Locating QCD's critical end point (with functional methods)

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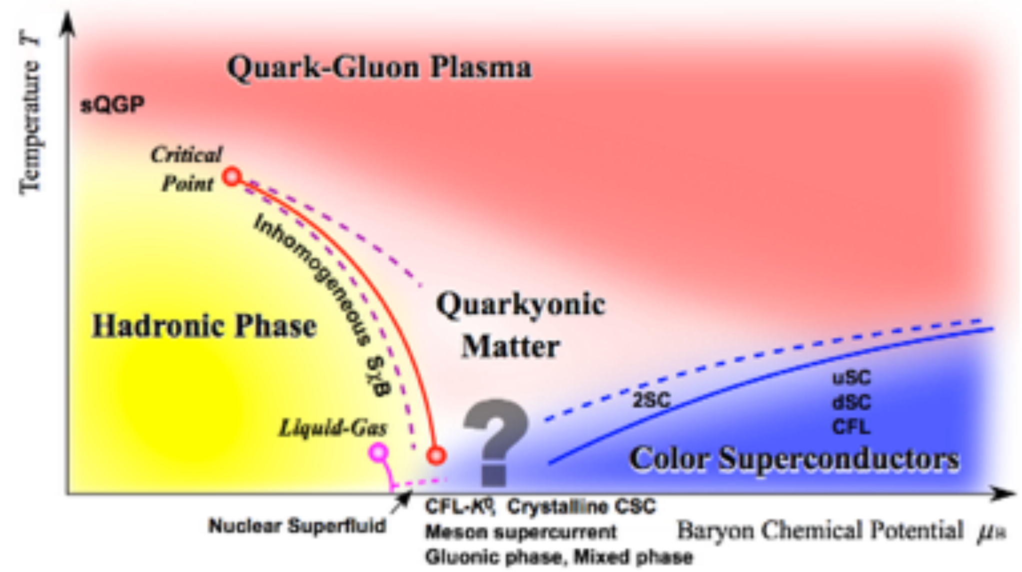
HIC for FAIR
Helmholtz International Center

