Chiral magnetic effect and chiral kinetic theory

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QCD matter: dense and hot

Hirschegg, January 17 - 23, 2016

Ref:

•J.H. Gao, Z.T. Liang, SP, Q. Wang, X.N. Wang, PRL 109 (2012) 232301

•J.W. Chen, SP, Q. Wang, X.N. Wang, PRL 110 (2013) 262301

•SP, S.Y. Wu, D.L. Yang, Phys.Rev. D89 (2014) 8, 085024; Phys.Rev. D91 (2015) 2, 025011


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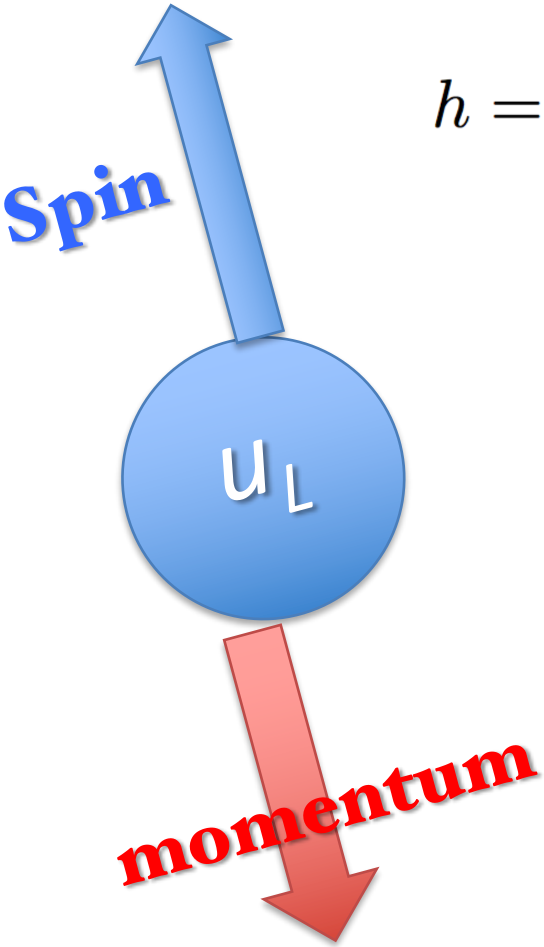
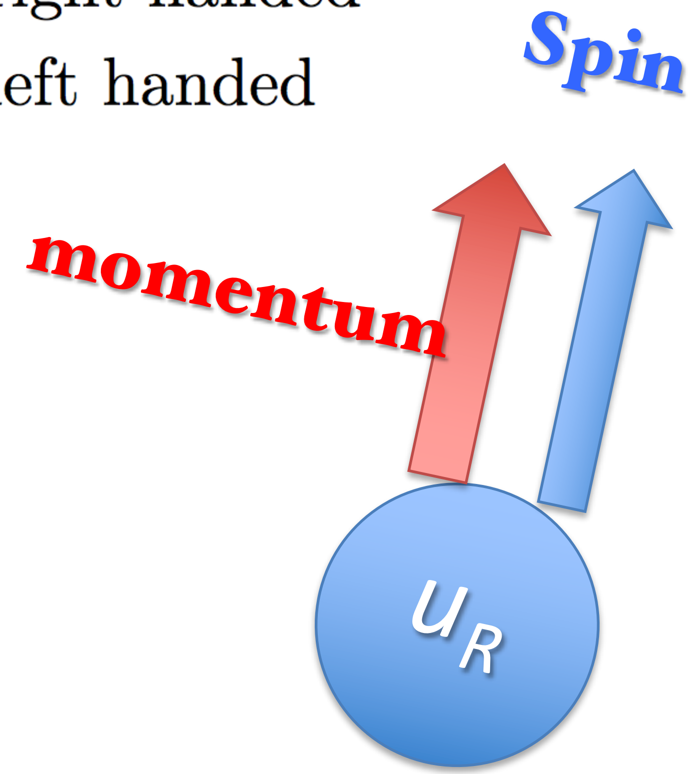
Outline

- Chiral magnetic and vortical effects
- Quantum kinetic theory and Berry phase
- Summary

