Possible medium effect on η-π^0 mixing angle and $\eta \to \pi^0\pi^+\pi^-$ decay in asymmetric nuclear matter

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- Brief history of $\eta \to \pi^0\pi^+\pi^-$ decay in free space
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  - $\eta \to \pi^0\pi^+\pi^-$ width

    in asymmetric nuclear matter
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Hadron physics and symmetry

- The various symmetries of QCD Lagrangian

$$\mathcal{L}_{QCD} = \bar{q}\left(i\not{D} - m_q\right)q - \frac{1}{4}\,\mathrm{tr}F_{\mu\nu}F^{\mu\nu}$$

  - Lorentz invariance, $SU(3)_{color}$ gauge symmetry, **Chiral symmetry**,…
- Spontaneous breaking of chiral symmetry
  - $SU(N_f)_L \times SU(N_f)_R \rightarrow SU(N_f)_f$
    - ✓ No parity doubling of hadrons in low energy region (ρ(770)↔a_1(1260))
  - Nambu-Goldstone (NG) bosons appear
    - ✓ Non-zero mass due to the current quark mass (chiral symmetry : approximate symmetry of QCD)
  - The important dynamical degrees of freedom in low energy QCD are "NG bosons with light masses"
    - ✓ **Chiral perturbation theory** (χPT) works well in low energy region degree of freedom : NG bosons

Brief history of $\eta \to \pi^0\pi^+\pi^-$ decay in free space

G parity:
η:even, π:odd
Ex.) ρ and ω decay
dominant decay mode
ρ→2π, ω→3π
(ρ:even, ω:odd)

- Prohibited by the G parity conservation
  (Isospin symmetry is exact→prohibited)
  - $O(e^2)$ correction vanishes
    D.G.Sutherland,Phys.Lett.23(1966)384.

  ➡ The intrinsic ***isospin-symmetry breaking*** in QCD:
  The difference of the u, d quark mass is responsible.
  - Too small value from the tree-level result of non-linear sigma model
    H.Osborn,D.J.Wallace,Nucl. Phys.B20(1970)23.
- Discussed in terms of the $U_A(1)$ problem S. Weinberg, Phys. Rev. D11 (1975) 3583.
  - Goldstone dipole solution, σ model analysis,…
    J.Kogut,L.Susskind,PRD11(1975)3549. K.Kawarabayashi,N.Ohta,NPB175(1980)477.
- The significant effect of $\pi^+\pi^-$ final state interaction on $\eta \to \pi^0\pi^+\pi^-$
  C.Roiesnel,N.Truong,NPB187(1981)293;J.Gasser,H.Leutwyler,NPB250(1985)465.
- Recent two-loop calculation of χPT with some assumptions
  shows good agreement with experimental data
  J.Bijnens,K.Ghorbani,JHEP11(2007)030.

➡ Attract our attention for a long time

Asymmetric nuclear matter and isospin-symmetry breaking

In asymmetric nuclear matter, $\langle \bar{d}d \rangle^* - \langle \bar{u}u \rangle^* = \langle \bar{q}q \rangle_{\rho=0} \frac{4c_5}{f^2}(\rho_n - \rho_p)$

(f, c_5:Low energy constant of chiral Lagrangian)

Ulf-G. Meißner, J.A. Oller, A. Wirzba, Ann. Phys.297 (2002) 27

The isospin-symmetry breaking is enhanced in asymmetric nuclear matter.

Asymmetric nuclear matter

- Heavy nuclei (N>Z)
- Neutron halo or skin structure of light nuclei (experiments @ e.g. RIBF/RIKEN, FAIR/GSI)
- Inside of the compact star

Does the asymmetric nuclear matter affect $\eta \to \pi^0 \pi^+ \pi^-$?

change the property of isospin symmetry

isospin-symmetry breaking is responsible

✓ Possible change of hadron properties in ***Environment***

Heat bath, baryon density, external magnetic field,…

T.Hatsuda,R.Hayano,Rev.Mod.Phys.82(2010)2942;R.Rapp,J.Wambach,H.van Hees,arXiv:0901.3289.
U.-G.Meissner,et al.,Eur.Phys.J.A32(2007)299.

Purpose

We explore the effect of the **asymmetric nuclear matter** on the **$\eta \rightarrow \pi^0 \pi^+ \pi^-$ decay**.

- **Estimate the effect of the asymmetric nuclear matter on**
  - **η-π^0 mixing angle**
  - **Partial width of the $\eta \rightarrow \pi^0 \pi^+ \pi^-$ decay**

Method:

Chiral perturbation in nuclear medium

Ulf-G.Meißner,J.A.Oller, A.Wirzba,Ann.Phys.297(2002)27.
N.Kaiser, S.Fritsch,W.Weise,Nucl.Phys.697(2002)255.

Chiral Perturbation Theory (χPT)

S. Weinberg,Physica96A(1979)327;J.Gasser, H. Leutwyler,NPB250(1985)465.,...

Lagrangian of χPT:

- constructed to have the same global symmetry as QCD
- containing the infinite number of the interaction terms

Power counting :

- ☐ mass of NG boson, 3-momenta of hadrons ~ O(q)