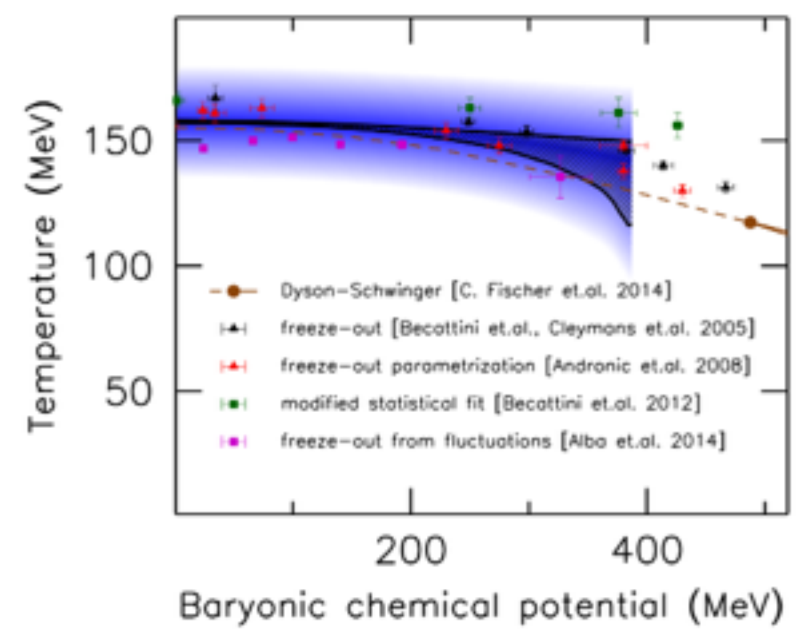
20th of January 2016

Bundesministerium für Bildung und Forschung

Eichmann, CF, Welzbacher, PRD in press, arXiv:1509.02082

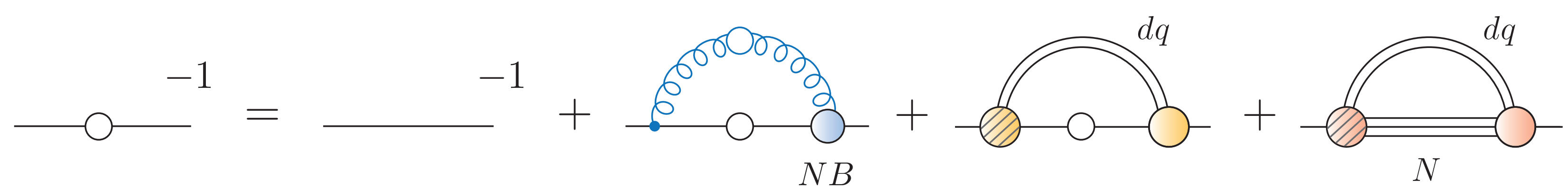


1.Introduction

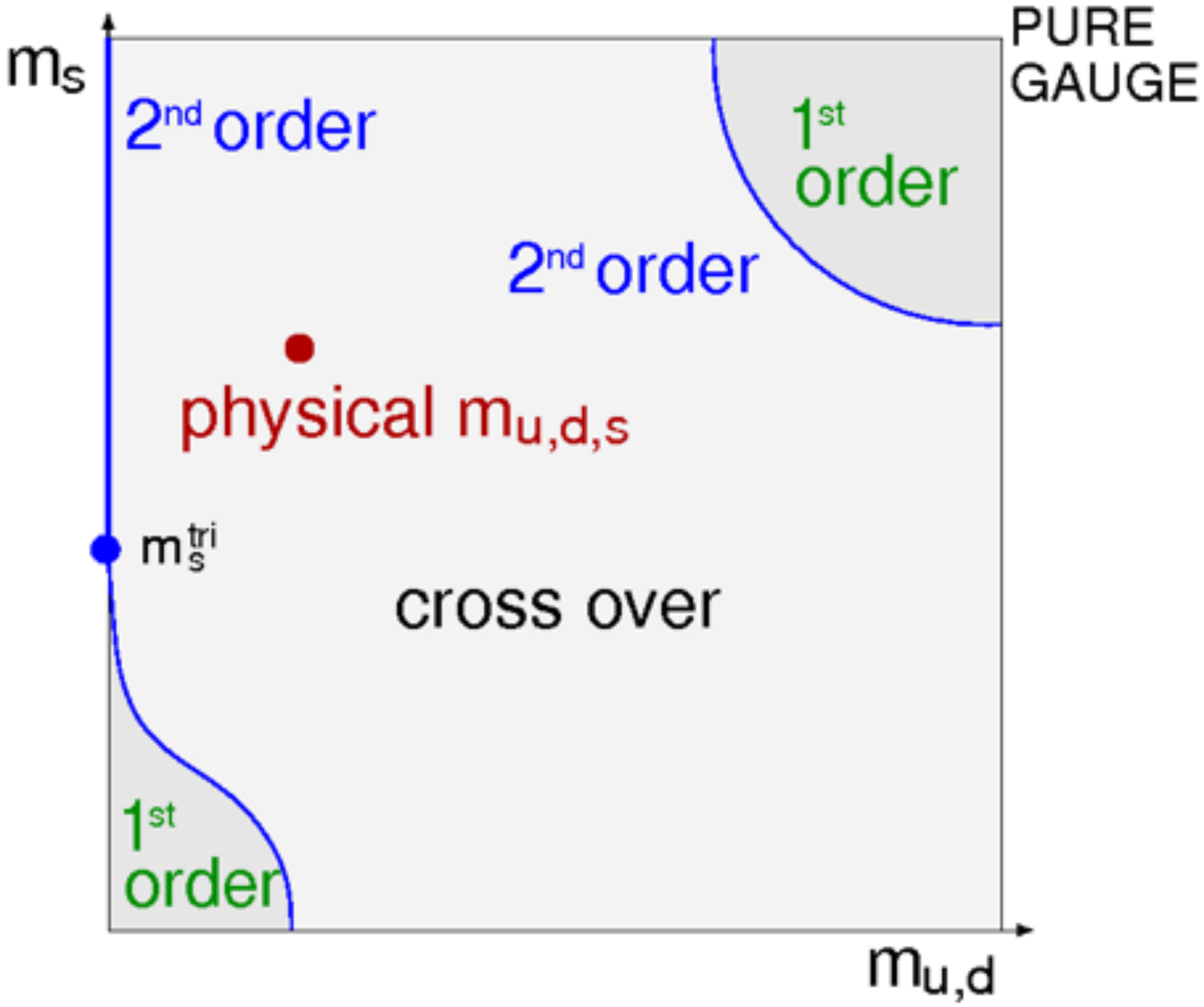


2.Gluons, quarks and the CEP

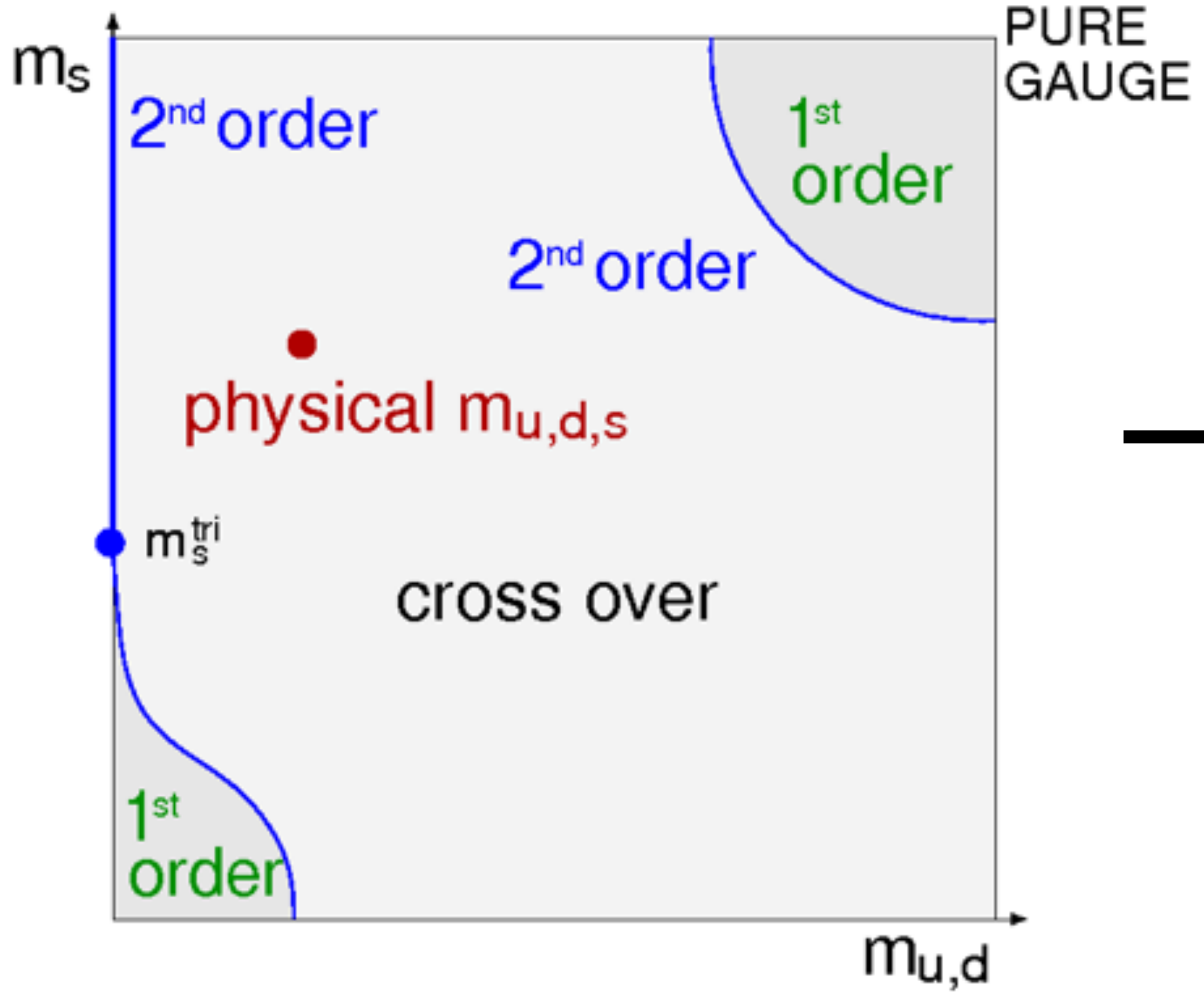
3.Baryon effects on the CEP

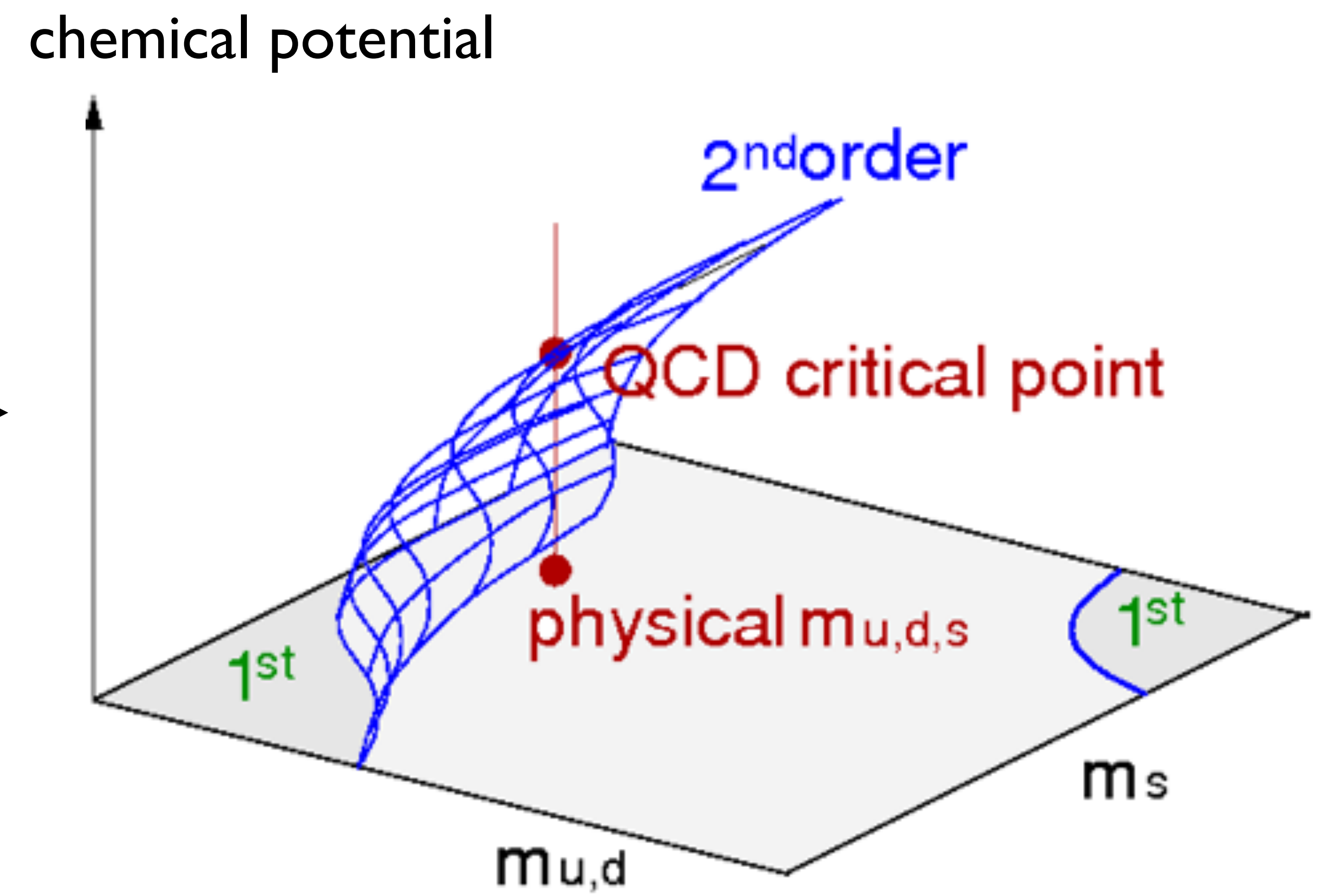


QCD phase transitions

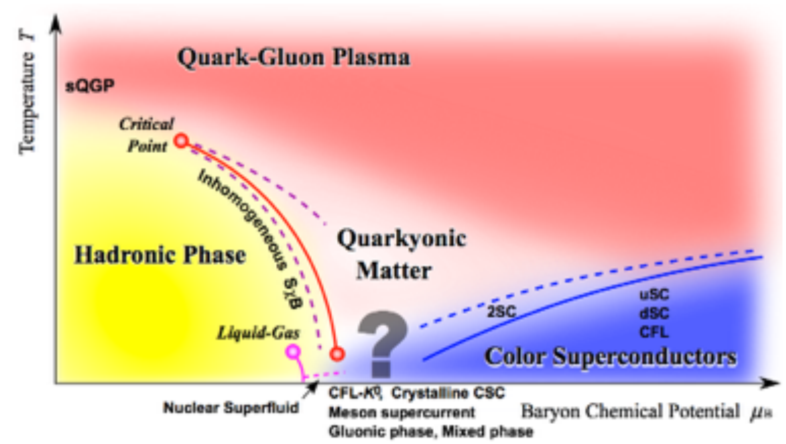


QCD phase transitions

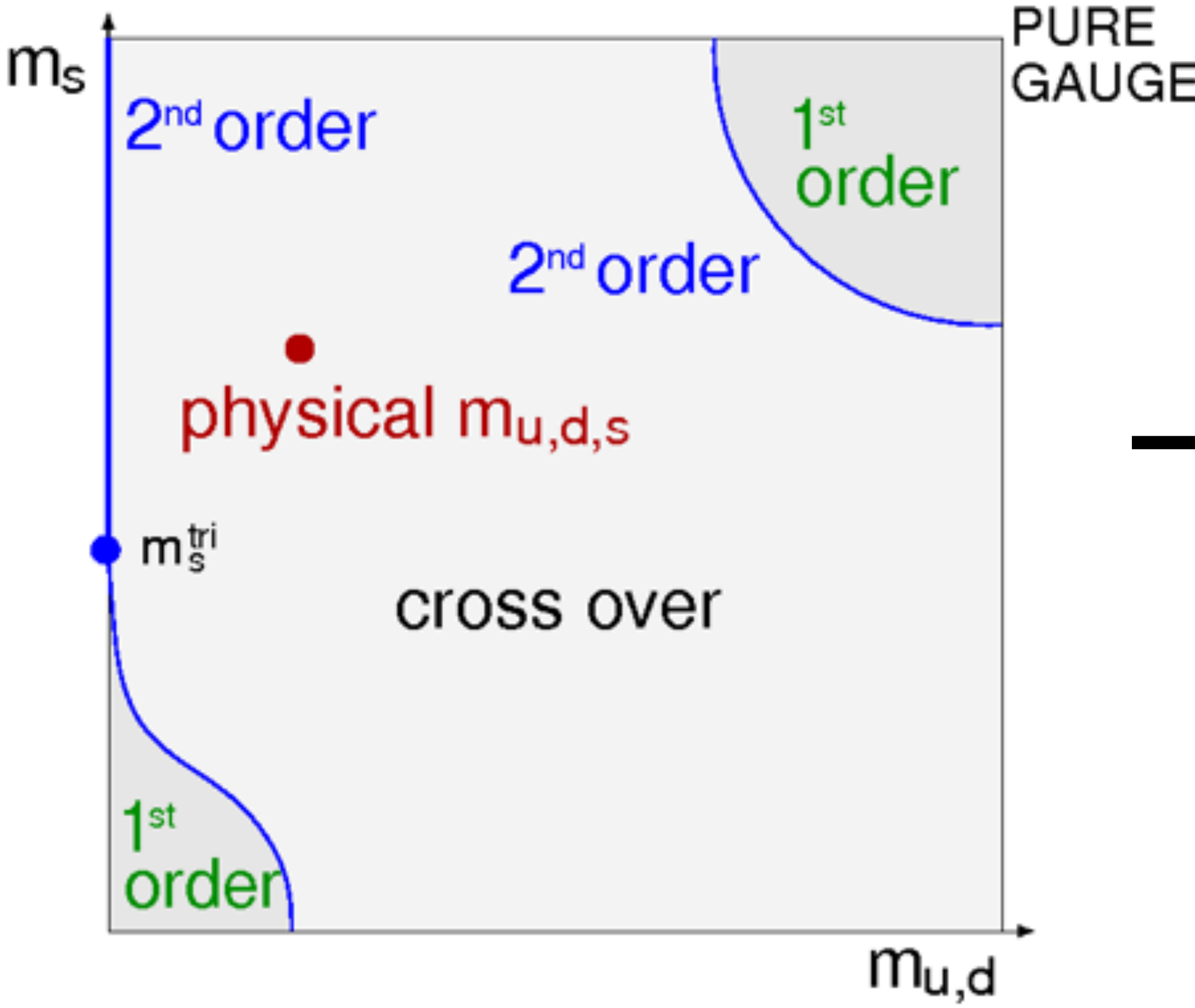


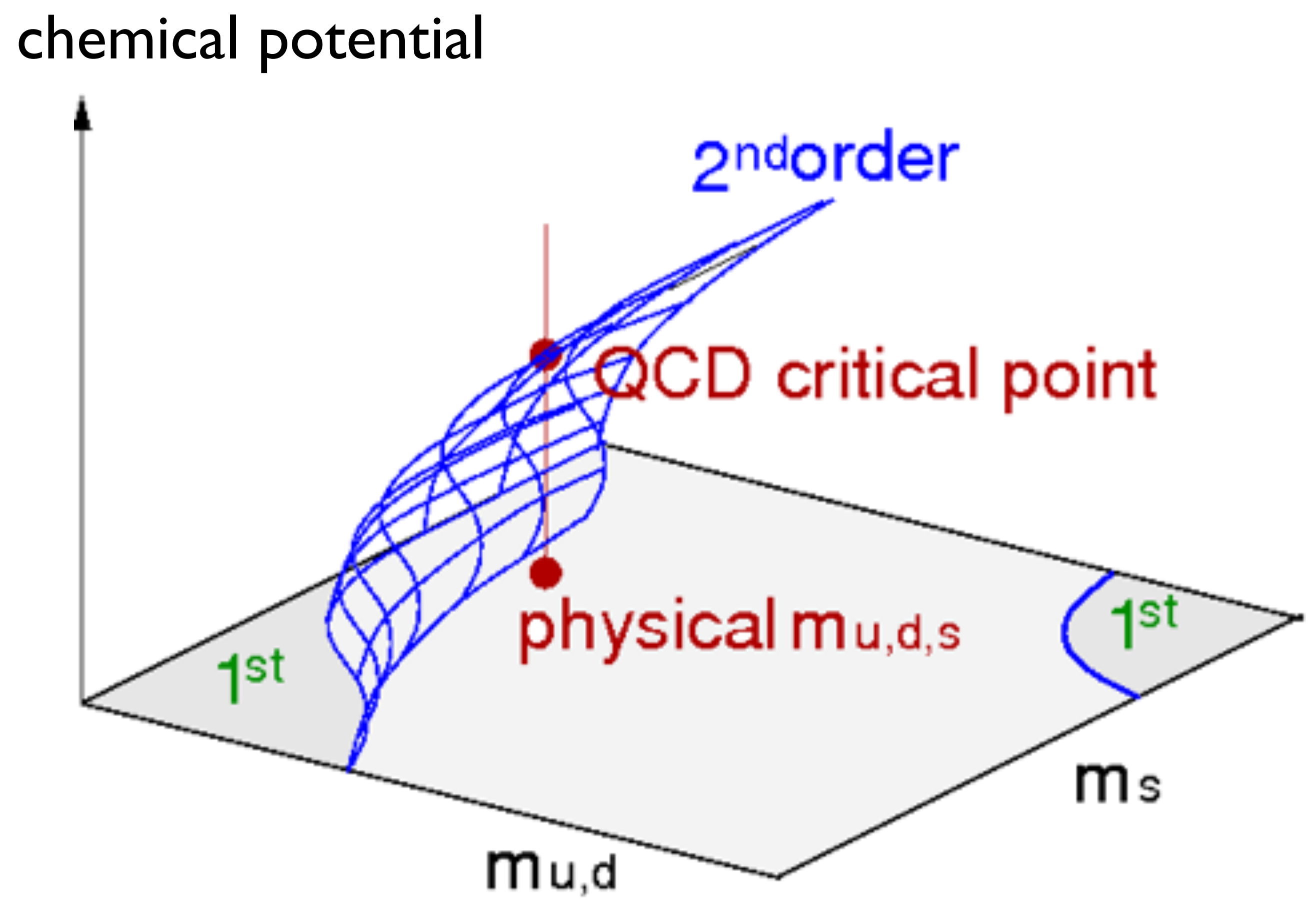


Is this happening ??



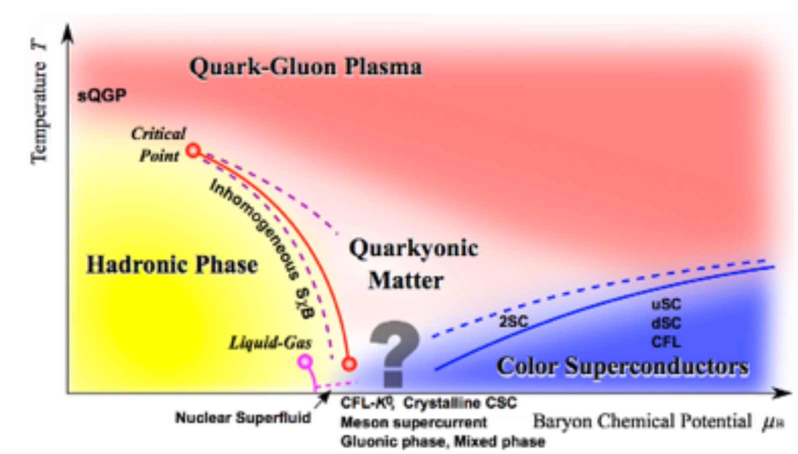
QCD phase transitions





- Lattice-QCD
  - present: extrapolation
  - future: exact methods ?
- DSE/FRG
  - not exact, but allow for '10%-physics'

Is this happening ??



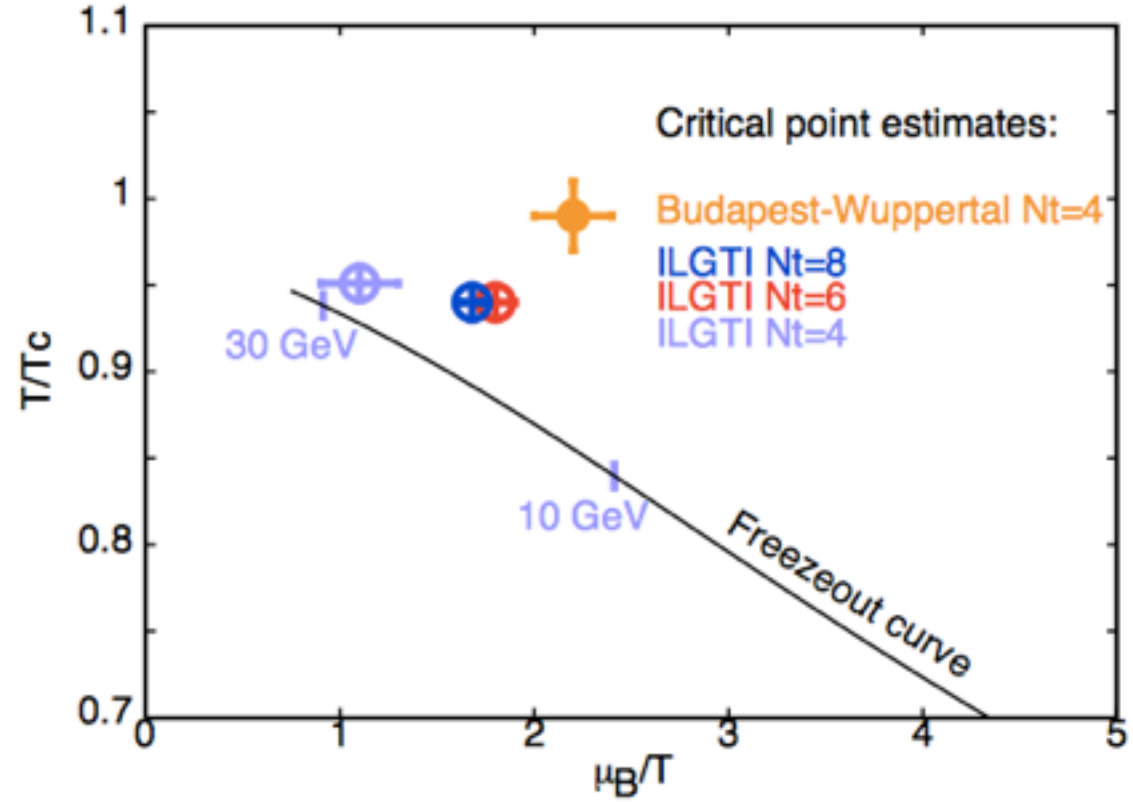
Search for the CEP

- Taylor expansion (N_f=2):

  Datta, Gavai and Gupta, NPA 904-905 (2013) 883c
  Gavai, Gupta, PRD 71 (2005) 114014

