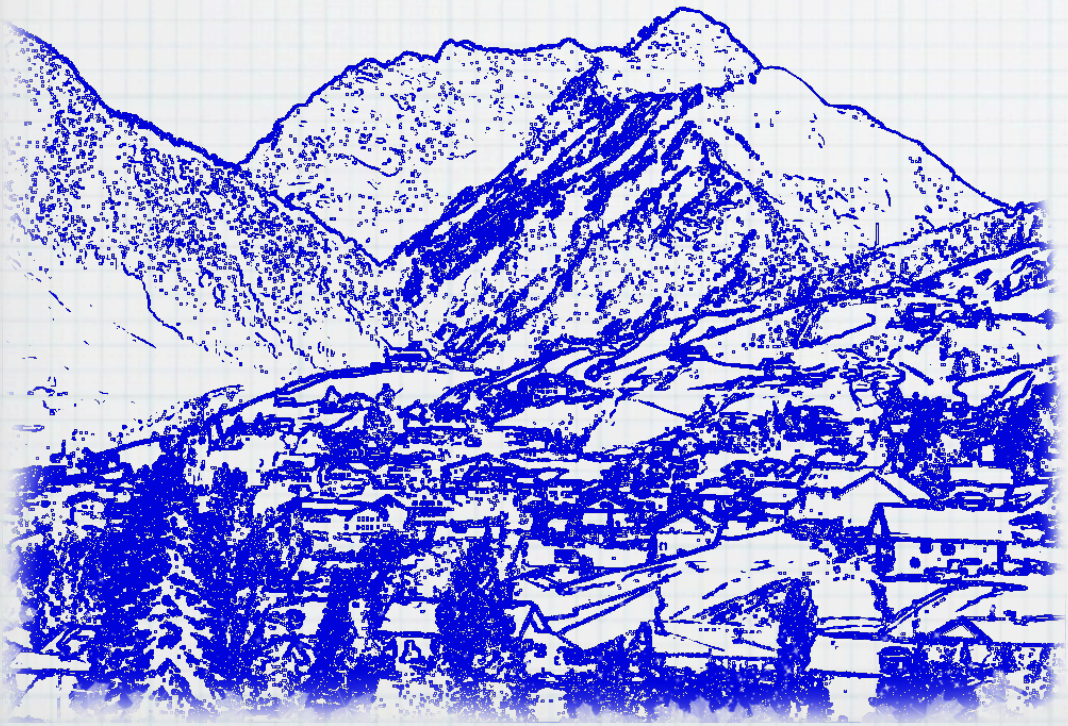
Novel approach to the hadronic LbL contribution to $(g-2)_\mu$



Vladyslav Pauk

Johannes Gutenberg University

Mainz, Germany

Hadrons from Quarks and Gluons,

Hirschegg, Austria

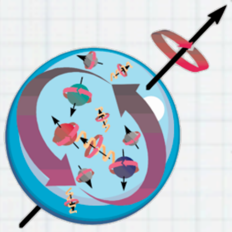
January 12-18, 2014

The muon's anomalous magnetic moment

The muon's

anomalous magnetic moment

The anomalous magnetic moment



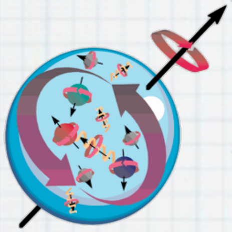
magnetic
dipole moment

$$\vec{\mu} = g\,Q\,\mu_0\,\frac{\vec{\sigma}}{2}$$

μ_0: Bohr magneton

Q: charge

The anomalous magnetic moment



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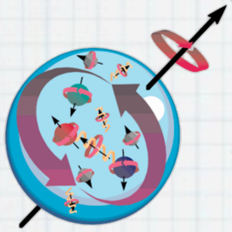
The anomalous magnetic moment



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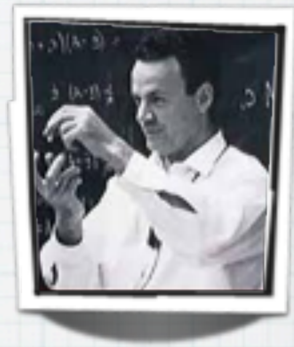


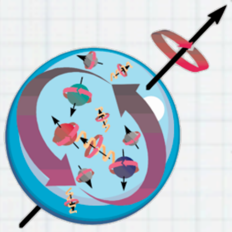
gyromagnetic factor

$$g = 2$$

Dirac theory (1928)

free electron

The anomalous magnetic moment



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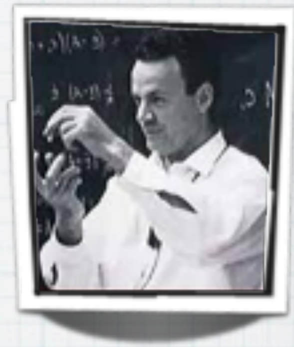

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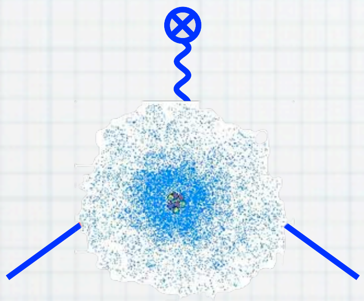
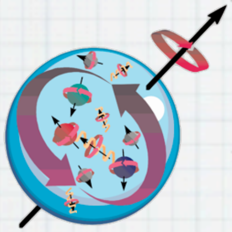


$$a_l \equiv \frac{g-2}{2}$$

anomalous moment

The anomalous magnetic moment



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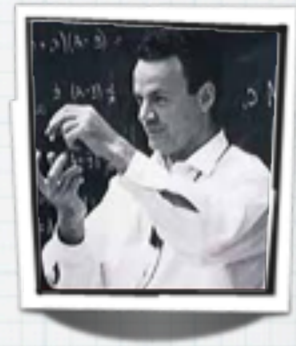

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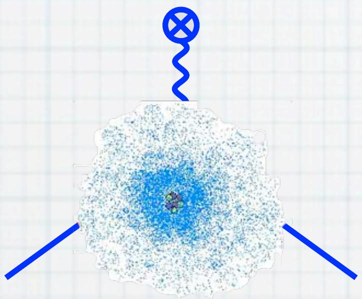

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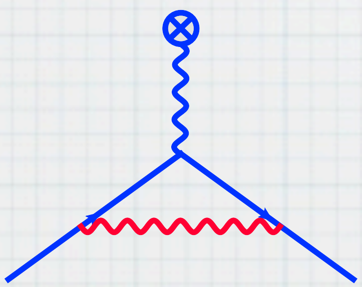
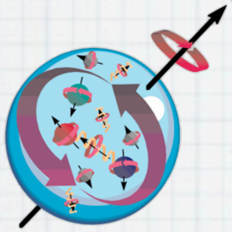
Schwinger (1948)



$a_\mu^{QED(1)} = \alpha_{em} / 2\pi = 0.001161$

The anomalous magnetic moment



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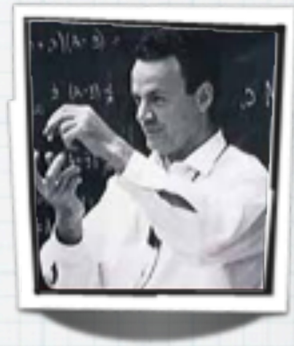

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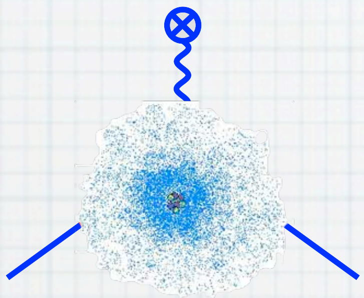

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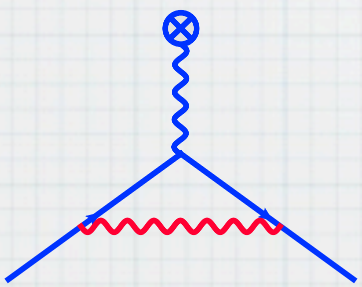
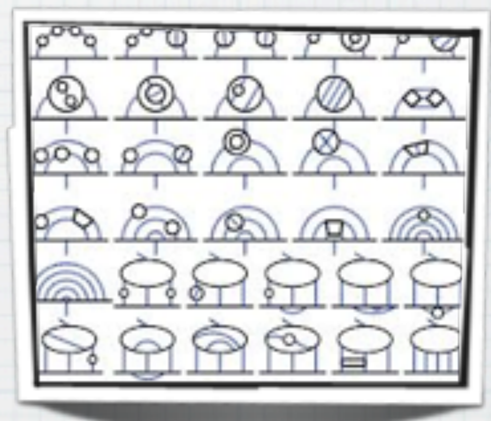
Kinoshita (2012)





$a_\mu^{QED(1)} = \alpha_{em} / 2\pi = 0.001161$

$a_\mu^{QED(5)} = (11\ 658\ 471.896 \pm 0.008)\cdot 10^{-10}$

up to α_{em}^5 !

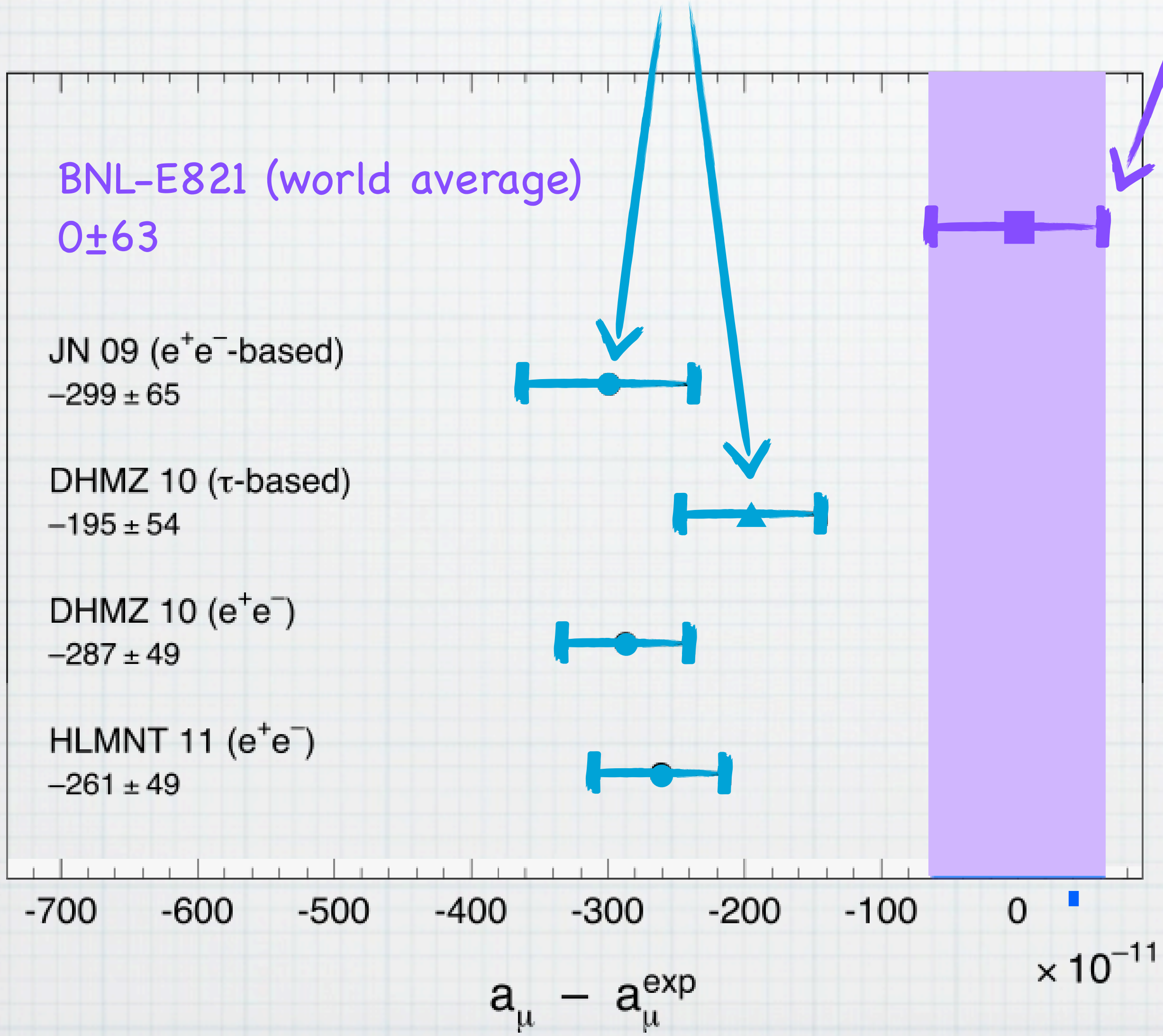
$(g-2)_\mu$: theory vs experiment

present theoretical SM value

$a_\mu^{SM} = (11\ 659\ 184.0 \pm 5.9) \times 10^{-10}$

E821 measurement of $(g-2)_\mu$ (2009)

$a_\mu^{exp} = (11\ 659\ 208.9 \pm 6.3) \times 10^{-10}$



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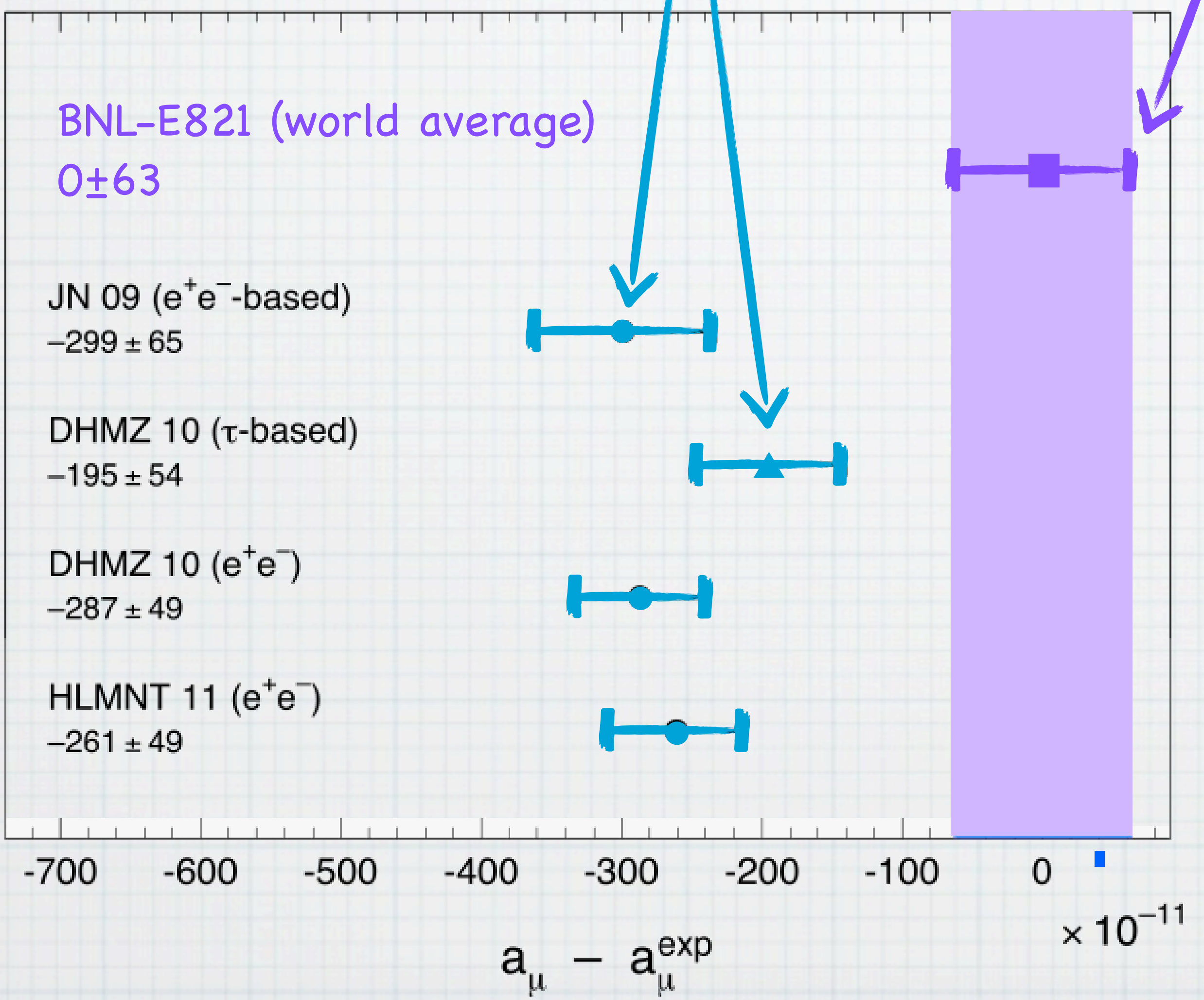

discrepancy between theory and experiment

$a_\mu^{exp} - a_\mu^{th} = (24.9 \pm 8.7) \times 10^{-10}$

(2.9 σ)

New Physics?

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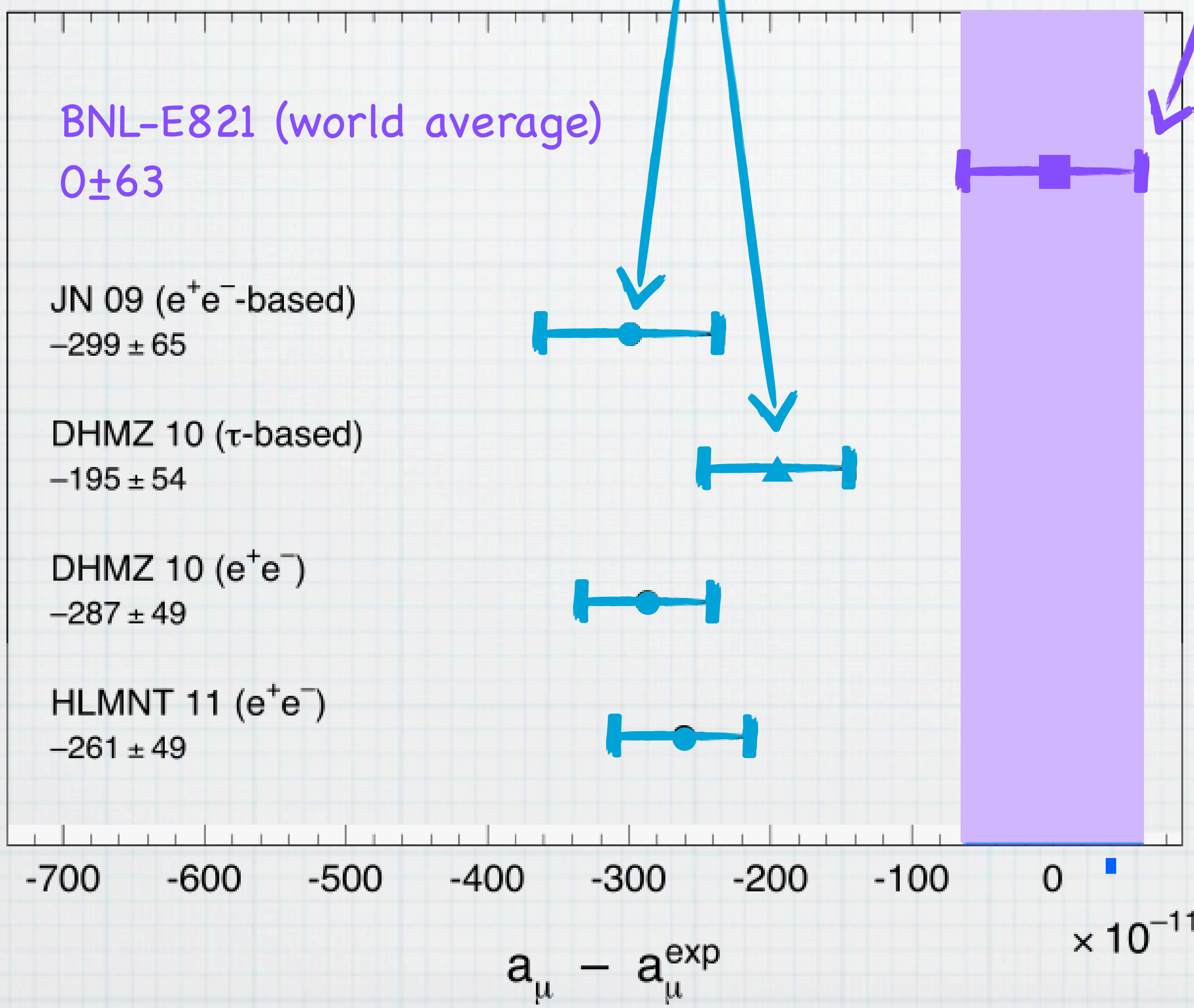

