Neutron matter from chiral effective field theory interactions

Thomas Krüger

International Workshop XLI on Gross Properties of Nuclei and Nuclear Excitations: Astrophysics and nuclear structure, Hirschegg

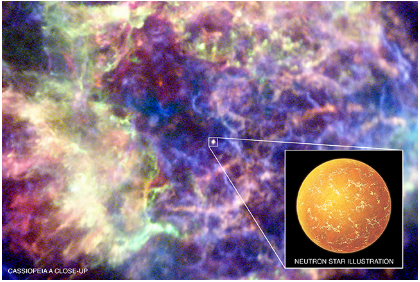
I. Tews, TK, K. Hebeler, and A. Schwenk, Phys. Rev. Lett. **110**, 032504 (2013)
TK, I. Tews, K. Hebeler, and A. Schwenk, *arXiv:1301.xxxx*

Outline

- Introduction: chiral EFT for nuclear interactions
- 3N and 4N interactions at N^3LO
- Complete neutron matter calculation at N^3LO
- From neutron matter to neutron stars
- Summary

Short motivation

- Neutron matter constrains properties of neutron stars
- All calculations so far:
  NN forces at N^3LO and
  3N forces at N^2LO
- First inclusion of 3N and 4N interactions at N^3LO



[NASA/CXC/M.Weiss]

Orientation

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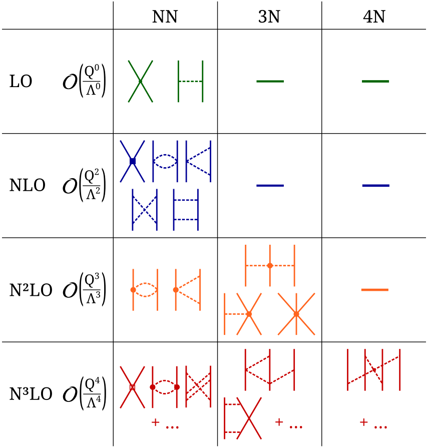
Chiral effective field theory for nuclear interactions



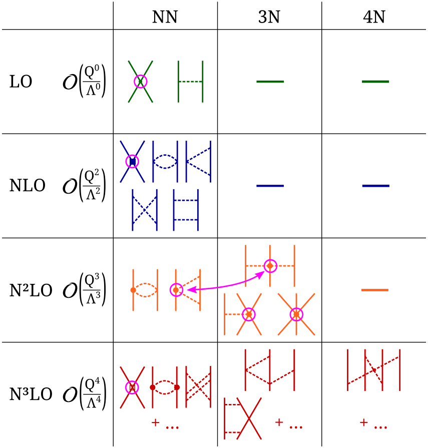
- Separation of scales:
  low mom. $Q \ll$ breakdown scale Λ
- Write most general Lagrangian
  and expand in powers of

$$\left(\frac{Q \sim m_\pi}{\Lambda \sim 500\,\mathrm{MeV}} \sim \frac{1}{3}\right)^\nu$$

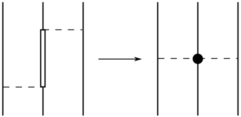
- Systematic: can work to desired
  accuracy and obtain error estimates

[Weinberg, van Kolck, Kaplan, Savage, Wise, Epelbaum, Kaiser, Meißner, . . .]

Chiral effective field theory for nuclear interactions



- Explicit degrees of freedom: pions and nucleons
- Long-range physics explicitly, short-range physics expanded in general operator basis
- High-momentum physics absorbed into few short-range couplings, fit to experiment



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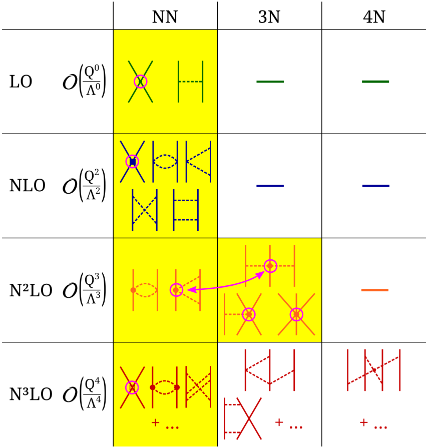
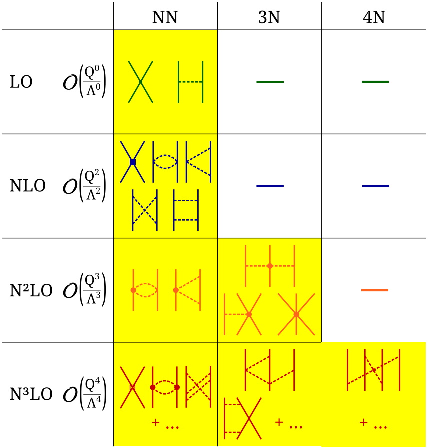


- Many-body forces are crucial
- Consistent interactions: same couplings for NN and many-body sector
- So far: only leading 3N forces included

[Weinberg, van Kolck, Kaplan, Savage, Wise, Epelbaum, Kaiser, Meißner, . . .]

