Recent developments within the empirical Shell Model framework

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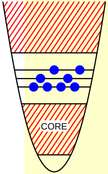
Shell model approach

Calculations Ab Initio

- Realistic NN interactions
- Diagonalization in $N\hbar\omega$ h.o.space



- define valence space
- $H_{\text{eff}}\Psi_{\text{eff}} = E\Psi_{\text{eff}}$

$\rightsquigarrow$ INTERACTIONS

- build and diagonalize Hamiltonian matrix

$\rightsquigarrow$ CODES

Weak processes:

- β decays
- $\beta\beta$ decays

$[T^{0\nu}_{1/2}(0^+ \to 0^+)]^{-1} = G_{0\nu}|M^{0\nu}|^2\langle m_\nu\rangle^2$

☞ASTROPHYSICS

☞PARTICLE PHYSICS

Collective excitations:

- deformation, superdeformation
- superfluidity
- symmetries

Shell evolution far from stability:

- Shell quenching
- New magic numbers

☞ASTROPHYSICS

Shell Model applications in astrophysics

- Electron capture rates for supernovae
- Inelastic neutrino-nucleus reactions: for supernovae dynamics and neutrino detection
- Half-lives for r-process nuclei

- Method of choice to describe correlations in medium-mass and heavier nuclei
- What is needed: high-precision effective interactions and SM codes

Shell Model: giant computations

- Problem dimension in the m-scheme:

$$D \sim \begin{pmatrix} d_\pi \\ p \end{pmatrix} \cdot \begin{pmatrix} d_\nu \\ n \end{pmatrix}$$

In the *pf*-shell ($1f_{7/2}$, $2p_{3/2}$, $1f_{5/2}$):

^{48}Cr	1,963,461
^{56}Ni	1,087,455,228

- Current diagonalization limit in m-scheme 10^{10}

- The largest SM diagonalization up to date has been achieved by the Strasbourg group (using very modest computing resources):
  Phys. Rev. C82 (2010) 054301, ibidem 064304

- ANTOINE can be adapted to calculations of any, up to two fluid systems of fermions or bosons:
  J. Navarro et al., Phys. Rev. A69 (2004) 023202

• m scheme CODE ANTOINE

$$|\Psi_\alpha\rangle = \Pi_{i=nljm\tau} a_i^\dagger |0\rangle = a_{i1}^\dagger \cdots a_{iA}^\dagger |0\rangle$$

Huge dimensions of the matrices (10^9-10^{10})

storage of Lanczos vectors on disk

Very large cases:
splitting of the initial and final vectors

$$\Psi_{i,f} = \bigcup_m \Psi_{i,f}^m$$
$$\Psi_f^{(m)} = \sum_n \mathscr{H}^{(m,n)} \Psi_i^{(n)}$$

• coupled scheme CODE NATHAN

• coupled scheme SVD FIT CODE

E. Caurier et al., Rev. Mod. Phys. 77 (2005) 427; ANTOINE website

Effective interactions for SM calculations

- realistic (Argonne, Bonn, ...) or chiral EFT (N3LO) V_{NN} interaction
  ☞problems with saturation and shell closures
- Nuclear structure challenge: inclusion of 3N forces
  - exactly ...
  - semi-empirically: keep V_{NN} and add empirical 3N forces
  - empirically: fit H_{eff} to data:
    -fit all TBME using linear combination method (SVD)
    -adjust only monopole Hamiltonian

$$H = H_{monopole} + H_{multipole}$$

$$V = \sum_{JT} V_{ijkl}^{JT} \left[(a_i^+ a_j^+)^{JT} (\tilde{a}_k \tilde{a}_l)^{JT} \right]^{00}$$

In order to express the number of particles operators $n_i = a_i^+ a_i \propto (a_i^+ \tilde{a}_i)^0$,

→ particle-hole recoupling :

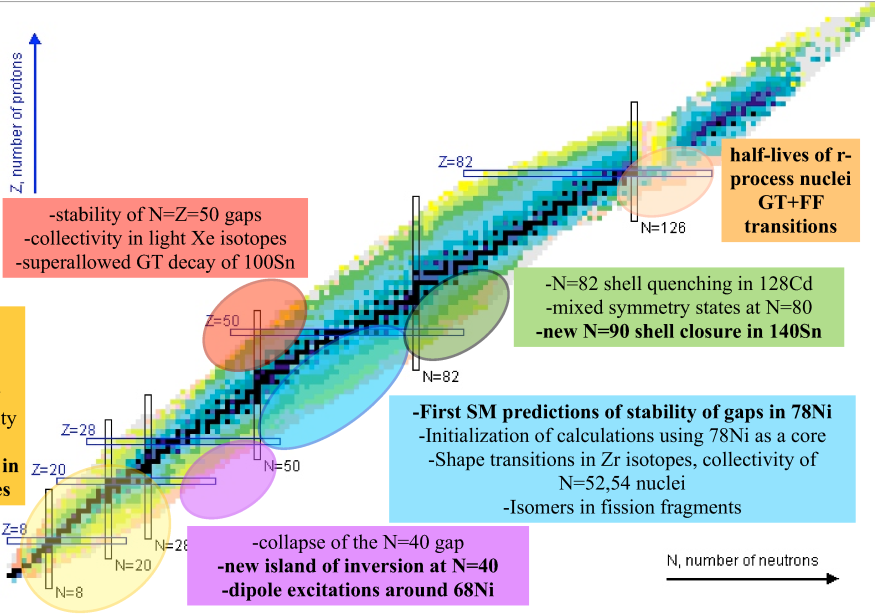
$$V = \sum_{\lambda\tau} \omega_{ikjl}^{\lambda\tau} \left[(a_i^+ \tilde{a}_k)^{\lambda\tau} (a_j^+ \tilde{a}_l)^{\lambda\tau} \right]^{00}$$

$H_{monopole}$ corresponds to $\boldsymbol{\lambda}\tau$**=00** and **01** which implies that $i = j$ and $k = l$

$$H_{monopole} = \sum_i n_i \varepsilon_i + \sum_{i \le j} n_i . n_j V_{ij}$$

$H_{multipole}$ corresponds to all other combinations of $\boldsymbol{\lambda}\tau$.

