

# Baryon Properties from a Relativistic Constituent-Quark Model

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Low-energy

QCD

RCQM

GBE RCQM

Spectroscopy

Light, strange,  
charm, bottom

Structure

Nucleon E.m.

Baryon E.m.

Axial FFs

Gravitational FF

Summary

# Motivation for Resorting to Quark Models

To be able to describe/understand

- in a consistent manner
- on the microscopic level
- in accordance with the properties of low-energy QCD such phenomena like
  - ▶ **hadron spectra**: ground states & excitations
  - ▶ **hadron structure**:  $r_E, \mu, g_A; G_E, G_M, G_A, G_P, \dots$   
i.e. electroweak form factors etc.
  - ▶ **resonance excitations**:  $\gamma N \rightarrow N^*, e^- N \rightarrow N^*, \dots$
  - ▶ **resonance decays**:  
 $\rho \rightarrow \pi\pi, \omega \rightarrow \pi\pi\pi, N^* \rightarrow N\pi, \Delta \rightarrow N\pi, \Lambda^* \rightarrow KN, \dots$
  - ▶ **meson-baryon interactions**:  $\pi - N, K - N, \dots$
  - ▶ **hyperon-hyperon interactions**:  $N - N, N - Y, \dots$   
etc. etc.

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Low-Energy QCD / Relevant Degrees of Freedom

Relativistic Constituent-Quark Model (RCQM)

Interacting mass operator with GBE dynamics

Baryon Spectroscopy

Light, strange, charm, bottom

Baryon Structure

Nucleon e.m. form factors - Flavor analysis

Baryon electromagnetic form factors

Nucleon and baryon axial form factors / charges

Nucleon gravitational form factors

Summary and Conclusions

Low-energy QCD of  $N_f$  flavors is characterized by:

- spontaneous breaking of chiral symmetry ( $SB_{\chi}S$ ):

$$SU(N_f)_L \times SU(N_f)_R \rightarrow SU(N_f)_V$$

→ appearance of  $(N_f^2 - 1)$  **Goldstone bosons**  $\vec{\phi}$

→ generation of quasiparticles with dynamical mass,  
i.e. **constituent quarks**  $\psi$

- thus (effective) interaction Lagrangian:

$$\mathcal{L}_{\text{int}} \sim ig \bar{\psi} \gamma_5 \vec{\lambda}^f \cdot \vec{\phi} \psi$$

A. Manohar and H. Georgi: Nucl. Phys. B 234 (1984) 189

E.V. Shuryak: Phys. Rep. **115**, 151 (1984)

L.Ya. Glozman and D.O. Riska: Phys. Rep. **268**, 263 (1996)

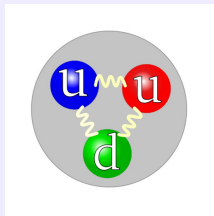
see also:

S. Weinberg: Phys. Rev. Lett. **105**, 261601 (2010)

# Baryons

Baryons are considered as colorless bound states of three constituent quarks.

Here the proton:



- ▶ 'Constituent' quarks are quasiparticles with **dynamical mass**, NOT the original QCD d.o.f. (i.e. 'current' quarks).
- ▶ 'Constituent' quarks are confined and interact via hyperfine interactions associated with  $SB_{\chi}S$ , i.e. **Goldstone-boson exchange**.

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## Relativistic quantum mechanics (RQM)

i.e. **quantum theory** respecting **Poincaré invariance**

(theory on a Hilbert space  $\mathcal{H}$  corresponding to a finite number of particles, not a field theory)

### Invariant mass operator

$$\hat{M} = \hat{M}_{free} + \hat{M}_{int}$$

### Eigenvalue equations

$$\hat{M} |P, J, \Sigma\rangle = M |P, J, \Sigma\rangle \quad , \quad \hat{M}^2 = \hat{P}^\mu \hat{P}_\mu$$

$$\hat{P}^\mu |P, J, \Sigma\rangle = P^\mu |P, J, \Sigma\rangle \quad , \quad \hat{P}^\mu = \hat{M} \hat{V}^\mu$$

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## Interacting mass operator

$$\hat{M} = \hat{M}_{free} + \hat{M}_{int}$$

$$\hat{M}_{free} = \sqrt{\hat{H}_{free}^2 - \hat{\vec{P}}_{free}^2}$$

$$\hat{M}_{int}^{rest\ frame} = \sum_{i < j}^3 \hat{V}_{ij} = \sum_{i < j}^3 [\hat{V}_{ij}^{conf} + \hat{V}_{ij}^{hf}]$$

fulfilling the **Poincaré algebra**

$$\begin{aligned} [\hat{P}_i, \hat{P}_j] &= 0, & [\hat{J}_i, \hat{H}] &= 0, & [\hat{P}_i, \hat{H}] &= 0, \\ [\hat{K}_i, \hat{H}] &= -i\hat{P}_i, & [\hat{J}_i, \hat{J}_j] &= i\epsilon_{ijk}\hat{J}_k, & [\hat{J}_i, \hat{K}_j] &= i\epsilon_{ijk}\hat{K}_k, \\ [\hat{J}_i, \hat{P}_j] &= i\epsilon_{ijk}\hat{P}_k, & [\hat{K}_i, \hat{K}_j] &= -i\epsilon_{ijk}\hat{J}_k, & [\hat{K}_i, \hat{P}_j] &= -i\delta_{ij}\hat{H} \end{aligned}$$

$\hat{H}, \hat{P}_i$  ... time and space translations,

$\hat{J}_i$  ... rotations,  $\hat{K}_i$  ... Lorentz boosts

Phenomenologically, baryons with 5 flavors:  $u, d, s, c, b$

$$\Rightarrow H_{free} = \sum_{i=1}^3 \sqrt{m_i^2 + \vec{k}_i^2}$$

$$V^{conf}(\vec{r}_{ij}) = B + C r_{ij}$$

$$V^{hf}(\vec{r}_{ij}) = \left[ V_{24}(\vec{r}_{ij}) \sum_{f=1}^{24} \lambda_i^f \lambda_j^f + V_0(\vec{r}_{ij}) \lambda_i^0 \lambda_j^0 \right] \vec{\sigma}_i \cdot \vec{\sigma}_j$$

- ▶ i.e., for  $N_f = 5$ , we have the exchange of a **24-plet** plus a **singlet** of Goldstone bosons.

L.Ya. Glozman, W. Plessas, K. Varga, and R.F. Wagenbrunn: Phys. Rev. D **58**, 094030 (1998)

J.P. Day, K.-S. Choi, and W. Plessas: arXiv:1205.6918

J.P. Day, K.-S. Choi, and W. Plessas: Few-Body Syst. **54**, 329 (2013)





