Spectral Functions and Transport Coefficients with the Functional Renormalization Group

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HGS-HIRe for FAIR
Helmholtz Graduate School for Hadron and Ion Research

H-QM Helmholtz Research School
Quark Matter Studies

Outline

I) Introduction and motivation

II) Theoretical setup

- Functional Renormalization Group (FRG)
- quark-meson model
- analytic continuation procedure

III) Results

- quark and meson spectral functions
- mesonic contributions to the shear viscosity and to η/s

IV) Summary and outlook

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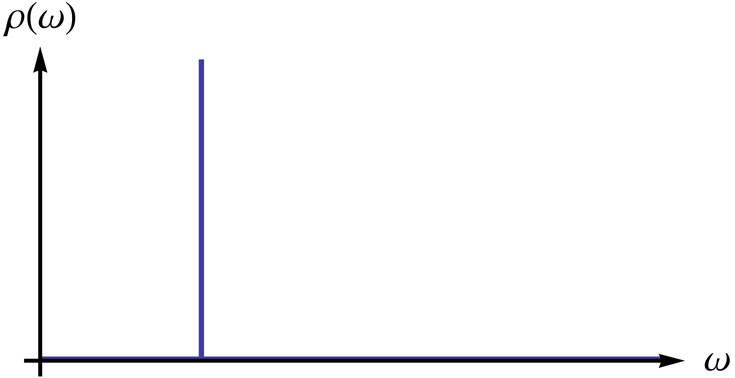
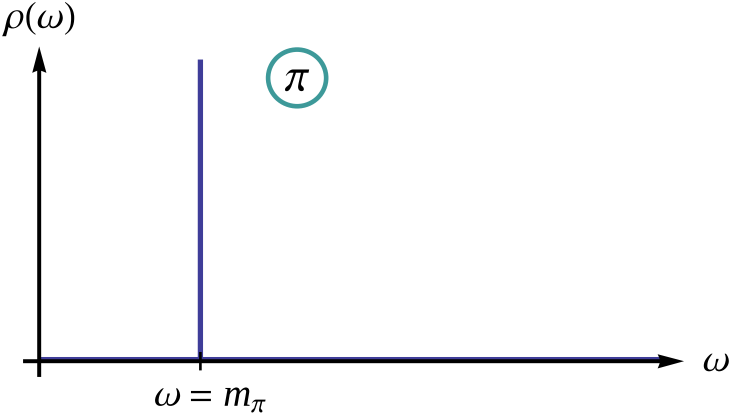
I) Introduction and motivation

[courtesy L. Holicki]

What is a spectral function?



What is a spectral function?



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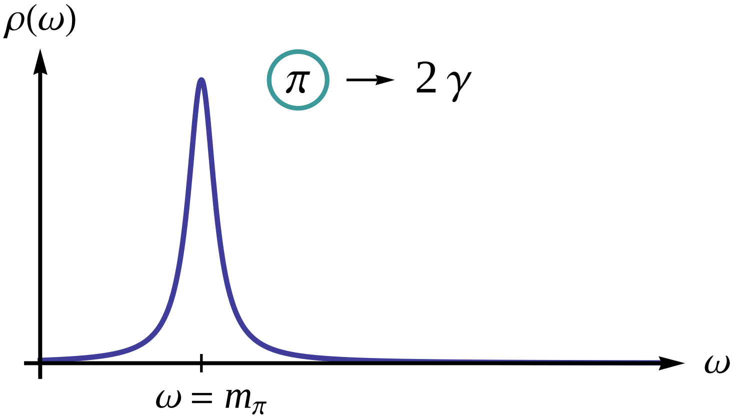
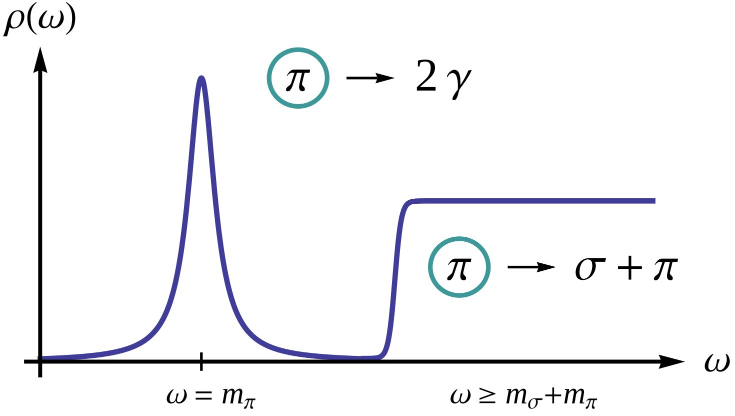


What is a spectral function?