Regions of our interest



-stability of N=Z=50 gaps
-collectivity in light Xe isotopes
-superallowed GT decay of 100Sn

half-lives of r-process nuclei GT+FF transitions

-3-body forces
-effective forces far from stability
-dipole excitations in Ne isotopes

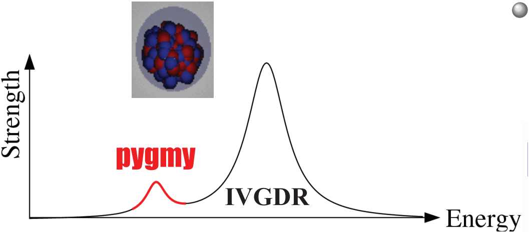
-N=82 shell quenching in 128Cd
-mixed symmetry states at N=80
-new N=90 shell closure in 140Sn

-First SM predictions of stability of gaps in 78Ni
-Initialization of calculations using 78Ni as a core
-Shape transitions in Zr isotopes, collectivity of N=52,54 nuclei
-Isomers in fission fragments

-collapse of the N=40 gap
-new island of inversion at N=40
-dipole excitations around 68Ni

Dipole excitations in the islands of inversion

E1 excitations on nuclei



- the pygmy part enhances the low lying E1 strength in the astrophysically relevant range.

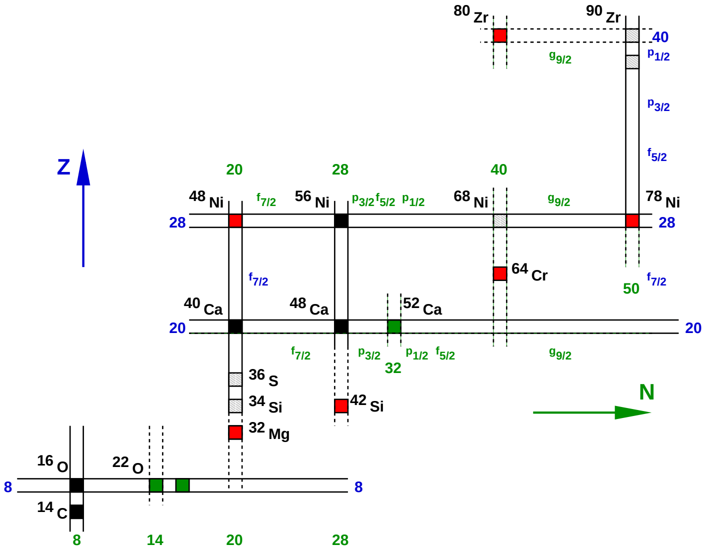
- Beyond mean-field approaches (QRPA, RQTBA)

  ☺ good reproduction of giant and pygmy resonances
  ☹ limited to even systems

- Shell model

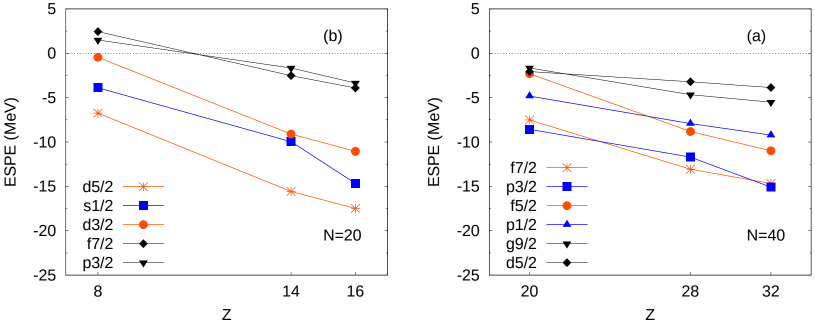
  ☺ ODD NUCLEI
  ☺☹ low lying strength only
  ☺ precise description of spectroscopy, transition rates, shell structure...

Islands of inversion at N=20 and N=40



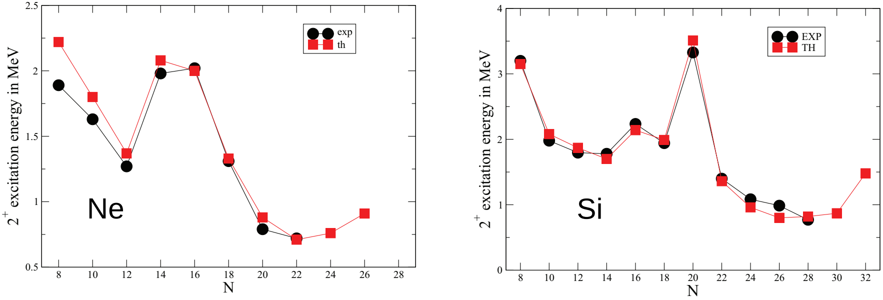
Islands of inversion at N=20 and N=40

Phys. Rev. C82 (2010) 054301



- Intruders $\rightarrow$ low lying E1 strength
- How the low lying strength evolves toward the island of inversion?

SM spectroscopy: Ne (Z=10) vs Si (Z=14)



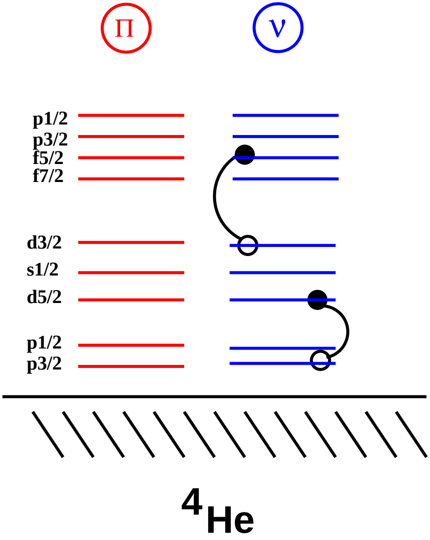
- N=20 gap washed out in Ne but present in Si

E1 calculations in psdpf SM

$$Q_{\mu}^{\lambda=1} = \frac{Z}{A} e \sum_{k=1}^{N} r_k Y_{1\mu}(r_k) - \frac{N}{A} e \sum_{k=1}^{Z} r_k Y_{1\mu}(r_k)$$

- SM in *p-sd-pf* model space
- full *sd* diagonalization + full $1\hbar\omega$ excitations
- Exact removal of COM components
- Interaction: PSDPF

