The EOS of neutron matter, and the effect of Λ hyperons to neutron star structure

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NUCLEI
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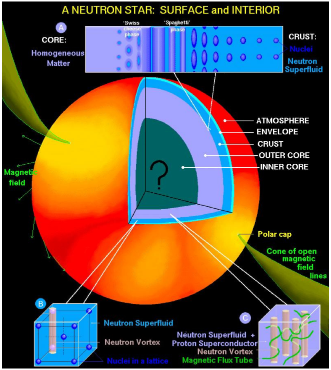
Neutron stars

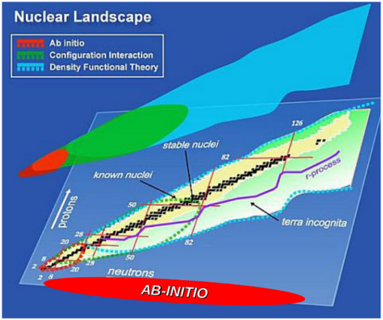
Neutron star is a wonderful natural laboratory

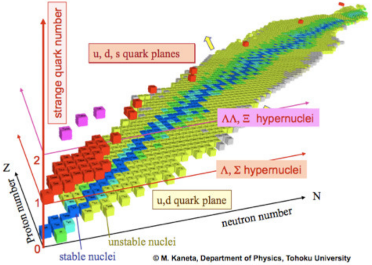


D. Page

- Atmosphere: atomic and plasma physics
- Crust: physics of superfluids (neutrons, vortex), solid state physics (nuclei)
- Inner crust: deformed nuclei, pasta phase
- Outer core: nuclear matter
- Inner core: hyperons? quark matter? π or K condensates?

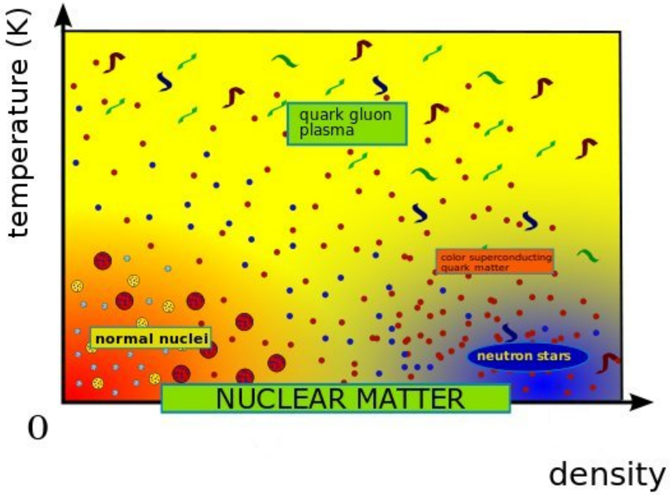
Nuclei and hypernuclei





Several thousands of binding energies for normal nuclei.
Only few, $\sim$50, for hypernuclei.

Homogeneous neutron matter



Outline

- The model and the method
- Equation of state of neutron matter, role of three-neutron force
- Symmetry energy
- Neutron star structure (I)
- Λ-hypernuclei
- Λ-neutron matter
- Neutron star structure (II)
- Conclusions

Nuclear Hamiltonian

Model: non-relativistic nucleons interacting with an effective nucleon-nucleon force (NN) and three-nucleon interaction (TNI).

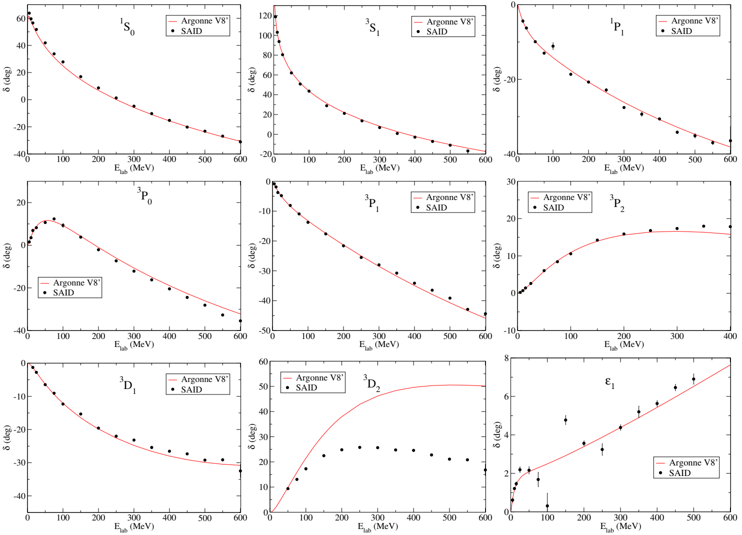
$$H = -\frac{\hbar^2}{2m}\sum_{i=1}^{A}\nabla_i^2 + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk}$$