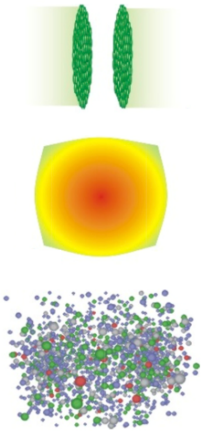
Why are spectral functions interesting?

Spectral functions determine both
real-time and imaginary-time propagators,

- $D^R(\omega) = -\int d\omega' \frac{\rho(\omega')}{\omega' - \omega - i\varepsilon}$
- $D^A(\omega) = -\int d\omega' \frac{\rho(\omega')}{\omega' - \omega + i\varepsilon}$
- $D^E(p_0) = \int d\omega' \frac{\rho(\omega')}{\omega' + ip_0}$

and thus allow access to many observables,
e.g. transport coefficients like the shear viscosity:

- $\eta = \frac{1}{24} \lim_{\omega \to 0} \lim_{|\vec{p}| \to 0} \frac{1}{\omega} \int d^4x \; e^{ipx} \left\langle \left[T_{ij}(x), T^{ij}(0) \right] \right\rangle$



[B. Mueller, arXiv: 1309.7616]

Why are spectral functions interesting?

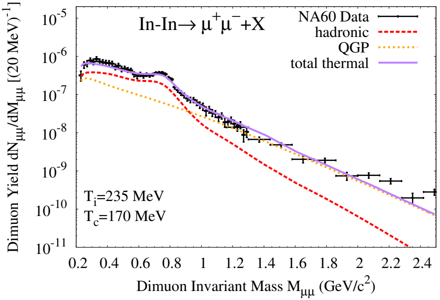
Calculation of dilepton excess spectra requires in-medium spectral function:

$$\frac{dN_{ll}}{d^4x d^4q} = -\frac{\alpha^2}{3\pi^3}\frac{L(M)}{M^2}\,\mathrm{Im}\Pi^{\mu\mu}_{\mathrm{EM}}(M,q)\; f_{\mathrm{B}}(q_0;T),$$

$f_{\mathrm{B}}(q_0;T)\ldots$ thermal Bose function,
$\alpha = e^2/4\pi\ldots$ EM coupling constant,
$L(M)\ldots$ final-state lepton phase space factor,
$M = \sqrt{q_0^2 - \vec{q}^{\,2}}\ldots$ dilepton invariant mass,

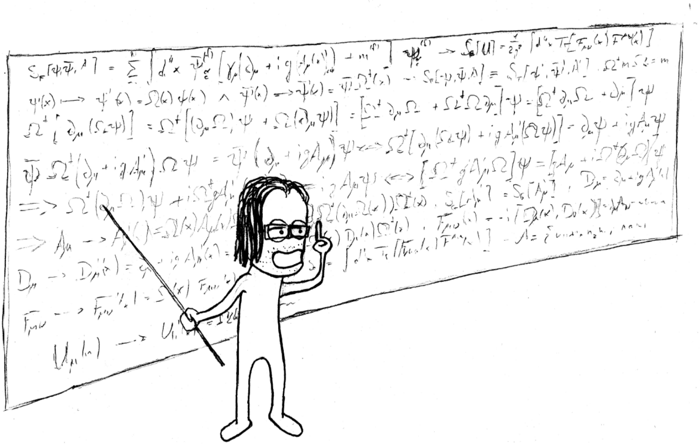
and the EM spectral function ($M \leq 1\mathrm{GeV}$):

$$\mathrm{Im}\Pi^{\mu\nu}_{\mathrm{EM}} \sim \mathrm{Im}D^{\mu\nu}_{\rho} + \frac{1}{9}\mathrm{Im}D^{\mu\nu}_{\omega} + \frac{2}{9}\mathrm{Im}D^{\mu\nu}_{\phi}$$



[R. Rapp, H. van Hees, Phys.Lett. B **753** (2016) 586-590]

II) Theoretical setup



[courtesy L. Holicki]

