Nucleon polarizabilities in the Dyson-Schwinger approach

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Hirschegg 2014: Hadrons from Quarks and Gluons
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Introduction

Goal: compute **nucleon's Compton scattering amplitude**
(and other things) from **quark-gluon substructure in QCD.**

- **Handbag** vs. nucleon (s- and u-channel) and meson (t-channel) **resonances?**
- **Quark core** vs. **pion cloud?**
- **Electromagnetic gauge invariance** at the quark-gluon level?
- **Tensor decomposition** for (on- and offshell) fermion two-photon vertex?

QCD's Green functions $\leftrightarrow$ "Dyson-Schwinger approach":
Nonperturbative, covariant, low and high energies, light and heavy quarks. But: **truncations!**

- **Baryon spectroscopy** from three-body Faddeev equation
  GE, Alkofer, Krassnigg, Nicmorus, PRL 104 (2010)
- **Elastic & transition form factors** for N and Δ
  GE, PRD 84 (2011); GE, Fischer, EPJ A48 (2012); GE, Nicmorus, PRD 85 (2012); Sanchis-Alepuz et al., PRD 87 (2013), ...
- **Tetraquark** interpretation for σ meson
  Heupel, GE, Fischer, PLB 718 (2012)
- **Compton scattering**
  GE, Fischer, PRD 85 (2012) & PRD 87 (2013)

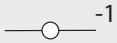
Dyson-Schwinger equations

QCD Lagrangian:
quarks, gluons (+ ghosts)

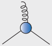
$$\mathcal{L} = \bar{\psi}(x)\,(i\partial\!\!\!/ + g A\!\!\!/ - M)\,\psi(x) - \frac{1}{4}\,F^a_{\mu\nu}\,F^{\mu\nu}_a$$

QCD & hadron properties are encoded in **QCD's Green functions.**
Their quantum equations of motion are the **Dyson-Schwinger equations (DSEs):**

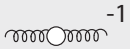
- **Quark propagator:**



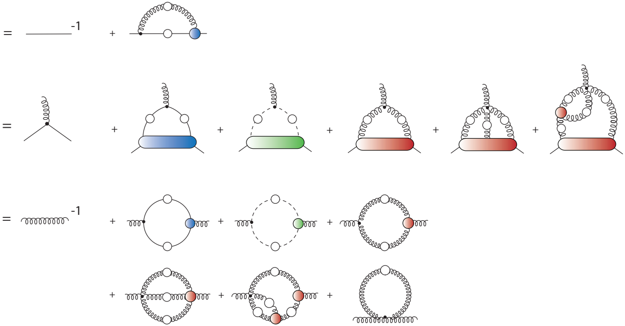
- **Quark-gluon vertex:**



- **Gluon propagator:**



- **Gluon self-interactions, ghosts, . . .**



Dyson-Schwinger equations

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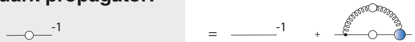

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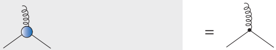

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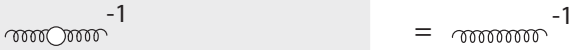


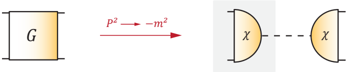
- **Gluon self-interactions, ghosts, . . .**

- **Truncation** ⇒ closed system, solveable.
  Ansätze for Green functions that are **not** solved (based on pQCD, lattice, FRG, ...)

- **Applications:**
  Origin of confinement,
  QCD phase diagram,
  Hadron physics

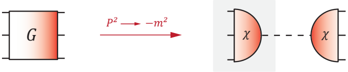
Hadrons: poles in Green functions

- **Quark four-point function:**
  $\langle 0|\mathrm{T}\,\psi(x_1)\,\bar{\psi}(x_2)\,\psi(x_3)\,\bar{\psi}(x_4)|0\rangle$



Bethe-Salpeter WF:
$\langle 0|\,T\,\psi(x_1)\,\bar{\psi}(x_2)\,|H\rangle$

- **Quark six-point function:**



Faddeev WF

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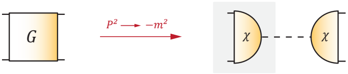

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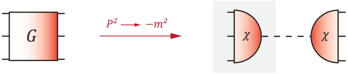
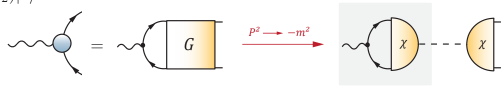


Faddeev WF

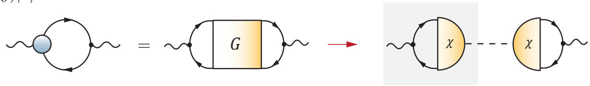
- **Quark-antiquark vertices:** (Currents: $J^\mu = \bar{\psi}\,\Gamma^\mu\psi$)
  $\langle 0|\mathrm{T}\,J^\mu(x)\,\psi(x_1)\,\bar{\psi}(x_2)|0\rangle$



Decay constant:
$\langle 0|\,J^\mu\,|H\rangle$

Quark-photon vertex
has ρ-meson poles:
'vector-meson dominance'

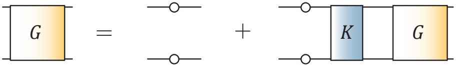
- **Current correlators:**
  $\langle 0|\mathrm{T}\,J^\mu(x)\,J^\nu(y)|0\rangle$



(→ Lattice QCD)

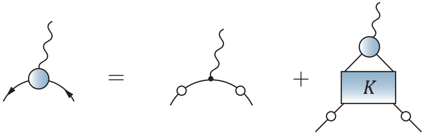
Bethe-Salpeter equations

- Inhomogeneous BSE
  for **quark four-point function:**



- Homogeneous BSE
  for **bound-state wave function:**

- Inhomogeneous BSE
  for **quark-antiquark vertices:**



Analogy: geometric series

$$f(x) = 1 + x f(x) \quad \Rightarrow \quad f(x) = \tfrac{1}{1-x}$$

$$|x| < 1 \quad \Rightarrow \quad f(x) = 1 + x + x^2 + \dots$$

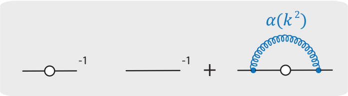
What's the kernel K?