# Universal GBE RCQM Parametrization

$$V^{conf}(\vec{r}_{ij}) = B + C r_{ij}$$

$$V_{\beta}(\vec{r}_{ij}) = \frac{g_{\beta}^2}{4\pi} \frac{1}{12m_i m_j} \left\{ \mu_{\beta}^2 \frac{e^{-\mu_{\beta} r_{ij}}}{r_{ij}} - 4\pi \delta(\vec{r}_{ij}) \right\}$$

$$= \frac{g_{\beta}^2}{4\pi} \frac{1}{12m_i m_j} \left\{ \mu_{\beta}^2 \frac{e^{-\mu_{\beta} r_{ij}}}{r_{ij}} - \Lambda_{\beta}^2 \frac{e^{-\Lambda_{\beta} r_{ij}}}{r_{ij}} \right\}$$

$$B = -402 \text{ MeV}, \quad C = 2.33 \text{ fm}^{-2}$$

$$\beta = 24 : \quad \frac{g_{24}^2}{4\pi} = 0.7, \quad \mu_{24} = \mu_{\pi} = 139 \text{ MeV}, \quad \Lambda_{24} = 700.5 \text{ MeV}$$

$$\beta = 0 : \quad \left( \frac{g_0}{g_{24}} \right)^2 = 1.5, \quad \mu_0 = \mu_{\eta'} = 958 \text{ MeV}, \quad \Lambda_0 = 1484 \text{ MeV}$$

$$m_u = m_d = 340 \text{ MeV}, \quad m_s = 480 \text{ MeV},$$

$$m_c = 1675 \text{ MeV}, \quad m_b = 5055 \text{ MeV}$$

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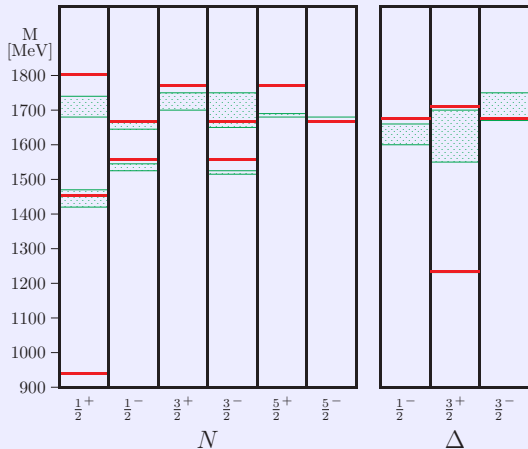
Summary

Baryon **Excitation Spectra**

and

Mass-Operator **Eigenstates**

# Light Baryon Spectra



red Universal GBE RCQM

green PDG 2013 (experiment)

Low-energy QCD

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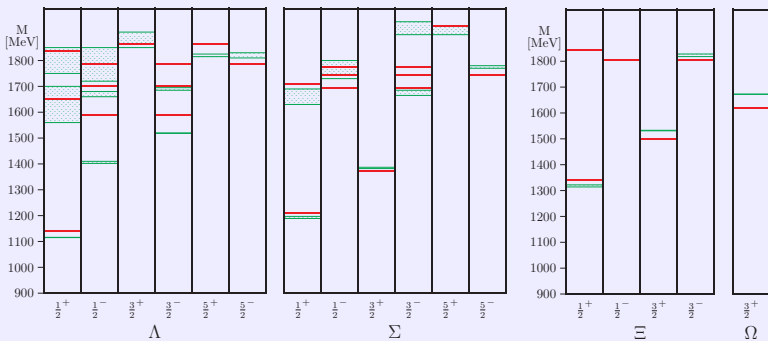
Nucleon E.m.

Baryon E.m.

Axial FFs

Gravitational FF

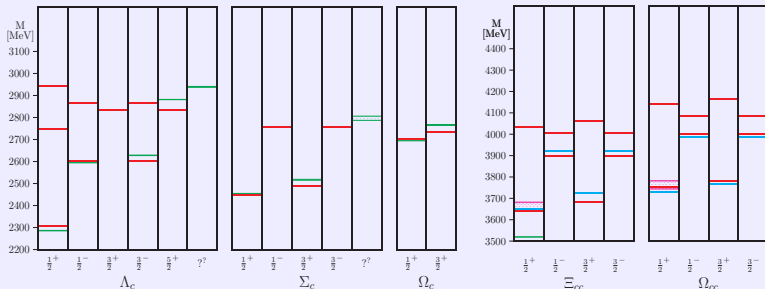
Summary



red Universal GBE RCQM

green PDG 2013 (experiment)

# Charm Baryon Spectra



## Left panel – single charm:

red Universal GBE RCQM prediction

green PDG 2013 (experiment)

## Right panel – double charm:

green M. Mattson et al.: Phys. Rev. Lett. 89 (2002) 112001 (SELEX experiment)

cyan S. Migura, D. Merten, B. Metsch, and H.-R. Petry: Eur. Phys. J. A 28 (2006) 41 (Bonn RCQM)

magenta L. Liu et al.: Phys. Rev. D 81 (2010) 094505 (Lattice QCD)

# Bottom Baryon Spectra

Low-energy QCD

RCQM

GBE RCQM

Spectroscopy

Light, strange, charm, bottom

Structure

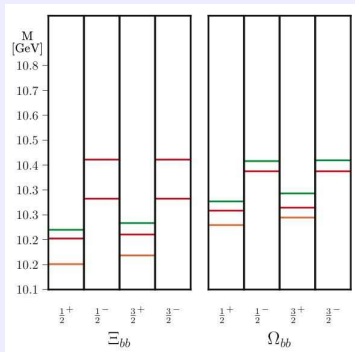
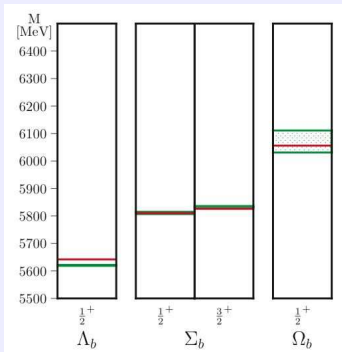
Nucleon E.m.

Baryon E.m.

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## Left panel – single bottom:

red Universal GBE RCQM prediction

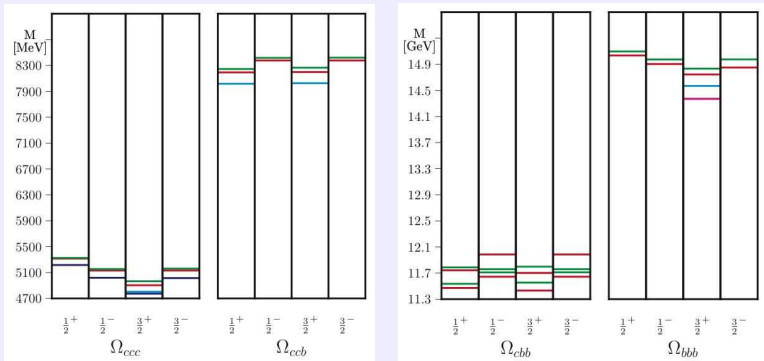
green PDG 2013 (experiment)

## Right panel – double bottom:

green W. Roberts and M. Pervin: Int. J. Mod. Phys. A 23 (2008) 2817 (nonrel. one-gluon-exchange CQM)

orange D. Ebert, R.N. Faustov, V.O. Galkin, and A.P. Martynenko: Phys. Rev. D 66 (2002) 014008 (RCQM)

# Triple-Heavy Baryon Spectra



red Universal GBE RCQM

green W. Roberts and M. Pervin: Int. J. Mod. Phys. A 23 (2008) 2817  
(nonrelativistic one-gluon-exchange CQM)

blue S. Migura, D. Merten, B. Metsch, and H.-R. Petry: Eur. Phys. J. A 28 (2006) 41 (Bonn RCQM)

cyan A.P. Martynenko: Phys. Lett. B 663 (2008) 317 (RCQM)

magenta S. Meinel: Phys. Rev. D 82 (2010) 114502 (lattice QCD)

# Influence of Light-Heavy Q-Q Interaction



Low-energy QCD

RCQM

GBE RCQM

Spectroscopy

Light, strange, charm, bottom

Structure

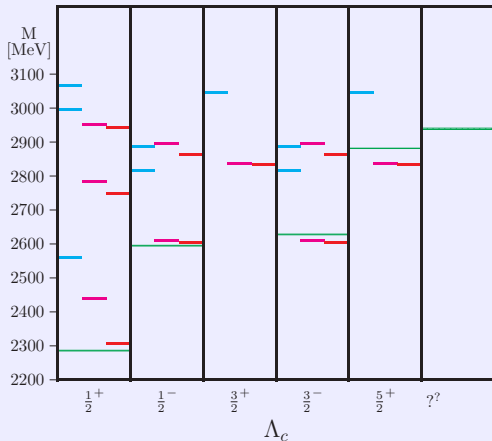
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Summary



leftmost cyan levels

confinement only

middle magenta levels

including only light-light GBE

rightmost red levels

including full GBE RCQM



## Mass operator eigenstates

$$\hat{M} |P, J, \Sigma, T, M_T\rangle = M |P, J, \Sigma, T, M_T\rangle$$

represented in configuration space

$$\langle \vec{\xi}, \vec{\eta} | P, J, \Sigma, T, M_T \rangle = \Psi_{PJ\Sigma TM_T}(\vec{\xi}, \vec{\eta})$$

with  $\vec{\xi}$  and  $\vec{\eta}$  the usual Jacobi coordinates.