v_{ij} NN fitted on scattering data. Sum of operators:

$$v_{ij} = \sum O_{ij}^{p=1,8} v^p(r_{ij})\,, \quad O_{ij}^p = (1, \vec{\sigma}_i \cdot \vec{\sigma}_j, S_{ij}, \vec{L}_{ij} \cdot \vec{S}_{ij}) \times (1, \vec{\tau}_i \cdot \vec{\tau}_j)$$

NN interaction - Argonne AV8'.

Phase shifts, AV8'



Scattering data and neutron matter

Two neutrons have

$$k \approx \sqrt{E_{lab}\, m/2}\,, \qquad \rightarrow k_F$$

that correspond to

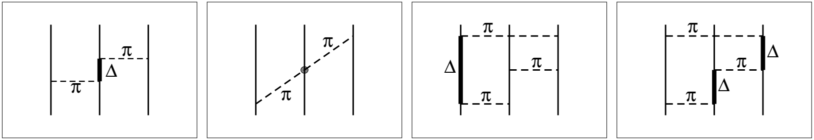
$$k_F \rightarrow \rho \approx (E_{lab}\, m/2)^{3/2}/2\pi^2\,.$$

E_{lab}=150 MeV corresponds to about 0.12 fm^{-3}.

E_{lab}=350 MeV to 0.44 fm^{-3}.

Argonne potentials useful to study dense matter above ρ_0=0.16 fm^{-3}

Urbana–Illinois V_{ijk} models processes like



$+$ short-range correlations (spin/isospin independent).

Urbana UIX: Fujita-Miyazawa plus short-range.

Quantum Monte Carlo

$$H\,\psi(\vec{r}_1\ldots\vec{r}_N) = E\,\psi(\vec{r}_1\ldots\vec{r}_N) \qquad \psi(t) = e^{-(H-E_T)t}\psi(0)$$

Ground-state extracted in the limit of $t \to \infty$.

Propagation performed by

$$\psi(R,t) = \langle R|\psi(t)\rangle = \int dR'\,G(R,R',t)\psi(R',0)$$

- Importance sampling: $G(R,R',t) \to G(R,R',t)\,\Psi_I(R')/\Psi_I(R)$
- Constrained-path approximation to control the sign problem. Unconstrained calculation possible in several cases (exact).

Ground–state obtained in a **non-perturbative way.** Systematic uncertainties within 1-2 %.

Overview

Recall: propagation in imaginary-time

$$e^{-(T+V)\Delta\tau}\psi \approx e^{-T\Delta\tau}e^{-V\Delta\tau}\psi$$

Kinetic energy is sampled as a diffusion of particles:

$$e^{-\nabla^2\Delta\tau}\psi(R) = e^{-(R-R')^2/2\Delta\tau}\psi(R) = \psi(R')$$

The (scalar) potential gives the weight of the configuration:

$$e^{-V(R)\Delta\tau}\psi(R) = w\psi(R)$$

Algorithm for each time-step:

- do the diffusion: $R' = R + \xi$
- compute the weight w
- compute observables using the configuration R' weighted using w over a trial wave function ψ_T.

For spin-dependent potentials things are much worse!

GFMC and AFDMC

Because the Hamiltonian is state dependent, all spin/isospin states of nucleons must be included in the wave-function.