  M. Bouhelal, F. Haas, E. Caurier, F. Nowacki and A. Bouldjedri, Nucl. Phys. A864 (2011) 113.



E1 strength in even neon isotopes

- SM in *psdpf* model space
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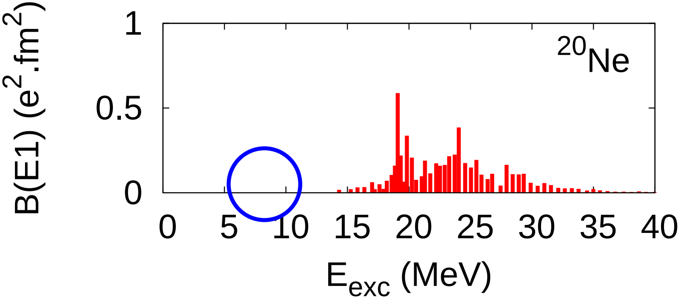

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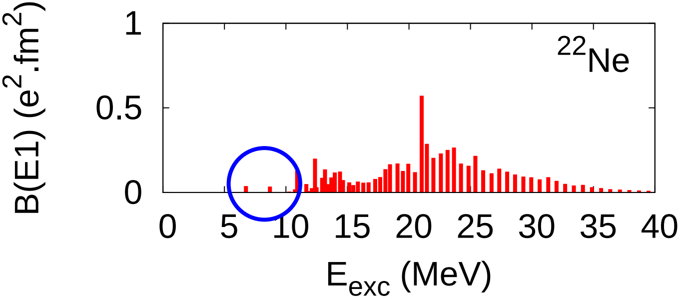

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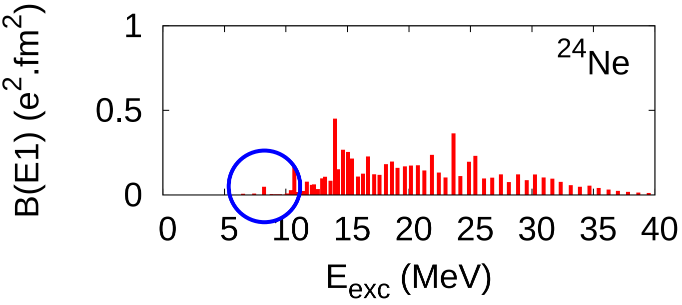

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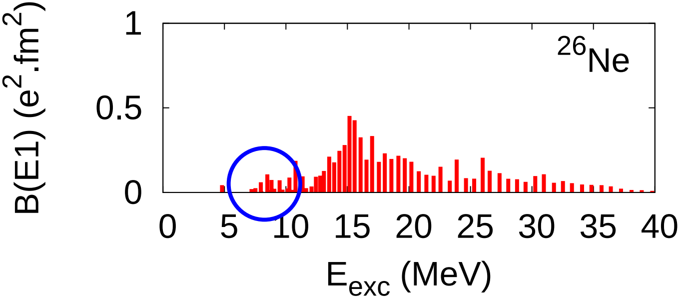
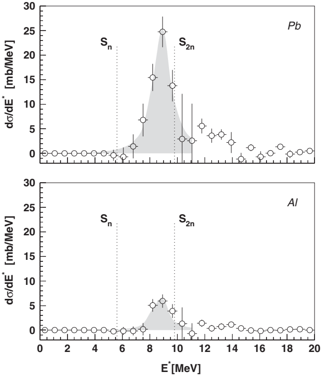
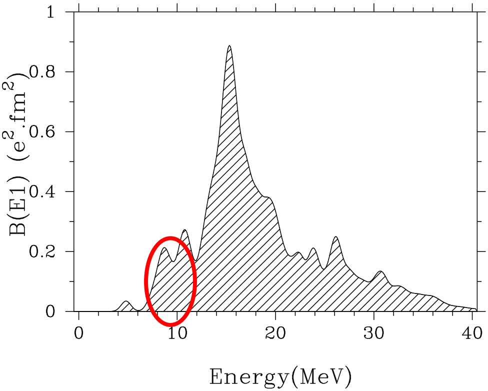


The case of ^{26}Ne

EXP: $\sum$B(E1)=0.49$\pm$0.16 e^{2}fm^2 (6-10MeV) THEO: $\sum$ B(E1)=0.485 e^{2}fm^2 (0-10MeV)





J. Gibelin et al., Phys. Rev. Lett. 101 (2008) 212503

Low peaks structure: $\nu s^{-1}_{1/2} p^1_{3/2}$, $\nu s^{-1}_{1/2} p^1_{1/2}$

SM: Complex wave functions (major contributions $\leq$ 30%)

QRPA main contribution: 70% of $\nu s^{-1}_{1/2} p^1_{3/2}$ *M. Martini, S. Péru, and M. Dupuis, Phys. Rev. C 83, 034309 (2011)*

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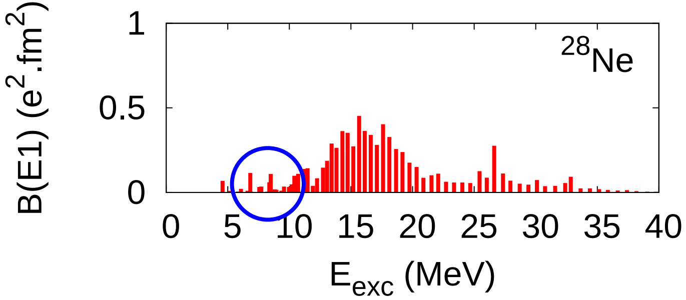
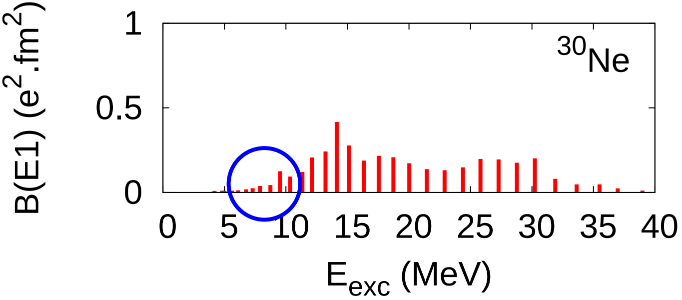


E1 strength in even neon isotopes

- SM in *sd-pf* model space
- t=4 *sd-pf* diagonalization for GS + 1p1h
- COM spuriosity $\sim 1\%$