Chirality of massless fermions

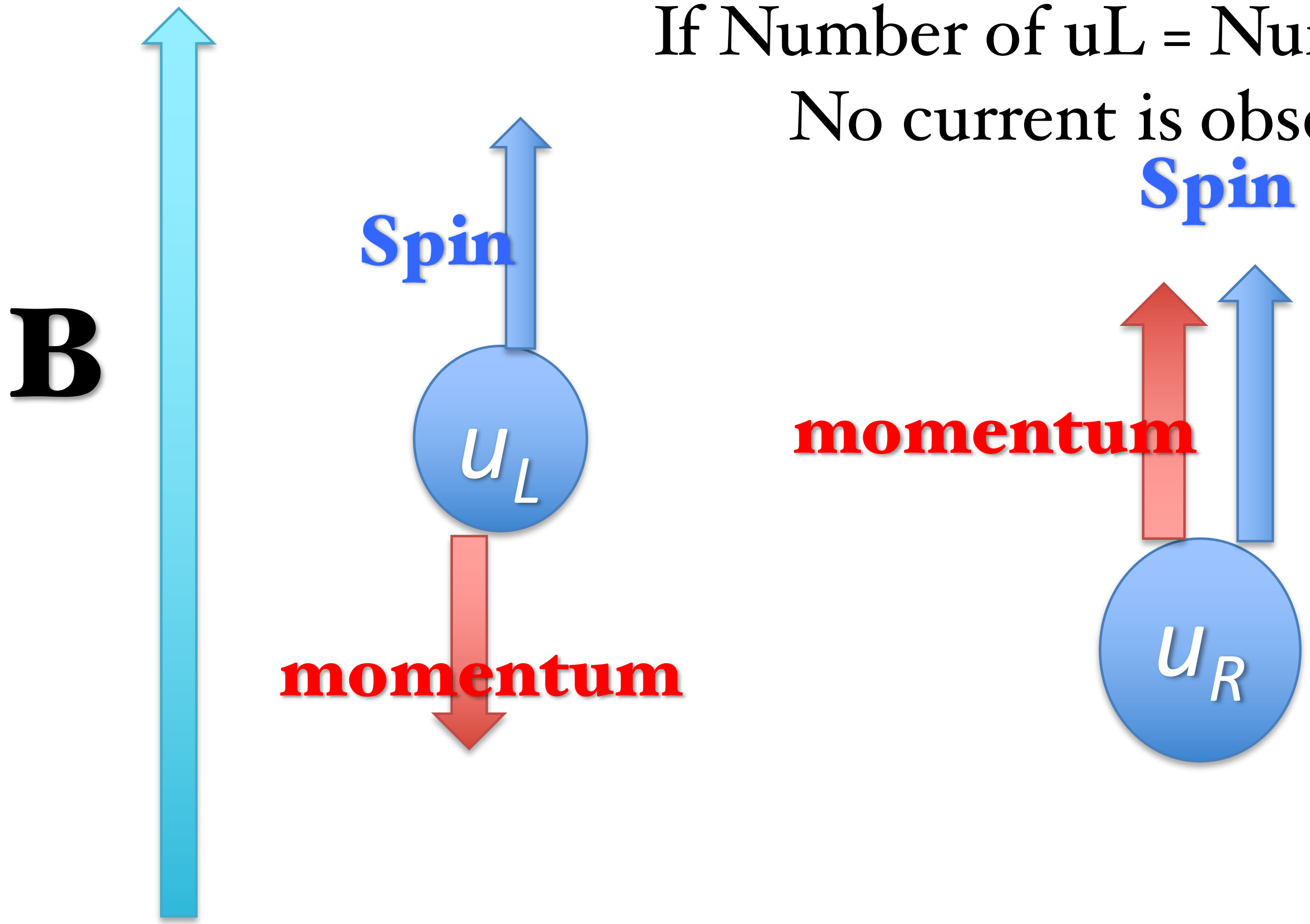
$$h = \frac{\sigma \cdot p}{|\mathbf{p}|} = \begin{cases} +1, & \text{right handed} \\ -1, & \text{left handed} \end{cases}$$