➡ **Systematic expansion with respect of the order of q containing the loop effect**

$$\mathcal{L} = \mathcal{L}_{\pi\pi}^{(2)} + \mathcal{L}_{\pi\pi}^{(4)} + \mathcal{L}_{\pi N}^{(1)} + \mathcal{L}_{\pi N}^{(2)} + \cdots$$

$$\mathcal{L}_{\pi\pi}^{(2)} = \frac{f^2}{4}\left(\langle \partial_\mu U \partial^\mu U \rangle + \langle \chi U^\dagger \rangle + \langle U \chi^\dagger \rangle\right)$$

$$\mathcal{L}_{\pi N}^{(1)} = \bar{N}\left(i\not{D} - m + \frac{g_A}{2}\gamma^\mu \gamma_5 u_\mu\right) N$$

$$U = u^2 = e^{i\pi/f}$$

$$u_\mu = u\partial_\mu u^\dagger - u^\dagger \partial_\mu u$$

$$D_\mu = \partial_\mu + \Gamma_\mu$$

$$\Gamma_\mu = u\partial_\mu u^\dagger + u^\dagger \partial_\mu u$$

$$\chi = 2B_0 \begin{pmatrix} m_u & & \\ & m_d & \\ & & m_s \end{pmatrix}$$

$\langle \ldots \rangle$: trace of flavor space

Chiral perturbation theory in medium

- The effect of the nuclear matter comes from the nucleon propagator

The Pauli blocking effect on the nucleon propagation

$$(\not{p}+m)\left\{\frac{i}{p^2-m^2+i\epsilon}-2\pi\theta(p_0)\delta(p^2-m^2)\theta(k_f-|\vec{p}|)\right\}$$

Free propagation — Pauli Blocking effect

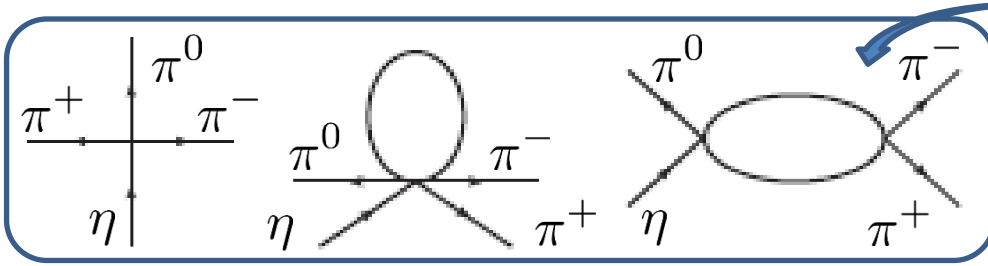
- The expansion with the order of $q \sim m_{NG} \sim k_f$ (k_f:Fermi momentum of nucleon)
- No additional inputs needed to calculate in-medium values

⇒ Calculation up to $O(q^5)$

(Calculation up to $\underline{\mathbf{k_f^3 \sim \rho}}$ with respect of the density order)

leading-order density correction

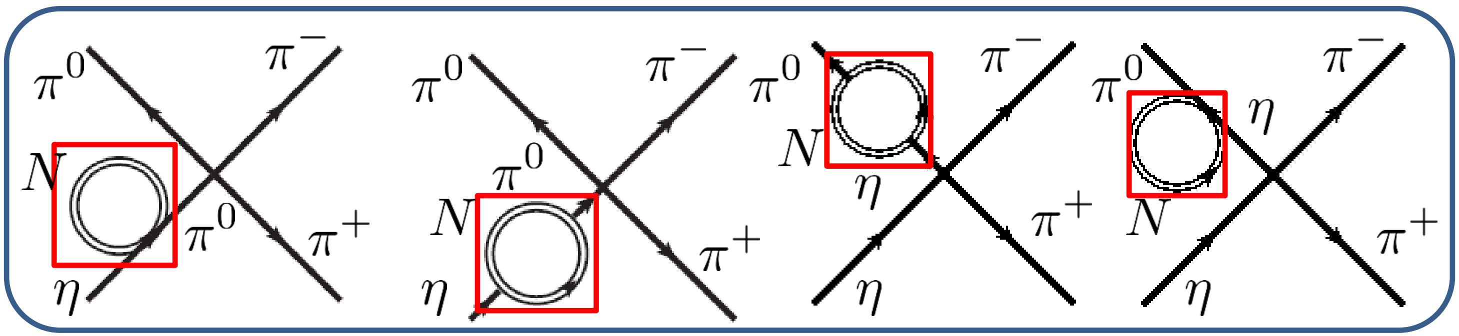
Diagrams (up to $O(q^5)$)



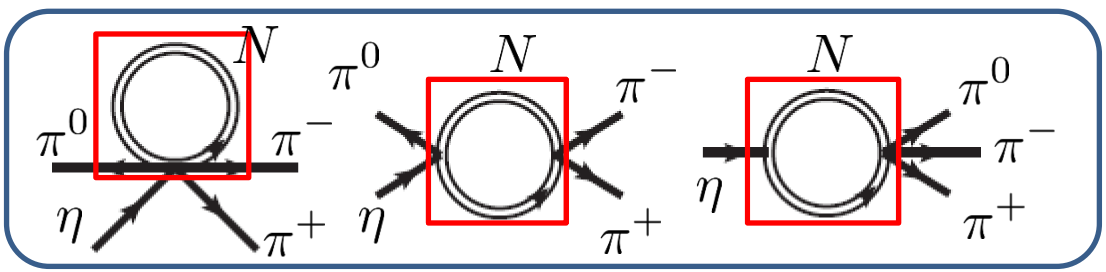
Contribution to the free space (no density dependence)

Significant effect
on the $\eta \to \pi^0\pi^+\pi^-$ decay in free space
($\pi^+\pi^-$ final state interaction)

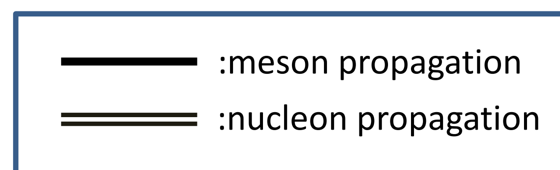
J.Gasser,H.Leutwyler,NPB250(1985)539.



Contribution to the η-π^0 mixing angle



Contribution to the $\eta \to \pi^0\pi^+\pi^-$ decay in nuclear matter



※The enclosed nucleon loops give the effect of the nuclear matter.