Chiral effective field theory for nuclear interactions



This work: take into account all contributions to N^3LO

N^3LO 3N forces have been derived only recently

[Bernard *et al.*, PRC **77**, 064004 (2008) and PRC **84**, 054001 (2011); Epelbaum, PLB **639**, 456, (2006)]

In neutron matter:

- simpler, only certain parts of the many-body forces contribute
- chiral 3- and 4-neutron forces are predicted to N^3LO

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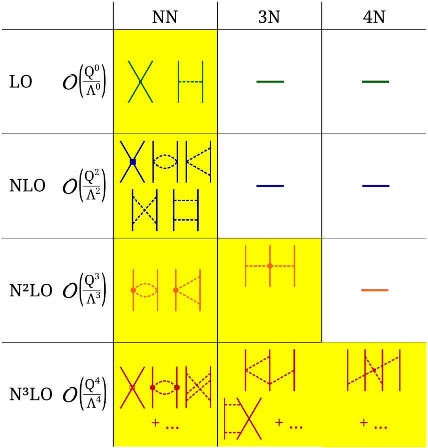

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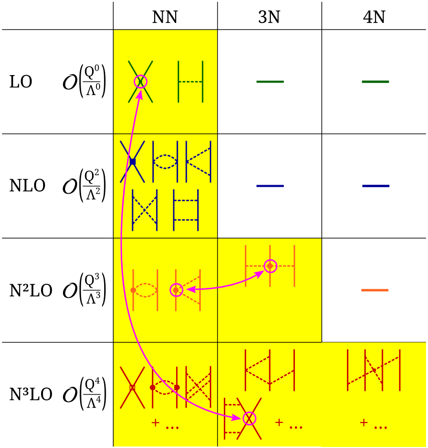

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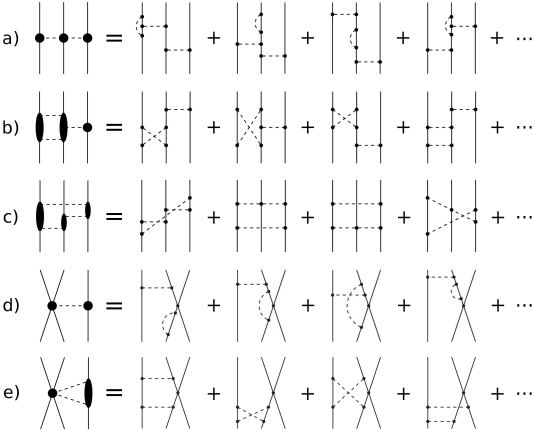
3N interactions at N^3LO



2π exchange

2π-1π exchange

pion ring (involved)

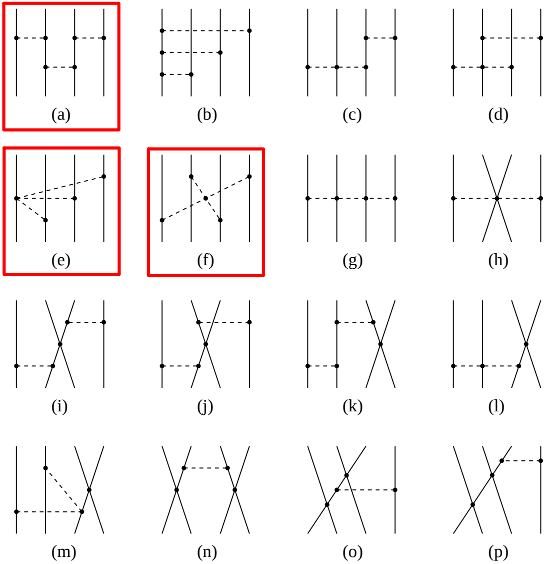
1π-exchange contact
(vanishes)

2π-exchange contact
(2-body-contact C_T)

relativistic corrections
(2-body-contacts C_T, C_S)

[Bernard *et al.*, PRC **77**, 064004 (2008) and PRC **84**, 054001 (2011)]

4N interactions at N^3LO

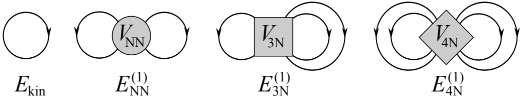


[Epelbaum, PLB **639**, 456 (2006) & EPJ A **34**, 197 (2007)]

Energy per particle in the Hartree-Fock approximation

3N forces are perturbative at $\mathrm{N^2LO}$ (for smaller c_i) [Hebeler, Schwenk]
Hartree-Fock is a reliable approximation at this order!

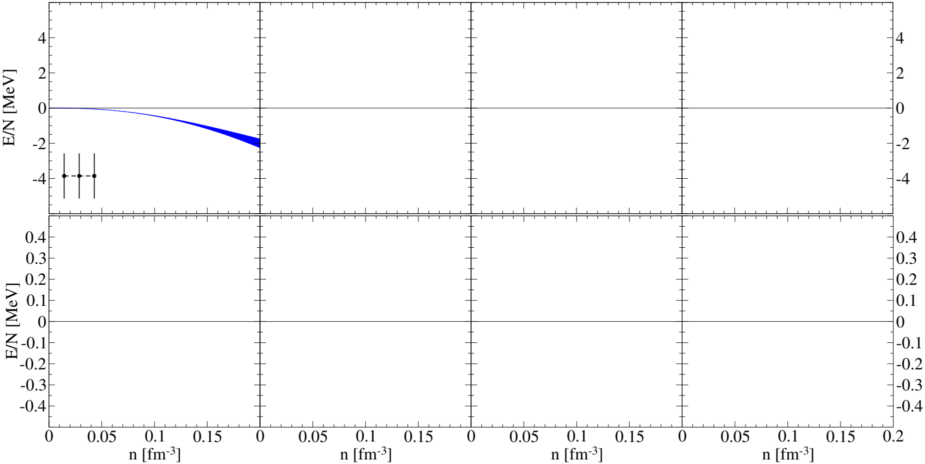
$$\frac{E}{N} = \frac{1}{n}\frac{1}{A!}\sum_{\boldsymbol{\sigma}_1,\ldots,\boldsymbol{\sigma}_A}\int\frac{d^3k_1}{(2\pi)^3}\cdots\int\frac{d^3k_A}{(2\pi)^3} f_R^2\, n_{\mathbf{k}_1}\cdots n_{\mathbf{k}_A}$$
$$\times \langle\, 1\cdots A \,|\, \mathcal{A}_A \sum_{\pi\in S_A} V\big(\pi(1),\ldots,\pi(A)\big)\,|\, 1\cdots A\,\rangle$$



