# Chiral effective field theory of hyperon-nucleon interactions

Norbert Kaiser

Technische Universität München

in collaboration with:

| | |
|---|---|
| Stefan Petschauer | TU München |
| Wolfram Weise | ECT* Trento, TU München |
| Johann Haidenbauer | FZ Jülich |
| Andreas Nogga | FZ Jülich |
| Ulf-G. Meißner | Universität Bonn, FZ Jülich |


Nuclear structure and reactions: weak, strange and exotic
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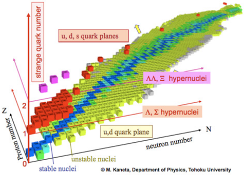
2. Hyperon-nucleon interaction at NLO
3. Chiral three-baryon forces
4. Summary / Outlook

# Motivation

- Goal: determine interactions between hyperons (Y) and nucleons (N), e.g. important for:
  - hyperon-nucleon scattering
  - hypernuclei
  - strange baryons in nuclear matter




- accurate description of nuclear interactions with SU(2) chiral effective field theory [Epelbaum, Glöckle, Meißner, Entem, Machleidt, . . .]

  extend SU(2) $\chi$EFT to include strangeness
  $\Rightarrow$ SU(3) chiral effective field theory

- Advantages:
  - can improve results systematically
  - can derive consistently two- and three-baryon forces

# Motivation

- systematic *NLO* analysis of chiral *contact terms* and *one*- and *two-meson exchange* contributions to baryon-baryon interactions using SU(3) $\chi$EFT

  Leading order (LO):

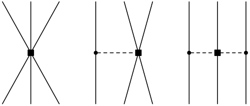
  [Polinder, Haidenbauer, Meißner, Nucl.Phys. A779, 2006]

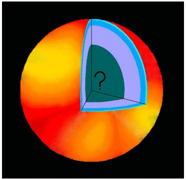
  Next-to-leading order (NLO):

  [Haidenbauer, Petschauer, Kaiser, Meißner, Nogga, Weise, Nucl.Phys. A915, 2013]

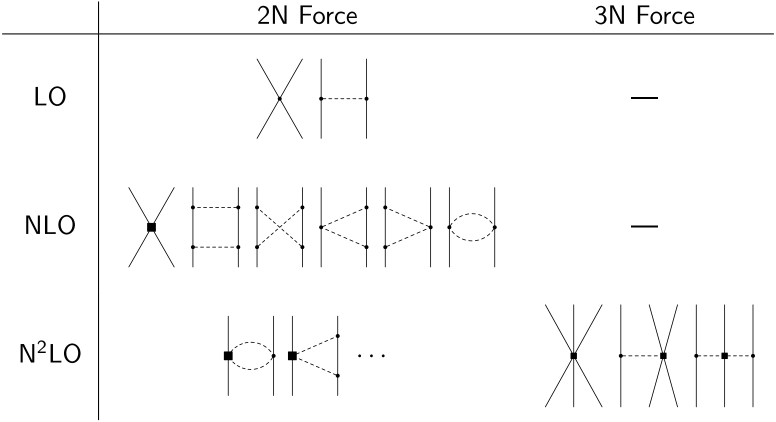
- repulsive $\Lambda NN$ force suggested to get stiffer equation of state for neutron stars and to describe hypernuclei

  [Gal et al.,Ann.Phys.63,1971] [Lonardoni et al.,Phys.Rev.C87,2013]





[ http://www.astro.umd.edu/~miller/nstar.html ]

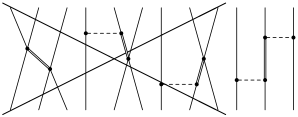
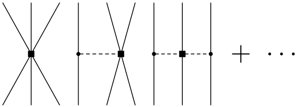



[van Kolck, Phys.Rev.C49, 1994]

[Epelbaum, Nogga, Glöckle, Kamada, Meißner, Witała, Phys.Rev.C66, 2002]

[Epelbaum, Hammer, Meißner, Rev.Mod.Phys.81, 2008]

# Three-nucleon force including delta resonance

| | 3N Force with $\Delta$ isobar |
|---|---|
| LO | — |
| NLO |  |
| $N^2LO$ |  |