Picture the baryon wave functions through  
**spatial probability density distributions**

$$\rho(\xi, \eta) = \xi^2 \eta^2 \int d\Omega_\xi d\Omega_\eta \Psi_{PJ\Sigma TM_T}^*(\xi, \Omega_\xi, \eta, \Omega_\eta) \Psi_{PJ\Sigma TM_T}(\xi, \Omega_\xi, \eta, \Omega_\eta)$$

Low-energy  
QCD

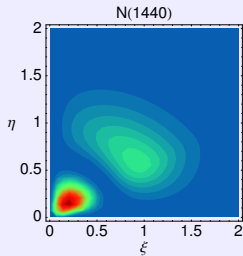
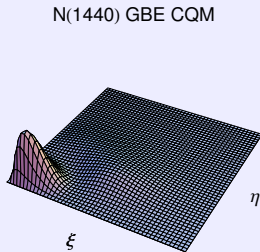
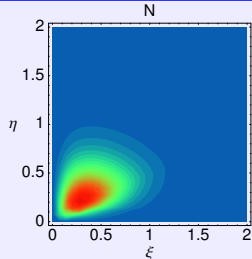
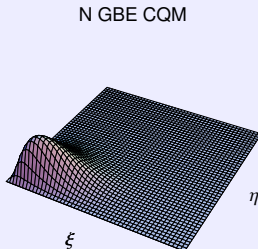
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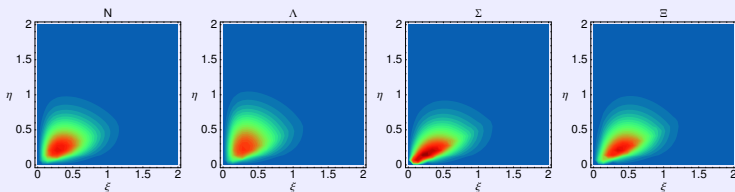
Summary

# Pictures of Baryons (rest frame)

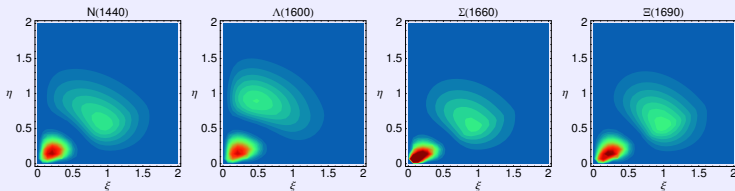


# Spatial Probability Density Distributions

$\rho(\xi, \eta)$  for the  $\frac{1}{2}^+$  octet baryon ground states  $N(939)$ ,  $\Lambda(1116)$ ,  $\Sigma(1193)$ ,  $\Xi(1318)$ :



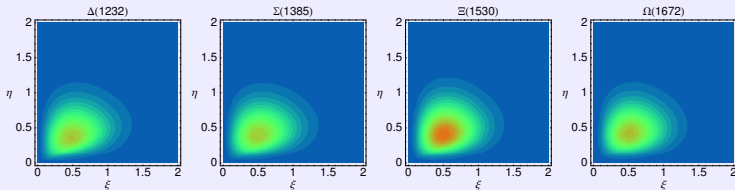
$\rho(\xi, \eta)$  for the  $\frac{1}{2}^+$  octet baryon states  $N(1440)$ ,  $\Lambda(1600)$ ,  $\Sigma(1660)$ ,  $\Xi(1690)$ :



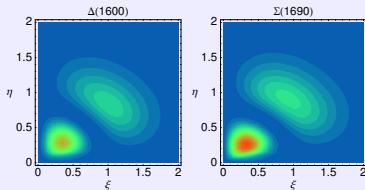
T. Melde, W. Plessas, and B. Sengl: Phys. Rev. D **77** (2008) 114002

# Spatial Probability Density Distributions

$\rho(\xi, \eta)$  for the  $\Sigma_c^{3+}$  decuplet baryon states  $\Delta(1232)$ ,  $\Sigma(1385)$ ,  $\Xi(1530)$ ,  $\Omega(1672)$ :



$\rho(\xi, \eta)$  for the  $\Sigma_c^{3+}$  decuplet baryon states  $\Delta(1600)$ ,  $\Sigma(1690)$ :



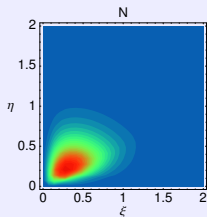
# Root-Mean-Square Radii

The **root-mean-square radius** (in the rest frame):

$$r_{\text{rms}} = \sqrt{\langle r_i^2 \rangle} = \left( \int d^3 r_i \langle P = 0, J, \Sigma | \hat{r}_i^2 | P = 0, J, \Sigma \rangle \right)^{\frac{1}{2}}$$

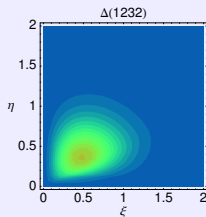
Is NOT an **observable**! Is NOT **relativistically invariant**!

→ Idea about the **spatial distribution** of constituent quarks.



$$r_{\text{rms}}^N = 0.304 \text{ fm}$$

$$r_E^p = 0.905 \text{ fm}, (r_E^n)^2 = -0.128 \text{ fm}^2$$



$$r_{\text{rms}}^\Delta = 0.390 \text{ fm}$$

$$r_E^{\Delta^{++}} = r_E^{\Delta^+} = r_E^{\Delta^-} = 0.656 \text{ fm}, r_E^{\Delta^0} = 0 \text{ fm}$$

See: K. Berger, R.F. Wagenbrunn, and W. Plessas: Phys. Rev. D **70**, 094027 (2004)

multiplet	$(LS)J^P$				
octet	$(0 \frac{1}{2} \frac{1}{2})^+$	$N(939)^{100}$	$\Lambda(1116)^{100}$	$\Sigma(1193)^{100}$	$\Xi(1318)^{100}$
octet	$(0 \frac{1}{2} \frac{1}{2})^+$	$N(1440)^{100}$	$\Lambda(1600)^{96}$	$\Sigma(1660)^{100}$	$\Xi(1690)^{100}$
octet	$(0 \frac{1}{2} \frac{1}{2})^+$	$N(1710)^{100}$		$\Sigma(1880)^{99}$	
octet	$(1 \frac{1}{2} \frac{1}{2})^-$	$N(1535)^{100}$	$\Lambda(1670)^{72}$	$\Sigma(1560)^{94}$	
octet	$(1 \frac{1}{2} \frac{1}{2})^-$	$N(1650)^{100}$	$\Lambda(1800)^{100}$	$\Sigma(1620)^{100}$	
octet	$(1 \frac{1}{2} \frac{1}{2})^-$	$N(1520)^{100}$	$\Lambda(1690)^{72}$	$\Sigma(1670)^{94}$	$\Xi(1820)^{97}$
octet	$(1 \frac{1}{2} \frac{1}{2})^-$	$N(1700)^{100}$		$\Sigma(1940)^{100}$	
octet	$(1 \frac{1}{2} \frac{1}{2})^-$	$N(1675)^{100}$	$\Lambda(1830)^{100}$	$\Sigma(1775)^{100}$	$\Xi(1950)^{100}$
decuplet	$(0 \frac{3}{2} \frac{3}{2})^+$	$\Delta(1232)^{100}$	$\Sigma(1385)^{100}$	$\Xi(1530)^{100}$	$\Omega(1672)^{100}$
decuplet	$(0 \frac{3}{2} \frac{3}{2})^+$	$\Delta(1600)^{100}$	$\Sigma(1690)^{99}$		
decuplet	$(1 \frac{1}{2} \frac{1}{2})^-$	$\Delta(1620)^{100}$	$\Sigma(1750)^{94}$		
decuplet	$(1 \frac{1}{2} \frac{1}{2})^-$	$\Delta(1700)^{100}$			
singlet	$(1 \frac{1}{2} \frac{1}{2})^-$	$\Lambda(1405)^{71}$			
singlet	$(1 \frac{1}{2} \frac{1}{2})^-$	$\Lambda(1520)^{71}$			
singlet	$(0 \frac{1}{2} \frac{1}{2})^+$	$\Lambda(1810)^{92}$			