Functional Renormalization Group

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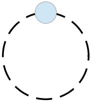
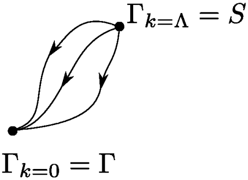
Flow equation for the effective average action Γ_k:

$$\partial_k \Gamma_k = \frac{1}{2} \mathrm{STr} \left(\partial_k R_k \left[\Gamma_k^{(2)} + R_k \right]^{-1} \right)$$

[C. Wetterich, Phys. Lett. **B 301** (1993) 90]

$$\partial_k \Gamma_k \quad = \quad \frac{1}{2}$$ 



[wikipedia.org/wiki/Functional_renormalization_group]

- Γ_k interpolates between bare action S at $k = \Lambda$ and effective action Γ at $k = 0$
- regulator R_k acts as a mass term and suppresses fluctuations with momenta smaller than k
- the use of 3D regulators allows for a simple analytic continuation procedure

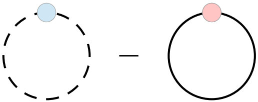
Quark-meson model

Ansatz for the scale-dependent effective average action:

$$\Gamma_k[\overline{\psi}, \psi, \phi] = \int d^4x \left\{ \overline{\psi} \left(\partial\!\!\!/ + h(\sigma + i\vec{\tau}\vec{\pi}\gamma_5) - \mu\gamma_0 \right) \psi + \frac{1}{2}(\partial_\mu \phi)^2 + U_k(\phi^2) - c\sigma \right\}$$

- effective low-energy model for QCD with two flavors
- describes spontaneous and explicit chiral symmetry breaking
- flow equation for the effective average action:

$$\partial_k \Gamma_k = \frac{1}{2}$$



Flow of the Effective Potential at μ = 0 and T = 0

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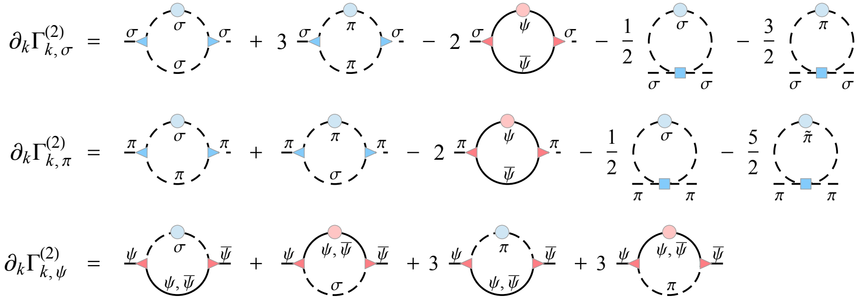
Flow equations for two-point functions

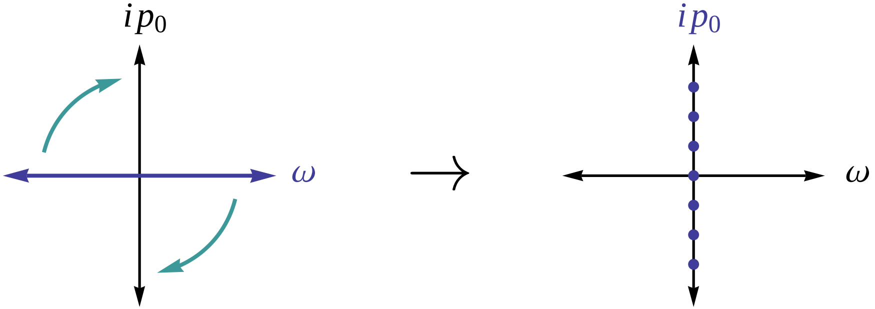


- quark-meson vertices are given by $\Gamma^{(3)}_{\bar{\psi}\psi\sigma} = h$, $\Gamma^{(3)}_{\bar{\psi}\psi\vec{\pi}} = ih\gamma^5\vec{\tau}$
- mesonic vertices from scale-dependent effective potential: $U^{(3)}_{k,\phi_i\phi_j\phi_m}$, $U^{(4)}_{k,\phi_i\phi_j\phi_m\phi_n}$
- one-loop structure and 3D regulators allow for a simple analytic continuation!

[R.-A. Tripolt, L. von Smekal, and J. Wambach, Phys. Rev. **D 90**, 074031 (2014)]

The analytic continuation problem

Calculations at finite temperature are often performed using imaginary energies:



The analytic continuation problem

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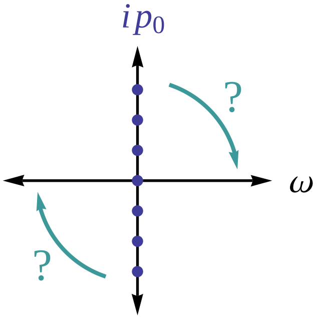
Analytic continuation problem: How to get back to real energies?



