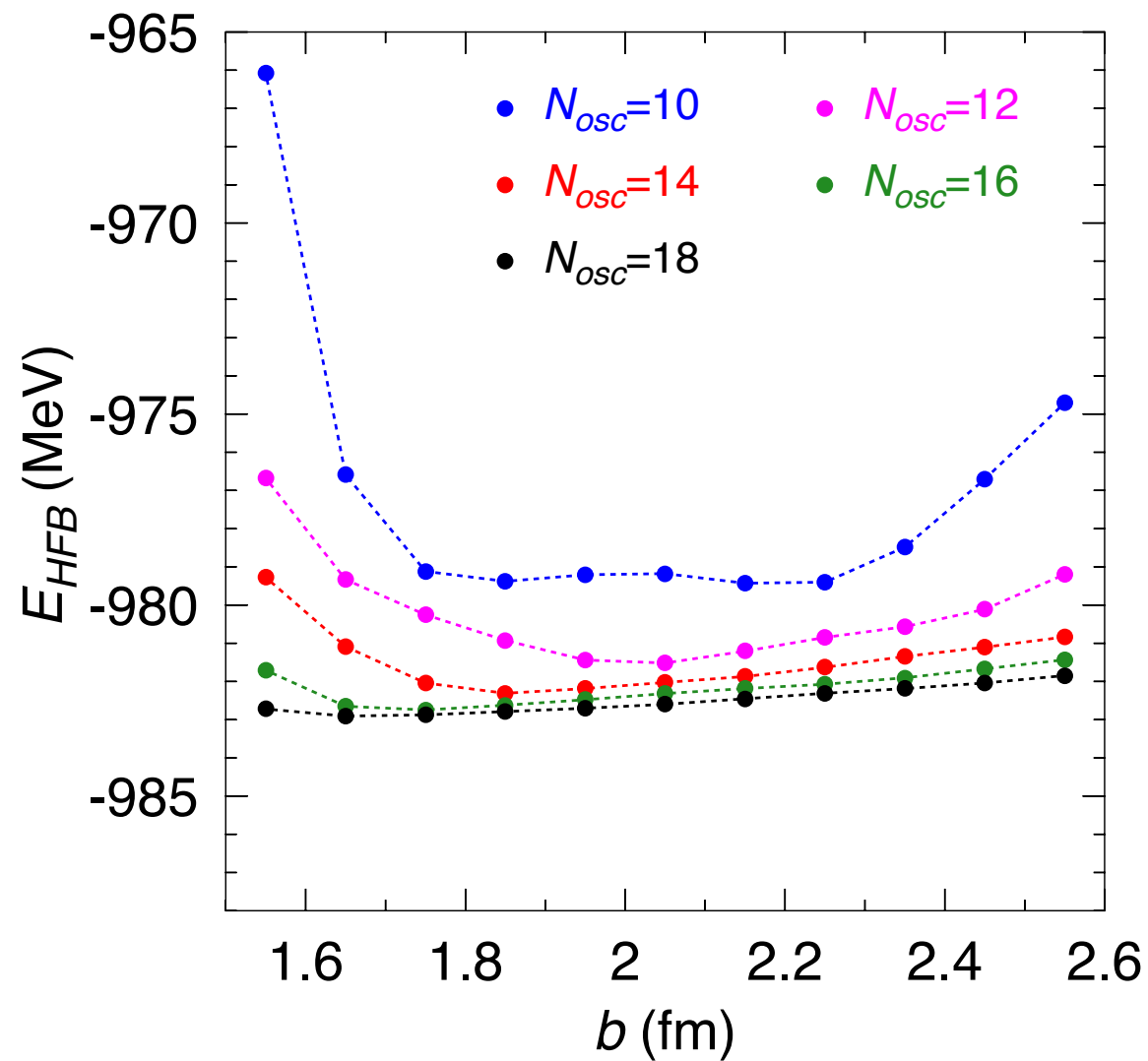


Convergence

Final convergence

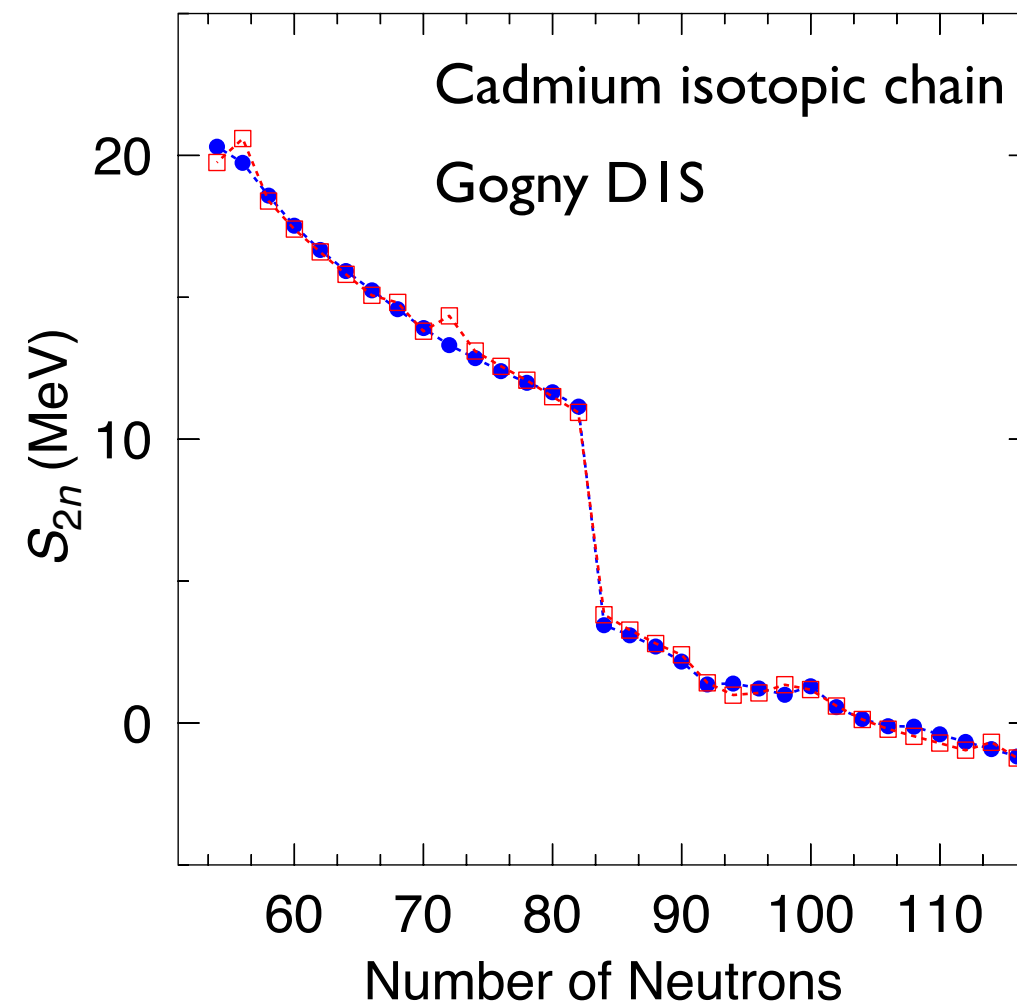
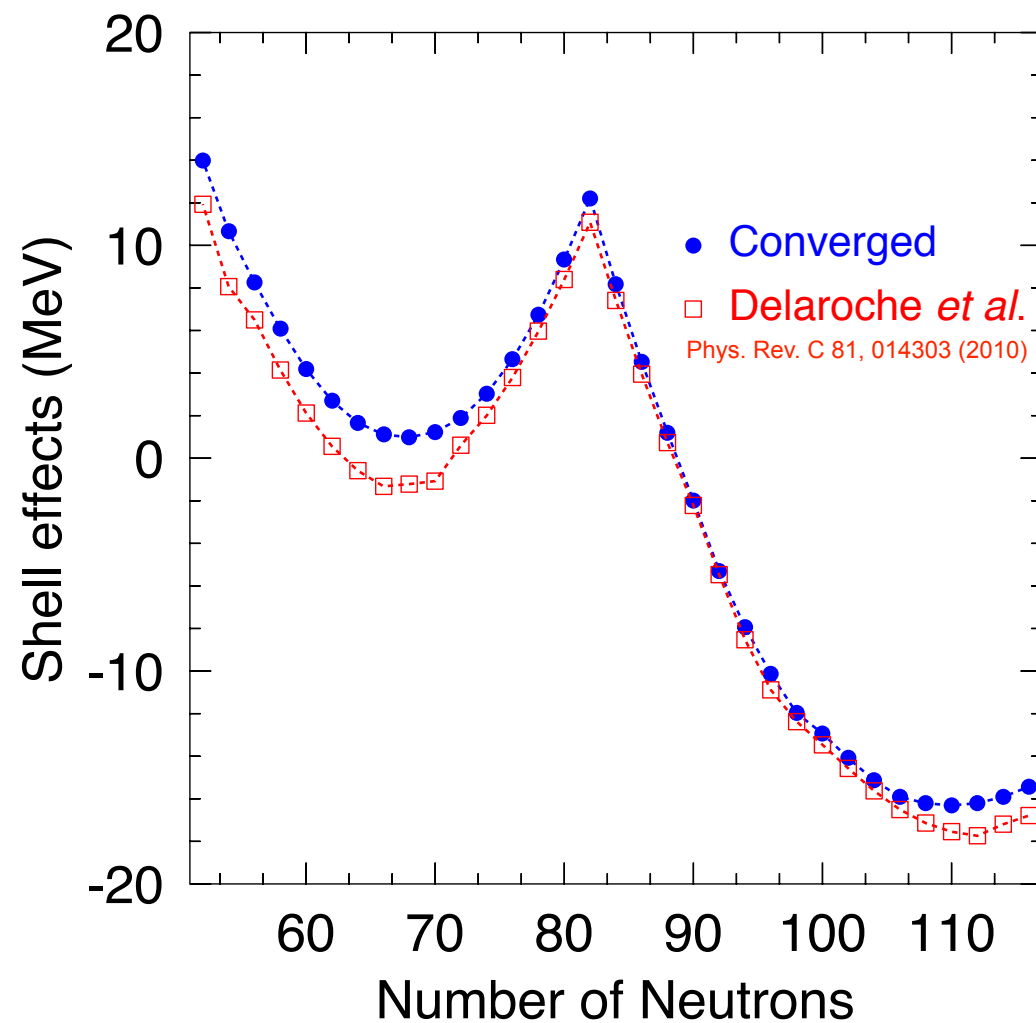
Example:

β^-	
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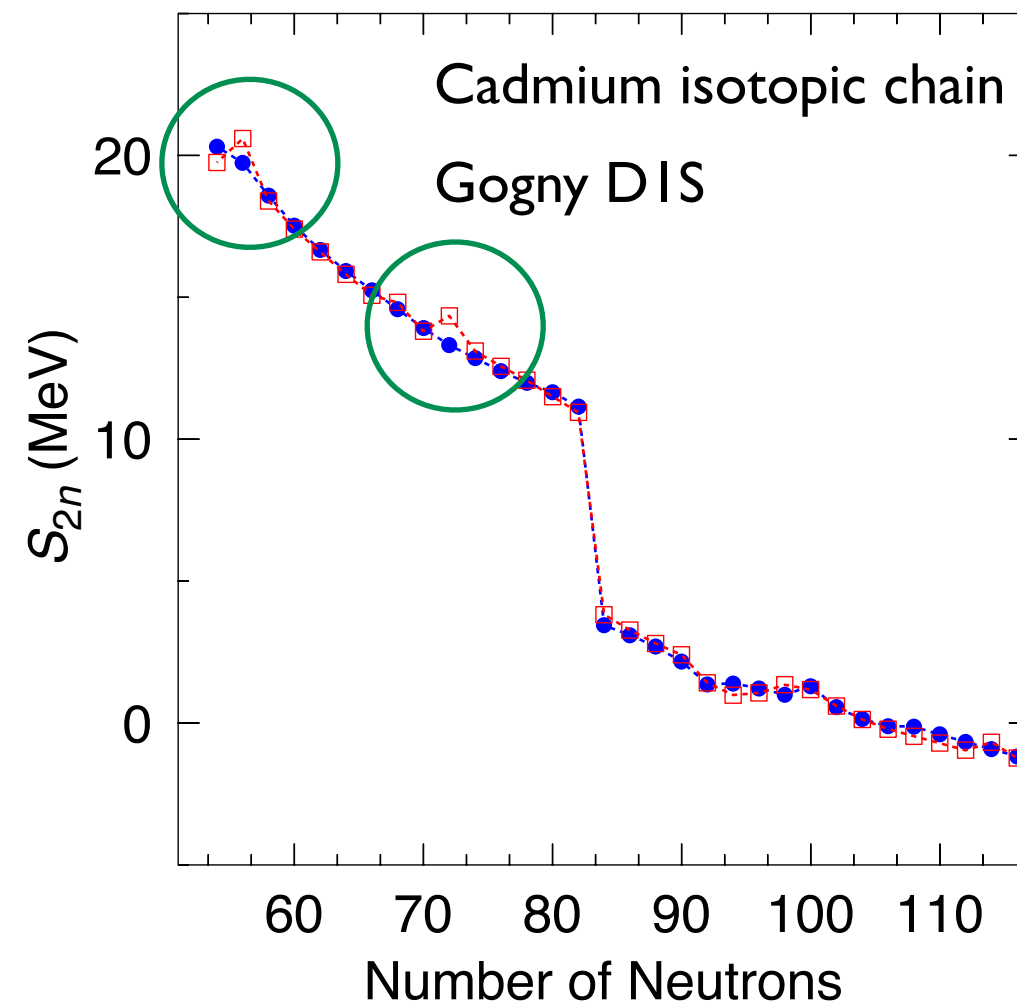
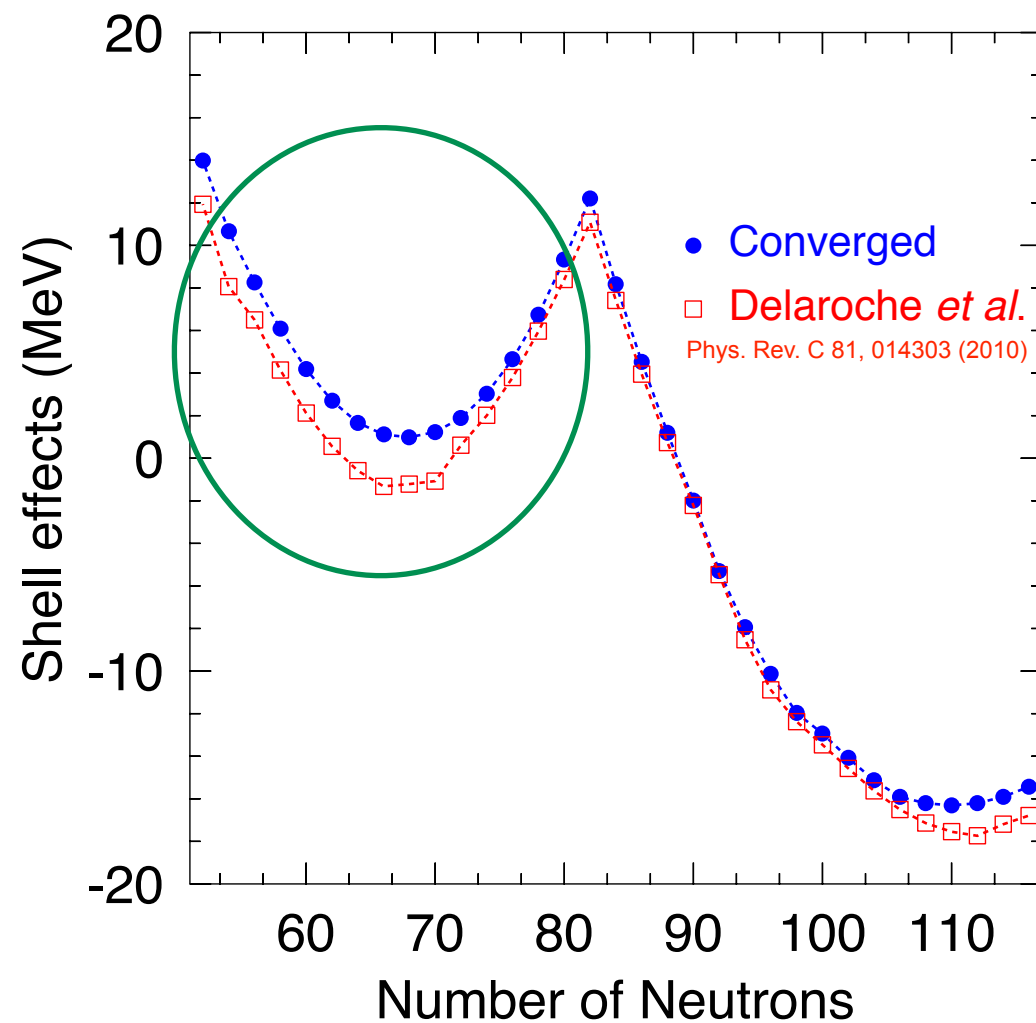
Convergence

