Recent developments within the empirical Shell Model framework

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Shell model approach

Calculations Ab Initio

- Realistic NN interactions
- Diagonalization in $N\hbar \omega$ h.o.space

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Define valence space

- $H_{\text{eff}} \Psi_{\text{eff}} = E \Psi_{\text{eff}}$

→ INTERACTIONS

- build and diagonalize Hamiltonian matrix

→ CODES

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Weak processes:

- $\beta$ decays
- $\beta \beta$ decays

$[\tau_{1/2}^{0+} (0^+ \rightarrow 0^+)]^{-1} = G_{0\nu} |M_{0\nu}|^2 \langle m_\nu \rangle^2$

= ASTROPHYSICS

= PARTICLE PHYSICS

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Collective excitations:

- deformation, superdeformation
- superfluidity
- symmetries

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Shell evolution far from stability:

- Shell quenching
- New magic numbers

= ASTROPHYSICS
Shell Model applications in astrophysics

- Electron capture rates for supernovae
- Inelastic neutrino-nucleus reactions: for supernovae dynamics and neutrino detection
- Half-lives for r-process nuclei
- Method of choice to describe correlations in medium-mass and heavier nuclei
- What is needed: high-precision effective interactions and SM codes
Shell Model: giant computations

Problem dimension in the m-scheme:

\[ D \sim \left( \frac{d_\pi}{p} \right) \cdot \left( \frac{d_\nu}{n} \right) \]

In the pf-shell (1\( f_7/2 \), 2\( p_3/2 \), 1\( f_5/2 \)):

- \( ^{48}\text{Cr} \) 1,963,461
- \( ^{56}\text{Ni} \) 1,087,455,228

Current diagonalization limit in m-scheme \( 10^{10} \)

The largest SM diagonalization up to date has been achieved by the Strasbourg group (using very modest computing resources):


ANTOINE can be adapted to calculations of any, up to two fluid systems of fermions or bosons:


- **m scheme** CODE ANTOINE

\[ |\Psi_\alpha\rangle = \prod_{i=nljm} a_i^\dagger |0\rangle = a_{i_1}^\dagger \cdots a_{i_A}^\dagger |0\rangle \]

Huge dimensions of the matrices \( (10^9 - 10^{10}) \)

storage of Lanczos vectors on disk

**Very large cases:**
splitting of the initial and final vectors

\[ \Psi_{i,f} = \bigcup_{m} \Psi_{i,f}^{m} \]

\[ \Psi_f^{(m)} = \sum_{n} H_{(m,n)} \Psi_i^{(n)} \]

- **coupled scheme** CODE NATHAN
- **coupled scheme** SVD FIT CODE

*E. Caurier et al., Rev. Mod. Phys. 77 (2005) 427; ANTOINE website*
Effective interactions for SM calculations

- realistic (Argonne, Bonn, ...) or chiral EFT (N3LO) $V_{NN}$ interaction
- problems with saturation and shell closures
- Nuclear structure challenge: inclusion of 3N forces
  - exactly ...
  - semi-empirically: keep $V_{NN}$ and add empirical 3N forces
  - empirically: fit $H_{\text{eff}}$ to data:
    - fit all TBME using linear combination method (SVD)
    - adjust only monopole Hamiltonian

\[
H = H_{\text{monopole}} + H_{\text{multipole}}
\]

\[
V = \sum_{JT} V_{ijkl}^{JT} \left[ (a_i^+ a_j^+)^{JT} (\tilde{a}_k \tilde{a}_l)^{JT} \right]^{00}
\]

In order to express the number of particles operators $n_i = a_i^+ a_i \propto (a_i^+ \tilde{a}_i)^0$,

- particle-hole recoupling:

\[
V = \sum_{\lambda \tau} \omega_{ij}^{\lambda \tau} \left[ (a_i^+ \tilde{a}_k)^{\lambda \tau} (a_j^+ \tilde{a}_l)^{\lambda \tau} \right]^{00}
\]