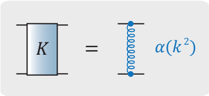
Related to Green functions
via **symmetries:** CVC, PCAC
$\Rightarrow$ vector, axialvector WTIs

Relate **K** with quark propagator
and quark-gluon vertex

Structure of the kernel

Rainbow-ladder: tree-level vertex + effective coupling





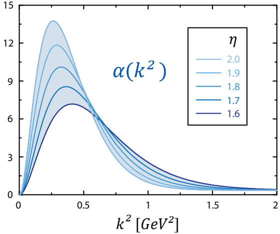
√ **DCSB, CVC, PCAC**

⇒ mass generation

⇒ Goldstone theorem, massless pion in χL

⇒ em. current conservation

⇒ Goldberger-Treiman



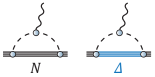
Ansatz for effective coupling:

Maris, Roberts, Tandy, PRC 56 (1997), PRC 60 (1999)

$$\alpha(k^2) = \alpha_{\text{IR}}\left(k^2/\Lambda^2, \eta\right) + \alpha_{\text{UV}}(k^2)$$

Adjust infrared scale Λ to physical observable, keep width η as parameter

No pion cloud, no flavor dependence, no $U_A(1)$ anomaly, no dynamical decay widths



, etc.

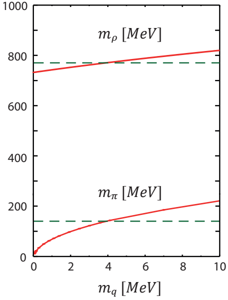
Pion cloud: need infinite summation of t-channel gluons

Mesons

- **Pseudoscalar & vector mesons:** rainbow-ladder is good. Masses, form factors, decays, $\pi\pi$ scattering lengths, PDFs

  Maris, Roberts, Tandy, PRC 56 (1997), PRC 60 (1999); Bashir et al., Commun. Theor. Phys. 58 (2012)

  Pion is Goldstone boson, satisfies GMOR: $m_\pi^2 \sim m_q$

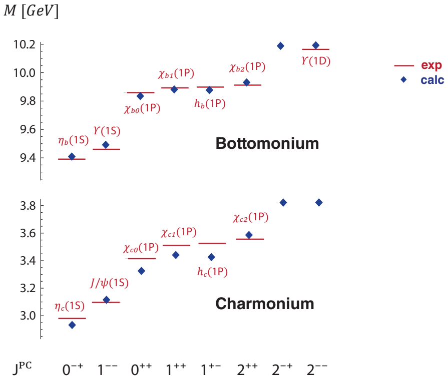


Fischer, J Phys G 32 (2006)

- Need to go **beyond rainbow-ladder** for excited, scalar, axialvector mesons, η-η', etc.

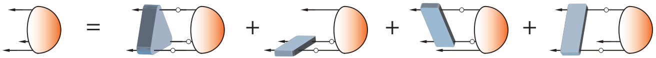
  Fischer, Williams & Chang, Roberts, PRL 103 (2009)
  Alkofer et al., EPJ A38 (2008), Bhagwat et al., PRC 76 (2007)

- **Heavy mesons** Blank, Krassnigg, PRD 84 (2011)



Baryons

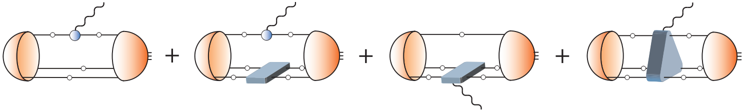
Covariant Faddeev equation: kernel contains 2PI and 3PI parts



Current matrix element: $\langle H|J^\mu|H\rangle = \bar{\chi}\left(G^{-1}\right)^\mu \chi$

'Gauging of equations':
Kvinikhidze, Blankleider, PRC 60 (1999)
Oettel, Pichowsky, von Smekal, EPJ A 8 (2000)

- Impulse approximation + gauged kernel $\left(G^{-1}\right)^\mu = (G_0^{-1})^\mu - K^\mu$



Truncation:

- **Quark-quark correlations** only (dominant structure in baryons?)
- Rainbow-ladder **gluon exchange**
- But **full Poincaré-covariant structure** of Faddeev amplitude retained

$\longrightarrow$ Same input as for mesons, quark from DSE, no additional parameters!

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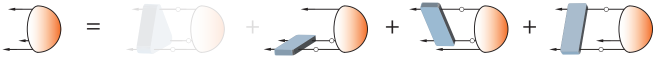

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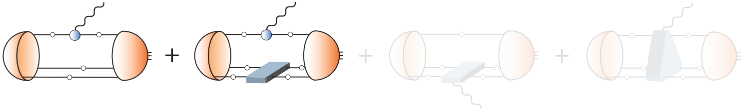

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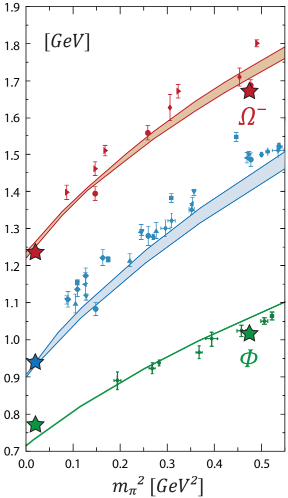
Baryon masses

- Good agreement with experiment & lattice. Pion mass is also calculated.
- Same kernel as for mesons, scale set by f_π. Full covariant wave functions, no further parameters or approximations.
- Masses not sensitive to effective interaction.
- **Diquark clustering in baryons:** similar results in quark-diquark approach
  Oettel, Alkofer, von Smekal, EPJ A8 (2000)
  GE, Cloet, Alkofer, Krassnigg, Roberts, PRC 79 (2009)
- **Excited baryons** (e.g. Roper): also quark-diquark structure?
  Chen, Chang, Roberts, Wan, Wilson, FBS 53 (2012)
- Role of **pion cloud:** see talk by C. Fischer
- Role of **three-gluon vertex?**
  Williams, Vujinovic, GE, Alkofer, in preparation



Delta mass:
Sanchis-Alepuz et al., PRD 84 (2011)

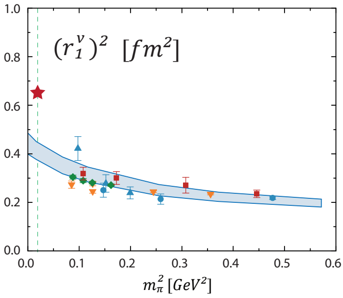
Nucleon mass:
GE, Alkofer, Krassnigg, Nicmorus, PRL 104 (2010);
GE, PRD 84 (2011)

ρ–meson mass:
Maris & Tandy, PRC 60 (1999)

Electromagnetic form factors

Nucleon charge radii:

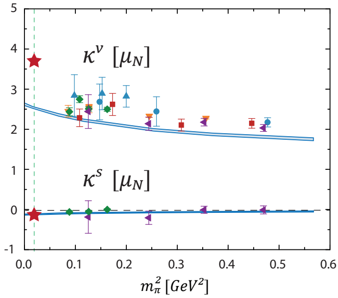
isovector (p-n) Dirac (F1) radius



- **Pion-cloud effects** missing in chiral region (⇒ divergence!), agreement with lattice at larger quark masses.