[Epelbaum, Krebs and Meißner, Nucl.Phys.A806, 2008]

[Epelbaum, Hammer, Meißner, Rev.Mod.Phys.81, 2008]

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# Chiral meson-baryon Lagrangian

## Meson Lagrangian (in isospin limit $m_u = m_d \neq m_s$)

$$\mathscr{L}_{\mathrm{M}}^{(2)} = \frac{f_0^2}{4} \operatorname{tr}\left(\partial_\mu U \partial^\mu U^\dagger\right) + \frac{1}{2} B_0 f_0^2 \operatorname{tr}\left(M U^\dagger + U M\right)$$

$$U(x) = \exp\left(\mathrm{i}\frac{\phi(x)}{f_0}\right), \quad \phi = \begin{pmatrix} \pi^0 + \frac{\eta}{\sqrt{3}} & \sqrt{2}\pi^+ & \sqrt{2}K^+ \\ \sqrt{2}\pi^- & -\pi^0 + \frac{\eta}{\sqrt{3}} & \sqrt{2}K^0 \\ \sqrt{2}K^- & \sqrt{2}\bar{K}^0 & -\frac{2\eta}{\sqrt{3}} \end{pmatrix}$$

Goldstone boson octet

$M \equiv \operatorname{diag}(m_u, m_d, m_s) \quad \Rightarrow \quad$ explicit SU(3)-breaking

## Meson-baryon interaction

$$\mathscr{L}_{\mathrm{MB}}^{(1)} = \operatorname{tr}\left(\bar{B}\left(\mathrm{i}\not{D} - M_0\right) B - \frac{D}{2}\bar{B}\gamma^\mu\gamma_5\{u_\mu, B\} - \frac{F}{2}\bar{B}\gamma^\mu\gamma_5\left[u_\mu, B\right]\right)$$

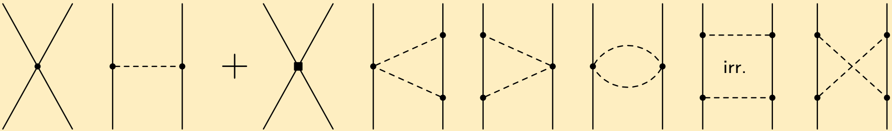
axial vector couplings:
$D \approx 0.8, \; F \approx 0.5$

$$B = \begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & -\frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & n \\ \Xi^- & \Xi^0 & -\frac{2\Lambda}{\sqrt{6}} \end{pmatrix}$$

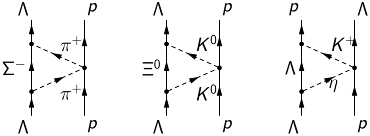
baryon octet

# Deriving the T-matrix

## Weinberg power counting for baryon-baryon potential




e.g.

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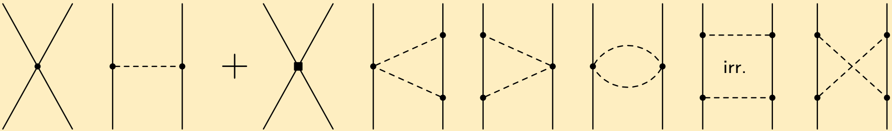
# Deriving the T-matrix

## Weinberg power counting for baryon-baryon potential



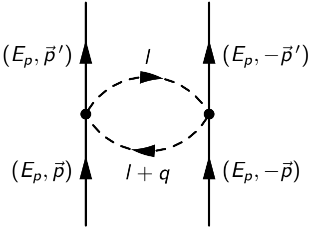

## Coupled-channel Lippmann-Schwinger equation

$$T^{\rho''\rho',J}_{\nu''\nu'}(p'',p';\sqrt{s}) = V^{\rho''\rho',J}_{\nu''\nu'}(p'',p') + \\ + \sum_{\rho,\nu}\int_0^\infty \frac{\mathrm{d}p\, p^2}{(2\pi)^3}\, V^{\rho''\rho,J}_{\nu''\nu}(p'',p)\frac{2\mu_\nu}{q_\nu^2 - p^2 + i\eta}\, T^{\rho\rho',J}_{\nu\nu'}(p,p';\sqrt{s})$$