Result

- Mixing angle :

$$\tan 2\theta = \frac{2m^2_{\eta\pi^0}}{m^2_\eta - m^2_{\pi^0}}$$

effect of asymmetric nuclear matter

$$m^2_{\eta\pi^0} = \frac{m_1^2}{\sqrt{3}} + \frac{g_A^2 m_\eta^2}{4\sqrt{3} f^2 m_N} \delta\rho + \frac{4c_1 m_1^2}{\sqrt{3} f^2}\rho + \frac{2c_5}{\sqrt{3} f^2} m^2_{\pi^0} \delta\rho$$

$$m_1^2 = m^2_{K^0} - m^2_{K^\pm} - m^2_{\pi^0} + m^2_{\pi^\pm}$$

$$\rho = \rho_n + \rho_p, \ \delta\rho = \rho_n - \rho_p$$

- Matrix element of $\eta \rightarrow \pi^0\pi^+\pi^-$ decay :

$$\mathcal{M}_{\eta\to\pi^0\pi^+\pi^-} = \frac{\sin\theta}{3f^2}\left((m_\eta^2 - m^2_{\pi^0}) + 3(s - s_0)\right) - \frac{g_A^2}{3\sqrt{3} f^4 m_N} m_\eta E_0 \delta\rho$$

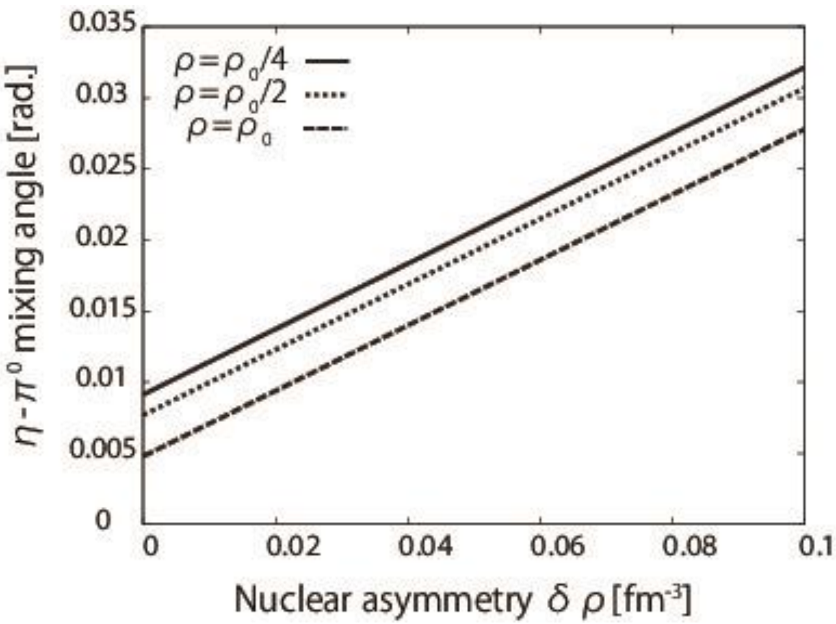
$$+ vacuum \ contribution.$$

$(s_0 = m_\eta^2 + 3m_\pi^2)$

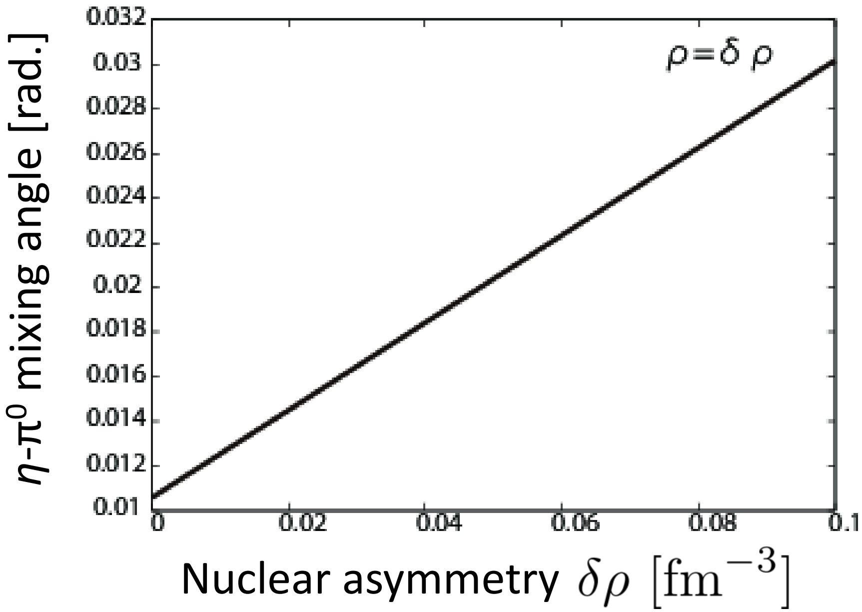
※Isospin-symmetry breaking effect comes from m_1^2 and $\delta\rho$.