- Published tables could contain some lack of convergence in the total binding energy.
- Two neutron separation energies are better converged.
- Artificial ‘jumps’ or ‘noise’ could appear in the S_{2n} due to lack of convergence.

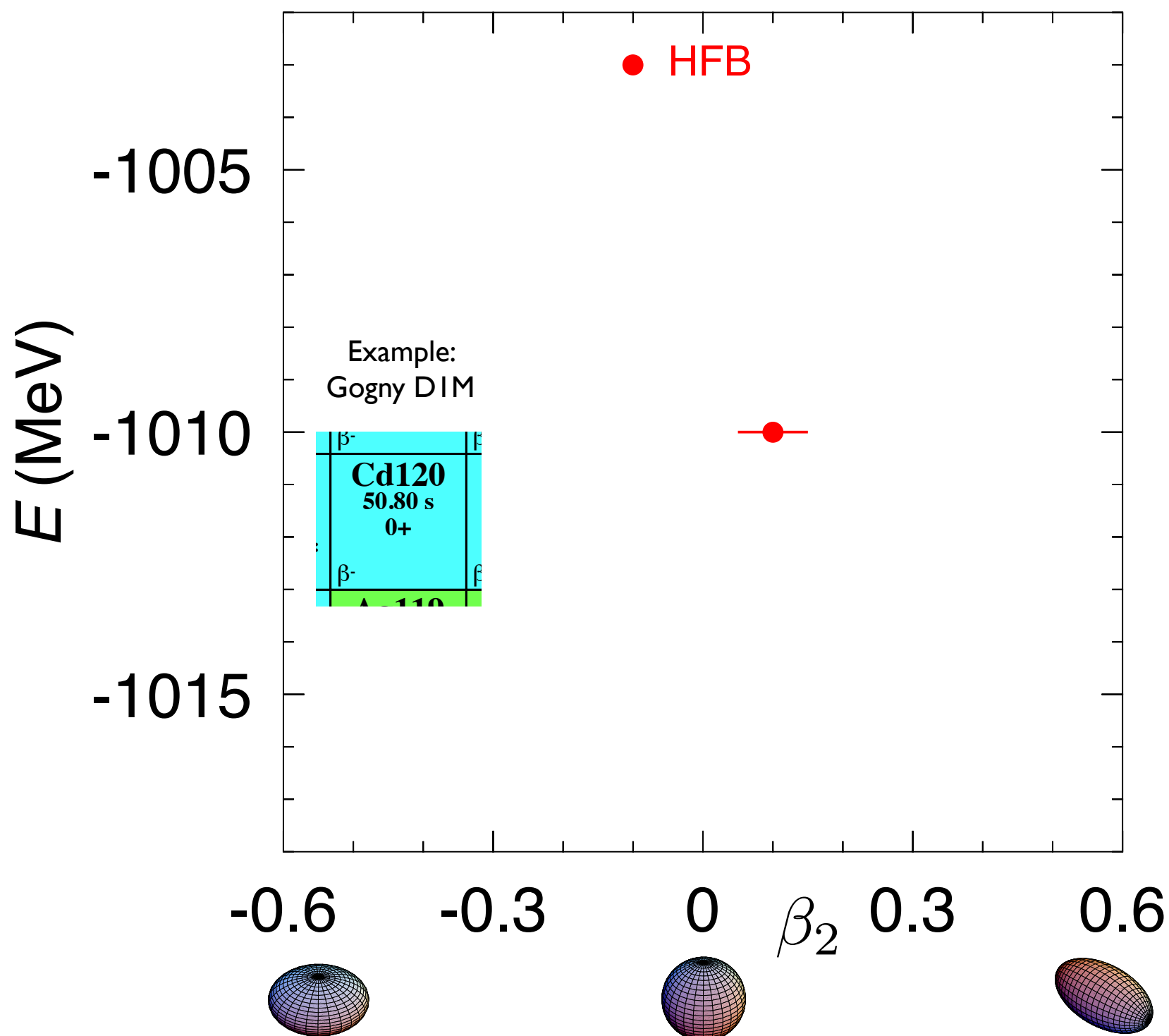


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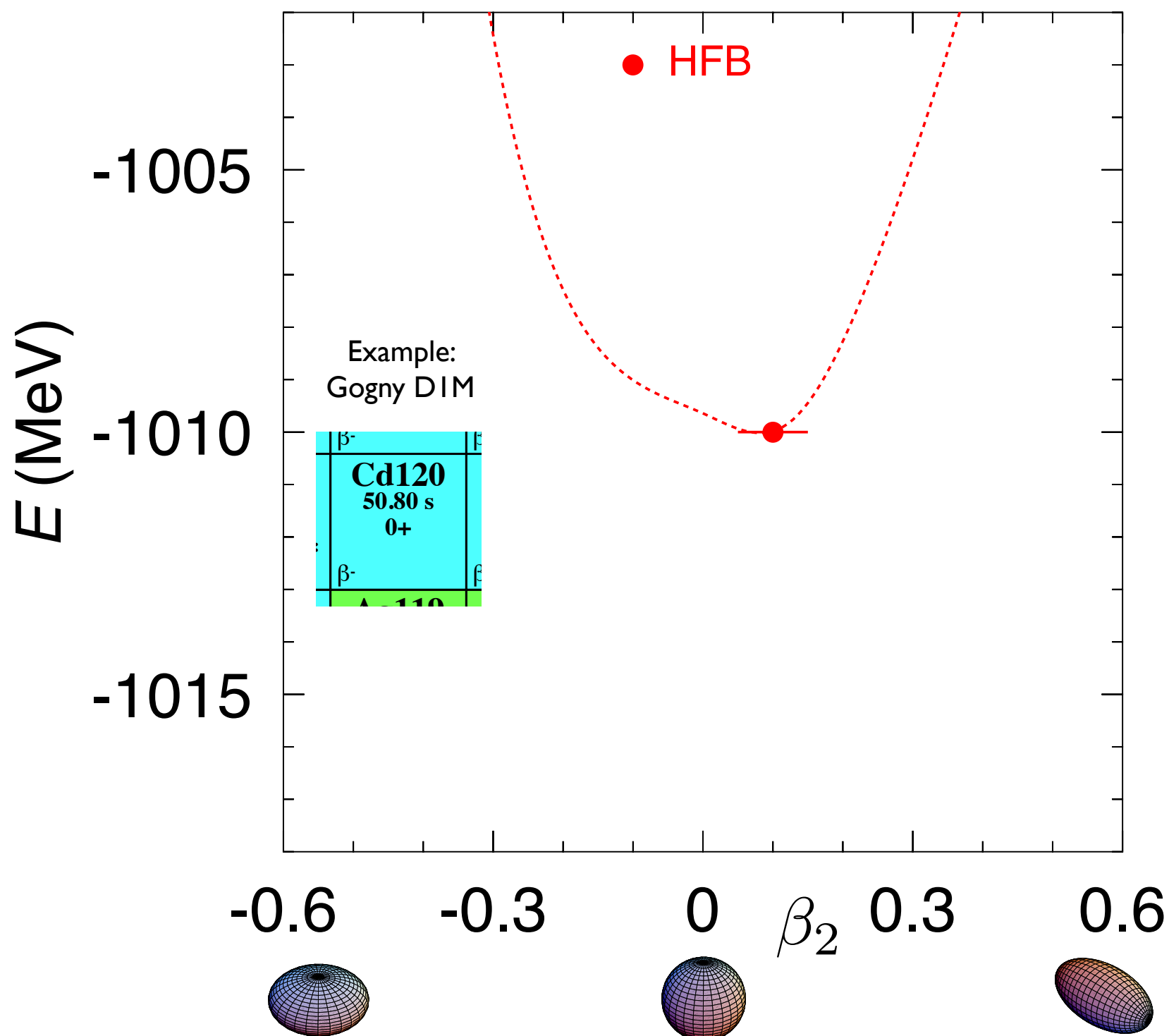


Self-consistent beyond mean field description



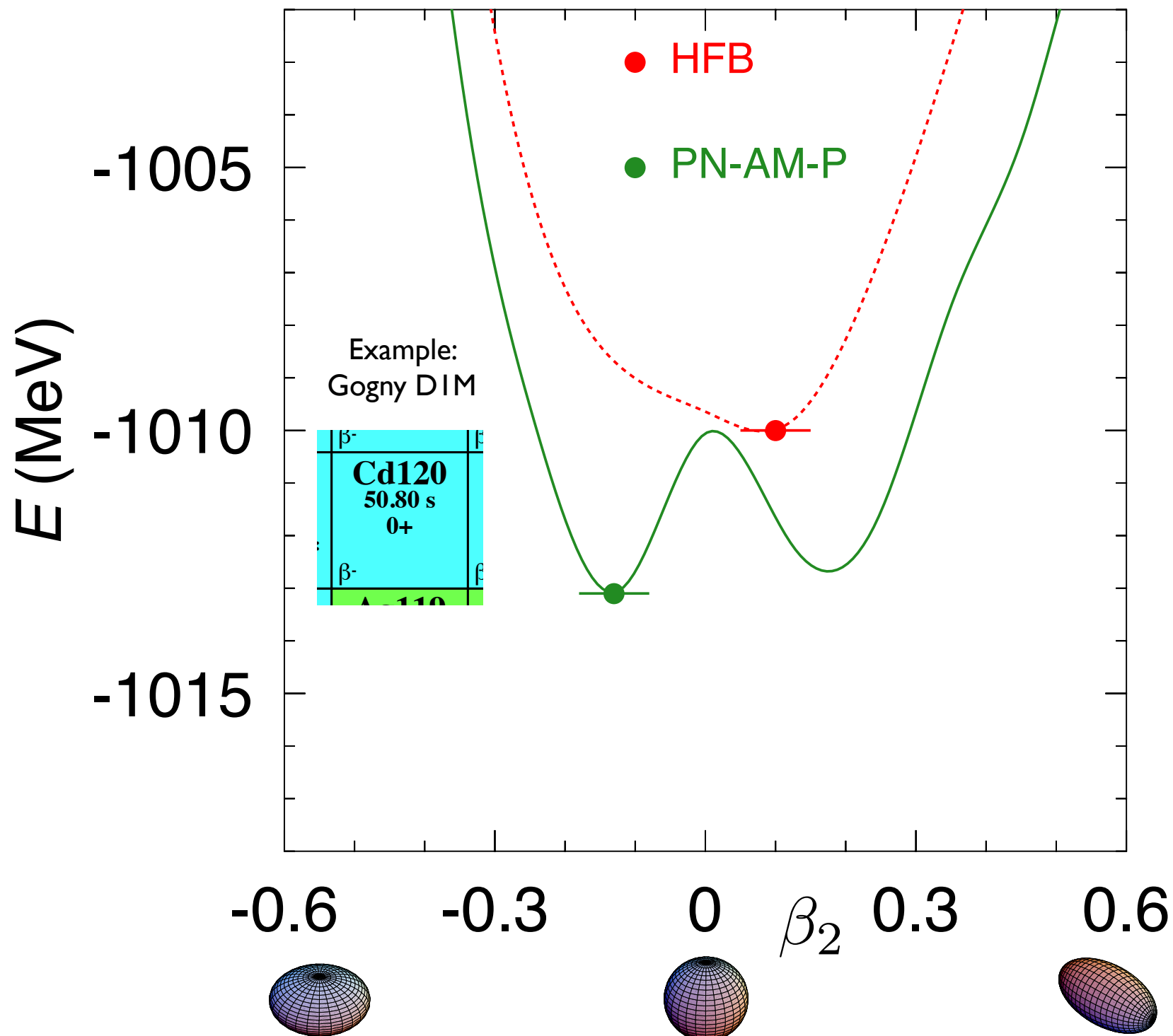
- Mean field: Hartree-Fock-Bogoliubov (HFB). No symmetry conservation and no configuration mixing.