$$f_R = \exp\left[-\left(\frac{k_1^2+\ldots+k_A^2-\mathbf{k}_1\cdot\mathbf{k}_2-\ldots-\mathbf{k}_{A-1}\cdot\mathbf{k}_A}{A\Lambda^2}\right)^{n_{\exp}}\right] \qquad n_{\exp}=4$$

cutoff variation: $\Lambda = 2 - 2.5\,\mathrm{fm}^{-1}$

Results: individual N^{3}LO contributions



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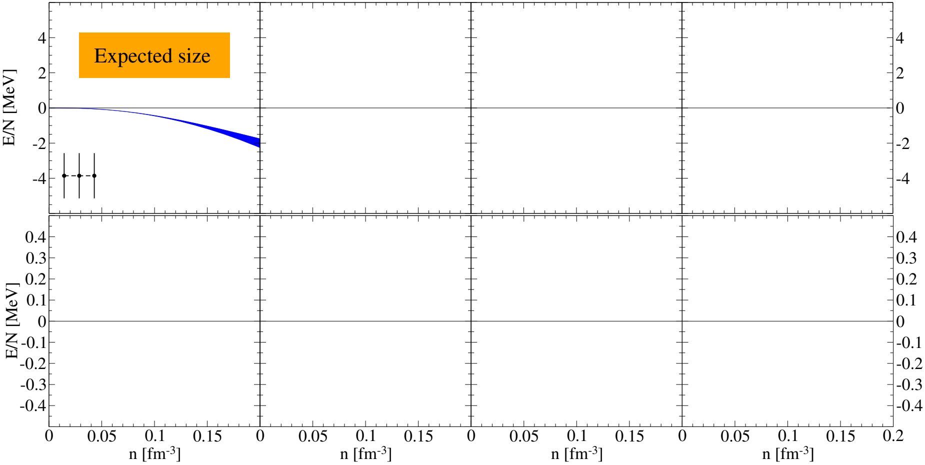

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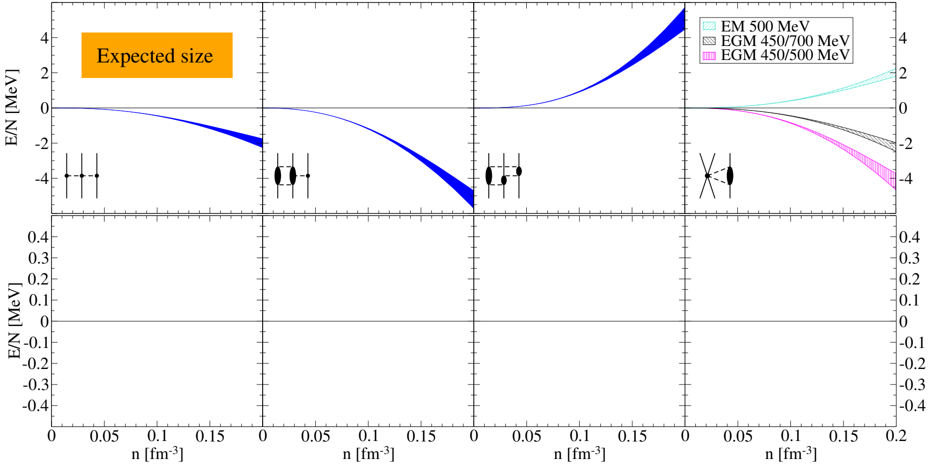

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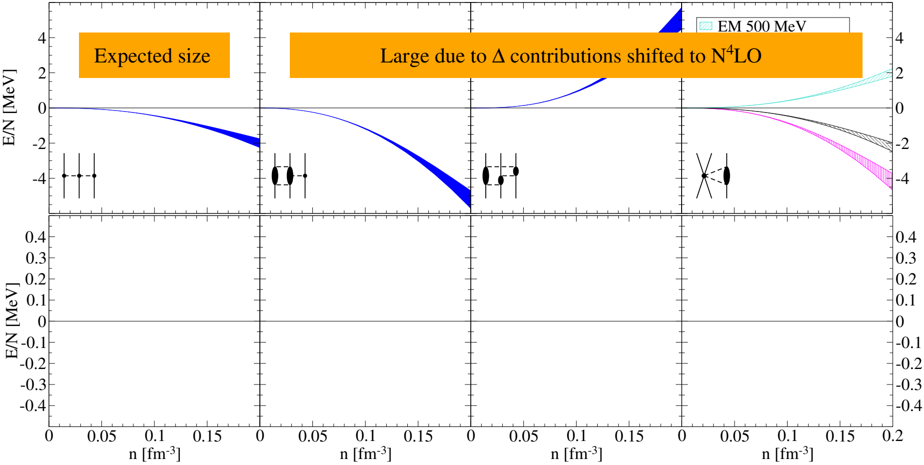