$\rho$: partial wave $\qquad$ $\nu$: particle channel

Coulomb interaction included via Vincent-Phatak method

# Example: Football Diagram




- $m_1, m_2$ masses of exchanged mesons
- $q = |\vec{q}|$ momentum transfer, $(\vec{q} = \vec{p}\,' - \vec{p})$
- $R = \frac{2}{d-4} + \gamma - 1 - \ln(4\pi)$
- scale $\lambda$ introduced in dim. regularization

$$V_C(q) = -\frac{N}{3072\pi^2 f_0^4}\left\{ -2\left(m_1^2+m_2^2\right) - \frac{5}{6}q^2 - \frac{m_1^2-m_2^2}{2q^4} + w^2(q)L(q) + \left[\left(m_1^2-m_2^2\right)^2 + 3\left(m_1^2+m_2^2\right)q^2\right]\ln\frac{m_1}{m_2} + \left(3\left(m_1^2+m_2^2\right)+q^2\right)\left[\frac{1}{2}R + \ln\frac{\sqrt{m_1 m_2}}{\lambda}\right]\right\}$$

$$w(q) = \frac{1}{q}\sqrt{\left(q^2+(m_1+m_2)^2\right)\left(q^2+(m_1-m_2)^2\right)}, \quad L(q) = \frac{w(q)}{2q}\ln\frac{\left(qw(q)+q^2\right)^2 - \left(m_1^2-m_2^2\right)^2}{4m_1 m_2 q^2}$$

# SU(3) symmetry and contact terms

- poor database for YN interaction (36 data points)
- use SU(3) symmetric contact terms for reduction of LECs
- LO+NLO contact terms of NN interaction [Epelbaum, 2000] generalized by SU(3) flavor symmetry

$$\mathbf{8} \otimes \mathbf{8} = \mathbf{27} \oplus \mathbf{8_s} \oplus \mathbf{1} \oplus \mathbf{10} \oplus \mathbf{10}^* \oplus \mathbf{8_a}$$

flavor symmetric $\mathbf{27}, \mathbf{8_s}, \mathbf{1}$
$\Rightarrow$ space-spin antisymmetric states

$$V^i(^1\mathrm{S}_0) = \tilde{C}^i_{^1\mathrm{S}_0} + C^i_{^1\mathrm{S}_0}(p^2 + p'^2)$$
$$V^i(^3\mathrm{P}_0) = C^i_{^3\mathrm{P}_0}(pp')$$
$$V^i(^3\mathrm{P}_1) = C^i_{^3\mathrm{P}_1}(pp')$$
$$V^i(^3\mathrm{P}_2) = C^i_{^3\mathrm{P}_2}(pp')$$

flavor antisymmetric $\mathbf{10}, \mathbf{10}^*, \mathbf{8_a}$
$\Rightarrow$ space-spin symmetric states

$$V^i(^3\mathrm{S}_1) = \tilde{C}^i_{^3\mathrm{S}_1} + C^i_{^3\mathrm{S}_1}(p^2 + p'^2)$$
$$V^i(^1\mathrm{P}_1) = C^i_{^1\mathrm{P}_1}(pp')$$
$$V^i(^3\mathrm{D}_1 - {}^3\mathrm{S}_1) = C^i_{^3\mathrm{D}_1 - {}^3\mathrm{S}_1}\, p'^2$$
$$V^i(^3\mathrm{S}_1 - {}^3\mathrm{D}_1) = C^i_{^3\mathrm{D}_1 - {}^3\mathrm{S}_1}\, p^2$$

singlet-triplet mixing from $\mathbf{8_s} \leftrightarrow \mathbf{8_a}$ neglected (antisym. spin-orbit)

$$V^i(^3\mathrm{P}_1 - {}^1\mathrm{P}_1) = C^i_{^3\mathrm{P}_1 - {}^1\mathrm{P}_1}(pp')$$