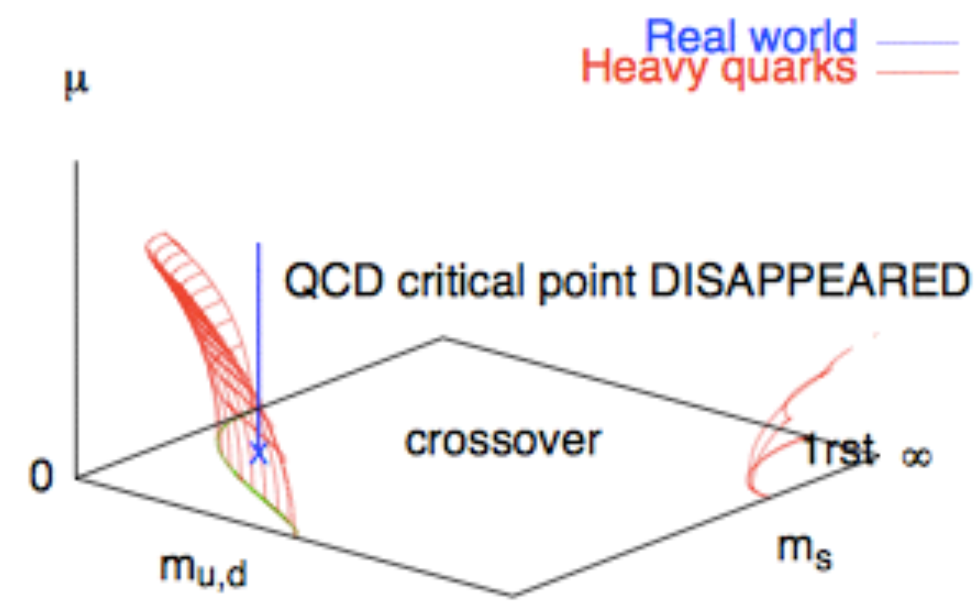
- Reweighting (N_f=2+1):

  Fodor, Katz, JHEP 0404 (2004) 050

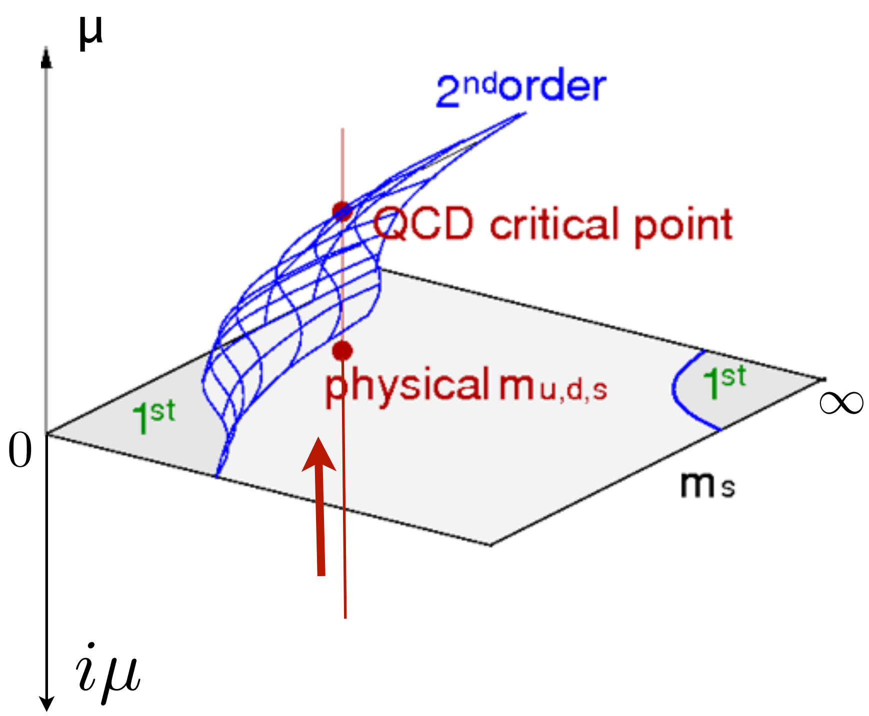


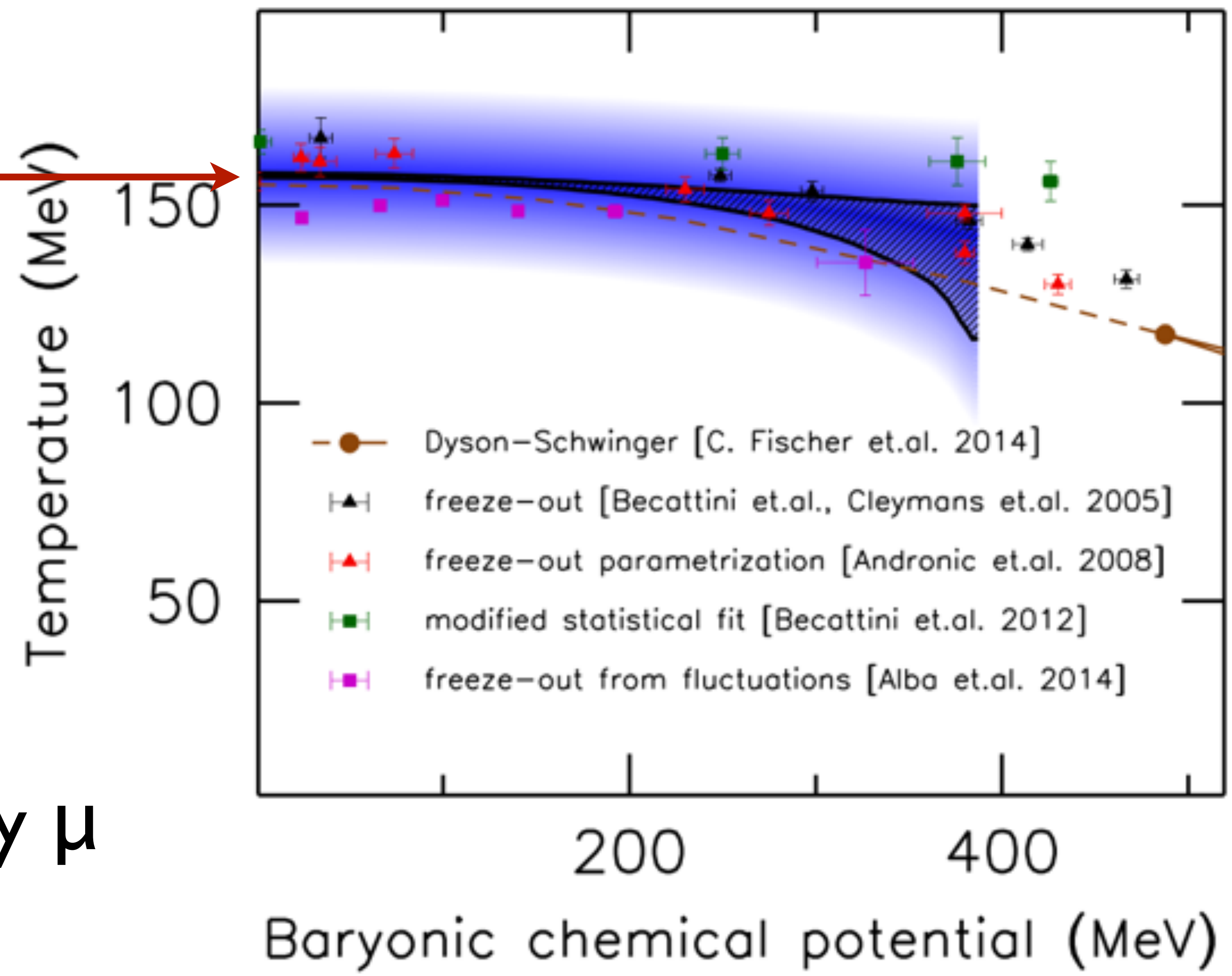
- Analytic continuation (N_f=3):

  de Forcrand, Philipsen, JHEP 0811 (2008) 012;
  NPB 642 (2002) 290



Chiral transition line from analytic continuation





Bellwied, Borsanyi, Fodor, Günther, Katz, Ratti and Szabo, PLB B 751 (2015) 559

Lattice method:

- Calc. boundary at imaginary μ and extrapolate to real μ
- Control systematics

Results:

- Larger curvature than previous results (but: different definitions and error budget)

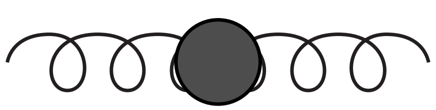
QCD in covariant gauge

Imaginary time formulation:

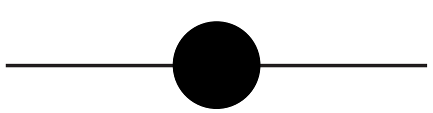
M_{weak}

$$\mathcal{Z}_{QCD} = \int \mathcal{D}[\Psi, A]\; \exp\left\{ -\int_0^{1/T} dt \int d^3x \left(\overline{\Psi}\,(i\not{D} + \gamma_4 \mu - m)\,\Psi - \frac{1}{4}\left(F^a_{\mu\nu}\right)^2 + \text{gauge fixing} \right) \right\}$$

Landau gauge propagators in momentum space, $p = (\vec{p}, \omega_p)$:

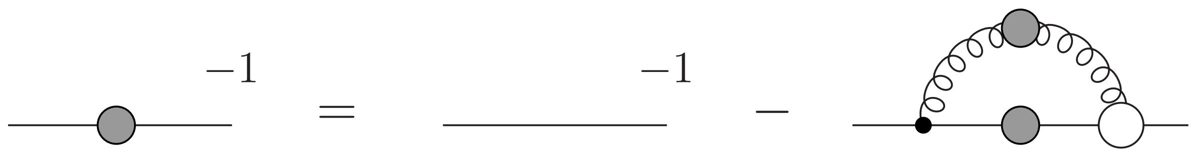


$$D^{\text{Gluon}}_{\mu\nu} = \frac{Z_T(p)}{p^2} P^T_{\mu\nu}(p) + \frac{Z_L(p)}{p^2} P^L_{\mu\nu}(p)$$



$$S^{\text{Quark}}(p) = \left[i\,\vec{\gamma}\vec{p}\, A(p) + i\,\gamma_4 \tilde{\omega}_n\, C(p) + B(p) \right]^{-1}$$

The Goal: gauge invariant information in a gauge fixed approach.



Chiral order parameter:

$$\langle \bar{\Psi}\Psi \rangle = Z_2 N_c Tr_D \frac{1}{T} \sum_\omega \int \frac{d^3p}{(2\pi)^3} S(\vec{p},\omega)$$

Deconfinement:

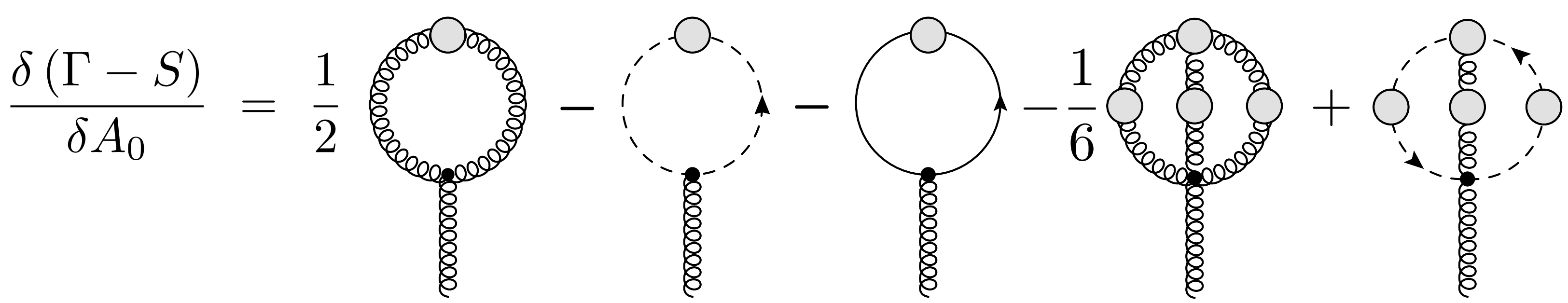
- dressed Polyakov loop

$$\Sigma = -\int_0^{2\pi} \frac{d\varphi}{2\pi} e^{-i\varphi} \langle \bar{\Psi}\Psi \rangle_\varphi$$

Synatschke, Wipf, Wozar, PRD 75, 114003 (2007)
Bilgici, Bruckmann, Gattringer, Hagen, PRD 77 094007 (2008)
CF, PRL 103 052003 (2009)

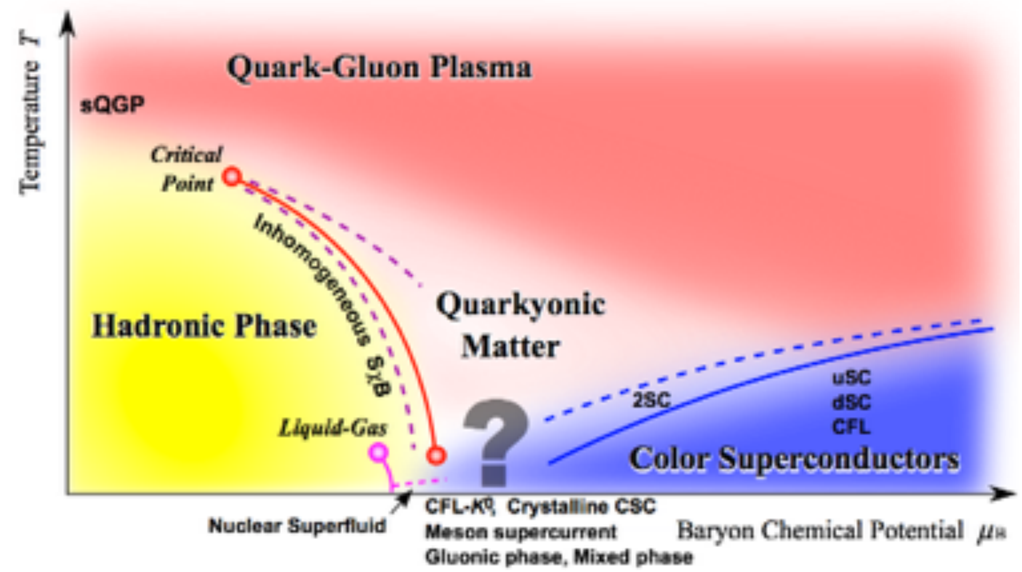
- Polyakov loop potential

$$L = \frac{1}{N_c} Tr\, e^{ig\beta A_0}$$



Braun, Gies, Pawlowski, PLB 684, 262 (2010)
Braun, Haas, Marhauser, Pawlowski, PRL 106 (2011)
Fister, Pawlowski, PRD 88 045010 (2013)
CF, Fister, Luecker, Pawlowski, PLB 732 (2014) 273