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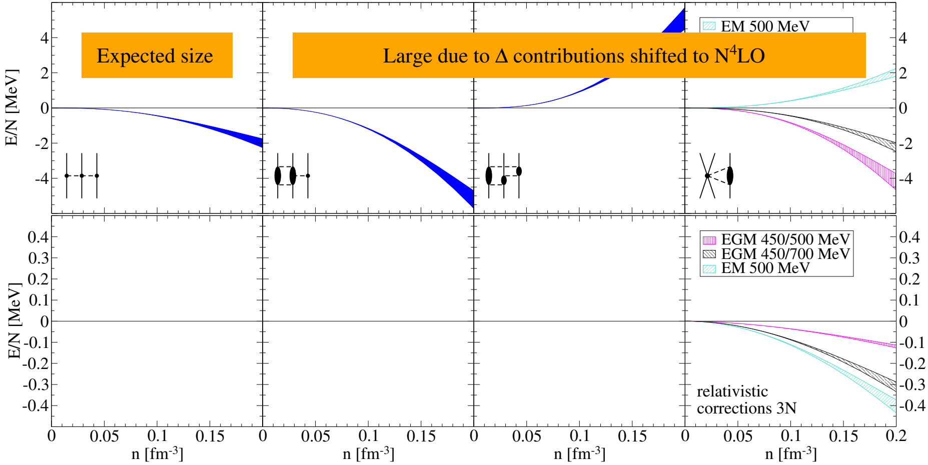

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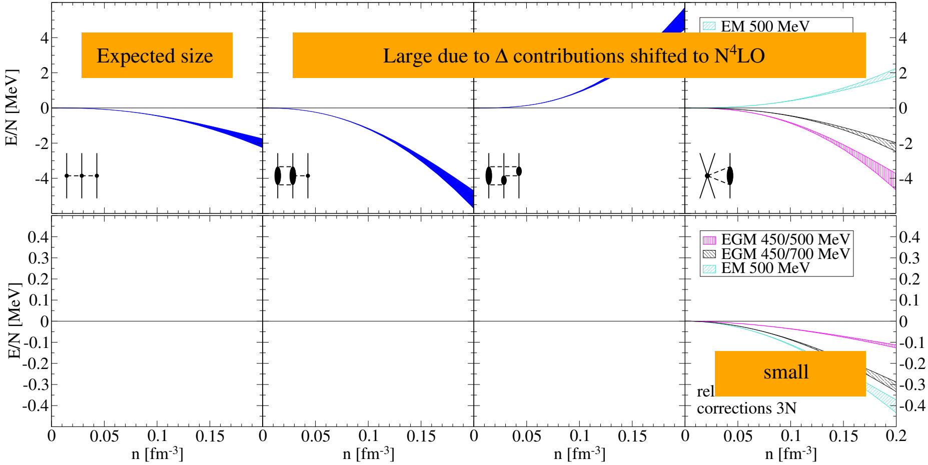

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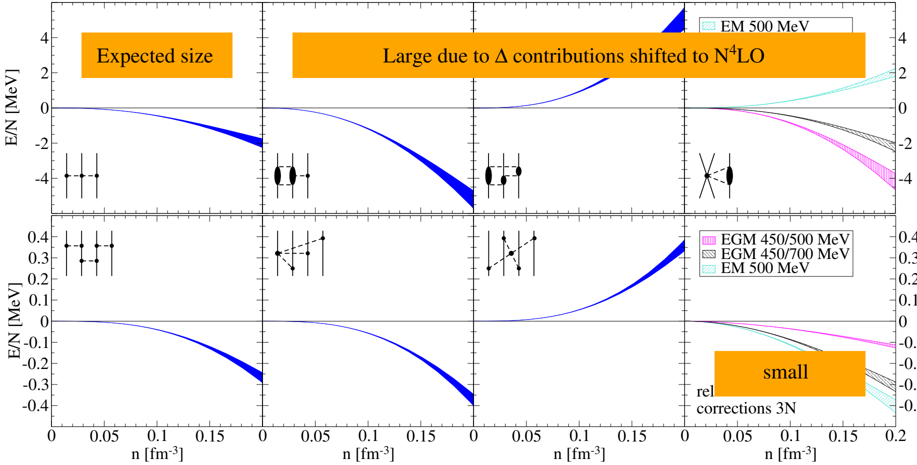

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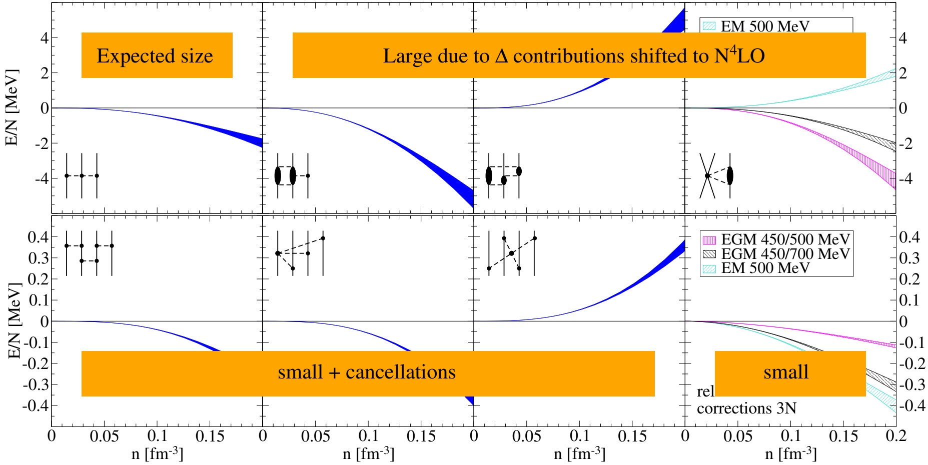