Self-consistent beyond mean field description



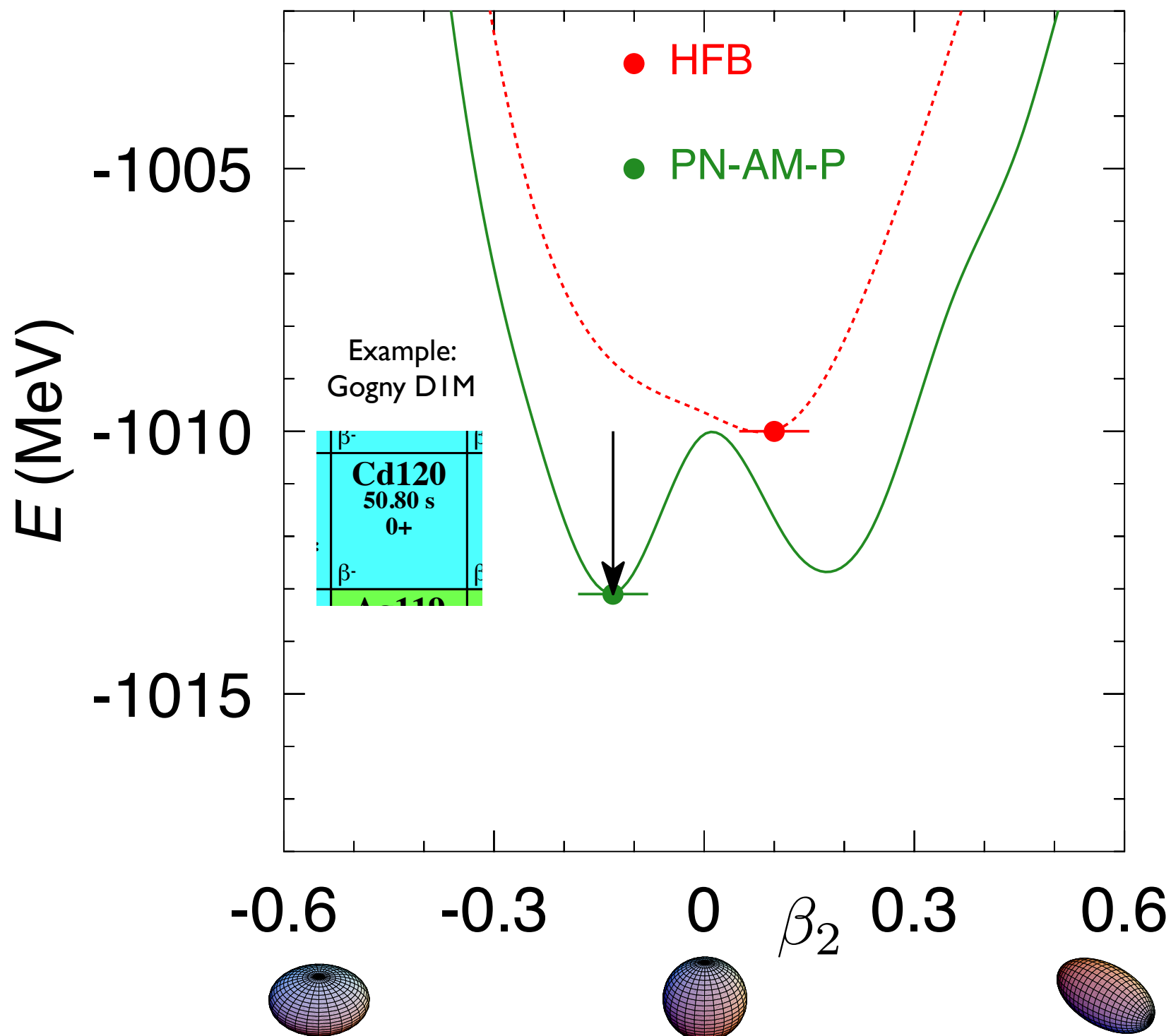
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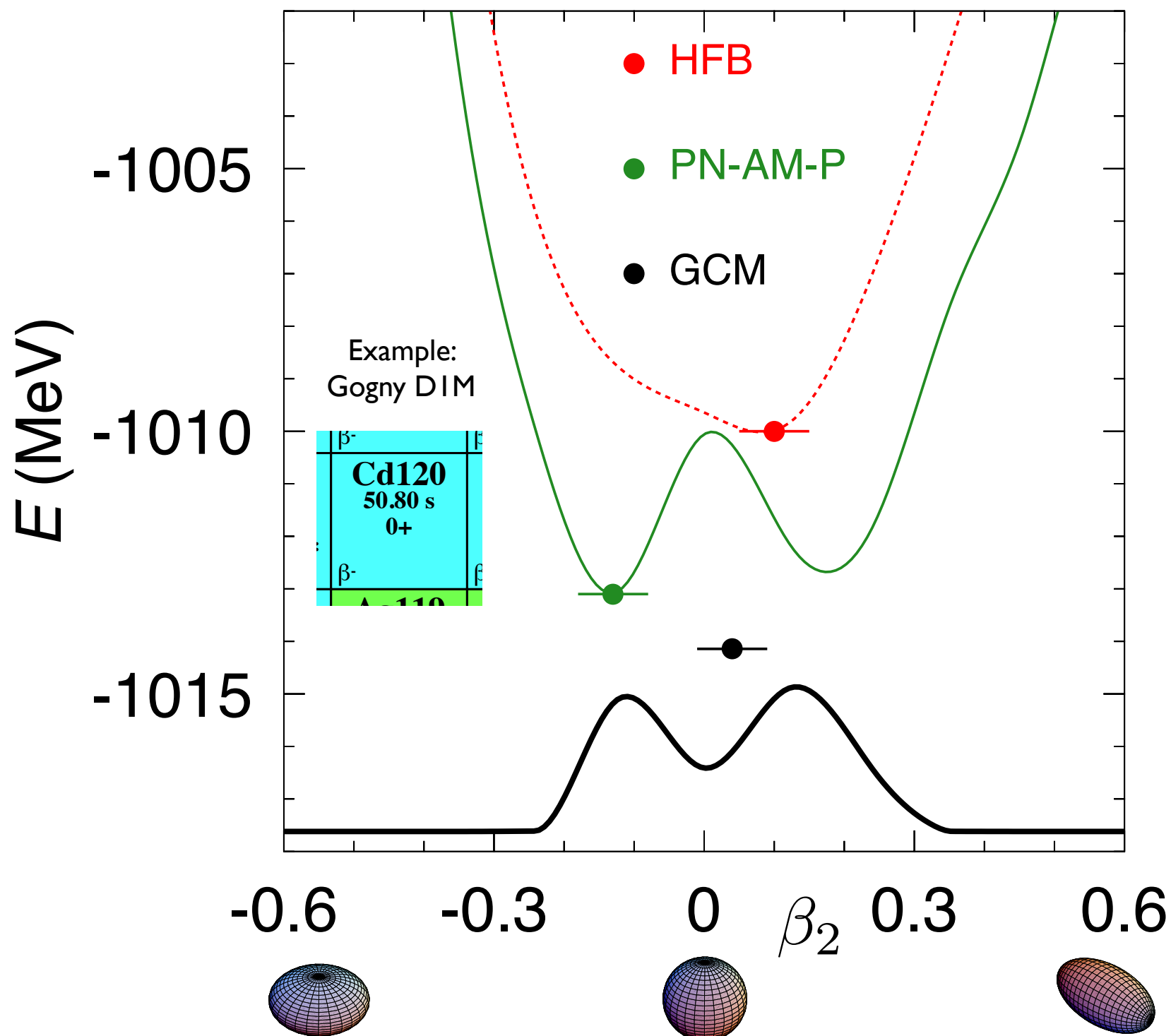
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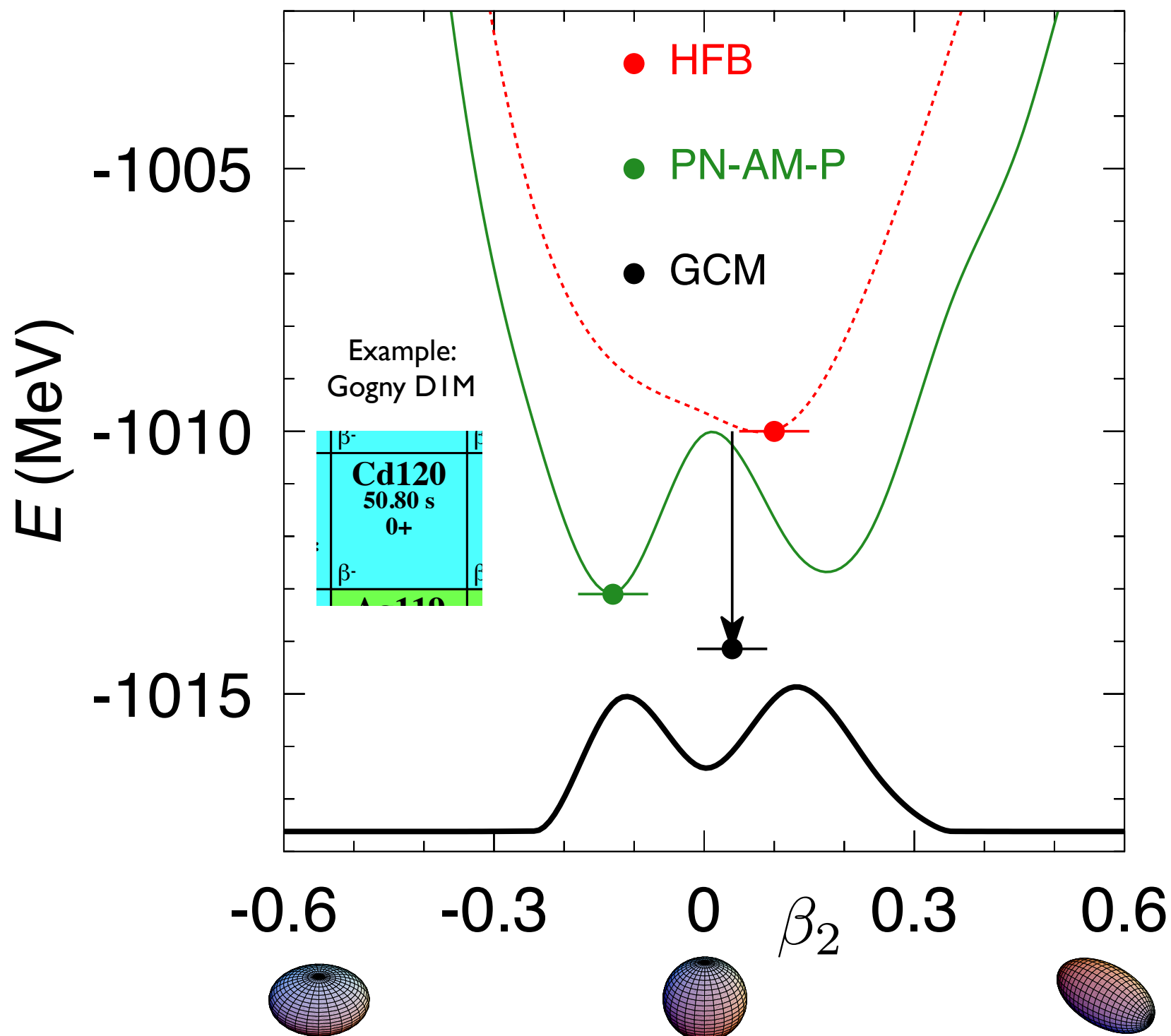


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Self-consistent beyond mean field description