Example: spin for 3 neutrons (radial parts also needed in real life):

GFMC wave-function:

$$\psi = \begin{pmatrix} a_{\uparrow\uparrow\uparrow} \\ a_{\uparrow\uparrow\downarrow} \\ a_{\uparrow\downarrow\uparrow} \\ a_{\uparrow\downarrow\downarrow} \\ a_{\downarrow\uparrow\uparrow} \\ a_{\downarrow\uparrow\downarrow} \\ a_{\downarrow\downarrow\uparrow} \\ a_{\downarrow\downarrow\downarrow} \end{pmatrix}$$

A correlation like

$$1 + f(r)\sigma_1 \cdot \sigma_2$$

can be used, and the variational wave function can be very good. Any operator accurately computed.

AFDMC wave-function:

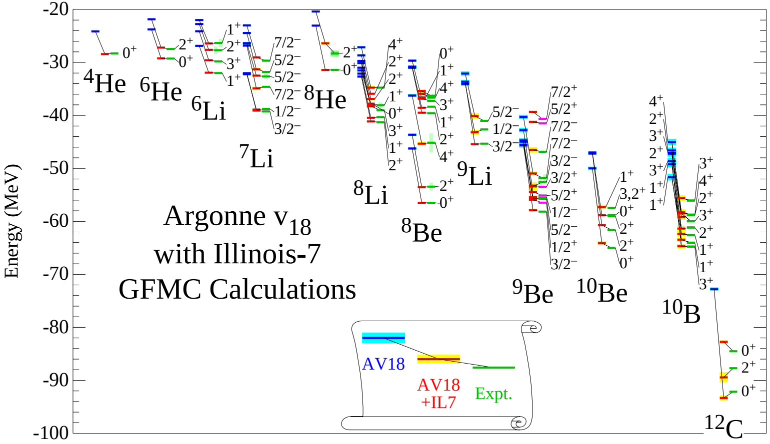
$$\psi = \mathcal{A}\left[\xi_{s_1}\begin{pmatrix} a_1 \\ b_1 \end{pmatrix}\xi_{s_2}\begin{pmatrix} a_2 \\ b_2 \end{pmatrix}\xi_{s_3}\begin{pmatrix} a_3 \\ b_3 \end{pmatrix}\right]$$

We must change the propagator by using the Hubbard-Stratonovich transformation:

$$e^{\frac{1}{2}\Delta t O^2} = \frac{1}{\sqrt{2\pi}}\int dx e^{-\frac{x^2}{2}+x\sqrt{\Delta t}O}$$

Auxiliary fields x must also be sampled. The wave-function is pretty bad, but we can simulate larger systems (up to $A \approx 100$). Operators (except the energy) are very hard to be computed, but in some case there is some trick!

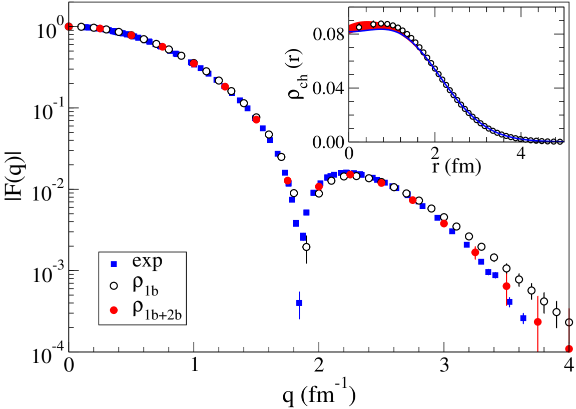
Light nuclei spectrum computed with GFMC



Carlson, *et al.*, arXiv:1412.3081

Charge form factor of ^{12}C

$$|F(q)| = \langle\psi|\rho_q|\psi\rangle \qquad \rho_q = \sum_i \rho_q(i) + \sum_{i<j} \rho_q(ij)$$



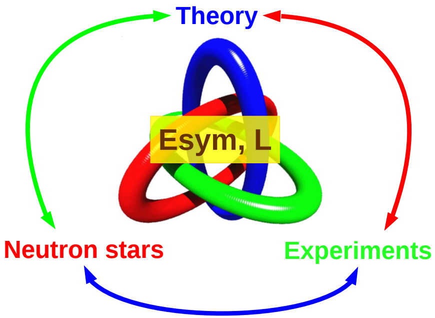
Lovato, Gandolfi, Butler, Carlson, Lusk, Pieper, Schiavilla, PRL (2013)