$H_{\text{monopole}}$ corresponds to $\lambda \tau = 00$ and $01$ which implies that $i = j$ and $k = l$

\[
H_{\text{monopole}} = \sum_i n_i \varepsilon_i + \sum_{i \leq j} n_i n_j V_{ij}
\]

$H_{\text{multipole}}$ corresponds to all other combinations of $\lambda \tau$. 
Regions of our interest

- Stability of N=Z=50 gaps
- Collectivity in light Xe isotopes
- Superallowed GT decay of 100Sn

- 3-body forces
  - Effective forces far from stability
  - Dipole excitations in Ne isotopes

- N=82 shell quenching in 128Cd
  - Mixed symmetry states at N=80
  - New N=90 shell closure in 140Sn

- First SM predictions of stability of gaps in 78Ni
  - Initialization of calculations using 78Ni as a core
  - Shape transitions in Zr isotopes, collectivity of N=52,54 nuclei
  - Isomers in fission fragments

- Collapse of the N=40 gap
  - New island of inversion at N=40
  - Dipole excitations around 68Ni
Dipole excitations in the islands of inversion
E1 excitations on nuclei

- Beyond mean-field approaches (QRPA, RQTBA)
  - ☺ good reproduction of giant and pygmy resonances
  - ☹ limited to even systems

- Shell model
  - ☺ ODD NUCLEI
  - ☹️ low lying strength only
  - ☺️ precise description of spectroscopy, transition rates, shell structure...

- the pygmy part enhances the low lying E1 strength in the astrophysically relevant range.
Islands of inversion at N=20 and N=40
Intruders $\rightarrow$ low lying E1 strength

How the low lying strength evolves toward the island of inversion?
N=20 gap washed out in Ne but present in Si
E1 calculations in psdpf SM

$$Q_{\mu}^{1=1} = \frac{Z}{A} e \sum_{k=1}^{N} r_k Y_{1\mu}(r_k) - \frac{N}{A} e \sum_{k=1}^{Z} r_k Y_{1\mu}(r_k)$$

- SM in $p$-$sd$-$pf$ model space
- full $sd$ diagonalization + full $1\hbar\omega$ excitations
- Exact removal of COM components
- Interaction: PSDPF

E1 strength in even neon isotopes

- SM in $psdpf$ model space
- full $sd$ diagonalization + full $1\hbar\omega$ excitations
- Exact removal of COM components

![Graph showing $B(E1)$ vs $E_{exc}$ for $^{20}\text{Ne}$]
E1 strength in even neon isotopes

- SM in $psd pf$ model space
- full $sd$ diagonalization + full $1\hbar\omega$ excitations
- Exact removal of COM components

![Graph showing B(E1) vs E_{exc} for 22Ne]
E1 strength in even neon isotopes

- SM in $psdpf$ model space
- full $sd$ diagonalization + full $1\hbar\omega$ excitations
- Exact removal of COM components
E1 strength in even neon isotopes

- SM in \textit{psdpt} model space
- full $sd$ diagonalization + full $1\hbar\omega$ excitations
- Exact removal of COM components
The case of $^{26}$Ne

**EXP:** $\sum B(E1) = 0.49 \pm 0.16 \, e^2 fm^2 \ (6-10 \text{MeV})$

**THEO:** $\sum B(E1) = 0.485 \, e^2 fm^2 \ (0-10 \text{MeV})$

Low peaks structure: $\nu s^{-1}_{1/2} p^1_{3/2}, \nu s^{-1}_{1/2} p^1_{1/2}$

SM: Complex wave functions (major contributions $\leq 30\%$)

QRPA main contribution: $70\%$ of $\nu s^{-1}_{1/2} p^1_{3/2}$

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E1 strength in even neon isotopes

- SM in $psd pf$ model space
- full $sd$ diagonalization + full $1\hbar\omega$ excitations
- Exact removal of COM components
E1 strength in even neon isotopes