1. Introduction



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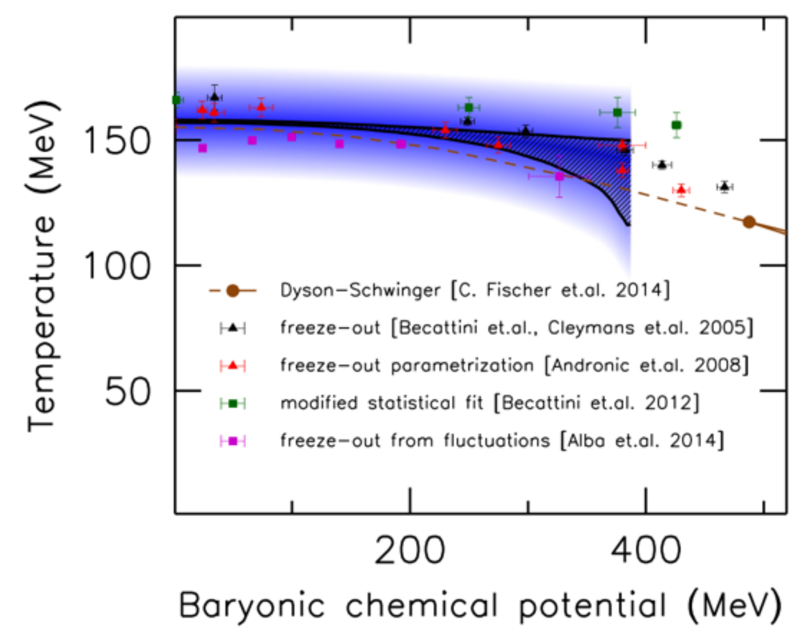

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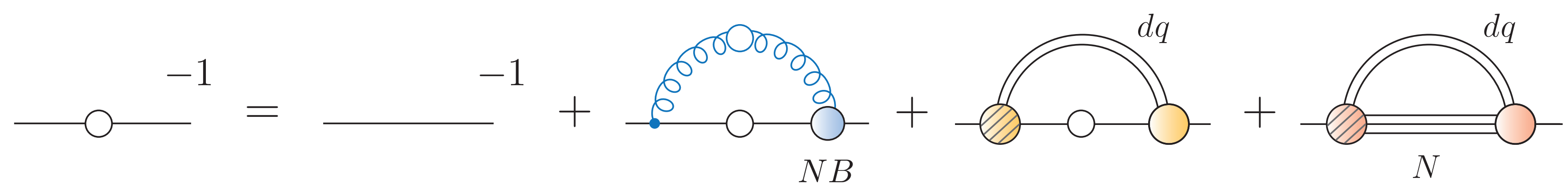
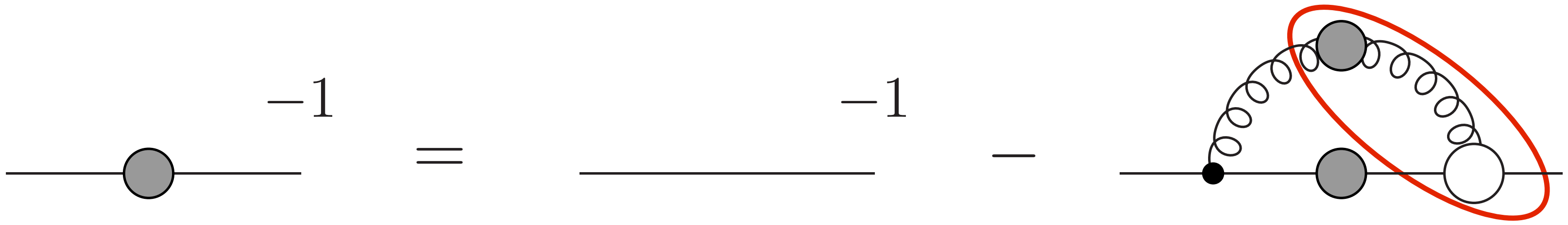


The DSE for the quark propagator



$$[S(p)]^{-1} = [-i\not{p} + M(p^2)]/Z_f(p^2)$$

Input:
- dressed Gluon propagator
- dressed Quark-Gluon-Vertex

Two strategies: I. use model for gluon and vertex

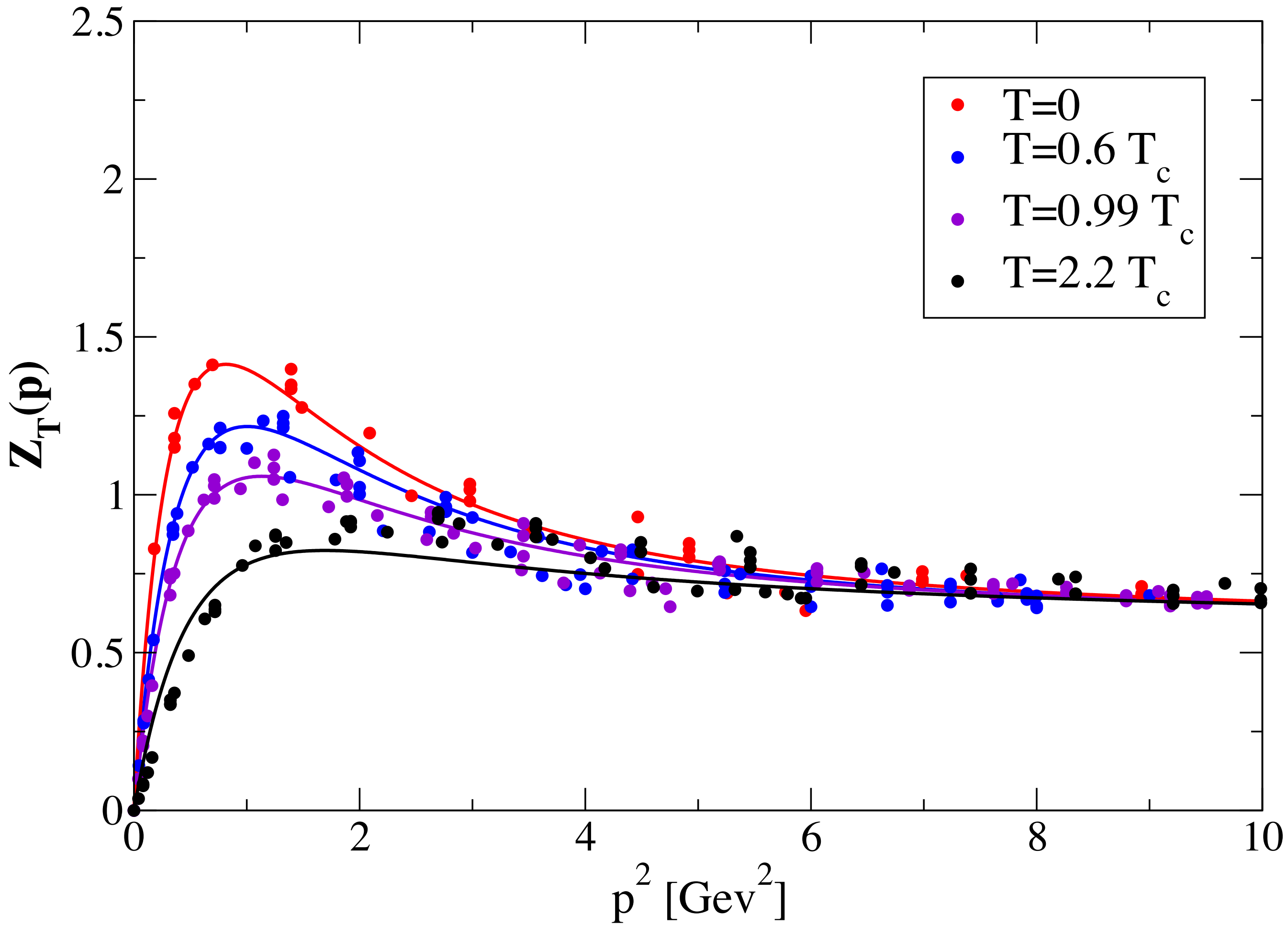
Qin, Chang, Chen, Liu and Roberts, PRL 106 (2011) 172301

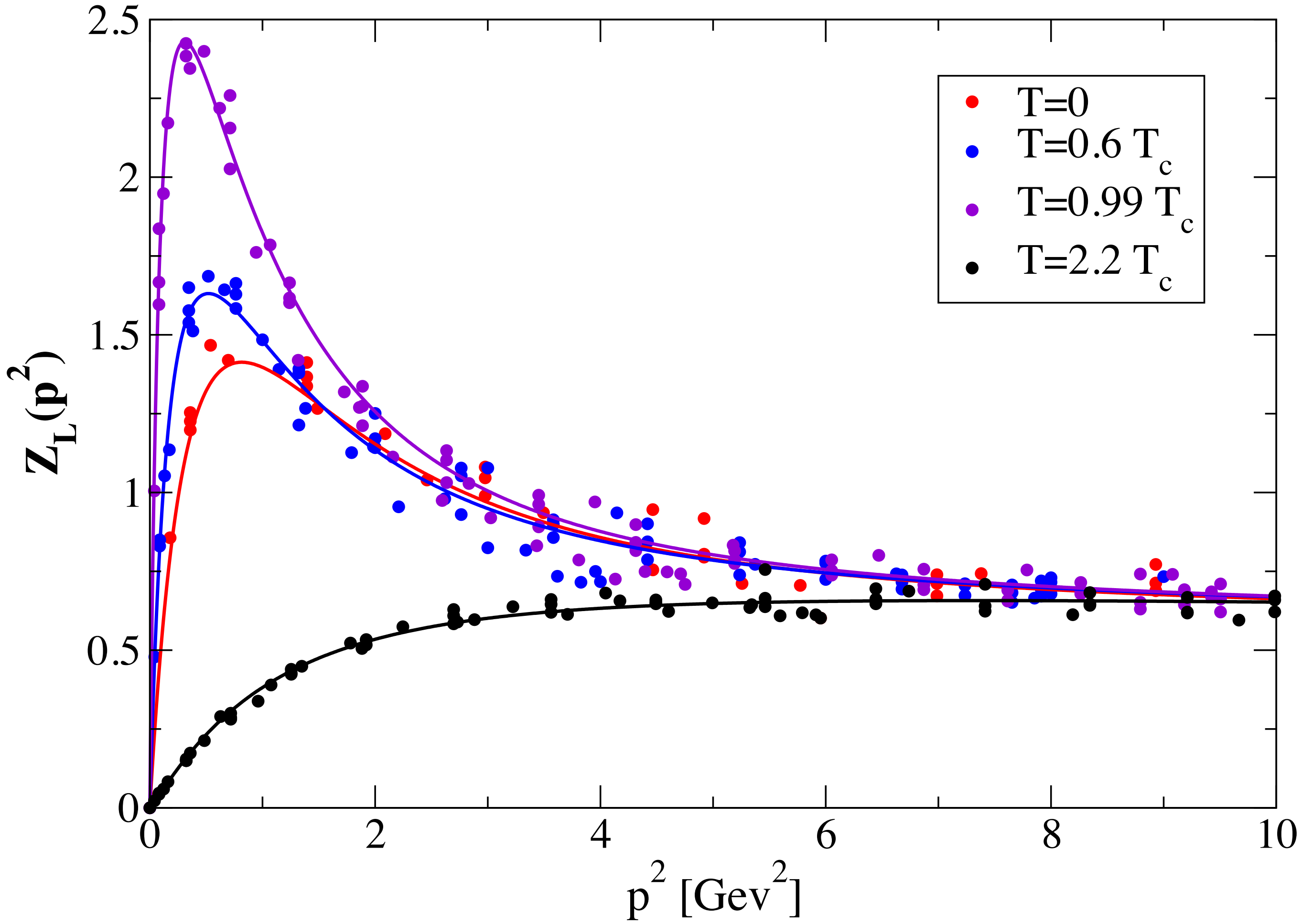
→ ok for first insights
→ not good enough for systematic study

II. determine gluon and vertex explicitly

Glue at finite temperature (T≠0)

T-dependent gluon propagator from quenched lattice simulations:



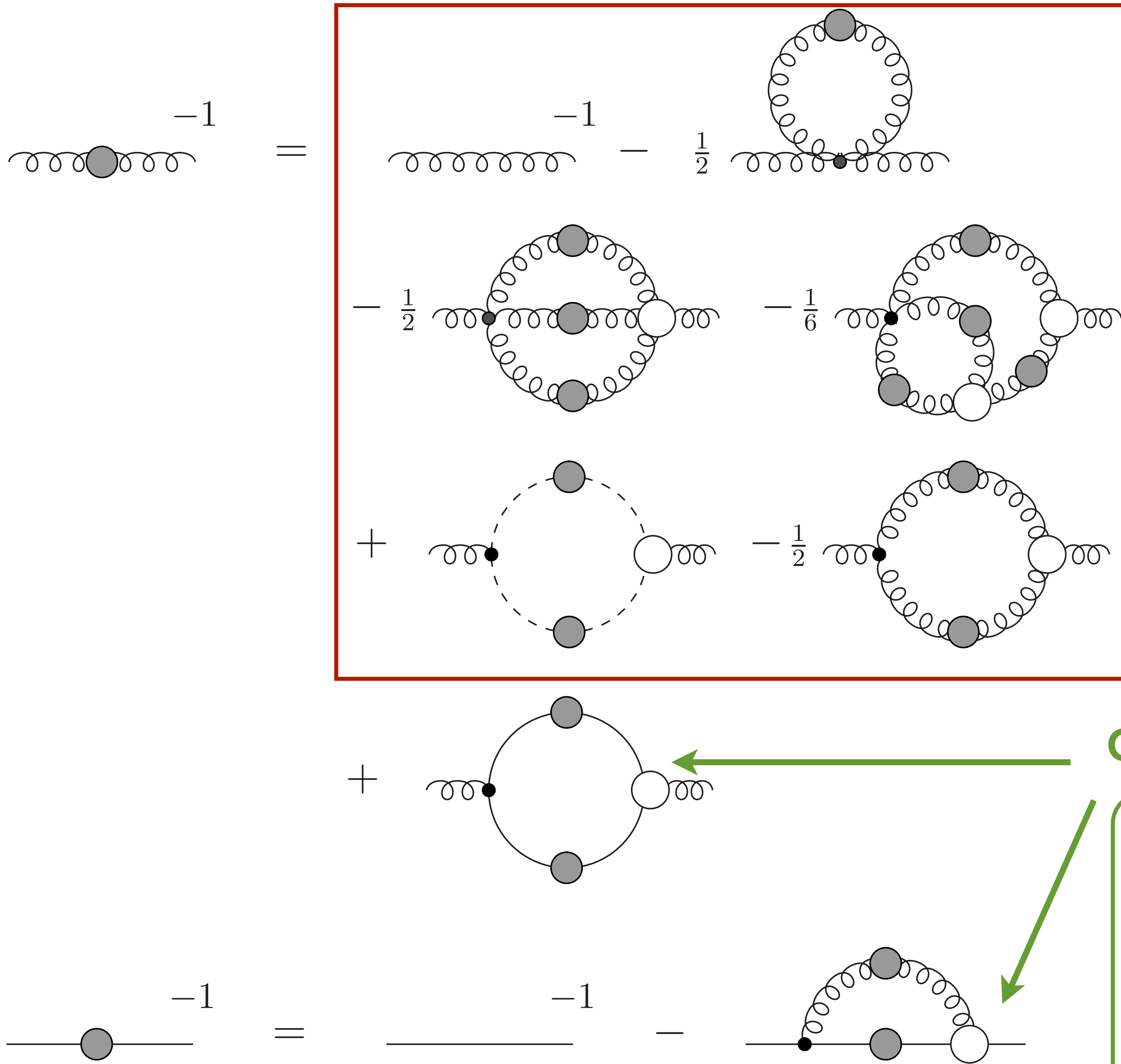


- Nonperturbative gluon is massive
- Crucial difference between magnetic and electric gluon
- Maximum of electric gluon near Tc