Neutron matter equation of state

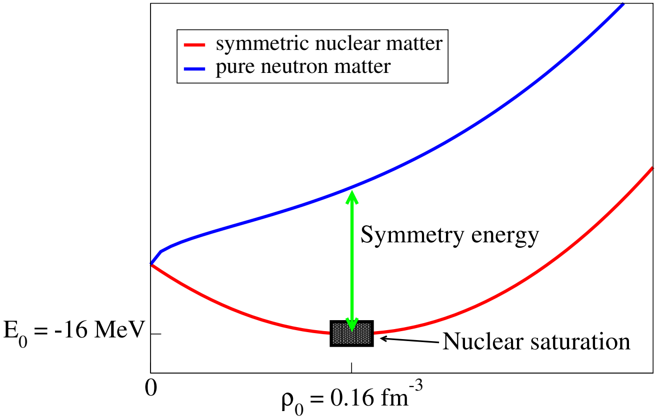
Why to study neutron matter at nuclear densities?

- EOS of neutron matter gives the symmetry energy and its slope.
- The three-neutron force ($T = 3/2$) very weak in light nuclei, while $T = 1/2$ is the dominant part. No direct $T = 3/2$ experiments available.

Why to study symmetry energy?



What is the Symmetry energy?



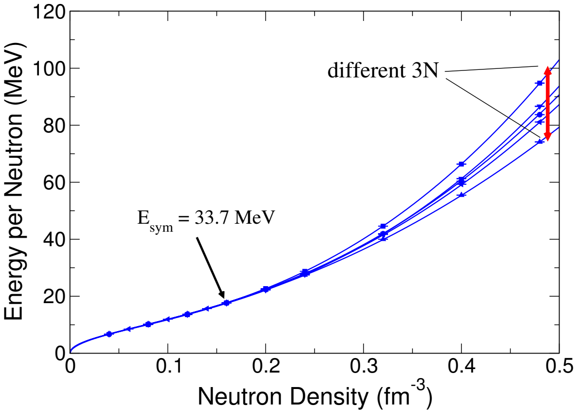
Assumption from experiments:

$$E_{SNM}(\rho_0) = -16 MeV \,, \quad \rho_0 = 0.16 fm^{-3} \,, \quad E_{sym} = E_{PNM}(\rho_0) + 16$$

At ρ_0 we access E_{sym} by studying PNM.

Neutron matter

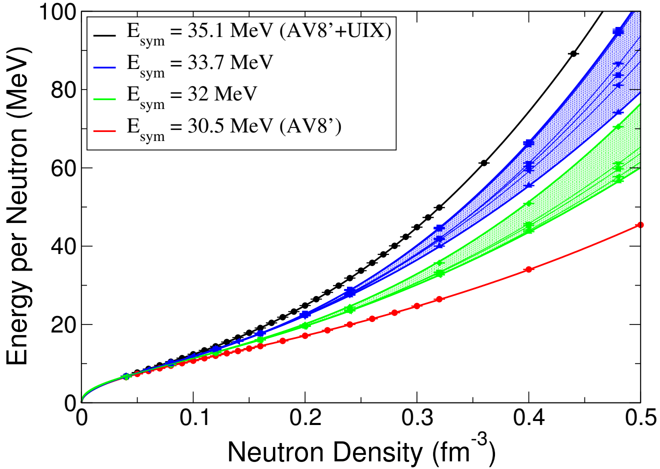
We consider different forms of three-neutron interaction by only requiring a particular value of E_{sym} at saturation.



different 3N:

- $V_{2\pi} + \alpha V_R$
- $V_{2\pi} + \alpha V_R^{\mu}$ (several μ)
- $V_{2\pi} + \alpha \tilde{V}_R$
- $V_{3\pi} + \alpha V_R$

Equation of state of neutron matter using Argonne forces:

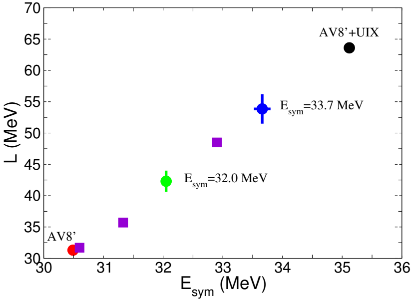


Gandolfi, Carlson, Reddy, PRC (2012)