Nucleon magnetic moments:

isovector (p-n), isoscalar (p+n)



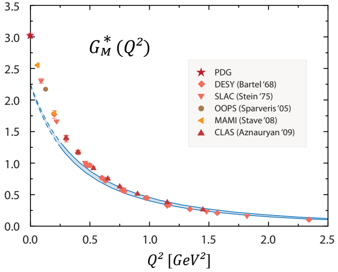
- **But:** pion-cloud **cancels** in κ^s ⇔ **quark core**

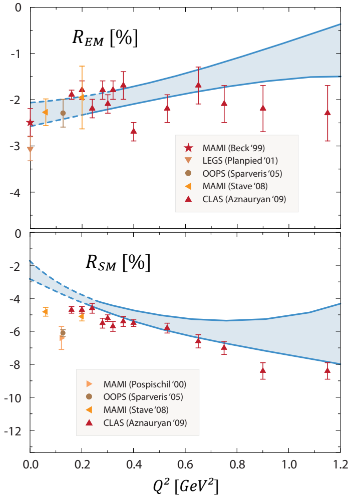
Exp: $\kappa^s = -0.12$

Calc: $\kappa^s = -0.12(1)$ **!!**

GE, PRD 84 (2011)

Nucleon-Δ-γ transition

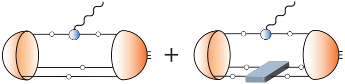


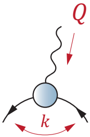


- **Magnetic dipole transition (G_M^*)** dominant: quark spin flip (s wave). "Core + 25% pion cloud"
- **Electric & Coulomb quadrupole transitions** small & negative, encode deformation.

Ratios reproduced without pion cloud:
OAM from **relativistic p waves** in the quark core!
Eichmann & Nicmorus, PRD 85 (2012)

Quark-photon vertex

Current matrix element: $\langle H|J^\mu|H\rangle =$ 



Vector WTI $Q^\mu\,\Gamma^\mu(k,Q) = S^{-1}(k_+) - S^{-1}(k_-)$
determines vertex up to transverse parts:

$$\Gamma^\mu(k,Q) = \Gamma^\mu_{\text{BC}}(k,Q) + \Gamma^\mu_{\text{T}}(k,Q)$$

- **Ball-Chiu vertex**, completely specified by dressed fermion propagator: Ball, Chiu, PRD 22 (1980)

$$\Gamma^\mu_{\text{BC}}(k,Q) = i\gamma^\mu\,\Sigma_A + 2k^\mu(i\not{k}\,\Delta_A + \Delta_B)$$

$$\Sigma_A := \frac{A(k_+^2)+A(k_-^2)}{2}, \quad \Delta_A := \frac{A(k_+^2)-A(k_-^2)}{k_+^2-k_-^2}, \quad \Delta_B := \frac{B(k_+^2)-B(k_-^2)}{k_+^2-k_-^2}$$

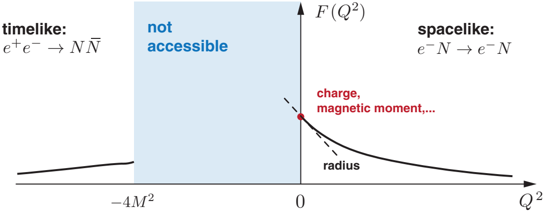
- **Transverse part:** free of kinematic singularities, tensor structures $\sim Q, Q^2, Q^3$, contains meson poles
Kizilersu, Reenders, Pennington, PRD 92 (1995); GE, Fischer, PRD 87 (2013)

$$t^{\mu\nu}_{ab} := a\cdot b\,\delta^{\mu\nu} - b^\mu a^\nu$$

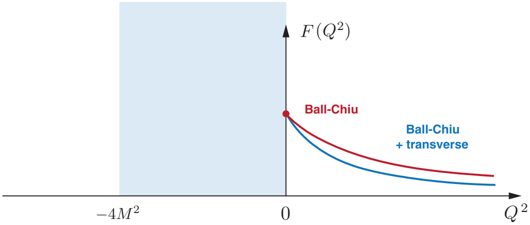
Dominant	$\tau_1^\mu = t^{\mu\nu}_{QQ}\,\gamma^\nu\,,$	$\tau_5^\mu = t^{\mu\nu}_{QQ}\,ik^\nu\,,$
	$\tau_2^\mu = t^{\mu\nu}_{QQ}\,k\cdot Q\,\frac{i}{2}[\gamma^\nu,\not{k}]\,,$	$\tau_6^\mu = t^{\mu\nu}_{QQ}\,k^\nu\not{k}\,,$
Anomalous magnetic moment	$\tau_3^\mu = \frac{i}{2}\,[\gamma^\mu,\not{Q}]\,,$	$\tau_7^\mu = t^{\mu\nu}_{Qk}\,k\cdot Q\,\gamma^\nu\,,$ Curtis, Pennington, PRD 42 (1990)
	$\tau_4^\mu = \frac{1}{6}\,[\gamma^\mu,\not{k},\not{Q}]\,,$	$\tau_8^\mu = t^{\mu\nu}_{Qk}\,\frac{i}{2}\,[\gamma^\nu,\not{k}]\,.$

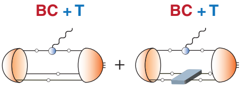
Quark-photon vertex

Structure of quark-photon vertex is reflected in form factors.
Experimentally (sketch):



Calculated:
(Sketch)





- Ball-Chiu part is dominant **(em. gauge invariance):** charge, magnetic moments
- Transverse part changes slope and charge radii. No pion cloud in RL ⇒ timelike ρ-**meson poles**