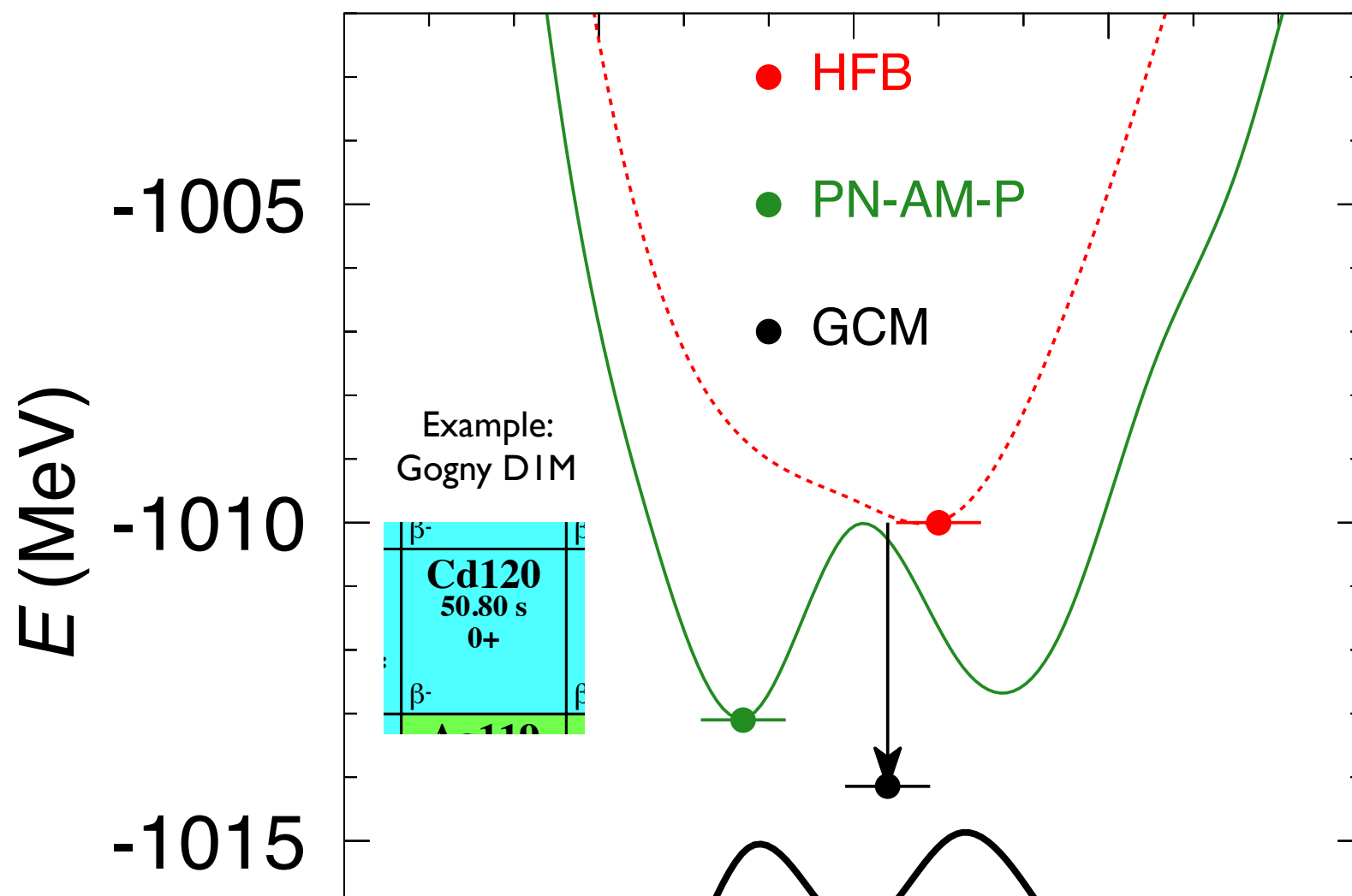


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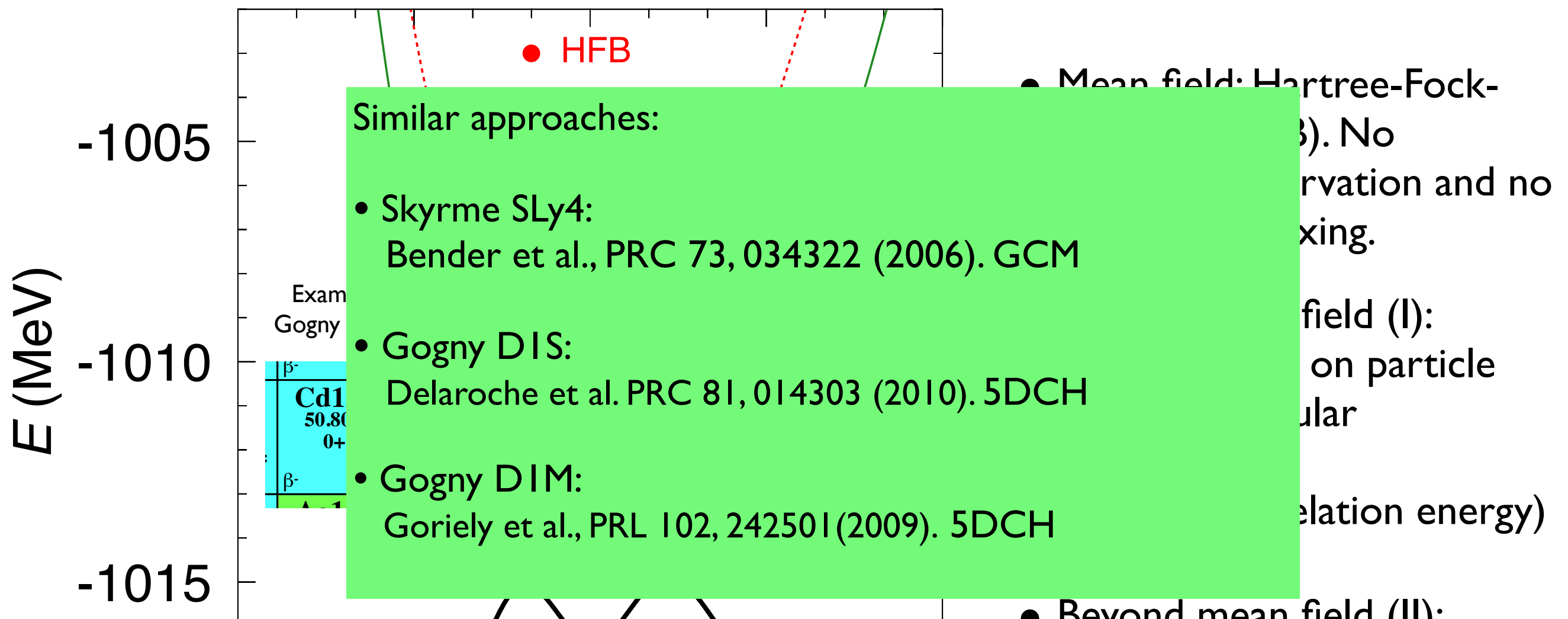
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- **Correlations:** Energy difference between the binding energies calculated with the mean-field approximation (HFB) and any other method, using the same underlying interaction.

If the beyond mean field method is **variational, correlations must give extra binding energy.**

Self-consistent beyond mean field description

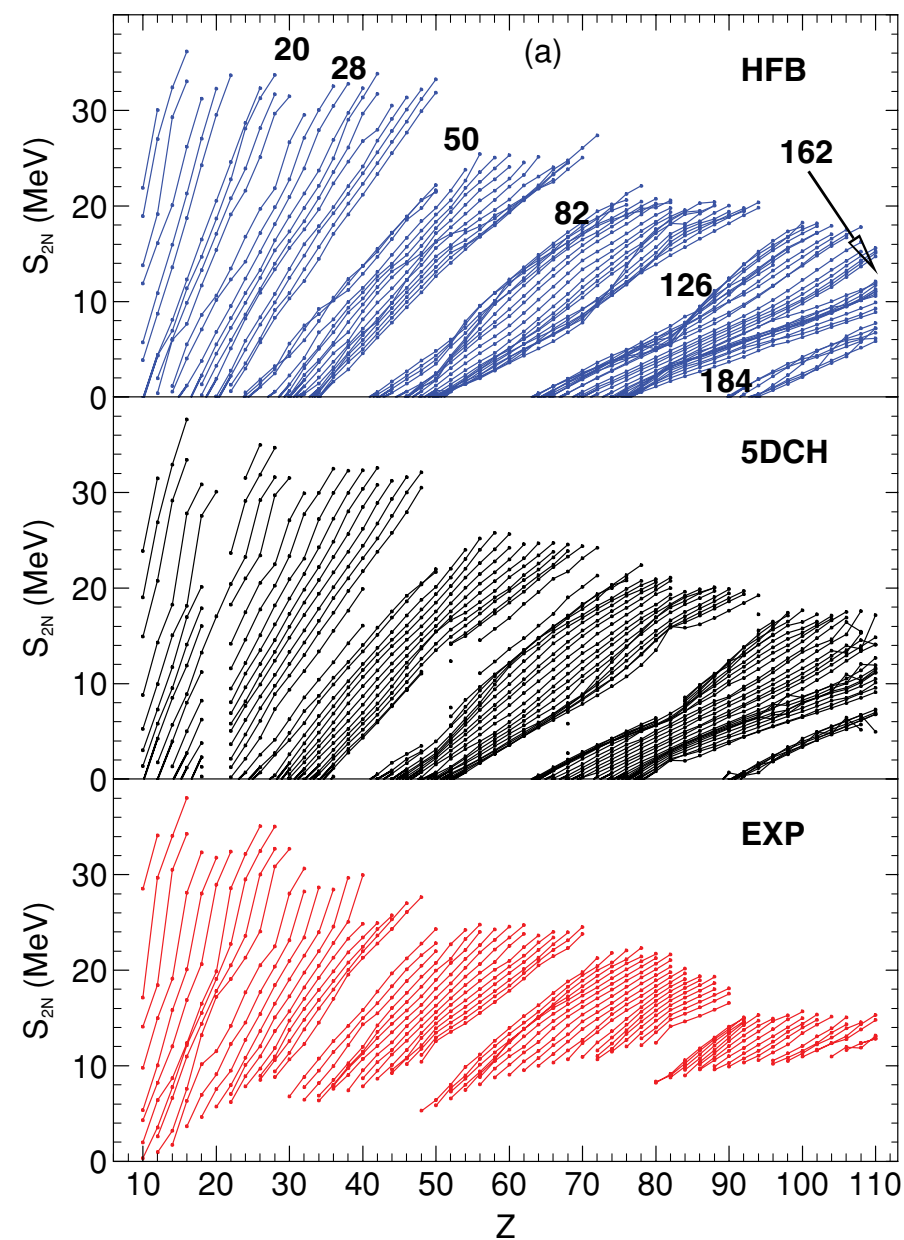


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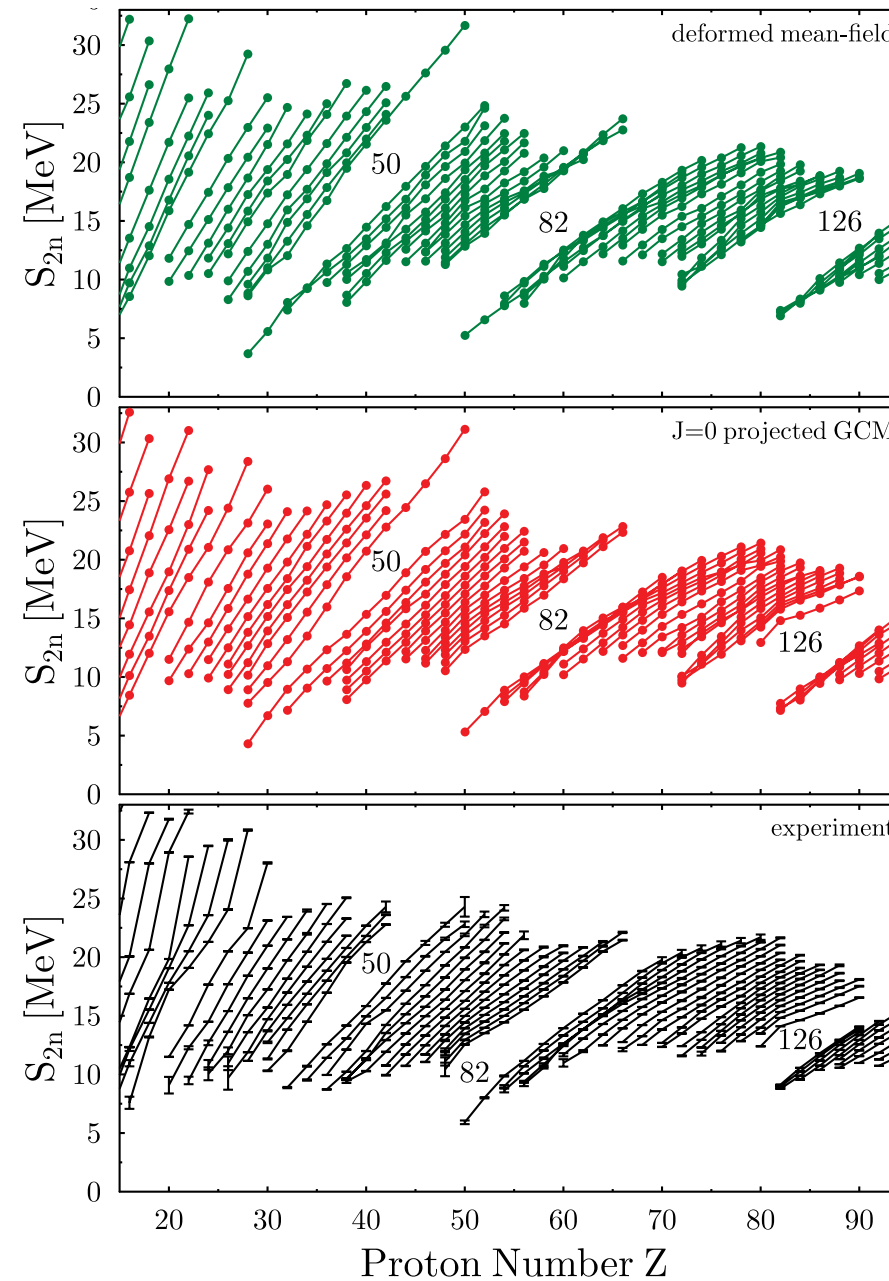
Mean field vs. Beyond mean field. Global systematics

Gogny DIS



Delaroche et al. PRC 81, 014303 (2010)