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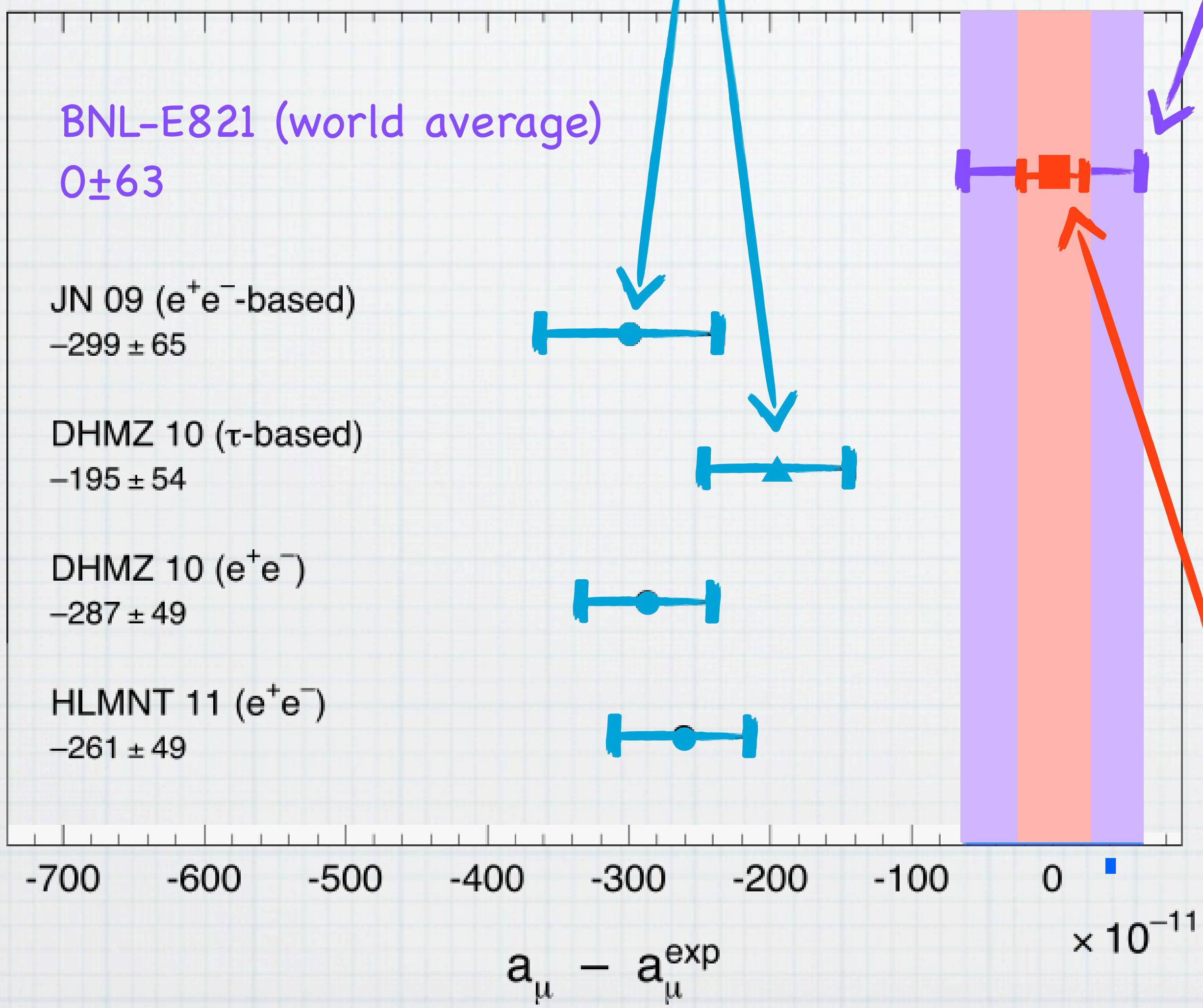

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new FNAL $(g-2)_\mu$ measurement (2015):

factor 4 precision improvement

$\pm 1.6 \cdot 10^{-10}$

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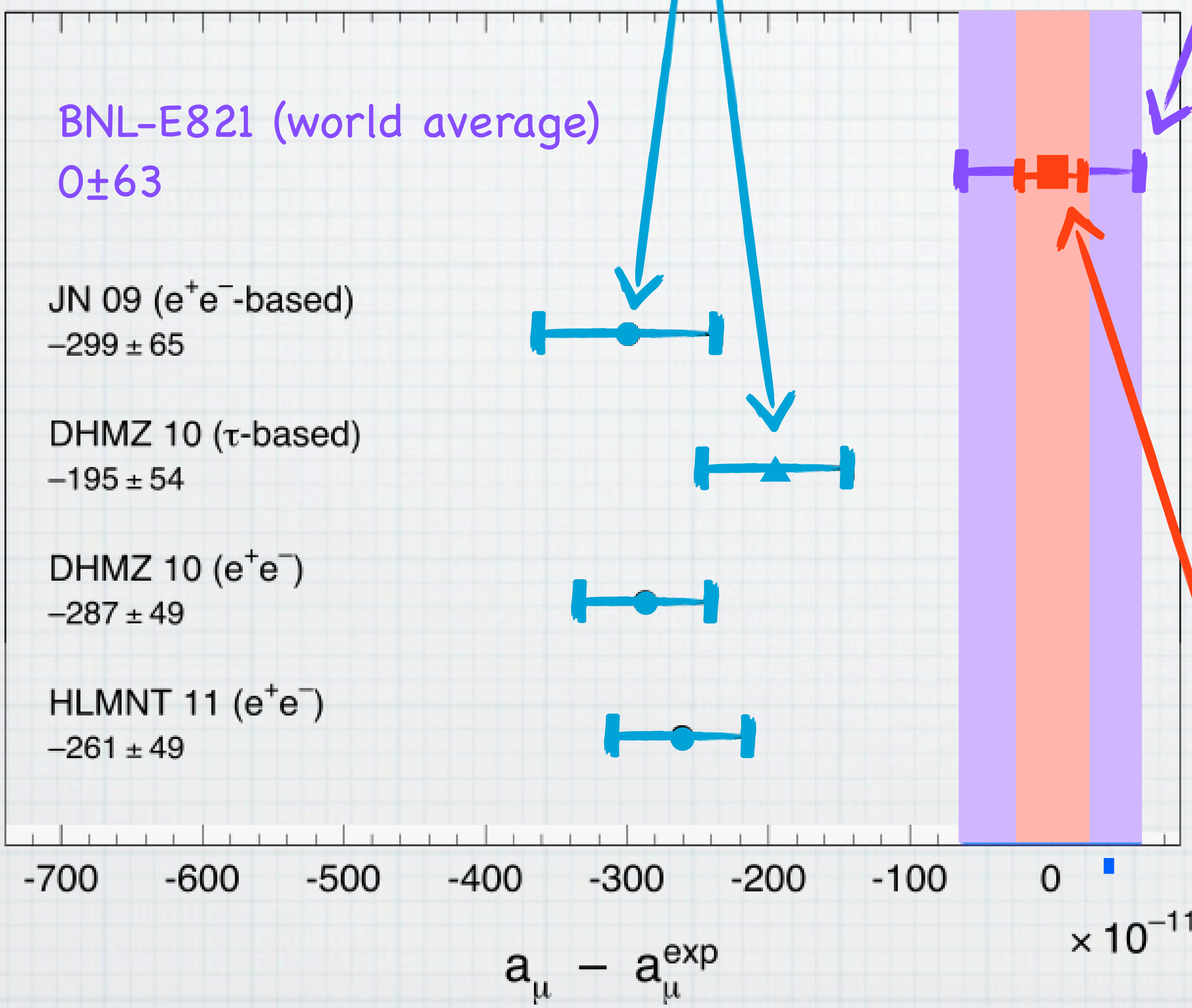

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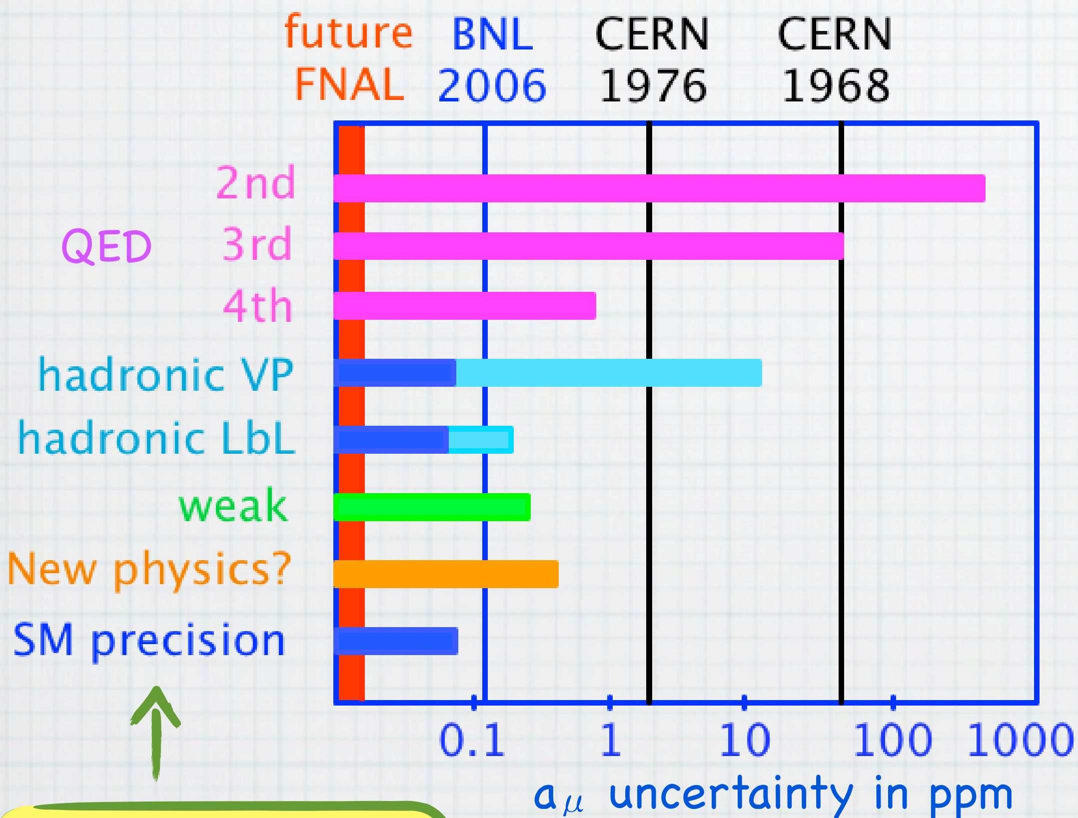
improve theory!

$(g-2)_\mu$: SM predictions & uncertainties

sensitivity of $(g-2)_\mu$ experiments to various corrections

experiments



theory corrections

$(g\text{-}2)_\mu$: SM predictions & uncertainties

sensitivity of $(g\text{-}2)_\mu$ experiments to various corrections

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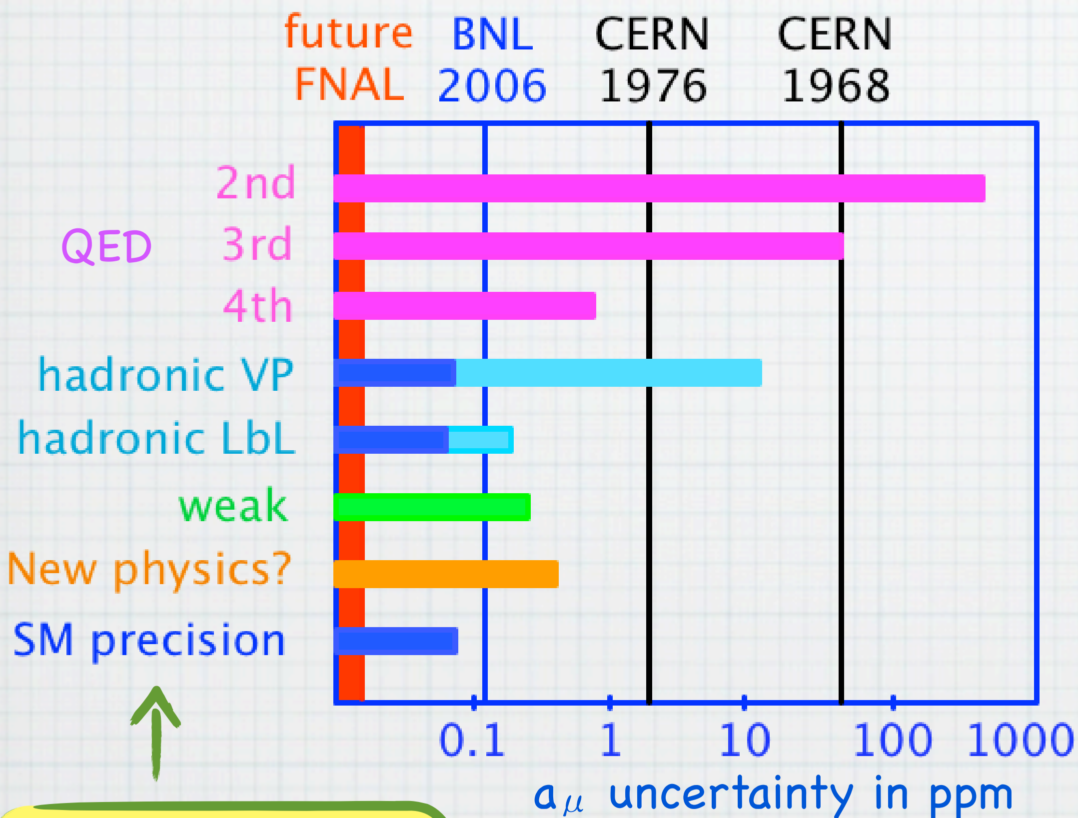
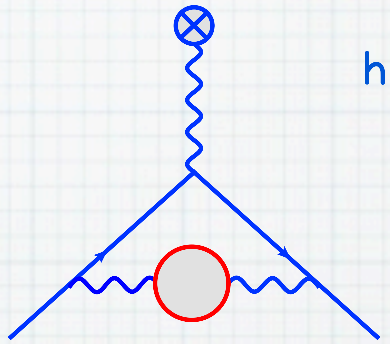


theory corrections

hadronic vacuum polarization (VP)



hadronic VP determined by cross section measurements of $e^+e^- \to$ hadrons

$a_\mu^{had,\, VP} = (692.3 \pm 4.2) \times 10^{-10}$

$(g-2)_\mu$: SM predictions & uncertainties

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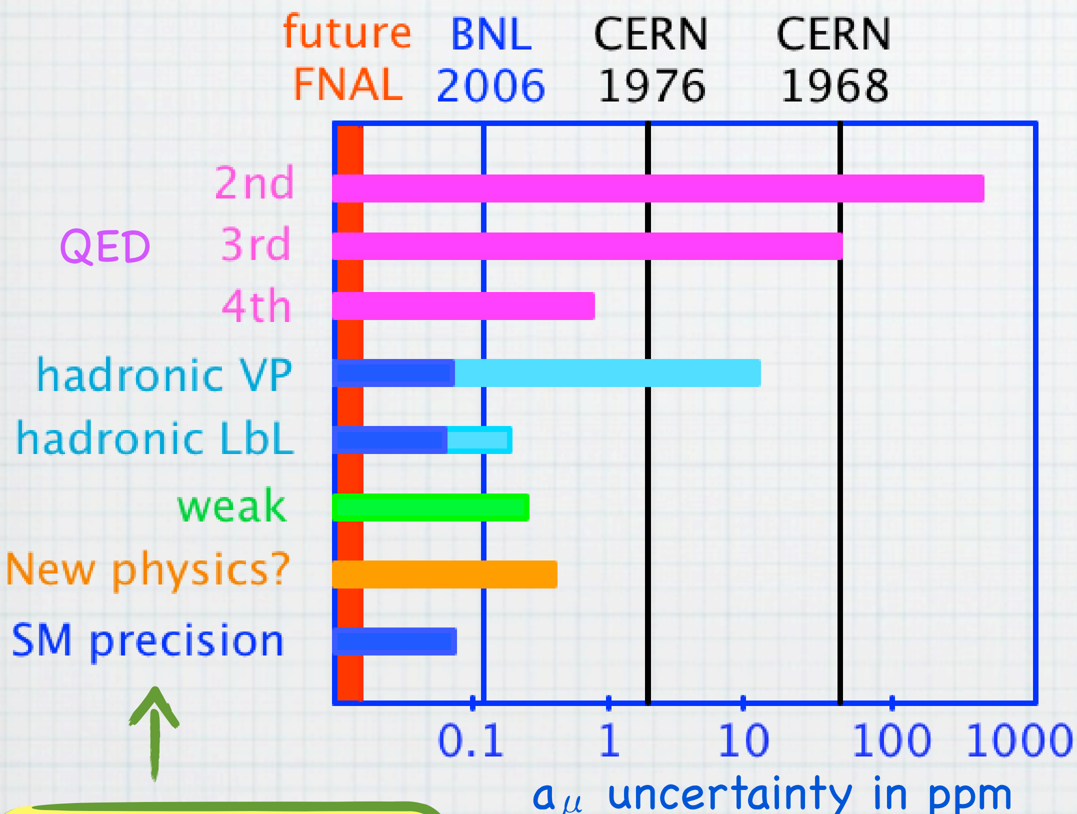

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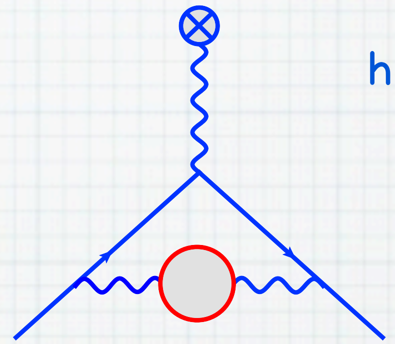

hadronic VP determined by cross section measurements of e^+e^- -> hadrons

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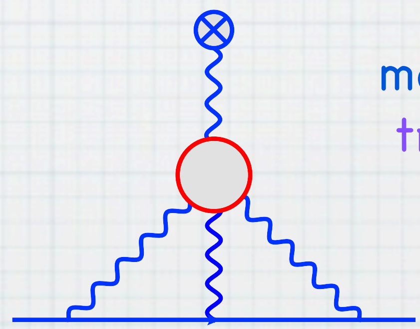
hadronic light-by-light scattering (LbL)

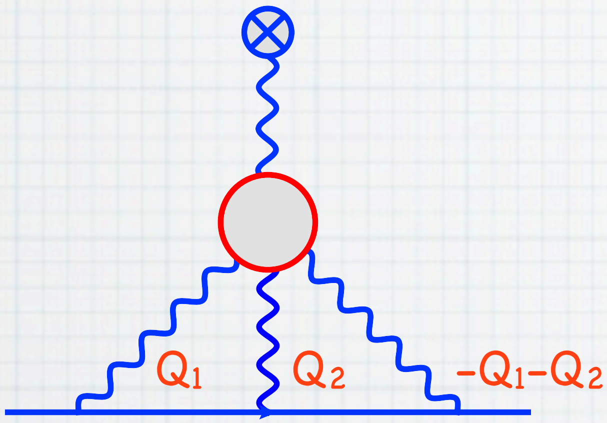


measurements of meson transition form factors required as input to reduce uncertainty

$a_\mu^{\text{had, LbL}} = (11.6 \pm 4.0) \times 10^{-10}$

Models of hadronic LbL scattering

hadronic LbL correction



Models of hadronic LbL scattering

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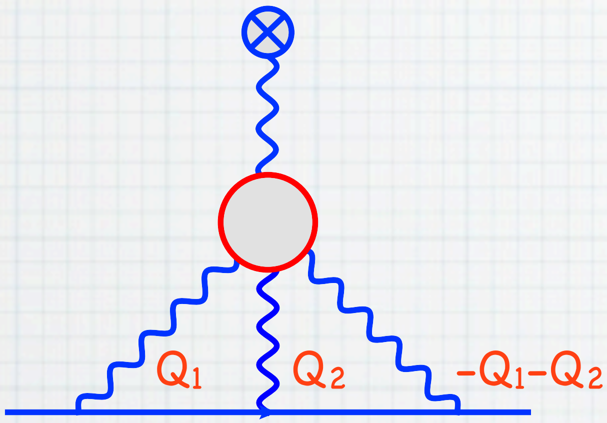
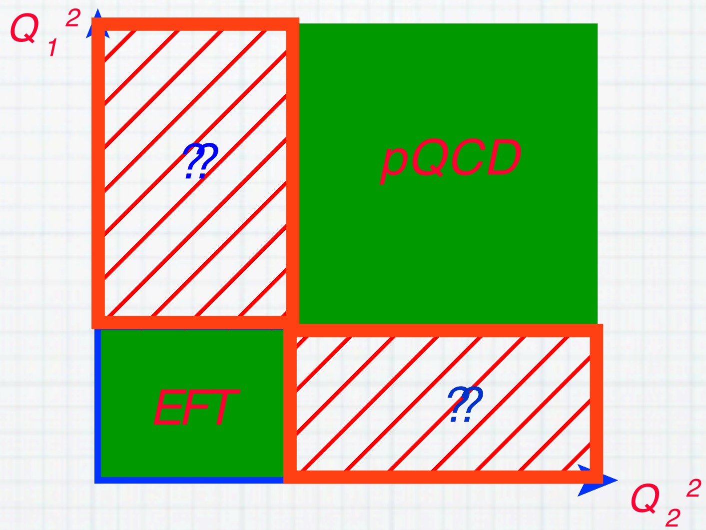
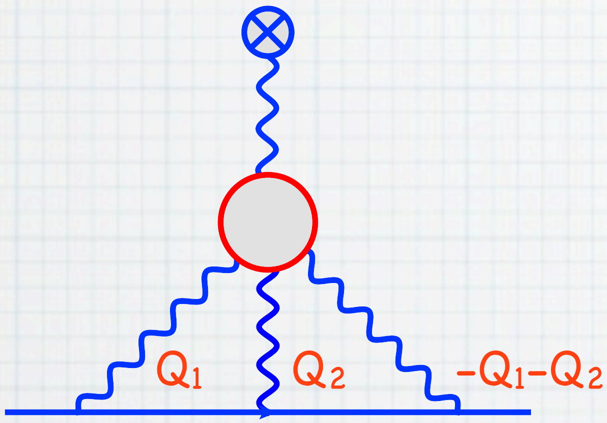