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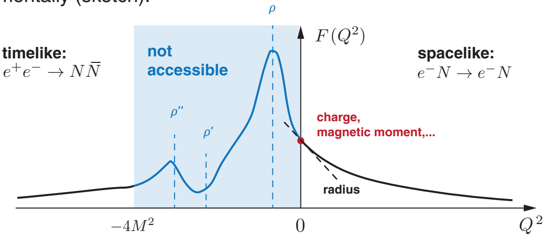

Calculated:
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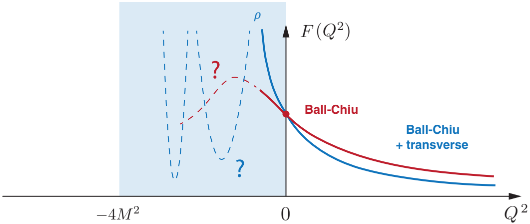

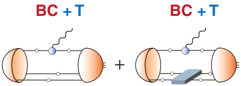

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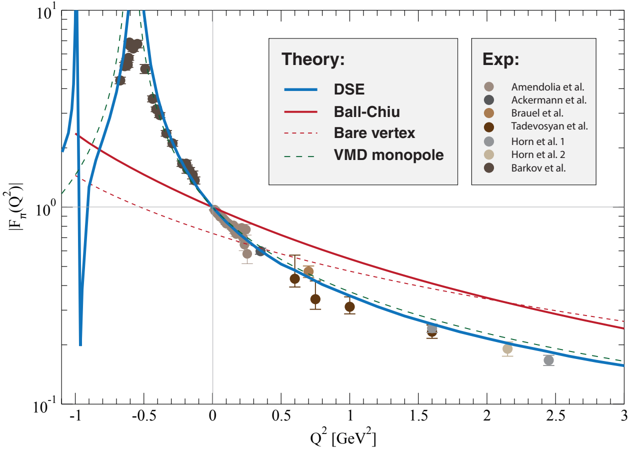
Pion form factor



Spacelike and timelike region:

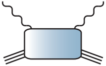
A. Krassnigg (Schladming 2010)
extension of Maris & Tandy,
Nucl. Phys. Proc. Suppl. 161 (2006)

Include **pion cloud:**

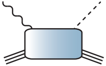
Kubrak et al., in preparation
(see talk by C. Fischer)

Hadron scattering

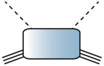
Can we extend this to **four-body scattering** processes?
GE, Fischer, PRD 85 (2012)



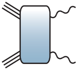
Compton scattering, DVCS, 2γ physics



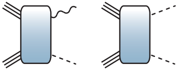
Meson photo- and electroproduction



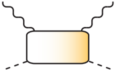
Nucleon-pion scattering



$\bar{p}p \to \gamma\gamma^*$
annihilation



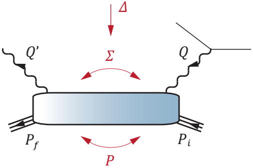
Meson production



Pion Compton scattering

⇒ Nonperturbative description of hadron-photon and hadron-meson scattering

Nucleon Compton scattering

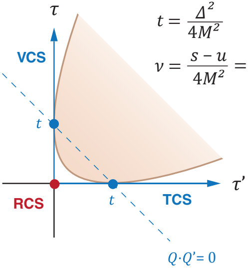


$$\tau = \frac{Q^2}{4M^2}$$

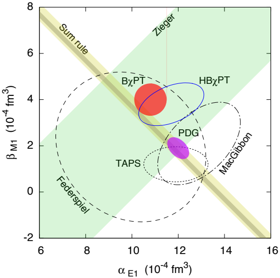
$$\tau' = \frac{Q'^2}{4M^2}$$

$$t = \frac{\Delta^2}{4M^2}$$

$$\nu = \frac{s-u}{4M^2} = -\frac{\Sigma\cdot P}{M^2}$$



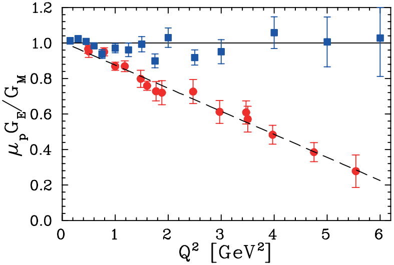
- **RCS, VCS:** nucleon polarizabilities



Krupina & Pascalutsa, PRL 110 (2013)

- **DVCS:** handbag dominance, GPDs
- **Forward limit:** structure functions in DIS
- **Timelike region:** $p\bar{p}$ annhihilation at PANDA
- **Spacelike region:** two-photon corrections to nucleon form factors, proton radius puzzle?

Two-photon corrections



Arrington et al., Prog. Part. Nucl. Phys. 66 (2011)

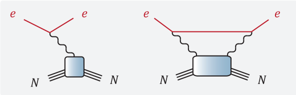
- **Proton form factor ratio:**

  Rosenbluth extraction suggested $G_E/G_M = \text{const.}$, in agreement with perturbative scaling

  Polarization data from JLAB showed falloff in G_E/G_M with possible **zero crossing**

  Modified pQCD predictions: OAM

  Difference likely due to two-photon corrections
  Blunden, Melnitchouk, Tjon & Guichon, Vanderhaeghen, PRL 91 (2003)



- **Proton radius puzzle:**

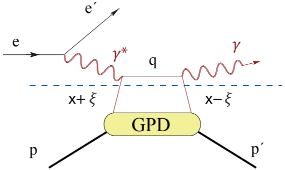
  Proton radius extracted from Lamb shift in μH 4% smaller than that from eH, would need additional ΔE ~ 300 μeV to agree Pohl et al., Nature 466, 213 (2010)

  Can two-photon offshell corrections explain discrepancy?
  Miller, Thomas, Carroll, Rafelski; Carlson, Vanderhaeghen; Birse, McGovern; ...

Handbag dominance

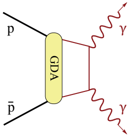
- **Handbag dominance in DVCS**

  large Q^2 & s, small t: factorization,
  extract **GPDs** from handbag diagram



- **p$\bar{\text{p}}$ annihilation at PANDA@FAIR**

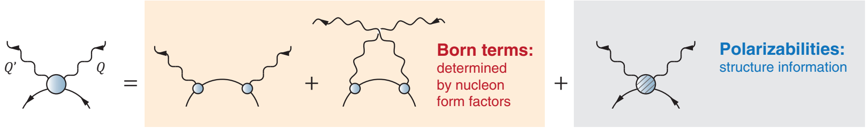
  Are the concepts developed for lepton scattering
  (factorization, handbag dominance, GPDs) applicable?