- SM in $sd$-$pf$ model space
- $t=4$ $sd$-$pf$ diagonalization for GS + 1p1h
- COM spuriosity $\sim 1\%$
E1 strength in even neon isotopes

- SM in $sd-pf$ model space
- $t=4$ $sd-pf$ diagonalization for GS + 1p1h
- COM spuriosity $\sim 1\%$

![Graph showing B(E1) vs. E_{exc} (MeV) for $^{32}$Ne]
The low lying strength moves up in energy in the island of inversion.
Evolution of dipole strength along the Ne chain: SM vs QRPA

Sum of $B(E1)$ strength up to 14MeV

Energy of the first peak with $B(E1) \geq 0.01 e^2 fm^2$

QRPA from *M. Martini, S. Péru, and M. Dupuis, Phys. Rev. C 83, 034309 (2011)*
E1 strength in odd neon isotopes

- SM in $psd pf$ model space
- full $sd$ diagonalization + full $1\hbar\omega$ excitations
- Exact removal of COM components

![Graph showing E1 strength in odd neon isotopes](image)
E1 strength in odd neon isotopes

- SM in \textit{psdpf} model space
- full \textit{sd} diagonalization + full \(1\hbar\omega\) excitations
- Exact removal of COM components

\[ B(E1) \ (e^2\cdot fm^2) \]

\( \begin{align*}
B(3/2^-) & \quad \text{red} \\
B(5/2^-) & \quad \text{blue} \\
B(7/2^-) & \quad \text{yellow} \\
\end{align*} \)

\( ^{23}\text{Ne} \)

\( E_{\text{exc}} \ (\text{MeV}) \)
E1 strength in odd neon isotopes

- SM in *psd*pf model space
- full *sd* diagonalization + full $1\hbar\omega$ excitations
- Exact removal of COM components

![Graph showing B(E1) vs. E_{exc} for 25Ne isotopes](image)
E1 strength in odd neon isotopes

- SM in $psdpf$ model space
- full $sd$ diagonalization + full $1\hbar\omega$ excitations
- Exact removal of COM components

![Graph showing $B(E1)$ vs $E_{exc}$ for $^{27}$Ne]
E1 strength in odd neon isotopes

- SM in psd$pf$ model space
- full $sd$ diagonalization + full $1\hbar\omega$ excitations
- Exact removal of COM components

![Graph showing B(E1) vs E_exc](image-url)

- $B(E1)$ in units of $e^2 \cdot fm^2$
- $E_{exc}$ in units of MeV
- Isotopes: $^{29}$Ne

- Levels: $1/2^-$, $3/2^-$, $5/2^-$
SM challenge: Dipole excitations in the Ni chain

- \( ^{67}\text{Ni} @ 600 \text{ AMeV} \)
- \( ^{68}\text{Ni} @ 600 \text{ AMeV} \)

**Graphs:**
- \( \text{B(E1)} \) vs. \( \gamma \)-energy for \( ^{67}\text{Ni} \) and \( ^{68}\text{Ni} \).

- **fp-gd model space**
- **truncated calculations** (t=6 GS+ 1p1h)
- **COM \( \sim 5\% \)**

Astrophysics applications
Impact of the realistic M1 fragmentation on the neutron capture cross sections

- State-by-state cross section 2 times larger than using Brink hypothesis
- Using SF of $2^+$ state instead of $0^+$ leads to larger cross sections

$\sigma$ [b]  
$E_n$ [MeV]

$\sigma$ [b]  
$E_n$ [MeV]

- M1 is 1% – 50% of the total contribution. Taking into account E1 is essential.

Half-lives of r-process nuclei with FF transitions
Making gold in nature: r-process nucleosynthesis

The r-process requires the knowledge of properties of extremely neutron-rich nuclei:

- Masses (DZ10, DZ31)
- Beta-decay half-lives.
- Neutron capture rates.
- Fission rates and yields.