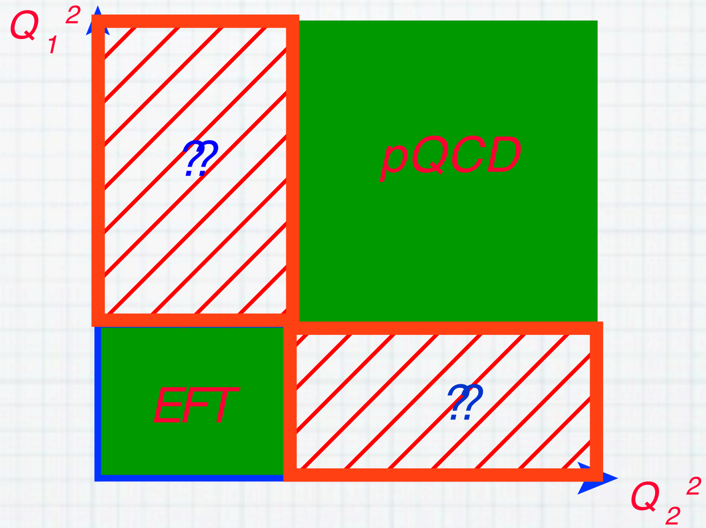


multi-scale problem -
mixed soft - hard regions

Models of hadronic LbL scattering

hadronic LbL correction





multi-scale problem -
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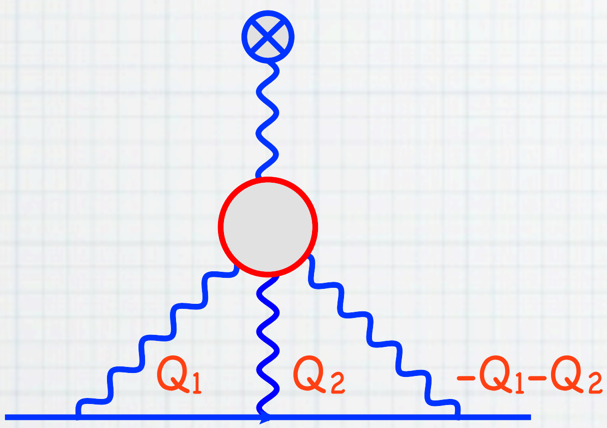
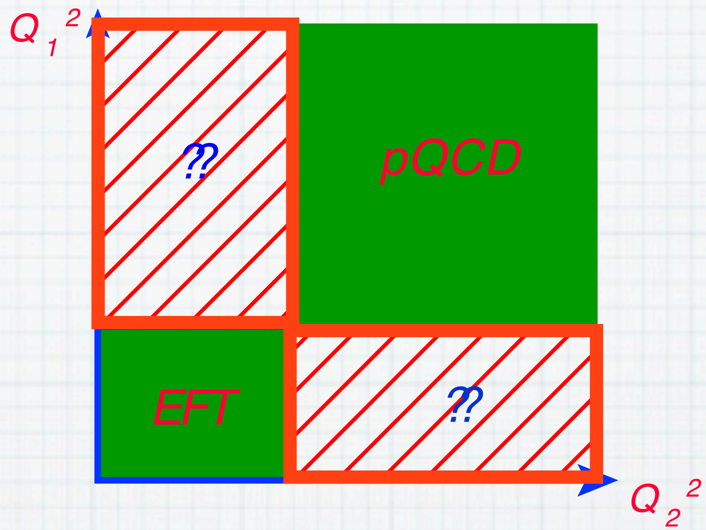
general solution from the first principles is not available

Models of hadronic LbL scattering

hadronic LbL correction





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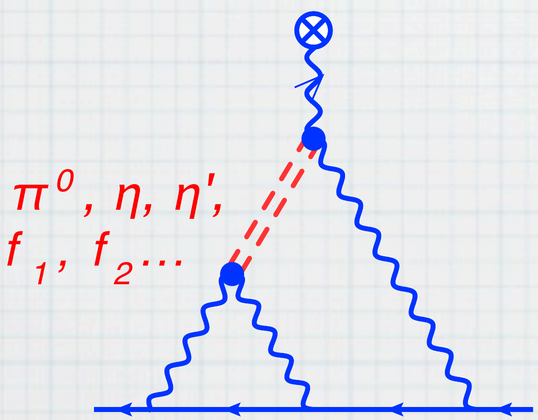

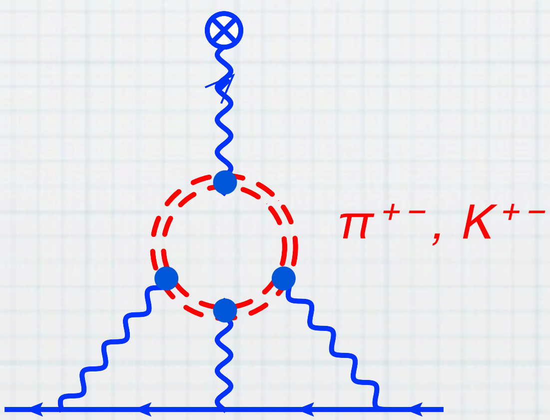

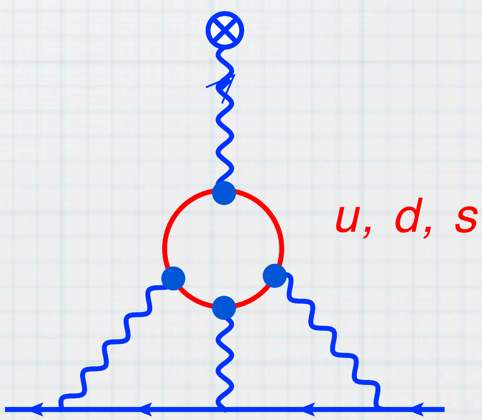

Models of hadronic LbL scattering

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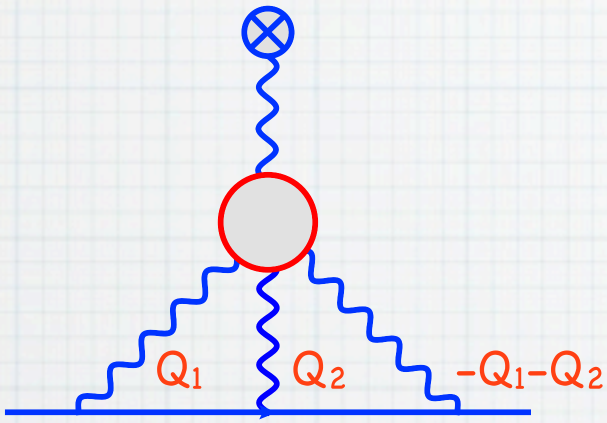

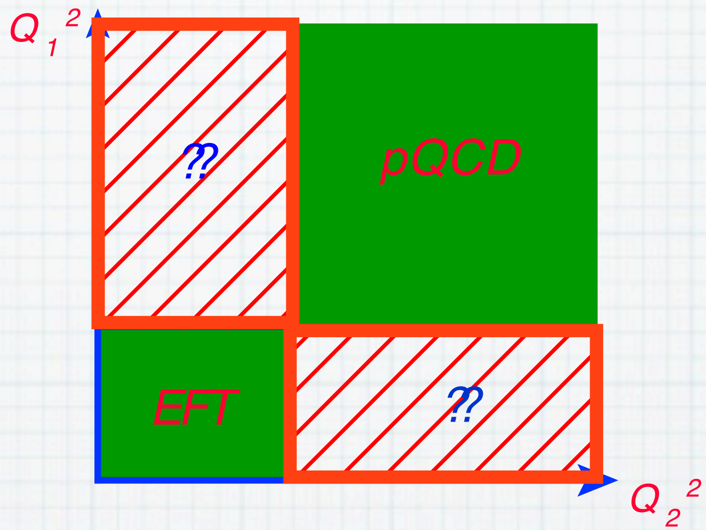

multi-scale problem -
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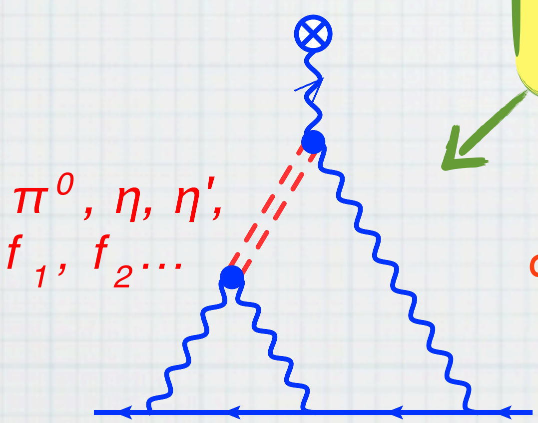
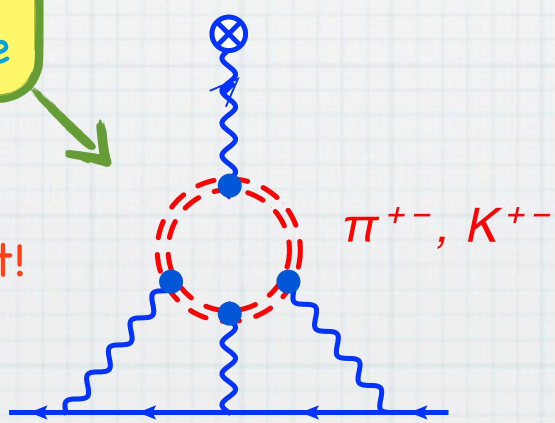
general solution from the first principles is not available

long distance

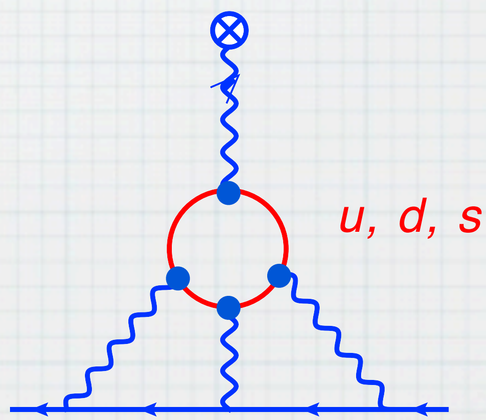


dominant!

1/N_c - expansion

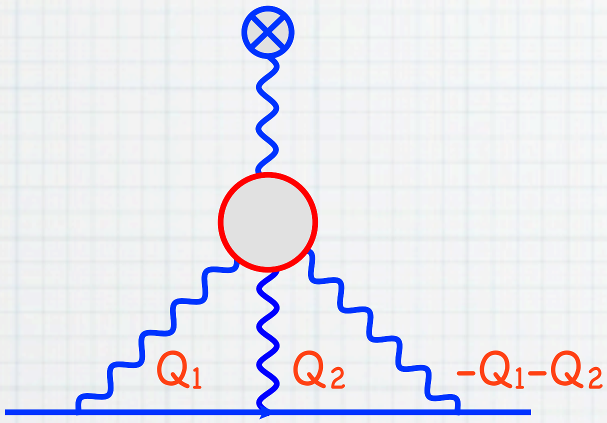


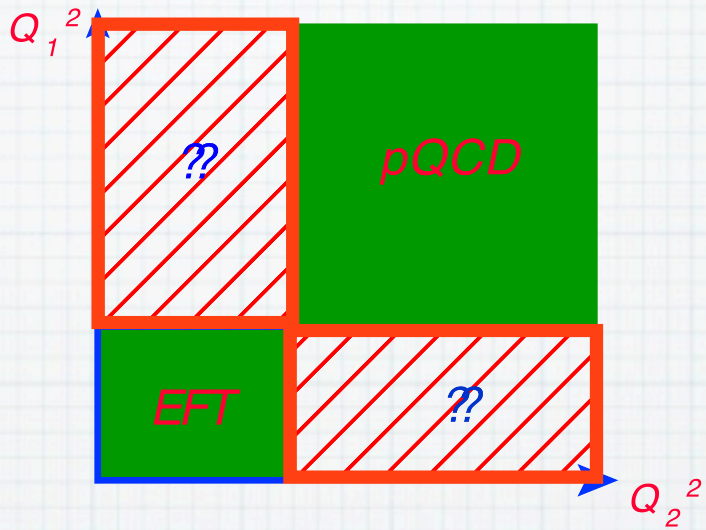
chiral expansion



Models of hadronic LbL scattering

hadronic LbL correction



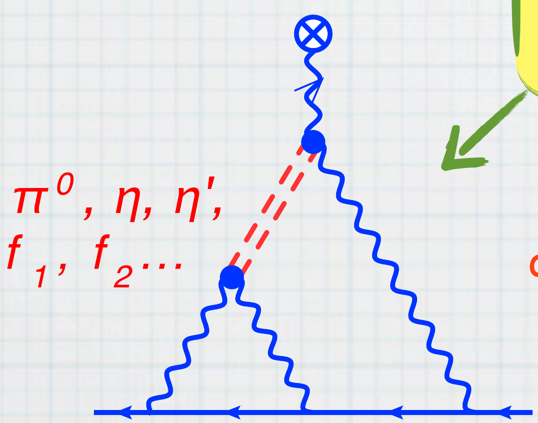


multi-scale problem -
mixed soft - hard regions

general solution from
the first principles is
not available

long
distance

short
distance



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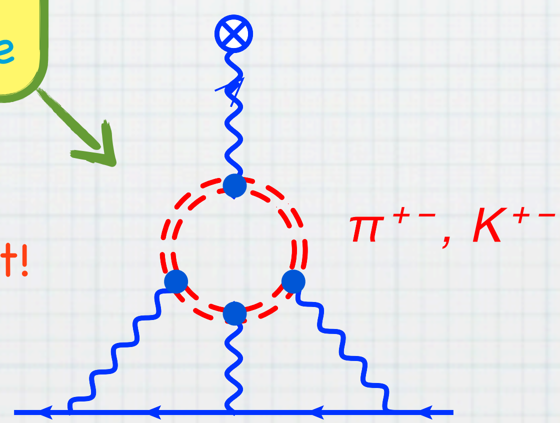
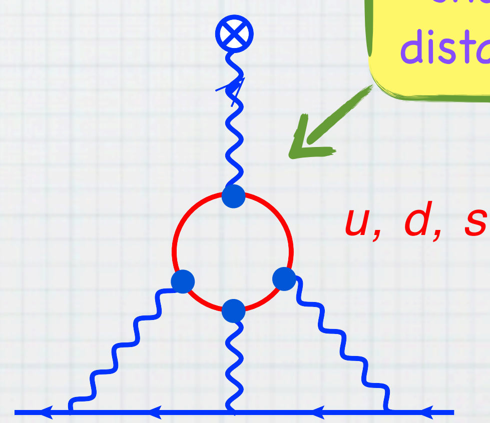




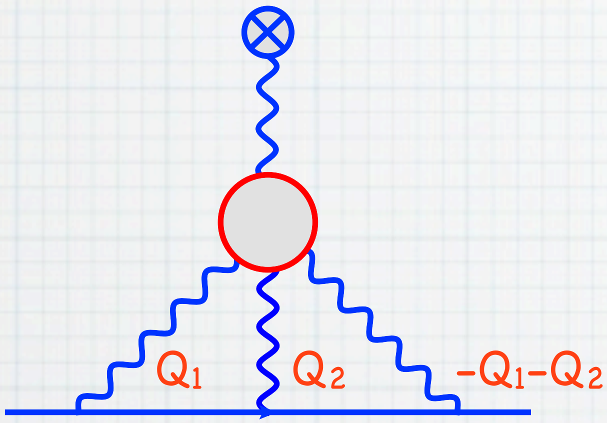
$1/N_c$ - expansion

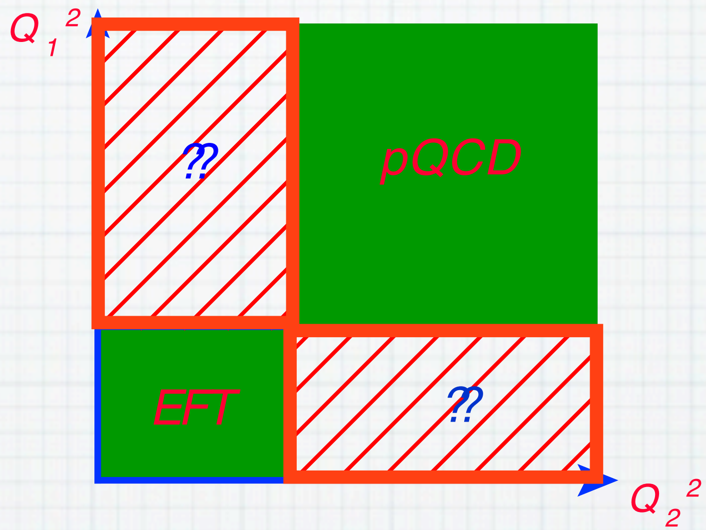
chiral expansion

perturbative regime

Models of hadronic LbL scattering

hadronic LbL correction





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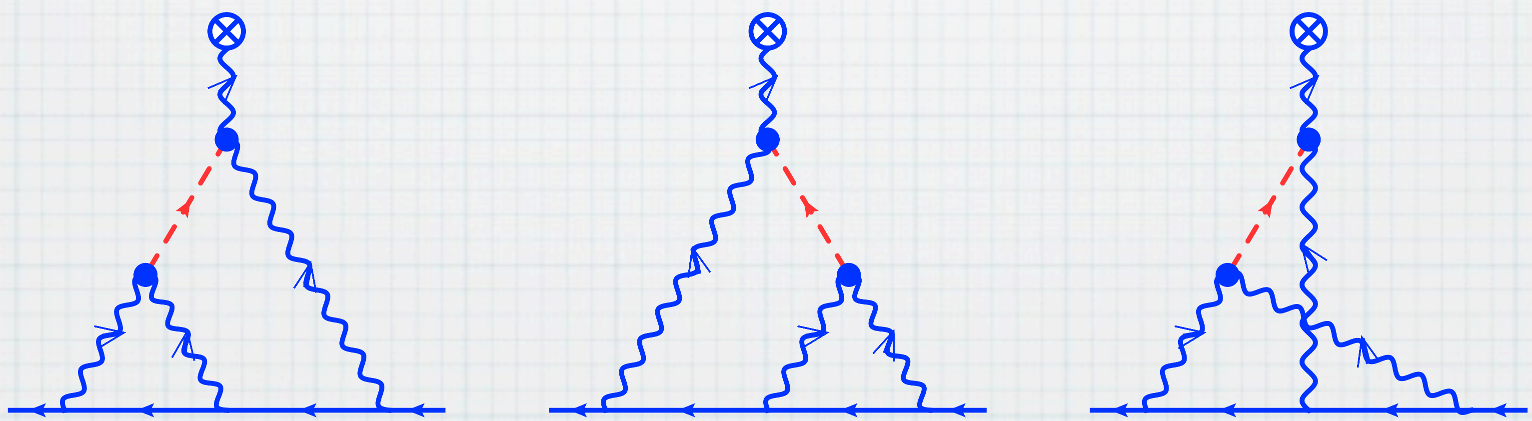
multi-scale problem – mixed soft – hard regions

Contribution	BPP	HKS	KN	MV	BP	PdRV	N/JN
π^0, η, η'	85±13	82.7±6.4	83±12	114±10	–	114±13	99±16
π, K loops	−19±13	−4.5±8.1	–	–	–	−19±19	−19±13
π, K loops + other subleading in N_c	–	–	–	0±10	–	–	–
axial vectors	2.5±1.0	1.7±1.7	–	22± 5	–	15±10	22± 5
scalars	−6.8±2.0	–	–	–	–	−7± 7	−7± 2
quark loops	21± 3	9.7±11.1	–	–	–	2.3	21± 3
total	83±32	89.6±15.4	80±40	136±25	110±40	105±26	116±39

Single-meson contributions to the $(g\text{-}2)_\mu$



Single-meson contribution

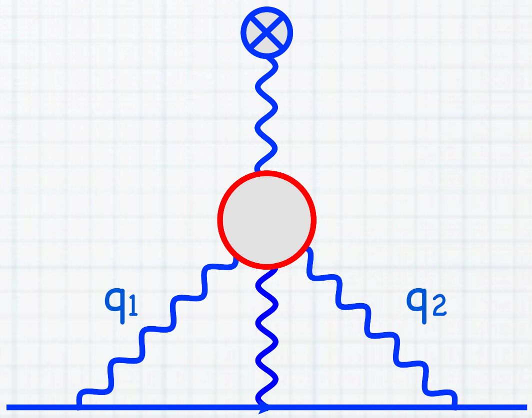
Single-meson contribution

$$a_\mu = \lim_{k \to 0} F_2(k^2)$$

Pauli form factor

two-loop
Feynman integral

$$\int \mathrm{d}^4 q_1 \int \mathrm{d}^4 q_2$$



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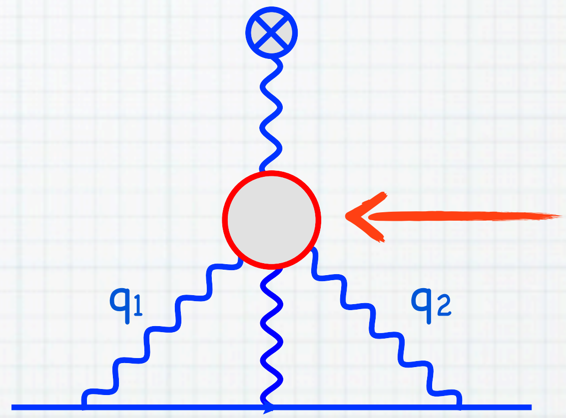
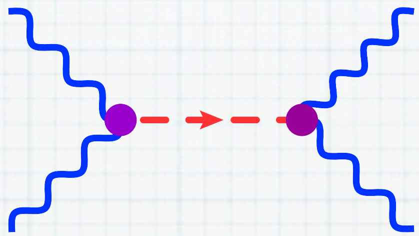


non-perturbative
LbL scattering tensor

Single-meson contribution

$F(Q_1, Q_2, P)$

non-perturbative dynamics



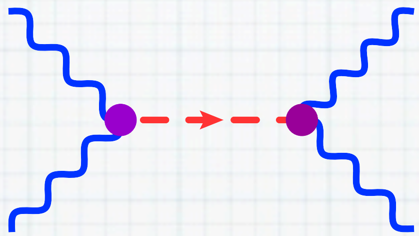
off-shell information?