T. Melde, W. Plessas, and B. Sengl: Phys. Rev. D **77**, 114002 (2008)

See also the PDG: Phys. Rev. D **86**, 010001 (2012)

# Various Baryon Reactions

Matrix elements of a transition operator  $\hat{O}$  between baryon eigenstates  $|P, J, \Sigma, T, T_3, Y\rangle$

$$\langle P', J', \Sigma', T', T'_3, Y' | \hat{O} | P, J, \Sigma, T, T_3, Y \rangle$$

$\hat{O} \dots \hat{J}_{\text{em}}^\mu \rightarrow$  electromagnetic FF's

$\dots \hat{A}_{\text{axial}}^\mu \rightarrow$  axial FF's

$\dots \hat{S} \rightarrow$  scalar FF

$\dots \hat{\Theta}^{\mu\nu} \rightarrow$  gravitational/tensor FF's

$\dots \hat{D}_\lambda^\mu \rightarrow$  hadronic decays

To be calculated from microscopic three-quark ME's

$$\langle p'_1, p'_2, p'_3; \sigma'_1, \sigma'_2, \sigma'_3; f'_{i'_1}, f'_{i'_2}, f'_{i'_3} | \hat{O} | p_1, p_2, p_3; \sigma_1, \sigma_2, \sigma_3; f_{i_1}, f_{i_2}, f_{i_3} \rangle$$

$\uparrow$   
boosted 3-body states

$\uparrow$   
boosted 3-body states

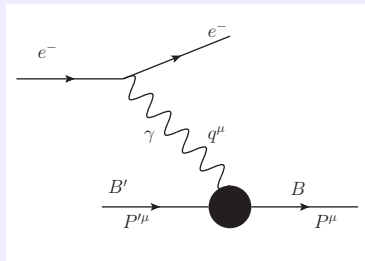
Covariant predictions for:

- ▶ **Electromagnetic** nucleon form factors  
 $G_E^p(Q^2), G_M^p(Q^2); G_E^n(Q^2), G_M^n(Q^2)$
- ▶ **Electric radii** and **magnetic moments**  
 $r_E^p, \mu^p; r_E^n, \mu^n$

→ Comparison to experiment



## Elastic electron scattering:



## Invariant form factors:

$$F_{\Sigma'\Sigma}^{\nu}(Q^2) = \langle P', J, \Sigma', T, M_T | \hat{J}_{\text{em}}^{\nu} | P, J, \Sigma, T, M_T \rangle$$

$$\text{with } Q^2 = -q^2; \quad q^{\mu} = P^{\mu} - P'^{\mu}$$

# Transition Matrix Elements in Point Form

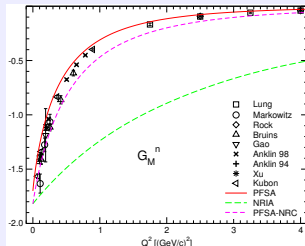
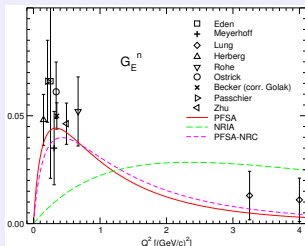
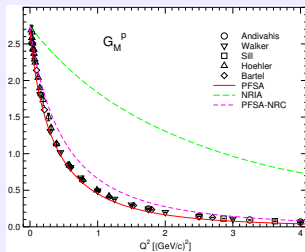
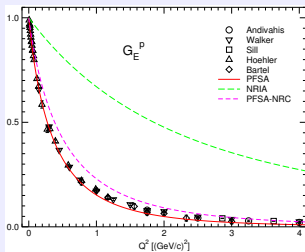
Incoming baryon state:  $|V, M, J, \Sigma\rangle \hat{=} |P, J, \Sigma\rangle$   
 Outgoing baryon state:  $|V', M', J', \Sigma'\rangle \hat{=} |P', J', \Sigma'\rangle$   
 Transition operator:  $\hat{O} = \hat{J}_{em}^\mu$

$$\begin{aligned}
 & \langle V', M', J', \Sigma' | \hat{J}_{em}^\mu | V, M, J, \Sigma \rangle = \\
 & = \frac{2}{MM'} \sum_{\sigma_i \sigma'_i} \sum_{\mu_i \mu'_i} \int d^3 \vec{k}_2 d^3 \vec{k}_3 d^3 \vec{k}'_2 d^3 \vec{k}'_3 \\
 & \times \sqrt{\frac{(\sum_i \omega'_i)^3}{\prod_i 2\omega'_i}} \prod_{\sigma'_i} D_{\sigma'_i \mu'_i}^{* \frac{1}{2}} \{R_W [k'_i; B(V')]\} \Psi_{M' J' \Sigma'}^* (\vec{k}'_1, \vec{k}'_2, \vec{k}'_3; \mu'_1, \mu'_2, \mu'_3) \\
 & \times \langle p'_1, p'_2, p'_3; \sigma'_1, \sigma'_2, \sigma'_3 | \hat{J}_{rd}^\mu | p_1, p_2, p_3; \sigma_1, \sigma_2, \sigma_3 \rangle \\
 & \times \sqrt{\frac{(\sum_i \omega_i)^3}{\prod_i 2\omega_i}} \prod_{\sigma_i} D_{\sigma_i \mu_i}^{\frac{1}{2}} \{R_W [k_i; B(V)]\} \Psi_{MJ\Sigma} (\vec{k}_1, \vec{k}_2, \vec{k}_3; \mu_1, \mu_2, \mu_3) \\
 & \times 2MV_0 \delta^3 (M\vec{V} - M'\vec{V}' - \vec{q})
 \end{aligned}$$

where  $p_i = B_c(V)k_i$ ,  $p'_i = B_c(V')k'_i$ , and  $\omega_i = \sqrt{\vec{k}_i^2 + m_i^2}$

# Electromagnetic Nucleon Form Factors

## Covariant predictions of the GBE CQM:



R.F. Wagenbrunn, S. Boffi, W. Klink, W. Plessas, and M. Radici: Phys. Lett. **B511** (2001) 33

Low-energy  
QCD

RCQM

GBE RCQM

Spectroscopy

Light, strange,  
charm, bottom

Structure

Nucleon E.m.

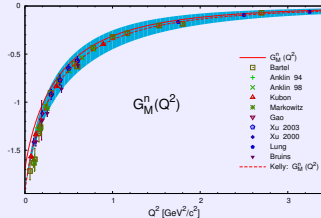
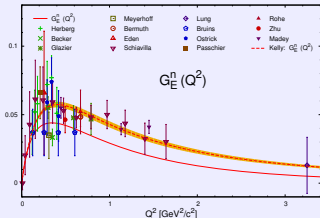
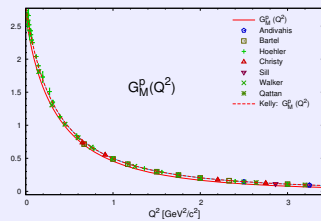
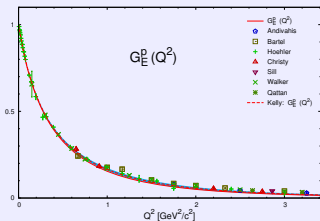
Baryon E.m.

