Outline

1. Introduction
   - Hadron Resonance Gas

2. HRG vs LQCD - Missing strange resonances
   - light resonances: $K_0^*(800)$ meson & S-matrix approach
   - heavy resonances: Hagedorn mass spectrum

3. Conclusions
QCD phase diagram

source: Brookhaven National Laboratory
Additional strange resonances from lattice QCD

Introduction

Hadron Resonance Gas

EFFECTIVE EQUATION OF STATE

relevant degrees of freedom + hadrons and their resonances

interactions + point-like and independent species

HADRON RESONANCE GAS MODEL
Idea of Hadron Resonance Gas

- Resonance production dominates the interactions
- Information about the interactions $\rightarrow$ medium composition

$$\ln Z \approx \sum_{i \in \text{mes}} \ln Z_i^M + \sum_{i \in \text{bar}} \ln Z_i^B, \quad \rho(m) = \sum_{i \in \text{had}} d_i \delta(m - m_i)$$

Pressure (Boltzmann approximation)

$$\hat{P} \equiv \frac{P}{T^4} = \frac{1}{2\pi^2} \sum_{i \in \text{had}} d_i \frac{m_i^2}{T^2} K_2 \left( \frac{m_i}{T} \right) e^{\hat{\mu}_i} \bigg|_{\hat{\mu}_B = \hat{\mu}_S = \hat{\mu}_Q = 0}$$

$$\hat{\mu}_i = B_i \hat{\mu}_B + S_i \hat{\mu}_S + Q_i \hat{\mu}_Q, \quad \hat{\mu} = \mu / T$$
Fluctuations in Hadron Resonance Gas

$2^{\text{nd}}$ order correlations $\rightarrow$ generalized susceptibilities

Generalized susceptibilities

$$\hat{\chi}_{xy} = \frac{\partial^2 \hat{P}}{\partial \hat{\mu}_x \partial \hat{\mu}_y}, \quad x, y = B, S, Q$$

$$\hat{\chi}_{BB} = \sum_i \frac{d_i}{\pi^2} \frac{m_i^2}{T^2} K_2 \left( \frac{m_i}{T} \right) B_i^2$$  

baryons

$$\hat{\chi}_{BS} = \sum_i \frac{d_i}{\pi^2} \frac{m_i^2}{T^2} K_2 \left( \frac{m_i}{T} \right) S_i B_i$$  

strange baryons

$$\hat{\chi}_{SS} = \sum_i \frac{d_i}{\pi^2} \frac{m_i^2}{T^2} K_2 \left( \frac{m_i}{T} \right) S_i^2$$  

strange hadrons
**HRG vs Lattice QCD**

**total thermodynamic pressure**

- HotQCD
- Wuppertal
- PDG

**net-baryon number fluctuations**

- HotQCD
- Wuppertal
- PDG


HRG vs Lattice QCD

baryon-strangeness correlations

net-strangeness fluctuations


Missing strangeness

- $\hat{P}, \hat{\chi}_{BB} \rightarrow$ match LQCD results
- $\hat{\chi}_{BS} \rightarrow$ missing resonances in the strange-baryonic sector
- $\hat{\chi}_{SS} \rightarrow$ missing resonances in the strange sector

\[ \downarrow \]
Known states are not sufficient


\[ \downarrow \]
Goal:
Identify the possible origins of the discrepancies in the strange-hadronic sector of the HRG model
Low-mass resonances $\rightarrow K_0^*(800)$ a.k.a. $\kappa$ meson

- $\kappa$ is a broad resonance with $m \sim 682$ MeV, $\Gamma \sim 550$ MeV
- zero-width approximation IS NOT an accurate description
- one needs a consistent treatment of all interaction effects
Related phase shift $\rightarrow K\pi$ S-wave, $I = \frac{1}{2}$ channel

- The total phase shift
  $$\delta^{1/2}_0 = \delta_\kappa + \delta_{K_0^*} + \delta_{BG}$$

- Resonances $\rightarrow$ Breit-Wigner
  $$\delta(s) = \text{arc tg} \left( \frac{-\sqrt{s} \Gamma(s)}{s - M_0^2} \right)$$

- Hard core background term
  $$\delta_{BG}(s) = -r_c P_{CM}(s)$$

- Near threshold
  $$\delta^{1/2}_0 (\sqrt{s} \approx m_{th}) \approx a^{1/2}_0 P_{CM}(s)$$
  $$a^{1/2}_0 \approx (0.18 - 0.39) m_\pi^{-1}$$

**S-matrix approach**


**Thermodynamic potential and pressure**

\[ \Omega = \Omega_\pi + \Omega_K + \Omega_{\text{int}}, \quad P = -\frac{\Omega}{V} \]

\[ \Omega_{\text{int}} \approx 2TV \int_{m_{\text{th}}}^\infty \frac{dM}{2\pi} \int \frac{d^3p}{(2\pi)^3} B(M) \sum_{\gamma=\pm1} \ln \left[ 1 - e^{-(\hat{E} + \gamma\hat{\mu}_s)} \right] \]