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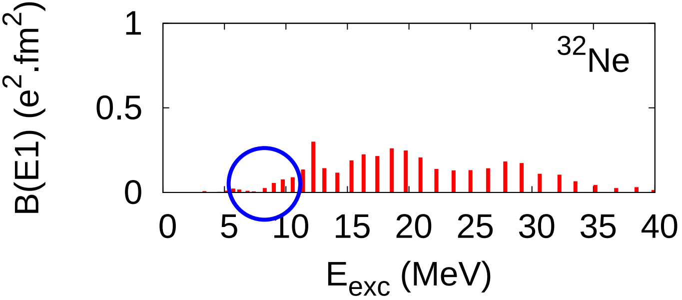
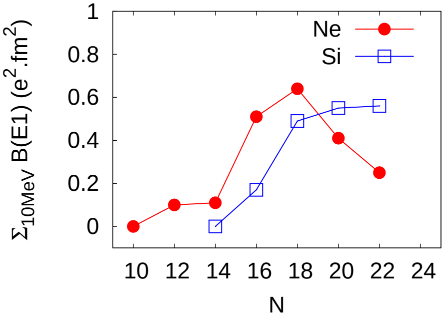
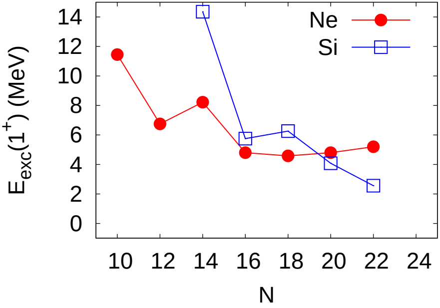


Evolution of dipole strength along Ne & Si chains



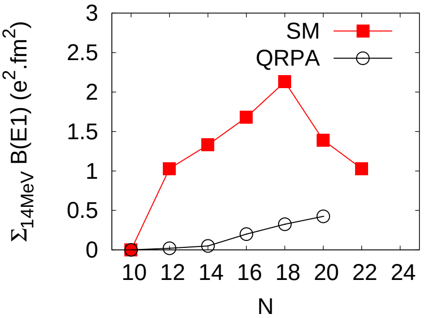
Sum of B(E1) strength up to 10MeV



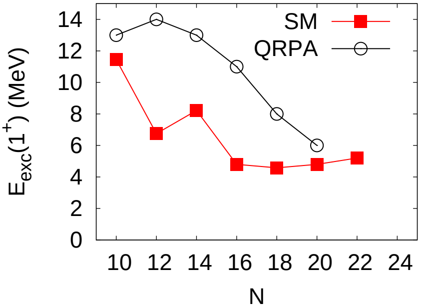
Energy of the first peak with $B(E1) \geq 0.01e^2\text{fm}^2$

- The low lying strength moves up in energy in the island of inversion

Evolution of dipole strength along the Ne chain: SM vs QRPA



Sum of B(E1) strength up to 14MeV

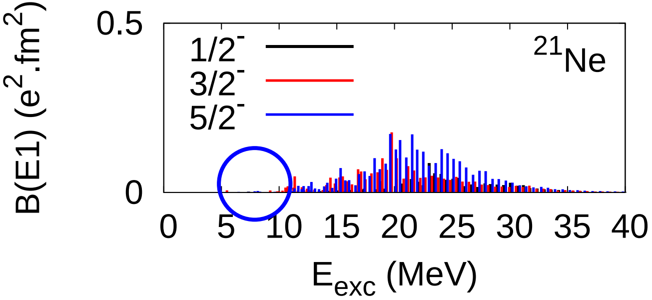


Energy of the first peak with $B(E1) \geq 0.01e^2\text{fm}^2$

QRPA from *M. Martini, S. Péru, and M. Dupuis, Phys. Rev. C 83, 034309 (2011)*

E1 strength in odd neon isotopes

- SM in *psdpf* model space
- full *sd* diagonalization + full $1\hbar\omega$ excitations
- Exact removal of COM components



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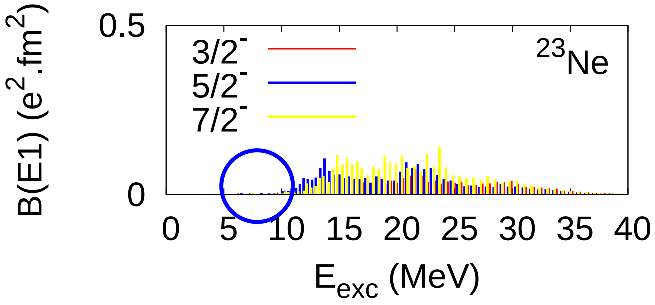

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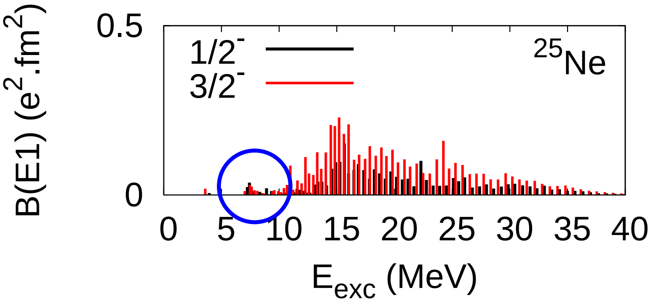

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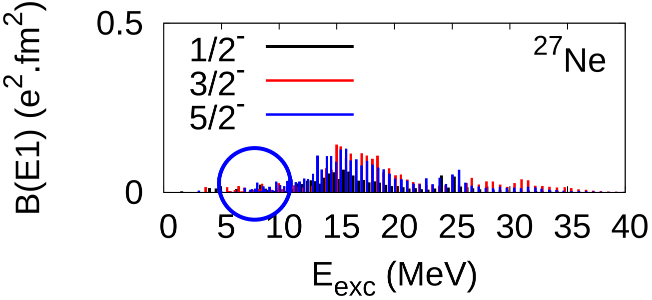