Mixing angle in asymmetric matter

Fixed ρ:



Neutron matter:



Asymmetric nuclear matter enhances the η-π^0 mixing angle

Ex.1)ρ=0.06, δρ=0.01[fm^{-3}] : 2.6 × 10^{-3} rad. enhancement of η-π^0 mixing angle

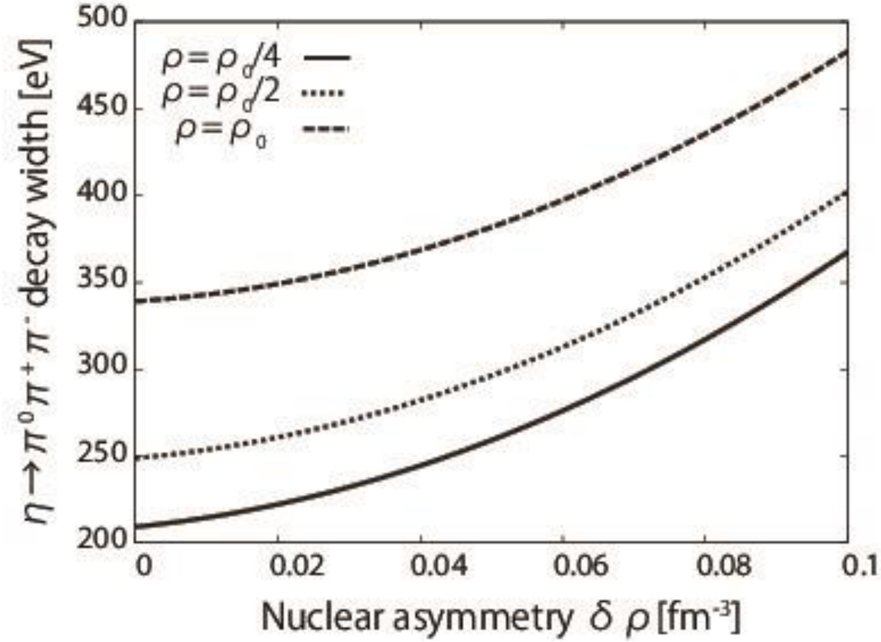
(about 20% enhancement compared with ρ=0)

Ex.2)ρ=δρ=0.04[fm^{-3}] : 7 × 10^{-2} rad. enhancement of η-π^0 mixing angle

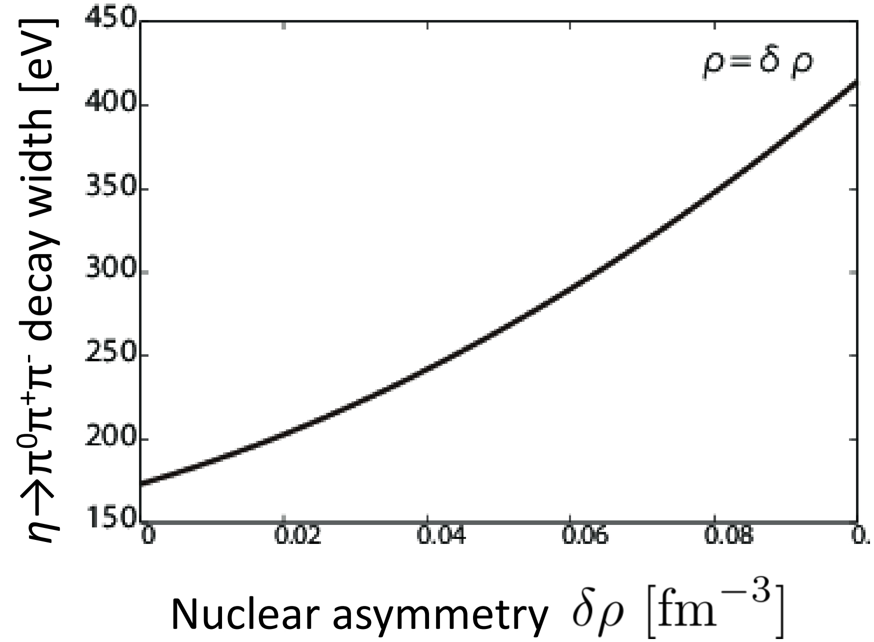
(about 60% enhancement compared with ρ=0)

Partial width of $\eta \to \pi^0\pi^+\pi^-$ in asymmetric matter

Fixed ρ:



Neutron matter:



※$\Gamma_{\eta\to\pi0\pi+\pi-}$~170eV @ρ=0

Asymmetric nuclear matter enhances the $\eta \to \pi^0\pi^+\pi^-$ decay

Ex.1) ρ=0.06, δρ=0.01[fm^{-3}] : 50 eV enhancement of the decay width

(about 30% enhancement compared with ρ=0)

Ex.2) ρ=δρ=0.04[fm^{-3}] : 80 eV enhancement of the decay width

(about 50% enhancement compared with ρ=0)

Conclusions

We have examined the effect of the asymmetric nuclear matter on the η-π^0 mixing angle and $\eta \to \pi^0\pi^+\pi^-$ width

- The mixing angle and the width of $\eta \to \pi^0\pi^+\pi^-$ become larger with the larger nuclear asymmetric density $\delta\rho$.
- The total density ρ also affects the $\eta \to \pi^0\pi^+\pi^-$ decay.
- Pure neutron matter causes the large enhancement of the η-to-$\pi^0\pi^+\pi^-$ decay width.
  - The neutron-rich nuclei may be good for the observation.

(Roughly 30-50% enhancement @ surface of the heavy nuclei)

Future prospects

- The $\pi^+\pi^-$ final state interaction is important for the decay.

  C.Roiesnel,T.N.Truong,NPB187(1981)293,, J.Gasser,H.Leutwyler,NPB250(1985)539.
  - related to the S-wave σ resonance
    (m_σ may reduce in the nuclear matter.)
- The effect of the N^*(1535) ?
  - strong coupling with ηN channel
  - It may affect the possibility of the observation of the $\eta\to\pi^0\pi^+\pi^-$ enhancement
- Other possible isospin-symmetry breaking source?
  - External magnetic field
    (The different electromagnetic charge of u, d quarks)

Thank you for your attention!