Two-step analytic continuation procedure

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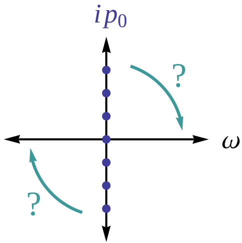
1) Use periodicity in external imaginary energy $ip_0 = i2n\pi T$:

$$n_{B,F}(E + ip_0) \to n_{B,F}(E)$$

2) Substitute p_0 by continuous real frequency ω:

$$\Gamma^{(2),R}(\omega, \vec{p}) = -\lim_{\epsilon \to 0} \Gamma^{(2),E}(ip_0 \to -\omega - i\epsilon, \vec{p})$$



Spectral function is then given by

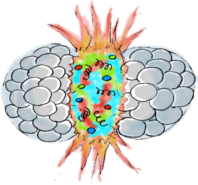
$$\rho(\omega, \vec{p}) = -\text{Im}(1/\Gamma^{(2),R}(\omega, \vec{p}))/\pi$$

Quark spectral function is parametrized as

$$\rho_{k,\psi}(\omega, \vec{p}) = -i\vec{\gamma}\vec{p}\, \rho^{(A)}_{k,\psi}(\omega, \vec{p}) - \rho^{(B)}_{k,\psi}(\omega, \vec{p}) - \gamma_0 \rho^{(C)}_{k,\psi}(\omega, \vec{p})$$

[R.-A. T., N. Strodthoff, L. v. Smekal, and J. Wambach, Phys. Rev. **D 89**, 034010 (2014)]
[J. M. Pawlowski, N. Strodthoff, Phys. Rev. **D 92**, 094009 (2015)]
[N. Landsman and C. v. Weert, Physics Reports 145, 3&4 (1987) 141]

III) Results



[courtesy L. Holicki]

Phase diagram of the quark-meson model

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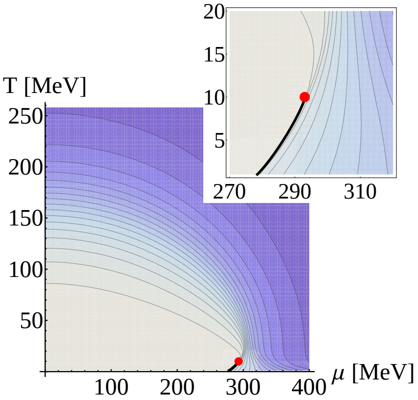
- chiral order parameter σ_0 decreases towards higher T and μ
- a crossover is observed at $T \approx 175$ MeV and $\mu = 0$
- critical endpoint (CEP) at $\mu \approx 292$ MeV and $T \approx 10$ MeV
- we will study spectral functions along $\mu = 0$ and $T \approx 10$ MeV



[R.-A. T., N. Strodthoff, L. v. Smekal, and J. Wambach, Phys. Rev. **D 89**, 034010 (2014)]

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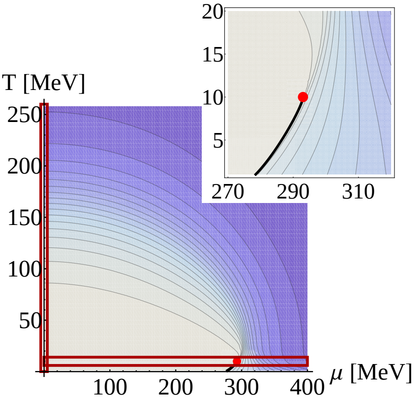
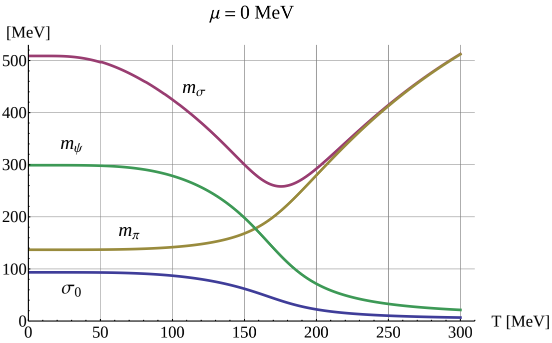