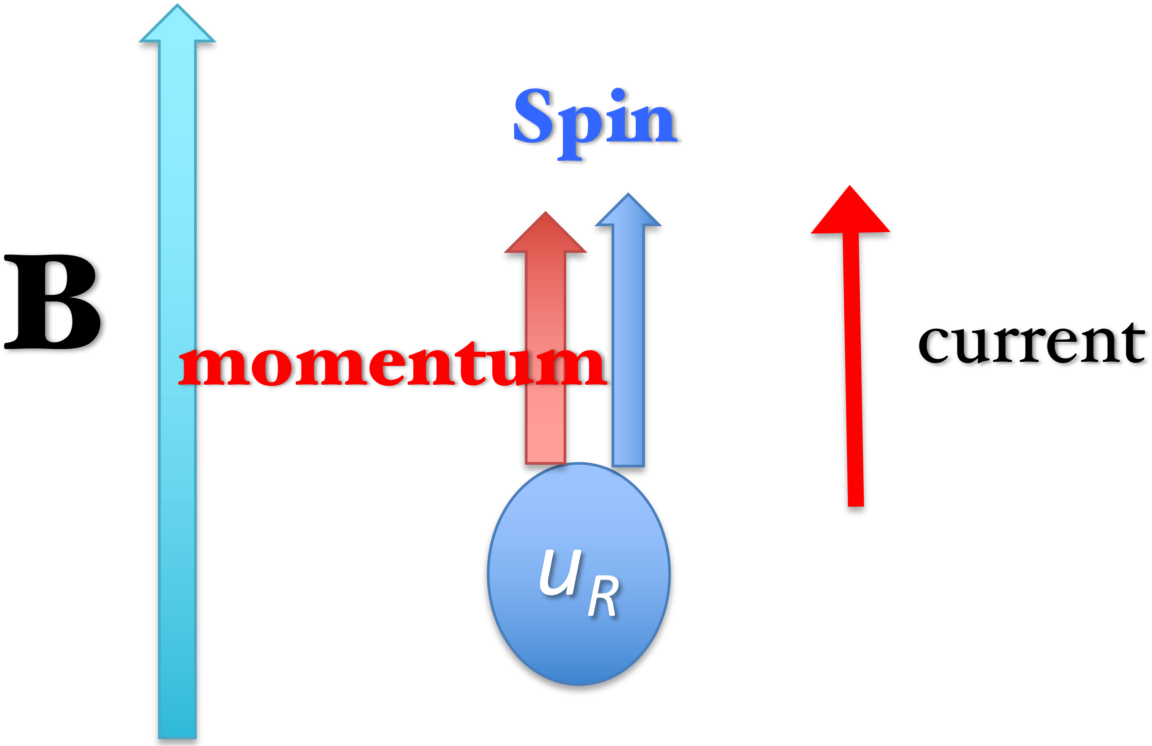
Chirality

If Number of uL = Number of uR
No current is observed.

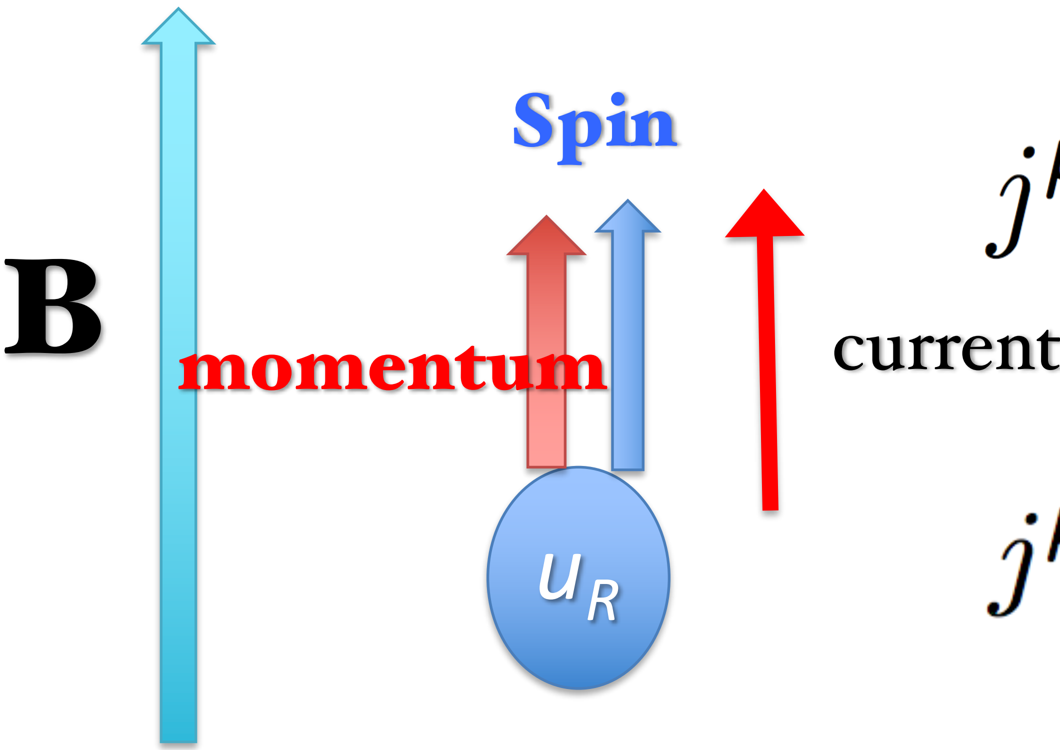


Chiral Magnetic Effect

If Number of uL ≠ Number of uR
A electric current will be observed.