PANDA Physics Book

- **Is it possible to calculate these processes directly within nonperturbative QCD?** Wishlist:
  - Em. gauge invariance
  - Crossing symmetry
  - Poincare invariance
  - Recover parton picture (handbag, ...)
  - Recover hadronic structure (s, u, t-channel resonances)

Compton scattering



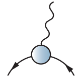
- All direct measurements in kinematic limits
  (RCS, VCS, forward limit).

- Em. gauge invariance $\Rightarrow$ Compton amplitude is **fully transverse.**
  Analyticity constrains 1PI part in these limits (low-energy theorem).

- Polarizabilities = coefficients of tensor structures
  that vanish like $\sim Q^\mu Q'^\nu,\ Q^\mu Q^\nu,\ Q'^\mu Q'^\nu, \dots$

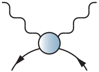
- Need tensor basis free of kinematic singularities (18 elements). Complicated...
  Bardeen, Tung, Phys. Rev. 173 (1968)
  Perrottet, Lett. Nuovo Cim. 7 (1973)
  Tarrach, Nuovo Cim. 28 A (1975)
  Drechsel et al., PRC 57 (1998)
  L'vov et al., PRC 64 (2001)
  Gorchtein, PRC 81 (2010)
  Belitsky, Mueller, Ji, 1212.6674 [hep-ph]
  ...

Tensor basis?



$$\tau_1 = \gamma^\mu$$
$$\tau_2 = \sigma^{\mu\nu}\, Q^\nu$$

Transversality, analyticity and **Bose symmetry** makes the construction extremely difficult...



Tarrach,
Nuovo Cim. 28 (1975)

$$\begin{aligned}
T_1 &= g_{\nu\mu}\,, & T_{18} &= (P_\nu k_\mu - P_\mu k'_\nu)\,\hat{K}\,,\\
T_2 &= k_\nu k'_\mu\,, & T_{19} &= (P_\nu k'_\mu + P_\mu k_\nu)\,\hat{K}\,,\\
T_3 &= k'_\nu k_\mu\,, & T_{20} &= (P_\nu k'_\mu - P_\mu k_\nu)\,\hat{K}\,,\\
T_4 &= k_\nu k_\mu + k'_\nu k'_\mu\,, & T_{21} &= P_\nu\gamma_\mu + P_\mu\gamma_\nu\,,\\
T_5 &= k_\nu k_\mu - k'_\nu k'_\mu\,, & T_{22} &= P_\nu\gamma_\mu - P_\mu\gamma_\nu\,,\\
T_6 &= P_\nu P_\mu\,, & T_{23} &= k_\nu\,\gamma_\mu + k'_\mu\gamma_\nu\,,\\
T_7 &= P_\nu k_\mu + P_\mu k'_\nu\,, & T_{24} &= k_\nu\,\gamma_\mu - k'_\mu\gamma_\nu\,,\\
T_8 &= P_\nu k_\mu - P_\mu k'_\nu\,, & T_{25} &= k'_\nu\,\gamma_\mu + k_\mu\gamma_\nu\,,\\
T_9 &= P_\nu k'_\mu + P_\mu k_\nu\,, & T_{26} &= k'_\nu\,\gamma_\mu - k_\mu\gamma_\nu\,,\\
T_{10} &= P_\nu k'_\mu - P_\mu k_\nu\,, & T_{27} &= (P_\nu\gamma_\mu + P_\mu\gamma_\nu)\,\hat{K} - \hat{K}(P_\nu\gamma_\mu + P_\mu\gamma_\nu)\,,\\
T_{11} &= g_{\nu\mu}\hat{K}\,, & T_{28} &= (P_\nu\gamma_\mu - P_\mu\gamma_\nu)\,\hat{K} - \hat{K}(P_\nu\gamma_\mu - P_\mu\gamma_\nu)\,,\\
T_{12} &= k_\nu k'_\mu\hat{K}\,, & T_{29} &= (k_\nu\,\gamma_\mu + k'_\mu\,\gamma_\nu)\,\hat{K} - \hat{K}(k_\nu\,\gamma_\mu + k'_\mu\,\gamma_\nu)\,,\\
T_{13} &= k'_\nu k_\mu\hat{K}, & T_{30} &= (k_\nu\,\gamma_\mu - k'_\mu\,\gamma_\nu)\,\hat{K} - \hat{K}(k_\nu\,\gamma_\mu - k'_\mu\,\gamma_\nu)\,,\\
T_{14} &= (k_\nu k_\mu + k'_\nu k'_\mu)\,\hat{K}\,, & T_{31} &= (k'_\nu\,\gamma_\mu + k_\mu\,\gamma_\nu)\,\hat{K} - \hat{K}(k'_\nu\,\gamma_\mu + k_\mu\,\gamma_\nu)\,,\\
T_{15} &= (k_\nu k_\mu - k'_\nu k'_\mu)\,\hat{K}\,, & T_{32} &= (k'_\nu\,\gamma_\mu - k_\mu\,\gamma_\nu)\,\hat{K} - \hat{K}(k'_\nu\,\gamma_\mu - k_\mu\,\gamma_\nu)\,,\\
T_{16} &= P_\nu P_\mu\hat{K}\,, & T_{33} &= \gamma_\nu\,\gamma_\mu - \gamma_\mu\,\gamma_\nu\,,\\
T_{17} &= (P_\nu k_\mu + P_\mu k'_\nu)\,\hat{K}\,, & T_{34} &= (\gamma_\nu\,\gamma_\mu - \gamma_\mu\,\gamma_\nu)\,\hat{K} + \hat{K}(\gamma_\nu\,\gamma_\mu - \gamma_\mu\,\gamma_\nu)\,,
\end{aligned}$$