Skyrme SLy4

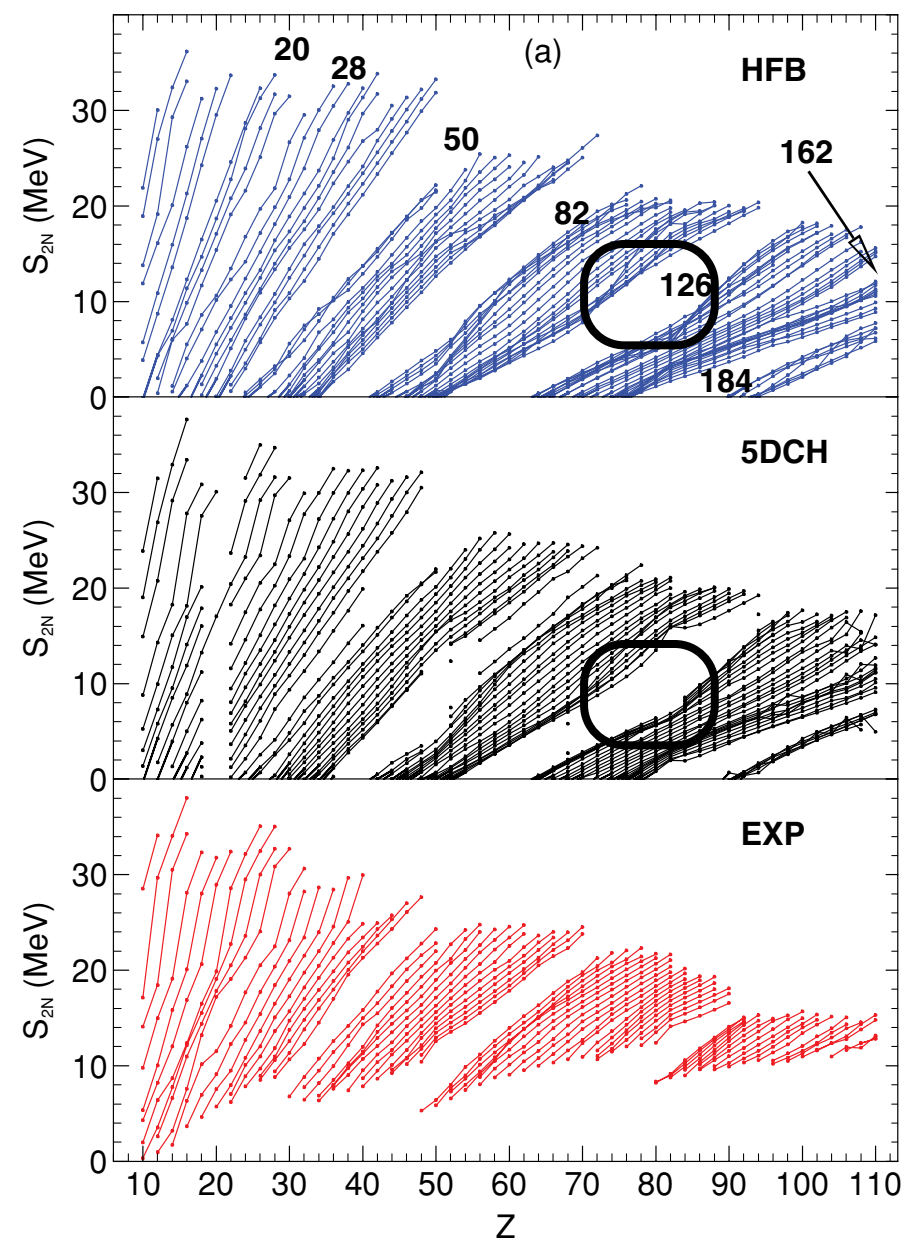


Bender et al., PRC 73, 034322 (2006)

- Beyond mean field effects tend to reduce the shell gaps
- Separation energies are smoother when beyond mean field are included.

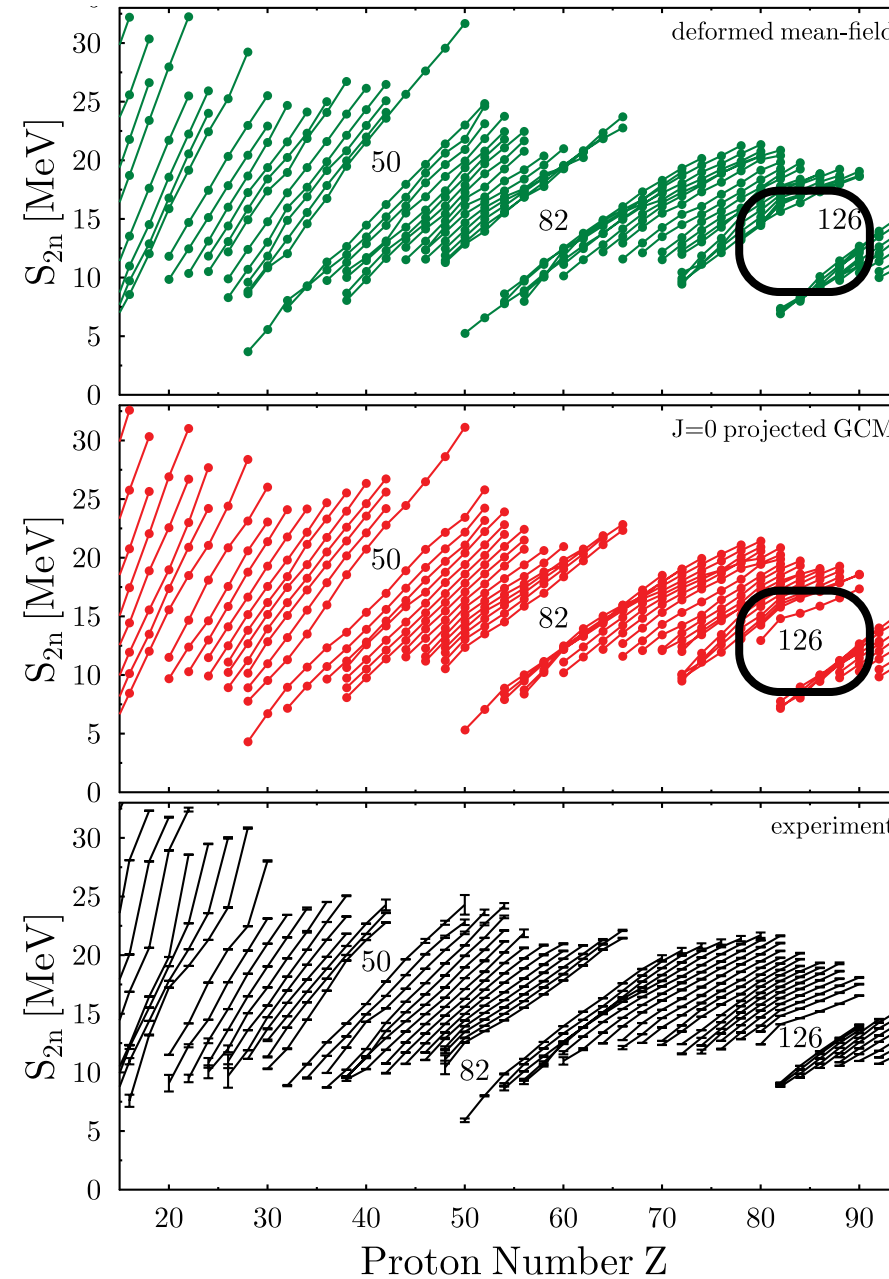
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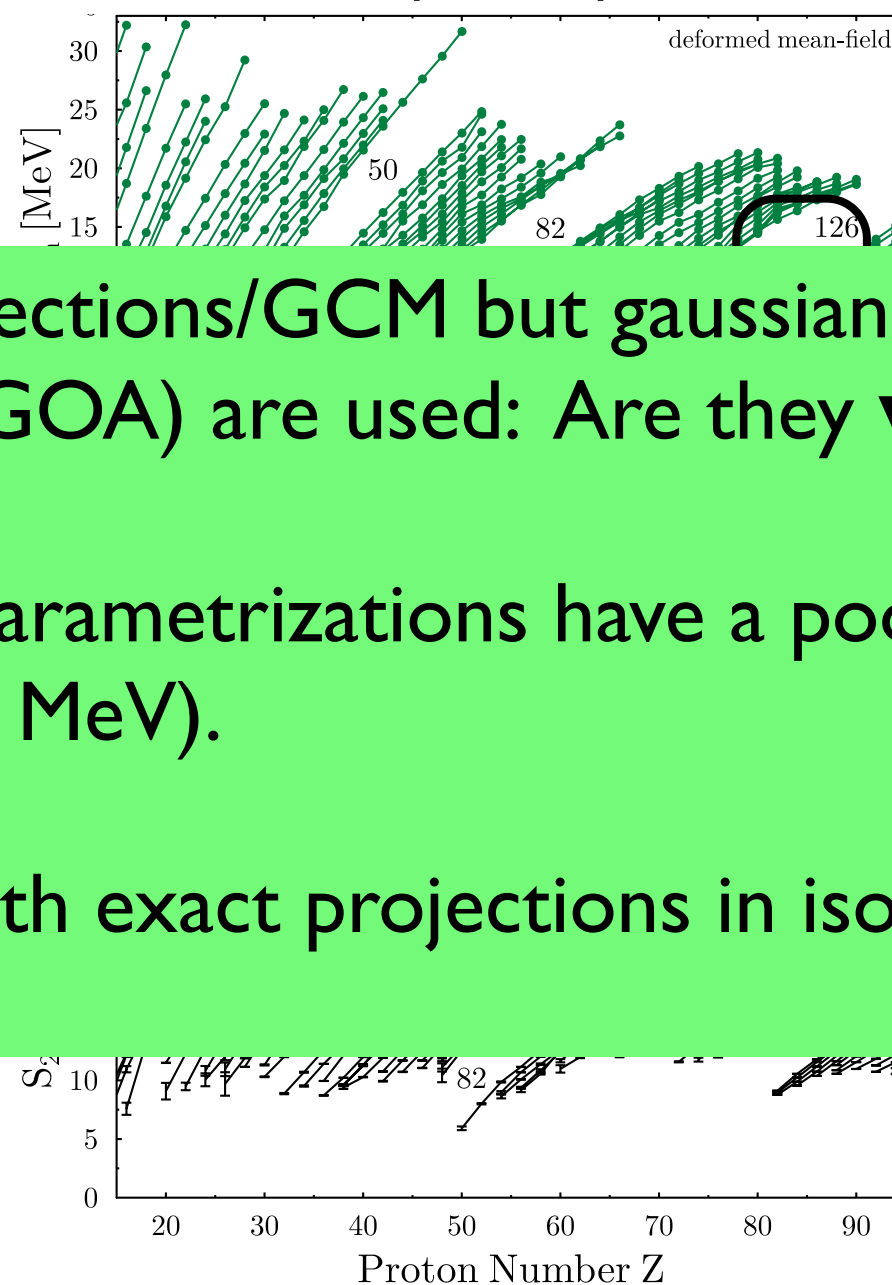
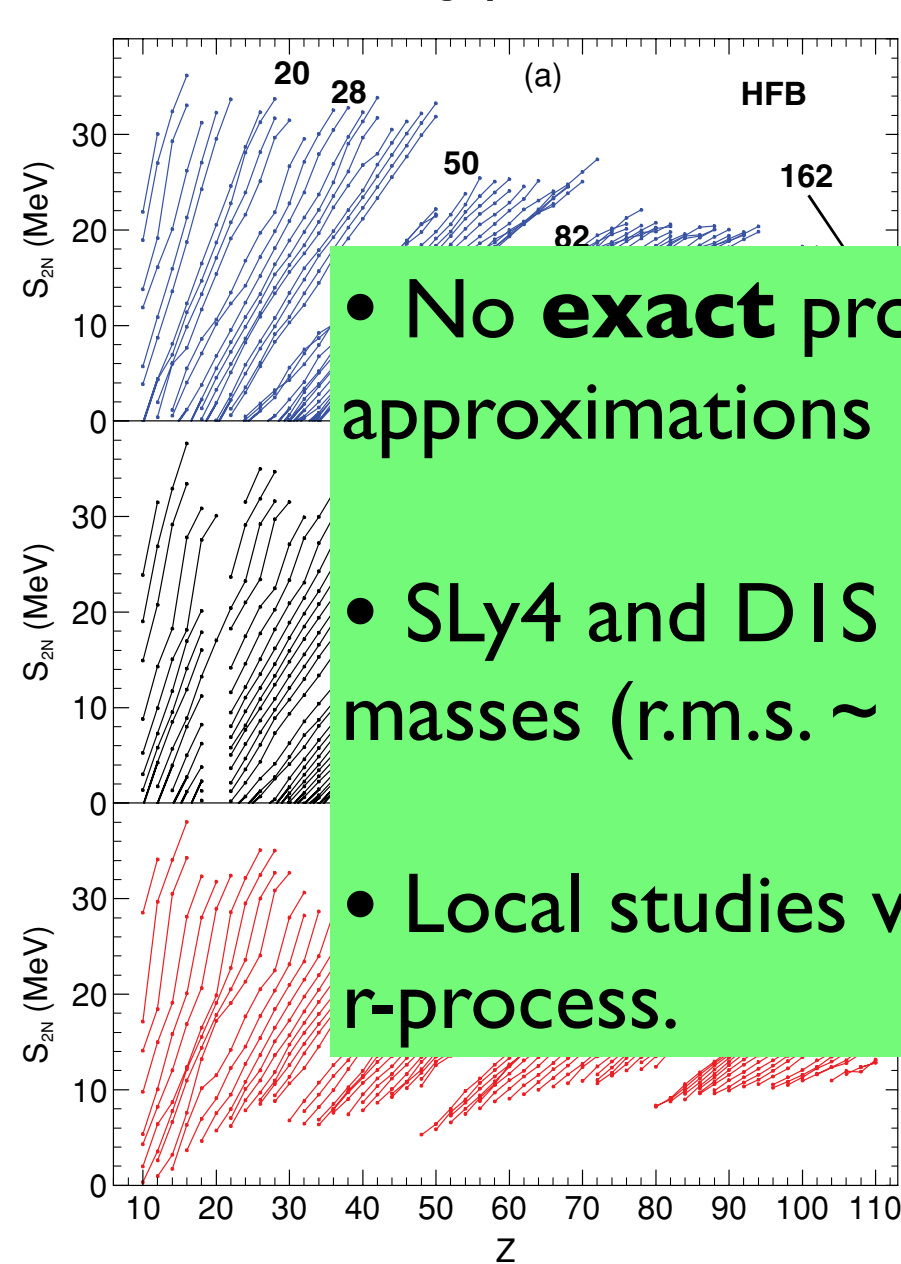
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Mean field vs. Beyond mean field. Global systematics

Gogny DIS

Skyrme SLy4



- No **exact** projections/GCM but gaussian overlap approximations (GOA) are used: Are they **variational**?

- SLy4 and DIS parametrizations have a poor performance for masses (r.m.s. ~ 5 MeV).