Chiral Magnetic Effect (CME)

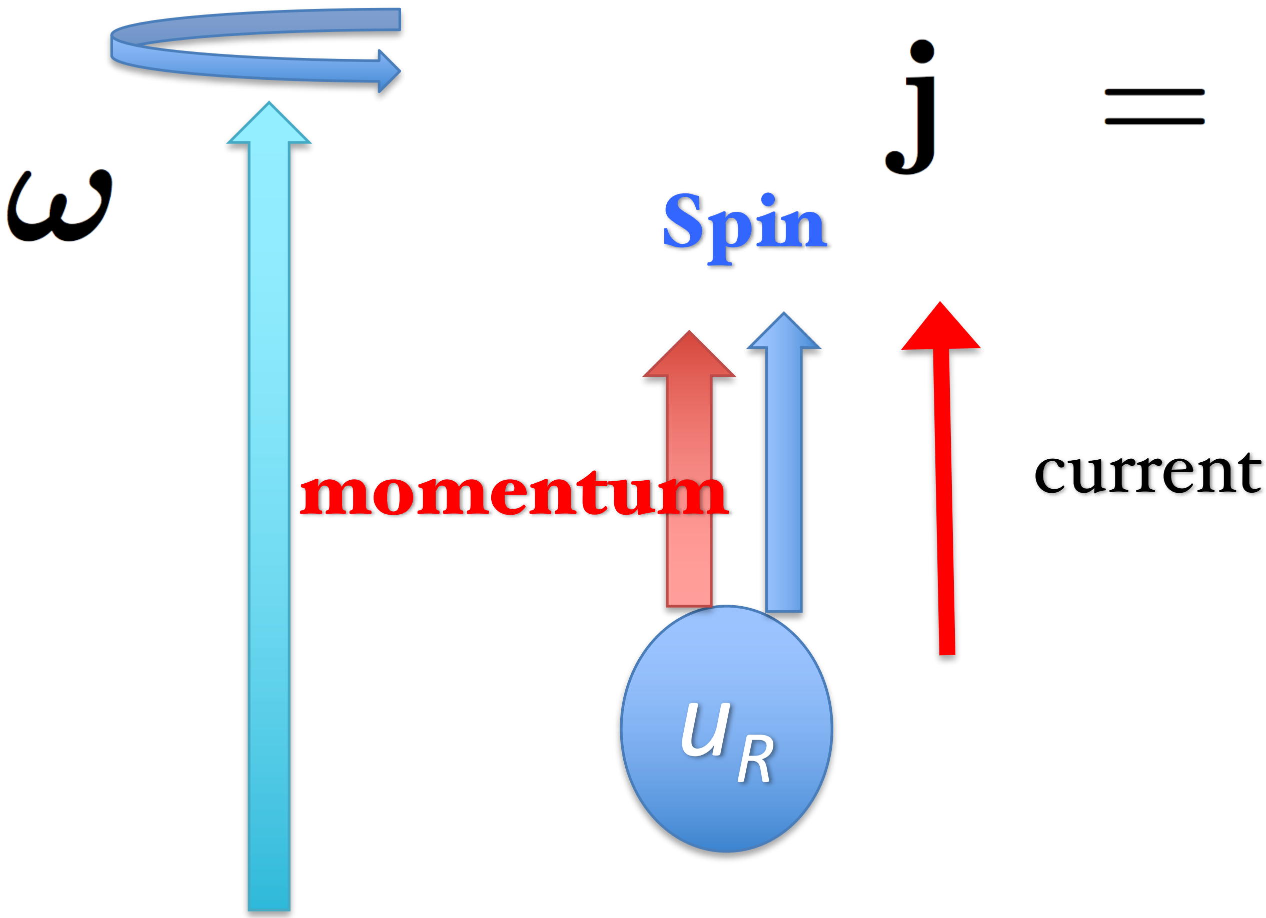


$$j^{\mu} = \xi_B B^{\mu},$$

$$j^{\mu} = \sigma E^{\mu},$$

Chiral Vortical Effect (CVE)

$$\mathbf{j} = \xi\boldsymbol{\omega}$$



Chiral Magnetic and Vortical Effect

Charge current

Magnetic field

Vorticity

$$
\begin{aligned}
j^\mu &= \xi_B B^\mu + \xi \omega^\mu, \\
j_5^\mu &= \xi_{5B} B^\mu + \xi_5 \omega^\mu,
\end{aligned}
$$

Axial current

Anomalous fluid dynamics

- We do not have those chiral transport terms in a normal fluid.