$F(Q_1, Q_2)$

Single-meson contribution

$F(Q_1, Q_2, P)$

non-perturbative dynamics



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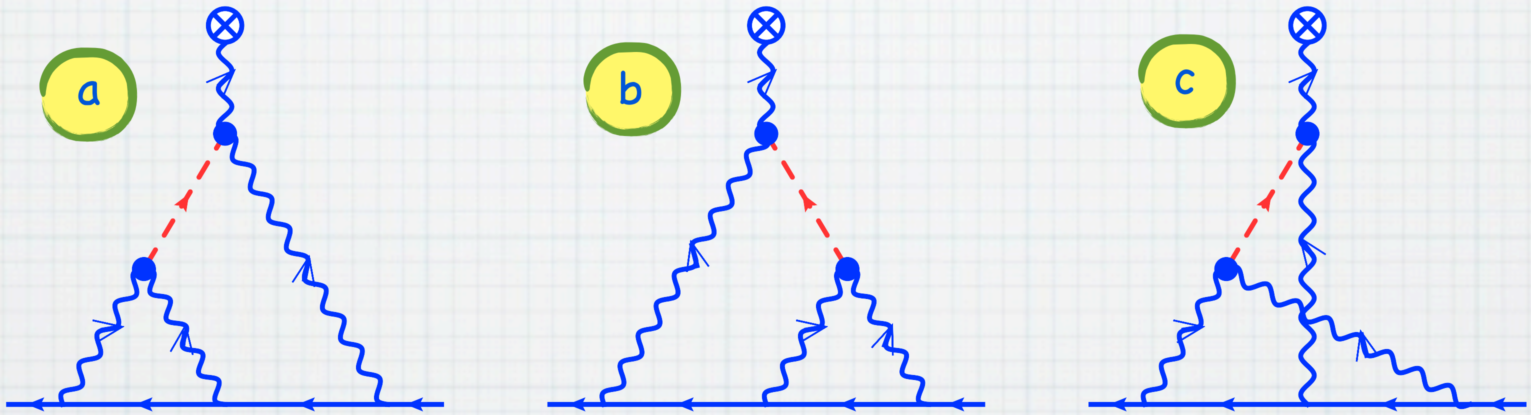

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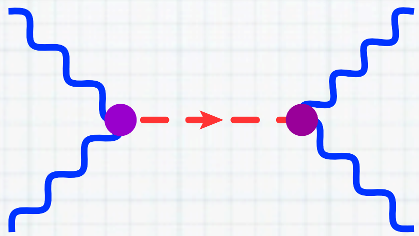

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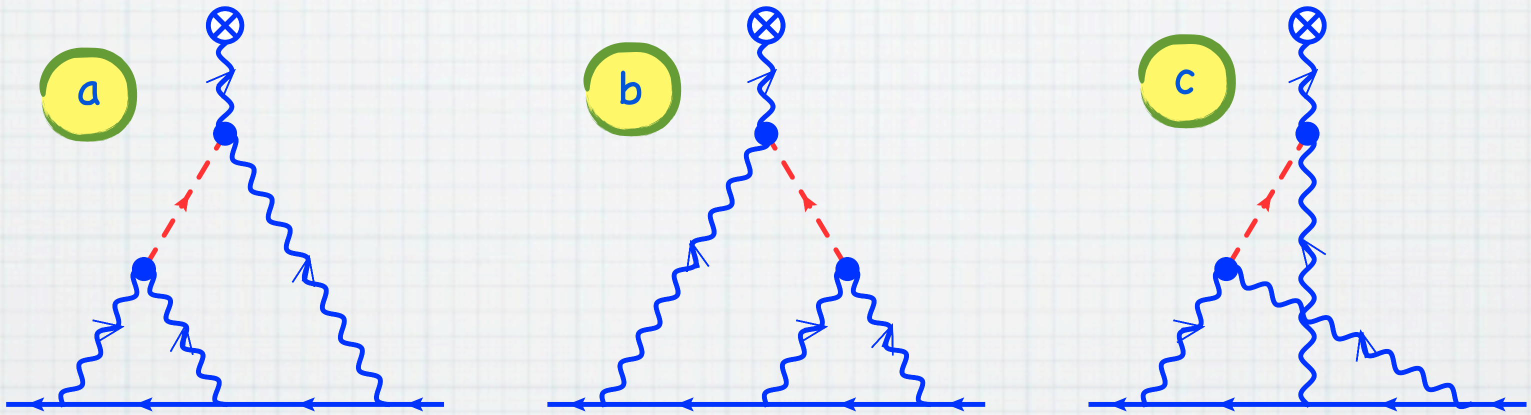
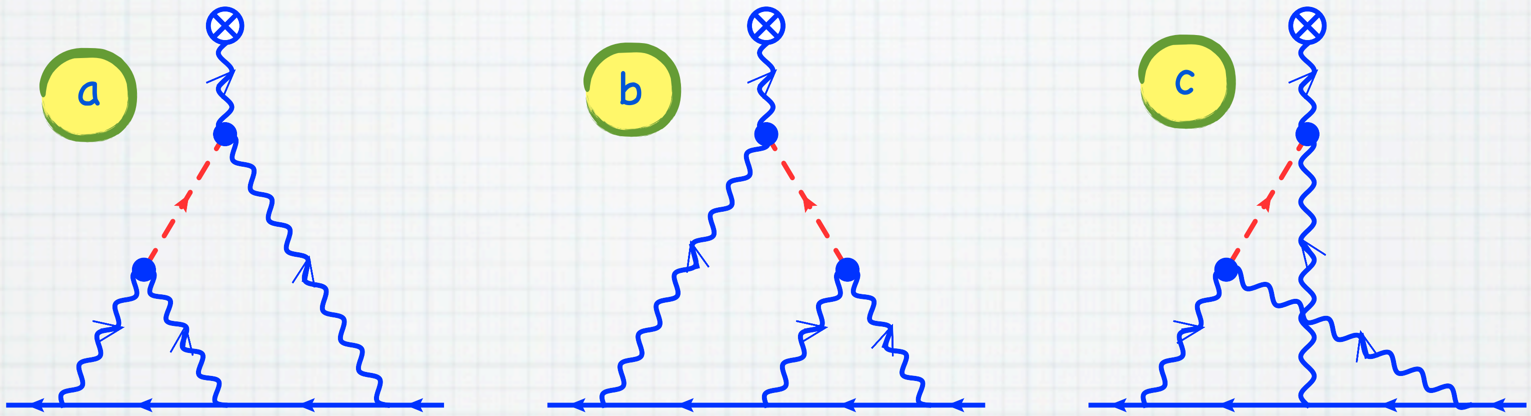


$$a_\mu^{LbL} = \frac{-e^6}{48m}\int\frac{\mathrm{d}^4q_1}{(2\pi)^4}\int\frac{\mathrm{d}^4q_2}{(2\pi)^4}\frac{1}{q_1^2q_2^2(q_1+q_2)^2}\frac{1}{(p+q_1)^2-m^2}\frac{1}{(p-q_2)^2-m^2}$$
$$\times\left[\frac{F\left(q_1^2,(q_1+q_2)^2\right)F\left(q_2^2,0\right)}{q_2^2-m_P^2}T_{ab}(q_1,q_2,p)+\frac{F\left(q_1^2,q_2^2\right)F\left((q_1+q_2)^2,0\right)}{(q_1+q_2)^2-m_P^2}T_c(q_1,q_2,p)\right]$$

Single-meson contribution



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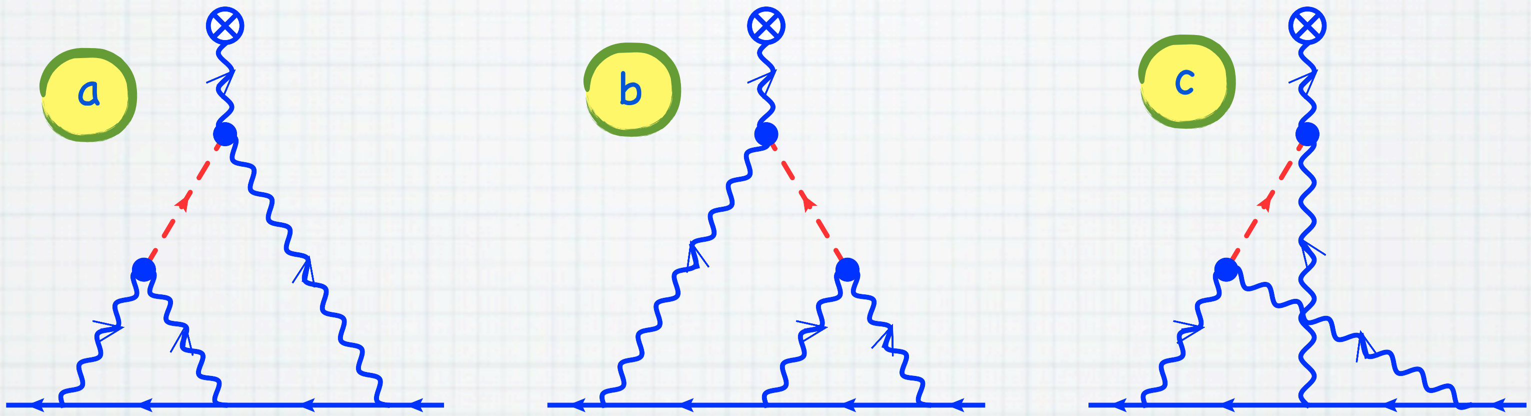

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pseudoscalar poles: π^0, η, η'

large-N_c

short-distance QCD constraints →

$a_\mu^{LbL,PS}$ = +8.3 (1.2) × 10^{-10}

Knecht, Nyffeler (2001)

$A\gamma^*\gamma$ transition amplitude

dipole parametrization

$A \rightarrow \gamma\gamma$ transition FF:

$$\frac{A(Q_1^2,0)}{A(0,0)} = \frac{1}{\left(1+Q_1^2/\Lambda_A^2\right)^2}$$

$$[A(0,0)]^2 = \frac{12}{\pi\alpha^2}\frac{1}{m_A^2}\Gamma_{\gamma\gamma}$$

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dipole parametrization
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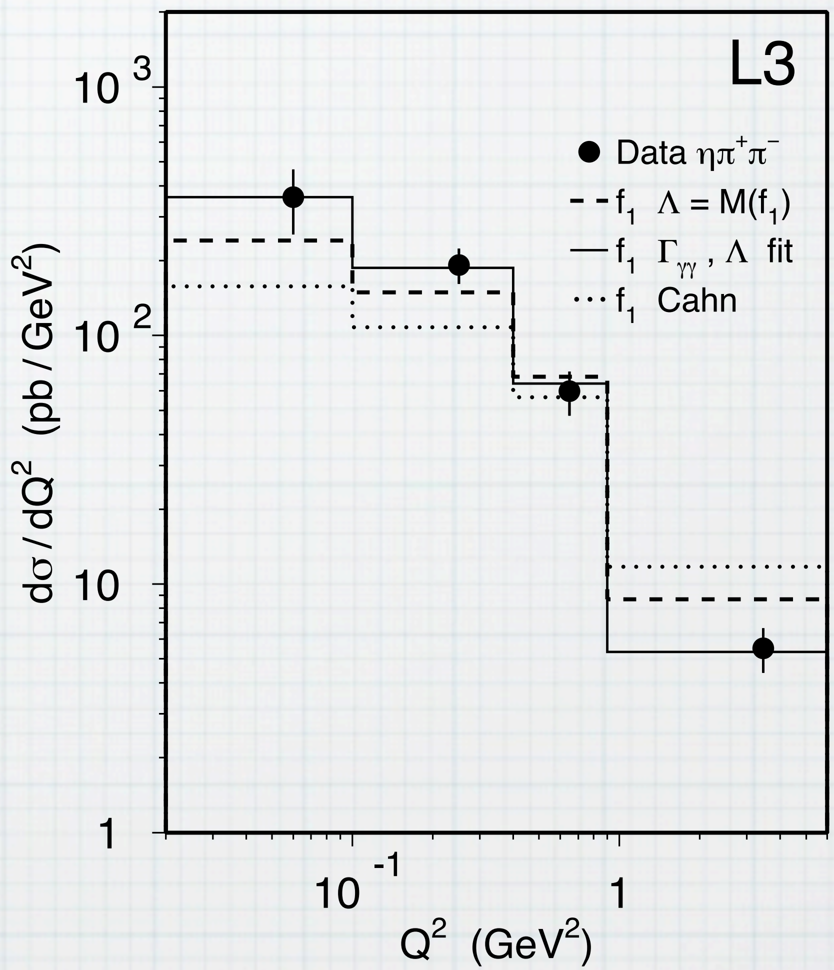
for 2γ decay widths $\Gamma_{\gamma\gamma}$ and dipole masses Λ_A entering the FF, we use the experimental results from the L3 Collaboration.

	m_A [MeV]	$\tilde{\Gamma}_{\gamma\gamma}$ [keV]	Λ_A [MeV]
$f_1(1285)$	1281.8 ± 0.6	3.5 ± 0.8	1040 ± 78
$f_1(1420)$	1426.4 ± 0.9	3.2 ± 0.9	926 ± 78

L3 Collaboration



Two-dimensional representation

$$a_\mu^{LbL} = \frac{\alpha}{(2\pi)^2} \frac{\Lambda_{A1}^6 \Lambda_{A2}^6 \tilde{\Gamma}_{\gamma\gamma}(A)}{m\, m_A^5} \int \mathrm{d}Q_1 \int \mathrm{d}Q_2 \left[2 w_a(Q_1, Q_2) + w_c(Q_1, Q_2)\right]$$

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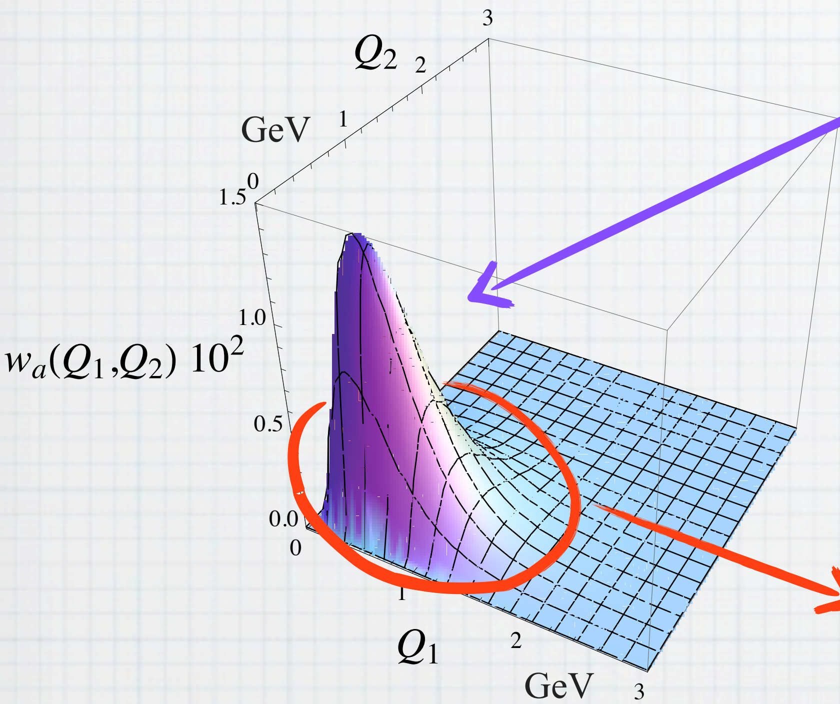
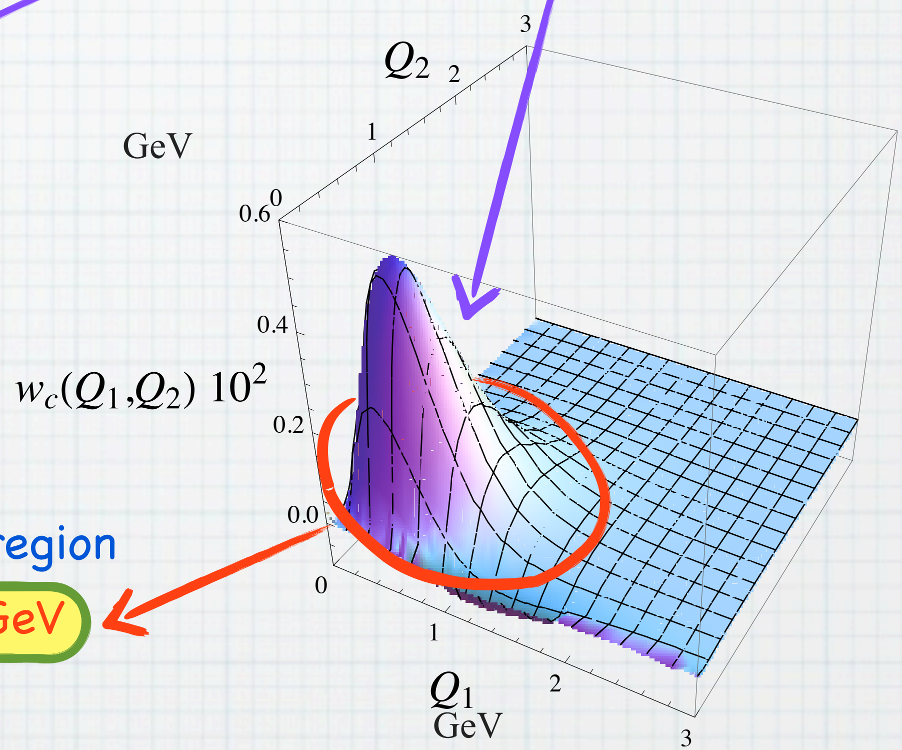


dominating region

$Q_1 \sim Q_2 \sim 1$ GeV



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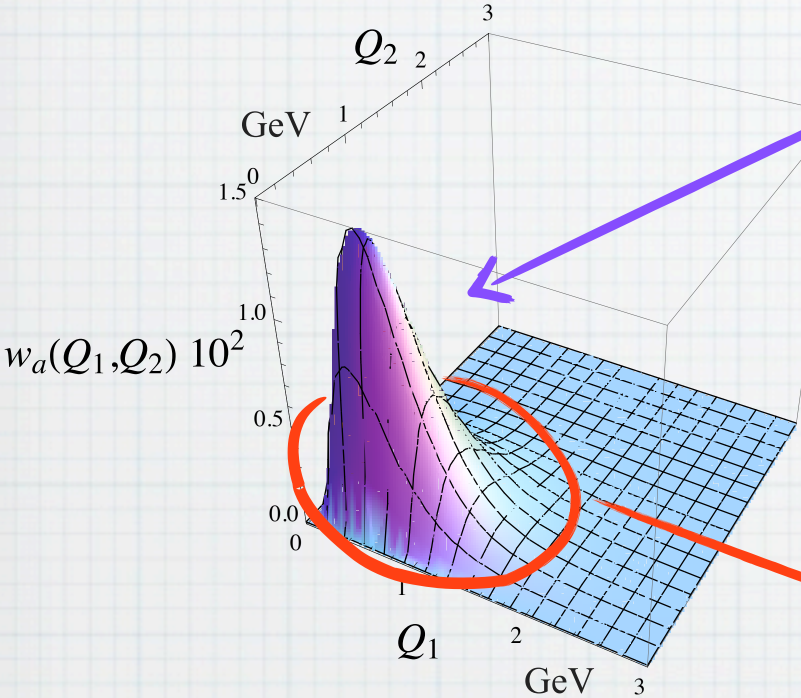

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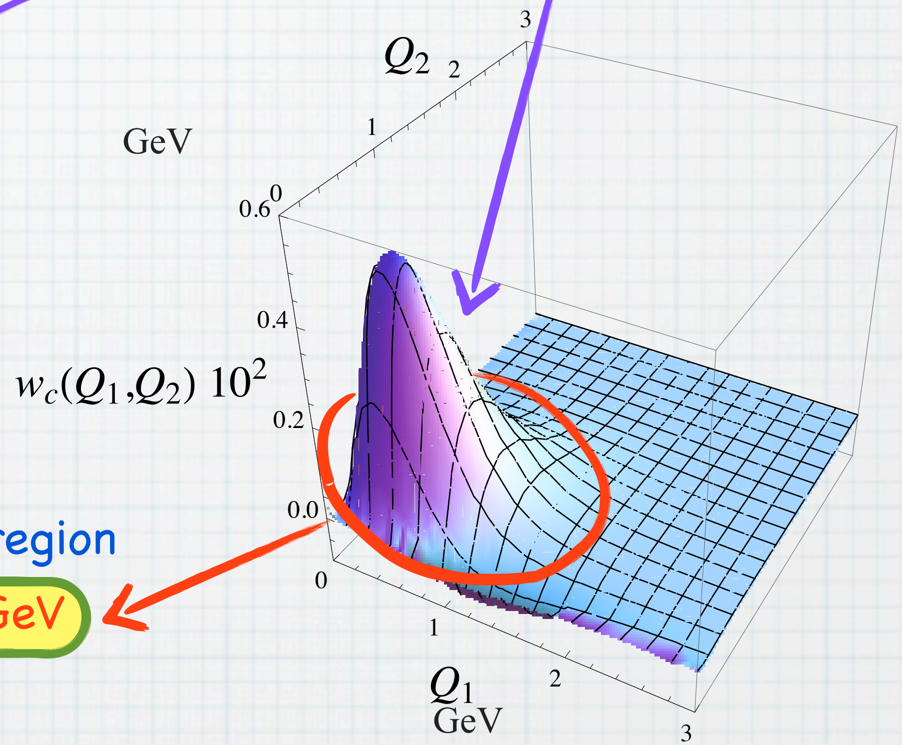


	m_A [MeV]	$\tilde{\Gamma}_{\gamma\gamma}$ [keV]	Λ_A [MeV]	$a_\mu^{LbL;A} \times 10^{10}$
$f_1(1285)$	1281.8 ± 0.6	3.5 ± 0.8	1040 ± 78	$0.50^{+0.20}_{-0.17}$
$f_1(1420)$	1426.4 ± 0.9	3.2 ± 0.9	926 ± 78	$0.14^{+0.07}_{-0.06}$

the contribution of the axial-vector pole to the $(g-2)_\mu$

V.P., M. Vanderhaeghen (2014)

...scalar and tensor mesons? f_0, f_2, a_0, a_2, etc.