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Energy per particle beyond the Hartree-Fock approximation

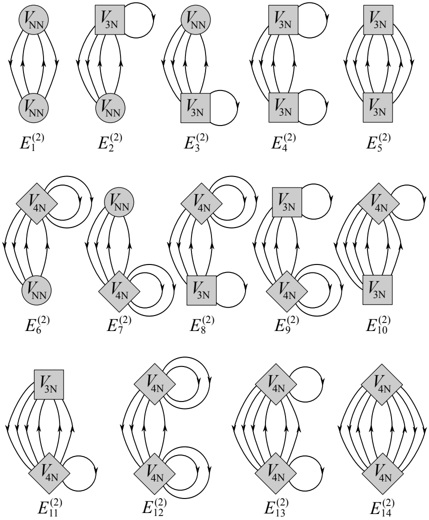
NN and leading 3N (with large c_i) need to be evaluated beyond the HF approximation [Hebeler, Schwenk, PRC **82**, 014314 (2010)]

Use density-dependent NN forces

$$\bar{V}_{3N} = \sum_{\sigma_3} \int \frac{d^3 k_3}{(2\pi)^3} n_{\mathbf{k}_3} \mathcal{A}_3 V_{3N} \Big|_{nnn}$$

Density dependent NN forces from many-body forces at N^3LO are currently developed.

Second order



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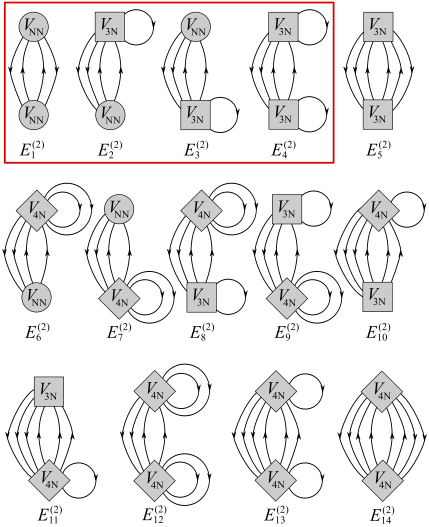
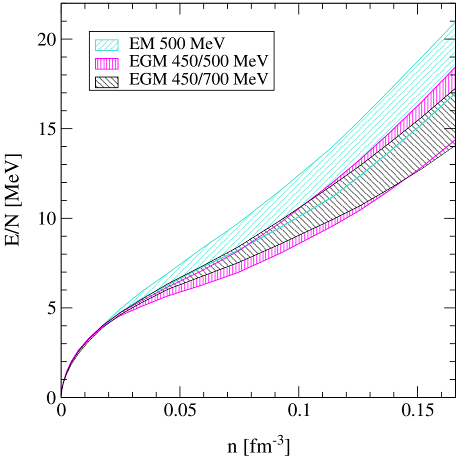


Total neutron matter energy



Bands include:

- $\Lambda = 2 - 2.5\,\mathrm{fm}^{-1}$
- many-body uncertainties
- **3N uncertainties mainly:**
  $c_1 = -(0.75 - 1.13)\,\mathrm{GeV}^{-1}$
  $c_3 = -(4.77 - 5.51)\,\mathrm{GeV}^{-1}$
  [Krebs, Gasparyan, Epelbaum, PRC **85**, 054006 (2012)]
- Final N^3LO result:

$$\frac{E}{N}(n_0) = (14.1 - 21.0)\,\mathrm{MeV}$$

Good agreement with other approaches!

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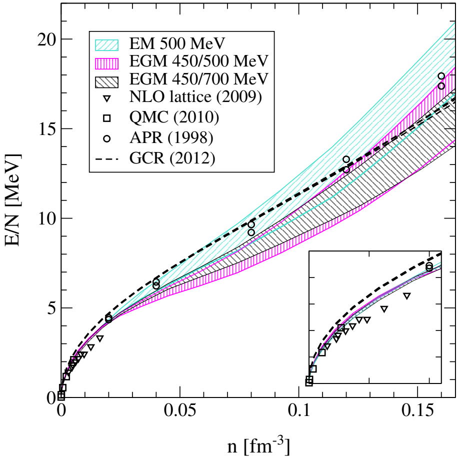

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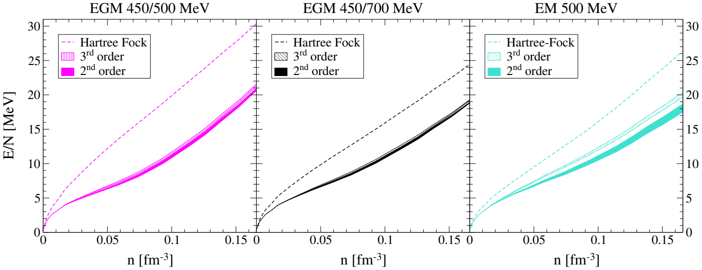