- Son and Suro´wka ('09) pointed out these terms are crucial to cancel the production of negative entropy in an anomalous fluid.

$$\partial_\mu T^{\mu\nu} = QF^{\nu\rho} j_\rho,$$

$$\partial_\mu j^\mu = 0, \qquad \partial_\mu j_5^\mu = -\frac{Q^2}{2\pi^2} E_\rho B^\rho,$$

New Transport coefficients

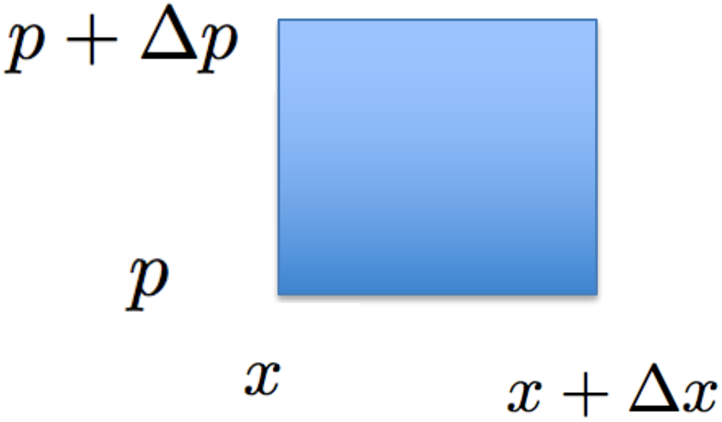
$$j^\mu = \xi_B B^\mu + \xi \omega^\mu,$$
$$j_5^\mu = \xi_{5B} B^\mu + \xi_5 \omega^\mu,$$

- Strong coupling, AdS/CFT duality,
  (Erdmenger('09), Banerjee('11), Torabian('11), ...)
- Weakly coupling, Kubo formula
  (Fukushima('08), Kharzeev('11), Landsteiner('11), Hou('12), ...)

Kinetic theory

- Kinetic theory: a microscopic dynamic theory for many-body system, to compute transport coefficients.

- distribution function, e.g. Fermi-Dirac distribution f(x,p)



Boltzmann equations

- We try to study these chiral phenomena by Boltzmann equations, but we failed…
- It seems that one has to modify the Boltzmann equations
- SP, J.H. Gao, Q. Wang, Phys.Rev. D83 (2011) 094017

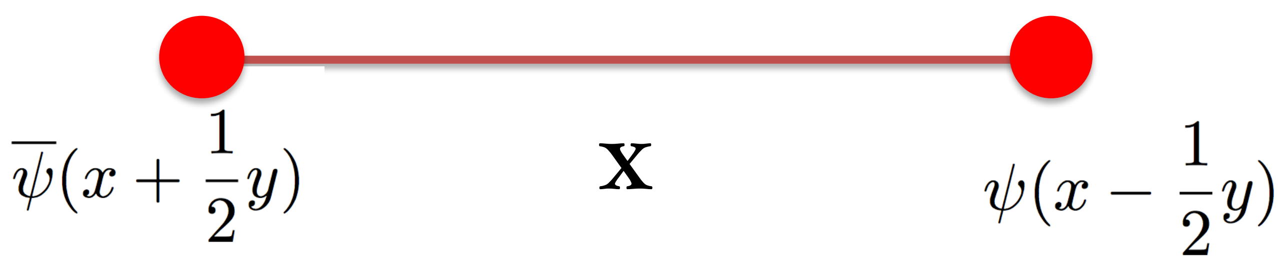
Wigner function for fermions

- Wigner function: a quantum distribution function, ensemble average, normal ordering

Vasak, Gyulassy and Elze ('86,'87,'89)

$$W(x,p) = \left< : \int \frac{d^4y}{(2\pi)^4} e^{-ipy} \overline{\psi}(x+\frac{1}{2}y) \otimes \mathcal{P}U(x,y) \psi(x-\frac{1}{2}y) : \right>$$

Gauge link



Macroscopic quantities

Charge current

$$j^{\mu}(x) \equiv <: \overline{\psi}(x)\gamma^{\mu}\psi(x) :>= \int d^4p \mathrm{Tr}\ (\gamma^{\mu}W),$$

Axial (chiral) current

$$j_5^{\mu}(x) \equiv <: \overline{\psi}(x)\gamma^5\gamma^{\mu}\psi(x) :>= \int d^4p \mathrm{Tr}\ (\gamma^5\gamma^{\mu}W),$$

Master equation from Dirac Eq.