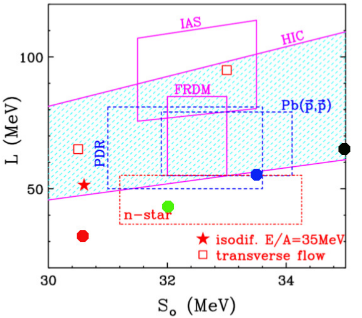
Neutron matter and symmetry energy

From the EOS, we can fit the symmetry energy around ρ_0 using

$$E_{sym}(\rho) = E_{sym} + \frac{L}{3}\frac{\rho - 0.16}{0.16} + \cdots$$



Gandolfi *et al.*, EPJ (2014)



Tsang *et al.*, PRC (2012)

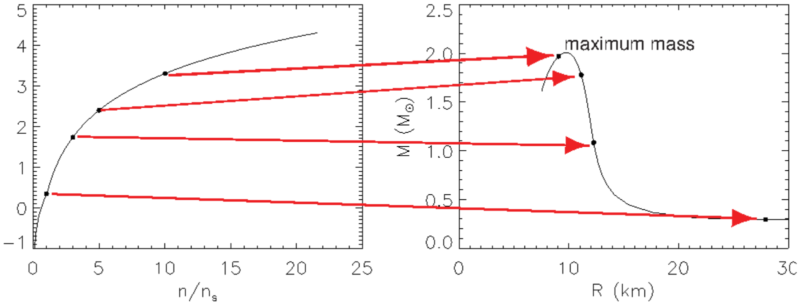
Very weak dependence to the model of 3N force for a given E_{sym}.
Chiral Hamiltonians give compatible results.

Neutron matter and neutron star structure

TOV equations:

$$\frac{dP}{dr} = -\frac{G[m(r) + 4\pi r^3 P/c^2][\epsilon + P/c^2]}{r[r - 2Gm(r)/c^2]} ,$$

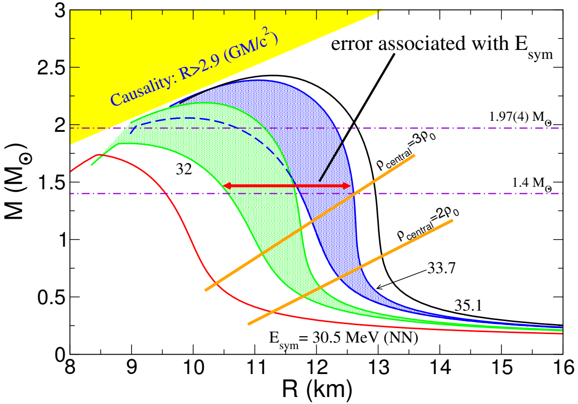
$$\frac{dm(r)}{dr} = 4\pi\epsilon r^2 ,$$



J. Lattimer

Neutron star structure

EOS used to solve the TOV equations.

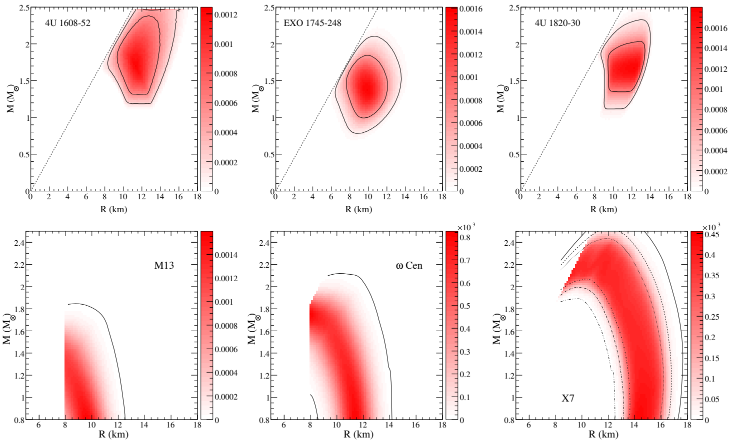


Gandolfi, Carlson, Reddy, PRC (2012).