**Relation to the phase shift**  

\[ B(M) = 2 \frac{d}{dM} \delta(M) \]
Additional strange resonances from lattice QCD

- HRG vs LQCD - Missing strange resonances
- light resonances: $K_0^*$ (800) meson & S-matrix approach

Result $\rightarrow$ enhancement of $\hat{\chi}_{SS}$

- $\mathcal{B}(M)$ diverges at the threshold unlike standard Breit-Wigner
- overestimate from standard Breit-Wigner

- finally, the enhancement of $\hat{\chi}_{SS}$ due to $\kappa$ is reduced by $\sim 80\%$

The effect of $I = \frac{3}{2} K\pi$ scattering

- $\kappa$-contribution almost fully cancels!
- $\kappa$ alone is only a part of missing contribution!
- similar result for the $\sigma$ meson


Contribution from heavy (unobserved?) resonances

Our key assumptions:

- $T_H > T_c$ for the observables to be consistent with LQCD;
- the same $T_H \sim 180$ MeV in all sectors;

Two possible sources:

- Heavy resonances beyond current experimental reach;
- resonances excluded from PDG;
Fit to PDG cumulants

\[ N(m) = \int_{0}^{m} dm' \rho(m') \text{ such that } \rho = \partial N/\partial m \]
Heavy resonances capture the difference only for high $T$


Implication of observables on medium composition

Only $\hat{\chi}_{BS}$ and $\hat{\chi}_{SS}$ to constrain four spectra

\[ \rho_{B}^{|S|=1}, \rho_{B}^{|S|=2}, \rho_{B}^{|S|=3}, \rho_{M}^{|S|=1} \]

- we can only constrain their linear combination!
- more data needed to constrain individual sectors, e.g. kurtosis;

- $\hat{\chi}_{BS}$ is dominated by $|S| = 1$ sector (Boltzmann suppression);
- $\hat{\chi}_{SS}$ is dominated by mesons (Boltzmann suppression);

Assumption:

- missing strength comes solely from $|S| = 1$ sector;
- $|S| = 2, 3$ sectors $\rightarrow$ fit to PDG;
Results → mass spectrum

**Strange Baryons**

- PDG
- PDG+unconf.

<table>
<thead>
<tr>
<th>S</th>
<th>N(m)</th>
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**Strange Mesons**

- PDG
- PDG+unconf.

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$m$ [GeV]
Additional strange resonances from lattice QCD

- HRG vs LQCD - Missing strange resonances
- Heavy resonances: Hagedorn mass spectrum

Results → mass spectrum

**Strange Baryons**

- PDG
- PDG + unconf.
- Fit to PDG
- LQCD

**Strange Mesons**

- PDG
- PDG + unconf.
- Fit to PDG
- Fit to LQCD
Magnitude of the interaction strength in the strange sector
Results → observables

**baryon-strangeness correlations**

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<tr>
<th>T [GeV]</th>
<th>-X_{bs}/T^2</th>
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<td>0.19</td>
<td>0.22</td>
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**net-strangeness fluctuations**

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- HotQCD
- Wuppertal
- fit to PDG
- fit to LQCD
- PDG

**HRG vs LQCD - Missing strange resonances**

**Heavy resonances: Hagedorn mass spectrum**
Conclusions

We addressed the problem of missing strange resonances utilizing the HRG model

- HRG is not a good approximation for broad resonances
  - S-matrix approach shows consistently importance of the width and non-resonant background;
  - $\kappa$ meson alone is insufficient to fix the discrepancy in $\hat{\chi}_{SS}$.
- Substantial contribution from intermediate states to the fluctuations near $T_c$
  - Spectra for strange baryons are consistent with unconfirmed states in the PDG;
  - Spectrum for mesons exceeds that of the PDG $\rightarrow$ new extra states?
The End
Hagedorn mass spectrum

Continuous mass spectrum

\[ \rho^H(m) = \frac{A e^{m/T_H}}{(m^2 + m_0^2)^{5/4}} \]

Previous fits → different \( T_H \) for mesons and baryons:
- \( T_H^M = 197 \) MeV
- \( T_H^B = 141 \) MeV < \( T_c \sim 155 \) MeV ⇒ disfavored by LQCD


Our key assumptions:
- \( T_H > T_c \) for the observables to be consistent with LQCD;
- the same \( T_H \sim 180 \) MeV in all sectors;
Results → mass spectrum

Strange baryons

Strange mesons

Different functional form → similar conclusions

Functional form

\[ \rho^H(m) = Ae^{m/B} \]