[R.-A. T., N. Strodthoff, L. v. Smekal, and J. Wambach, Phys. Rev. **D 89**, 034010 (2014)]

Masses and Order Parameter vs. T

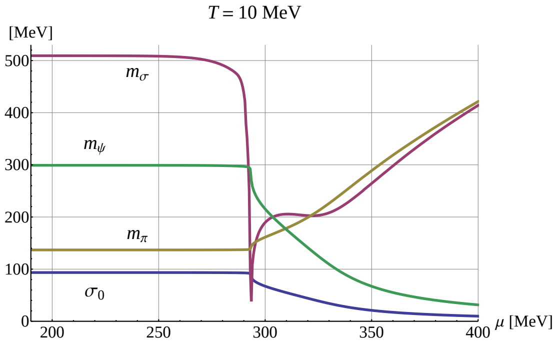


[R.-A. T., N. Strodthoff, L. v. Smekal, J. Wambach, Phys. Rev. D **89**, 034010 (2014)]

Screening masses determine thresholds in spectral functions, e.g. at T = 10 MeV:

$$\sigma^* \to \pi + \pi, \quad \omega \geq 2\, m_\pi \approx 280 \text{ MeV}$$

Masses and Order Parameter vs. μ

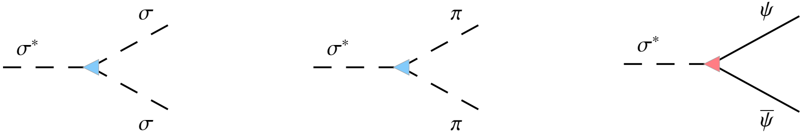


[R.-A. T., N. Strodthoff, L. v. Smekal, J. Wambach, Phys. Rev. D **89**, 034010 (2014)]

Screening masses determine thresholds in spectral functions, e.g. at $\mu = 0$:

$$\sigma^* \to \bar{\psi} + \psi, \quad \omega \geq 2\, m_\psi \approx 600 \text{ MeV}$$

Decay channels of the sigma mesons

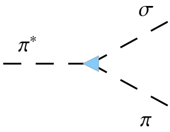


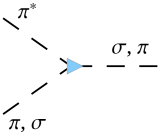
$\sigma^* \to \sigma + \sigma$ | $\sigma^* \to \pi + \pi$ | $\sigma^* \to \psi + \overline{\psi}$

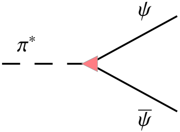
$\omega \geq \sqrt{(2m_\sigma)^2 + \vec{p}^{\,2}}$ | $\omega \geq \sqrt{(2m_\pi)^2 + \vec{p}^{\,2}}$ | $\omega \geq \sqrt{(2m_\psi)^2 + \vec{p}^{\,2}}$

[R.-A. Tripolt, L. von Smekal, and J. Wambach, Phys. Rev. **D 90**, 074031 (2014)]

Decay channels of the pions







$$\pi^* \to \sigma + \pi$$

$$\pi^* + \pi \to \sigma$$

$$\pi^* + \sigma \to \pi$$

$$\pi^* \to \psi + \overline{\psi}$$

$$\omega \geq \sqrt{(m_\sigma + m_\pi)^2 + \vec{p}^{\,2}}$$

$$\omega \leq (m_\sigma - m_\pi)\sqrt{1 + \frac{\vec{p}^{\,2}}{\Delta m^2}}$$

$$\omega \leq (m_\pi - m_\sigma)\sqrt{1 + \frac{\vec{p}^{\,2}}{\Delta m^2}}$$

$$\omega \geq \sqrt{(2m_\psi)^2 + \vec{p}^{\,2}}$$

[R.-A. Tripolt, L. von Smekal, and J. Wambach, Phys. Rev. **D 90**, 074031 (2014)]

Decay channels of the (anti-)quarks

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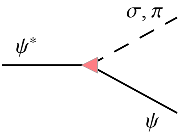

