WEAK INTERACTIONS IN SUPERNOVA ENVIRONMENTS

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Nuclear Structure and Reactions: Weak, Strange and Exotic

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Massive stars ($M \geq 10M_\odot$) at the end of their life evolve to an onion like structure;

Iron core can be stabilized by the pressure of degenerate electron gas as along as $M_{\text{core}} < M_{\text{Ch}} \approx 1.44(2Y_e)^2 M_\odot$;

There are two processes that make the situation unstable:

1. as the silicon burning proceeds, the iron core approaches $M_{\text{Ch}}$ and contracts ($\mu_e \sim \rho^{1/3}$);
2. when $\mu_e \approx 2$ MeV electron capture reduces the electron gas pressure.

Finally the core collapses ($\sim 1$ sec.) under its own gravity.

When $\rho \sim 10^{14} g/cm^3$ a shock-wave is produced that triggers supernova explosion.
Motivation

Prompt mechanism is unable to trigger supernova explosion!

Two reasons for energy loss in a shock-wave:

1. Dissociation nuclei into nucleons ($\sim 8.8$ MeV per nucleon)

2. Neutrino emission:

$$e^- + p \rightarrow n + \nu_e$$

Motivation

Possible mechanisms for the shock-wave revival:

1. Neutrino-heating mechanism
3. Acoustic mechanism (A. Burrows, 2006)
\[ \Delta M = M_{\text{iron core}} - M_{\text{inner core}} \]

- \( M_{\text{iron core}} \approx M_{\text{Ch}} \sim Y_e^2 \), therefore
  
  \[ e^- + (A, Z) \rightarrow (A, Z - 1) + \nu_e \]
  
  \( (A, Z) \rightarrow (A, Z + 1) + e^- + \bar{\nu}_e \)

  determine the \textit{iron core} mass.

- \( \nu \)-nucleus reactions become important at \( \rho \geq 10^{11} \text{ g cm}^{-3} \):
  
  \[ \nu + (A, Z) \rightarrow (A, Z) + \nu \]
  
  \[ \nu + (A, Z) \rightarrow (A, Z) + \nu' \]
  
  \[ \nu_e + (A, Z) \rightarrow (A, Z - 1) + e^- \]

  trap neutrinos in the \textit{inner core} and determine its mass:

  \[ M_{\text{inner core}} \sim Y_L^2 \], where \( Y_L = Y_e + Y_\nu \)
GT transitions

In supernova $E_{e,\nu} \leq 30$ MeV and Gamow-Teller ($1^+$) transitions dominate the nuclear weak-interaction processes:

- $e^- + A(N, Z) \rightarrow A(Z - 1, N + 1) + \nu_e$ (GT$^+ = \sum_i \sigma_i t_i^+$);
- $\nu_e + A(Z, N) \rightarrow A(Z + 1, N - 1) + e^-$ (GT$^- = \sum_i \sigma_i t_i^-$);
- $\nu + A(Z, N) \rightarrow A(Z, N) + \nu'$ (GT$^0 = \sum_i \sigma_i t_i^0$).

How to determine strength distribution for GT transitions (from the nuclear ground state)?

- Experiment: $(p, n), (n, p), (d, ^2\text{He})$ reactions, M1 resonance, etc
- Theoretical calculations: QRPA, shell-model calculations.

Modern large-scale shell-model calculations in a huge configuration space ($\sim 10^9 \div 10^{12}$) very well reproduce experimental data for iron group nuclei ($A = 45 - 65$).

Nuclear statistical equilibrium

For $T > 0.1$ MeV all electromagnetic and strong reactions

$$\begin{align*}
(A, Z) + p &\rightleftharpoons (A + 1, Z + 1) + \gamma \\
(A, Z) + n &\rightleftharpoons (A + 1, Z) + \gamma
\end{align*}$$

as well as $(\alpha, \gamma)$, $(\alpha, n)$, $(\alpha, p)$, $(p, n)$ are in equilibrium.

Saha equation:

$$Y(A, Z) = \frac{G(A, Z)A^{3/2}}{2^A} \frac{Y_p^Z Y_n^N}{2^A} \left(\frac{2\pi\hbar^2}{m_u kT}\right)^{3/2(A-1)} e^{B(A, Z)/kT},$$

where $\sum_i Y_i A_i = 1$ and $\sum_i Y_i Z_i = Y_e$. 