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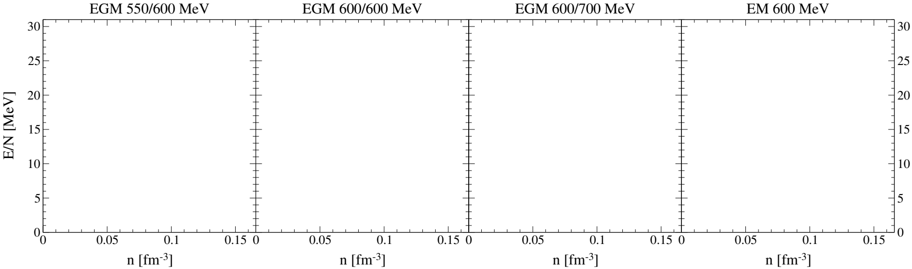

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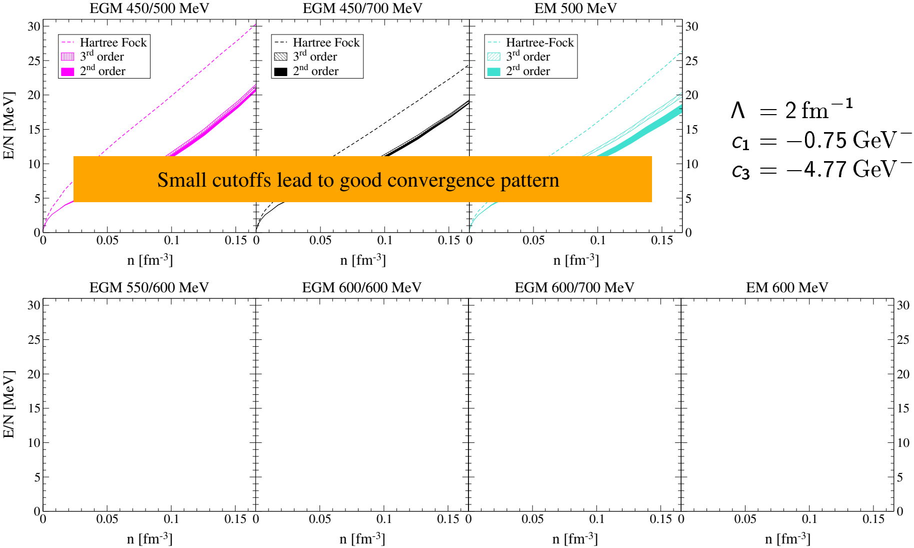

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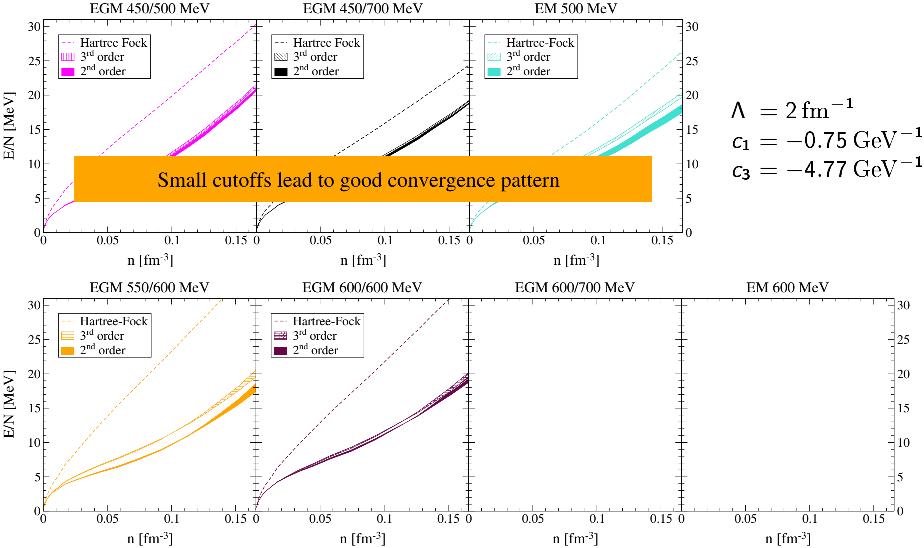

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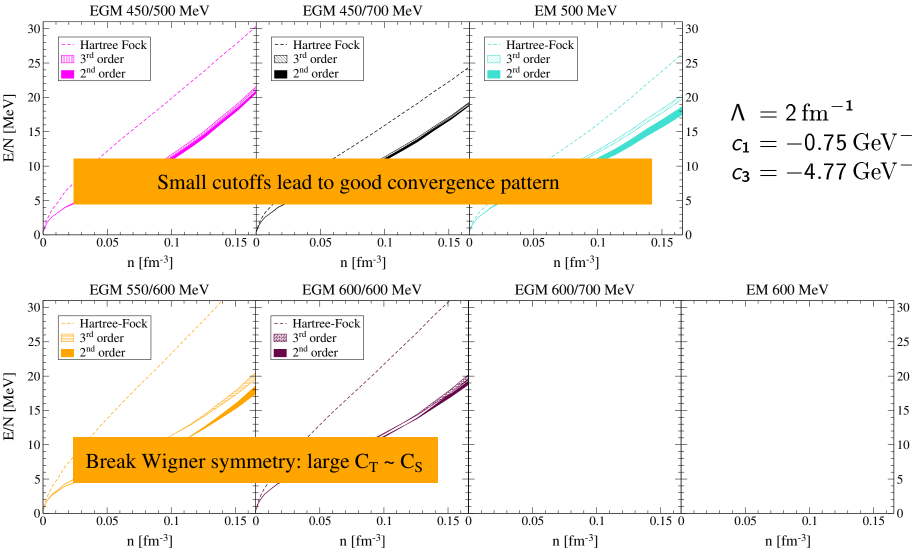