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To calculate beta decay between two states one needs:

- accurate value of the decay energy 
  \[(T_{1/2} \sim \Delta E^{-5})\]

- matrix elements of Gamow-Teller \((\Delta J^\pi = 0^+, 1^+)\)
  first forbidden \((\Delta J^\pi = 0^-, 1^-, 2^-)\)
  transition operators

Nuclear model has to provide good description of masses, spectra and wavefunctions

In r-process nuclei, GT is not enough, as protons and neutrons occupy different parity levels:

\[
\begin{pmatrix}
H.O. \\
\end{pmatrix}
\]

The beta half-life to a given state is given as: \(ft = 6146\), where the space phase factor \(f\) is given as

\[
f = \int_{W_0}^1 C(W)F(Z, W)(W^2 - 1)^{1/2} W(W_0 - W)^2 dW
\]

For allowed transitions: \(C(W) = B(F) + B(GT)\)

For forbidden \(C(W)\) is a function of the energy, and we have to calculate the integral

\[
C(W) = k(1 + aW + \frac{b}{W} + cW^2)
\]


\[
\begin{align*}
  k &= \left[ \zeta_0^2 + \frac{1}{9} w^2 \right]^{(0)} + \left[ \zeta_1^2 + \frac{1}{9} (x + u)^2 - \frac{4}{9} \mu_1 \gamma_1 u(x + u) ight]^{(1)} + \left[ \frac{1}{12} z^2 (W_0^2 - \lambda_2) \right]^{(2)}, \\
  ka &= \left[ -\frac{4}{3} u Y - \frac{1}{9} W_0 (4 x^2 + 5 u^2) \right]^{(1)} - \left[ \frac{1}{6} z^2 W_0 \right]^{(2)}, \\
  kb &= \frac{2}{3} \mu_1 \gamma_1 \left\{ -[\zeta_0 w]^{(0)} + [\zeta_1 (x + u)]^{(1)} \right\}, \\
  kc &= \frac{1}{18} \left[ 8 u^2 + (2x + u)^2 + \lambda_2 (2x - u)^2 \right]^{(1)} + \frac{1}{12} \left[ z^2 (1 + \lambda_2) \right]^{(2)}.
\end{align*}
\]

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Matrix elements

\[ w = - R^A F^0_{011} \]
\[ = \lambda \sqrt{3} \langle J_f T_f || ir [C_1 \times \sigma]^0 \tau || J_i T_i \rangle C, \]
\[ x = - \frac{1}{\sqrt{3}} R^V F^0_{110} \]
\[ = - \langle J_f T_f || ir C_1 \tau || J_i T_i \rangle C, \]
\[ u = - \sqrt{\frac{2}{3}} R^A F^0_{111} \]
\[ = \lambda \sqrt{2} \langle J_f T_f || ir [C_1 \times \sigma]^1 \tau || J_i T_i \rangle C, \]
\[ z = \frac{2}{\sqrt{3}} R^A F^0_{211} \]
\[ = - 2 \lambda \langle J_f T_f || ir [C_1 \times \sigma]^2 \tau || J_i T_i \rangle C, \]
\[ w' = - \frac{2}{3} R^A F^0_{011}(1,1,1,1) \]
\[ = \lambda \sqrt{3} \langle J_f T_f || ir \frac{2}{3} I(1,1,1,1,r) [C_1 \times \sigma]^0 \tau || J_i T_i \rangle C, \]
\[ x' = - \frac{2}{3 \sqrt{3}} R^V F^0_{110}(1,1,1,1) \]
\[ = - \langle J_f T_f || ir \frac{2}{3} I(1,1,1,1,r) C_1 \tau || J_i T_i \rangle C, \]
\[ u' = - \frac{2 \sqrt{2}}{3 \sqrt{3}} R^A F^0_{111}(1,1,1,1) \]
\[ = \lambda \sqrt{2} \langle J_f T_f || ir \frac{2}{3} I(1,1,1,1,r) [C_1 \times \sigma]^1 \tau || J_i T_i \rangle C, \]
Results: half-lives of r-process nuclei

- Accurate description of $Q_\beta$, neutron emission probabilities and half-lives in the known region
- Universal quenching factors on GT and FF operators in all mass regions

Toward a generalized monopole interaction
Realistic NN potentials fail in reproducing spin-orbit shell closures

Empirical approaches are necessary: fit all TBME or monopoles only
Universal Monopole

Can we build a simple model of monopole interaction?