Accurate measurement of E_{sym} put a constraint to the radius of neutron stars, **OR** observation of M and R would constrain E_{sym}!

Neutron stars

Observations of the mass-radius relation are becoming available:



Steiner, Lattimer, Brown, ApJ (2010)

Neutron star observations can be used to 'measure' the EOS and constrain E_{sym} and L.

Neutron star matter

Neutron star matter model:

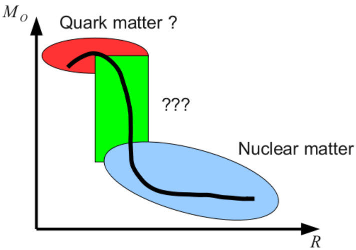
$$E_{NSM} = a\left(\frac{\rho}{\rho_0}\right)^{\alpha} + b\left(\frac{\rho}{\rho_0}\right)^{\beta}, \quad \rho < \rho_t$$

(form suggested by QMC simulations),

and a high density model for $\rho > \rho_t$

i) two polytropes

ii) polytrope+quark matter model



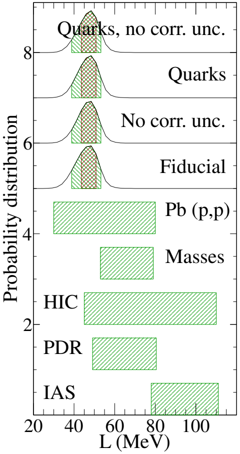
Neutron star radius sensitive to the EOS at nuclear densities!

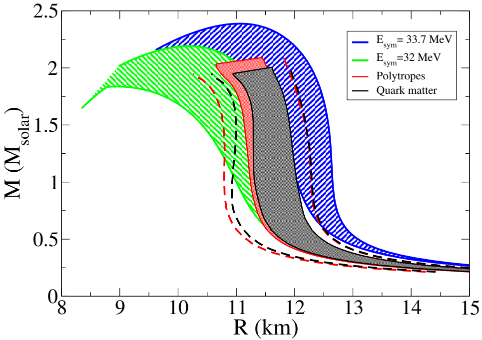
Direct way to extract E_{sym} and L from neutron stars observations:

$$E_{sym} = a + b + 16\,, \quad L = 3(a\alpha + b\beta)$$

Neutron star matter really matters!

Here an 'astrophysical measurement'





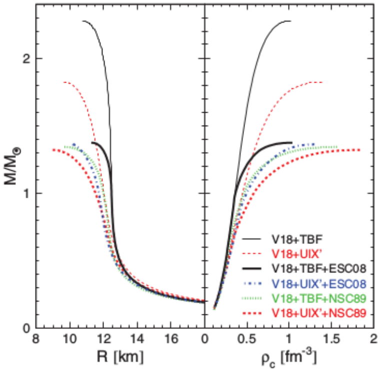
$$32 < E_{sym} < 34\ MeV,\ \ 43 < L < 52\ MeV$$

Steiner, Gandolfi, PRL (2012).

High density neutron matter

If chemical potential large enough ($\rho \sim 2 - 3\rho_0$), heavier particles form, i.e. Λ, Σ, ...

Non-relativistic BHF calculations suggest that none of the available hyperon-nucleon Hamiltonians support an EOS with $M > 2M_\odot$:



Schulze and Rijken PRC (2011).

(Some) other relativistic model support $2M_\odot$ neutron stars.

Λ-systems, outline

Hypernuclei and hypermatter:

$$H = H_N + \frac{\hbar^2}{2m_\Lambda}\sum_{i=1}^{A}\nabla_i^2 + \sum_{i<j} v_{ij}^{\Lambda N} + \sum_{i<j<k} V_{ijk}^{\Lambda NN}$$

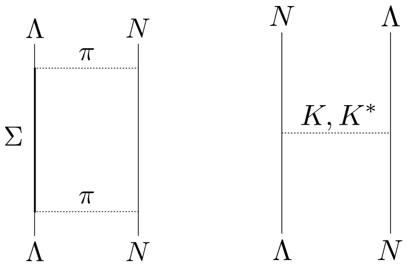
Λ-binding energy calculated as the difference between the system with and without Λ.