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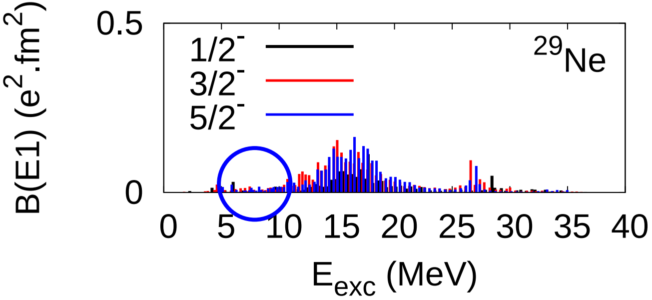
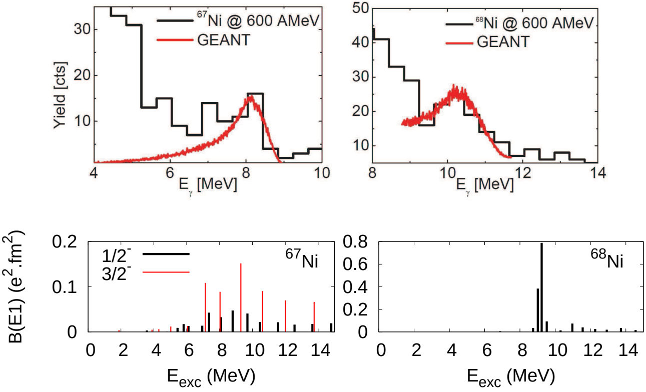


SM challenge: Dipole excitations in the Ni chain

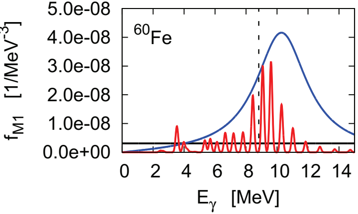


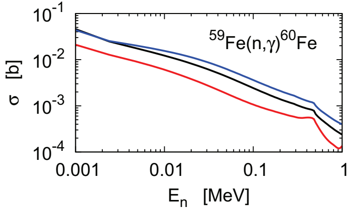
A. Bracco and O. Wieland, Acta Phys. Pol. B40 (2009) 545.

- *fp-gd* model space
- truncated calculations (t=6 GS+ 1p1h)
- COM $\sim$ 5%

Astrophysics applications

Impact of the realistic M1 fragmentation on the neutron capture cross sections





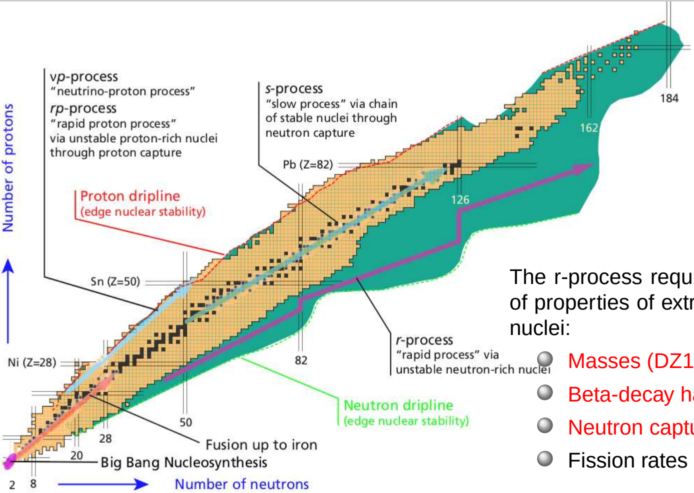
- State-by-state cross section 2 times larger than using Brink hypothesis
- Using SF of 2^+ state instead of 0^+ leads to larger cross sections

Eur. Phys. J A48 (2012) 34.

- M1 is $1\% - 50\%$ of the total contribution. Taking into account E1 is essential.

Half-lives of r-process nuclei with FF transitions

Making gold in nature: r-process nucleosynthesis



The r-process requires the knowledge of properties of extremely neutron-rich nuclei:

- Masses (DZ10,DZ31)
- Beta-decay half-lives.
- Neutron capture rates.
- Fission rates and yields.

Eur. Phys. J. A48 (2012) 34;
Nucl. Phys. A859 (2011) 172.

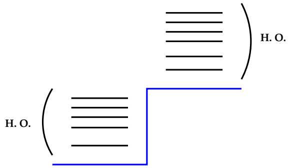
Beta-decay calculations

☞To calculate beta decay between two states one needs:

- accurate value of the decay energy ($T_{1/2} \sim \Delta E^{-5}$)
- matrix elements of
  Gamow-Teller ($\Delta J^{\pi} = 0^{+}, 1^{+}$)
  first forbidden ($\Delta J^{\pi} = 0^{-}, 1^{-}, 2^{-}$)
  transition operators

☞Nuclear model has to provide good description of masses, spectra and wave-functions

In r-process nuclei, GT is not enough, as protons and neutrons occupy different parity levels:



The beta half-life to a given state is gives as: $ft = 6146$, where the space phase factor f is given as

$$f = \int_1^{W_0} C(W) F(Z,W)(W^2-1)^{1/2} W (W_0 - W)^2 dW$$

For allowed transitions: $C(W) = B(F) + B(GT)$
For forbidden $C(W)$ is a function of the energy, and we have to calculate the integral