Example: Shell evolution in N=51 nuclei

\[ j_+ = l + \frac{1}{2}, \quad j_- = l - \frac{1}{2} \]

\[ V_C = \Sigma_{S,T} f_{S,T} P_{S,T} \exp\left(-\frac{r}{\mu}\right) \]

(2 new parameters: \( f_{1,0} = -166 \text{MeV}, \mu = 1.0\text{fm} \))
Monopole Hamiltonian and invariant representation

\[ H_{\text{mono}} = \sum_i e_i n_i + \sum_{i \leq j} (V_{ij} n_{ij}) + \sum_{i \leq j \leq k} (V_{ijk} n_{ijk}) \]

\[ V_{ij} = \frac{\sum_J V_{ijj}^J (2J + 1)(1 + (-1)^J \delta_{ij})}{(2j_i + 1)(2j_j + 1 - \delta_{ij})} \]

3-body interaction produces also 2-body interactions in the valence space:

\[ \sum_c V_{ijc} n_{ijc} = N_c \sum_{ij} V_{ij} n_{ij} \]

In a first step one can consider 2-body terms modulated by total particle number \( n \).
Invariant representation: 2-shell example

\[ H_{\text{mono}} = \varepsilon_1 n_1 + \varepsilon_2 n_2 + \frac{n_1(n_1 - 1)}{2} V_{11} + \frac{n_2(n_2 - 1)}{2} V_{22} + n_1 n_2 V_{12} \]

Jacobi construction to separate a global term \( H_0 \) (depending only on the total number of particles \( n = n_1 + n_2 \)) from a linear term \( H_1 \) and a quadratic term \( H_2 \) in \( n_1 \) and \( n_2 \). For example, the first term can be transformed as:

\[ n_1 \varepsilon_1 + n_2 \varepsilon_2 = (n_1 + n_2) \left( \frac{D_1 \varepsilon_1 + D_2 \varepsilon_2}{D_1 + D_2} \right) + \left( \frac{n_1}{D_1} - \frac{n_2}{D_2} \right) \frac{D_1 D_2}{D_1 + D_2} (\varepsilon_1 - \varepsilon_2) \]

\[ H_m = n \bar{\varepsilon}_0 + \frac{n(n-1)}{2} W_0 + \Gamma_{12}[\bar{\varepsilon}_1 + (n-1) W_1] + \Gamma_{12}^{(2)} W_2 \]

with

\[ \Gamma_{12} = \frac{D_2 n_1 - D_1 n_2}{D_1 + D_2} \quad \Gamma_{12}^{(2)} = \frac{D_1 D_2}{2} \left( \frac{2n_1 n_2}{D_1 D_2} - \frac{n_1(n_1 - 1)}{D_1(D_1 - 1)} - \frac{n_2(n_2 - 1)}{D_2(D_2 - 1)} \right) \]
Invariant decomposition

\[ H_{\text{mono}} = \varepsilon n + \frac{W_n(n-1)}{2} + \sum_i \Gamma_1^i (e_i - \varepsilon + (n-1)W_i) + \sum_{i \leq j} \Gamma_2^i j W_{ij} \]

\[ V_{ij} = \frac{\sum_J (2J+1)(2T+1) V_{ijj}^{JT}}{\sum_J (2J+1)(2T+1)} \]

\[ \varepsilon = \sum_i \frac{D_i e_i}{D} \quad \text{W} = 2 \sum_{ij} \frac{D_{ij} V_{ij}}{D(D-1)} \]

\[ W_i = \sum_i (D_i - \delta_{ij})(V_{ij} - W) \quad W_{ij} = V_{ij} - W_i - W_j - W \]