$$\begin{aligned}
\tau_1 &= k\cdot k'\,T_1 - T_3\,,\\
\tau_2 &= k^2k'^2T_1 + k\cdot k'\,T_2 - \frac{k^2+k'^2}{2}\,T_4 + \frac{k^2-k'^2}{2}\,T_5\,,\\
\tau_3 &= (P\cdot K)^2T_1 + k\cdot k'\,T_6 - P\cdot KT_7\,,\\
\tau_4 &= P\cdot K(k^2+k'^2)T_1 - P\cdot KT_4 - \frac{k^2+k'^2}{2}\,T_7 + \frac{k^2-k'^2}{2}\,T_8 + k\cdot k'\,T_9\,,\\
\tau_5 &= -P\cdot K(k^2-k'^2)T_1 + P\cdot KT_5 + \frac{k^2-k'^2}{2}\,T_7 - \frac{k^2+k'^2}{2}\,T_8 + k\cdot k'\,T_{10}\,,\\
\tau_6 &= P\cdot KT_2 - \frac{k^2+k'^2}{4}\,T_9 - \frac{k^2-k'^2}{4}\,T_{10} - MT_{12} + M\frac{k^2+k'^2}{4}\,T_{23} -\\
&\qquad - M\frac{k^2-k'^2}{4}\,T_{24} + \frac{k^2-k'^2}{8}\,T_{29} - \frac{k^2+k'^2}{8}\,T_{30} - \frac{k^2k'^2}{4}\,T_{33}\,,\\
\tau_7 &= 8T_{16} - 4P\cdot KT_{21} + P\cdot KT_{34}\,,\\
\tau_8 &= T_{19} + \frac{k^2-k'^2}{2}\,T_{22} - P\cdot KT_{23} + \frac{k^2+k'^2}{8}\,T_{34}\,,\\
\tau_9 &= T_{20} - \frac{k^2+k'^2}{2}\,T_{22} + P\cdot KT_{24} - \frac{k^2-k'^2}{8}\,T_{34}\,,\\
\tau_{10} &= -8k\cdot k'\,T_6 + 4P\cdot KT_7 + 4Mk\cdot k'\,T_{21} - 4MP\cdot KT_{25} -\\
&\qquad - 2P\cdot KT_{32} - 2k\cdot k'\,P\cdot KT_{33} + Mk\cdot k'\,T_{34}\,,\\
\tau_{11} &= T_{18} - k\cdot k'\,T_{22} + P\cdot KT_{26}\,,\\
\tau_{12} &= P\cdot KT_4 - \frac{k^2-k'^2}{2}\,T_8 - k\cdot k'\,T_9 - MT_{14} + Mk\cdot k'\,T_{23} -\\
&\qquad - M\frac{k^2-k'^2}{2}\,T_{26} - \frac{k^2+k'^2}{4}\,T_{32} - k\cdot k'\,\frac{k^2+k'^2}{4}\,T_{33}\,,\\
\tau_{13} &= P\cdot KT_5 - \frac{k^2+k'^2}{2}\,T_8 + k\cdot k'\,T_{10} - MT_{15} + Mk\cdot k'\,T_{24} -\\
&\qquad - M\frac{k^2+k'^2}{2}\,T_{26} - \frac{k^2-k'^2}{4}\,T_{32} - k\cdot k'\,\frac{k^2-k'^2}{4}\,T_{33}\,,
\end{aligned}$$

$$\begin{aligned}
\tau_{14} &= 2P\cdot KT_8 - 2Mk\cdot k'\,T_{22} + 2MP\cdot KT_{26} - k\cdot k'\,T_{27} + P\cdot KT_{31}\,,\\
\tau_{15} &= -(k^2-k'^2)T_7 + (k^2+k'^2)T_8 - 2k\cdot k'\,T_{10} - 2Mk\cdot k'\,T_{24} +\\
&\quad + M(k^2-k'^2)T_{25} + M(k^2+k'^2)T_{26} - k\cdot k'\,T_{29} +\\
&\qquad + \frac{k^2+k'^2}{2}\,T_{31} + \frac{k^2-k'^2}{2}\,T_{32}\,,\\
\tau_{16} &= -(k^2+k'^2)T_7 + (k^2-k'^2)T_8 + 2k\cdot k'\,T_9 - 2Mk\cdot k'\,T_{23} +\\
&\quad + M(k^2+k'^2)T_{25} + M(k^2-k'^2)T_{26} - k\cdot k'\,T_{30} +\\
&\qquad + \frac{k^2-k'^2}{2}\,T_{31} + \frac{k^2+k'^2}{2}\,T_{32}\,,\\
\tau_{17} &= -4P\cdot KT_1 + 2T_7 + 4MT_{11} - 2MT_{25} + T_{32} + k\cdot k'\,T_{33}\,,\\
\tau_{18} &= 4T_{17} - 4P\cdot KT_{25} + k\cdot k'\,T_{34}\,.\\
\tau_{19} &= \frac{1}{k\cdot k'}\,[2(P\cdot K)^2\tau_2 + 2k^2k'^2\tau_3 - P\cdot K(k^2+k'^2)\tau_4 - P\cdot K(k^2-k'^2)\tau_5] =\\
&= 2(P\cdot K)^2T_2 + 2k^2k'^2T_6 - P\cdot K(k^2+k'^2)T_9 - P\cdot K(k^2-k'^2)T_{10}\,,\\
\tau_{20} &= \frac{1}{4k\cdot k'}\,[(k^2-k'^2)\tau_{10} - 2(k^2+k'^2)\tau_{14} + 4P\cdot K\tau_{15}] =\\
&= -2(k^2-k'^2)T_6 - 2P\cdot KT_{10} + M(k^2-k'^2)T_{21} + M(k^2+k'^2)T_{22} -\\
&\quad - 2MP\cdot KT_{24} + \frac{k^2+k'^2}{2}\,T_{27} - P\cdot KT_{29} -\\
&\qquad - P\cdot K\frac{k^2-k'^2}{2}\,T_{33} + M\frac{k^2-k'^2}{4}\,T_{34}\,,\\
\tau_{21} &= \frac{1}{4k\cdot k'}\,[(k^2+k'^2)\tau_{10} - 2(k^2-k'^2)\tau_{14} + 4P\cdot K\tau_{16}] =\\
&= -2(k^2+k'^2)T_6 + 2P\cdot KT_9 + M(k^2+k'^2)T_{21} + M(k^2-k'^2)T_{22} -\\
&\quad - 2MP\cdot KT_{23} + \frac{k^2-k'^2}{2}\,T_{27} - P\cdot KT_{30} -\\
&\qquad - P\cdot K\frac{k^2+k'^2}{2}\,T_{33} + M\frac{k^2+k'^2}{4}\,T_{34}\,.
\end{aligned}$$