$$C(W) = k(1 + aW + \frac{b}{W} + cW^2)$$

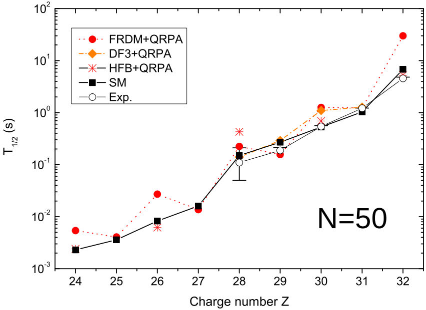
Coeficients a,b,c

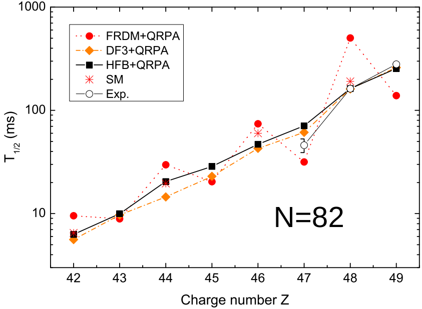
$$
\begin{aligned}
k &= \left[\zeta_0^2 + \frac{1}{9}w^2\right]^{(0)} + \left[\zeta_1^2 + \frac{1}{9}(x+u)^2 - \frac{4}{9}\mu_1\gamma_1 u(x+u)\right. \\
&\left. + \frac{1}{18}W_0^2(2x+u)^2 - \frac{1}{18}\lambda_2(2x-u)^2\right]^{(1)} + \left[\frac{1}{12}z^2(W_0^2 - \lambda_2)\right]^{(2)}, \\
ka &= \left[-\frac{4}{3}uY - \frac{1}{9}W_0(4x^2+5u^2)\right]^{(1)} - \left[\frac{1}{6}z^2 W_0\right]^{(2)}, \\
kb &= \frac{2}{3}\mu_1\gamma_1\left\{-[\zeta_0 w]^{(0)} + [\zeta_1(x+u)]^{(1)}\right\}, \\
kc &= \frac{1}{18}\left[8u^2 + (2x+u)^2 + \lambda_2(2x-u)^2\right]^{(1)} + \frac{1}{12}\left[z^2(1+\lambda_2)\right]^{(2)}.
\end{aligned}
$$

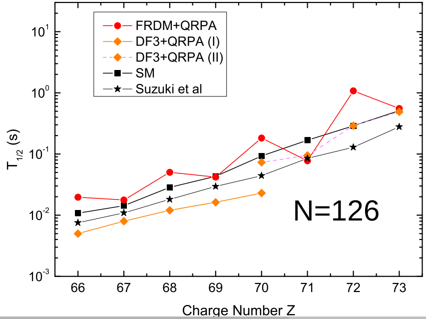
Matrix elements

$$
\begin{aligned}
w &= -R\,{}^{A}F^{0}_{011} \\
&= \lambda\sqrt{3}\langle J_f T_f|||ir\,[C_1\times\boldsymbol{\sigma}]^0\,\boldsymbol{\tau}|||J_i T_i\rangle C, \\
x &= -\frac{1}{\sqrt{3}}R\,{}^{V}F^{0}_{110} \\
&= -\langle J_f T_f|||ir C_1\boldsymbol{\tau}|||J_i T_i\rangle C, \\
u &= -\sqrt{\frac{2}{3}}R\,{}^{A}F^{0}_{111} \\
&= \lambda\sqrt{2}\langle J_f T_f|||ir\,[C_1\times\boldsymbol{\sigma}]^1\,\boldsymbol{\tau}|||J_i T_i\rangle C, \\
z &= \frac{2}{\sqrt{3}}R\,{}^{A}F^{0}_{211} \\
&= -2\lambda\langle J_f T_f|||ir\,[C_1\times\boldsymbol{\sigma}]^2\,\boldsymbol{\tau}|||J_i T_i\rangle C, \\
w' &= -\frac{2}{3}R\,{}^{A}F^{0}_{011}(1,1,1,1) \\
&= \lambda\sqrt{3}\langle J_f T_f|||ir\,\frac{2}{3}I(1,1,1,1,r)\,[C_1\times\boldsymbol{\sigma}]^0\,\boldsymbol{\tau}|||J_i T_i\rangle C, \\
x' &= -\frac{2}{3\sqrt{3}}R\,{}^{V}F^{0}_{110}(1,1,1,1) \\
&= -\langle J_f T_f|||ir\,\frac{2}{3}I(1,1,1,1,r)C_1\boldsymbol{\tau}|||J_i T_i\rangle C, \\
u' &= -\frac{2\sqrt{2}}{3\sqrt{3}}R\,{}^{A}F^{0}_{111}(1,1,1,1) \\
&= \lambda\sqrt{2}\langle J_f T_f|||ir\,\frac{2}{3}I(1,1,1,1,r)\,[C_1\times\boldsymbol{\sigma}]^1\,\boldsymbol{\tau}|||J_i T_i\rangle C,
\end{aligned}
$$

Results: half-lives of r-process nuclei





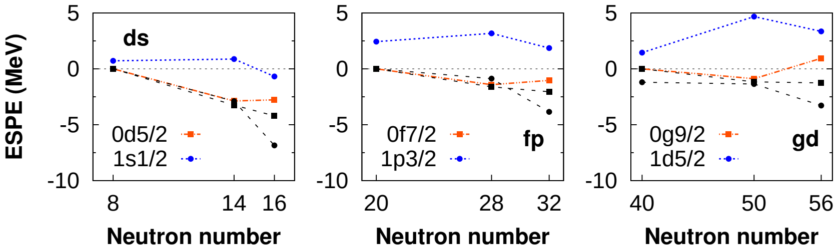


- Accurate description of Q_β, neutron emission probabilities and half-lives in the known region
- Universal quenching factors on GT and FF operators in all mass regions

to appear in Phys. Rev. C (2013)

Toward a generalized monopole interaction

NN interactions and spin-orbit shell closures

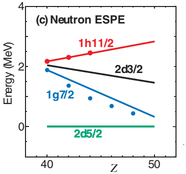


Phys. Rev. C85, 051301R (2012).

- Realistic NN potentials fail in reproducing spin-orbit shell closures
- Empirical approaches are necessary: fit all TBME or monopoles only

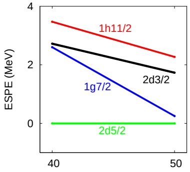
Can we build a simple model of monopole interaction?

Example: Shell evolution in N=51 nuclei



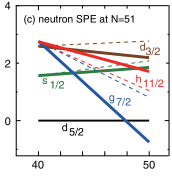
Otsuka et al. 2004

$(\pi + \rho)$ tensor force



Sieja et al. 2008

CD-Bonn+monopole correction



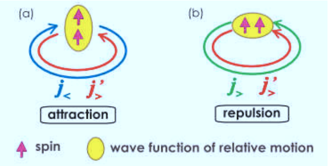
Otsuka et al. 2010

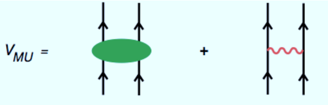
tensor+gaussian central

$j_> = l + 1/2, \;\; j_< = l - 1/2$

$V_C = \Sigma_{S,T} f_{S,T} P_{S,T} exp(-(r/\mu)^2)$

(2 new parameters: $f_{1,0}$=-166MeV, $\mu = 1.0 fm$)





Monopole Hamiltonian and invariant representation

$$H_{mono} = \sum_i e_i n_i + \sum_{i \le j} (V_{ij} n_{ij}) + \sum_{i \le j \le k} (V_{ijk} n_{ijk})$$

$$V_{ij} = \frac{\sum_J V_{ijij}^J (2J+1)(1+(-1)^J \delta_{ij})}{(2j_i+1)(2j_j+1-\delta_{ij})}$$

3-body interaction produces also 2-body interactions in the valence space:

$$\sum_c V_{ijc} n_{ijc} = N_c \sum_{ij} V_{ij} n_{ij}$$

In a first step one can consider 2-body terms modulated by total particle number *n*.