# SU(3) symmetry and contact terms

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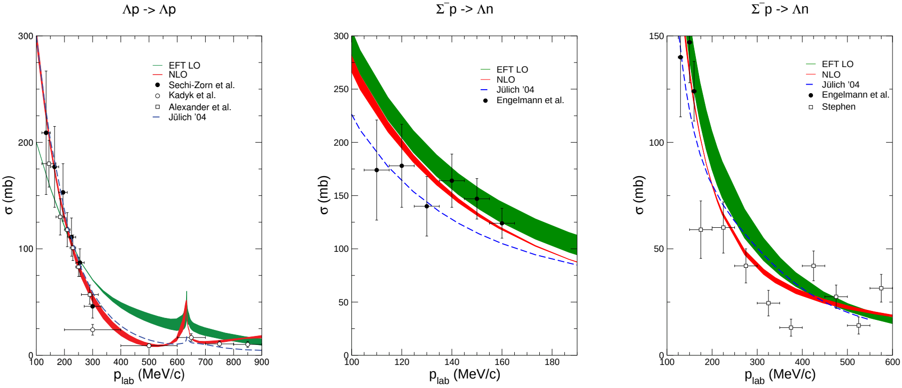
| S | Channel | I | $V_{^1S_0,\,^3P_0,\,^3P_1,\,^3P_2}$ | $V_{^3S_1,\,^3S_1\text{-}^3D_1,\,^1P_1}$ | $V_{^1P_1\text{-}^3P_1}$ |
|---|---|---|---|---|---|
| 0 | $NN \to NN$ | 0 | – | $C^{10^*}$ | – |
| | $NN \to NN$ | 1 | $C^{27}$ | – | – |
| $-1$ | $\Lambda N \to \Lambda N$ | $\frac{1}{2}$ | $\frac{1}{10}\left(9C^{27} + C^{8_s}\right)$ | $\frac{1}{2}\left(C^{8_a} + C^{10^*}\right)$ | $\frac{-1}{\sqrt{20}} C^{8_s 8_a}$ |
| | $\Lambda N \to \Sigma N$ | $\frac{1}{2}$ | $\frac{3}{10}\left(-C^{27} + C^{8_s}\right)$ | $\frac{1}{2}\left(-C^{8_a} + C^{10^*}\right)$ | $\frac{3}{\sqrt{20}} C^{8_s 8_a}$ |
| | $\Sigma N \to \Lambda N$ | | | | $\frac{-1}{\sqrt{20}} C^{8_s 8_a}$ |
| | $\Sigma N \to \Sigma N$ | $\frac{1}{2}$ | $\frac{1}{10}\left(C^{27} + 9C^{8_s}\right)$ | $\frac{1}{2}\left(C^{8_a} + C^{10^*}\right)$ | $\frac{3}{\sqrt{20}} C^{8_s 8_a}$ |
| | $\Sigma N \to \Sigma N$ | $\frac{3}{2}$ | $C^{27}$ | $C^{10}$ | – |

[Polinder, Haidenbauer, Meißner, Nucl.Phys. A779, 2006]

- does not include SU(3) breaking effects from quark masses $m_{u,d} \neq m_s$; full Lagrangian with SU(3) breaking available [Petschauer, Kaiser, NPA916, 2013]

# Results for integrated cross sections




## Included:

- one- and two-meson exchange; physical meson masses $\rightarrow$ SU(3) breaking
- LO and NLO contact terms
- Cutoff: 500 - 650 MeV
- LECs satisfy SU(3)

[Haidenbauer, Petschauer, Kaiser, Meißner, Nogga, Weise, Nucl.Phys. A915, 2013]