Sutherland's theorem

Matrix element of η→3π in QED

$$\left\langle \pi_1\pi_2\pi_3 \left| d_{\mu\nu}(-y)\mathbf{s}\cdot\mathbf{J}(0)J^0_\nu(y)\right| \eta\right\rangle$$

photon propagator | iso-vector current | iso-scalar current

Soft pion theorem ⇩

$$\left\langle \pi_1\pi_2 \left| d_{\mu\nu}(-y)\left[\mathbf{Q_5}\cdot\pi_\mathbf{3}, \mathbf{s}\cdot\mathbf{J}(0)J^0_\nu(y)\right]\right| \eta\right\rangle$$

$(\pi_\mathbf{1}\times\pi_\mathbf{2})\cdot(\pi_\mathbf{3}\times\mathbf{s}\cdot A_\mu)$:only the possibility

$\pi_{1,2,3}$ is not symmetric : not consistent with Bose statistics

Lagrangian

$$\mathcal{L} = \mathcal{L}_{\pi\pi}^{(2)} + \mathcal{L}_{\pi\pi}^{(4)} + \mathcal{L}_{\pi N}^{(1)} + \mathcal{L}_{\pi N}^{(2)}$$

$$\mathcal{L}_{\pi\pi}^{(2)} = \frac{f^2}{4}\left(\langle \partial_\mu U \partial^\mu U \rangle + \langle \chi U^\dagger \rangle + \langle U \chi^\dagger \rangle\right)$$

$$\mathcal{L}_{\pi N}^{(1)} = \bar{N}\left(i\not{D} - m + \frac{g_A}{2}\gamma^\mu \gamma_5 u_\mu\right) N \qquad ,...$$

$$U = u^2 = e^{i\pi/f}$$
$$u_\mu = u\partial_\mu u^\dagger - u^\dagger \partial_\mu u$$
$$D_\mu = \partial_\mu + \Gamma_\mu$$
$$\Gamma_\mu = u\partial_\mu u^\dagger + u^\dagger \partial_\mu u$$
$$\chi = B_0 \begin{pmatrix} m_u & & \\ & m_d & \\ & & m_s \end{pmatrix}$$
$\langle \ldots \rangle$: trace of flavor space

- The field χ represents the effect of the current quark mass

Lagrangian

$$U = u^2 = e^{i\pi/f}$$
$$u_\mu = u\partial_\mu u^\dagger - u^\dagger \partial_\mu u$$
$$D_\mu = \partial_\mu + \Gamma_\mu$$
$$\Gamma_\mu = u\partial_\mu u^\dagger + u^\dagger \partial_\mu u$$
$$\chi = B_0 \begin{pmatrix} m_u & & \\ & m_d & \\ & & m_s \end{pmatrix}$$
$\langle \ldots \rangle$: trace of flavor space

$$\mathcal{L} = \mathcal{L}_{\pi\pi}^{(2)} + \mathcal{L}_{\pi\pi}^{(4)} + \mathcal{L}_{\pi N}^{(1)} + \mathcal{L}_{\pi N}^{(2)}$$

$$\mathcal{L}_{\pi\pi}^{(2)} = \frac{f^2}{4} \left(\langle \partial_\mu U \partial^\mu U \rangle + \left\langle \chi U^\dagger \right\rangle + \left\langle U \chi^\dagger \right\rangle \right)$$

$$\mathcal{L}_{\pi N}^{(1)} = \bar{N} \left(i\not{D} - m + \frac{g_A}{2} \gamma^\mu \gamma_5 u_\mu \right) N$$

$$\begin{aligned}
\mathcal{L}_{\pi\pi}^{(4)} \quad = \quad & L_1 \left\langle D_\mu U^\dagger D^\mu U \right\rangle^2 + L_2 \left\langle D_\mu U^\dagger D_\nu U \right\rangle \left\langle D^\mu U^\dagger D^\nu U \right\rangle \\
& + L_3 \left\langle D_\mu U^\dagger D^\mu U D^\nu U^\dagger D_\nu U \right\rangle + L_4 \left\langle D^\mu U^\dagger D_\mu U \right\rangle \left\langle \chi^\dagger U + \chi U^\dagger \right\rangle \\
& + L_5 \left\langle D^\mu U^\dagger D_\mu U (\chi^\dagger U + U^\dagger chi) \right\rangle + L_6 \left\langle \chi^\dagger U + \chi U^\dagger \right\rangle^2 \\
& + L_7 \left\langle \chi^\dagger U - \chi U^\dagger \right\rangle^2 + L_8 \left\langle \chi^\dagger U \chi^\dagger U + \chi U^\dagger \chi U^\dagger \right\rangle \\
& - i L_9 \left\langle F_{\mu\nu}^R D^\mu U D^\nu U^\dagger + F_{\mu\nu}^L D^\mu U D^\nu U^\dagger \right\rangle \\
& + L_{10} \left\langle U^\dagger F_{\mu\nu}^R U F^{L\mu\nu} \right\rangle + H_1 \left\langle F_{\mu\nu}^R F^{R\mu\nu} + F_{\mu\nu}^L F^{L\mu\nu} \right\rangle + H_2 \left\langle \chi^\dagger \chi \right\rangle
\end{aligned}$$

$$
\begin{aligned}
\mathcal{L}_{\pi N}^{(2)} &= c_1 \langle \chi_+ \rangle \bar{N} N - \frac{c_2}{4m_N^2} \langle u_\mu u_\nu \rangle (\bar{N} D^\mu D^\nu N + \text{ h.c.}) \\
&+ \frac{c_3}{2} \langle u^\mu u_\mu \rangle \bar{N} N - \frac{c_4}{4} \bar{N} \gamma^\mu \gamma^\nu [u_\mu, u_\nu] N + c_5 \bar{N} [\chi_+ - \frac{1}{2} \langle \chi_+ \rangle] N \\
&+ \bar{N} [\frac{c_6}{2} f_{\mu\nu}^+ + \mu_{\mu\nu}^{(s)}] N
\end{aligned}
$$

Definition of mixing angle

$$\pi^0 = \eta_8 \cos\theta - \pi_3 \sin\theta$$
$$\eta = \eta_8 \sin\theta + \pi_3 \cos\theta$$

π_0, η : physical basis
π_3, η_8: flavor basis

$$\tan 2\theta = \frac{2m^2_{\eta\pi^0}}{m^2_\eta - m^2_{\pi^0}}$$

LEC of the Lagrangian

- The πN coupling, meson masses and decay constant are physical one.