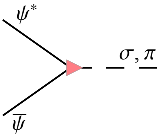

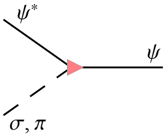

$$\psi^* \to \psi + \sigma$$
$$\psi^* \to \psi + \pi$$

$$\psi^* + \overline{\psi} \to \sigma$$
$$\psi^* + \overline{\psi} \to \pi$$

$$\psi^* + \sigma \to \psi$$
$$\psi^* + \pi \to \psi$$

$$\omega \geq \sqrt{(m_\psi + m_\sigma)^2 + \vec{p}^{\,2}}$$
$$\omega \geq \sqrt{(m_\psi + m_\pi)^2 + \vec{p}^{\,2}}$$

$$\omega \leq (m_\sigma - m_\psi)\sqrt{1 + \frac{\vec{p}^{\,2}}{\Delta m^2}}$$
$$\omega \leq (m_\pi - m_\psi)\sqrt{1 + \frac{\vec{p}^{\,2}}{\Delta m^2}}$$

$$\omega \leq (m_\psi - m_\sigma)\sqrt{1 + \frac{\vec{p}^{\,2}}{\Delta m^2}}$$
$$\omega \leq (m_\psi - m_\pi)\sqrt{1 + \frac{\vec{p}^{\,2}}{\Delta m^2}}$$

Flow of Sigma and Pion Spectral Function at μ = 0, T = 0 and $\vec{p}$ = 0

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Flow of Quark Spectral Function $\rho_\psi^{(C)}$ at μ = 0, T = 0 and $\vec{p}$ = 0

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Sigma and Pion Spectral Function with increasing T at μ = 0 and $\vec{p}$ = 0

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Sigma and Pion Spectral Function with increasing μ at T $\approx$ 10 MeV and $\vec{p}$ = 0

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Towards the shear viscosity

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Applying the Green-Kubo formula for the shear viscosity

$$\eta = \frac{1}{24} \lim_{\omega \to 0} \lim_{|\vec{p}| \to 0} \frac{1}{\omega} \int d^4x \; e^{ipx} \left\langle \left[T_{ij}(x), T^{ij}(0) \right] \right\rangle$$

to the quark-meson model with energy-momentum tensor

$$T^{ij}(x) = \frac{i}{2} \left(\overline{\psi} \gamma^i \partial^j \psi - \partial^j \, \overline{\psi} \gamma^i \psi \right) + \partial^j \sigma \partial^i \sigma + \partial^j \vec{\pi} \partial^i \vec{\pi}$$

gives (dominant contribution)

$$\eta \propto \int \frac{d\omega}{2\pi} \int \frac{d^3p}{(2\pi)^3} \; p_x^2 \, p_y^2 \, n_B'(\omega) \; \left(\rho_\sigma^2(\omega, \vec{p}) + 3\rho_\pi^2(\omega, \vec{p}) \right)$$

Space-like processes of the sigma mesons

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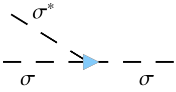

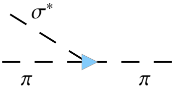

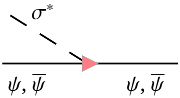

$\sigma^* + \sigma \to \sigma$

$\sigma^* + \pi \to \pi$

$\sigma^* + \psi \to \psi$
$\sigma^* + \bar{\psi} \to \bar{\psi}$

$0 \leq \omega \leq |\vec{p}|$

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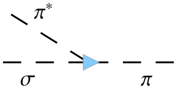
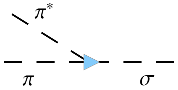
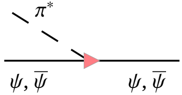
$0 \leq \omega \leq |\vec{p}|$

[R.-A. Tripolt, L. von Smekal, and J. Wambach, Phys. Rev. **D 90**, 074031 (2014)]

Space-like processes of the pions







$$\pi^* + \sigma \to \pi$$

$$\pi^* + \pi \to \sigma$$

$$\pi^* + \psi \to \psi$$
$$\pi^* + \bar{\psi} \to \bar{\psi}$$

$$0 \le \omega \le |\vec{p}|$$

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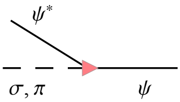
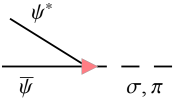
$$0 \le \omega \le |\vec{p}|$$

[R.-A. Tripolt, L. von Smekal, and J. Wambach, Phys. Rev. **D 90**, 074031 (2014)]