- Massless, constant external electromagnetic fields $F^{\mu\nu}_{ext}$, turn off all internal interactions

$$[\gamma^{\mu}p_{\mu} + \frac{1}{2}i\ \gamma^{\mu}(\partial^{x}_{\mu} - QF^{ext}_{\mu\nu}\partial^{p}_{\mu})]W = 0,$$

- First order differential equation, solve it order by order

Solve the Master equation

- Gradient expansion to Winger function W and its master equation,
  - expand all quantities at the power of derivatives

    $O(\partial_x^1), O(\partial_x^2),$
  - external fields are weak $F^{\mu\nu} \sim \partial_x^\mu A^\nu \sim O(\partial^1),$

Leading order

- 0^{th} order, non-interacting ideal gas
  – classical Fermi-Dirac distribution

- input
  – finite temperature T,
  – chemical potential $\mu = \mu_R + \mu_L$,
  – chiral chemical potential $\mu_5 = \mu_R - \mu_L$

1^{st} order, Chiral anomaly

- Remarkable, we obtain the chiral anomaly by Winger function!

Energy momentum conservation

$$\partial_\mu T^{\mu\nu} = Q F^{\nu\rho} j_\rho,$$

$$\partial_\mu j^\mu \ = \ 0,$$

Triangle anomaly

$$\partial_\mu j_5^\mu \ = \boxed{-\frac{Q^2}{16\pi^2}\epsilon^{\mu\nu\alpha\beta}F_{\mu\nu}F_{\alpha\beta}}$$

$$\propto E\cdot B$$

Chiral magnetic and vortical effect

$$j^\mu = \xi_B B^\mu + \xi \omega^\mu,$$
$$j_5^\mu = \xi_{5B} B^\mu + \xi_5 \omega^\mu,$$

Consistent with other approaches!

$$\xi = \frac{1}{\pi^2}\mu\mu_5,$$
$$\xi_B = \frac{Q}{2\pi^2}\mu_5,$$
$$\xi_5 = \frac{1}{6}T^2 + \frac{1}{2\pi^2}\left(\mu^2 + \mu_5^2\right),$$
$$\xi_{B5} = \frac{Q}{2\pi^2}\mu.$$