Axial FFs

Gravitational FF

Summary

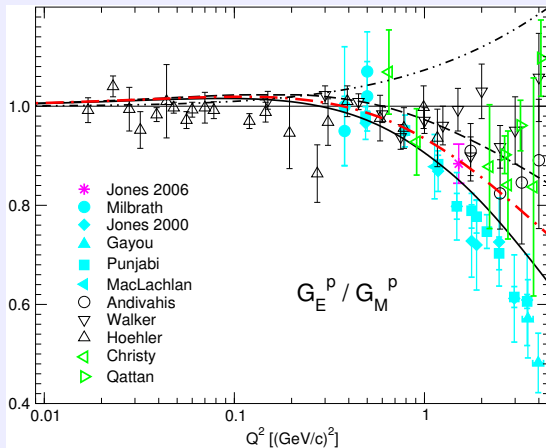
## Covariant predictions of the GBE CQM:



R.F. Wagenbrunn, S. Boffi, W. Klink, W. Plessas, and M. Radici: Phys. Lett. **B511** (2001) 33

M. Rohmoser: Diploma Thesis, Univ. of Graz, 2013

# Proton Electric/Magnetic Form Factor Ratio



solid: GBE RCQM PFSM

dash-double-dot: GBE RCQM IFSM

T. Melde, K. Berger, L. Canton, W. Plessas, and R. F. Wagenbrunn: Phys. Rev. D **76**, 074020 (2007)

Low-energy  
QCD

RCQM  
GBE RCQM

Spectroscopy  
Light, strange,  
charm, bottom

Structure  
Nucleon E.m.  
Baryon E.m.

Axial FFs  
Gravitational FF

Summary

# Nucleon Electric Radii and Magnetic Moments

Electric radii  $r_E^2$  [ $\text{fm}^2$ ]

Baryon	GBE PFSM	Experiment
$p$	0.82	$0.7692 \pm 0.0123$ <sup>1)</sup> $0.70870 \pm 0.00113$ <sup>2)</sup>
$n$	-0.13	$-0.1161 \pm 0.0022$

<sup>1)</sup> CODATA value (PDG)

<sup>2)</sup> Pohl et al.: Nature **466** (2010) 213

Magnetic moments  $\mu$  [n.m.]

Baryon	GBE PFSM	Experiment
$p$	2.70	2.792847356
$n$	-1.70	-1.9130427

K. Berger, R.F. Wagenbrunn, and W. Plessas: Phys. Rev. D **70**, 094027 (2004)

Low-energy  
QCD

RCQM  
GBE RCQM

Spectroscopy  
Light, strange,  
charm, bottom

Structure  
Nucleon E.m.  
Baryon E.m.  
Axial FFs  
Gravitational FF

Summary

# Nucleon $r_E^2$ and $\mu$ – Nonrelativistic !!!

Electric radii  $r_E^2$  [fm<sup>2</sup>]

Baryon	GBE PFSM	GBE NR1A	Experiment
$p$	0.82	0.10	$0.7692 \pm 0.0123$ <sup>1)</sup> $0.70870 \pm 0.00113$ <sup>2)</sup>
$n$	-0.13	-0.01	$-0.1161 \pm 0.0022$

<sup>1)</sup> CODATA value (PDG)

<sup>2)</sup> Pohl et al.: Nature **466** (2010) 213

Magnetic moments  $\mu$  [n.m.]

Baryon	GBE PFSM	GBE NR1A	Experiment
$p$	2.70	2.74	2.792847356
$n$	-1.70	-1.82	-1.9130427