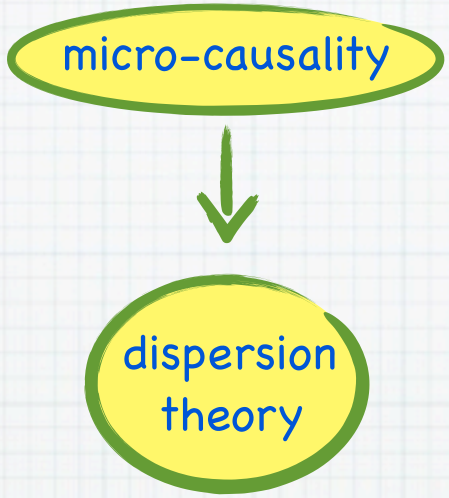
experimental information is very limited!

light-by-light scattering
sum rules

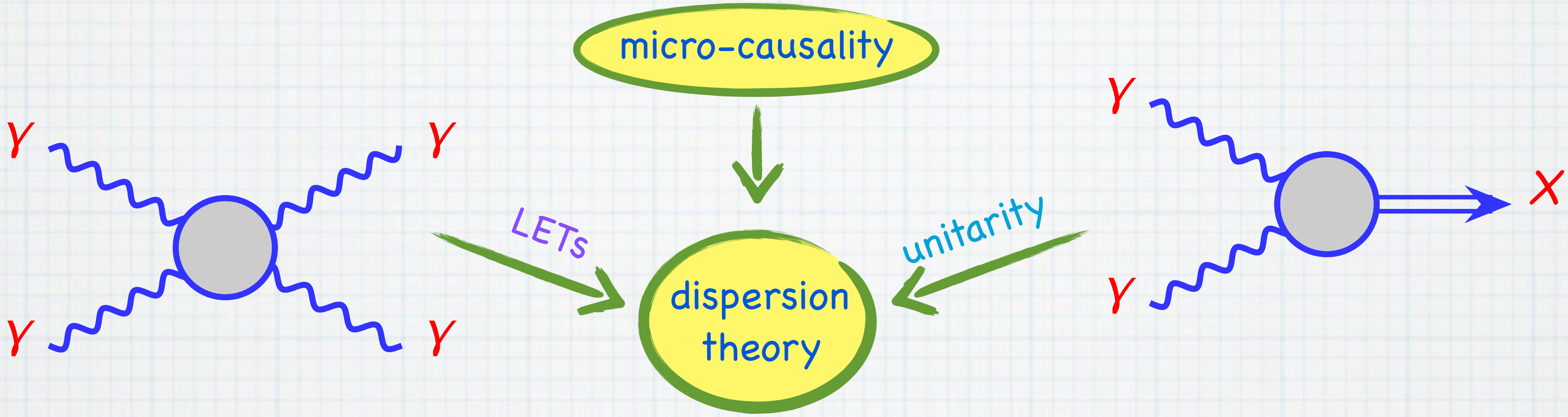
Sum rules

micro-causality

Sum rules

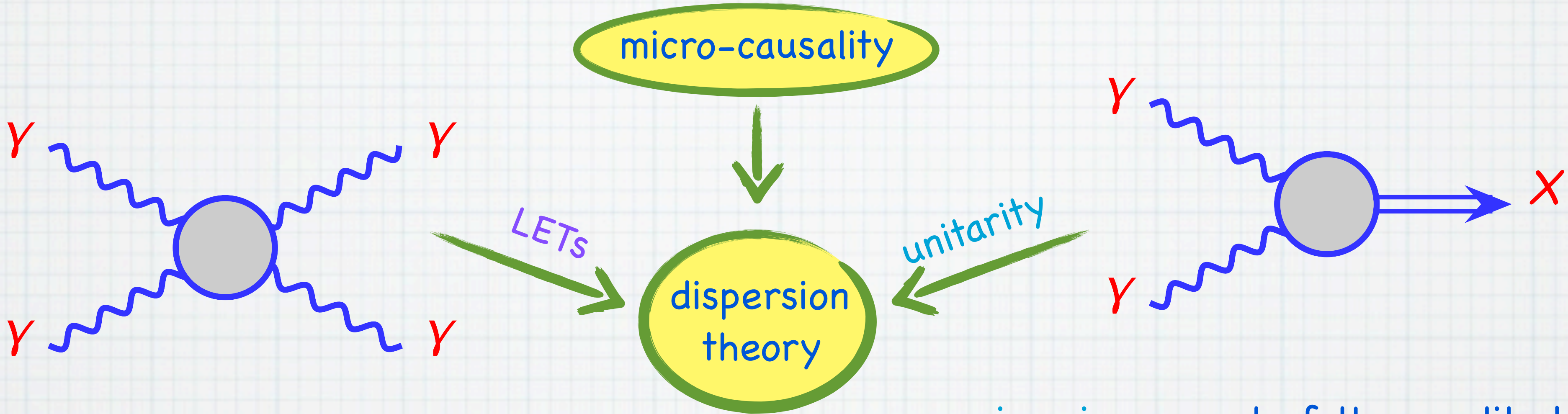


Sum rules



micro-causality
γ
γ
γ
γ
LETs
dispersion theory
unitarity
γ
γ
X

Sum rules

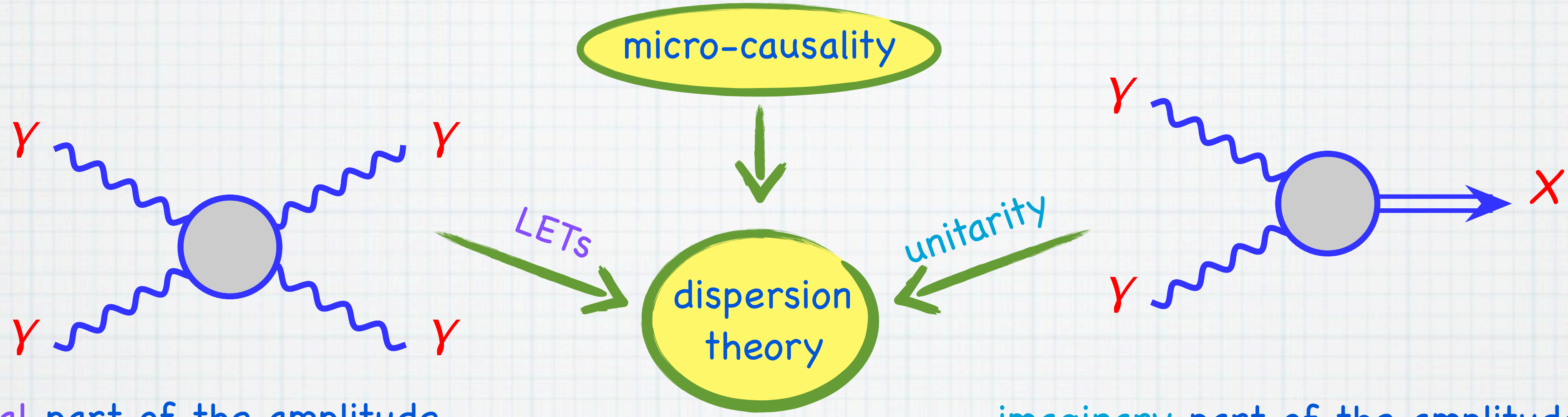


imaginary part of the amplitude - photon-photon fusion into leptons and hadrons:

$$\mathrm{Im} f^{(-)}(s) = -\frac{s}{8}[\sigma_2(s) - \sigma_0(s)]$$

$$\mathrm{Im} f^{(+)}(s) = -\frac{s}{8}[\sigma_{tot}(s)]$$

Sum rules



real part of the amplitude – gauge symmetry ⟶ low-energy structure of the elastic LbL scattering:

$$\mathcal{L}^{(8)} = c_1 (F_{\mu\nu} F^{\mu\nu})^2 + c_2 (F_{\mu\nu} \tilde{F}^{\mu\nu})^2$$

Euler, Heisenberg (1936)

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Sum rules

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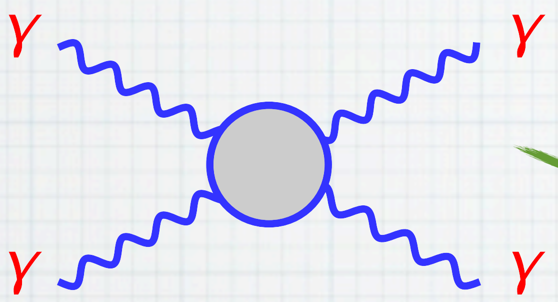
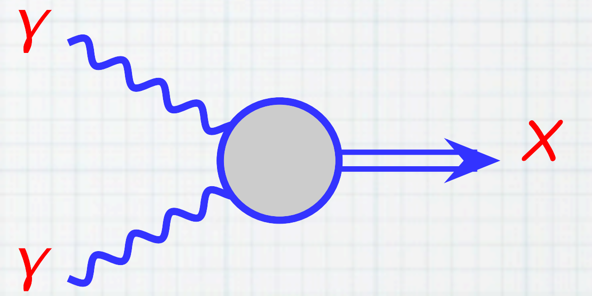


LETs

dispersion theory

unitarity



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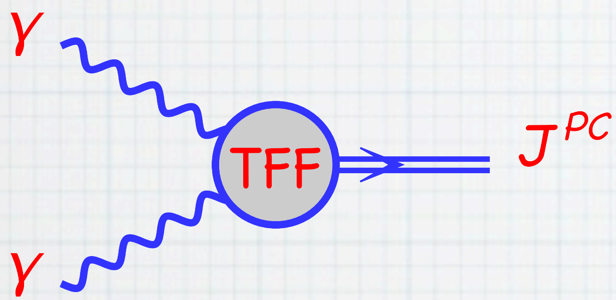
$$\mathrm{Im} f^{(+)}(s) = -\frac{s}{8}[\sigma_{tot}(s)]$$

$$\int_{s_0}^{\infty} \frac{ds}{s} [\sigma_2(s) - \sigma_0(s)] = 0$$

V. Pascalutsa, M. Vdh

$$c_1 \pm c_2 = \frac{1}{8\pi} \int_{s_0}^{\infty} \frac{ds}{s^2} [\sigma_{\parallel}(s) \pm \sigma_{\perp}(s)]$$

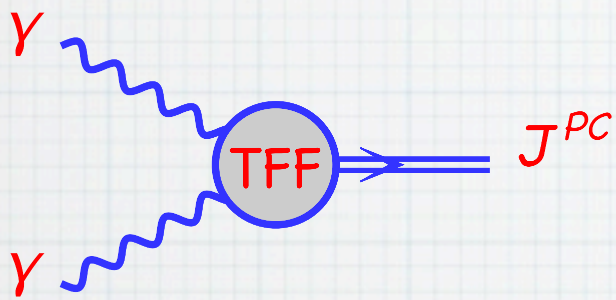
Meson production



- the SRs hold separately for channels of given intrinsic quantum numbers: isoscalar and isovector mesons, cc states

- input for the absorptive part of the SRs: $\gamma\,\gamma$ - hadrons response functions, can be expressed in terms of $\gamma\,\gamma \to M$ transition form factors

Meson production



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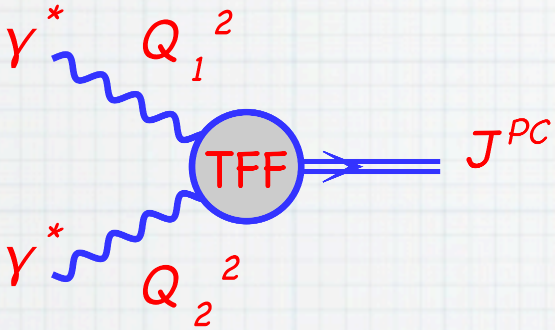
isoscalar light quark states:

the contribution of η, η'

is entirely compensated by $f_2(1270)$, $f_2(1565)$ and $f_2'(1525)$

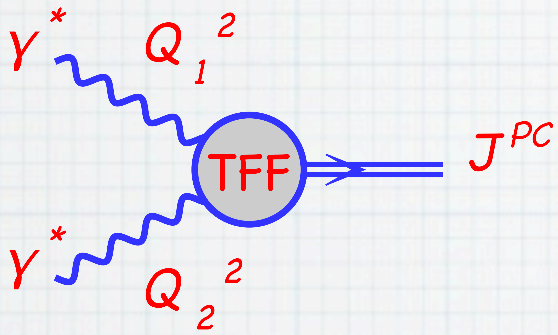
	$\int \frac{ds}{s}(\sigma_2 - \sigma_0)$ [nb]
η	-191 ± 10
η'	-300 ± 10
$f_0(980)$	-19 ± 5
$f_0'(1370)$	-91 ± 36
$f_2(1270)$	449 ± 52
$f_2'(1525)$	7 ± 1
$f_2(1565)$	56 ± 11
Sum	-89 ± 66

Meson production in $\gamma^* \gamma$ collision: TFF



at finite Q_1^2 the SRs imply information on meson transition form-factors:

Meson production in $\gamma^* \gamma$ collision: TFF



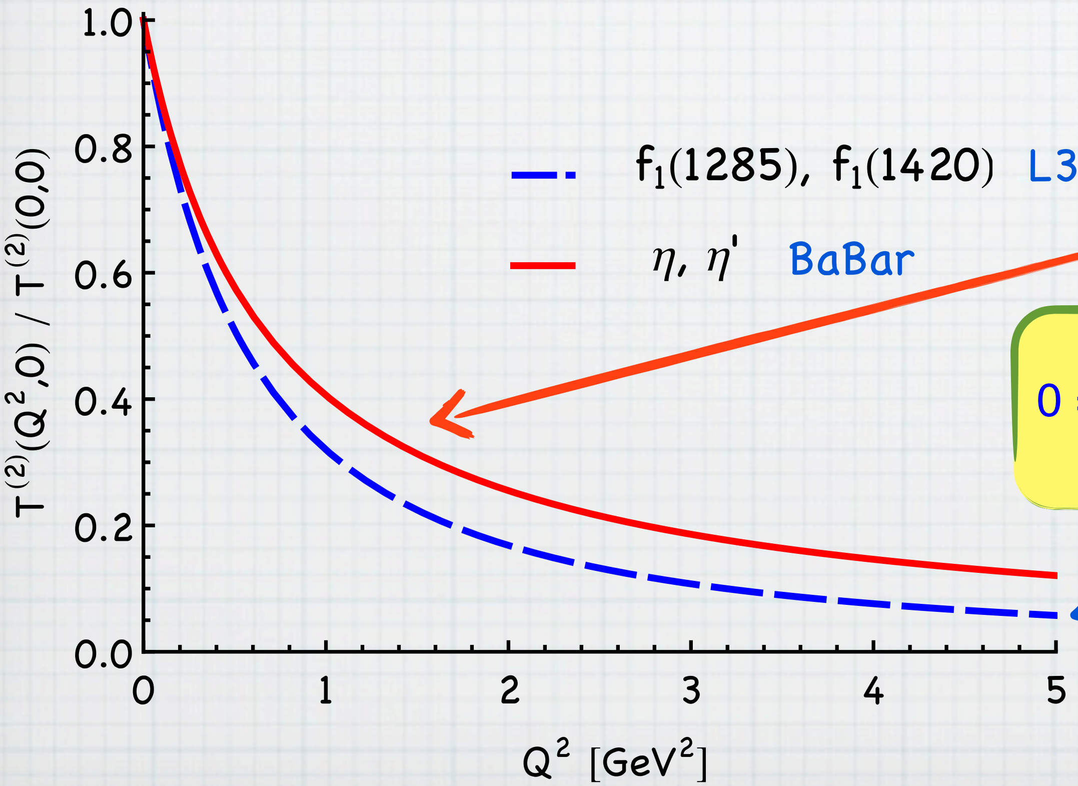
at finite Q_1^2 the SRs imply information on

meson transition form-factors:

estimate for the $f_2(1270)$ tensor FF in terms of

the η, η' and f_1 FFs

f₂(1270)



$$0 = \int_{s_0}^{\infty} ds \frac{1}{(s+Q_1^2)} \left[\sigma_2 - \sigma_0\right]_{Q_2^2=0}$$

$$0 = \int_{s_0}^{\infty} ds \frac{1}{(s+Q_1^2)^2} \left[\sigma_\parallel + \sigma_{LT} + \frac{(s+Q_1^2)}{Q_1 Q_2} \tau_{TL}^a \right]_{Q_2^2=0}$$

direct measurements → **BES III**

Scalars and tensors: results

contribution of the narrow scalar resonances

	m_M [MeV]	$\Gamma_{\gamma\gamma}$ [keV]	a_μ ($\Lambda_{mono} = 1$ GeV) [10^{-11}]	a_μ ($\Lambda_{mono} = 2$ GeV) [10^{-11}]
$f_0(980)$	980 ± 10	0.29 ± 0.07	-0.19 ± 0.05	-0.61 ± 0.15
$f_0'(1370)$	$1200 - 1500$	3.8 ± 1.5	-0.54 ± 0.21	-1.84 ± 0.73
$a_0(980)$	980 ± 20	0.3 ± 0.1	-0.20 ± 0.07	-0.63 ± 0.21
Sum			-0.9 ± 0.2	-3.1 ± 0.8

contribution of the narrow tensor resonances

	m_M [MeV]	$\Gamma_{\gamma\gamma}$ [keV]	a_μ ($\Lambda_{dip} = 1.5$ GeV) [10^{-11}]
$f_2(1270)$	1275.1 ± 1.2	3.03 ± 0.35	0.79 ± 0.09
$f_2(1565)$	1562 ± 13	0.70 ± 0.14	0.07 ± 0.01
$a_2(1320)$	1318.3 ± 0.6	1.00 ± 0.06	0.22 ± 0.01
$a_2(1700)$	1732 ± 16	0.30 ± 0.05	0.02 ± 0.003
Sum			1.1 ± 0.1

Scalars and tensors: results

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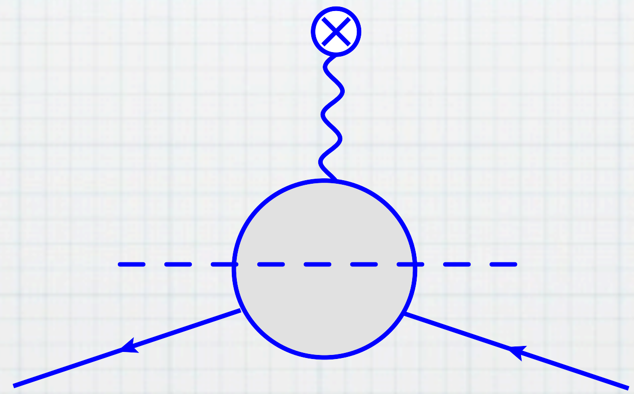
$(g\text{-}2)_\mu$ and dispersion relations

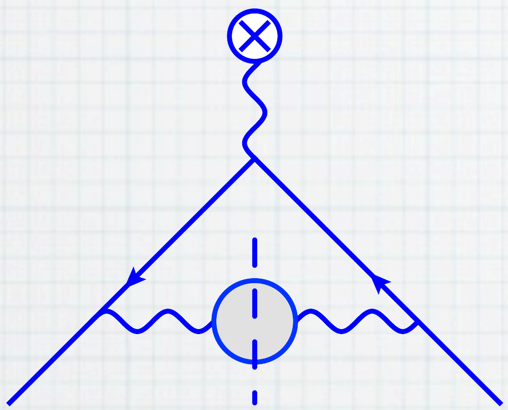


Dispersion approach

Dispersion approach

hadronic sector:

vacuum polarization in g-2



Dispersion approach

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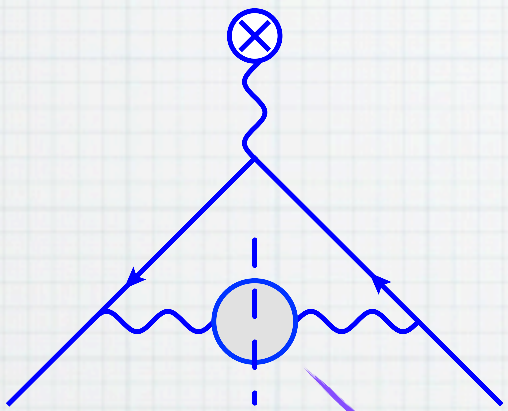
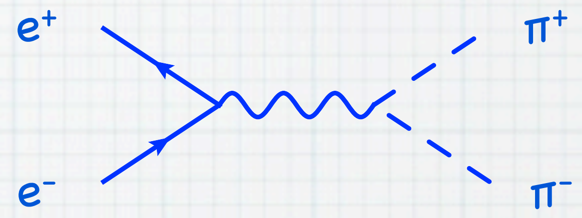


e^+e^- – production of hadrons



dispersion relations

$$\Pi(q^2) - \Pi(0) = \frac{q^2}{\pi} \int_0^\infty \mathrm{d}s \, \frac{\mathrm{Im}\,\Pi(s)}{s(s-q^2)}$$

Dispersion approach

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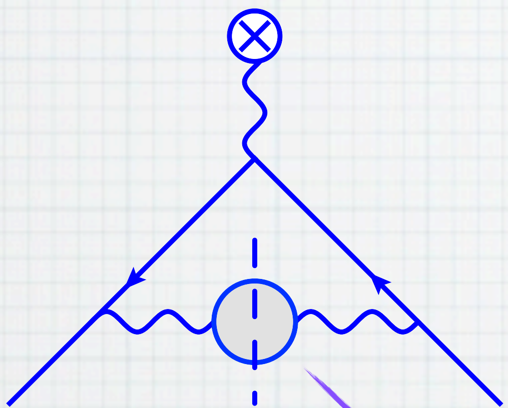

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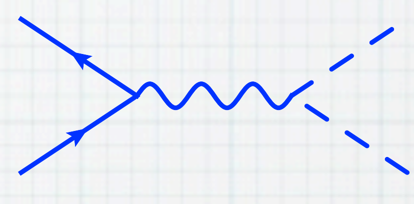

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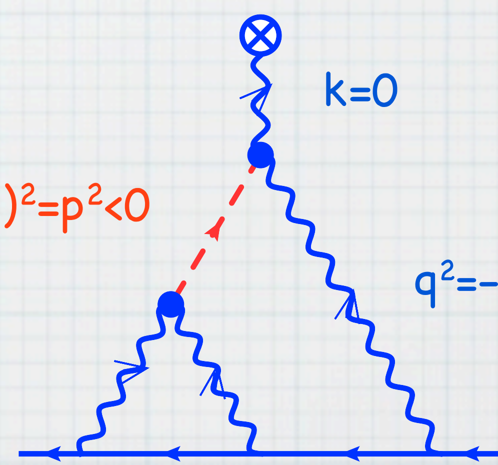
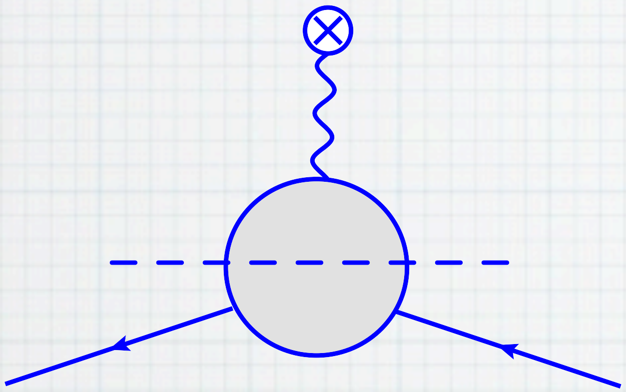
the hadronic state has negative invariant mass:
NO dispersion relation can be written!!