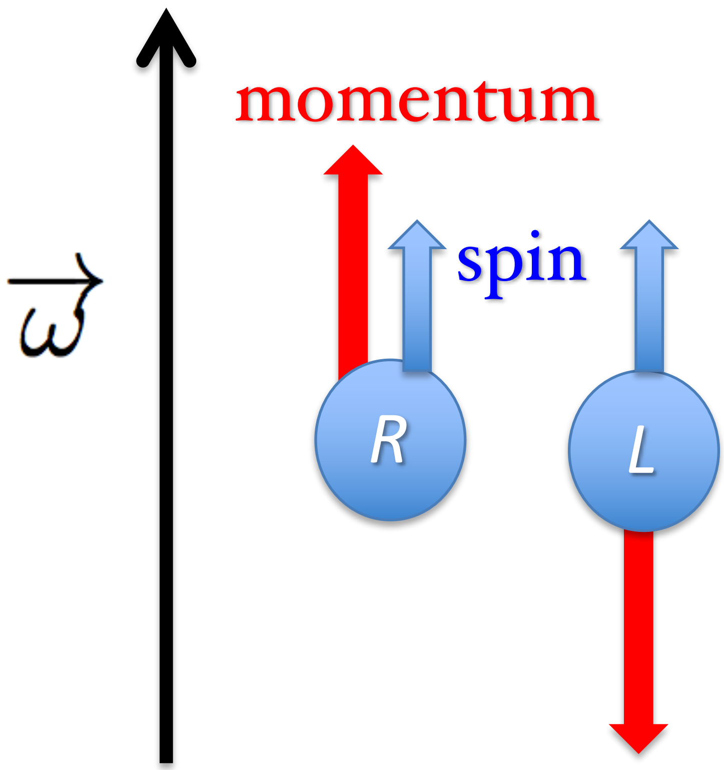
Q: charge
T: temperature

Chemical potentials

$$\mu = \mu_R + \mu_L,$$
$$\mu_5 = \mu_R - \mu_L,$$

Local Polarization Effect

Spin local polarization effect



Axial current

$$j_5^\mu \equiv j_R^\mu - j_L^\mu = \xi_5 \omega^\mu,$$

$$\xi_5 = \frac{1}{6}T^2 + \frac{1}{2\pi^2}(\mu^2 + \mu_5^2),$$

Can be observed in both high/low energy collisions

Connection to Berry phase: Modified Boltzmann equation

3-dimensional Chiral kinetic equation

Integral over p0

$$\frac{dt}{d\tau}\partial_t f_{R/L} + \frac{d\mathbf{x}}{d\tau}\cdot\nabla_{\mathbf{x}} f_{R/L} + \frac{d\mathbf{p}}{d\tau}\cdot\nabla_{\mathbf{p}} f_{R/L} = 0,$$

$$\frac{dt}{d\tau} = 1 \pm Q\boldsymbol{\Omega}\cdot\mathbf{B} \pm 4|\mathbf{p}|(\boldsymbol{\Omega}\cdot\boldsymbol{\omega}),$$

velocity

$$\frac{d\mathbf{x}}{d\tau} = \hat{\mathbf{p}} \pm Q(\hat{\mathbf{p}}\cdot\boldsymbol{\Omega})\mathbf{B} \pm Q(\mathbf{E}\times\boldsymbol{\Omega}) \pm \frac{1}{|\mathbf{p}|}\boldsymbol{\omega},$$

force

$$\frac{d\mathbf{p}}{d\tau} = Q(\mathbf{E}+\hat{\mathbf{p}}\times\mathbf{B}) \pm Q^2(\mathbf{E}\cdot\mathbf{B})\boldsymbol{\Omega} \mp Q|\mathbf{p}|(\mathbf{E}\cdot\boldsymbol{\omega})\boldsymbol{\Omega} \pm 3Q(\boldsymbol{\Omega}\cdot\boldsymbol{\omega})(\mathbf{p}\cdot\mathbf{E})\hat{\mathbf{p}},$$

3-dimensional Chiral kinetic equation

Integral over p0

$$\frac{dt}{d\tau}\partial_t f_{R/L} + \frac{d\mathbf{x}}{d\tau}\cdot\nabla_{\mathbf{x}} f_{R/L} + \frac{d\mathbf{p}}{d\tau}\cdot\nabla_{\mathbf{p}} f_{R/L} = 0,$$

$$\frac{dt}{d\tau} = 1$$

velocity $$\frac{d\mathbf{x}}{d\tau} = \hat{\mathbf{p}}$$

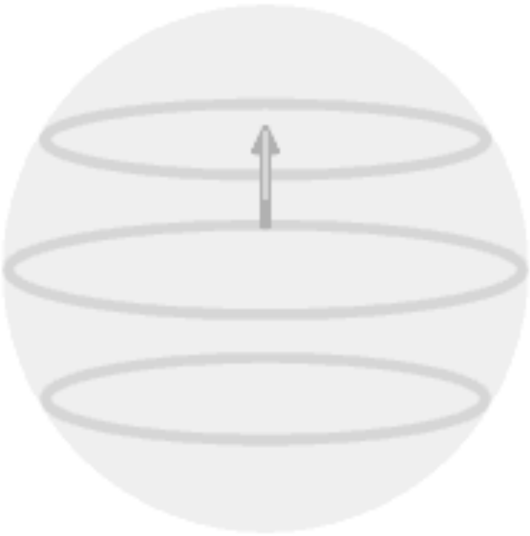
force $$\frac{d\mathbf{p}}{d\tau} = Q(\mathbf{E} + \hat{\mathbf{p}}\times\mathbf{B})$$

Berry Phase (1)

- For an adiabatic process, assume the Hamiltonian is time dependent H=H(t),
- After some evolution, the system goes back to its initial eigenstate, then the Hamiltonian will pick up a new phase factor.
- Widely-used in condensed matter,

Berry Phase (2)

- Analogous to Foucault Pendulum.



Berry Phase (3)

- According to Berry phase, there is additional term in Hamiltonian.

- In “classical” single particle description, the velocity and effective force will be modified.