K. Berger, R.F. Wagenbrunn, and W. Plessas: Phys. Rev. D **70**, 094027 (2004)

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Gravitational FF

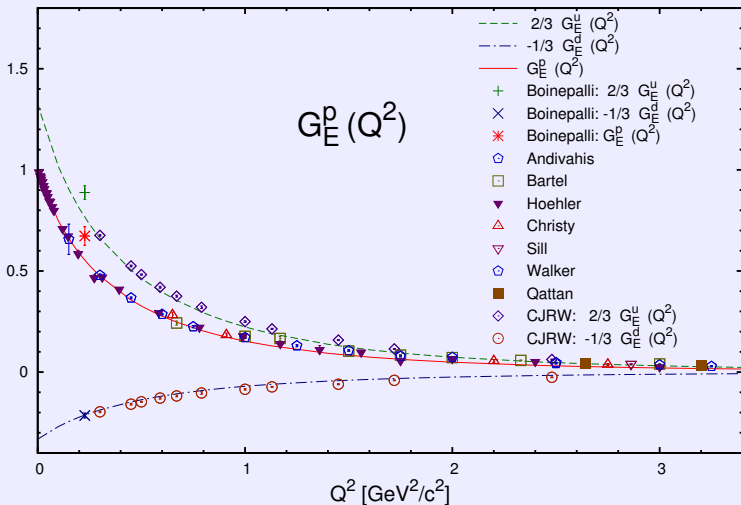
Summary

## Nucleons $N$



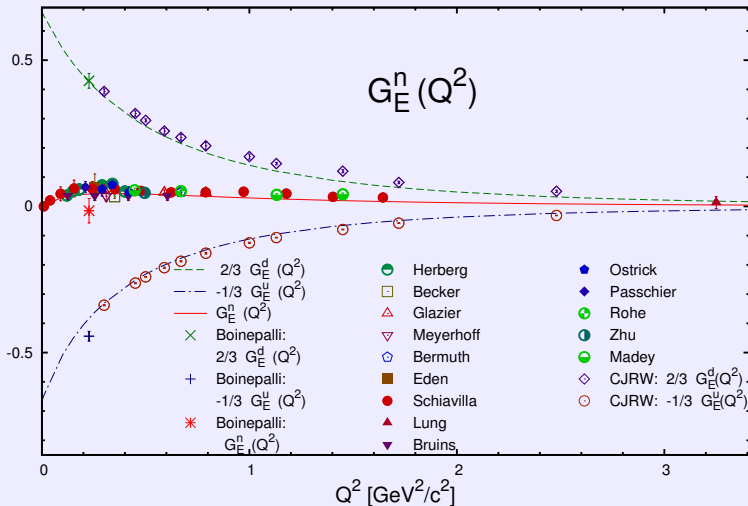
# Proton Electric Form Factor

$$G_E^p = \frac{2}{3} G_E^u - \frac{1}{3} G_E^d$$



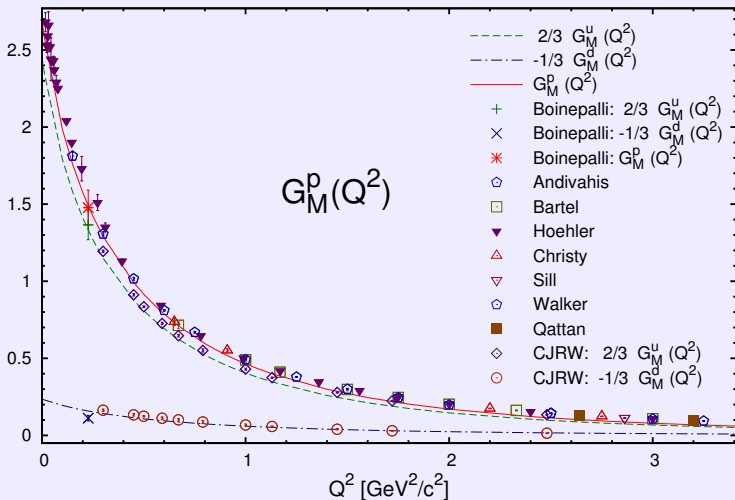
# Neutron Electric Form Factor

$$G_E^n = \frac{2}{3} G_E^d - \frac{1}{3} G_E^u$$



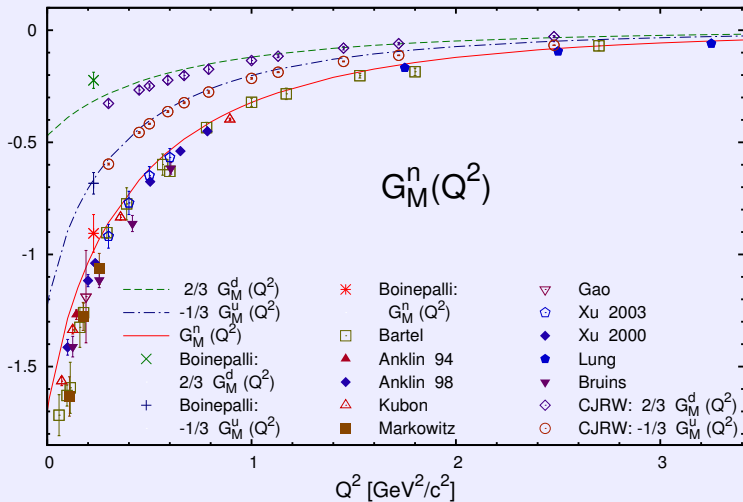
# Proton Magnetic Form Factor

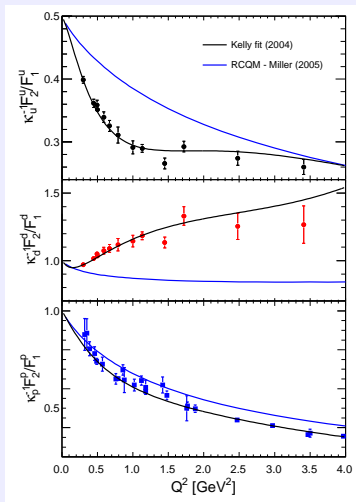
$$G_M^p = \frac{2}{3} G_M^u - \frac{1}{3} G_M^d$$



# Neutron Magnetic Form Factor

$$G_M^n = \frac{2}{3} G_M^d - \frac{1}{3} G_M^u$$

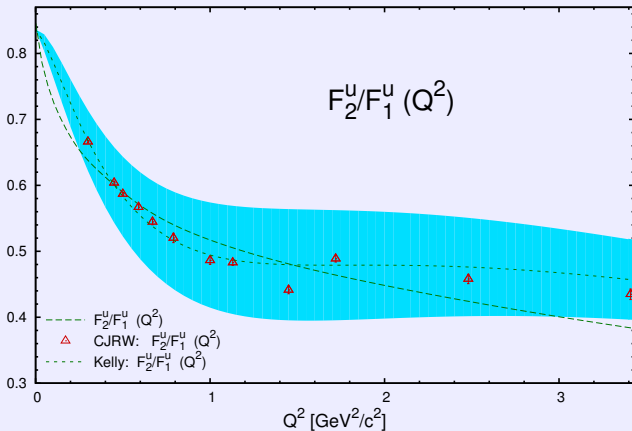




**3-Q vs. 5-Q components?**

From: G. D. Cates, C. W. de Jager, S. Riordan, B. Wojtsekhowski: Phys. Rev. Lett. **106**, 252003 (2011)

# Ratio $F_2^u/F_1^u$ of $u$ -Flavor Contr. to $F_1$ and $F_2$

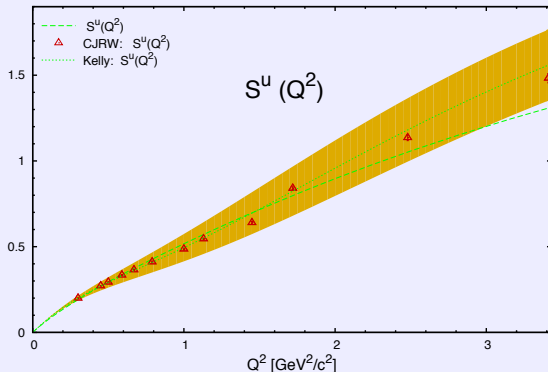


----- dashed green line: GBE RCQM      ..... dotted line and blue area: Kelly fit with  $\frac{1}{2}$  \*error

**No indication for 5- $Q$  components** in the nucleons!

# Ratio of $u$ -Flavor Contr. to $F_1$ and $F_2$ by $S^q$

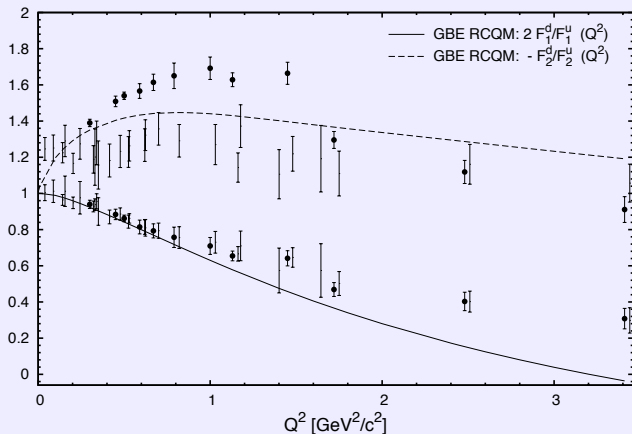
$$S^u(Q^2) = Q^2 \frac{F_2^u(Q^2)}{F_1^u(Q^2)}$$



----- dashed green line: GBE RCQM      ..... dotted line and orange area: Kelly fit with  $\frac{1}{2}$  \*error

**No indication for 5- $Q$  components in the nucleons!**

# Ratios $F_i^d/F_i^u$ of Flavor Contr. to $F_1$ and $F_2$



Fall-off is **no indication for diquark clustering** in the nucleons!

GBE RCQM prediction: M. Rohrmoser, Ki-Seok Choi, and W. Plessas: arXiv:1110.3665

Phenomenology: ● G. D. Cates et al.: Phys. Rev. Lett. **106**, 252003 (2011)

┌ M. Diehl and P. Kroll: arXiv:1302.4604



# Conclusions from Nucleon Flavor Analysis

- ▶ **Flavor analysis of nucleon e.m. form factors** in a relativistically invariant framework (point form).
- ▶ The **GBE RCQM** predicts flavor contributions in reasonable agreement with **experimental data**.
- ▶ The GBE RCQM relies on  $\{QQQ\}$  degrees of freedom only; no explicit  $\{QQQQ\bar{Q}\}$  etc.
- ▶ No explicit **meson-cloud effects** are included.
- ▶ No **strangeness content** in the nucleon for the low momentum transfers considered here.
- ▶ With respect to  $F_2^d/F_2^u$  three different phenomenological analyses give **distinct answers**.
- ▶ Details:  
M. Rohrmoser, Ki-Seok Choi, and W. Plessas: arXiv:1110.3665  
W. Plessas: Mod. Phys. Lett. A **28**, 136022 (2013)