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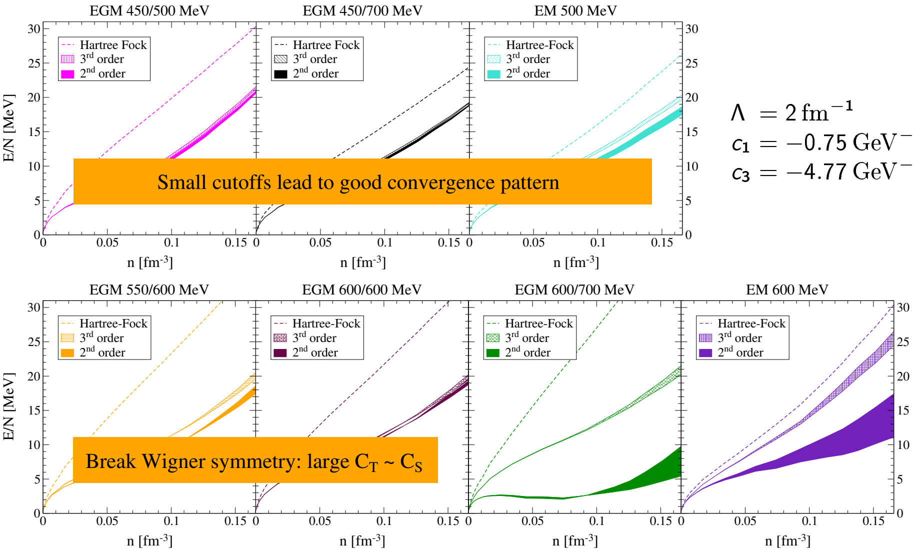

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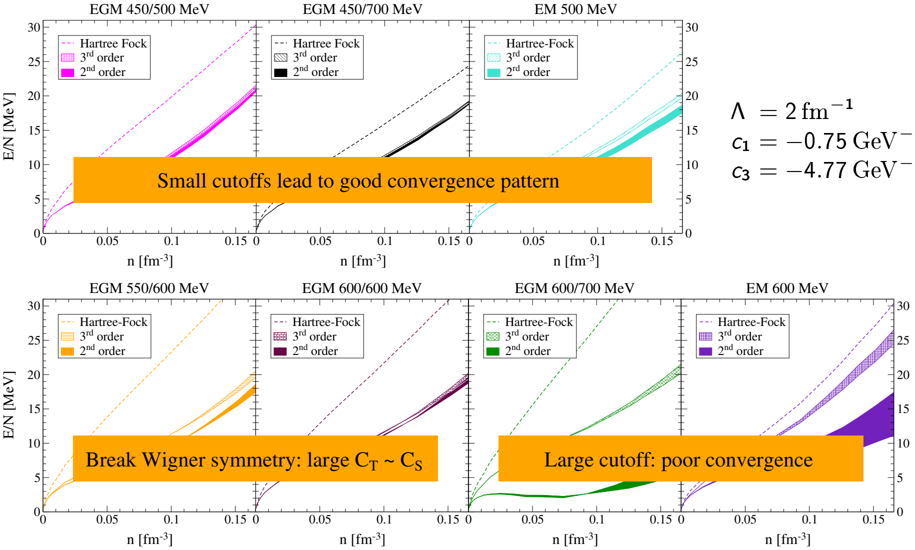

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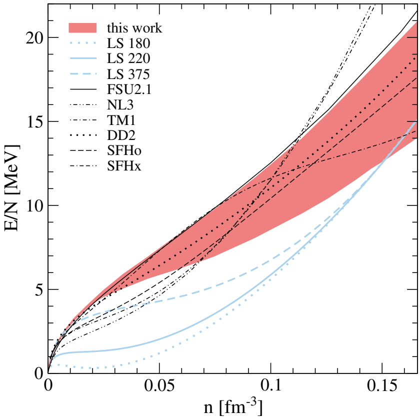
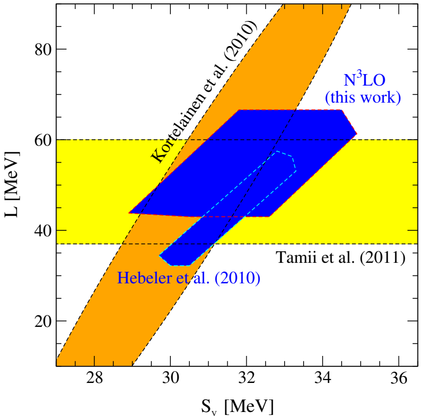
Neutron matter from chiral EFT vs. supernova EOS



[Lines from Hempel; Lattimer; G. Shen]

- Chiral EFT constrains neutron matter energy per particle
- N^3LO many-body forces add more density dependence
- Constrains many model equations of state

Constraining the symmetry energy



Neutron matter band puts constraints on symmetry energy and its density dependence

[Hebeler *et al.*, PRL **105**, 161102 (2010)]

- $S_v = 28.9 - 34.9\,\mathrm{MeV}$
- $L = 43.0 - 66.6\,\mathrm{MeV}$

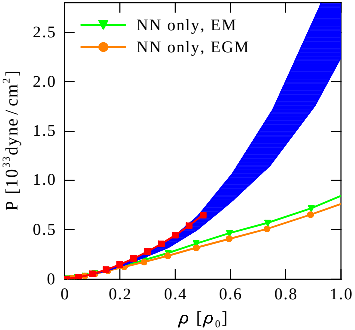
Good agreement with experimental constraints:

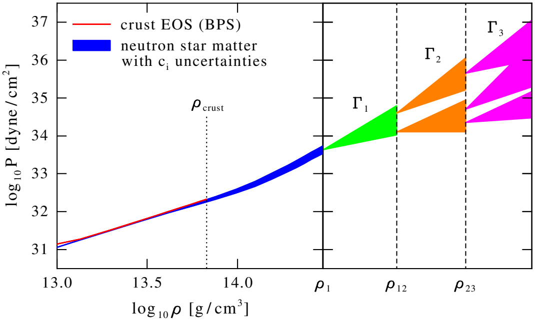
- Dipole polarizability [Tamii *et al.*, PRL **107**, 062502 (2011)]
- Nuclear masses [Kortelainen *et al.*, PRC **82**, 024313 (2010)]

Impact on neutron stars

Equation of state for neutron-star matter: extend results to small $Y_{e,p}$

[Hebeler et al., PRL **105**, 161102 (2010) and *in preparation*]





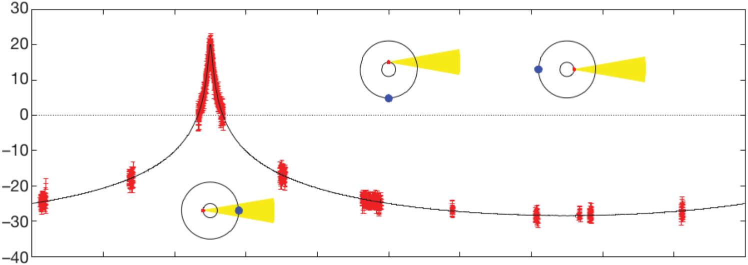
Agrees with standard crust EOS after inclusion of many-body forces

Extend to higher densities using polytropic expansion

Impact on neutron stars

A two-solar-mass neutron star measured using Shapiro delay

P. B. Demorest[1], T. Pennucci[2], S. M. Ransom[1], M. S. E. Roberts[3] & J. W. T. Hessels[4,5]

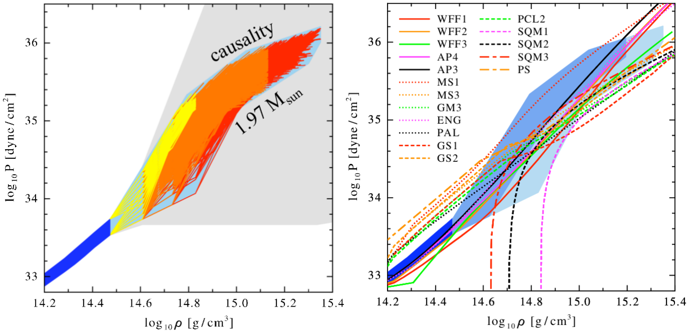


Heaviest neutron star: $M = 1.97 \pm 0.04 M_{\odot}$

[Nature **467**, 1081 (2010)]

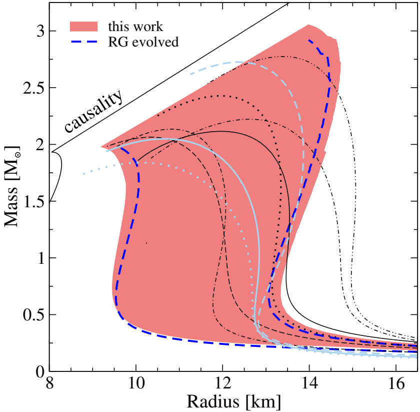
Impact on neutron stars

Constrain resulting EOS with causality and heaviest observed neutron star



- Chiral EFT interactions provide strong constraints for EOS
- Rule out many model equations of state

Impact on neutron stars



Radius for $M = 1.4 M_\odot$ neutron star:

- $R = 9.7 - 13.9\,\mathrm{km}$

Maximal supported mass:

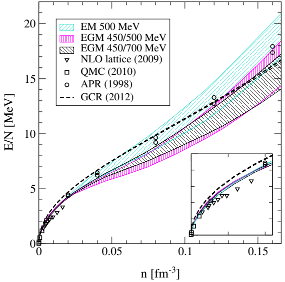
- $M_{\mathrm{max}} = 3.05 M_\odot$ $(14\,\mathrm{km})$

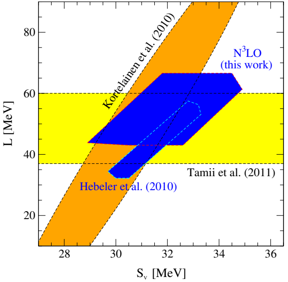
Uncertainties from many-body forces and polytropic expansion

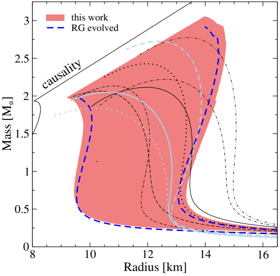
[TK, Tews, Hebeler, Schwenk *arXiv:1301.xxxx*]

Summary

- First consistent neutron matter calculation at N^{3}LO
- Neutron matter energy per particle at n_0: $14.1 - 21.0\,\mathrm{MeV}$
- Symmetry energy: $S_v = 28.9 - 34.9\,\mathrm{MeV}$, $L = 43.0 - 66.6\,\mathrm{MeV}$
- Neutron stars: $1.4 M_\odot$ neutron star $\rightarrow R = 9.7 - 13.9\,\mathrm{km}$







Thanks for Your Attention