Cucchieri, Maas, Mendes, PRD 75 (2007)
CF, Maas, Mueller, EPJC 68 (2010)
Maas, Pawlowski, von Smekal and Spielmann, PRD 85 (2012) 034037
Aouane, Bornyakov, Ilgenfritz, Mitrjushkin, Muller-Preussker and Sternbeck, PRD 85 (2012) 034501

FRG: Fister, Pawlowski, arXiv:1112.5440

DSEs of QCD



quenched, T-dependent lattice propagator

quark gluon vertex

- under study at T=0

  Skullerud, Kizilersu, JHEP 0209 (2002) 013
  CF, Williams PRL 103 (2009) 122001
  Mitter, Pawlowski and Strodthoff, PRD 91 (2015) 054035
  Williams, Fischer, Heupel, PRD in press, arXiv:1512.00455
  Sternbeck et al. in preparation

- T≠0: ansatz, T,m,μ dependent

Approximation for Quark-Gluon interaction

- T,μ,m-dependent vertex:

Abelian WTI

$$\Gamma_\nu(q,k,p) = \widetilde{Z}_3 \left(\delta_{4\nu}\gamma_4 \frac{C(k)+C(p)}{2} + \delta_{j\nu}\gamma_j \frac{A(k)+A(p)}{2} \right) \times$$
$$\times \left(\frac{d_1}{d_2+q^2} + \frac{q^2}{\Lambda^2+q^2} \left(\frac{\beta_0 \alpha(\mu) \ln[q^2/\Lambda^2+1]}{4\pi} \right)^{2\delta} \right)$$

perturbation theory

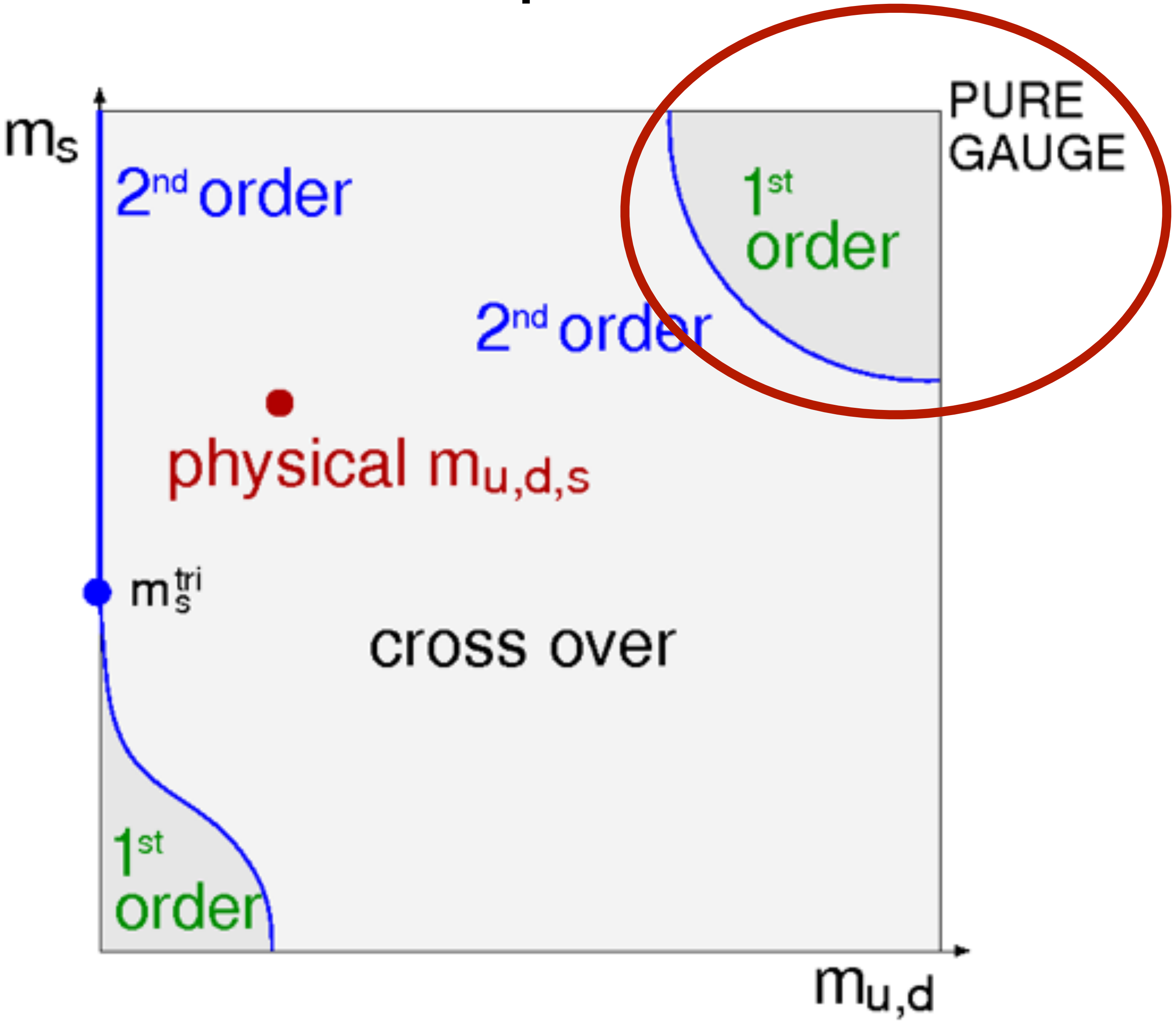
Infrared ansatz:

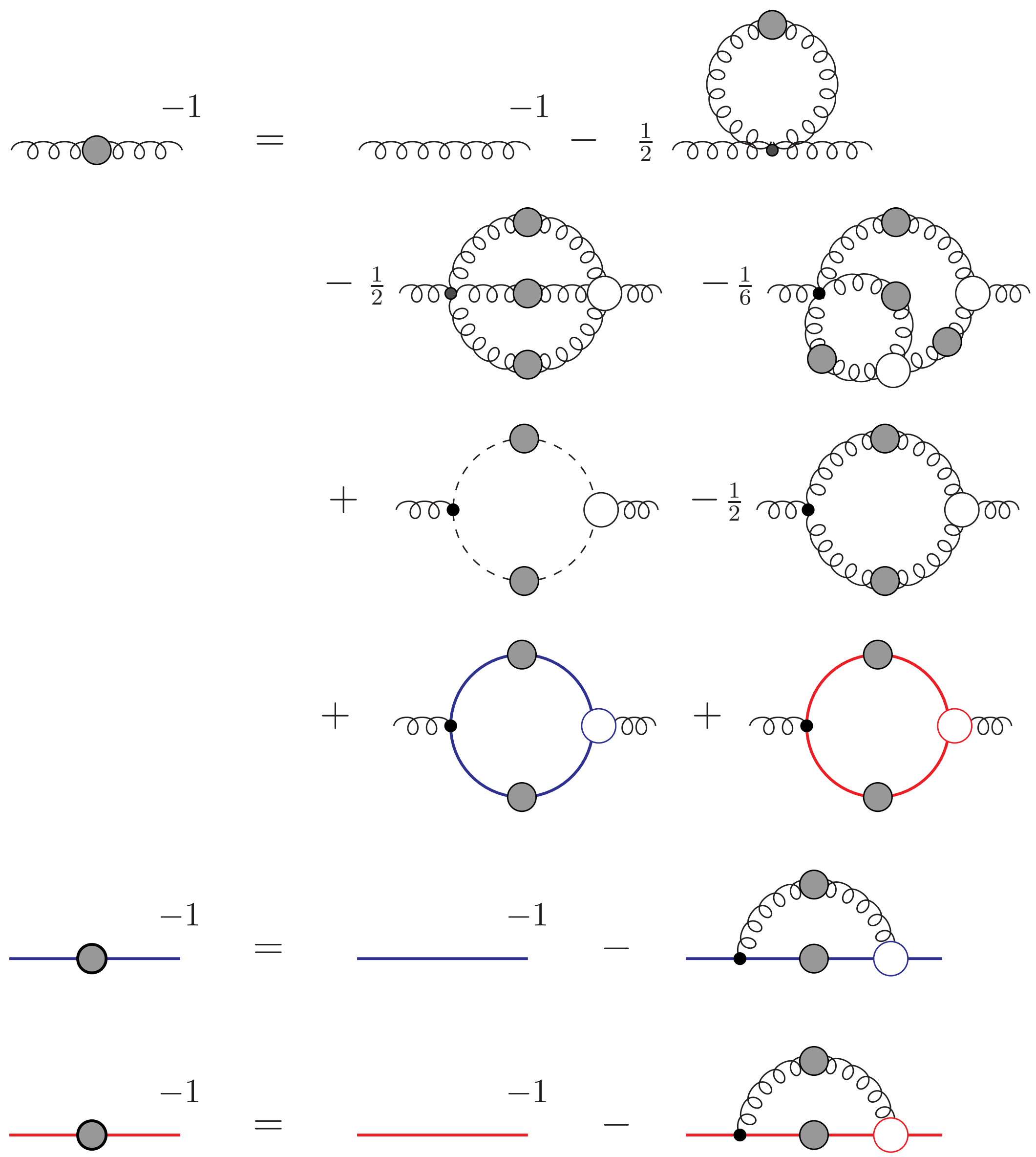
- d2 fixed to match gluon input
- d1 fixed via quark condensate (see later)
- correct UV (quant.) and IR-behavior (qual.)

CF, Pawlowski, PRD 80 (2009) 025023
Mitter, Pawlowski and Strodthoff, PRD 91 (2015) 054035
Williams, Fischer, Heupel, PRD in press, arXiv:1512.00455

QCD phase transition: heavy quark limit/quenched

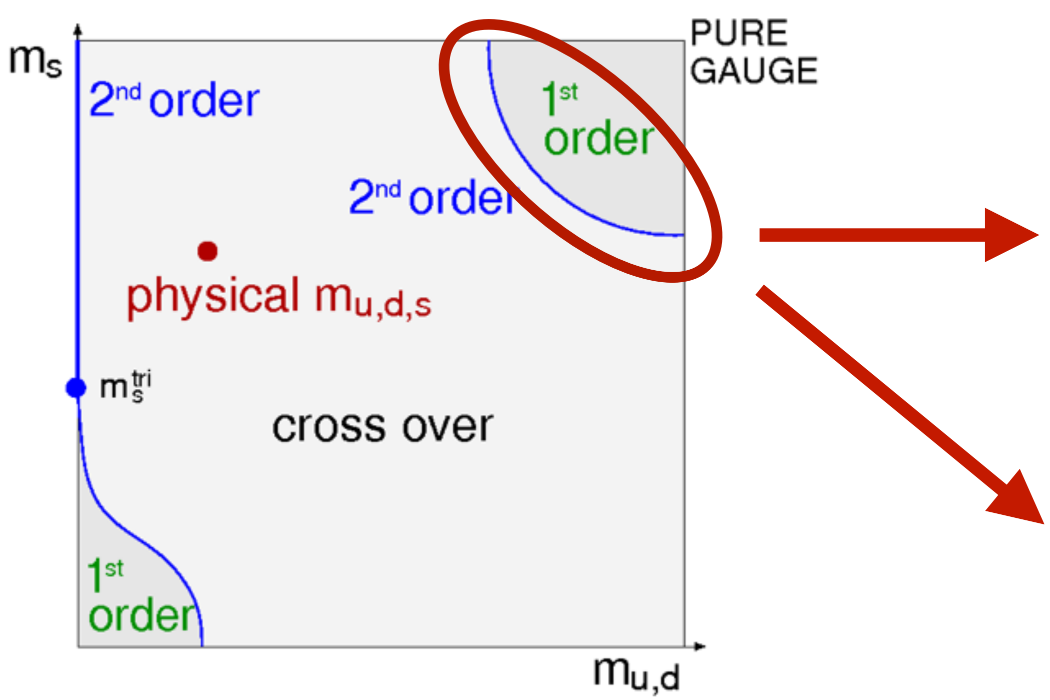
Quark mass dependence:



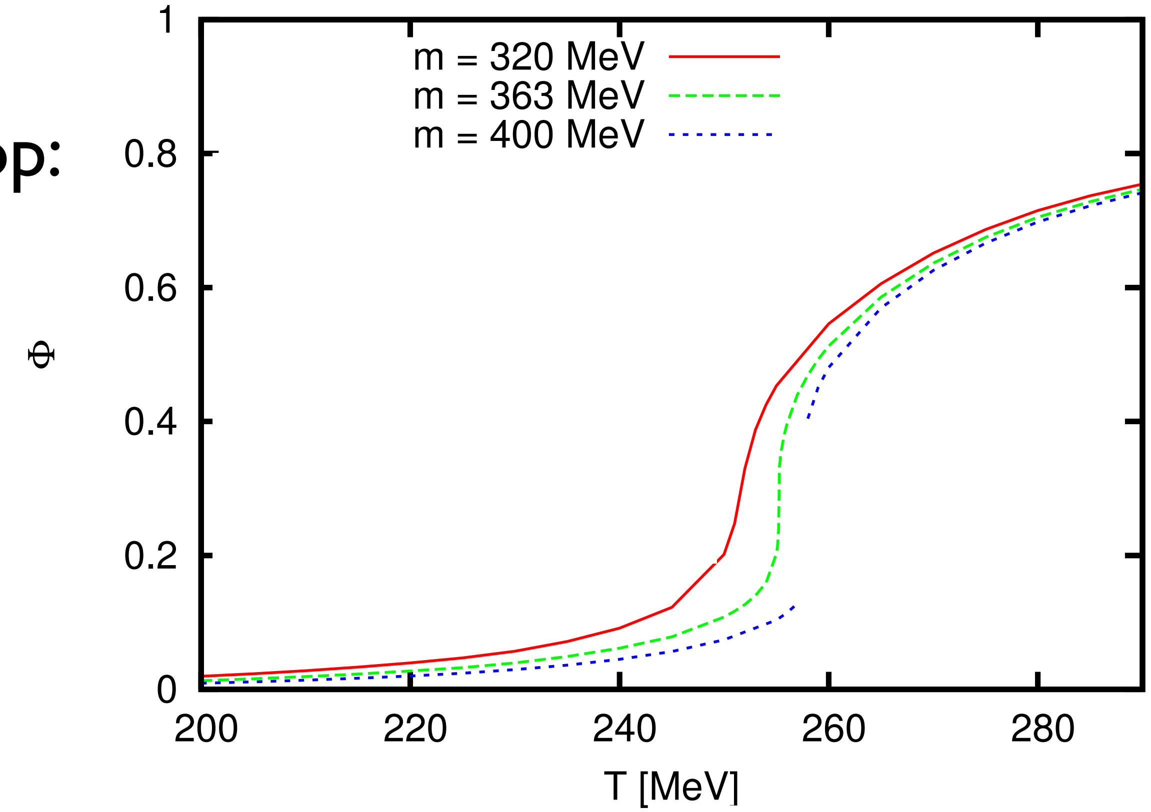


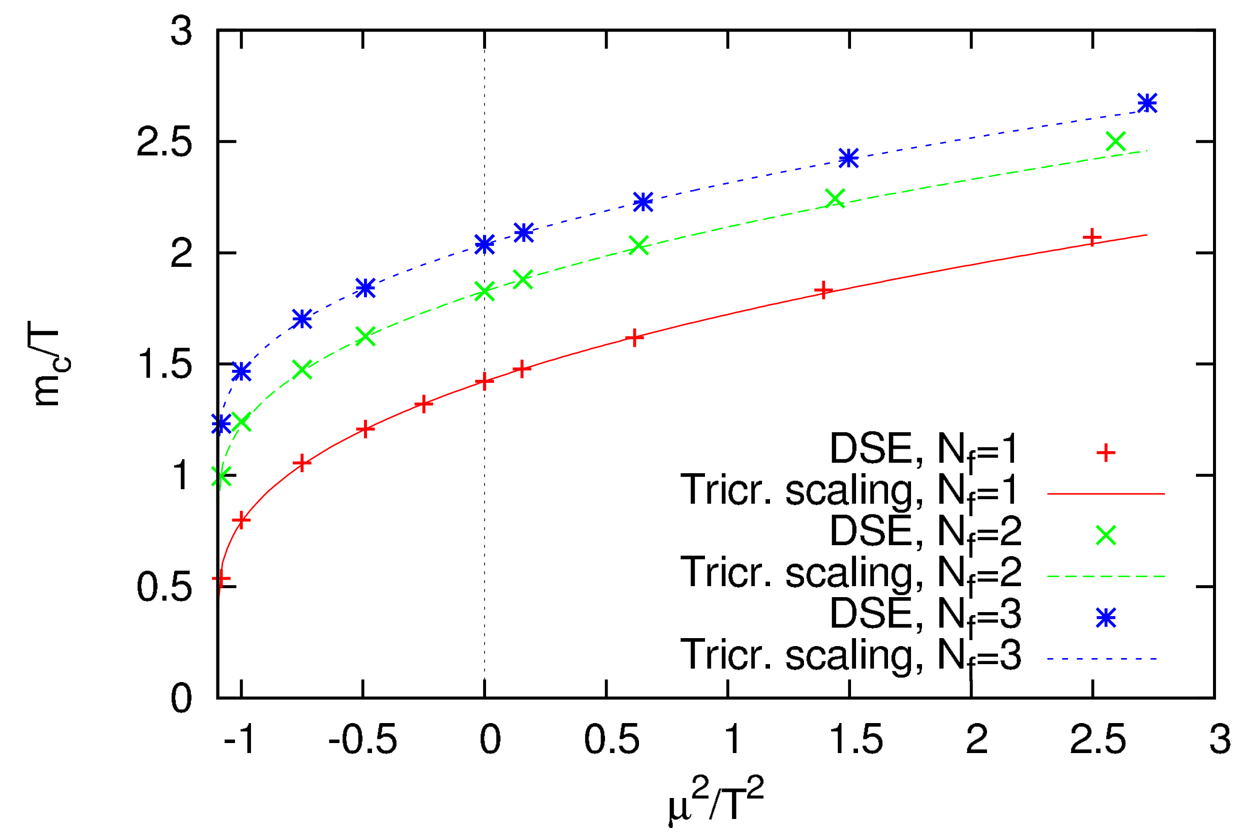
- Expect: Transitions controlled by deconfinement
- SU(2) second order, SU(3) first order

Critical line/surface for heavy quarks



Polyakov Loop:



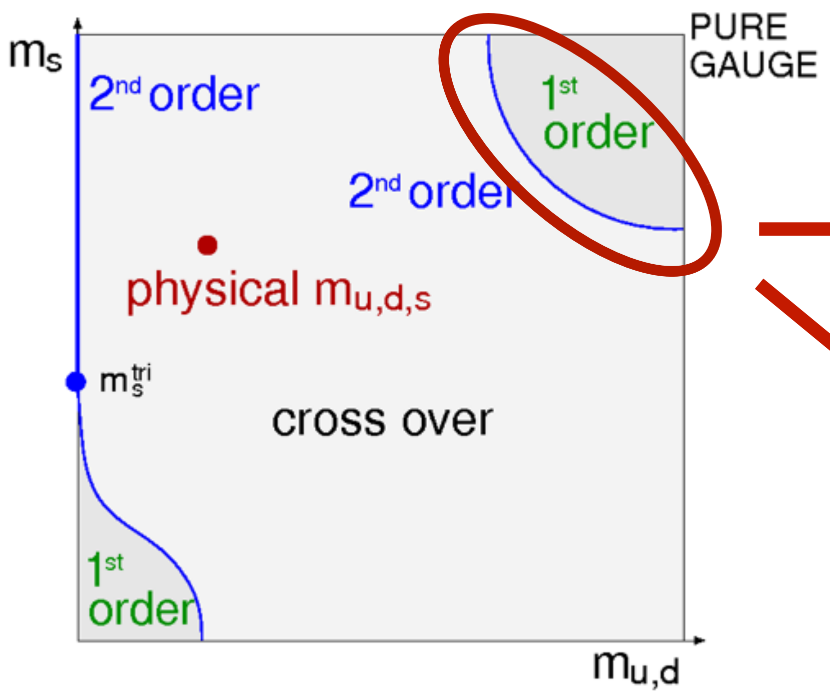


- Deconfinement transition in agreement with lattice QCD
- Correct tricritical scaling

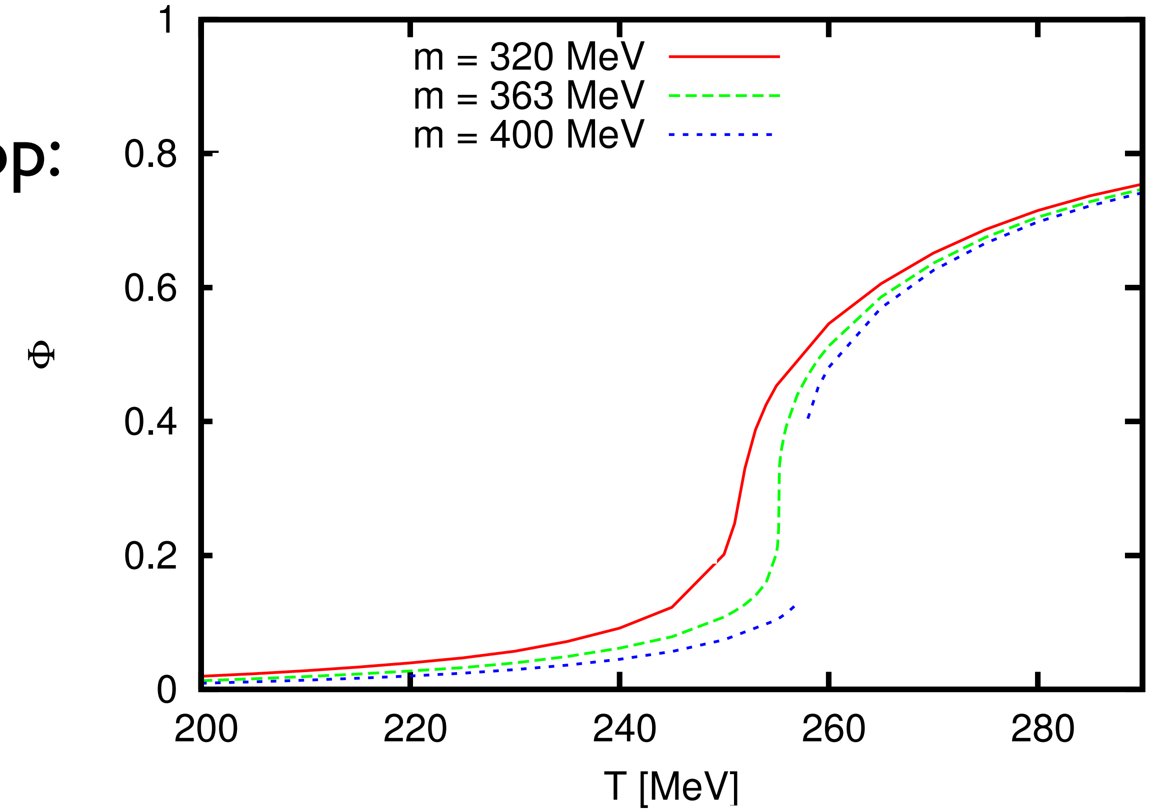
Fromm, Langelage, Lottini, Philipsen, JHEP 1201 (2012) 042

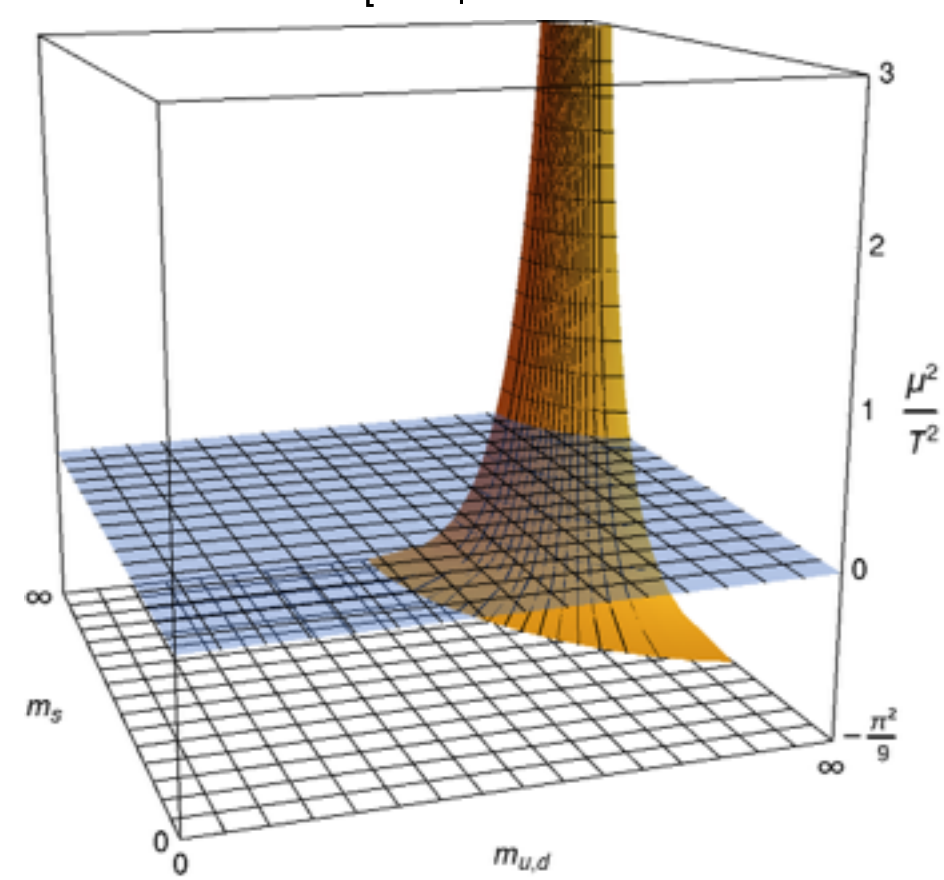
CF, Luecker, Pawlowski, PRD 91 (2015) 1

Critical line/surface for heavy quarks



Polyakov Loop:





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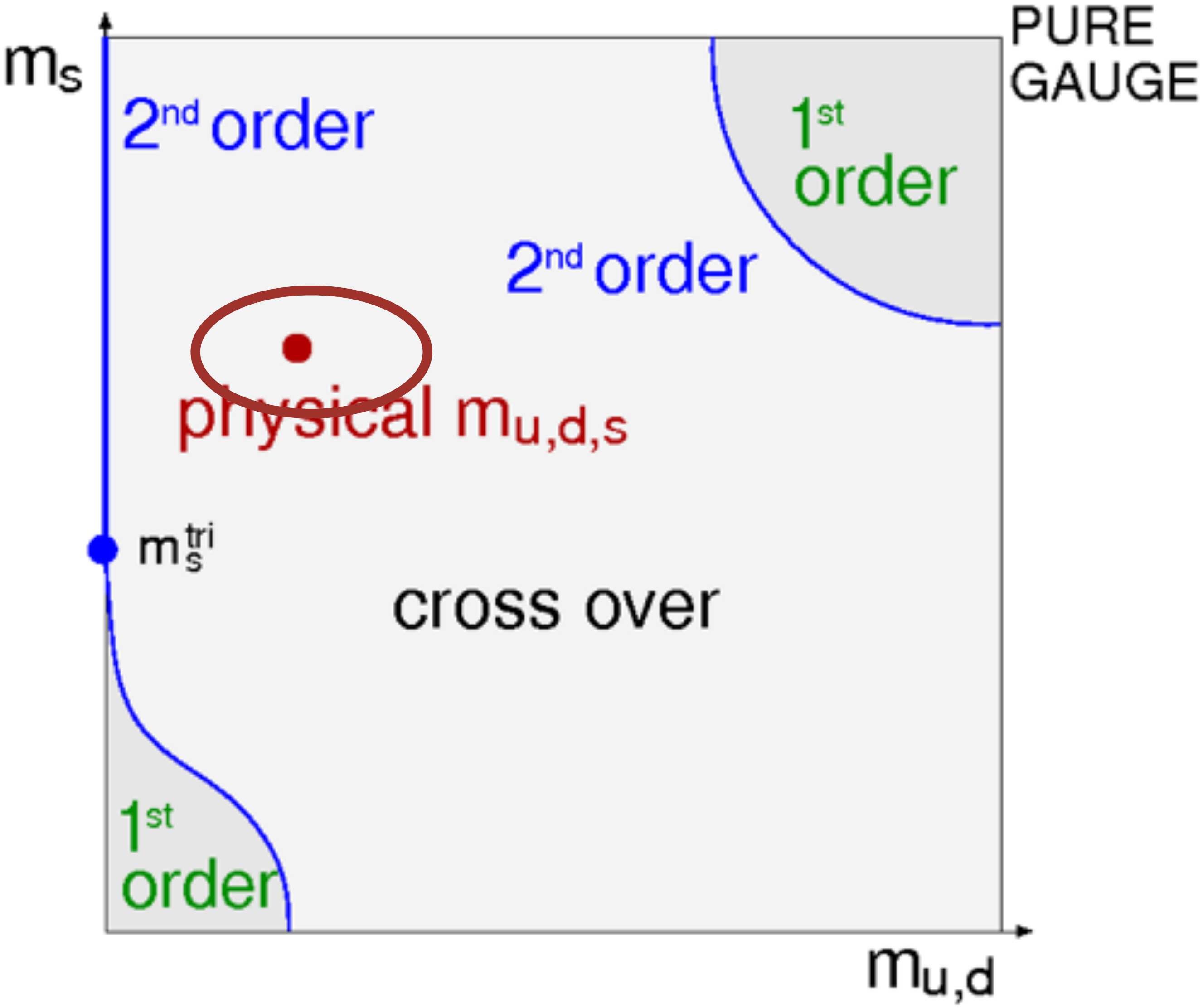
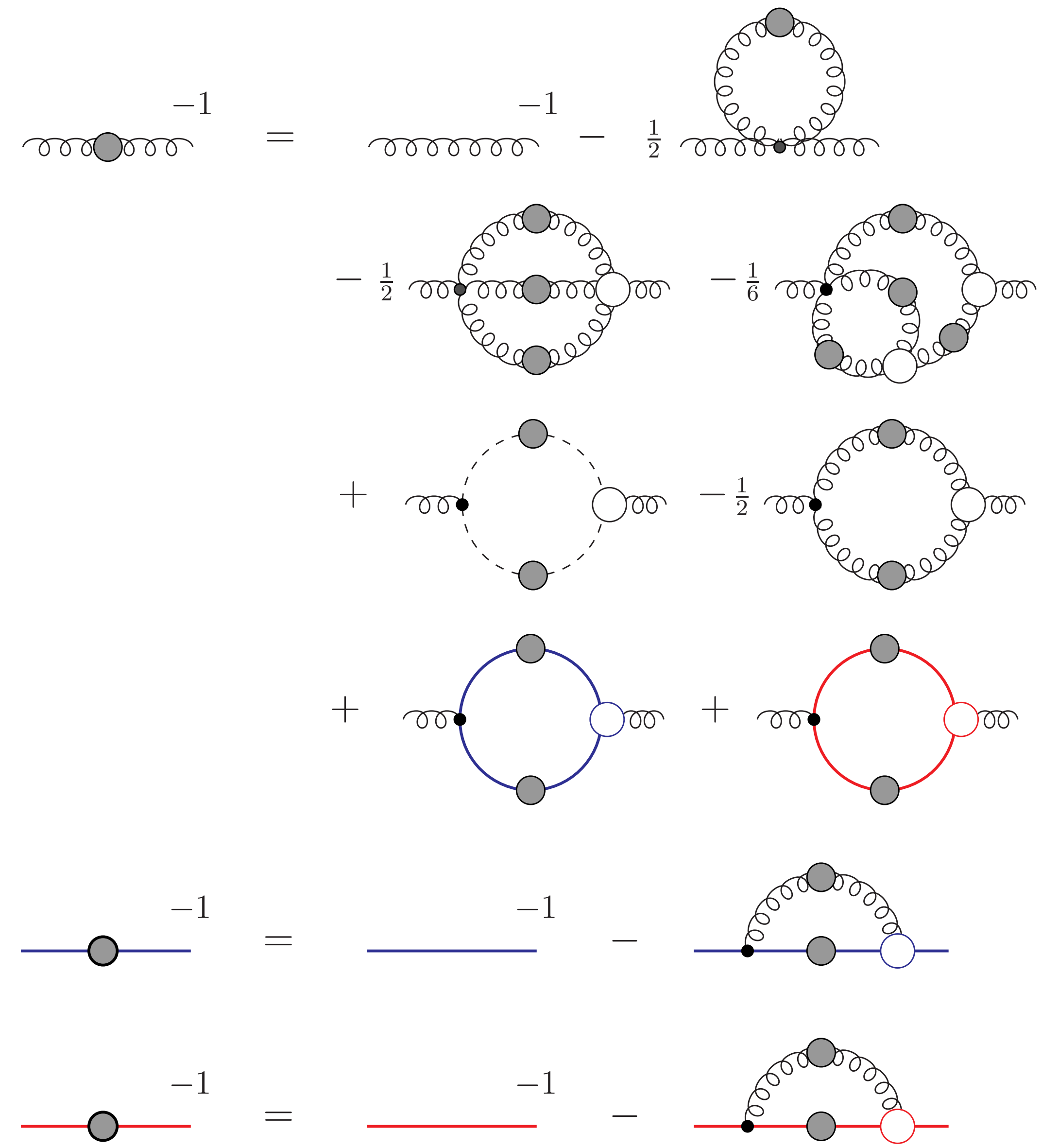
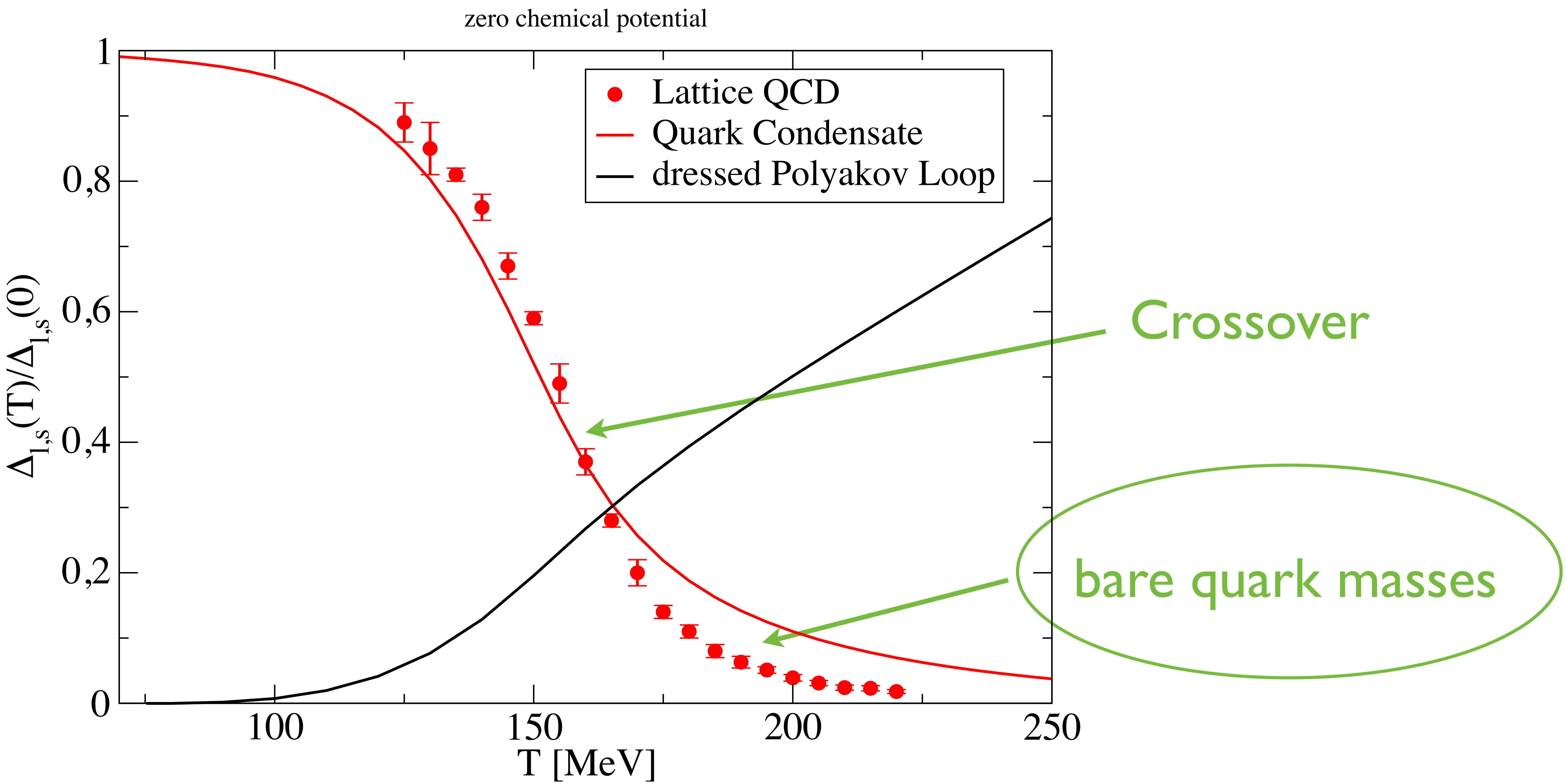
QCD phase transitions: N_f=2+1





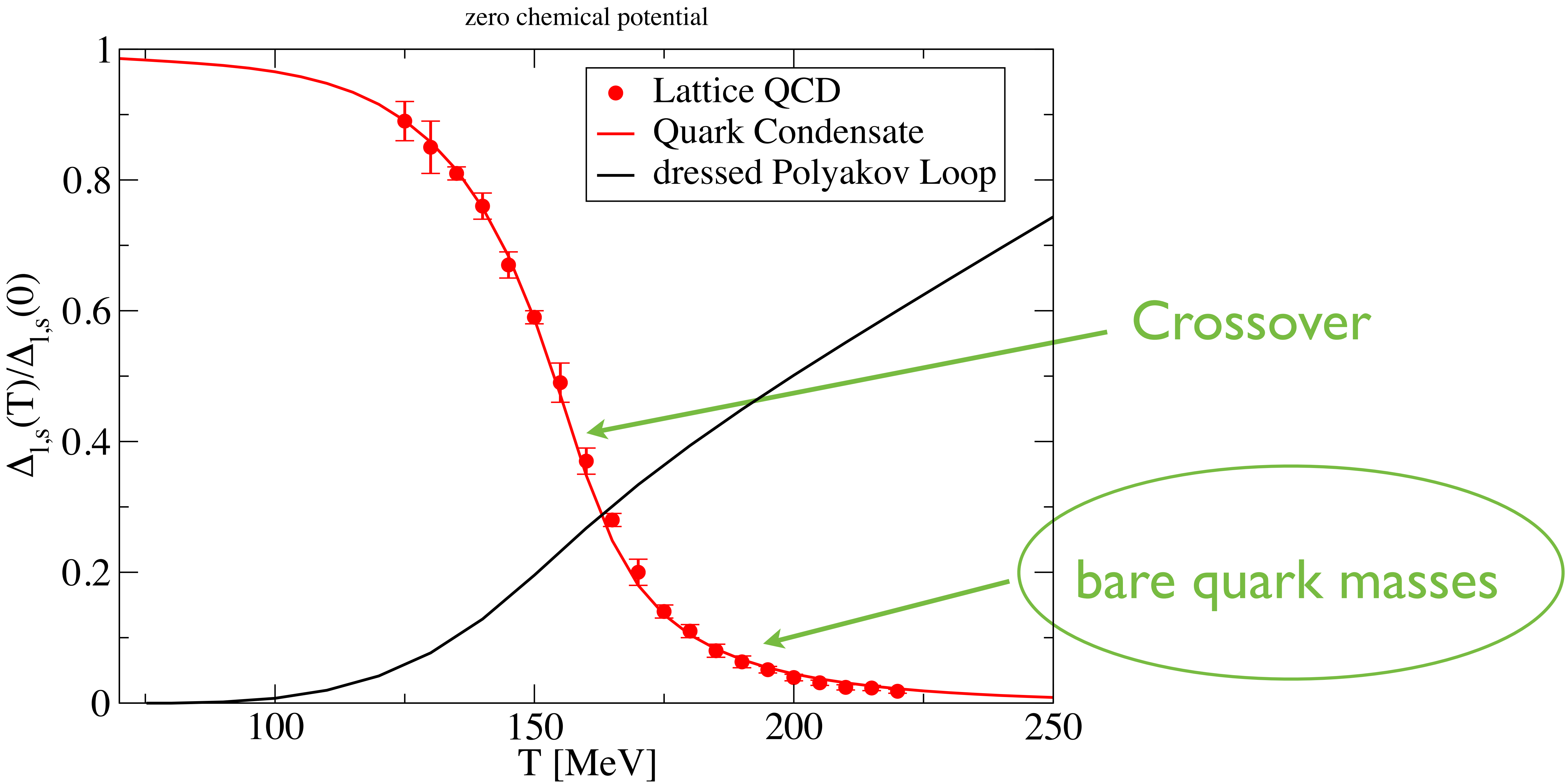
- Physical up/down and strange quark masses
- Transition controlled by chiral dynamics
- at μ=0: compare to available lattice results

N_f=2+1, zero chemical potential



Lattice: Borsanyi *et al.* [Wuppertal-Budapest Collaboration], JHEP 1009(2010) 073
DSE: CF, Luecker, PLB 718 (2013) 1036, CF, Luecker, Welzbacher, PRD 90 (2014) 034022

N_f=2+1, zero chemical potential



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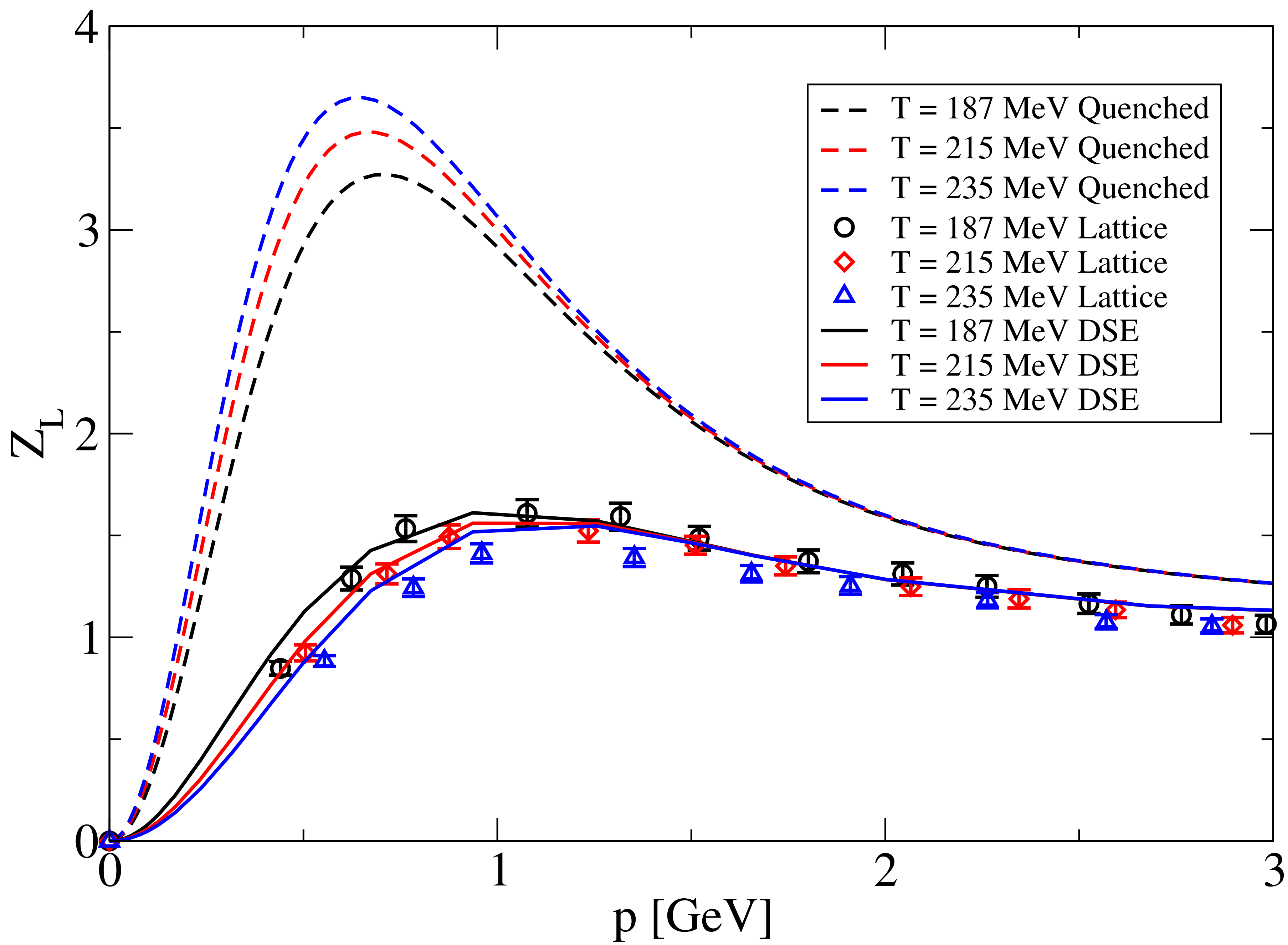
- quantitative agreement