# $\Delta$ and Hyperon E.m. Form Factors

Low-energy  
QCD

RCQM  
GBE RCQM

Spectroscopy  
Light, strange,  
charm, bottom

Structure  
Nucleon E.m.  
Baryon E.m.  
Axial FFs  
Gravitational FF

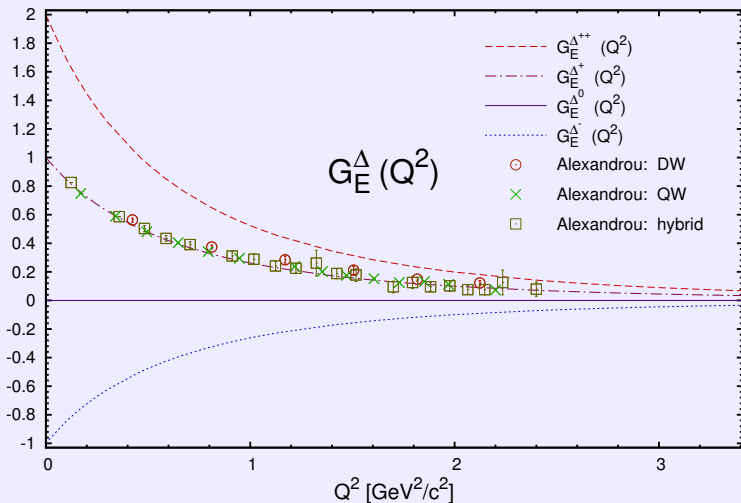
Summary

$\Delta$

$\Lambda, \Sigma, \Xi$

$\Sigma^*, \Xi^*, \Omega$

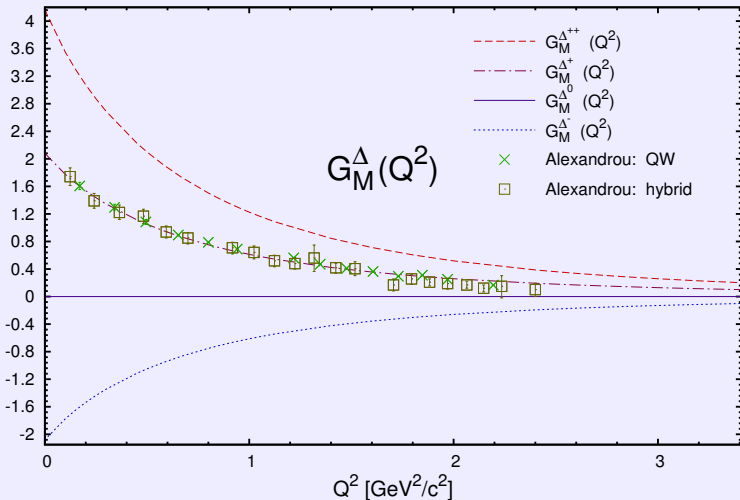
# Electric $\Delta$ Form Factors



GBE RCQM: Ki-Seok Choi: PhD Thesis, Univ. Graz, 2011

Lattice QCD: C. Alexandrou et al. Phys. Rev. D **79** (2009) 014507

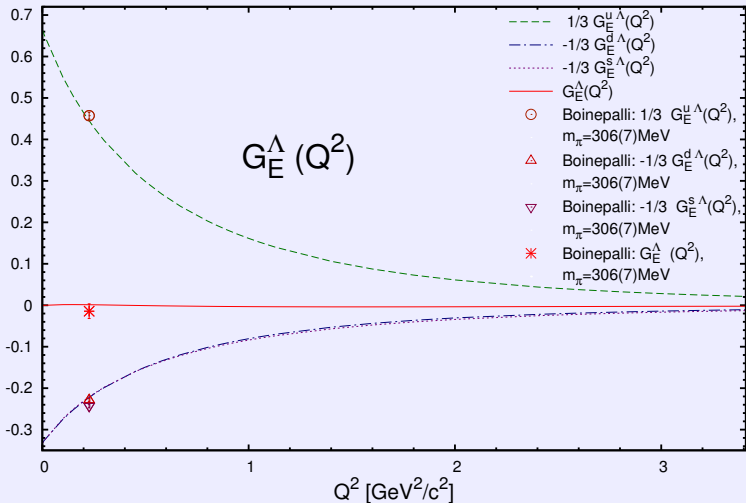
# Magnetic $\Delta$ Form Factors



GBE RCQM: Ki-Seok Choi: PhD Thesis, Univ. Graz, 2011

Lattice QCD: C. Alexandrou et al. Phys. Rev. D **79** (2009) 014507

# Octet $\Lambda(uds)$ Electric Form Factor



Low-energy QCD

RCQM

GBE RCQM

Spectroscopy

Light, strange, charm, bottom

Structure

Nucleon E.m.

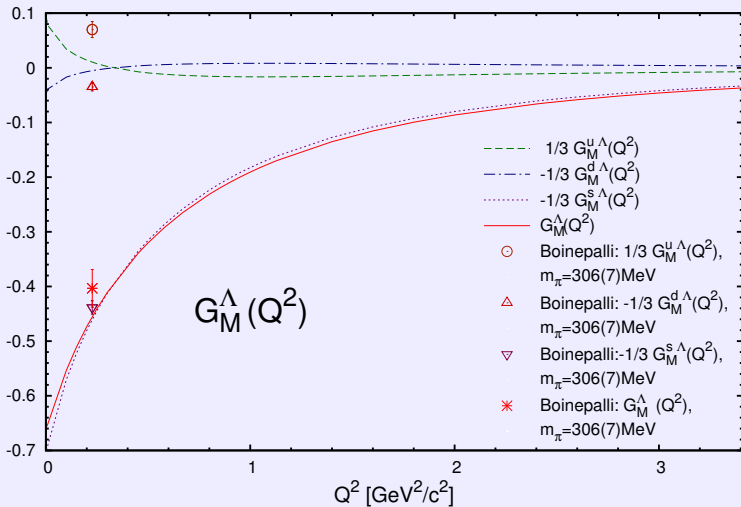
Baryon E.m.

Axial FFs

Gravitational FF

Summary

# Octet $\Lambda(uds)$ Magnetic Form Factor



Low-energy  
QCD

RCQM  
GBE RCQM

Spectroscopy  
Light, strange,  
charm, bottom

Structure  
Nucleon E.m.  
Baryon E.m.  
Axial FFs  
Gravitational FF

Summary

## Axial **Charges** and Axial **Form Factors**

of

**$N$**  Ground State and  **$N^*$**  Resonances

as well as

**$\Delta, \Sigma, \Xi, \Sigma^*, \Xi^*$**

Low-energy  
QCD

RCQM  
GBE RCQM

Spectroscopy  
Light, strange,  
charm, bottom

Structure  
Nucleon E.m.  
Baryon E.m.  
Axial FFs  
Gravitational FF

Summary

# Axial Nucleon Form Factors

Low-energy  
QCD

RCQM

GBE RCQM

Spectroscopy

Light, strange,  
charm, bottom

Structure

Nucleon E.m.

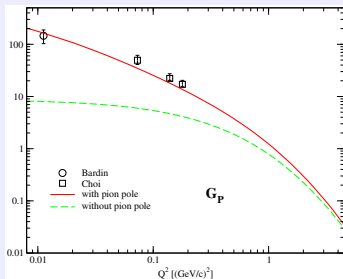
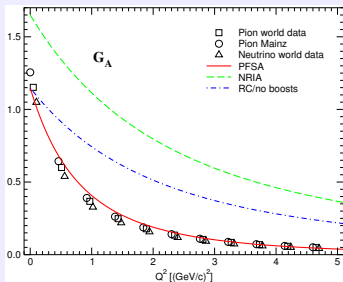
Baryon E.m.

Axial FFs

Gravitational FF

Summary

## Covariant predictions of the GBE RCQM:



$$g_A^{GBE} = 1.15 \quad \text{vs.}$$

$$g_A^{exp} = 1.2695 \pm 0.0029$$

L.Ya. Glozman, M. Radici, R.F. Wagenbrunn, S. Boffi, W. Klink, and W. Plessas: Phys. Lett. B **516**, 183 (2001)



# Axial Charges of $N$ and $N^*$ Resonances

State	$J^P$	EGBE	Lattice QCD	GN	NR
N(939)	$\frac{1}{2}^+$	1.15	1.23~1.26	1.66	1.65
N(1440)	$\frac{1}{2}^+$	1.16	?	1.66	1.61
N(1535)	$\frac{1}{2}^-$	0.02	~0.00	-0.11	-0.20
N(1710)	$\frac{1}{2}^+$	0.35	?	0.33	0.42
N(1650)	$\frac{1}{2}^-$	0.51	~0.55	0.55	0.64

EGBE      **E**xtended **G**BE RCQM covariant result

Lattice      **L**attice **Q**CD calculations by LHPC Collaboration and Takahashi-Kunihiro (Kyoto)

GN      **G**lozman-**N**efediev  $SU(6) \times O(3)$  nonrelativistic QM

NR      **N**on-**R**elativistic EGBE result

K.-S. Choi, W. Plessas, and R.F. Wagenbrunn: Phys. Rev. C **81**, 028201 (2010)

Low-energy  
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GBE RCQM

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Nucleon E.m.

