WEAK INTERACTIONS IN SUPERNOVA ENVIRONMENTS

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Nuclear Structure and Reactions: Weak, Strange and Exotic

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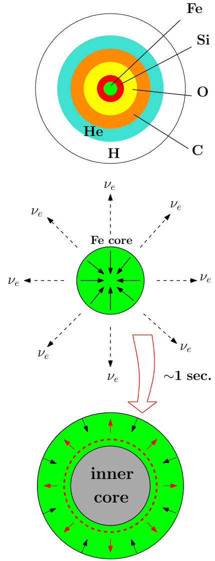
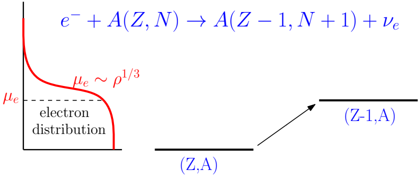
Motivation



- Massive stars ($M \geq 10 M_\odot$) at the end of their life evolve to an onion like structure;
- Iron core can be stabilized by the pressure of degenerate electron gas as along as $M_{core} < M_{Ch} \approx 1.44(2Y_e)^2 M_\odot$;
- There are two processes that make the situation unstable:
  1. as the silicon burning proceeds, the iron core approaches M_{Ch} and contracts ($\mu_e \sim \rho^{1/3}$);
  2. when $\mu_e \approx 2$ MeV electron capture reduces the electron gas pressure.



- Finally the core collapses ($\sim$ 1 sec.) under its own gravity.
- When $\rho \sim 10^{14} g/cm^3$ a shock-wave is produced that triggers supernova explosion.

Motivation

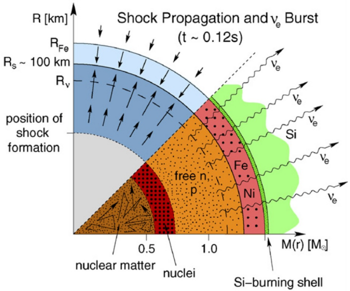
Prompt mechanism is unable to trigger supernova explosion!

Two reasons for energy loss in a shock-wave:

1. Dissociation nuclei into nucleons ($\sim 8.8\,\mathrm{MeV}$ per nucleon)
2. Neutrino emission:

$$e^- + p \to n + \nu_e$$

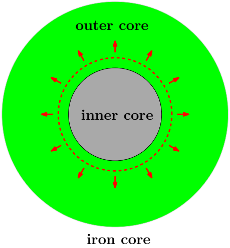
from H.-Th. Janka et al, Phys. Rep. **442** (2007) 38



Motivation

Possible mechanisms for the shock-wave revival:

1. Neutrino-heating mechanism
2. Magnetorotational mechanism (G. Bisnovatiy-Kogan, 1970)
3. Acoustic mechanism (A. Burrows, 2006)
4. Phase-transition mechanism (I. Sagert, 2009, T. Fischer, 2011)



$$\Delta M = M_{\text{iron core}} - M_{\text{inner core}}$$

- $M_{\text{iron core}} \approx M_{\text{Ch}} \sim Y_e^2$, therefore

$$e^- + (A,Z) \to (A, Z-1) + \nu_e$$
$$(A,Z) \to (A, Z+1) + e^- + \overline{\nu}_e$$

determine the **iron core** mass.

- ν-nucleus reactions become important at $\rho \geq 10^{11}\ \mathrm{g\,cm^{-3}}$:

$$\nu + (A,Z) \to (A,Z) + \nu$$
$$\nu + (A,Z) \to (A,Z) + \nu'$$
$$\nu_e + (A,Z) \to (A,Z-1) + e^-$$

trap neutrinos in the **inner core** and determine its mass:

$$M_{\text{inner core}} \sim Y_L^2, \text{ where } Y_L = Y_e + Y_\nu$$

GT transitions

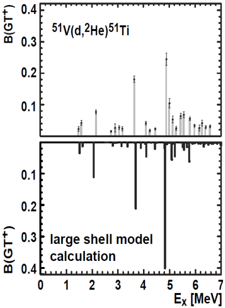
In supernova $E_{e,\nu} \leq 30$ MeV and **Gamow-Teller** (1^+) transitions dominate the nuclear weak-interaction processes:

- $e^- + A(N,Z) \to A(Z-1, N+1) + \nu_e \quad (GT_+ = \sum_i \sigma_i t_i^+)$;
- $\nu_e + A(Z,N) \to A(Z+1, N-1) + e^- \quad (GT_- = \sum_i \sigma_i t_i^-)$;
- $\nu + A(Z,N) \to A(Z,N) + \nu' \quad (GT_0 = \sum_i \sigma_i t_i^0)$.