  (meson and nucleon loop are

  renormalized into the LEC)
- The LEC of $\mathcal{L}_{\pi\pi}^{(4)}$ is given by

Result of LO in χPT

Decay amplicude of the $\eta \rightarrow \pi^0\pi^+\pi^-$ decay at leading order:

$$\mathcal{M}_{\eta\rightarrow\pi^0\pi^+\pi^-} = \boxed{\frac{m_1^2}{m_\eta^2 - m_{\pi^0}^2}} \cdot \frac{1}{f^2}\left(\frac{4}{3}m_\pi^2 - s\right)$$

η-π^0 mixing angle

$$m_1^2 = m_{K^0}^2 - m_{K^\pm} - m_{\pi^0}^2 + m_{\pi^\pm}^2$$ (u, d quark mass difference)

$$\Gamma_{\eta\rightarrow\pi^0\pi^+\pi^-} \sim 77\ \mathrm{e}V$$

References (proceeding works in $\eta \to \pi^0\pi^+\pi^-$)

- Linear sigma model
  - schechter and hadnall
- Non-linear sigma model @leading order
  - H. Osborn, D.J. Wallace, Nucl. Phys. B20 (1970) 23.
- effect of η' mixing

- Goldstone-dipole model (related with $U_A(1)$ problem)
  - S. Weinberg, Phys. Rev. D11 (1975) 3583.
  - J. Kogut, L. Susskind, Phys. Rev. D11 (1975) 3594.
  - S. Raby, Phys. Rev. D13 (1976) 2594.
- large N_c argument
  - K. Kawarabayashi, N. Ohta, Nucl. Phys. B175 (1980) 477.
- χPT @one loop
  - J. Gassser, H. Leutwyler, Nucl. Phys. B250 (1985) 539.
- χPT @two loop
  - J. Bijnens, K. Ghorbani, JHEP
- dispersive method

G parity

$$G = e^{i\pi I_y}\mathcal{C} \qquad \mathcal{C} : \text{charge conjugation operator}$$

$G^2=1 \rightarrow G=\pm 1$

$$e^{i\pi I_y}\left|\pi^+\right\rangle = \left|\pi^-\right\rangle \qquad \mathcal{C}\left|\pi^\pm\right\rangle = -\left|\pi^\mp\right\rangle$$

$$e^{i\pi I_y}\left|\pi^0\right\rangle = -\left|\pi^0\right\rangle \qquad \mathcal{C}\left|\pi^0\right\rangle = \left|\pi^0\right\rangle$$

$$e^{i\pi I_y}\left|\pi^-\right\rangle = \left|\pi^+\right\rangle$$

⇨ G parity of π is -.

Hadron properties in the nuclear matter

Partial restoration of chiral symmetry

@ normal nuclear density

– The reduction of the order parameter ⟨q^{bar}q⟩ of the spontaneous chiral symmetry breaking

Quark condensate @low density E.G. Drukarev, E.M. Levin, Nucl. Phys. A511 (1990) 679.

$$\langle \bar{q}q \rangle^* = \left(1 - \frac{\sigma_{\pi N}}{m_\pi^2 f_\pi^2}\rho\right) \langle \bar{q}q \rangle + \mathcal{O}(\rho^{n>1})$$

cf) Gell-Mann-Oakes-Renner relation

$$f_\pi^2 m_\pi^2 = -m_q \langle \bar{q}q \rangle$$

- Enhancement of the πN repulsion
- Enhancement of the ππ attraction , …

The nuclear matter causes the change of the hadron properties.

Measurement of the double differential cross section of $Sn(d,^3He)Sn'$

K.Suzuki, et.al. Phys.Rev.Lett.92(2004)072302.

$$U_s(r) = -\frac{2\pi}{m_\pi}\left[\epsilon_1\left\{b_0\rho(r) + \boxed{b_1}\left[\rho_n(r) - \rho_p(r)\right]\right\} + \epsilon_2 B_0\rho^2(r)\right]$$