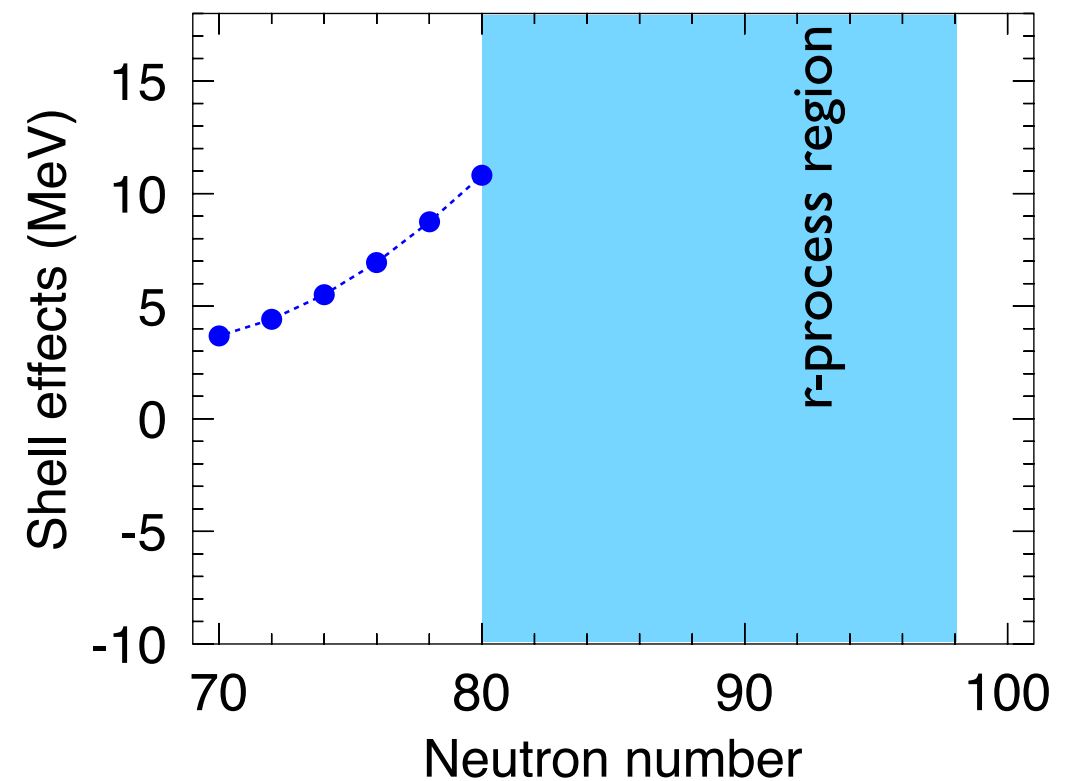
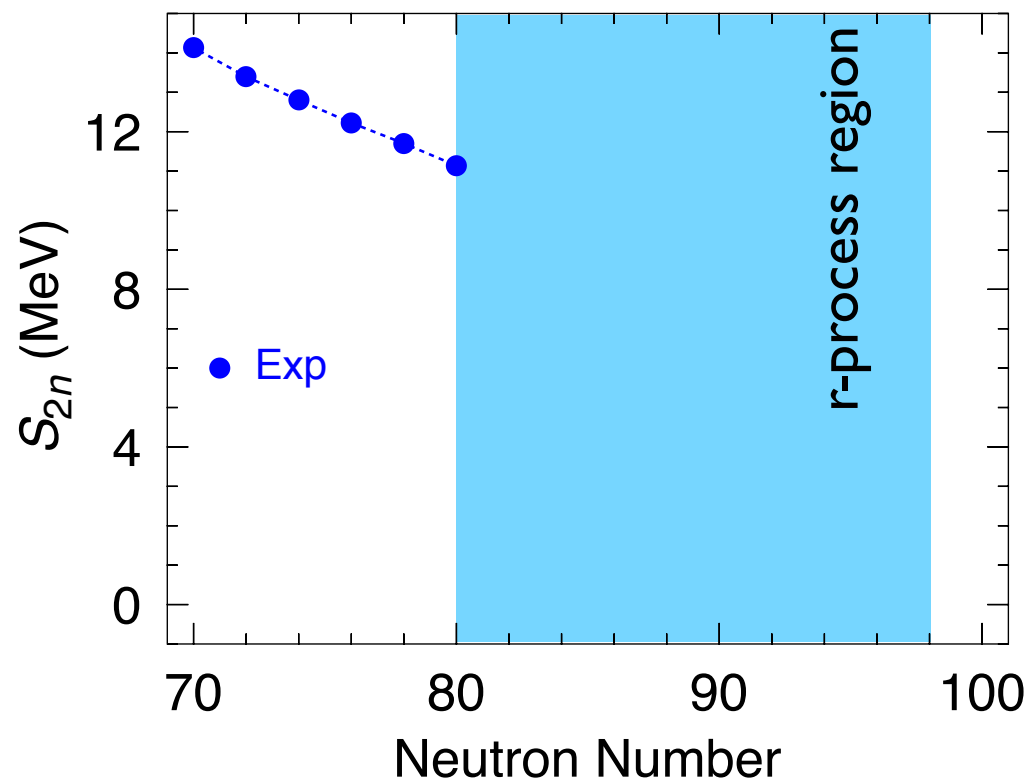
- Local studies with exact projections in isotopes relevant for r-process.

Delaroche et al. PRC 81, 014303 (2010)

Bender et al., PRC 73, 034322 (2006)

Mean field vs. Beyond mean field. Local systematics

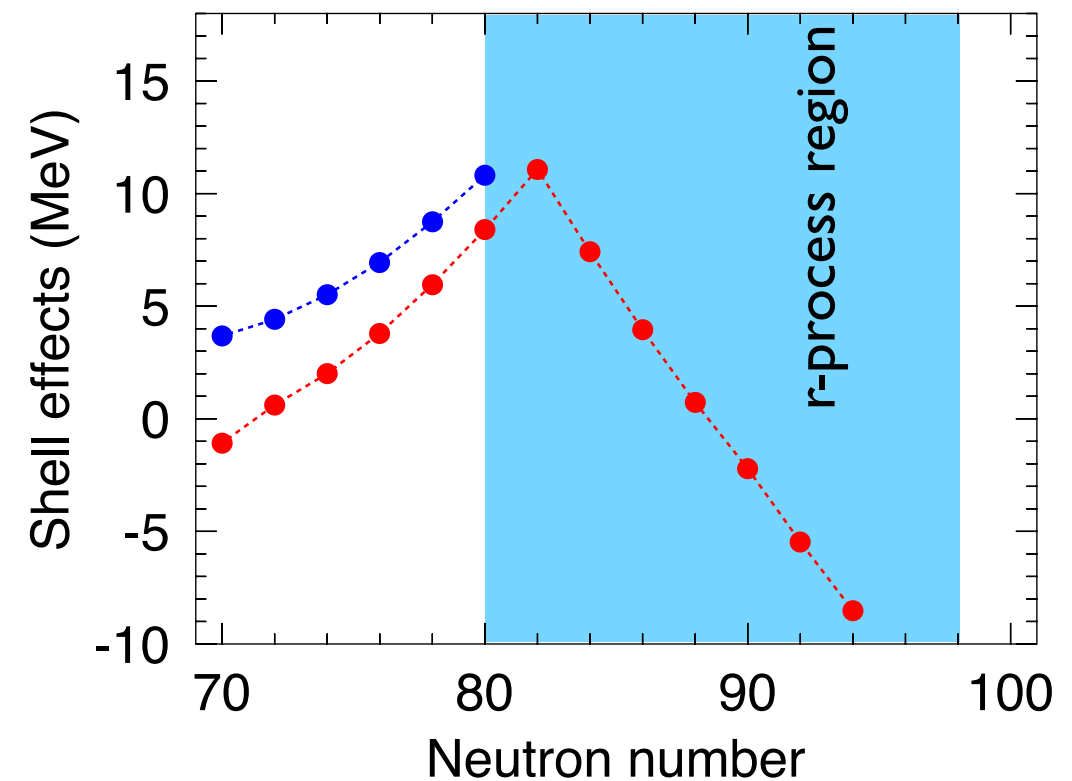
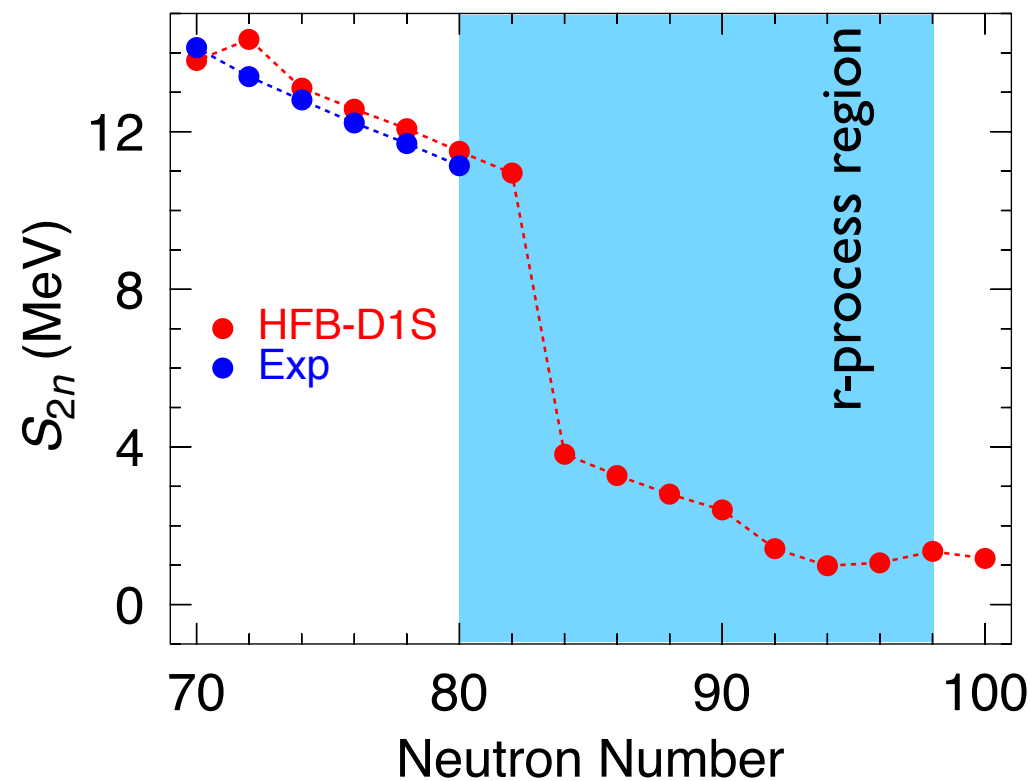
Cadmium isotopes. Gogny DIS parametrization



- Similar behavior of the two-neutron separation energies in all approaches and close to the experiment in the experimental region.
- GCM approach always includes correlation energies (variational) while 5DCH fails close to the shell closure.
- 5DCH approach removes the shell gap at $N=82$ while the others still give a sizable gap. This quenching is an artifact of the 5DCH and not an effect of including correlations beyond mean field (NOT VARIATIONAL).