How to determine strength distribution for **GT** transitions (from the nuclear ground state)?

- Experiment: **(p,n), (n,p), $(d, {}^2\mathrm{He})$ reactions**, **M1 resonance**, etc
- Theoretical calculations: **QRPA, shell-model calculations.**

Modern large-scale shell-model calculations in a huge configuration space ($\sim 10^9 \div 10^{12}$) very well reproduce experimental data for iron group nuclei ($A = 45 - 65$).



Bäumer *et al*, PRC **68**, 031303 (2003)

For $T > 0.1$ **MeV** all electromagnetic and strong reactions

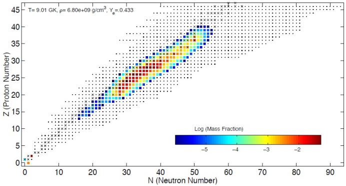
$$(A,Z) + p \rightleftarrows (A+1, Z+1) + \gamma$$
$$(A,Z) + n \rightleftarrows (A+1, Z) + \gamma$$

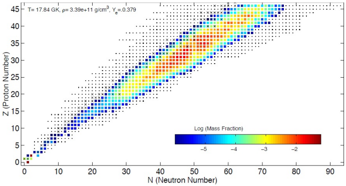
as well as (α,γ), (α,n), (α,p), (p,n) are in equilibrium.

Saha equation:

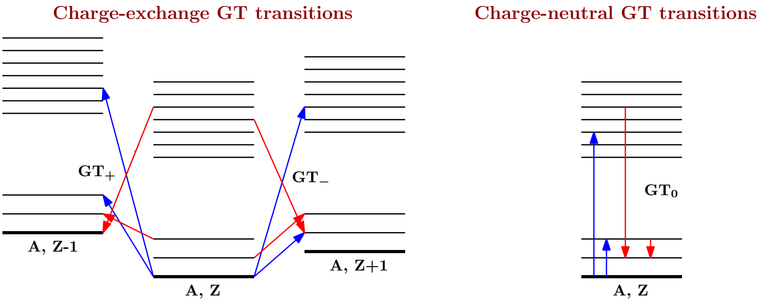
$$Y(A,Z) = \frac{G(A,Z)A^{3/2}}{2^A} Y_p^Z Y_n^N \left(\frac{2\pi\hbar^2}{m_u kT}\right)^{3/2(A-1)} e^{B(A,Z)/kT},$$

where $\sum_i Y_i A_i = 1$ and $\sum_i Y_i Z_i = Y_e$.





In the supernova environment nuclear excited states are thermally populated according to Boltzmann distribution: $g_i(T) \sim (J_i + 1)\exp(-E_i/T)$.



$$\sigma(E,T) = \sum_i g_i(T)\sigma_i(E), \quad \lambda(E,T) = \sum_i g_i(T)\lambda_i(E),$$

The temperature varies from **a few hundreds keV** to **a few MeV** ($0.86\ \mathrm{MeV} \approx 10^{10}\ \mathrm{K}$).

For $T = 1$ MeV the mean excitation energy is $\langle E\rangle_{Fermi\ gas} = AT^2/8 \approx 7 \div 8$ MeV for iron-group nuclei ($A = 45 - 65$).