Λ-nucleon interaction

The Λ-nucleon interaction is constructed similarly to the Argonne potentials (Usmani).

Argonne NN: $v_{ij} = \sum_p v_p(r_{ij}) O^p_{ij}$, $O_{ij} = (1, \sigma_i \cdot \sigma_j, S_{ij}, \vec{L}_{ij} \cdot \vec{S}_{ij}) \times (1, \tau_i \cdot \tau_j)$

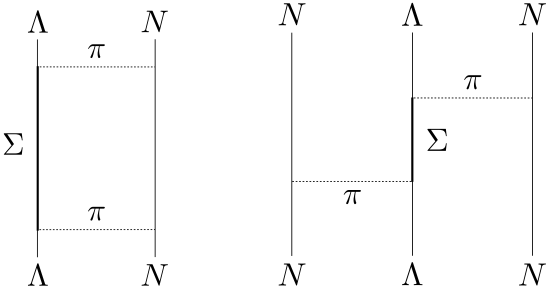
Usmani ΛN: $v_{ij} = \sum_p v_p(r_{ij}) O^p_{ij}$, $O_{\lambda j} = (1, \sigma_\lambda \cdot \sigma_j) \times (1, \tau^z_j)$



But... ∼4500 NN data, ∼50 of ΛN data.

ΛN and ΛNN interactions

ΛNN has the same range of ΛN



Hypernuclei

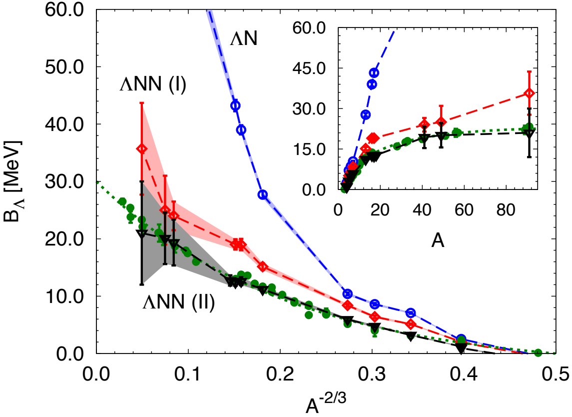
AFDMC for (hyper)nuclei is limited to simple interactions. We found that the Λ-binding energy is quite independent to the details of NN interactions.

NN potential	$V_{\Lambda N} + V_{\Lambda NN}$	
	$^{5}_{\Lambda}\mathrm{He}$	$^{17}_{\Lambda}\mathrm{O}$
Argonne V4'	5.1(1)	19(1)
Argonne V6'	5.2(1)	21(1)
Minnesota	5.2(1)	17(2)
Expt.	3.12(2)	13.0(4)

D. L., S. Gandolfi, F. Pederiva, Phys. Rev. C 87, 041303(R) (2013)

The inclusion of (simple) three-body forces gives very similar results (unpublished).

$v^{\Lambda N}$ and $V^{\Lambda NN}$(I) are phenomenological (Usmani).

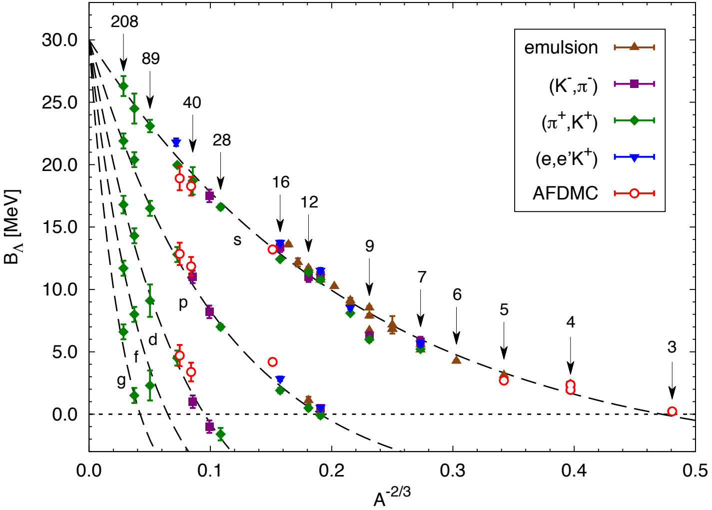


Lonardoni, Pederiva, SG, PRC (2013) and PRC (2014).