Related to $1/f_\pi^2$

$$\frac{b_1^{\text{free}}}{b_1^*(\rho)} \sim \frac{f_\pi^*(\rho)^2}{f_\pi^2}$$

Larger asymmetry enhances the PRCS

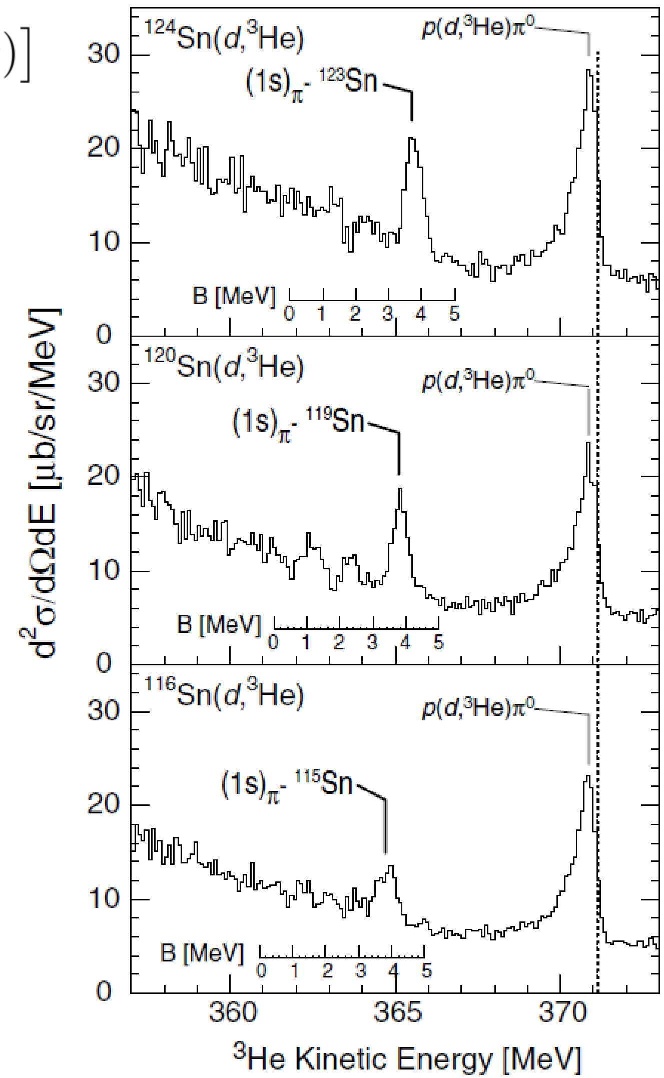
$Sn(d,^3He)Sn'$ reaction

Parameters b_0, b_1, B_0 to fit the experimental data

b_1^{free}(the value in free space) is determined in π-Hydrogen.

$$\frac{b_1^{\text{free}}}{b_1^*(\rho_e)} = 0.78 \pm 0.05 \quad (\rho_e = 0.6\rho_0)$$

Gell-Mann-Oakes-Renner relation in medium and the π mass in medium →about 30% reduction of chiral condensate.



Hadron physics and chiral symmetry

- The remaining flavor symmetry $SU(N_f)_f$ constrains hadron reactions.
  - G parity is almost conserved in the strong decay