Cross section for $\nu + A \to \nu' + A$:

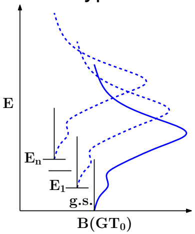
$$\sigma(E_\nu, T) = \sigma_d(E_\nu) + \sigma_{up}(E_\nu, T),$$

$$\sigma_d(E_\nu) \sim \sum_f E_{\nu'}^2 |\langle g.s.|\boldsymbol{\sigma} t_0|f\rangle|^2, \quad (E_{\nu'} = E_\nu - E_f);$$

$$\sigma_{up}(E_\nu, T) \sim \sum_{i,f} E_{\nu'}^2 |\langle i|\boldsymbol{\sigma} t_0|f\rangle|^2 \exp\left(-\frac{E_i}{T}\right), \quad (E_i > E_f)$$

$$|\langle i|\boldsymbol{\sigma} t_0|f\rangle|^2 = \frac{2J_f+1}{2J_i+1} |\langle f|\boldsymbol{\sigma} t_0|i\rangle|^2;$$

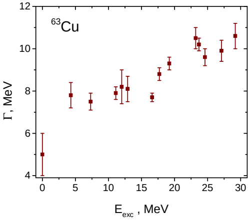
Brink hypothesis



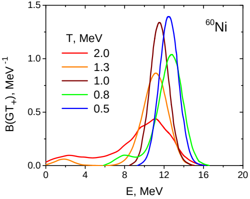
Shortcomings:

- Brink's hypothesis is applied;
- Detailed balance principle is violated

$$S(T, -E) \neq S(T, E) \exp\left(-\frac{E}{T}\right).$$

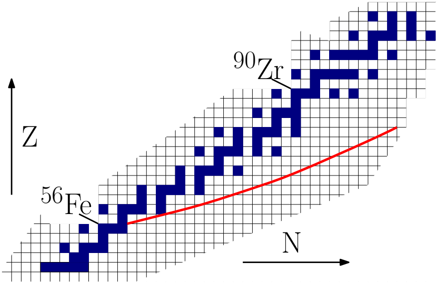


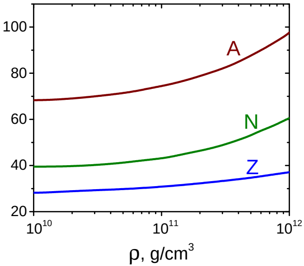
PRC36(1987)612



PRC56(1997)3079

LSSM calculations are limited (at present!) by iron group nuclei ($A = 45 - 65$)





Element trajectory (left figure) and evolution of the mean nucleus as a function of the matter density in the core (right figure).

Definition:

$$\boldsymbol{S_{\mathcal{T}}(E,T) = \sum_{i,f} |\langle f|\mathcal{T}|i\rangle|^2 \frac{e^{-E_i/T}}{W}\delta(E - E_f + E_i)}, \quad W = \sum_i e^{-E_i/T}.$$

Detailed balance: $\boldsymbol{S_{\mathcal{T}}(-E,T) = S_{\mathcal{T}}(E,T)\exp\left(-\frac{E}{T}\right)}$.

Applying $\delta(E) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} dte^{iEt}$ we find

$$\boldsymbol{S_{\mathcal{T}}(E,T) = \frac{1}{2\pi}\int dte^{iEt}\langle\langle\mathcal{T}(t)\mathcal{T}(0)\rangle\rangle}$$

where $\langle\langle\ldots\rangle\rangle = \sum_i \frac{e^{-E_i/T}}{W}\langle i|\ldots|i\rangle$ and $\mathcal{T}(t) = e^{iHt}\mathcal{T}e^{-iHt}$.

Two-point functions can be computed either by applying the thermal Green's function technique or by the Thermo Field Dynamics.

- Thermal Hamiltonian: $\mathcal{H} = H(a^\dagger, a) - H(\widetilde{a}^\dagger, \widetilde{a})$;
- Equilibrium state (thermal vacuum):

$$\mathcal{H}|0(T)\rangle = 0 \ \text{ and } \ \langle\langle A\rangle\rangle = \langle 0(T)|A|0(T)\rangle$$

- Nonequilibrium states:

$$\mathcal{H} \approx \sum_n \omega_n(T)(Q_n^\dagger Q_n - \widetilde{Q}_n^\dagger \widetilde{Q}_n)$$

Response function within TFD :

$$S_{\mathcal{T}}(E,T) \approx \sum_n \{ |\langle Q_n|\mathcal{T}|0(T)\rangle|^2 \delta(\omega_n - E) + |\langle \widetilde{Q}_n|\mathcal{T}|0(T)\rangle|^2 \delta(\omega_n + E) \};$$

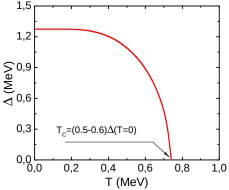
Therefore

- $|0(T)\rangle \to |Q_n\rangle$ – excitation process, $|0(T)\rangle \to |\widetilde{Q}_n\rangle$ – de-excitation process;
- Transition strengths $S_n = |\langle Q_n|\mathcal{T}|0(T)\rangle|^2$ and $\widetilde{S}_n = |\langle \widetilde{Q}_n|\mathcal{T}|0(T)\rangle|^2$ obey the detailed balance principle $\widetilde{S}_n = \exp\left(-\frac{\omega_n}{T}\right) S_n$.