$V^{\Lambda NN}$ (II) is a new form where the parameters have been fine tuned.

As expected, the role of ΛNN is crucial for saturation.

Λ in different states:



Lonardoni, SG, Pederiva, in preparation.

Hyper-neutron matter

Neutrons and Λ particles:

$$\rho = \rho_n + \rho_\Lambda \,, \qquad\qquad x = \frac{\rho_\Lambda}{\rho}$$

$$E_{\rm HNM}(\rho,x) = \Big[E_{\rm PNM}((1-x)\rho)+m_n\Big](1-x)+\Big[E_{\rm P\Lambda M}(x\rho)+m_\Lambda\Big]x+f(\rho,x)$$

where $E_{P\Lambda M}$ is the non-interacting energy (no $v_{\Lambda\Lambda}$ interaction),

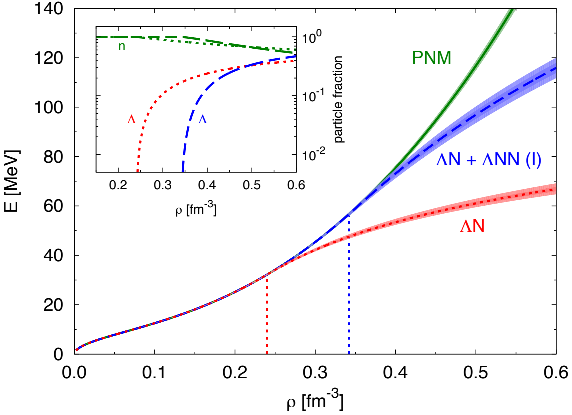
$$E_{PNM}(\rho) = a\left(\frac{\rho}{\rho_0}\right)^{\alpha} + b\left(\frac{\rho}{\rho_0}\right)^{\beta}$$

and

$$f(\rho,x) = c_1\frac{x(1-x)\rho}{\rho_0} + c_2\frac{x(1-x)^2\rho^2}{\rho_0^2}$$

All the parameters are fit to AFDMC results.

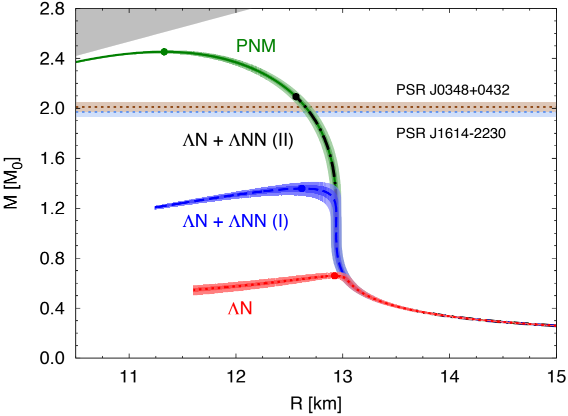
EOS obtained by solving for $\mu_\Lambda(\rho, x) = \mu_n(\rho, x)$



Lonardoni, Lovato, Pederiva, SG, arXiv:1407.4448.

No hyperons up to $\rho = 0.5$ fm^{-3} using ΛNN (II)!!!

Λ-neutron matter



Lonardoni, Lovato, Pederiva, SG, arXiv:1407.4448.

Drastic role played by ΛNN. Calculations can be compatible with neutron star observations.

Note: no $v_{\Lambda\Lambda}$, no protons, and no other hyperons included

Summary

QMC methods useful to study nuclear systems in a coherent framework:

- Three-neutron force is the bridge between E_{sym} and neutron star structure.
- Neutron star observations becoming competitive with experiments.
- Λ-nucleon data very limited, but ΛNN is very important. Role of Λ in neutron stars far to be understood. More ΛN data needed. Input from Lattice QCD?

  Conclusion? We cannot conclude **anything** with present models...

Acknowledgments

- J. Carlson (LANL)
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- F. Pederiva (Trento)
- A. Steiner (UT)