\[ \Gamma_1^i = D_i \left[ \frac{n_i}{D_i} - \frac{n_1}{D_1} \right] \quad \Gamma_2^i j = \frac{D_{ij}}{2} \left[ \frac{2n_{ij}}{D_{ij}} - \frac{n_{ii}}{D_{ii}} - \frac{n_{jj}}{D_{jj}} \right] \]
Universal monopole in invariant scheme

\[ H_{mono} = \varepsilon n + \frac{Wn(n-1)}{2} + \sum_i \Gamma_1^i (e_i - \varepsilon + (n-1)W_i) + \sum_{i<j} \Gamma_2^{ij} W_{ij} \]

- pin down the relevant operators & make them 3-body
  \[ \Gamma_1^i \rightarrow \kappa_1(n) \ast \Gamma_1^i \]
  \[ \Gamma_2^{ij} \rightarrow \kappa_2(n) \ast \Gamma_2^{ij} \]
  \[ + \kappa_3 \Gamma_3^{ijk} \]

- find their parameterizations in several shells
- establish the global mass dependence of these parameters
Invariant scheme and ESPE

Analysis of centroids in invariant scheme reveal that the major empirical corrections are mostly due to one term

\[ \Gamma_{fr}^{(2)} \cdot \kappa(n) ; \quad \kappa(n) = a + b \cdot n \]

\[ f = d_{5/2}, r = s_{1/2}, d_{3/2} \]
Invariant monopole: results

**rmsd error in** \( p \)-shell (55 states in 15 nuclei) **rmsd error in** \( sd \)-shell (309 states in 60 nuclei)

<table>
<thead>
<tr>
<th></th>
<th>( V_{\text{eff}}^{\text{NN}} )</th>
<th>( +\kappa_2(n)\Gamma_{fr}^2 )</th>
<th>fit-mono</th>
</tr>
</thead>
<tbody>
<tr>
<td>N3LO</td>
<td>1.34</td>
<td>0.44</td>
<td>0.43</td>
</tr>
<tr>
<td>AV18</td>
<td>1.41</td>
<td>0.47</td>
<td>0.47</td>
</tr>
</tbody>
</table>

Parameterization independent on the initial potential

- Invariant decomposition provides a method of including 3N corrections to the Hamiltonian at the cost of \( \sim 2 \) empirical parameters.
- The parameters have a clear physical meaning: they provide a missing 3N contribution to NN interactions from core and valence particles.
- Work in progress on generalization of this scheme for heavier nuclei.
Conclusion

- Shell Model remains the most accurate method in nuclear physics, flexible enough to be applied to even, odd, even-odd systems with various number of particles outside closed shells.

- It is a method of choice for structure calculations where a detailed spectroscopic description is needed and where the description of correlations is crucial.

- The problems of SM are related to computing possibilities and effective interactions. In both directions there is a considerable progress:
  - current diagonalization limit of $10^{10}$ should be overcome within a few years
  - EFT + ChPT give us hope to use in future SM calculations effective interactions which take consistently into account 3N contributions.
Thanks to:
E. Caurier, F. Nowacki, A. Zuker (IPHC Strasbourg)
A. Poves (UAM Madrid)
G. Martinez-Pinedo, K. Langanke (GSI/TU Darmstadt)
Q. Zhi (Gouizhou University, China)
Spin-tensor decomposition of NN interactions

\[ V = \sum_{k=0,1,2} \left( S^{(k)} \cdot Q^{(k)} \right) = \sum_{k=0,1,2} V^{(k)}, \]

\( Q^{(k)} \): operators in the coordinate space.
\( S^{(k)} \): spin-tensors constructed from nucleon spin-1/2 operators.