Invariant representation: 2-shell example

$$H_{mono} = \varepsilon_1 n_1 + \varepsilon_2 n_2 + \frac{n_1(n_1-1)}{2} V_{11} + \frac{n_2(n_2-1)}{2} V_{22} + n_1 n_2 V_{12}$$

Jacobi construction to separate a global term $\mathscr{H}_0$ (depending only on the total number of particles $n = n_1 + n_2$) from a linear term $\mathscr{H}_1$ and a quadratic term $\mathscr{H}_2$ in n_1 and n_2. For example, the first term can be transformed as:

$$n_1\varepsilon_1 + n_2\varepsilon_2 = (n_1+n_2)\left(\frac{D_1\varepsilon_1 + D_2\varepsilon_2}{D_1+D_2}\right) + \left(\frac{n_1}{D_1} - \frac{n_2}{D_2}\right)\frac{D_1D_2}{D_1+D_2}(\varepsilon_1 - \varepsilon_2)$$

$$\begin{aligned} \mathscr{H}_m &= n\,\bar{\varepsilon}_0 + \tfrac{n(n-1)}{2} W_0 + \Gamma_{12}[\bar{\varepsilon}_1 + (n-1)W_1] + \Gamma_{12}^{(2)} W_2 \\ &= \mathscr{H}_0 + \mathscr{H}_1 + \mathscr{H}_2 \end{aligned}$$

with

$$\Gamma_{12} = \frac{D_2 n_1 - D_1 n_2}{D_1 + D_2} \qquad \Gamma_{12}^{(2)} = \frac{D_1 D_2}{2}\left(\frac{2n_1n_2}{D_1D_2} - \frac{n_1(n_1-1)}{D_1(D_1-1)} - \frac{n_2(n_2-1)}{D_2(D_2-1)}\right)$$

Invariant decomposition

$$H_{mono} = \underbrace{\varepsilon n + \frac{Wn(n-1)}{2}}_{H_0} + \underbrace{\sum_i \Gamma_i^1 (e_i - \varepsilon + (n-1)W_i)}_{H_1} + \underbrace{\sum_{i\leq j} \Gamma_{ij}^2 W_{ij}}_{H_2}$$

$$V_{ij} = \frac{\sum_{JT}(2J+1)(2T+1)V_{ijij}^{JT}}{\sum_{JT}(2J+1)(2T+1)}$$

$$\varepsilon = \sum_i \frac{D_i e_i}{D} \qquad W = 2\sum_{ij} \frac{D_{ij} V_{ij}}{D(D-1)}$$

$$W_i = \sum_i (D_i - \delta_{ij})(V_{ij} - W) \qquad W_{ij} = V_{ij} - W_i - W_j - W$$

$$\Gamma_i^1 = D_i \left[\frac{n_i}{D_i} - \frac{n_1}{D_1}\right] \qquad \Gamma_{ij}^2 = \frac{D_{ij}}{2}\left[\frac{2n_{ij}}{D_{ij}} - \frac{n_{ii}}{D_{ii}} - \frac{n_{jj}}{D_{jj}}\right]$$

Universal monopole in invariant scheme

$$H_{mono} = \underbrace{\varepsilon n + \frac{Wn(n-1)}{2}}_{H_0} + \underbrace{\sum_i \Gamma_i^1 (e_i - \varepsilon + (n-1)W_i)}_{H_1} + \underbrace{\sum_{i \leq j} \Gamma_{ij}^2 W_{ij}}_{H_2}$$

- pin down the relevant operators & make them 3-body

$$\Gamma_i^1 \to \kappa_1(n) * \Gamma_i^1$$
$$\Gamma_{ij}^2 \to \kappa_2(n) * \Gamma_{ij}^2 + \kappa_3 \Gamma_{ijk}^3$$

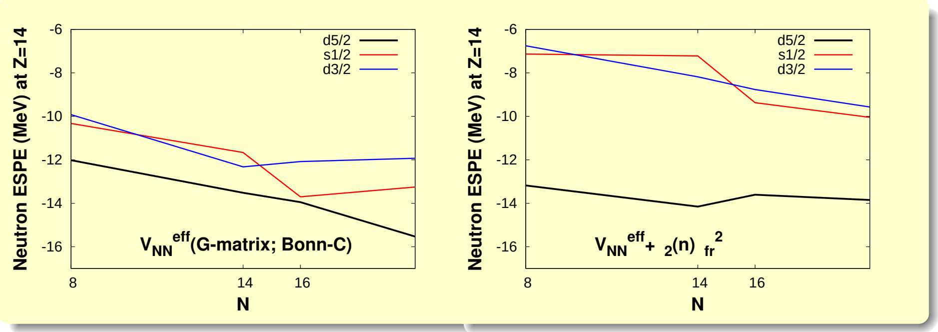
- find their parameterizations in several shells
- establish the global mass dependence of these parameters

Invariant scheme and ESPE

Analysis of centroids in invariant scheme reveal that the major empirical corrections are mostly due to one term

$$\Gamma_{fr}^{(2)} \cdot \kappa(n); \quad \kappa(n) = a + b \cdot n$$

$f = d_{5/2}, r = s_{1/2}, d_{3/2}$



Invariant monopole: results

rmsd error in *p*-shell (55 states in 15 nuclei)

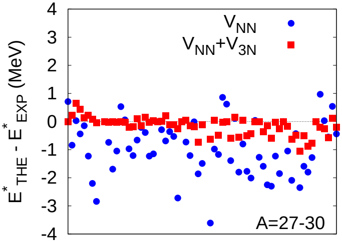
	V_{eff}^{NN} (0)	$+\kappa_2(n)\Gamma_{fr}^2$ (2)	fit-mono (3)
N3LO	1.34	0.44	0.43
AV18	1.41	0.47	0.47

rmsd error in *sd*-shell (309 states in 60 nuclei)

	V_{eff}^{NN} (0)	$+\kappa_2(n)\Gamma_{fr}^2$ (2)	fit-mono (6)
N3LO	0.95	0.38	0.35
CD-Bonn	0.95	0.41	0.37

☞Parameterization independent on the initial potential

- Invariant decomposition provides a method of including 3N corrections to the Hamiltonian at the cost of ~ 2 empirical parameters.
- The parameters have a clear physical meaning: they provide a missing 3N contribution to NN interactions from core and valence particles.
- Work in progress on generalization of this scheme for heavier nuclei.