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<insert graphs here>
Finite temperature

In the supernova environment nuclear excited states are thermally populated according to Boltzmann distribution: 

\[ g_i(T) \sim (J_i + 1) \exp(-E_i/T). \]

The temperature varies from a few hundreds keV to a few MeV (0.86 MeV \(\approx 10^{10}\) K).

For \( T = 1 \text{ MeV} \) the mean excitation energy is 

\[ \langle E \rangle_{\text{Fermi gas}} = AT^2/8 \approx 7 \div 8 \text{ MeV} \]

for iron-group nuclei (\( A = 45 - 65 \)).
Large-Scale Shell Model calculations at $T \neq 0$ (K. Langanke, et al.)

Cross section for $\nu + A \rightarrow \nu' + A$:

$$\sigma(E_\nu, T) = \sigma_d(E_\nu) + \sigma_{up}(E_\nu, T),$$

$$\sigma_d(E_\nu) \sim \sum_f E_{\nu'} f |\langle g.s. |\sigma_0 |f \rangle |^2, \quad (E_{\nu'} = E_\nu - E_f);$$

$$\sigma_{up}(E_\nu, T) \sim \sum_{i,f} E_{\nu'} f |\langle i |\sigma_0 |f \rangle |^2 \exp \left( -\frac{E_i}{T} \right), \quad (E_i > E_f)$$

$$|\langle i |\sigma_0 |f \rangle |^2 = \frac{2J_f + 1}{2J_i + 1} |\langle f |\sigma_0 |i \rangle |^2;$$

Shortcomings:

- Brink's hypothesis is applied;
- Detailed balance principle is violated

$$S(T, -E) \neq S(T, E) \exp \left( -\frac{E}{T} \right).$$
LSSM calculations are limited (at present!) by iron group nuclei ($A = 45 - 65$)

Element trajectory (left figure) and evolution of the mean nucleus as a function of the matter density in the core (right figure).
Thermal strength function for the transition operator $\mathcal{T}$

**Definition:**

$$S_{\mathcal{T}}(E, T) = \sum_{i,f} |\langle f | \mathcal{T} | i \rangle|^2 \frac{e^{-E_i/T}}{W} \delta(E - E_f + E_i), \quad W = \sum_i e^{-E_i/T}.$$ 

Detailed balance: $S_{\mathcal{T}}(-E, T) = S_{\mathcal{T}}(E, T) \exp\left(\frac{-E}{T}\right)$.

Applying $\delta(E) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} dt e^{iEt}$ we find

$$S_{\mathcal{T}}(E, T) = \frac{1}{2\pi} \int dt e^{iEt} \langle \mathcal{T}(t) \mathcal{T}(0) \rangle$$

where $\langle \ldots \rangle = \sum_i \frac{e^{-E_i/T}}{W} \langle i | \ldots | i \rangle$ and $\mathcal{T}(t) = e^{iHt} \mathcal{T} e^{-iHt}$.

Two-point functions can be computed either by applying the thermal Green’s function technique or by the Thermo Field Dynamics.
Thermo Field Dynamics (basics of the formalism)

- Thermal Hamiltonian: $\mathcal{H} = H(a^\dagger, a) - H(\tilde{a}^\dagger, \tilde{a})$;
- Equilibrium state (thermal vacuum):
  \[ \mathcal{H}|0(T)\rangle = 0 \text{ and } \langle\langle A\rangle\rangle = \langle 0(T)|A|0(T)\rangle \]
- Nonequilibrium states:
  \[ \mathcal{H} \approx \sum_n \omega_n(T)(Q_n^\dagger Q_n - \tilde{Q}_n^\dagger \tilde{Q}_n) \]

Response function within TFD:
\[ S_T(E, T) \approx \sum_n \left\{ |\langle Q_n|T|0(T)\rangle|^2 \delta(\omega_n - E) \right. \]
\[ + |\langle \tilde{Q}_n|T|0(T)\rangle|^2 \delta(\omega_n + E) \right\}; \]