<table>
<thead>
<tr>
<th>Operators</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scalar operators:</td>
<td></td>
</tr>
<tr>
<td>((\vec{\sigma}_1 \cdot \vec{\sigma}_2))</td>
<td>central</td>
</tr>
<tr>
<td>Vector operators:</td>
<td></td>
</tr>
<tr>
<td>(\vec{\sigma}_1 + \vec{\sigma}_2)</td>
<td>spin-orbit</td>
</tr>
<tr>
<td>([\vec{\sigma}_1 \times \vec{\sigma}_2]^{(1)})</td>
<td>ALS</td>
</tr>
<tr>
<td>(</td>
<td>\vec{\sigma}_1 - \vec{\sigma}_2</td>
</tr>
<tr>
<td>Tensor operators:</td>
<td></td>
</tr>
<tr>
<td>([\vec{\sigma}_1 \times \vec{\sigma}_2]^{(2)})</td>
<td>tensor force</td>
</tr>
</tbody>
</table>

In the \(LS\)-scheme, the matrix elements of each \(V^{(k)}\) can be obtained from \(V\):

\[
\langle (nl, n'l' : LS, JMTM_T | V^{(k)} | n''l'', n'''l''' : L'S', J'MTMT_T \rangle =
(2k+1)(-1)^J \left\{ \begin{array}{ccc} L & S & J \\ S' & L' & k \end{array} \right\} \sum_{J'} (-1)^{J'} (2J' + 1) \left\{ \begin{array}{ccc} L & S & J' \\ S' & L' & k \end{array} \right\} (1)
\times \langle (nl, n'l' : LS, J'MTM_T | V | n''l'', n'''l''' : L'S', J'MTMT_T \rangle.
\]
Spin-tensor decomposition of NN interactions

<table>
<thead>
<tr>
<th>Energy gap</th>
<th>$\nu(0d_{5/2}-1s_{1/2})$ MeV</th>
<th>$\nu(0f_{7/2}-0p_{3/2})$ MeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Filling orbital</td>
<td>$\pi 0d_{5/2}$</td>
<td>$\pi 0f_{7/2}$</td>
</tr>
<tr>
<td>$^{28}\text{O} \rightarrow ^{28}\text{Si}$</td>
<td>$^{48}\text{Ca} \rightarrow ^{56}\text{Ni}$</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>0.95</td>
<td>3.82</td>
</tr>
<tr>
<td>Central</td>
<td>2.58</td>
<td>3.59</td>
</tr>
<tr>
<td>Vector</td>
<td>−0.87</td>
<td>0.90</td>
</tr>
<tr>
<td>LS</td>
<td>−0.72</td>
<td>0.41</td>
</tr>
<tr>
<td>ALS</td>
<td>−0.15</td>
<td>0.49</td>
</tr>
<tr>
<td>Tensor</td>
<td>−0.76</td>
<td>−0.68</td>
</tr>
</tbody>
</table>

- Tensor force from realistic NN interaction is preserved in the empirical fits.
- Spin-orbit and central parts that needs to be modified to reproduce the gaps.
- No regular behaviour with mass.
- What about the 3N forces?

Lanczos Structure Function

Initial vector $|1\rangle = \frac{|\Omega\rangle}{\sqrt{\langle\Omega|\Omega\rangle}}$.

$E_{11} = \langle 1 | H | 1 \rangle$
$E_{12} |2\rangle = (H - E_{11}) |1\rangle$
$E_{23} |3\rangle = (H - E_{22}) |2\rangle - E_{12} |1\rangle$

$E_{NN+1} |N+1\rangle = (H - E_{NN}) |N\rangle - E_{N-1N} |N-1\rangle$

where $E_{NN} = \langle N | H | N \rangle$, $E_{NN+1} = E_{N+1N}$

Each Lanczos iteration gives information about two new moments of the distribution.

$E_{11} = \langle 1 | H | 1 \rangle = m_1 = \bar{E}$
$E_{12}^2 = \langle 1 | (H - E_{11})^2 | 1 \rangle = m_2$

$E_{22} = \frac{m_3}{m_2} + \bar{E}$

$E_{23}^2 = \frac{m_4}{m_2} - \frac{m_3^2}{m_2^2} - m_2$