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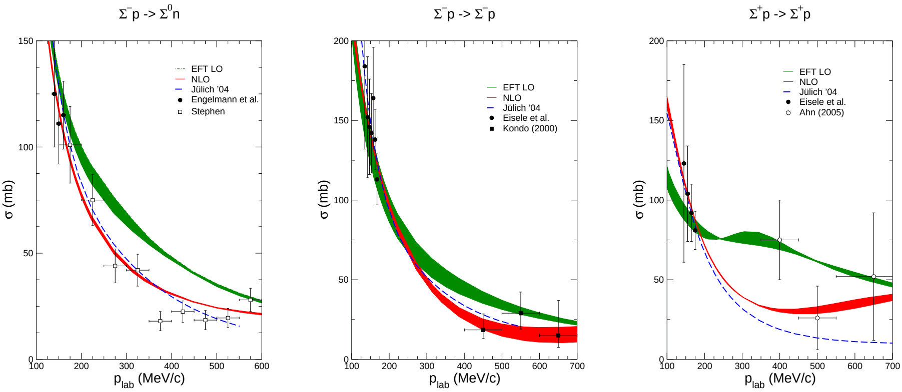


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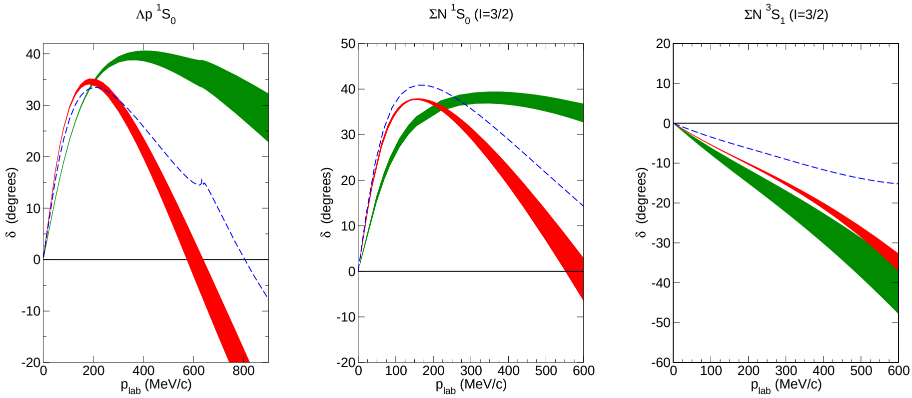
[Haidenbauer, Petschauer, Kaiser, Meißner, Nogga, Weise, Nucl.Phys. A915, 2013]

# Results for phase shifts




- $\Lambda N$: stronger repulsion for higher momenta
- $\Sigma N$: description of YN data possible with attractive or repulsive ${}^3S_1$,
  experimental hints for repulsive Σ-nuclear mean-field potential,
  $(\pi^-, K^+)$ inclusive spectra of $\Sigma^-$ formation in heavy nuclei

# Hyperons in symmetric nuclear matter

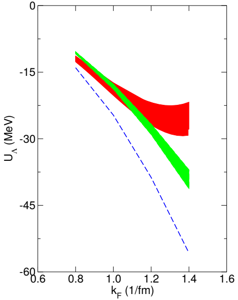
- results for conventional Brueckner calculation with gap choice in symmetric nuclear matter: Haidenbauer, Meißner, arXiv:1411.3114

- $U_\Lambda(p_\Lambda = 0)$ at $k_F = 1.35\,\mathrm{fm}^{-1}$: LO $\chi$EFT: -38 ...-34 MeV
  NLO $\chi$EFT: -28 ...-23 MeV

- $U_\Sigma(p_\Sigma = 0)$ at $k_F = 1.35\,\mathrm{fm}^{-1}$: LO $\chi$EFT: 11 ... 28 MeV
  NLO $\chi$EFT: 12 ... 17 MeV

- dominant contributions from ${}^1S_0$, ${}^3S_1 + {}^3D_1$ partial waves

- $\Lambda$ single-particle potential:
  empirical value: $U_\Lambda \approx -27$ MeV from binding energies of heavy $\Lambda$-hypernuclei, at NLO onset of repulsive effects