Mean field vs. Beyond mean field. Local systematics

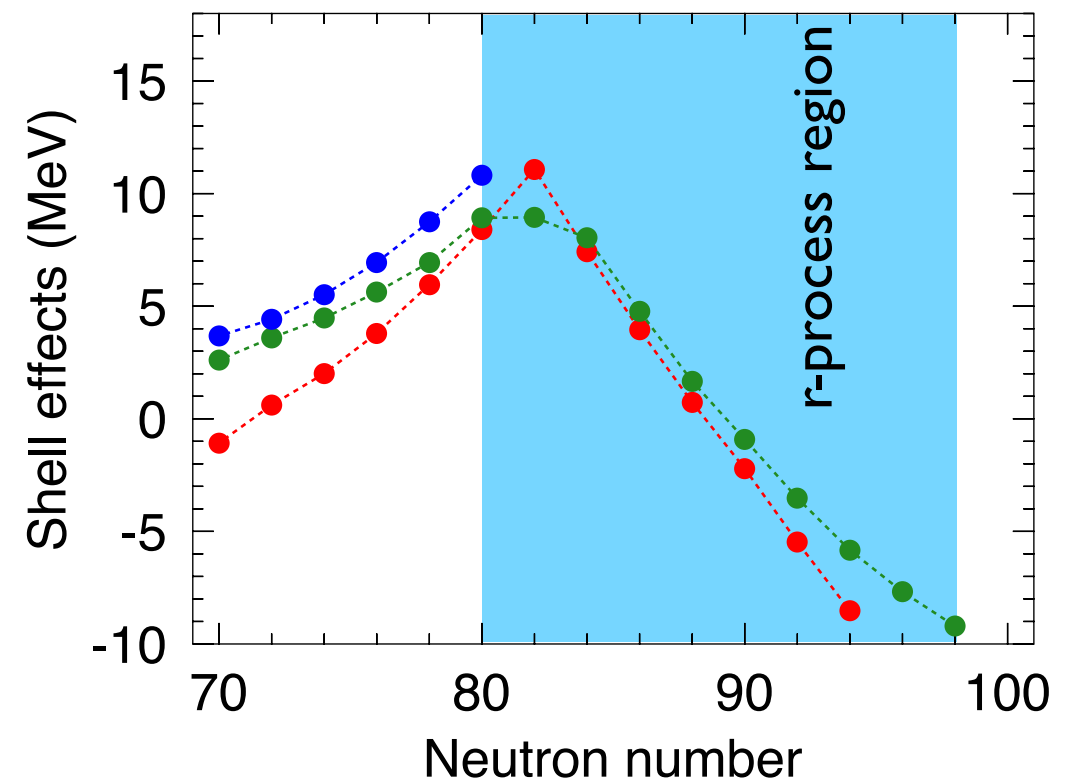
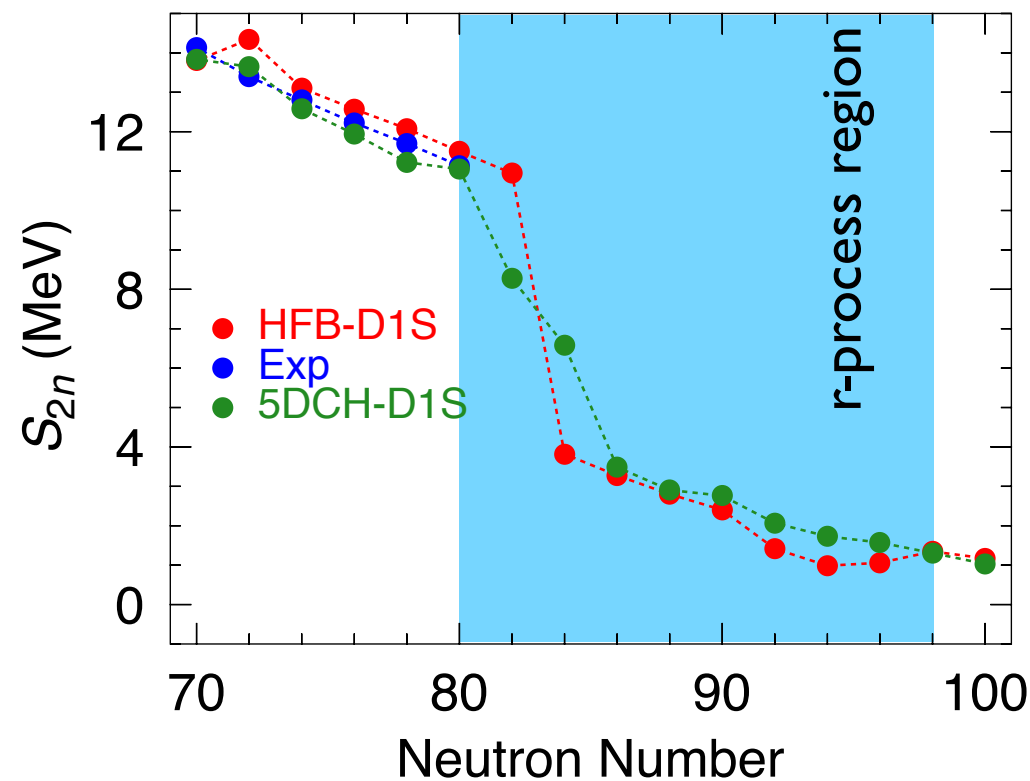
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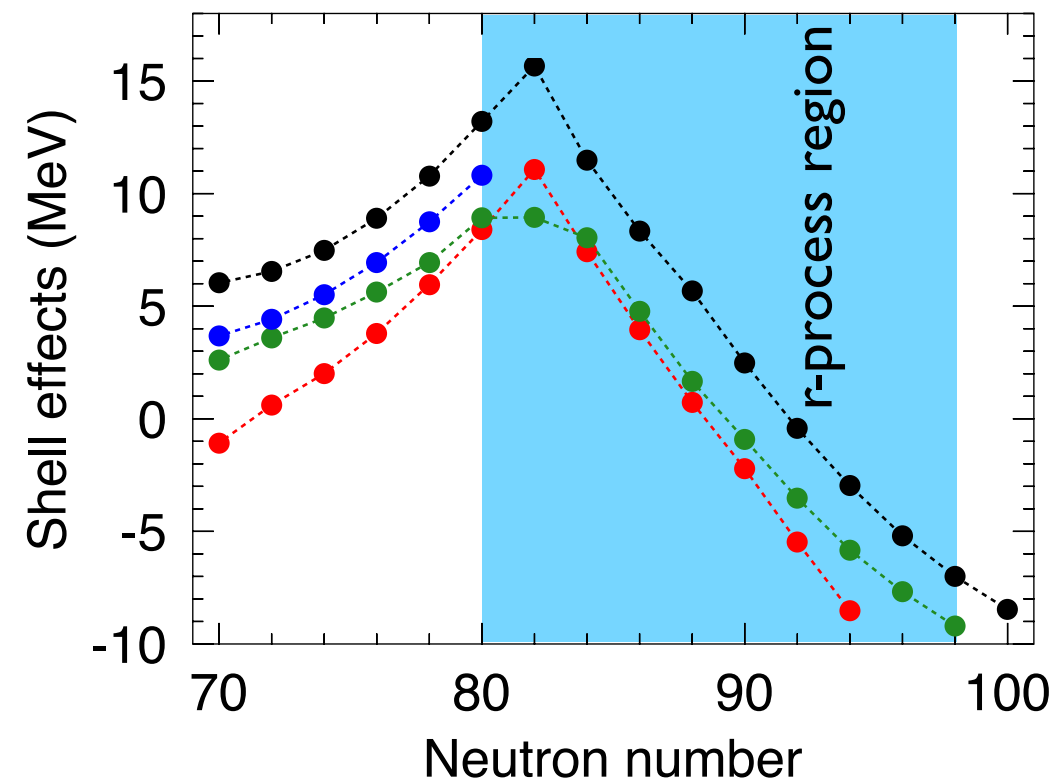
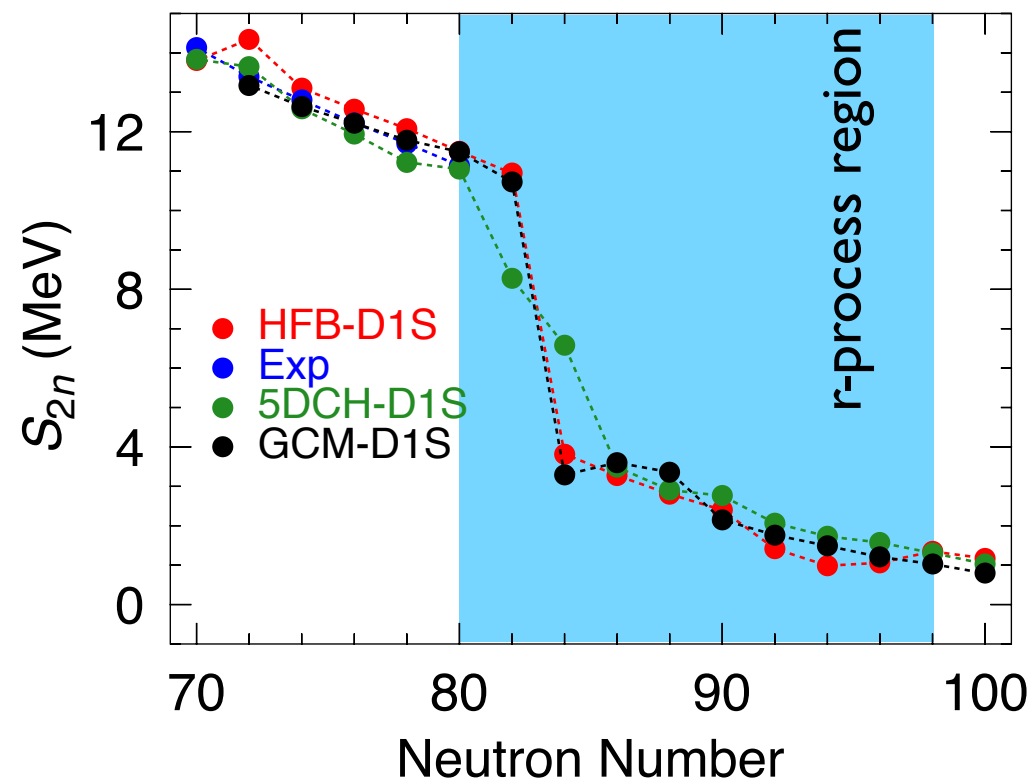
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Mean field vs. Beyond mean field. Local systematics

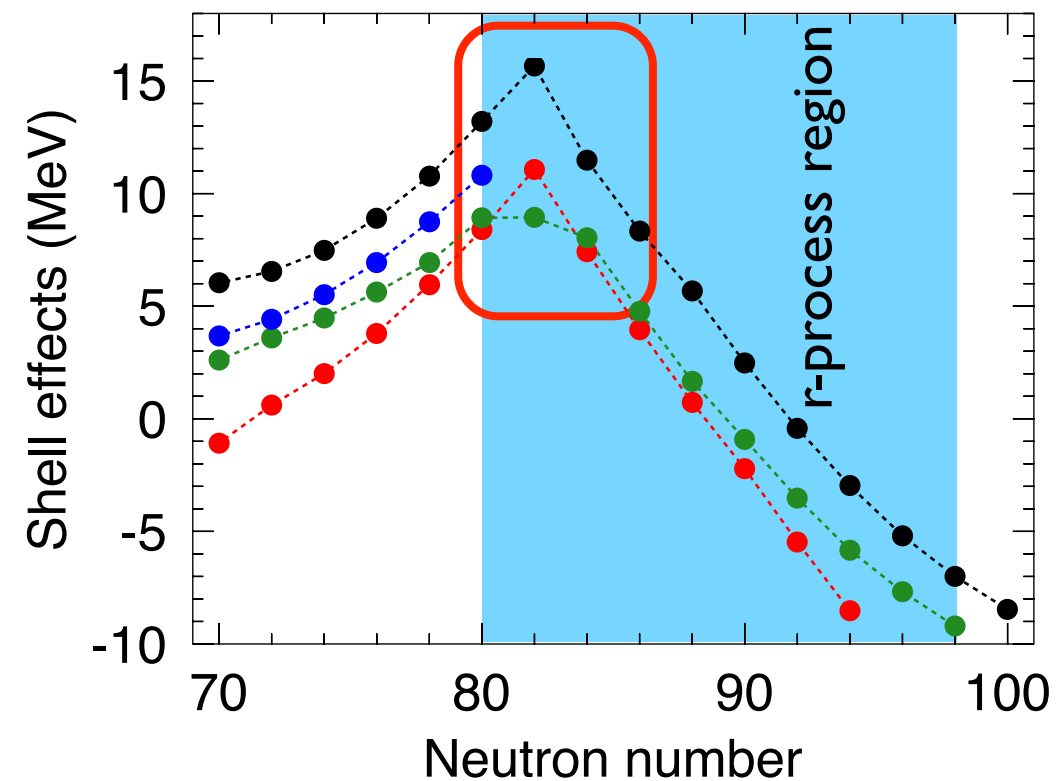
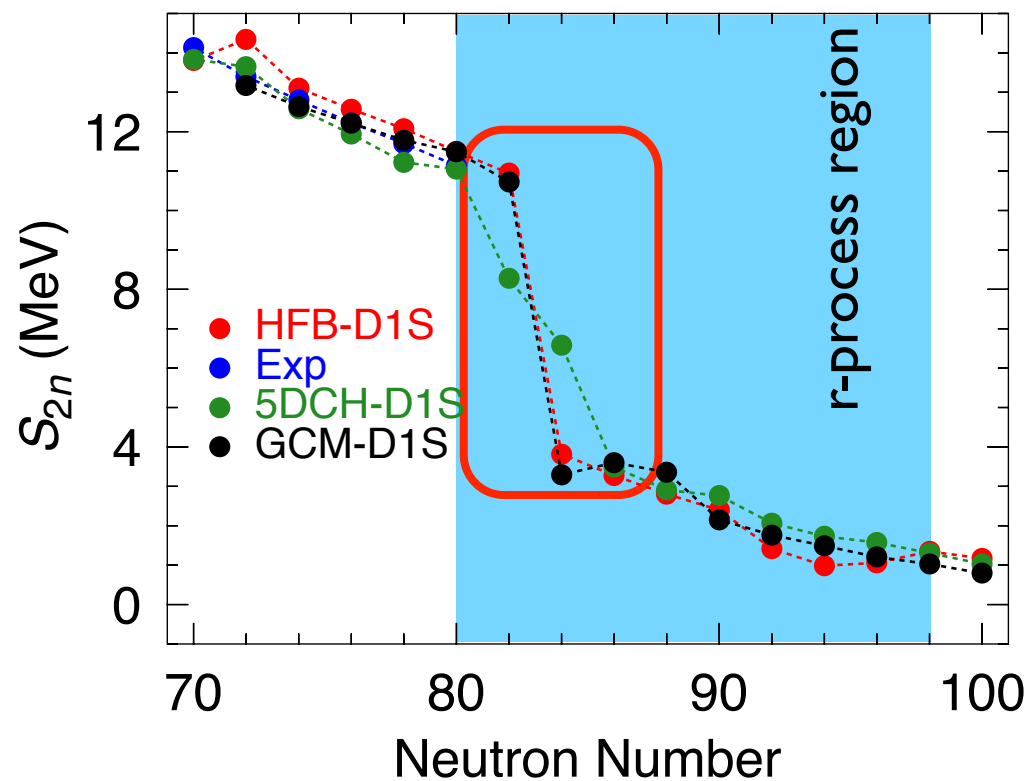
Cadmium isotopes. Gogny D1S parametrization