Baryon E.m.

Axial FFs

Gravitational FF

Summary

# Axial Charges of $\Delta, \Sigma, \Xi, \Sigma^*, \Xi^*$

	$J^P$	Exp	EGBE	LO	EOT	JT	NR
N	$\frac{1}{2}^+$	1.2695	1.15	1.18	1.314	1.18	1.65
$\Sigma$	$\frac{1}{2}^+$	-	0.65	0.636	0.686	0.73	0.93
$\Xi$	$\frac{1}{2}^+$	-	-0.21	-0.277	-0.299	-0.23	-0.32
$\Delta$	$\frac{3}{2}^+$	-	-4.48	-	-	$\sim -4.5$	-6.00
$\Sigma^*$	$\frac{3}{2}^+$	-	-1.06	-	-	-	-1.41
$\Xi^*$	$\frac{3}{2}^+$	-	-0.75	-	-	-	-1.00

Low-energy QCD

RCQM  
GBE RCQM

Spectroscopy

Light, strange, charm, bottom

Structure

Nucleon E.m.

Baryon E.m.

Axial FFs

Gravitational FF

Summary

EGBE      **E**xtended **G**BE RCQM covariant result  
 LO        **L**in and **O**rginos lattice-QCD calculation  
 EOT      **E**rkol, **O**ka, and **T**akahashi lattice-QCD calculation  
 JT        **J**iang and **T**iburzi  $\chi$ PT calculation  
 NR        **N**on-**R**elativistic EGBE result

K.-S. Choi, W. Plessas, and R.F. Wagenbrunn: Phys. Rev. D **82**, 014007 (2010)

Low-energy  
QCD

RCQM  
GBE RCQM

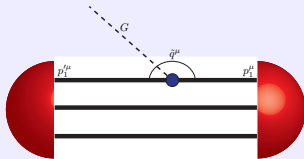
Spectroscopy  
Light, strange,  
charm, bottom

Structure  
Nucleon E.m.  
Baryon E.m.  
Axial FFs  
Gravitational FF

Summary

# Gravitational Form Factors of the Nucleon

# Gravitational Form Factors



Invariant ME of **energy-momentum tensor**  $\hat{\Theta}^{\mu\nu}$ :

$$\langle P' J \Sigma' | \hat{\Theta}^{\mu\nu} | P J \Sigma \rangle = \bar{U}(P') \left[ \gamma^{(\mu} \bar{P}^{\nu)} A(Q^2) + \frac{i}{2M} \bar{P}^{(\mu} \sigma^{\nu)} B(Q^2) + \frac{q^\mu q^\nu - q^2 g^{\mu\nu}}{M} C(Q^2) \right] U(P)$$

$$A(Q^2) \sim \langle P' J \Sigma' | \Theta^{00} | P J \Sigma \rangle$$

Low-energy

QCD

RCQM

GBE RCQM

Spectroscopy

Light, strange,  
charm, bottom

Structure

Nucleon E.m.

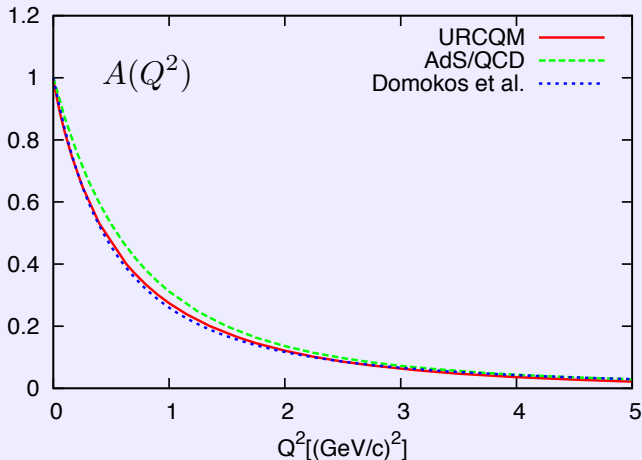
Baryon E.m.

Axial FFs

Gravitational FF

Summary

# Nucleon Gravitational Form Factor $A(Q^2)$



# Summary and Conclusions

- ▶ Surprisingly **good agreement** of predictions by GBE RCQM with experimental data (wherever such data are available)
- ▶ **Small deviations** left in some observables, such as electric radii and magnetic moments
- ▶ Surprisingly **good agreement** of predictions by GBE RCQM with lattice-QCD results
- ▶ Most important symmetries of GBE RCQM:
  - ▶ **SB $\chi$ S**
  - ▶ **Lorentz invariance**
  - ▶ **time-reversal invariance**
  - ▶ **current conservation**
- ▶ The **non-relativistic** quark model **does not work** in any instance

## Graz

K. Berger, J.P. Day, K.-S. Choi, L. Glozman, T. Melde,  
M. Rohrmoser, R.C. Schardmüller, B. Sengl,  
R.F. Wagenbrunn

(Theoretical Physics, University of Graz)

## Pavia

S. Boffi and M. Radici  
(INFN, Sezione di Pavia)

## Padova

L. Canton  
(INFN, Sezione di Padova)

## Iowa City

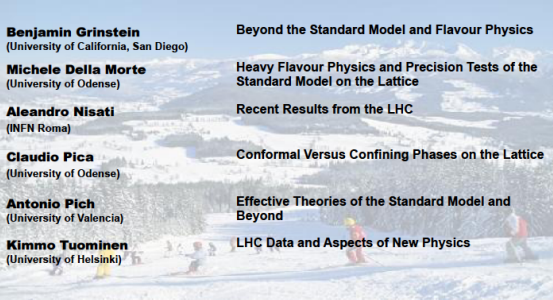
W. Klink  
(Department of Physics, University of Iowa, USA)

# Forthcoming Schladming Winter School

## 52. Internationale Universitätswochen für Theoretische Physik

### Physics Beyond the Higgs

Schladming, Styria, Austria, March 1 - 8, 2014



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<b>Michele Della Morte</b> (University of Odense)	<b>Heavy Flavour Physics and Precision Tests of the Standard Model on the Lattice</b>
<b>Aleandro Nisati</b> (INFN Roma)	<b>Recent Results from the LHC</b>
<b>Claudio Pica</b> (University of Odense)	<b>Conformal Versus Confining Phases on the Lattice</b>
<b>Antonio Pich</b> (University of Valencia)	<b>Effective Theories of the Standard Model and Beyond</b>
<b>Kimmo Tuominen</b> (University of Helsinki)	<b>LHC Data and Aspects of New Physics</b>

If you wish to apply, please access the web page and complete the registration form as soon as possible, but not later than **February 17, 2014**. More information about the school can be found on the web page as well.

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Director of the School:	Reinhard Alkofer	Institut für Physik, FB Theoretische Physik
Organizing Committee:	Natalia Alkofer	Karl-Franzens-Universität Graz
	Markus Pak	Universitätsplatz 5, A-8010 Graz, Austria
	Willibald Plessas	Phone: +43 316 380 5225
	Francesco Sannino (CP3-Origins)	Fax: +43 316 380 9820
		E-mail: <a href="mailto:theor.physik@uni-graz.at">theor.physik@uni-graz.at</a>
		<a href="http://physik.uni-graz.at/schladming2014/">http://physik.uni-graz.at/schladming2014/</a>

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Summary

Thank you very much  
for  
your attention!