# $\Lambda$-nuclear spin-orbit coupling

- very small spin-orbit splitting of $\Lambda$ single-particle levels in nuclei, E1-transition in ${}^{13}_{\Lambda}C$: $p_{3/2}$-$p_{1/2}$ splitting $\approx 150$ keV (6 MeV in ordinary nuclei)
- quantified by Scheerbaum factor $S_\Lambda$: $\quad U_\Lambda^{ls}(r) = -\frac{\pi}{2} S_\Lambda \frac{1}{r} \frac{\mathrm{d}\rho(r)}{\mathrm{d}r} \vec{l} \cdot \vec{\sigma}$
- Scheerbaum factor given by combination of G-matrix elements

$$
\begin{aligned}
S_Y(p_Y) = &- \frac{3\pi}{4k_F^3} \xi(1+\xi)^2 \sum_{I,J} \frac{2I_0+1}{2I_Y+1}(2J+1) \\
&\times \int_0^{p_{\max}} \frac{\mathrm{d}p}{8\pi^3} W(p,p_Y) \Big\{ (J+2) G^{J,I_0}_{Y1J+1,Y1J+1}(p,p;p_Y) \\
&+ G^{J,I_0}_{Y1J,Y1J}(p,p;p_Y) - (J-1) G^{J,I_0}_{Y1J-1,Y1J-1}(p,p;p_Y) \\
&- \sqrt{J(J+1)} \Big[ G^{J,I_0}_{Y1J,Y0J}(p,p;p_Y) + G^{J,I_0}_{Y0J,Y1J}(p,p;p_Y) \Big] \Big\}
\end{aligned}
$$

- $S_\Lambda \approx -3.7\,\mathrm{MeV\,fm^5}$ achieved by adjusting antisymmetric spin-orbit term (splitting of 5/2 and 3/2 states in ${}^{9}_{\Lambda}Be$)
- equally good description of YN scattering data, no refit of $S$-waves required, some change in ${}^1P_1$ to reproduce trend of $\Sigma^- p \to \Lambda n$ diff. cross sections

Haidenbauer, Meißner, arXiv:1411.3114

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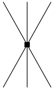
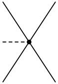
# Constructing the chiral Lagrangian

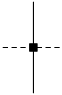
- symmetries of the effective Lagrangian:
  - chiral symmetry $\mathrm{SU(3)_L} \times \mathrm{SU(3)_R}$
  - C, P, T, Hermitian conjugation
  - Lorentz transformation
- degrees of freedom:
  - pseudoscalar Goldstone boson octet $(\pi, K, \eta)$
  - baryon octet $(N, \Lambda, \Sigma, \Xi)$
  - baryon decuplet $(\Delta, \Sigma^*, \Xi^*, \Omega)$
- antisymmetrized potential to respect generalized Pauli principle
- vertices:



18 low-energy constants (SU(3) symmetric)

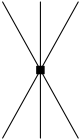


14 low-energy constants
[Petschauer, Kaiser, Nucl.Phys.A916, 2013]



10 low-energy constants
[Krause, Helv.Phys.Acta 63, 1990]

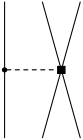
# Potentials for leading three-baryon forces



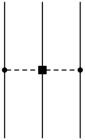
$$V^{\mathrm{ct}} = N_1\,\mathbb{1} + N_2\vec{\sigma}_1\cdot\vec{\sigma}_2 + N_3\vec{\sigma}_1\cdot\vec{\sigma}_3 + N_4\vec{\sigma}_2\cdot\vec{\sigma}_3 + N_5\,\mathrm{i}\,\vec{\sigma}_1\times\vec{\sigma}_2\cdot\vec{\sigma}_3$$

example: $V^{\mathrm{ct},\,I=0}_{\Lambda NN\to\Lambda NN} = c_1(\mathbb{1} + \frac{1}{3}\,\vec{\sigma}_2\cdot\vec{\sigma}_3) + c_2(\vec{\sigma}_1\cdot\vec{\sigma}_2 + \vec{\sigma}_1\cdot\vec{\sigma}_3)$

$V^{\mathrm{ct},\,I=1}_{\Lambda NN\to\Lambda NN} = c_3(\mathbb{1} - \vec{\sigma}_2\cdot\vec{\sigma}_3)$