Conclusion

- Shell Model remains the most accurate method in nuclear physics, flexible enough to be applied to even, odd , even-odd systems with various number of particles outside closed shells.
- It is a method of choice for structure calculations where a detailed spectroscopic description is needed and where the description of correlations is crucial.
- The problems of SM are related to compuitng possibilities and effective interactions. In both directions there is a considerable progress:
  -current diagonalization limit of 10^{10} should be overcome within a few years
  - EFT + ChPT give us hope to use in future SM calculations effective interactions which take consistently into account 3N contributions.

Thanks to:

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Spin-tensor decomposition of NN interactions

$$V = \sum_{k=0,1,2} \left(S^{(k)} \cdot Q^{(k)} \right) = \sum_{k=0,1,2} V^{(k)},$$

$Q^{(k)}$: operators in the coordinate space.
$S^{(k)}$: spin-tensors constructed from nucleon spin-1/2 operators.

scalar operators:	1	central
	$(\vec{\sigma}_1 \cdot \vec{\sigma}_2)$	central
vector operators:	$\vec{\sigma}_1 + \vec{\sigma}_2$	spin-orbit
	$[\vec{\sigma}_1 \times \vec{\sigma}_2]^{(1)}$	ALS
	$\vec{\sigma}_1 - \vec{\sigma}_2$	ALS
tensor operators:	$[\vec{\sigma}_1 \times \vec{\sigma}_2]^{(2)}$	tensor force

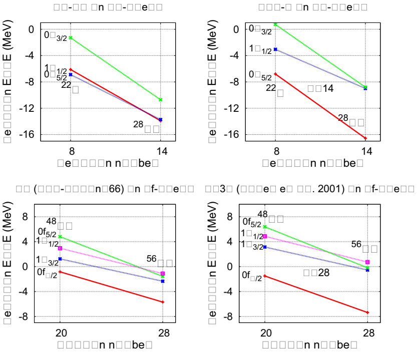
In the LS-scheme, the matrix elements of each $V^{(k)}$ can be obtained from V:

$$\langle (nl, n'l' : LS, JMTM_T | V^{(k)} | n''l'', n'''l''' : L'S', JMTM_T \rangle =$$

$$(2k+1)(-1)^J \left\{ \begin{array}{ccc} L & S & J \\ S' & L' & k \end{array} \right\} \sum_{J'} (-1)^{J'} (2J'+1) \left\{ \begin{array}{ccc} L & S & J' \\ S' & L' & k \end{array} \right\} \quad (1)$$

$$\times \langle nl, n'l' : LS, J'MTM_T | V | n''l'', n'''l''' : L'S', J'MTM_T \rangle.$$

Spin-tensor decomposition of NN interactions



Energy gap	$\nu(0d_{5/2}$-$1s_{1/2})$ MeV		$\nu(0f_{7/2}$-$0p_{3/2})$ MeV	
Filling orbital	$\pi 0d_{5/2}$ $^{22}O \rightarrow ^{28}Si$		$\pi 0f_{7/2}$ $^{48}Ca \rightarrow ^{56}Ni$	
	RG-SD	USD-B	KB	KB3G
Total	**0.95**	**3.82**	**1.27**	**2.39**
Central	**2.58**	**3.59**	**2.11**	**2.90**
Vector	$-$**0.87**	**0.90**	$-$**0.09**	**0.33**
LS	-0.72	0.41	-0.14	-0.07
ALS	-0.15	0.49	0.06	0.40
Tensor	$-$**0.76**	$-$**0.68**	$-$**0.74**	$-$**0.84**

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☞Tensor force from realistic NN interaction is preserved in the empirical fits.
☞Spin-orbit and central parts that needs to be modified to reproduce the gaps.
☞No regular behaviour with mass.

☞What about the 3N forces?

Lanczos Structure Function

Initial vector $|\mathbf{1}\rangle = \frac{|\Omega\rangle}{\sqrt{\langle\Omega|\Omega\rangle}}$.

$$\begin{aligned}
&E_{11} = \langle \mathbf{1}|H|\mathbf{1}\rangle \\
&E_{12}|\mathbf{2}\rangle = (H - E_{11})|\mathbf{1}\rangle \\
&E_{23}|\mathbf{3}\rangle = (H - E_{22})|\mathbf{2}\rangle - E_{12}|\mathbf{1}\rangle \\
&\dots \\
&E_{NN+1}|\mathbf{N+1}\rangle = (H - E_{NN})|\mathbf{N}\rangle \\
&\qquad\qquad\qquad - E_{N-1N}|\mathbf{N-1}\rangle
\end{aligned}$$

where

$$E_{NN} = \langle \mathbf{N}|H|\mathbf{N}\rangle, \quad E_{NN+1} = E_{N+1N}$$

Each Lanczos iteration gives information about two new moments of the distribution.

$$\begin{aligned}
&E_{11} = \langle \mathbf{1}|H|\mathbf{1}\rangle = m_1 = \bar{E} \\
&E_{12}^2 = \langle \mathbf{1}|(H - E_{11})^2|\mathbf{1}\rangle = m_2 \\
&E_{22} = \frac{m_3}{m_2} + \bar{E} \\
&E_{23}^2 = \frac{m_4}{m_2} - \frac{m_3^2}{m_2^2} - m_2
\end{aligned}$$

$$\begin{pmatrix}
E_{11} & E_{12} & 0 & 0 & 0 & 0 \\
E_{12} & E_{22} & E_{23} & 0 & 0 & 0 \\
0 & E_{32} & E_{33} & E_{34} & 0 & 0 \\
0 & 0 & E_{43} & E_{44} & E_{45} & 0 \\
 & & & & & \\
 & & & & &
\end{pmatrix}$$