- The QPM Hamiltonian: $\boldsymbol{H = H_{WS} + H_{BCS} + H^{M}_{ph} + H^{SM}_{ph}}$
- Thermal quasiparticles: $\boldsymbol{\mathcal{H}_{WS+BCS} \approx \sum_j \varepsilon_j(T)(\beta^\dagger_{jm}\beta_{jm} - \widetilde{\beta}^\dagger_{jm}\widetilde{\beta}_{jm})}$

$$\langle 0(T)|\alpha^\dagger_{jm}\alpha_{jm}|0(T)\rangle = \left[1 + \exp\left(\frac{\varepsilon_j}{T}\right)\right]^{-1}$$

$$\beta^\dagger|0(T)\rangle \sim \alpha^\dagger|0(T)\rangle, \qquad \widetilde{\beta}^\dagger|0(T)\rangle \sim \alpha|0(T)\rangle.$$

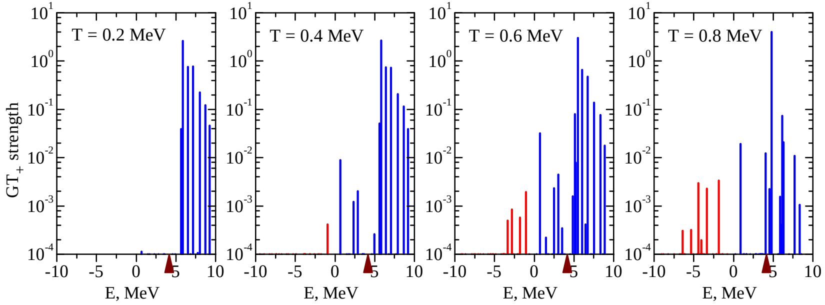


- Thermal phonons: $\boldsymbol{\mathcal{H} \approx \sum_{\lambda\mu k} \omega_{\lambda k}(T)(Q^\dagger_{\lambda\mu k}Q_{\lambda\mu k} - \widetilde{Q}^\dagger_{\lambda\mu k}\widetilde{Q}_{\lambda\mu k})}$

$$Q^\dagger_{\lambda\mu k} = \sum_{jj'}\Big(\psi^{\lambda k}_{jj'}[\beta^\dagger_j\beta^\dagger_{j'}]^\lambda_\mu + \widetilde{\psi}^{\lambda k}_{jj'}[\widetilde{\beta}^\dagger_{\overline{\jmath}}\widetilde{\beta}^\dagger_{\overline{\jmath'}}]^\lambda_\mu + \eta^{\lambda k}_{jj'}[\beta^\dagger_j\widetilde{\beta}^\dagger_{\overline{\jmath'}}]^\lambda_\mu$$

$$+\phi^{\lambda k}_{jj'}[\beta_{\overline{\jmath}}\beta_{\overline{\jmath'}}]^\lambda_\mu + \widetilde{\phi}^{\lambda k}_{jj'}[\widetilde{\beta}_j\widetilde{\beta}_{j'}]^\lambda_\mu + \xi^{\lambda k}_{jj'}[\beta_{\overline{\jmath}}\widetilde{\beta}_{j'}]^\lambda_\mu\Big)$$

GT_+ strength distribution in ^{56}Fe ($T \neq 0$)