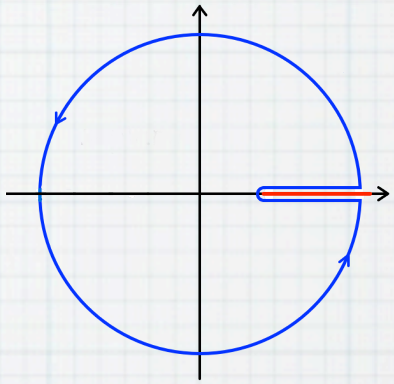
Scalar theory

Scalar theory

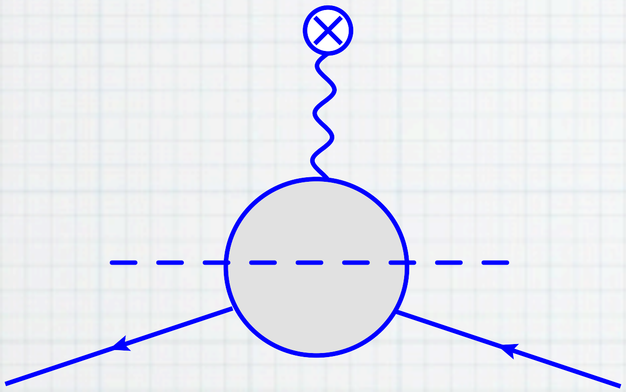


Dispersion relations

$$F(q^2) = \frac{1}{\pi i}\int \frac{q'\mathrm{d}q'}{q'^2 - q^2}\mathrm{Disc}F(q'^2)$$



Scalar theory



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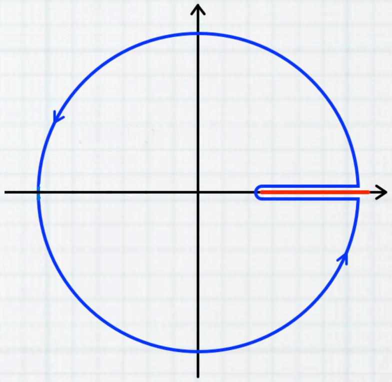
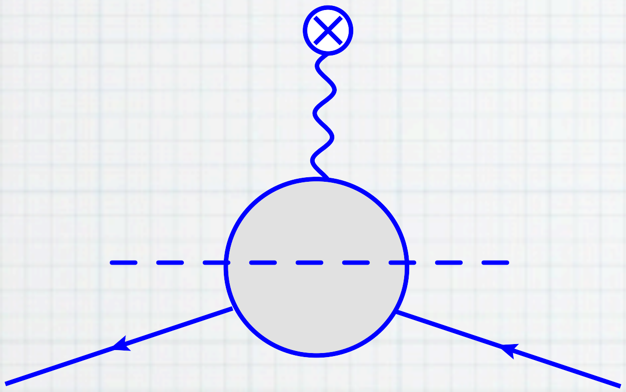


Discontinuity

Generalized unitarity
(Cutkosky rules)

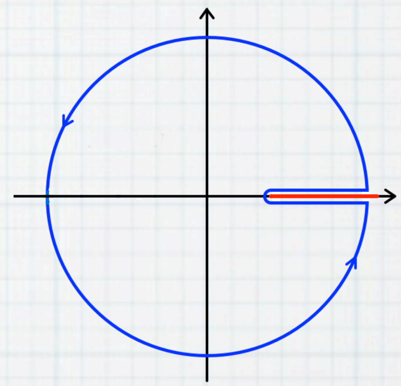
$$2\mathrm{Disc}\mathcal{M}_{if} = \sum_n \mathcal{M}_{in}\mathcal{M}^*_{nf}$$

Scalar theory



Dispersion relations

$$F(q^2) = \frac{1}{\pi i}\int \frac{q' \mathrm{d}q'}{q'^2 - q^2}\mathrm{Disc}F(q'^2)$$



Discontinuity

Generalized unitarity
(Cutkosky rules)

$$2\mathrm{Disc}\mathcal{M}_{if} = \sum_n \mathcal{M}_{in}\mathcal{M}^*_{nf}$$

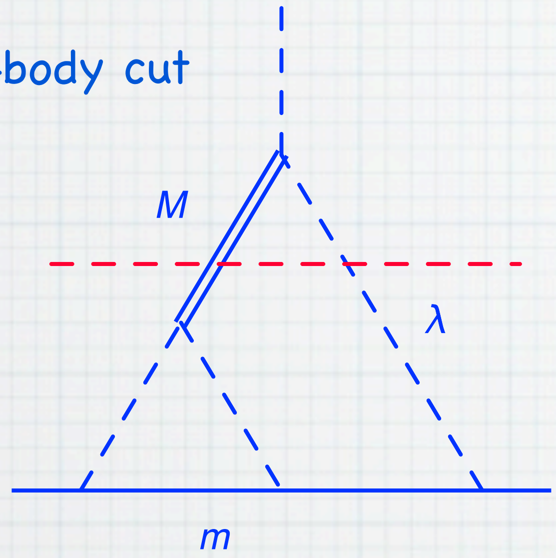
$$\mathrm{Disc}F(q'^2) = \mathrm{Disc}_2 F(q'^2) + \mathrm{Disc}_3 F(q'^2)$$

$$\left[\mathrm{Disc}_2 F(q'^2)\right] = \int \mathrm{d}\Phi_2 \mathcal{M}_{1\to 2}\mathcal{M}^*_{2\to 2}$$

$$\left[\mathrm{Disc}_3 F(q'^2)\right] = \int \mathrm{d}\Phi_3 \mathcal{M}_{1\to 3}\mathcal{M}^*_{3\to 2}$$

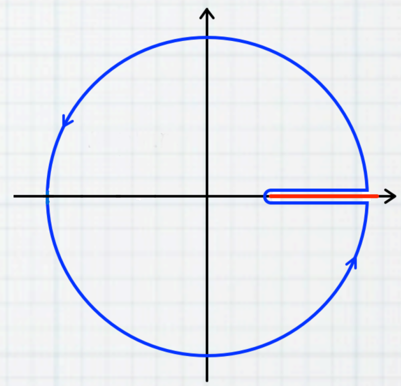
Scalar theory

2-body cut



Dispersion relations

$$F(q^2) = \frac{1}{\pi i} \int \frac{q' \mathrm{d}q'}{q'^2 - q^2} \mathrm{Disc} F(q'^2)$$



Discontinuity

Generalized unitarity
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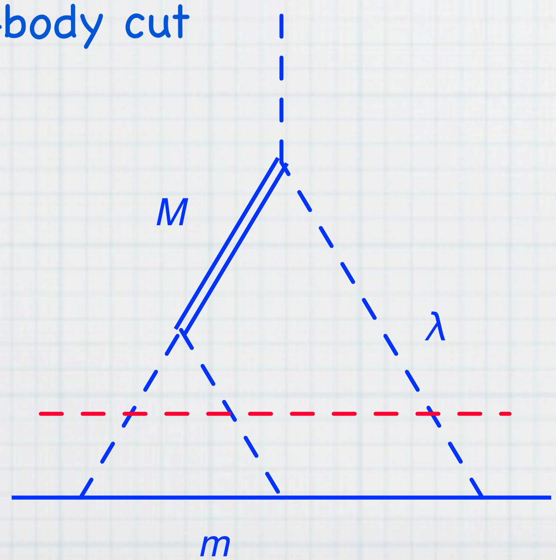
$$2 \mathrm{Disc} \mathcal{M}_{if} = \sum_n \mathcal{M}_{in} \mathcal{M}^*_{nf}$$

3-body cut



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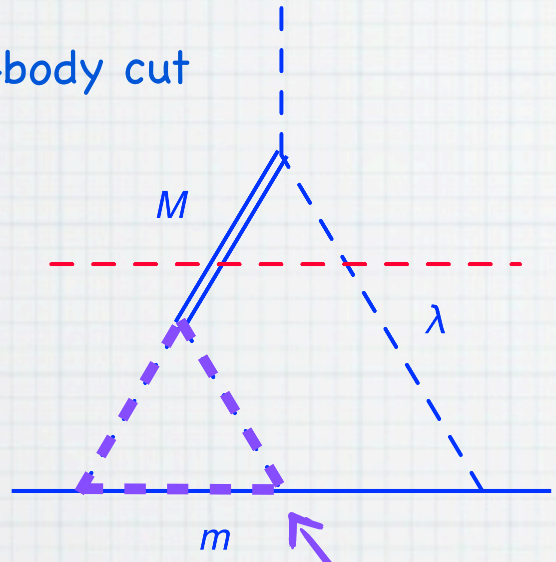
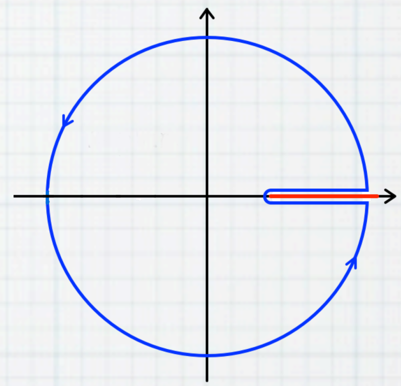
Scalar theory

2-body cut



Dispersion relations

$$F(q^2) = \frac{1}{\pi i}\int \frac{q' \mathrm{d}q'}{q'^2 - q^2}\mathrm{Disc}F(q'^2)$$



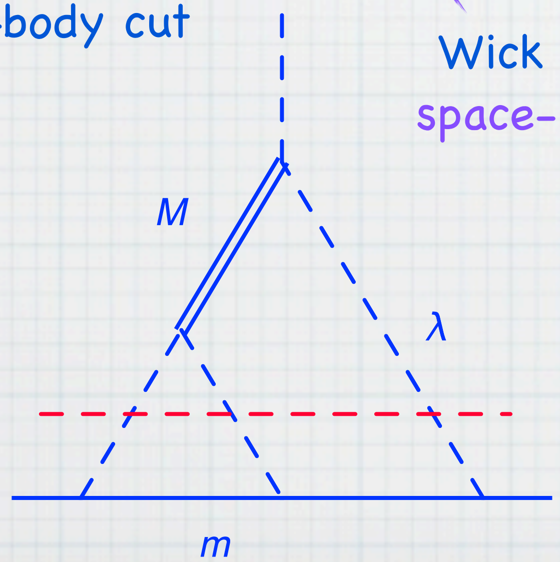
Discontinuity

Generalized unitarity
(Cutkosky rules)

$$2\mathrm{Disc}\mathcal{M}_{if} = \sum_n \mathcal{M}_{in}\mathcal{M}^*_{nf}$$

Wick rotation:
space-like region

3-body cut



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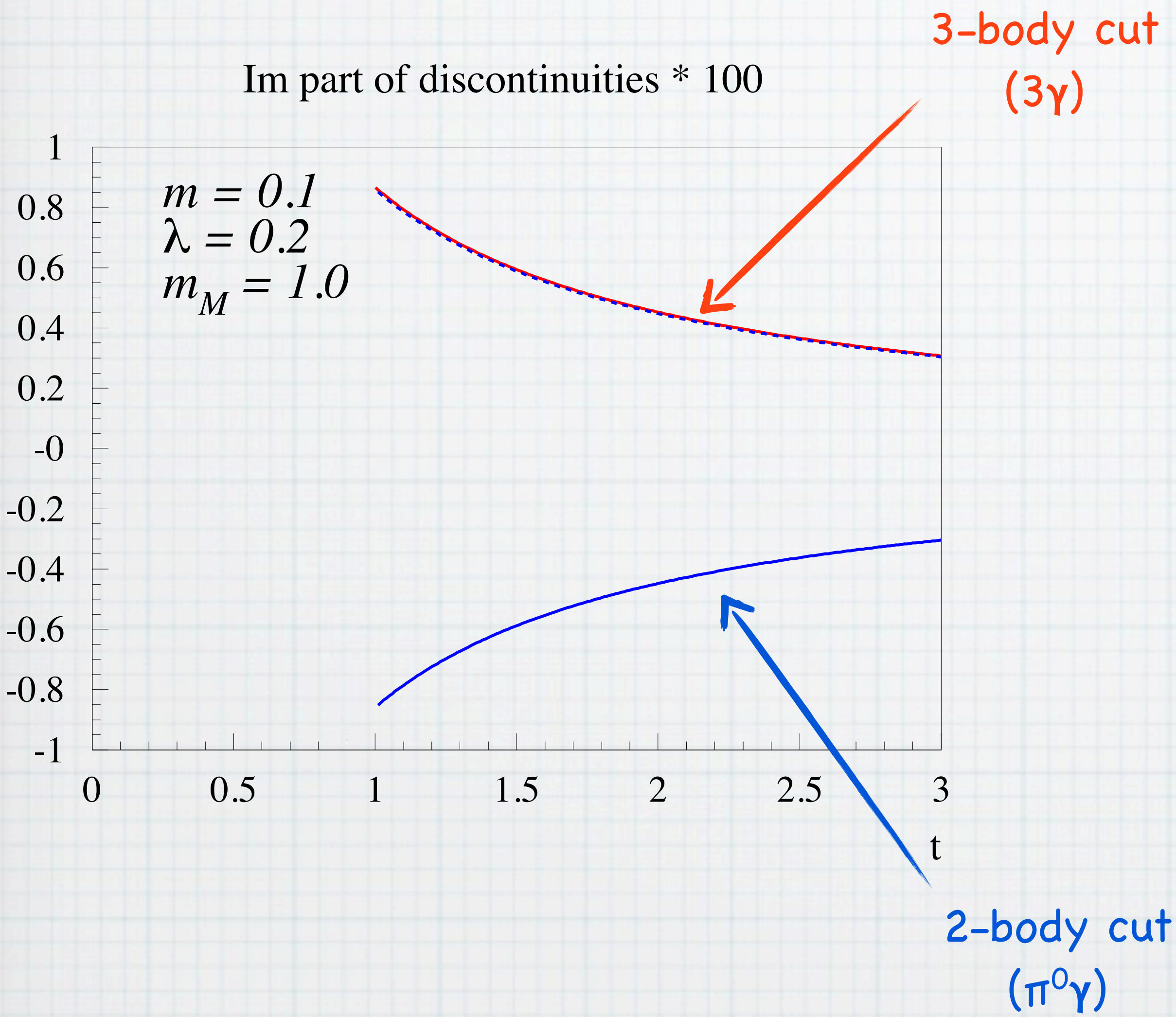
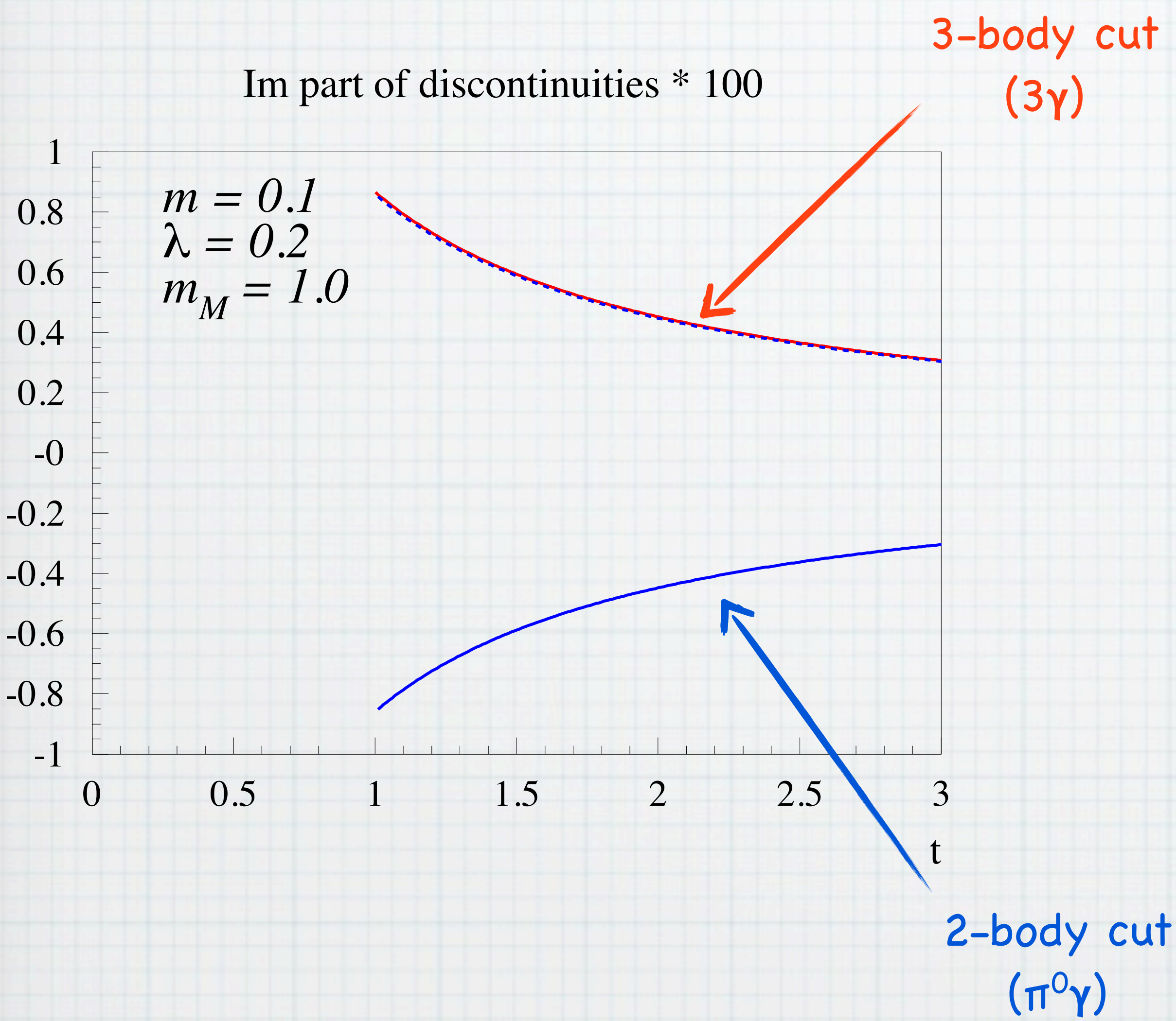
Discontinuity