Therefore
- $|0(T)\rangle \rightarrow |Q_n\rangle$ – excitation process, $|0(T)\rangle \rightarrow |\tilde{Q}_n\rangle$ – de-excitation process;
- Transition strengths $S_n = |\langle Q_n|T|0(T)\rangle|^2$ and $\tilde{S}_n = |\langle \tilde{Q}_n|T|0(T)\rangle|^2$ obey the detailed balance principle $\tilde{S}_n = \exp\left(-\frac{\omega_n}{T}\right)S_n$. 
Thermal quasiparticle RPA

- The QPM Hamiltonian: \( H = H_{WS} + H_{BCS} + H_{ph}^M + H_{ph}^{SM} \)

- Thermal quasiparticles: \( H_{WS+BCS} \approx \sum_j \epsilon_j(T)(\beta_{jm}\beta_{jm} - \tilde{\beta}_{jm}\tilde{\beta}_{jm}) \)

\[
\langle 0(T)|\alpha_j^\dagger\alpha_{jm}|0(T)\rangle = \left[ 1 + \exp \left( \frac{\epsilon_j}{T} \right) \right]^{-1}
\]

\( \beta^\dagger|0(T)\rangle \sim \alpha^\dagger|0(T)\rangle, \quad \tilde{\beta}^\dagger|0(T)\rangle \sim \alpha|0(T)\rangle. \)

- Thermal phonons: \( H \approx \sum_{\lambda\mu k} \omega_{\lambda k}(T)(Q_{\lambda\mu k}^\dagger Q_{\lambda\mu k} - \tilde{Q}_{\lambda\mu k}^\dagger \tilde{Q}_{\lambda\mu k}) \)

\[
Q_{\lambda\mu k}^\dagger = \sum_{jj'} \left( \psi_{jj'}^\lambda [\beta_{j}^\dagger \beta_{j'}^\dagger]_{\mu} + \tilde{\psi}_{jj'}^\lambda [\tilde{\beta}_{j}^\dagger \tilde{\beta}_{j'}^\dagger]_{\mu} + \eta_{jj'}^\lambda [\beta_{j}^\dagger \tilde{\beta}_{j'}^\dagger]_{\mu} 
+ \phi_{jj'}^\lambda [\beta_{j}^\dagger \beta_{j'}^\dagger]_{\mu} + \tilde{\phi}_{jj'}^\lambda [\tilde{\beta}_{j}^\dagger \tilde{\beta}_{j'}^\dagger]_{\mu} + \xi_{jj'}^\lambda [\beta_{j}^\dagger \tilde{\beta}_{j'}^\dagger]_{\mu} \right)
\]

\( T_c = (0.5-0.6)\Delta(T=0) \)
\[ \Delta \rho \approx 1.4 \text{ MeV} \quad \Rightarrow \quad T_{cr} \approx 0.5 \Delta (\approx 0.7 \text{ MeV}). \]

The brown arrows indicate the zero-temperature EC threshold:
\[ Q = M(^{56}\text{Mn}) - M(^{56}\text{Fe}) = 4.2 \text{ MeV}. \]
Electron capture rates for $^{56}\text{Fe}$

$$\lambda^{\text{ec}} = \frac{\ln 2}{6150\text{s}} \sum_k S_k (G T_+ ) F_k, \quad F_k = \int_{E_0}^{\infty} E^2 (E - E_k)^2 F(Z, E) f_e(E) dE$$

where $f_e(E) = \left[ 1 + \exp \left( -\frac{E - \mu_e}{T} \right) \right]^{-1}$ and $\mu_e \approx 11.1 (\rho_{10} Y_e)^{1/3} \text{ MeV}$. 

$T_9 = 10^9 \text{ Kelvin}, \rho_{10}$ is the density in units of $10^{10} \text{ g cm}^{-3}$

Electron capture on neutron-rich nuclei

Neutron-rich nuclei ($N > 40, \ Z < 40$)

Unblocking mechanisms: configuration mixing and thermal excitations

Hybrid model: SMMC + RPA (K. Langanke et al, PRC 63 (2001) 032801)
GT_+ strength distribution in $^{76}\text{Ge}$ ($T \neq 0$)

Electron capture cross sections for $^{76}$Ge

GT$_0$ strength distribution in $^{56}$Fe

$$\nu + ^{56}\text{Fe} \rightarrow ^{56}\text{Fe} + \nu'$$

$T = 0.86$ MeV ($10^{10}$ K) corresponds to the condition of a presupernova model for a $15M_{\odot}$ star; $T = 1.29$ MeV ($1.5 \times 10^{10}$ K) - relates to neutrino trapping, $T = 1.72$ MeV ($2 \times 10^{10}$ K) - to neutrino thermalization.