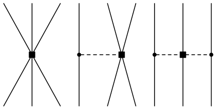
$$V^{1\phi} = -\frac{1}{2f_0^2}\,\frac{\vec{\sigma}_1\cdot\vec{q}_1}{\vec{q}_1^{\,2} + m_\phi^2}\left\{N_6\vec{\sigma}_2\cdot\vec{q}_1 + N_7\vec{\sigma}_3\cdot\vec{q}_1 + N_8\,\mathrm{i}\,(\vec{\sigma}_2\times\vec{\sigma}_3)\cdot\vec{q}_1\right\}$$



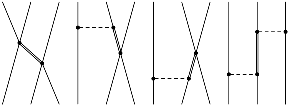
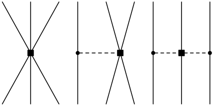
$$V^{2\phi} = \frac{1}{4f_0^2}\,\frac{\vec{\sigma}_1\cdot\vec{q}_1\;\vec{\sigma}_3\cdot\vec{q}_3}{(\vec{q}_1^{\,2} + m_{\phi_1}^2)(\vec{q}_3^{\,2} + m_{\phi_3}^2)} \times\left\{N_9\,m_\pi^2 + N_{10}\,m_K^2 + N_{11}\,\vec{q}_1\cdot\vec{q}_3 + N_{12}\,\mathrm{i}\,\vec{\sigma}_2\cdot(\vec{q}_1\times\vec{q}_3)\right\}$$

$p_i(p_i')$ are initial (final) momenta of the baryon i and $\vec{q}_i \equiv \vec{p}_i^{\,\prime} - \vec{p}_i$

# Hierarchy of three-baryon forces

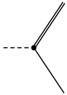
| | 3B Force |
|---|---|
| LO | — |
| NLO | — |
| $N^2LO$ |  |

# Hierarchy of three-baryon forces

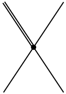
| | 3B Force with decuplet baryons |
|---|---|
| LO | — |
| NLO |  |
| $N^2LO$ |  $+ \cdots$ |

# Three-baryon forces and explicit decuplet baryons

- new vertices:



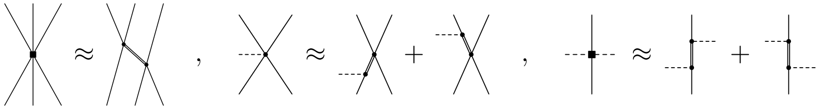
one constant ($C = \frac{3}{4} g_A \approx 1$ from $\Delta \to N\pi$)



two constants (Pauli-forbidden in nucleonic sector)

| | tensor products in *flavor* space | and in *spin* space |
|---|---|---|
| final state | $\mathbf{10} \otimes \mathbf{8} = \mathbf{35} \oplus \mathbf{27} \oplus \mathbf{10} \oplus \mathbf{8}$ | $\mathbf{3/2} \otimes \mathbf{1/2} = \mathbf{1} \oplus \mathbf{2}$ |
| initial state | $\mathbf{8} \otimes \mathbf{8} = \underbrace{\mathbf{27} \oplus \mathbf{8_s} \oplus \mathbf{1}}_{\text{symmetric}} \oplus \underbrace{\mathbf{10} \oplus \overline{\mathbf{10}} \oplus \mathbf{8_a}}_{\text{antisymmetric}}$ | $\mathbf{1/2} \otimes \mathbf{1/2} = \underbrace{\mathbf{0}}_{\text{a.sym.}} \oplus \underbrace{\mathbf{1}}_{\text{sym.}}$ |

- estimate chiral three-baryon forces via decuplet saturation:



- presently implemented into hypertriton calculations (A. Nogga)

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## Summary

- SU(3) heavy baryon chiral effective field theory
- Hyperon-nucleon potentials at NLO including one- and two-meson exchange and SU(3) symmetric contact terms
- good description of available YN scattering data; comparable to most advanced phenomenological models
- G-matrix calculation of hyperon-nuclear mean fields: $U_\Lambda$ attractive, $U_\Sigma$ repulsive, $\Lambda$ spin-orbit coupling small
- leading three-baryon forces constructed in general
- couplings estimated through decuplet exchange $\Rightarrow$ only 2 unknown low-energy constants left

## Outlook

- future applications of YN potential: hypernuclei, neutron star matter
- quantify effect of chiral three-baryon forces in light hypernuclei