    Reflect the isospin symmetry

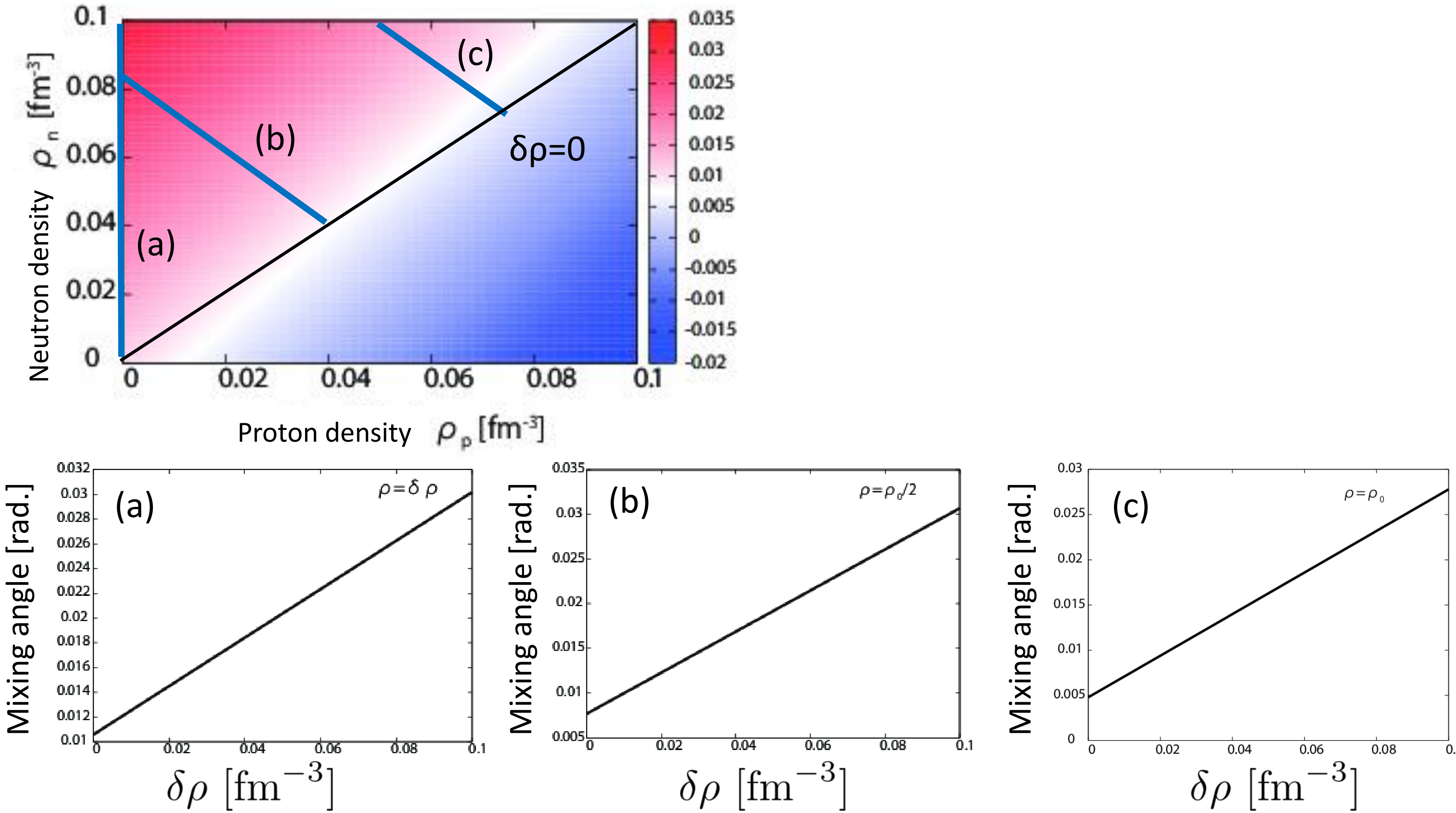
Ex.) ρ and ω decay

$\rho : I^G=1^+$
$\omega : I^G=0^-$
$\pi : I^G=1^-$

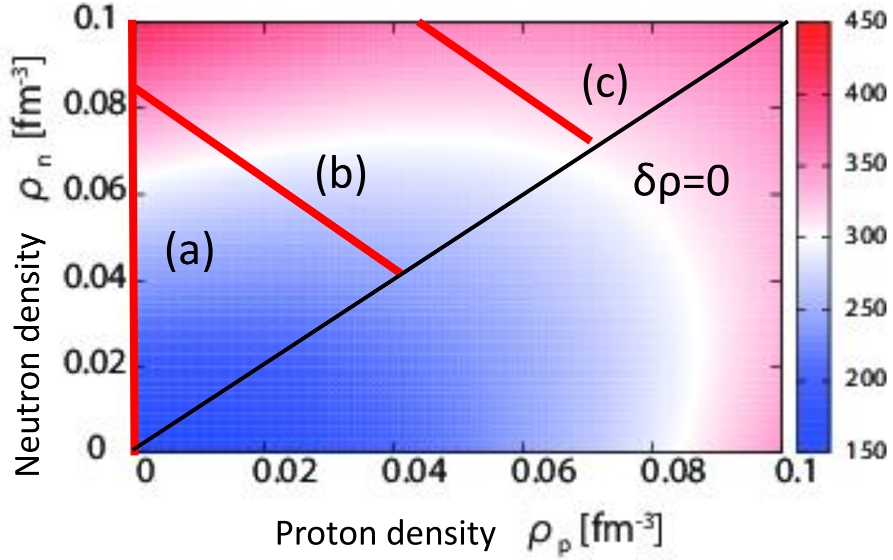
dominant decay mode

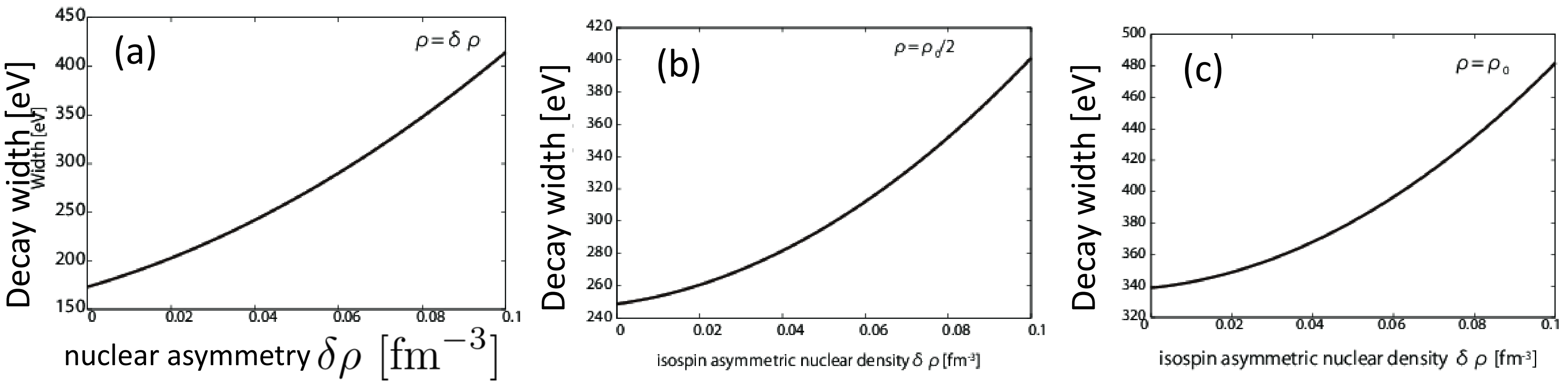
$\rho \rightarrow 2\pi$
$\omega \rightarrow 3\pi$

- The environment affect the properties of chiral symmetry
  - ✓ Partial restoration of chiral symmetry (PRCS) in nuclear matter
  - ✓ System under finite temperature, finite density, electromagnetic field,…



0.1
0.08
0.06
0.04
0.02
0
Neutron density ρ n [fm-3]
0
0.02
0.04
0.06
0.08
0.1
Proton density ρ p [fm-3]
(a)
(b)
(c)
δρ=0
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
-0.005
-0.01
-0.015
-0.02
Mixing angle [rad.]
ρ=δ ρ
ρ=ρ0/2
ρ=ρ0
δρ [fm−3]


0.1
0.08
0.06
0.04
0.02
0
Neutron density ρ_n [fm^{-3}]
(a)
(b)
(c)
δρ=0
0 0.02 0.04 0.06 0.08 0.1
Proton density ρ_p [fm^{-3}]
450
400
350
300
250
200
150


(a)
$\rho = \delta\rho$
Decay width [eV]
nuclear asymmetry $\delta\rho$ [fm^{-3}]
(b)
$\rho = \rho_0/2$
Decay width [eV]
isospin asymmetric nuclear density $\delta\rho$ [fm^{-3}]
(c)
$\rho = \rho_0$
Decay width [eV]
isospin asymmetric nuclear density $\delta\rho$ [fm^{-3}]

K. Suzuki, *et al*., Phys. Rev. Lett. 92 (2004) 72302.
E. Friedman, *et al*., Phys. Rev. Lett. 93 (2004) 122302.
P. Camerini,et al., Nucl. Phys. A735 (2004) 89.

$$\langle \bar{q}q \rangle^* = \left(1 - \frac{\sigma_{\pi N}}{m_\pi^2 f_\pi^2}\rho\right) \langle \bar{q}q \rangle + \mathcal{O}(\rho^{n>1}) \quad \Rightarrow$$