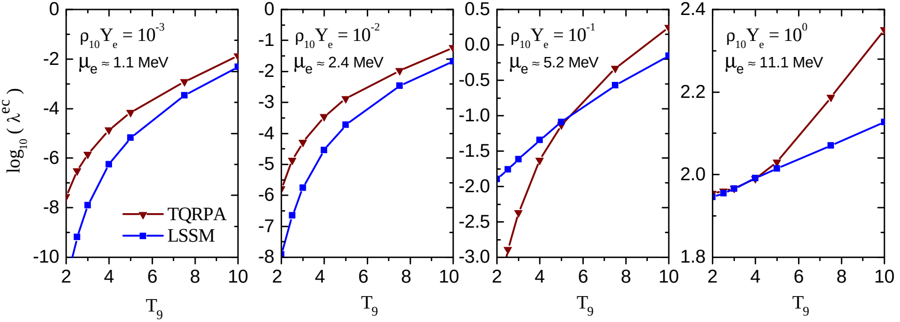


$\Delta_p \approx 1.4$ MeV $\Rightarrow$ $T_{\rm cr} \approx 0.5\Delta(\approx 0.7$ MeV$)$.

The brown arrows indicate the zero-temperature EC threshold:
$Q = M(^{56}\mathrm{Mn}) - M(^{56}\mathrm{Fe}) = 4.2\ \mathrm{MeV}$.

$$\lambda^{ec} = \frac{\ln 2}{6150 s} \sum_k S_k(GT_+) F_k, \quad F_k = \int_{E_0}^{\infty} E^2 (E - E_k)^2 F(Z, E) f_e(E) dE$$

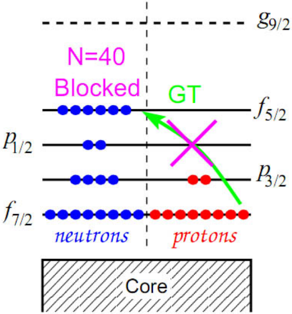
where $f_e(E) = \left[1 + \exp\left(-\frac{E-\mu_e}{T}\right)\right]^{-1}$ and $\mu_e \approx 11.1(\rho_{10} Y_e)^{1/3}$ MeV.

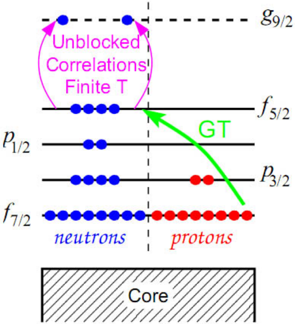


$T_9 = 10^9$ Kelvin, ρ_{10} is the density in units of 10^{10} g cm^{-3}

TQRPA - A. Dzhioev et al, Phys. Rev. C **81** (2010) 015804; **LSSM** - K. Langanke and G. Martínez-Pinedo, ADNDT **79** (2001) 1

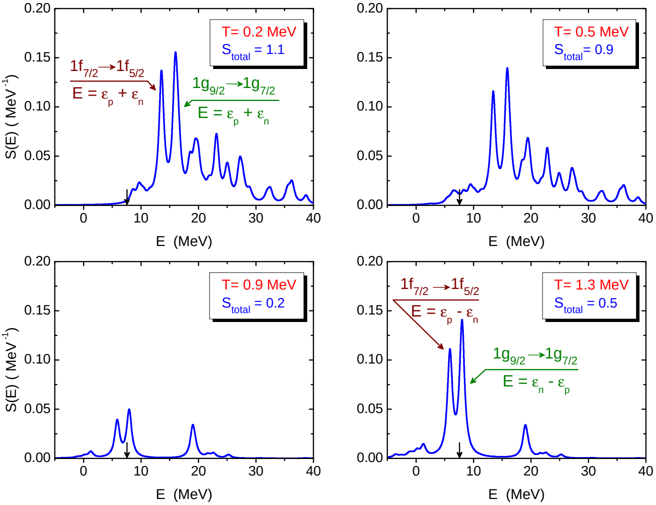
Neutron-rich nuclei ($N > 40, \quad Z < 40$)