Space-like processes of the quarks





$$\psi^* + \sigma \to \psi$$
$$\psi^* + \pi \to \psi$$

$$\psi^* + \overline{\psi} \to \sigma$$
$$\psi^* + \overline{\psi} \to \pi$$

$$0 \leq \omega \leq |\vec{p}|$$

$$0 \leq \omega \leq |\vec{p}|$$

Sigma Spectral Function vs. ω and $\vec{p}$ at μ = 0 and T = 0 MeV

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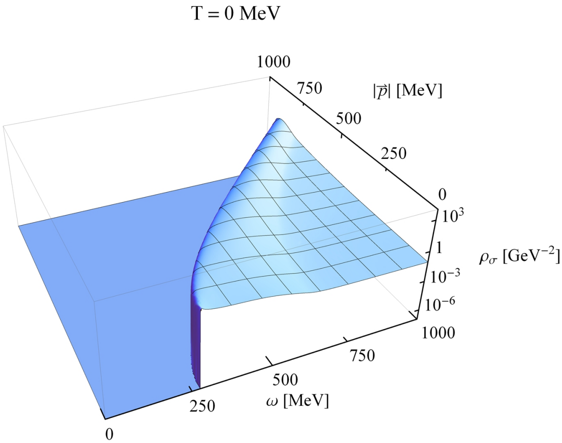
- time-like region ($\omega > \vec{p}$) is Lorentz-boosted to higher energies
- space-like region ($\omega < \vec{p}$) is non-zero at finite T due to space-like processes



Sigma Spectral Function
vs. ω and $\vec{p}$ at μ = 0 and increasing T

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- time-like region ($\omega > \vec{p}$) is Lorentz-boosted to higher energies
- space-like region ($\omega < \vec{p}$) is non-zero at finite T due to space-like processes

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Pion Spectral Function vs. ω and $\vec{p}$ at μ = 0 and T = 0 MeV

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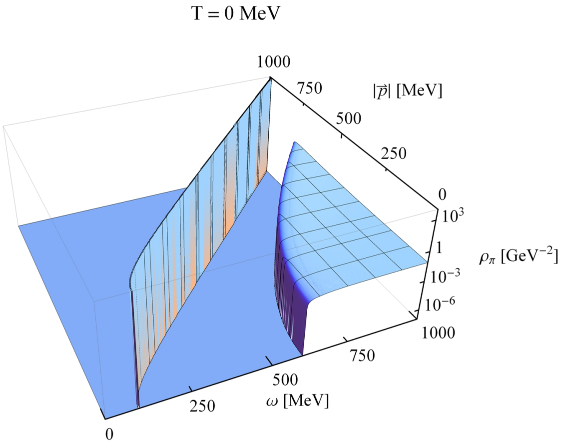
- time-like region ($\omega > \vec{p}$) is Lorentz-boosted to higher energies
- capture process $\pi^* + \pi \to \sigma$ is suppressed at large $\vec{p}$
- space-like region ($\omega < \vec{p}$) is non-zero at finite T due to space-like processes



Pion Spectral Function vs. ω and $\vec{p}$ at μ = 0 and increasing T

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- time-like region ($\omega > \vec{p}$) is Lorentz-boosted to higher energies
- capture process $\pi^* + \pi \rightarrow \sigma$ is suppressed at large $\vec{p}$
- space-like region ($\omega < \vec{p}$) is non-zero at finite T due to space-like processes

(Loading movie...)

Sigma Spectral Function
vs. ω and $\vec{p}$ at T $\approx$ 10 MeV and increasing μ

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- time-like region ($\omega > \vec{p}$) is Lorentz-boosted to higher energies
- space-like region ($\omega < \vec{p}$) is non-zero at finite T due to space-like processes
- sigma becomes stable near the critical endpoint for small momenta

(Loading movie...)

Pion Spectral Function
vs. ω and $\vec{p}$ at $T \approx 10$ MeV and increasing μ

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- time-like region ($\omega > \vec{p}$) is Lorentz-boosted to higher energies
- space-like region ($\omega < \vec{p}$) is non-zero at finite T due to space-like processes
- $\pi^* \to \pi + \sigma$ threshold moves to smaller energies due to decreasing sigma mass

(Loading movie...)