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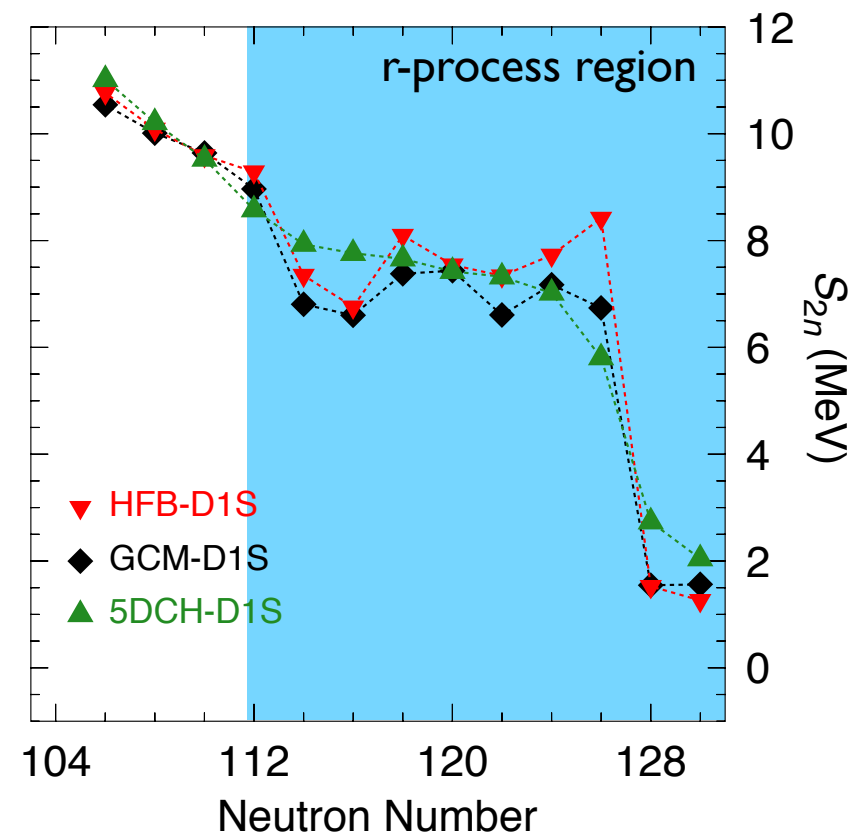
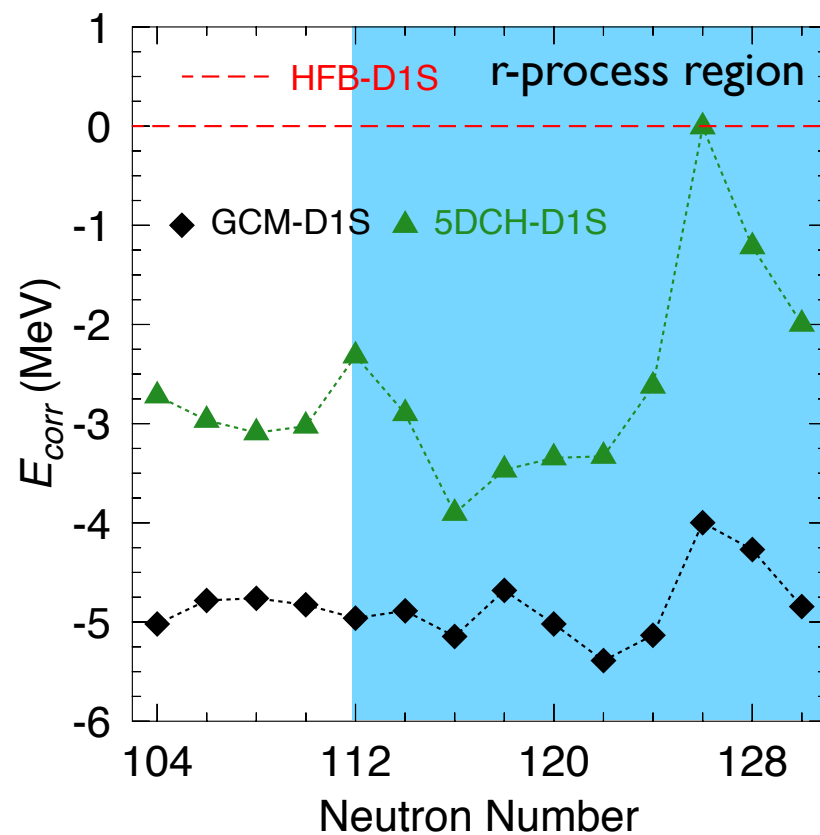
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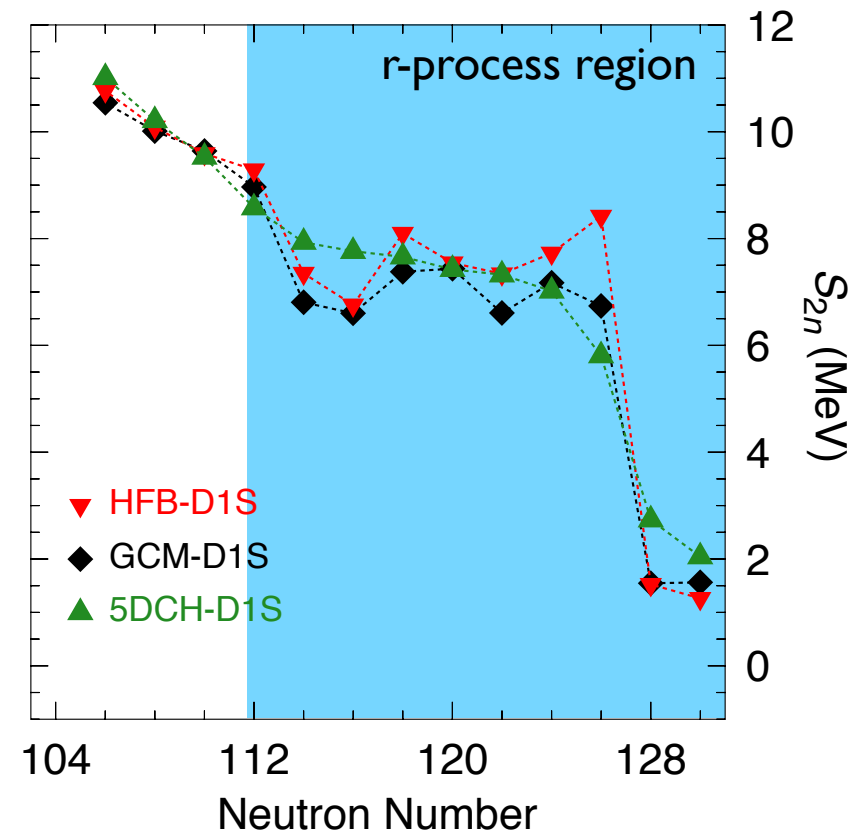
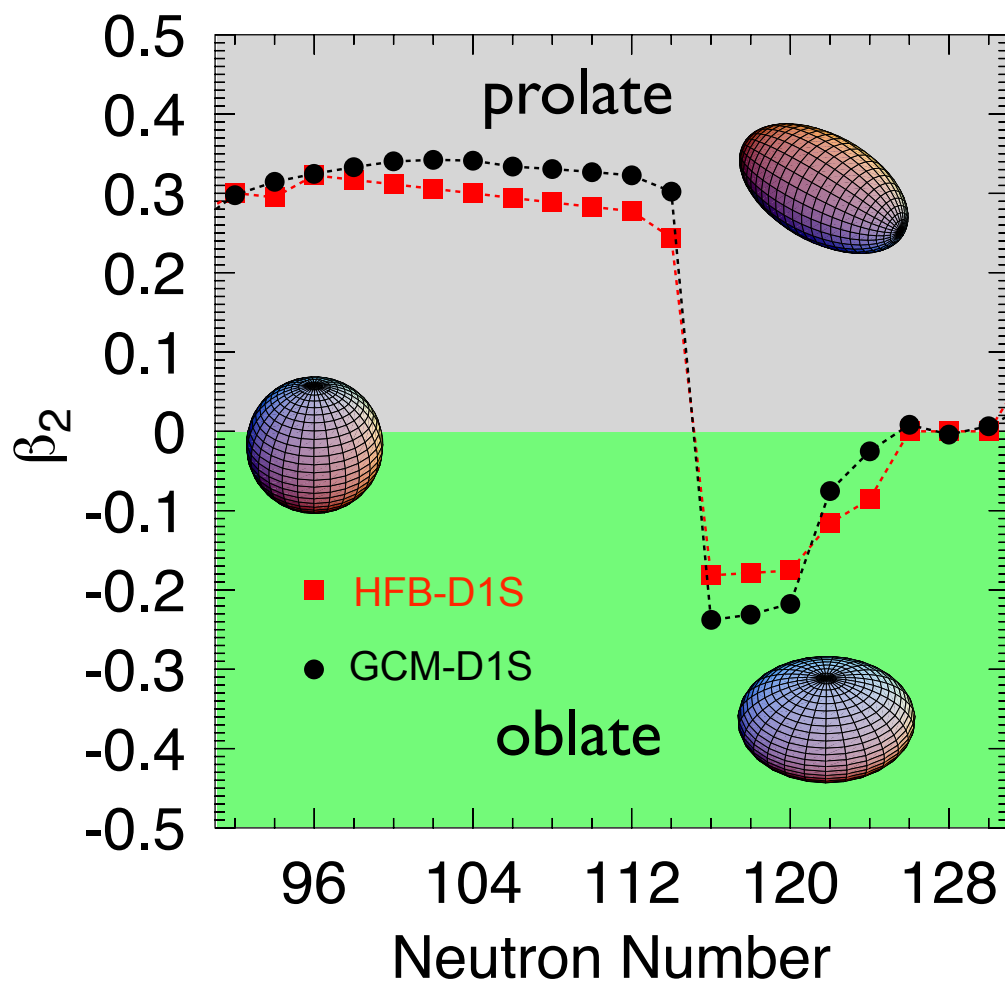
Erbium isotopes. Gogny D1S parametrization



- 5DCH fails in accounting for correlations at $N=126$ shell closure.
- 5DCH artificially smooths out the trough and the shell gap in the S_{2n} .
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Mean field vs. Beyond mean field. Local systematics

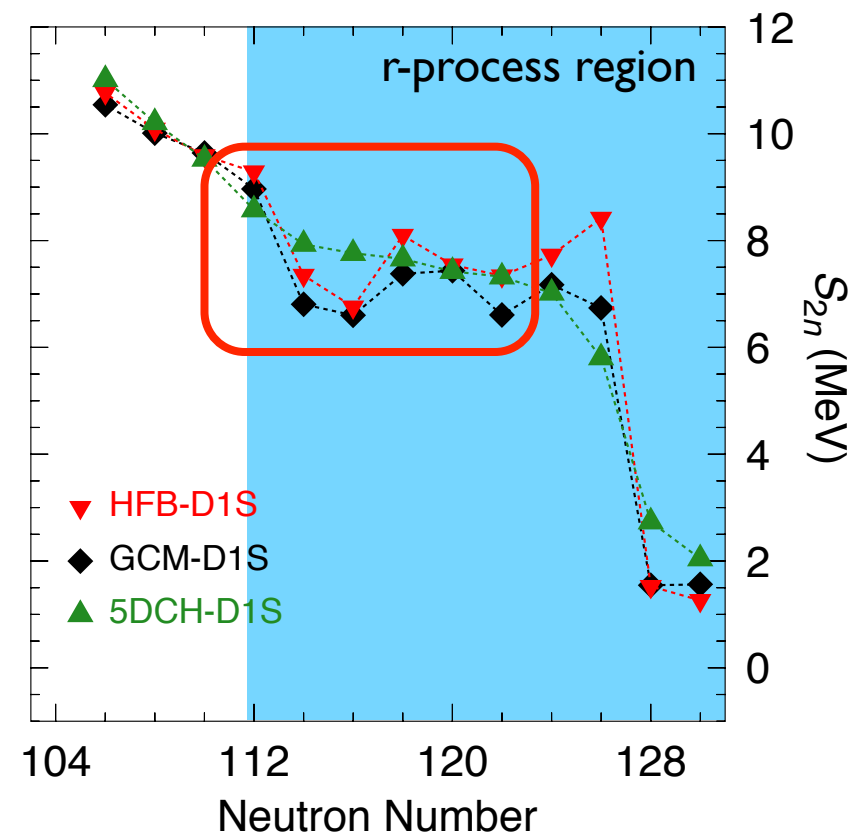
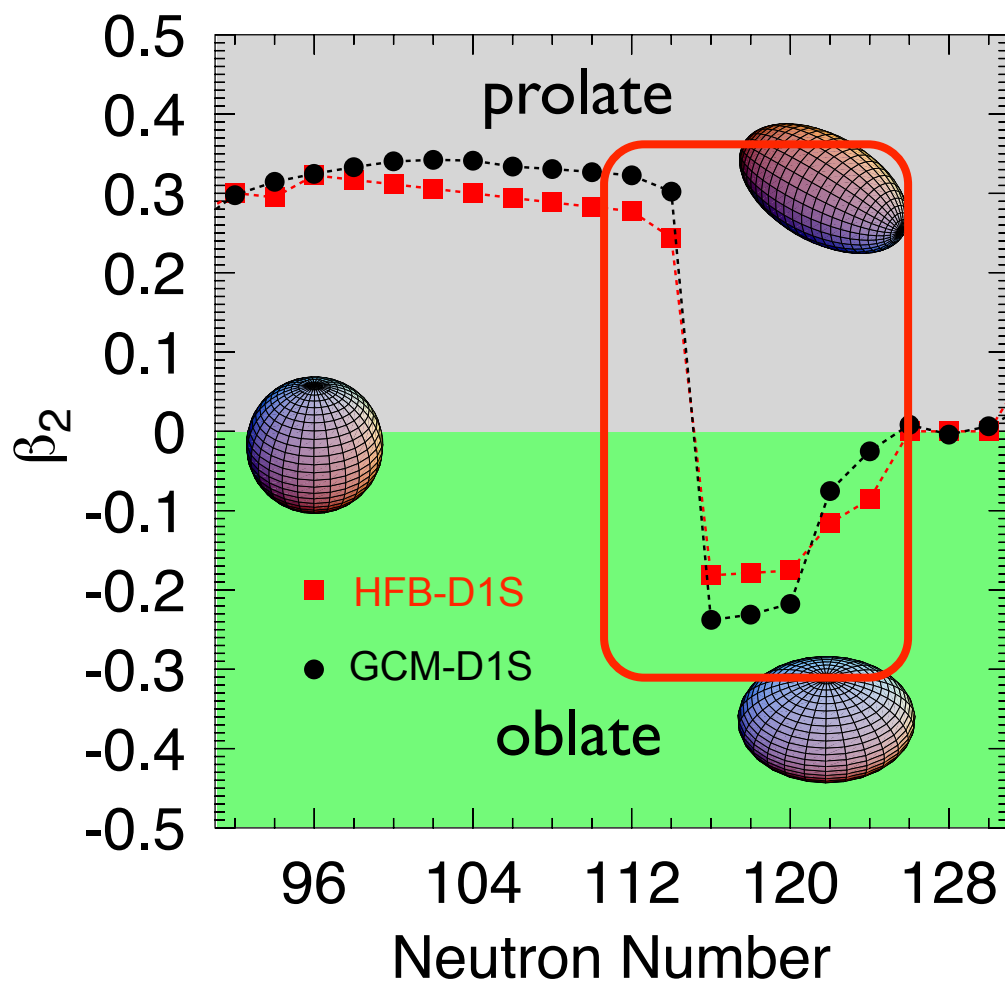
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2. Convergence and numerical noise

3. Beyond mean field effects

4. **Summary and outlook**

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- **Current global calculations including BMF effects** have assumed certain approaches/interactions that **could produce unphysical results** whenever local analyses are performed:
 - 5DCH is not always variational/consistent with the underlying mean-field and fails near the shell closures: spurious rather than BMF effects in these regions.
- Trough appearing both in S_{2n} and shell effects for Er isotopes is produced by an oblate-prolate shape transition and it is not smoothing out with the BMF model used here.

- Systematic analysis of the convergence/numerical noise.
- Perform global studies ensuring convergence of the results with the present variational BMF method.
- Study the impact on nucleosynthesis simulations.
- In the long-range plan:
 - Description of the odd systems at the same level of BMF approach.
 - Development of parametrizations of the interaction fitted with BMF functionals.

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