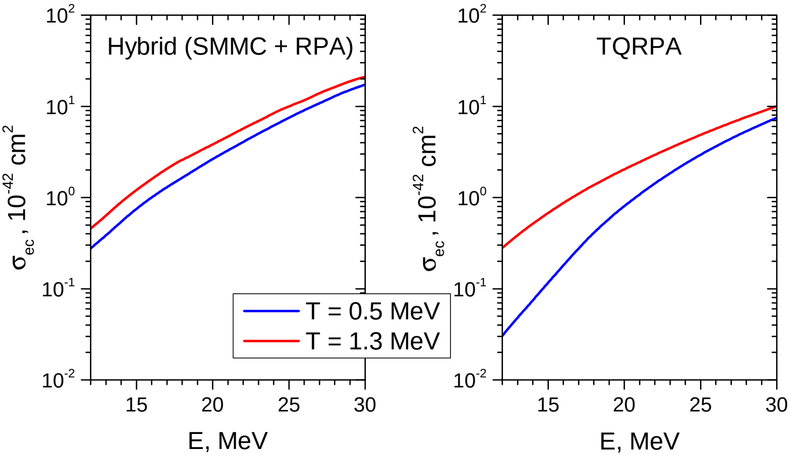
- Unblocking mechanisms: configuration mixing and thermal excitations
- Hybrid model: SMMC + RPA (K. Langanke et al, PRC**63** (2001) 032801)

GT$_+$ strength distribution in ^{76}Ge ($T \neq 0$)



A. Dzhioev et al, Phys. Rev. C **81** (2010) 015804

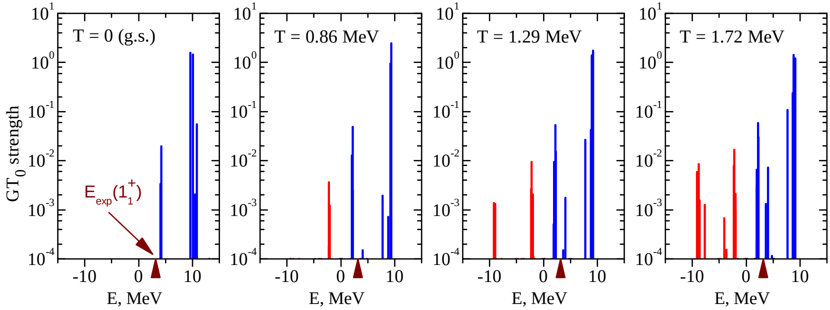
Electron capture cross sections for ^{76}Ge



- **Hybrid** - K. Langanke et al, Phys. Rev. C **63** (2001) 032801(R)
- **TQRPA** - A. Dzhioev et al, Phys. Rev. C **81** (2010) 015804

$$\nu + {}^{56}\text{Fe} \rightarrow {}^{56}\text{Fe} + \nu'$$

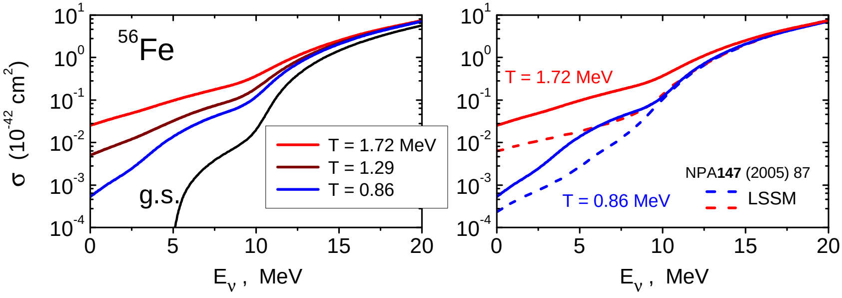
$T = 0.86$ MeV (10^{10} K) corresponds to the condition of a presupernova model for a $15M_\odot$ star; $T = 1.29$ MeV (1.5×10^{10} K) - relates to neutrino trapping, $T = 1.72$ MeV (2×10^{10} K) - to neutrino thermalization.

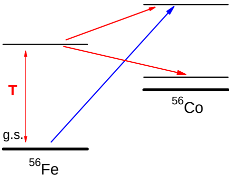


Detailed balance: $S_{GT}(-E, T) = S_{GT}(E, T)\exp\left(-\frac{E}{T}\right)$.

$$\sigma(E_\nu, T) = \frac{G_F^2}{\pi}\left\{\sum_k (E_\nu - \omega_k)^2 S_k + \sum_k (E_\nu + \omega_k)^2 \widetilde{S}_k\right\}$$
$$= \sigma_d(E_\nu, T) + \sigma_{up}(E_\nu, T),$$

- $S_k = |\langle Q_k|\boldsymbol{\sigma} t_0|0(T)\rangle|^2$ and $E'_\nu = E_\nu - \omega_k$ for down-scattering;
- $\widetilde{S}_k = |\langle \widetilde{Q}_k|\boldsymbol{\sigma} t_0|0(T)\rangle|^2$ and $E'_\nu = E_\nu + \omega_k$ for up-scattering.