Transverse tensor basis for $\Gamma^{\mu\nu}(p,Q,Q')$

- Generalize transverse projectors: $t^{\mu\nu}_{ab} := a\cdot b\,\delta^{\mu\nu} - b^\mu a^\nu$ $\quad a,b\in\{p,Q,Q'\}$
  $\varepsilon^{\mu\nu}_{ab} := \gamma_5\,\varepsilon^{\mu\nu\alpha\beta} a^\alpha b^\beta$ (exhausts all possibilities)

- Apply Bose-(anti-)symmetric combinations

$$\mathsf{E}^{\mu\alpha,\beta\nu}_{\pm}(a,b) := \tfrac{1}{2}\left(\varepsilon^{\mu\alpha}_{Q'a'}\,\varepsilon^{\beta\nu}_{bQ} \pm \varepsilon^{\mu\alpha}_{Q'b'}\,\varepsilon^{\beta\nu}_{aQ}\right)$$

$$\mathsf{F}^{\mu\alpha,\beta\nu}_{\pm}(a,b) := \tfrac{1}{2}\left(t^{\mu\alpha}_{Q'a'}\,t^{\beta\nu}_{bQ} \pm t^{\mu\alpha}_{Q'b'}\,t^{\beta\nu}_{aQ}\right)$$

$$\mathsf{G}^{\mu\alpha,\beta\nu}_{\pm}(a,b) := \tfrac{1}{2}\left(\varepsilon^{\mu\alpha}_{Q'a'}\,t^{\beta\nu}_{bQ} \pm t^{\mu\alpha}_{Q'b'}\,\varepsilon^{\beta\nu}_{aQ}\right)$$

to structures independent of Q, Q':

	$p^\alpha\gamma^\beta + \gamma^\alpha p^\beta$
$\delta^{\alpha\beta}$	$p^\alpha\gamma^\beta - \gamma^\alpha p^\beta$
$\delta^{\alpha\beta}\,\not{p}$	$[p^\alpha\gamma^\beta + \gamma^\alpha p^\beta, \not{p}]$
$[\gamma^\alpha,\gamma^\beta]$	$[p^\alpha\gamma^\beta - \gamma^\alpha p^\beta, \not{p}]$
$[\gamma^\alpha,\gamma^\beta,\not{p}]$	$p^\alpha p^\beta$
	$p^\alpha p^\beta\,\not{p}$

- obtain
  16 quadratic,
  40 cubic
  16 quartic terms
  $\Rightarrow$ **72 in total** √
- no kinematic singularities √

- **Transverse onshell basis:** GE, Fischer, PRD 87 (2013) & PoS Conf. X (2012)

$\mathsf{E}_+(P,P)$ $(++)$	$\widetilde{\mathsf{E}}_+(P,P)$ $(-+)$	$\mathsf{F}_+(P,Q)$ $(-+)$	$\widetilde{\mathsf{F}}_+(P,Q)$ $(++)$
$\mathsf{F}_+(P,P)$ $(++)$	$\widetilde{\mathsf{F}}_+(P,P)$ $(-+)$	$\mathsf{G}_+(P,Q)$ $(-+)$	$\widetilde{\mathsf{G}}_+(P,Q)$ $(+-)$
$\mathsf{G}_+(P,P)$ $(++)$	$\widetilde{\mathsf{G}}_+(P,P)$ $(--)$	$\mathsf{F}_-(P,Q)$ $(+-)$	$\widetilde{\mathsf{F}}_-(P,Q)$ $(--)$
$\mathsf{G}_-(P,P)$ $(--)$	$\widetilde{\mathsf{G}}_-(P,P)$ $(++)$	$\mathsf{G}_-(P,Q)$ $(+-)$	$\widetilde{\mathsf{G}}_-(P,Q)$ $(-+)$
		$\mathsf{F}_+(Q,Q)$ $(++)$	$\widetilde{\mathsf{F}}_+(Q,Q)$ $(-+)$

VCS (red dashed) · VVCS (blue dash-dotted) · RCS (light blue) · Scalar vertex (grey)

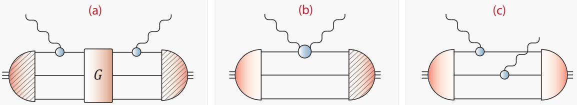
- Simple
- analytic in all limits
- manifest crossing and charge-conjugation symmetry
- scalar & pion pole only in a few Compton form factors
- Tarrach's basis can be cast in a similar form

Compton amplitude at quark level

Baryon's **Compton scattering amplitude,** consistent with Faddeev equation:
GE, Fischer, PRD 85 (2012)

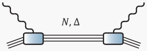
$$\langle H | J^\mu J^\nu | H \rangle = \bar{\chi} \left({G^{-1}}^\mu G \, {G^{-1}}^\nu + {G^{-1}}^\nu G \, {G^{-1}}^\mu - (G^{-1})^{\mu\nu} \right) \chi$$

In rainbow-ladder (+ crossing & permutation):



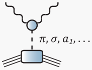
- √ crossing symmetry
- √ em. gauge invariance
- √ perturbative processes included
- √ s, t, u channel poles generated in QCD

- **Born (handbag) diagrams:** G = **1** + T
- all s- and u-channel **nucleon resonances:**



1PI **quark 2-photon vertex:**

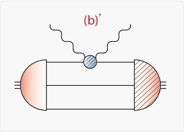
all t-channel **meson poles**



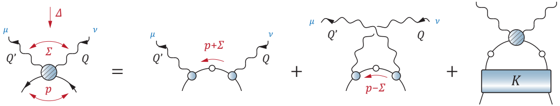
cat's ears diagrams

Compton amplitude at quark level

Collect all (nonperturbative!) **‘handbag’ diagrams**: no nucleon resonances, no cat’s ears

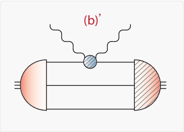


- **not electromagnetically gauge invariant**, but comparable to 1PI ‚structure part‘ at nucleon level?
- reduces to **perturbative handbag** at large photon momenta, but also all **t-channel poles** included! (scalar, pion, ...)
- represented by full **quark Compton vertex**, including Born terms. Satisfies inhomogeneous BSE, solved in RL (128 tensor structures)