3-dimensional Chiral kinetic equation

Integral over p0

$$\frac{dt}{d\tau}\partial_t f_{R/L} + \frac{d\mathbf{x}}{d\tau}\cdot\nabla_{\mathbf{x}} f_{R/L} + \frac{d\mathbf{p}}{d\tau}\cdot\nabla_{\mathbf{p}} f_{R/L} = 0,$$

$$\frac{dt}{d\tau} = 1 \pm Q\boldsymbol{\Omega}\cdot\mathbf{B} \pm 4|\mathbf{p}|(\boldsymbol{\Omega}\cdot\boldsymbol{\omega}),$$

velocity

$$\frac{d\mathbf{x}}{d\tau} = \hat{\mathbf{p}} \pm Q(\hat{\mathbf{p}}\cdot\boldsymbol{\Omega})\mathbf{B} \pm Q(\mathbf{E}\times\boldsymbol{\Omega}) \pm \frac{1}{|\mathbf{p}|}\boldsymbol{\omega},$$

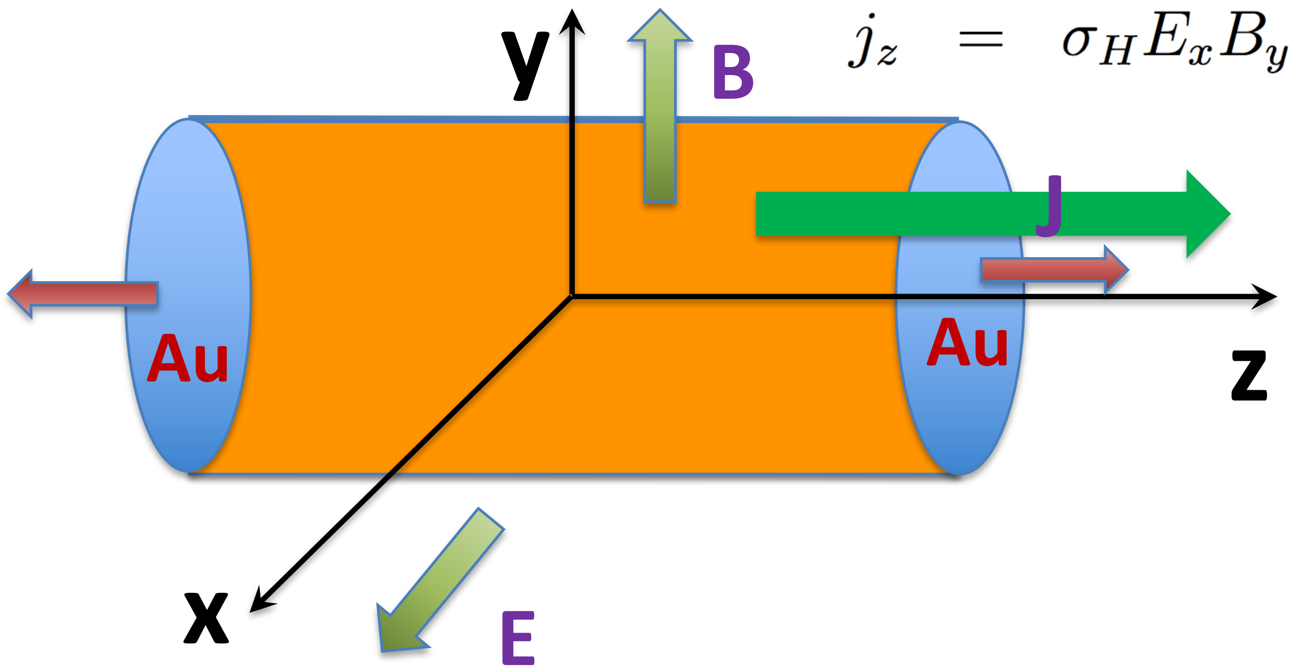
force

$$\frac{d\mathbf{p}}{d\tau} = Q(\mathbf{E} + \hat{\mathbf{p}}\times\mathbf{B}) \pm Q^2(\mathbf{E}\cdot\mathbf{B})\boldsymbol{\Omega} \mp Q|\mathbf{p}|(\mathbf{E}\cdot\boldsymbol{\omega})\boldsymbol{\Omega} \pm 3Q(\boldsymbol{\Omega}\cdot\boldsymbol{\omega})(\mathbf{p}\cdot\mathbf{E})\hat{\mathbf{p}},$$

Another “novel” chiral transport phenomena

Chiral Hall separation effect

- Assuming $E \perp B$, Hall effect:



- Charge and chirality separation in longitudinal direction
- SP, S.Y. Wu, D.L. Yang, Phys.Rev. D91 (2015) 2, 025011

Summary

- We obtain the chiral magnetic and vortical effect, chiral anomaly by Wigner function.
- We derive the chiral kinetic equation (modified Boltzmann equation) related to Berry phase.
- Chiral Hall separation effect might cause the charge and chirality separation in longitudinal direction.

Thank you!