Detailed balance: $S_{GT}(-E, T) = S_{GT}(E, T) \exp\left(-\frac{E}{T}\right)$. 
Cross section for inelastic neutrino scattering off $^{56}$Fe

$$\sigma(E_\nu, T) = \frac{G_F^2}{\pi} \left\{ \sum_k (E_\nu - \omega_k)^2 S_k + \sum_k (E_\nu + \omega_k)^2 \tilde{S}_k \right\}$$

$$= \sigma_d(E_\nu, T) + \sigma_{up}(E_\nu, T),$$

- $S_k = |\langle Q_k | \sigma t_0 | 0(T) \rangle|^2$ and $E'_\nu = E_\nu - \omega_k$ for down-scattering;
- $\tilde{S}_k = |\langle \tilde{Q}_k | \sigma t_0 | 0(T) \rangle|^2$ and $E'_\nu = E_\nu + \omega_k$ for up-scattering.
GT$^-$ strength distribution and neutrino capture on $^{56}$Fe

$$\nu_e + ^{56}\text{Fe} \rightarrow ^{56}\text{Co} + e^-$$

$$\sigma(E_\nu, T) = \frac{G_F^2}{\pi} \sum_k (E^k_e)^2 S_k (GT^-)[1 - f(E^k_e)]$$

$$f_e(E) = \left[ 1 + \exp\left( - \frac{E - \mu_e}{T} \right) \right]^{-1}$$

**Graphs:**
- $T = 0$ (g.s.)
- $T = 0.86$ MeV
- $T = 1.29$ MeV
- $T = 1.72$ MeV

**Equations:**
- $E_\nu$, MeV
- $\mu_e$, MeV
- $\sigma$, $10^{-42}$ cm$^2$

**Parameters:**
- $\mu_e = 8.3$ MeV
- $\mu_e = 18.1$ MeV
- $\mu_e = 36.2$ MeV
GT− strength distribution and neutrino capture on $^{82}\text{Ge}$

$\nu_e + ^{82}\text{Ge} \rightarrow ^{82}\text{As} + e^−$

\[
\sigma(E_\nu, T) = \frac{G_F^2}{\pi} \sum_k (E_{e_k}^L)^2 S_k(GT−)[1 − f(E_{e_k}^L)]
\]

\[
f_e(E) = \left[1 + \exp \left(- \frac{E - \mu_e}{T}\right) \right]^{-1}
\]

$T = 0 \text{ (g.s.)}$

$T = 0.86 \text{ MeV}$

$T = 1.29 \text{ MeV}$

$T = 1.72 \text{ MeV}$

No Pauli blocking

Pauli blocking

$E_\nu$, MeV

$E_e$, MeV

$\sigma$, $10^{-42}$ cm$^2$

$T = 0 \text{ (g.s.)}$

$T = 0.86 \text{ MeV}$

$T = 1.29 \text{ MeV}$

$T = 1.72 \text{ MeV}$

$\mu = 8.3 \text{ MeV}$

$\mu = 18.1 \text{ MeV}$

$\mu = 36.2 \text{ MeV}$

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WEAK INTERACTIONS IN ...

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Currently, there exist two main theoretical approaches in calculations of the rates and cross-sections for weak-interaction reactions with hot nuclei in stellar environments: the Large Scale Shell Model approach and the Thermal Quasiparticle Random Phase Approximation.

Both the approaches predict a strong thermal enhancement of the cross-section and rates at low lepton energies. This enhancement is due to thermal population of nuclear excited states.

The LSSM approach is fairly successful in calculations with the iron-group nuclei ($A = 45 – 65$), but it partially employs the Brink hypothesis when treating GT transitions from nuclear excited states.

The TQRPA method does not rely on the Brink hypothesis and it can be applied to massive neutron-rich nuclei. Moreover, the corresponding calculations are much less time consuming.
Credits to: A. Vdovin, J. Wambach, K. Langanke, G. Martínez-Pinedo and V. Ponomarev.

THANK YOU FOR ATTENTION!