Compton amplitude at quark level

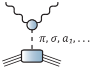
Collect all (nonperturbative!) **'handbag' diagrams**: no nucleon resonances, no cat's ears



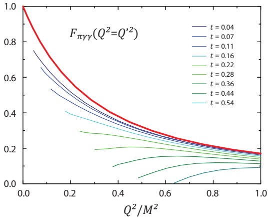
- **not electromagnetically gauge invariant**, but comparable to 1PI ‚structure part' at nucleon level?
- reduces to **perturbative handbag** at large photon momenta, but also all **t-channel poles** included! (scalar, pion, ...)
- represented by full **quark Compton vertex**, including Born terms. Satisfies inhomogeneous BSE, solved in RL (128 tensor structures)

Residues at pion pole recover
$\pi\gamma\gamma$ **transition form factor** √

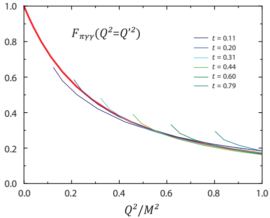
GE & Fischer, PRD 87 (2013)



Rainbow-ladder result:
Maris & Tandy, PRC 65 (2002)



(extracted from quark Compton vertex)



(extracted from nucleon Compton amplitude)

Compton amplitude at quark level

- Quark Compton vertex has **extremely** rich structure:

$$\Gamma^{\mu\nu}(p,Q,Q') = \sum_{i=1}^{72} f_i\,(p^2, Q^2, {Q'}^2, Q\cdot Q', p\cdot Q, p\cdot Q')\;\tau_i^{\mu\nu}(p,Q,Q')$$

- Exploit **em. gauge invariance:** general **offshell quark Compton vertex** can be written as

$$\Gamma^{\mu\nu} = \Gamma^{\mu\nu}_{\mathrm{B}} + \Gamma^{\mu\nu}_{\mathrm{BC}} + \Gamma^{\mu\nu}_{\mathrm{T}} + \Gamma^{\mu\nu}_{\mathrm{TT}}$$

Born — **WTI** — **WTI-T**

- 2-photon equivalent of **Ball-Chiu vertex,** fixed by quark propagator & quark-photon vertex
- **no kinematic singularities**

Transverse

- not constrained by WTI, calculated from BSE
- **no kinematic singularities**
- contains **t-channel poles**
- 72 elements offshell **(18 elements onshell)**

- All these will contribute to Compton form factors (⇒ polarizabilities, structure functions, GPDs, etc.) Dominant contributions?

  ⇒ Born **(pure handbag)?**

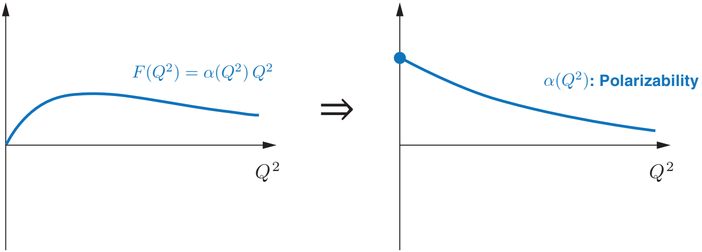
  ⇒ WTI, WTI-T **(em. gauge invariance) ?**

  ⇒ Fully transverse part **(t-channel poles) ?**

Here be dragons

- **Gauge invariance $\Leftrightarrow$ transversality:**
  when inserted in nucleon Compton amplitude,
  non-transverse terms in quark Compton vertex (in Born, WTI, WTI-T)
  must be cancelled by those in remaining diagrams (cat's ears, 6pt function)

- But handbag alone is **not gauge-invariant,**
  incomplete calculation can produce **singularities** in $Q^2, Q'^2, Q\cdot Q', P\cdot Q$



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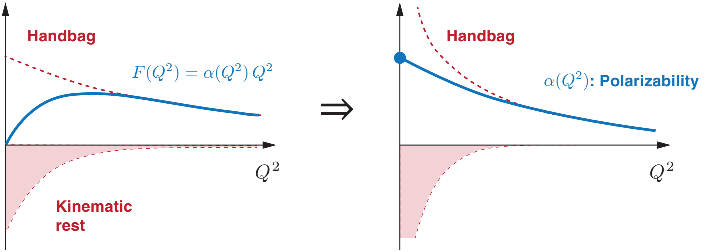
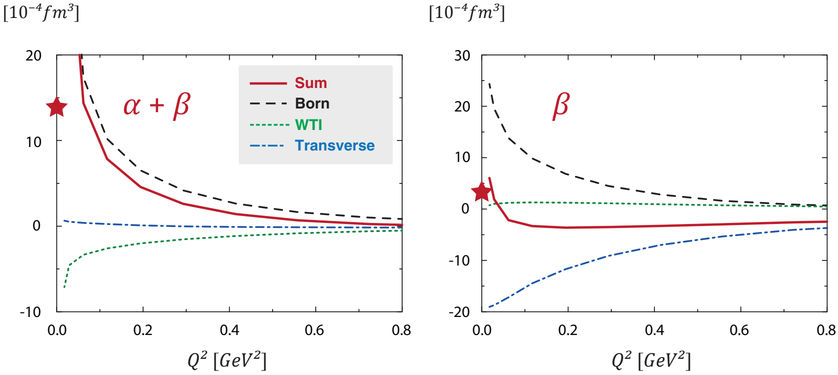


Polarizabilities: a first look



- $\alpha + \beta$: dominated by **quark Born terms (pure handbag)**
  (here: $1 / Q \cdot Q'$ singularity not yet removed)
- β: cancellation between **Born** and **t-channel poles?**
  no singularity in β

Summary

So far:

- Structure analysis of **Compton scattering**
- Nonperturbative calculation of **handbag part** (Born + t-channel)

Next:

- Extract **polarizabilities**
- **Two-photon exchange** contribution to form factors
- **GPDs & nucleon PDFs**
- **Pion electroproduction** at quark level
- **Nucleon resonances**
- **Timelike form factors & processes**

Need to improve:

- **Go beyond rainbow-ladder!** (Pion cloud, decay channels, higher n-point functions, ...)
- Deal with quark singularities ⇒ access high Q^2, timelike region etc.)

Thanks for your attention.

Cheers to my collaborators:

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