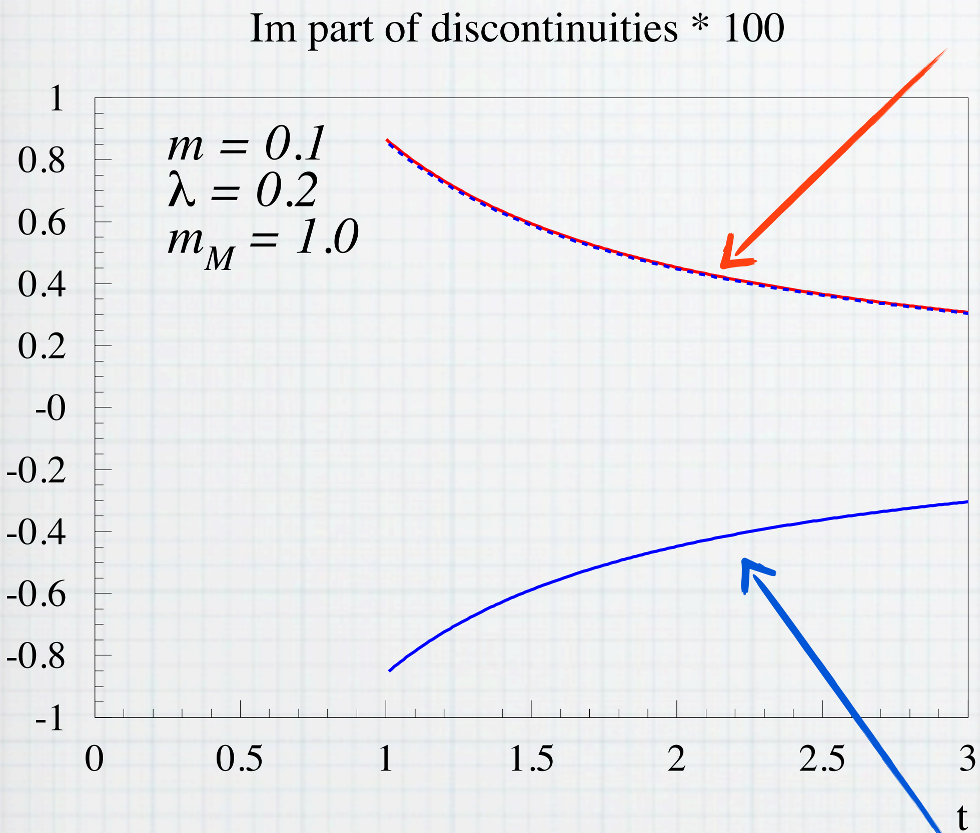


Discontinuity

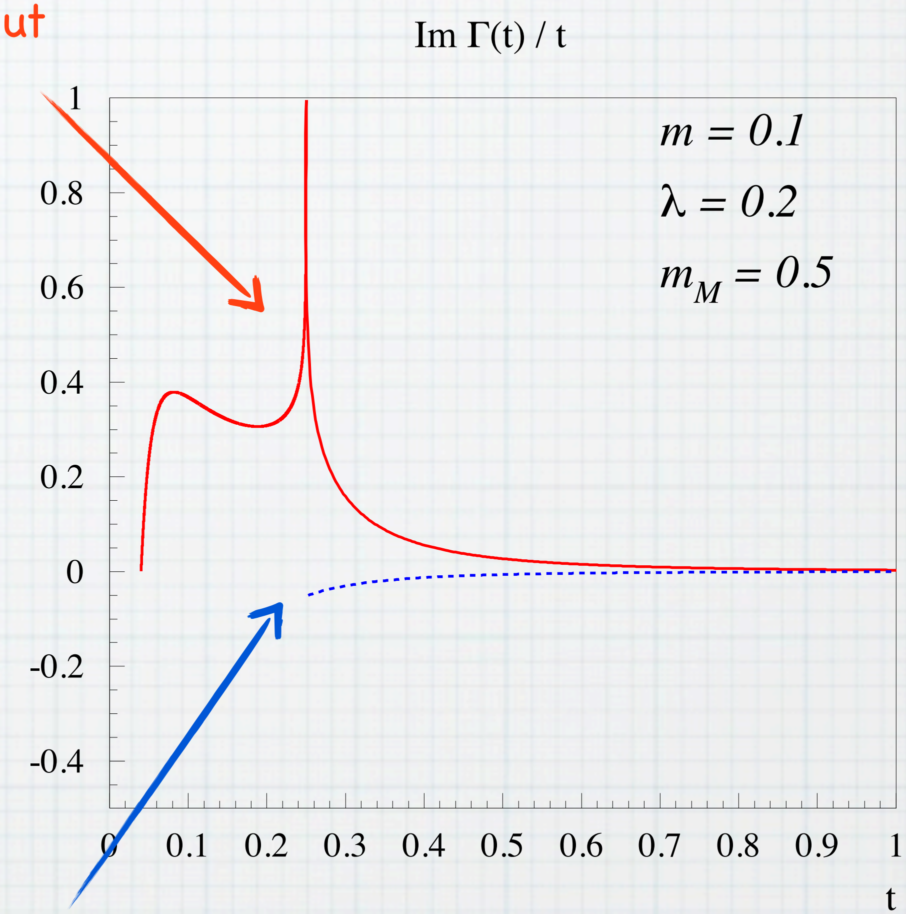


Imaginary parts cancel: $\mathrm{Im}\left[\mathrm{Disc}_2 F(q'^2)\right] + \mathrm{Im}\left[\mathrm{Disc}_3 F(q'^2)\right] = 0$

Discontinuity



3-body cut
(3γ)

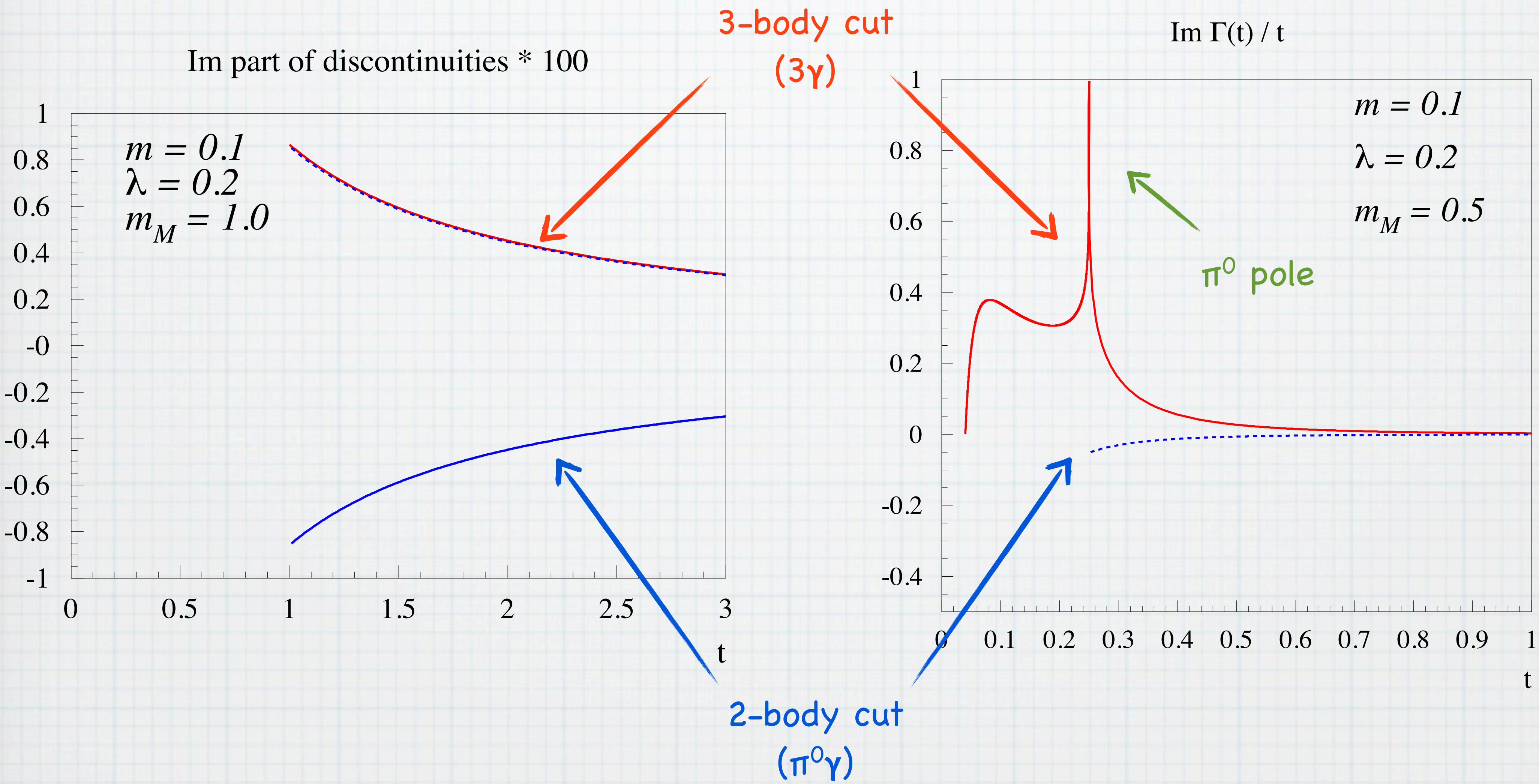


2-body cut
($\pi^0\gamma$)

Imaginary parts cancel:

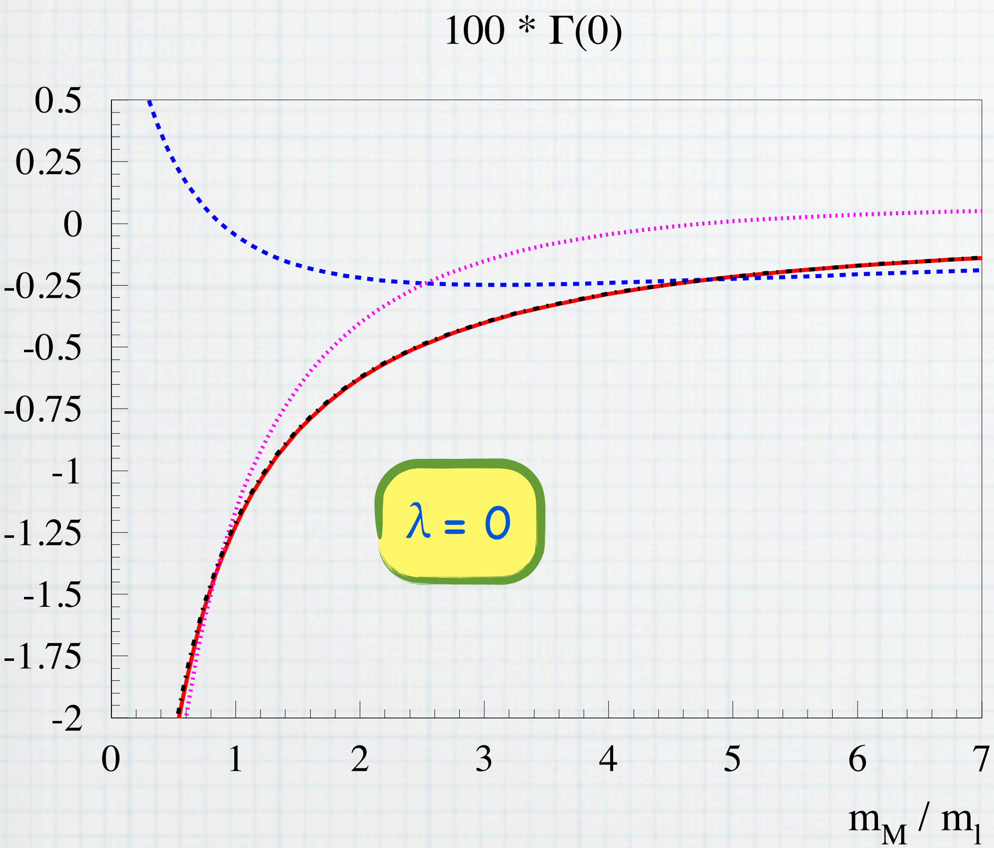
$$\mathrm{Im}\left[\mathrm{Disc}_2 F(q'^2)\right] + \mathrm{Im}\left[\mathrm{Disc}_3 F(q'^2)\right] = 0$$

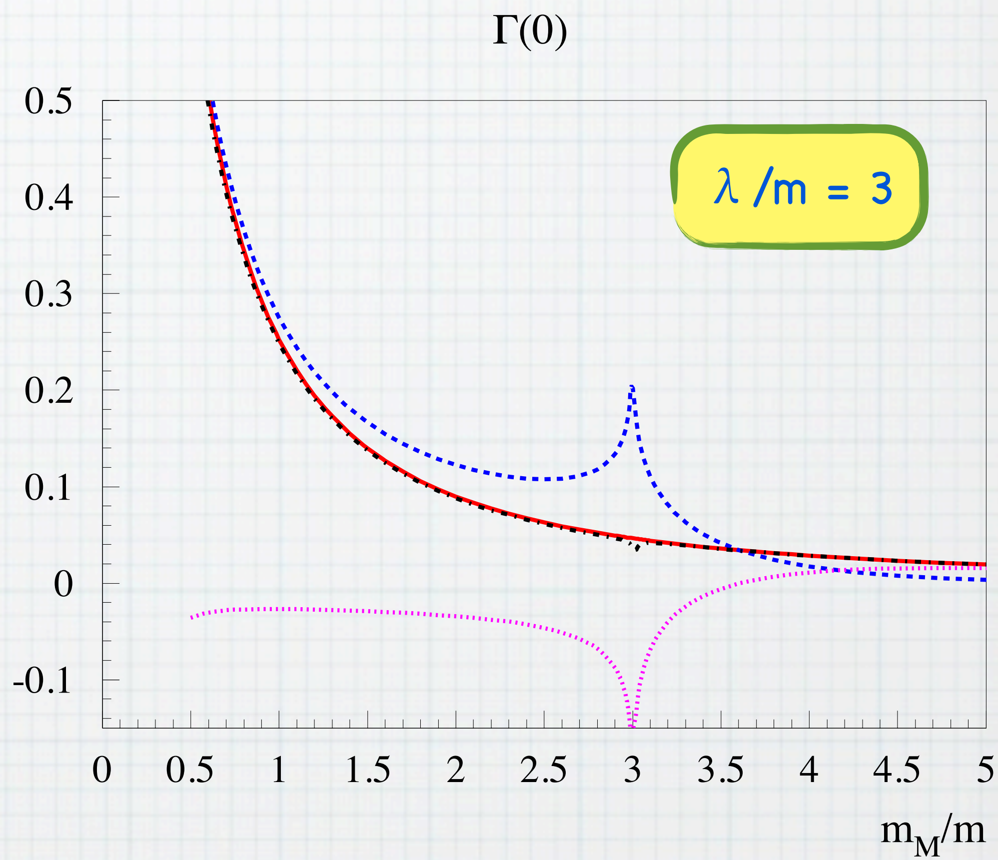
Discontinuity



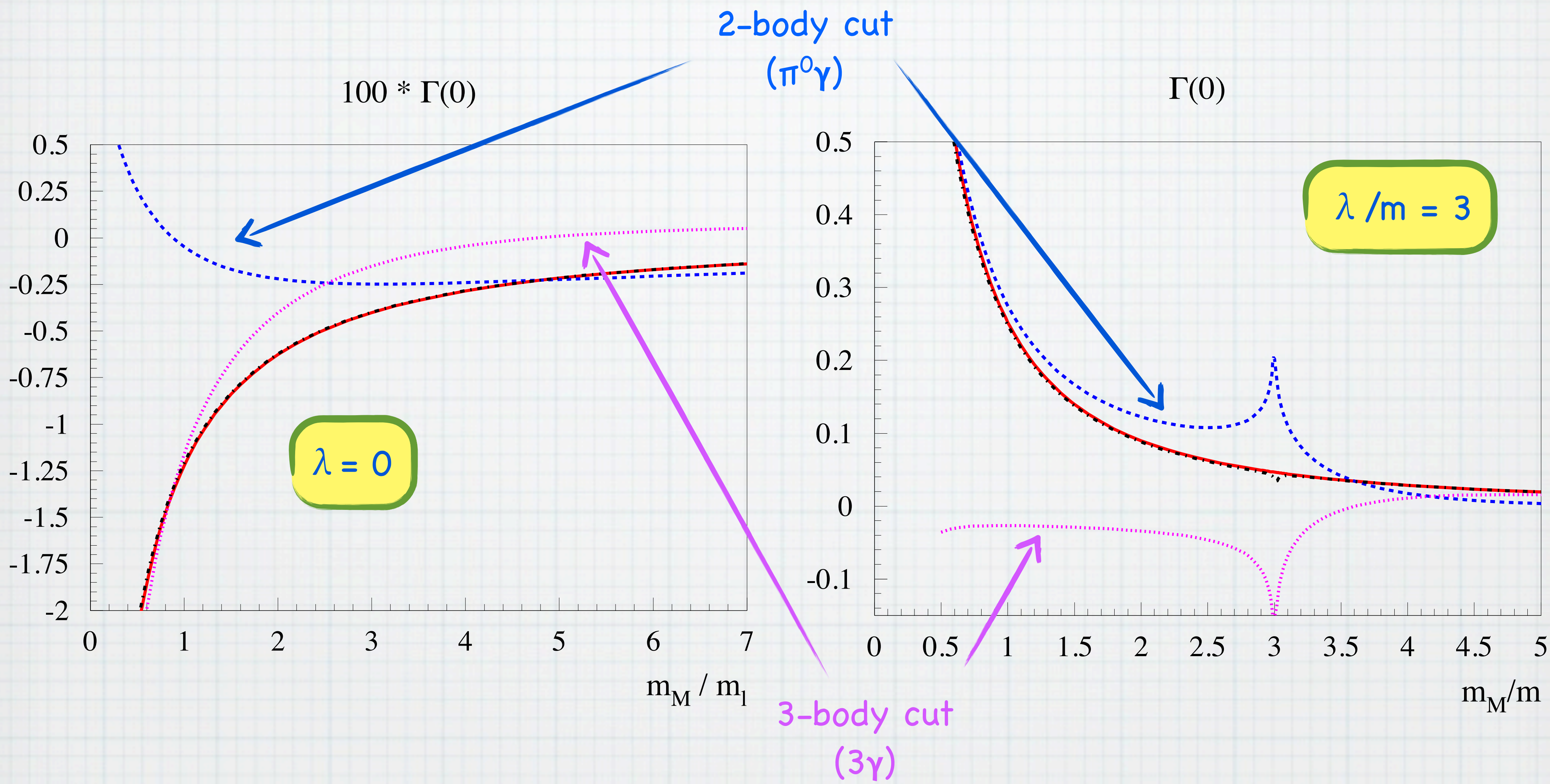
Imaginary parts cancel: $\mathrm{Im}\left[\mathrm{Disc}_2 F(q'^2)\right] + \mathrm{Im}\left[\mathrm{Disc}_3 F(q'^2)\right] = 0$

Real parts

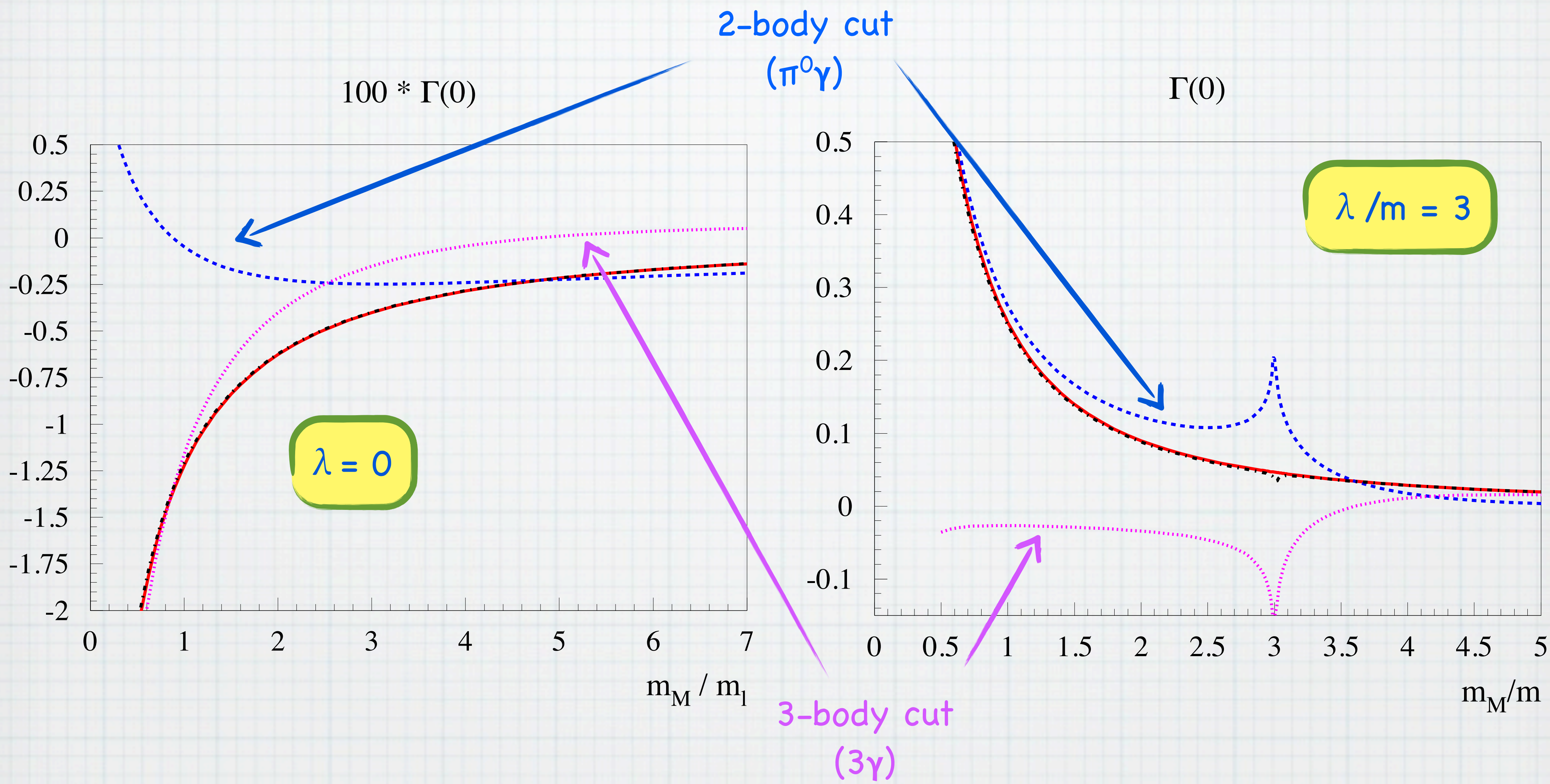

100 * Γ(0)
0.5
0.25
0
-0.25
-0.5
-0.75
-1
-1.25
-1.5
-1.75
-2
0
1
2
3
4
5
6
7
m_M / m_l
λ = 0


Γ(0)
0.5
0.4
0.3
0.2
0.1
0
-0.1
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
m_M/m
λ /m = 3