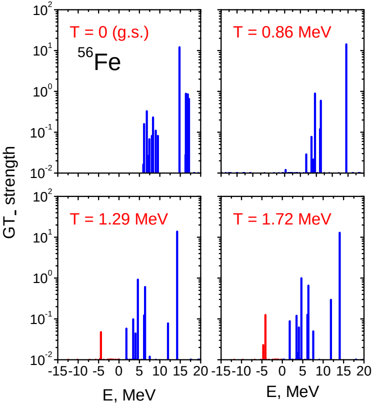


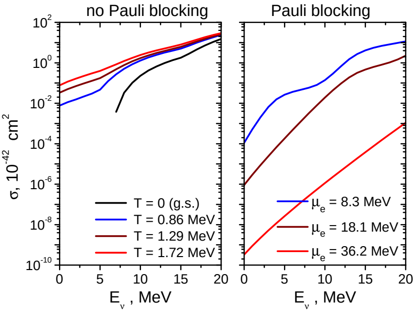


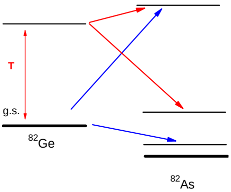
$$\nu_e + {}^{56}\mathrm{Fe} \rightarrow {}^{56}\mathrm{Co} + e^-$$

$$\sigma(E_\nu, T) = \frac{G_F^2}{\pi} \sum_k (E_e^k)^2 S_k(GT_-)[1 - f(E_e^k)]$$

$$f_e(E) = \left[1 + \exp\left(-\frac{E - \mu_e}{T}\right)\right]^{-1}$$



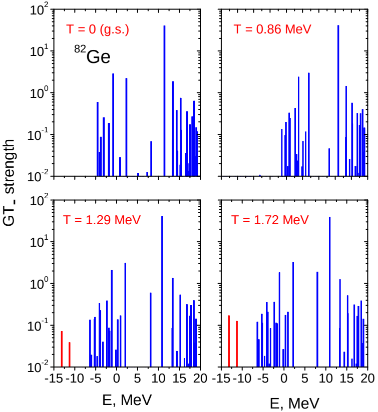


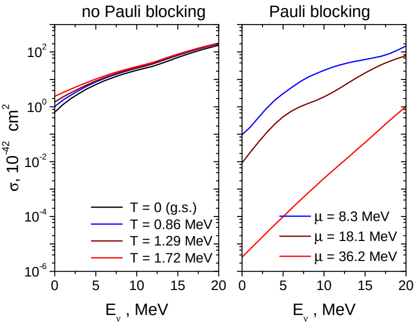


$$\nu_e + {}^{82}\mathrm{Ge} \rightarrow {}^{82}\mathrm{As} + e^-$$

$$\sigma(E_\nu, T) = \frac{G_F^2}{\pi}\sum_k (E_e^k)^2 S_k(GT_-)[1 - f(E_e^k)]$$

$$f_e(E) = \left[1 + \exp\left(-\frac{E-\mu_e}{T}\right)\right]^{-1}$$





Summary

- Currently, there exist two main theoretical approaches in calculations of the rates and cross-sections for weak-interaction reactions with hot nuclei in stellar environments: the Large Scale Shell Model approach and the Thermal Quasiparticle Random Phase Approximation.
- Both the approaches predict a strong thermal enhancement of the cross-section and rates at low lepton energies. This enhancement is due to thermal population of nuclear excited states.
- The LSSM approach is fairly successful in calculations with the iron-group nuclei ($A = 45 - 65$), but it partially employs the Brink hypothesis when treating GT transitions from nuclear excited states.
- The TQRPA method does not rely on the Brink hypothesis and it can be applied to massive neutron-rich nuclei. Moreover, the corresponding calculations are much less time consuming.

Credits to: A. Vdovin, J. Wambach, K. Langanke, G. Martínez-Pinedo and V. Ponomarev.

THANK YOU FOR ATTENTION !