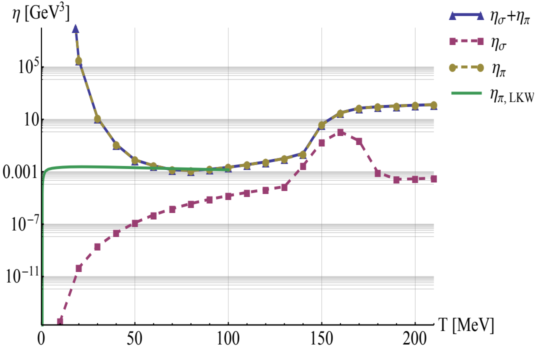
Shear viscosity at $\mu = 0$

- $\eta_{\pi,\mathrm{LKW}}$: result from chiral perturbation theory

[Lang, Kaiser, and Weise, Eur. Phys. J. **A 48**, 109 (2012)]

- large shear viscosity at low temperatures due to small width of the pion peak $\rightarrow 4\pi$ processes missing
- stable-particle delta functions are regularized by a Breit-Wigner shape

$$\rho = \frac{1}{\pi} \frac{2\omega\gamma}{(\omega^2 - \gamma^2 - \omega_0^2)^2 + 4\omega^2\gamma^2}$$



Entropy density at $\mu = 0$

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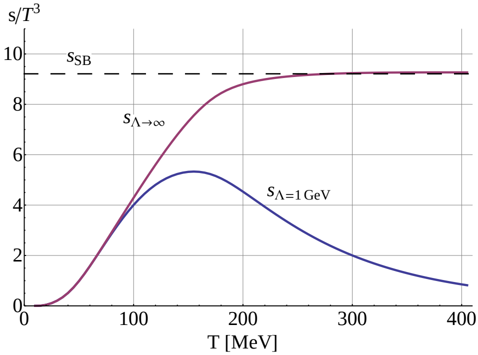
- entropy density can be extracted from the effective potential:

  $s = \partial p / \partial T = -\partial U_{k\to 0} / \partial T$

- it has been UV-corrected by taking quark fluctuations from higher scales into account

- Stefan-Boltzmann value is reproduced at high T:

  $s_{SB} / T^3 = 14\,\pi^2 / 15$



Shear viscosity over entropy density η/s at $\mu = 0$

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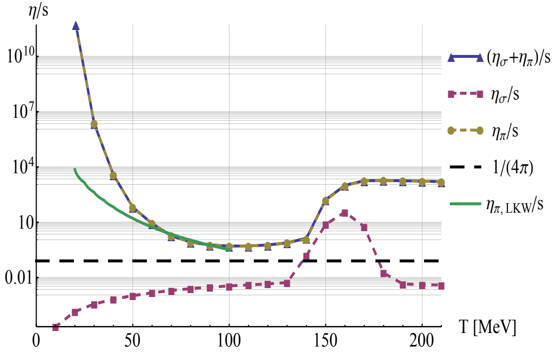
- $\eta_{\pi,\mathrm{LKW}}$: result from chiral perturbation theory

[Lang, Kaiser, and Weise, Eur. Phys. J. **A 48**, 109 (2012)]

- entropy density s contains quarks and mesons
- $(\eta_\pi + \eta_\sigma)/s$ large at low T due to large η_π and small s
- $(\eta_\pi + \eta_\sigma)/s$ is always larger than the AdS/CFT limiting value of $\eta/s \geq 1/4\pi$

[Kovtun, Son, and Starinets, Phys. Rev. Lett. **94**, 111601 (2005)]



Shear viscosity near the CEP

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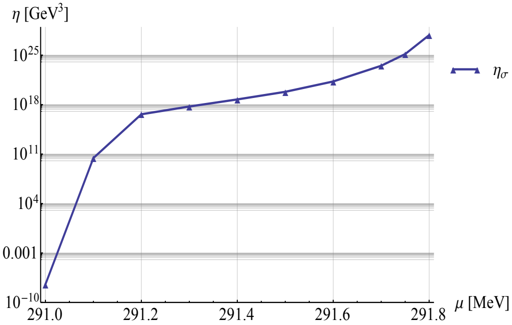
- stable-particle delta function is regularized by a Breit-Wigner shape

$$\rho = \frac{1}{\pi} \frac{2\omega\gamma}{(\omega^2 - \gamma^2 - \omega_0^2)^2 + 4\omega^2\gamma^2}$$

- shear viscosity strongly depends on the chosen value for γ
- at the CEP, shear viscosity of the sigma mesons η_σ diverges due to the massless σ excitation



Summary and outlook

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We presented a new method to obtain real-time quantities like spectral functions and transport coefficients at finite T and μ from the FRG:

- our method involves an analytic continuation from imaginary to real frequencies on the level of the flow equations
- it is thermodynamically consistent and symmetry-structure preserving
- feasibility of the method demonstrated by calculating quark and meson spectral functions and η/s for the quark-meson model

Outlook:

- calculation of the shear viscosity of the quarks
- extending the model by including vector and axial-vector mesons and improving the truncations