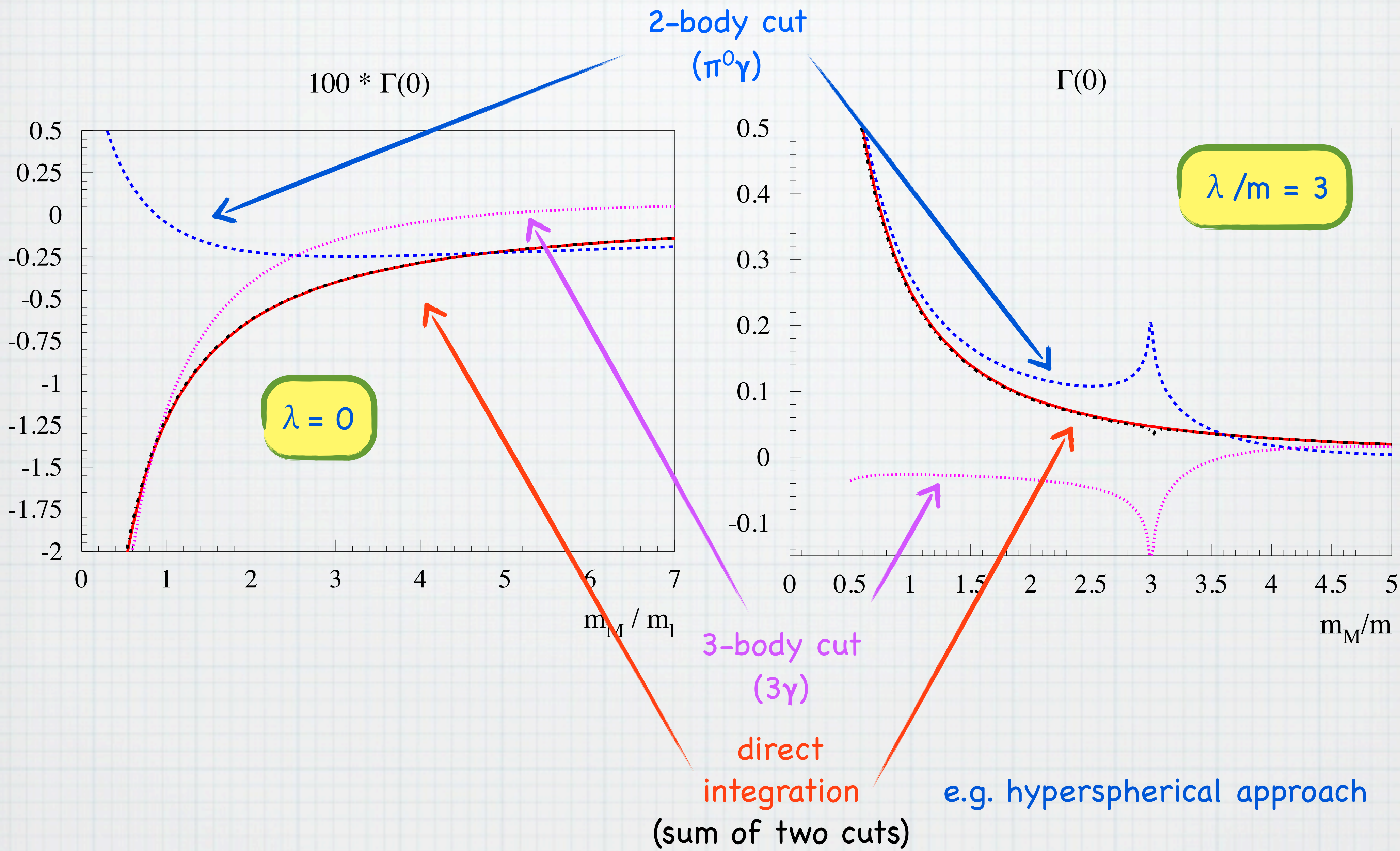
Real parts


2-body cut
(π⁰γ)
100 * Γ(0)
Γ(0)
0.5
0.25
0
-0.25
-0.5
-0.75
-1
-1.25
-1.5
-1.75
-2
0 1 2 3 4 5 6 7
m_M / m_l
λ = 0
0.5
0.4
0.3
0.2
0.1
0
-0.1
0 0.5 1 1.5 2 2.5 3 3.5 4 4.5 5
m_M/m
λ /m = 3
3-body cut
(3γ)

Real parts

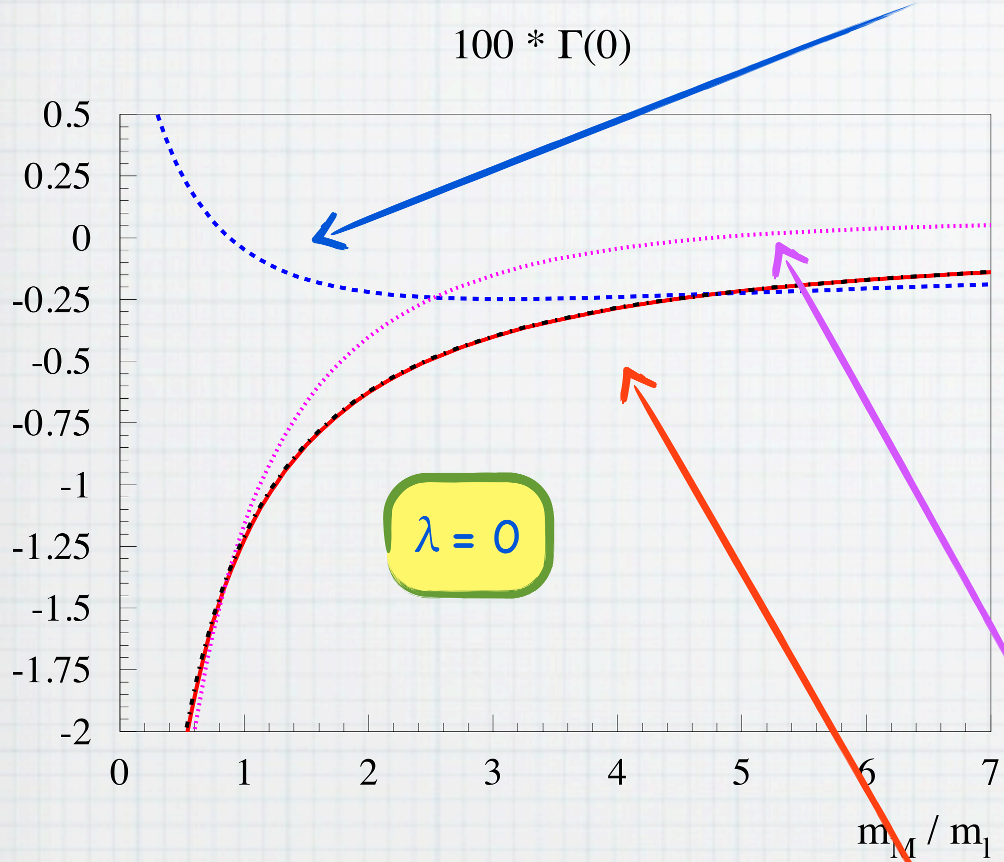

2-body cut
($\pi^0\gamma$)
100 * Γ(0)
Γ(0)
0.5
0.25
0
-0.25
-0.5
-0.75
-1
-1.25
-1.5
-1.75
-2
0 1 2 3 4 5 6 7
m_M / m_l
λ = 0
0.5
0.4
0.3
0.2
0.1
0
-0.1
0 0.5 1 1.5 2 2.5 3 3.5 4 4.5 5
m_M/m
λ /m = 3
3-body cut
(3γ)

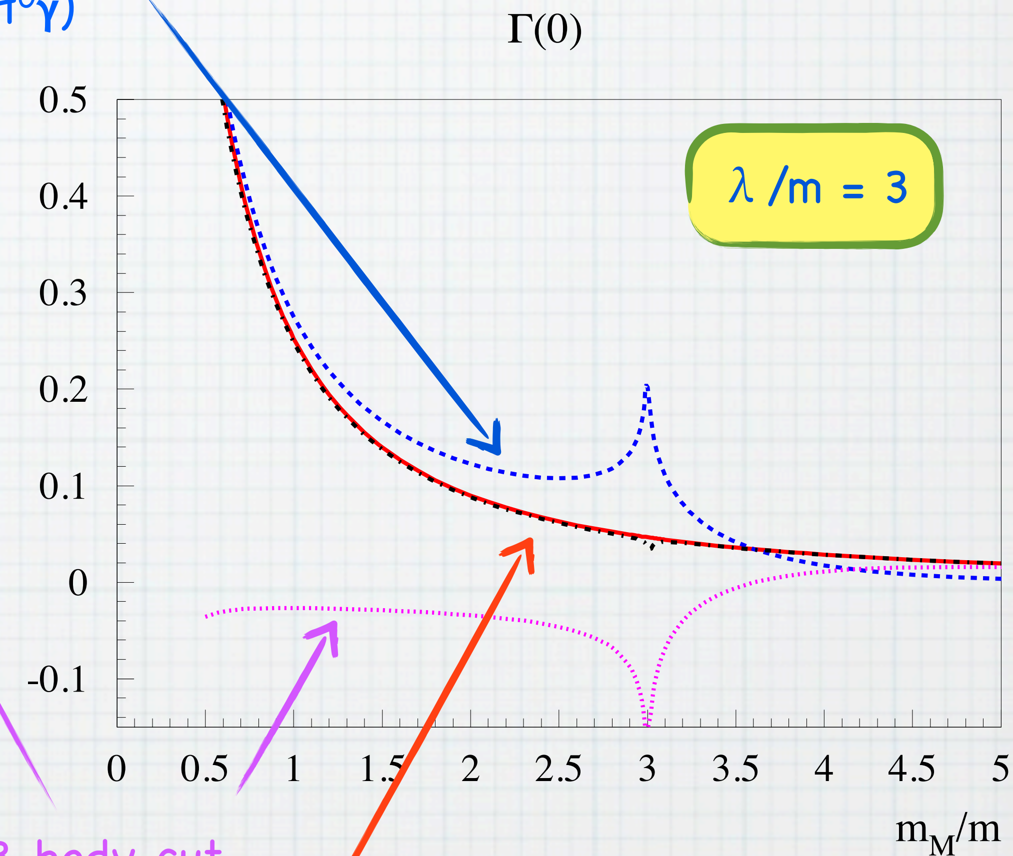
Real parts


2-body cut
($\pi^0\gamma$)
100 * Γ(0)
Γ(0)
λ = 0
λ /m = 3
0.5
0.25
0
-0.25
-0.5
-0.75
-1
-1.25
-1.5
-1.75
-2
0 1 2 3 4 5 6 7
m_M / m_l
0.5
0.4
0.3
0.2
0.1
0
-0.1
0 0.5 1 1.5 2 2.5 3 3.5 4 4.5 5
m_M/m
3-body cut
(3γ)
direct
integration
(sum of two cuts)
e.g. hyperspherical approach

Real parts

2-body cut ($\pi^0\gamma$)





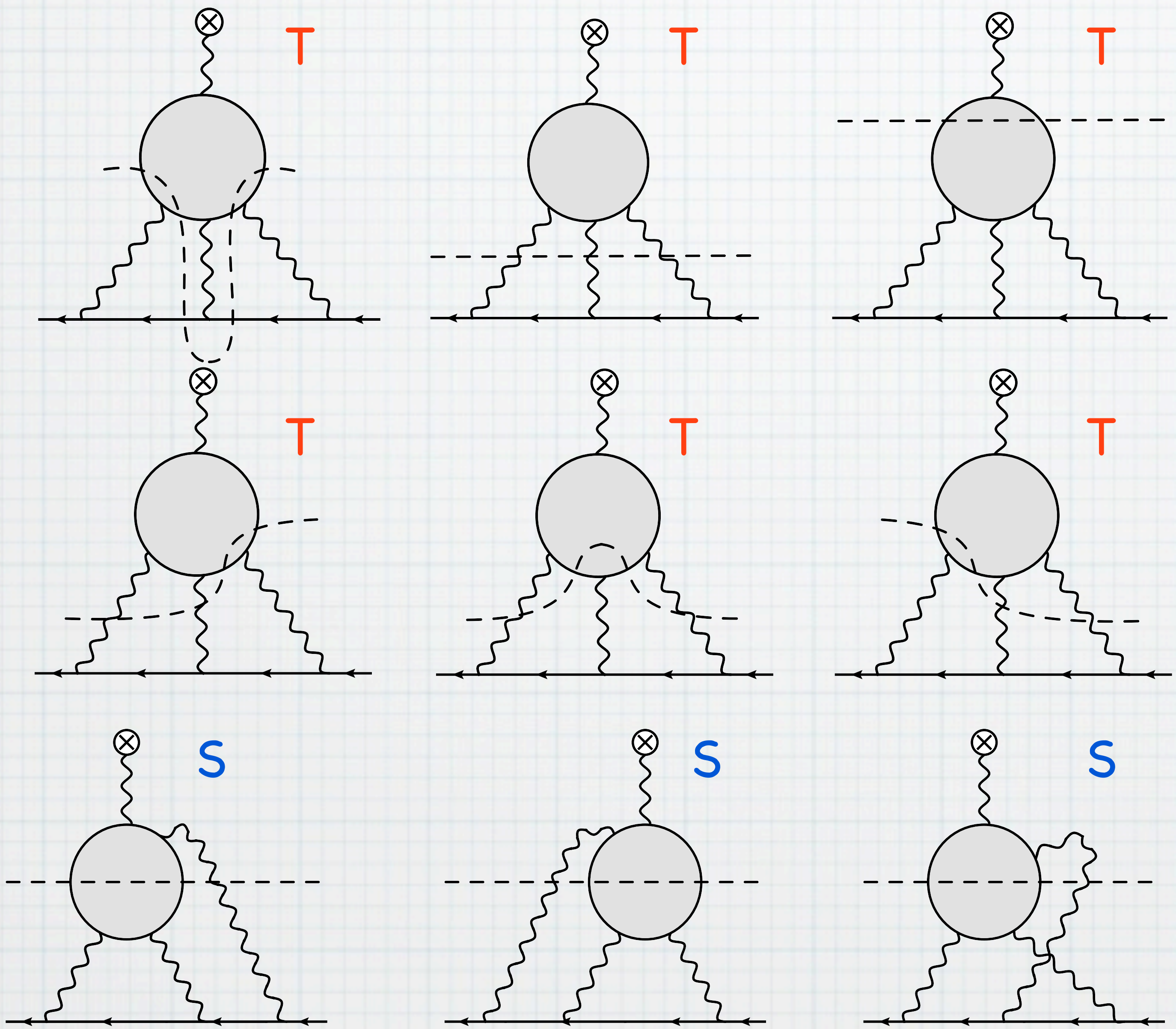
3-body cut (3γ)

complete agreement!

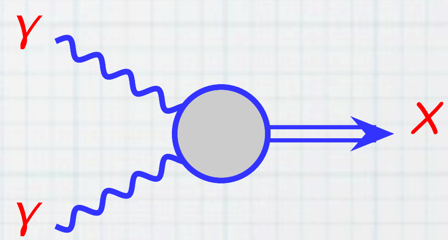
direct integration (sum of two cuts)

e.g. hyperspherical approach

LbL discontinuity



T - time-like information



S - time-like information

...towards a model independent evaluation

$\pi_0 \gamma^* \gamma^*$ transition FF

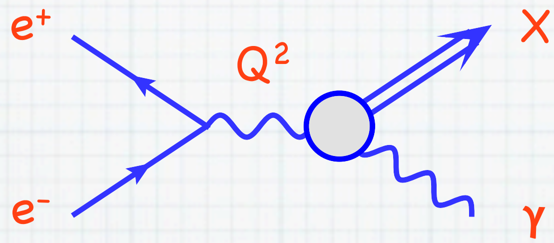
CMD-2

e^+e^- colliders

$\pi_0 \gamma^* \gamma^*$ transition FF

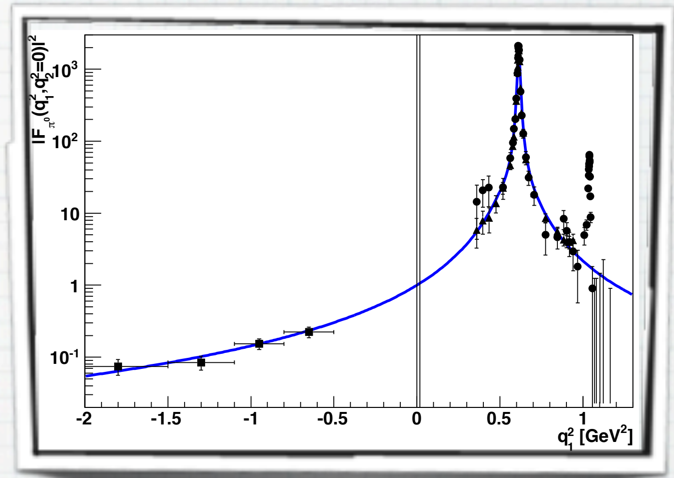
$\pi_0 \gamma \gamma$ transition FF

in the time-like region



e+e- colliders

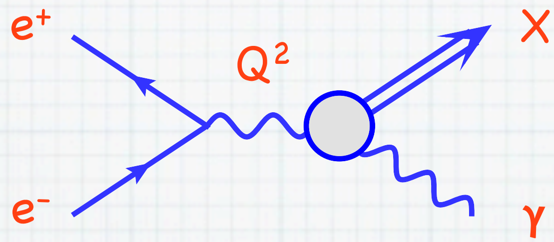
CMD-2



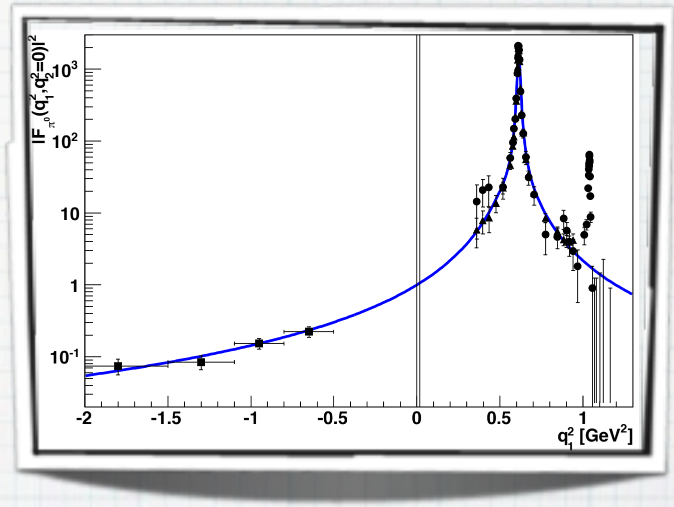
$\pi_0 \gamma^* \gamma^*$ transition FF

$\pi_0 \gamma\ \gamma$ transition FF

in the time-like region



CMD-2

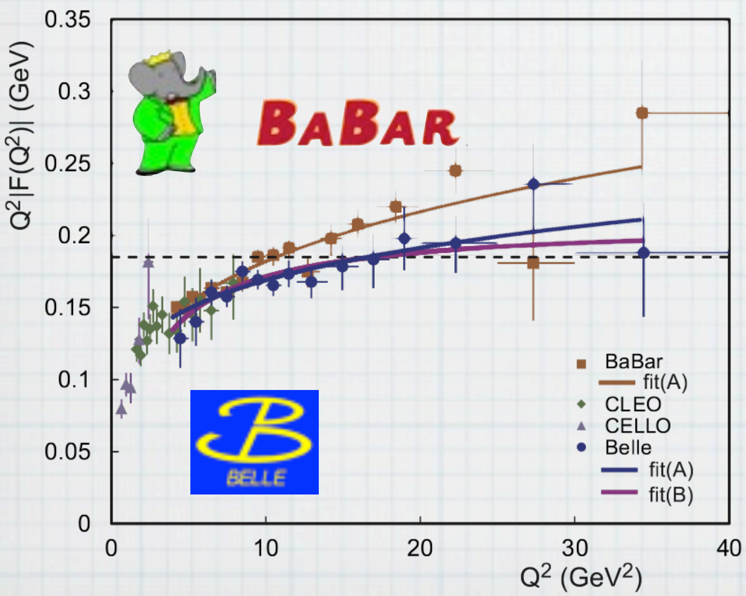


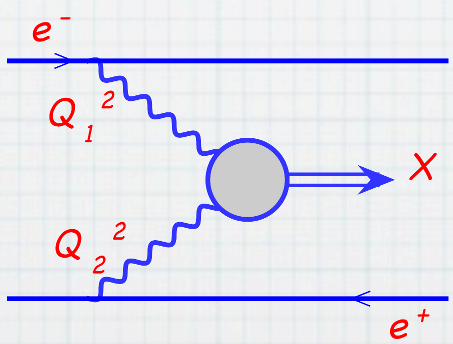
e^+e^- colliders

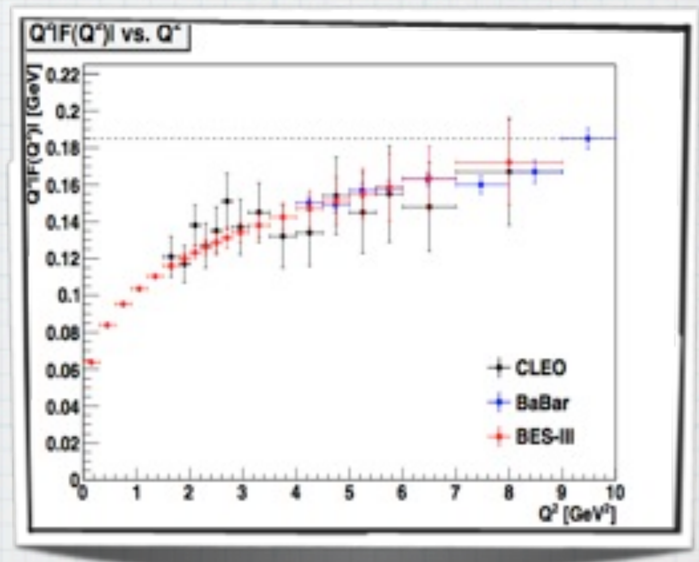
$\pi_0 \gamma\ \gamma$ transition FF

in the space-like region

BES III







space-like region

$Q_2^2 < 40\ GeV^2$

for $Q_1^2=0$

Perspectives & conclusions

direct calculation in field theory does NOT give needed precision!

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need of experimental input

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estimates based on data:

light-quark states are dominating:

π^0, $\pi^+ \pi^-$, η , η', a_1 ...

Perspectives & conclusions

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need of experimental input

estimates based on data:

model dependence

light-quark states are dominating:

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π^0, $\pi^+\pi^-$, η, η', a_1 ...

dispersion relations

Perspectives & conclusions

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need of experimental input

estimates based on data:

model dependence

light-quark states are dominating:
π^0, $\pi^+ \pi^-$, η , η', a_1 ...

dispersion relations

allow to incorporate
non-perturbative information in terms of
on-shell information -
can be measured in experiment!

Perspectives & conclusions

direct calculation in field theory does NOT give needed precision!

need of experimental input

estimates based on data:

light-quark states are dominating: π^0, $\pi^+ \pi^-$, η, η', a_1 ...

model dependence

dispersion relations

allow to incorporate non-perturbative information in terms of on-shell information - can be measured in experiment!

time- and space-like data are needed


Thank you!