Unquenched Gluon DSE vs Lattice

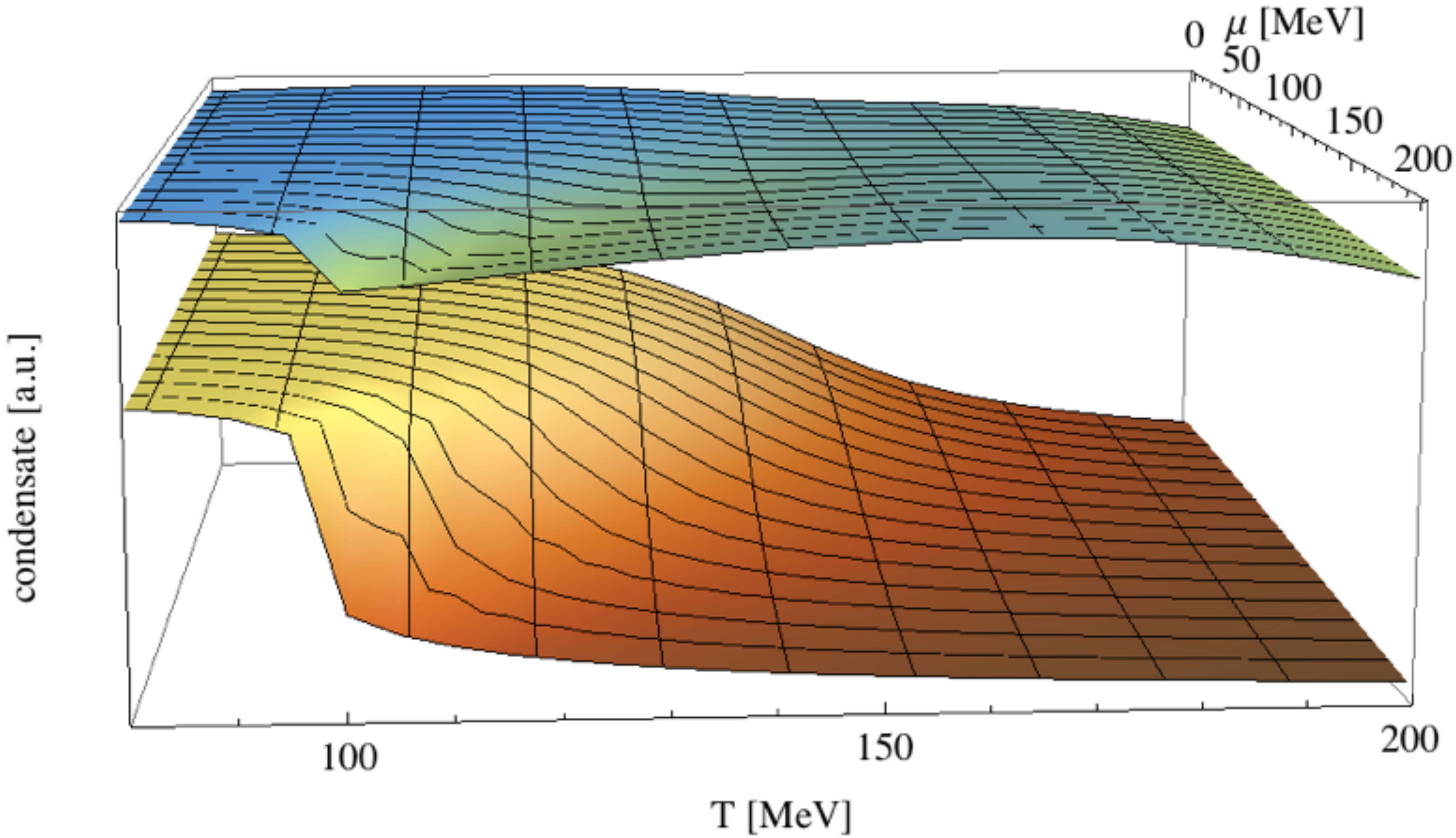


- quantitative agreement: DSE prediction verified by lattice

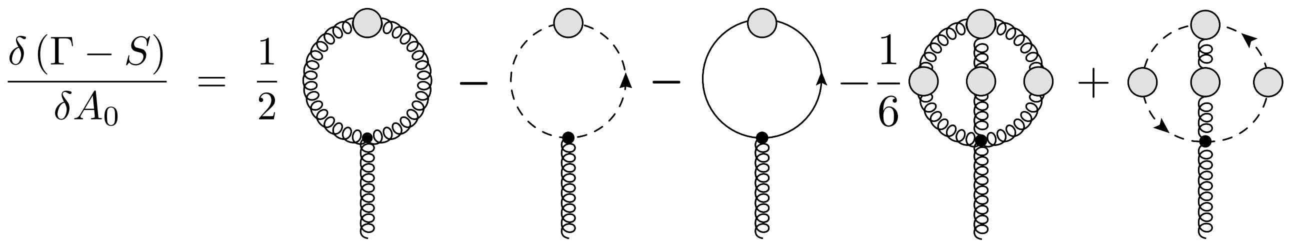
DSE: CF, Luecker, PLB 718 (2013) 1036 [arXiv:1206.5191]

Lattice: Aouane, Burger, Ilgenfritz, Muller-Preussker and Sternbeck, PRD 87 (2013) 11 [arXiv:1212.1102]

Nf=2+1: Condensate

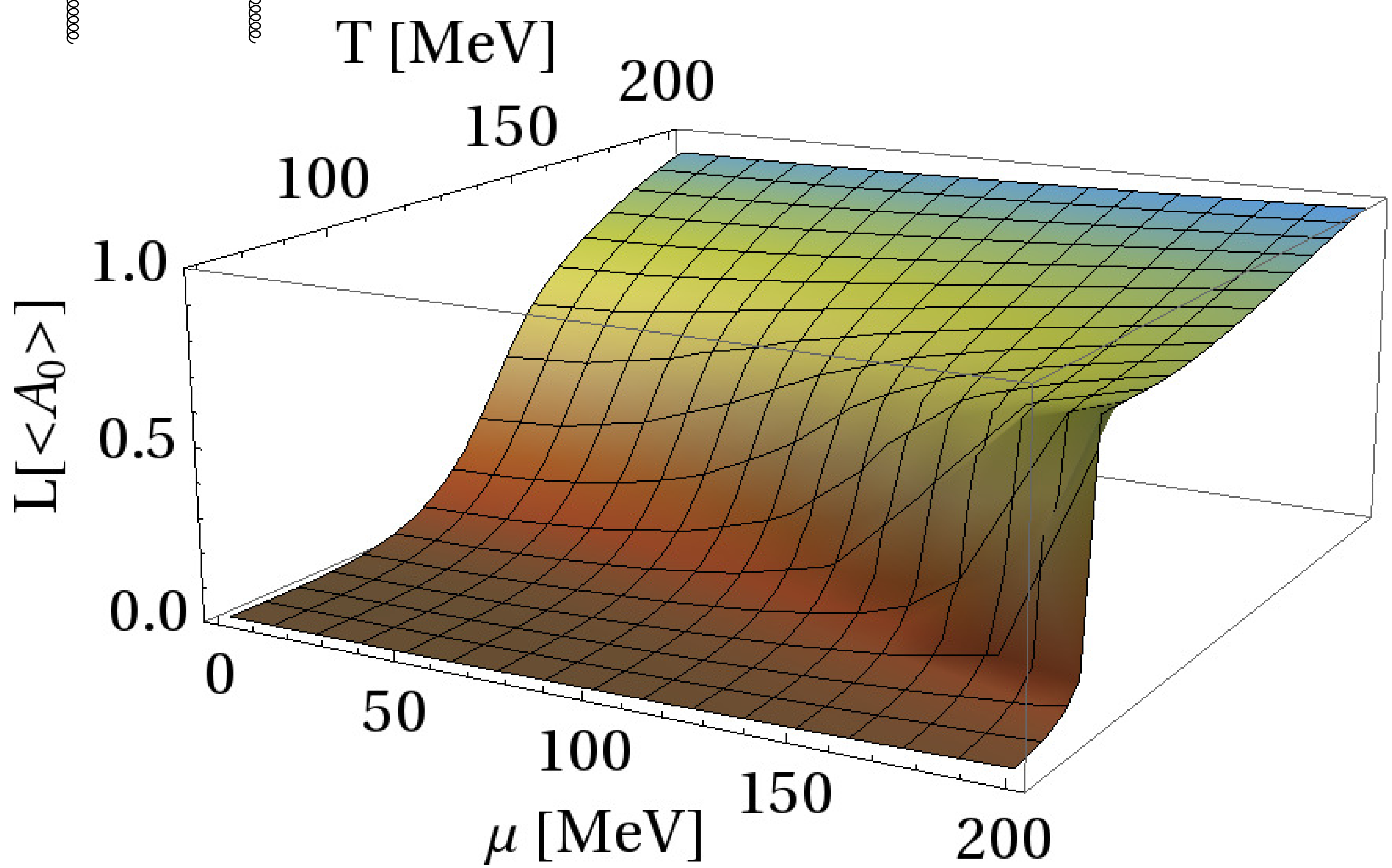


Quark condensate



Polyakov-Loop

$$L = \frac{1}{N_c} tr\ e^{ig \int A_0}$$

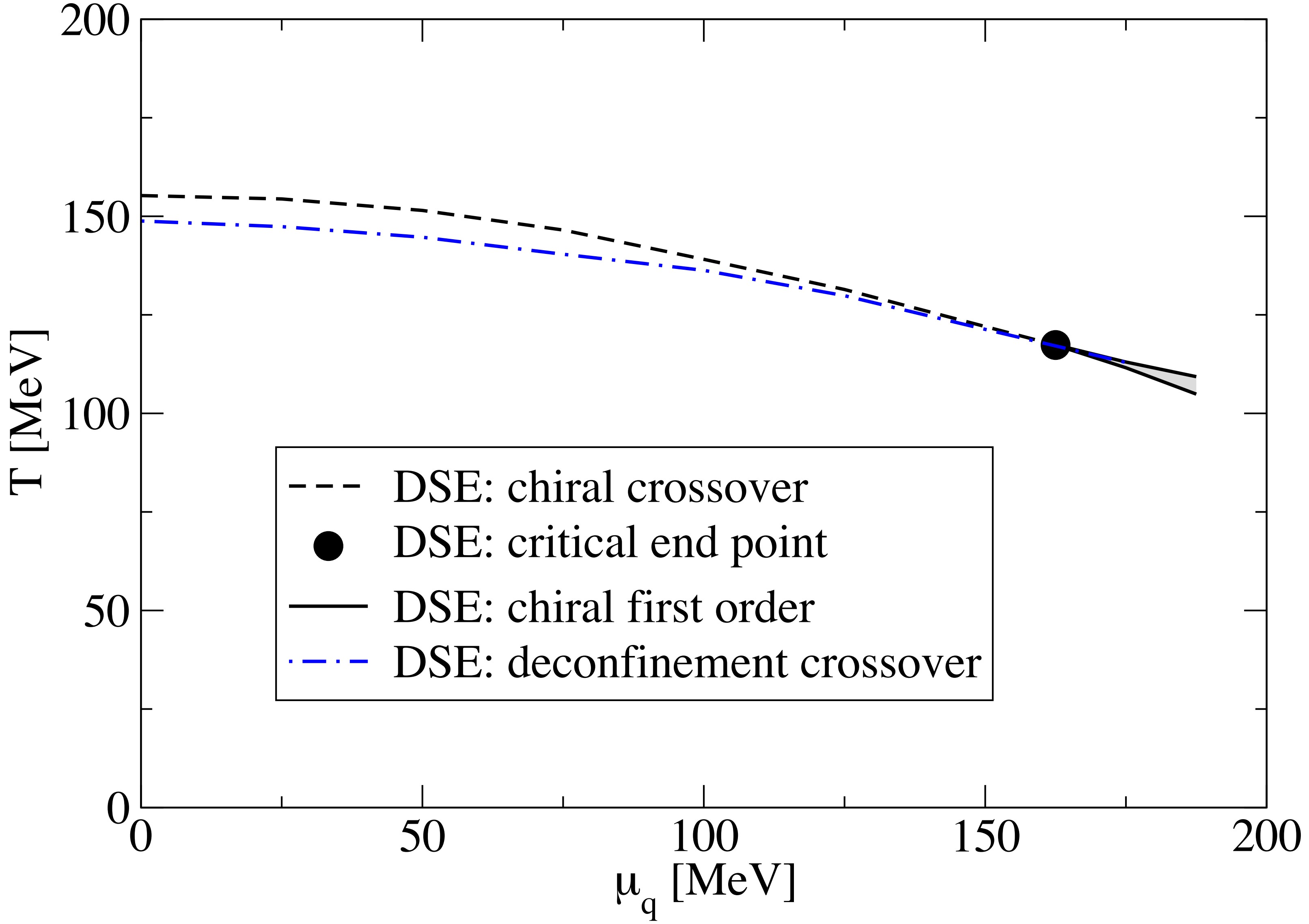


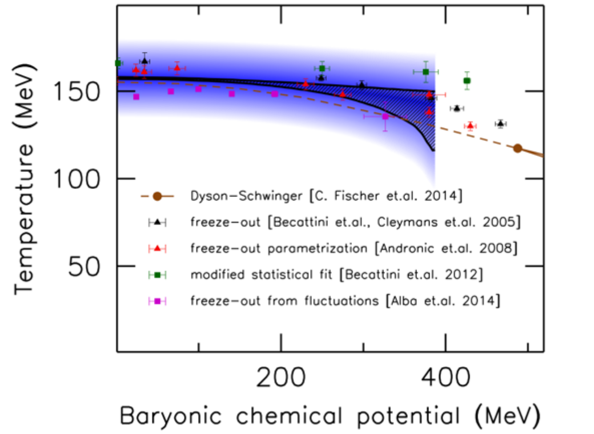
CF, Fister, Luecker, Pawlowski, PLB 732 (2014) 273

- evaluated from Polyakov-Loop potential
- important input for P-models: PQM, PNJL !

Herbst, Mitter, Pawlowski, Schaefer, Stiele, PLB 731 (2014) 248

N_f=2+1: Polyakov loop and phase diagram





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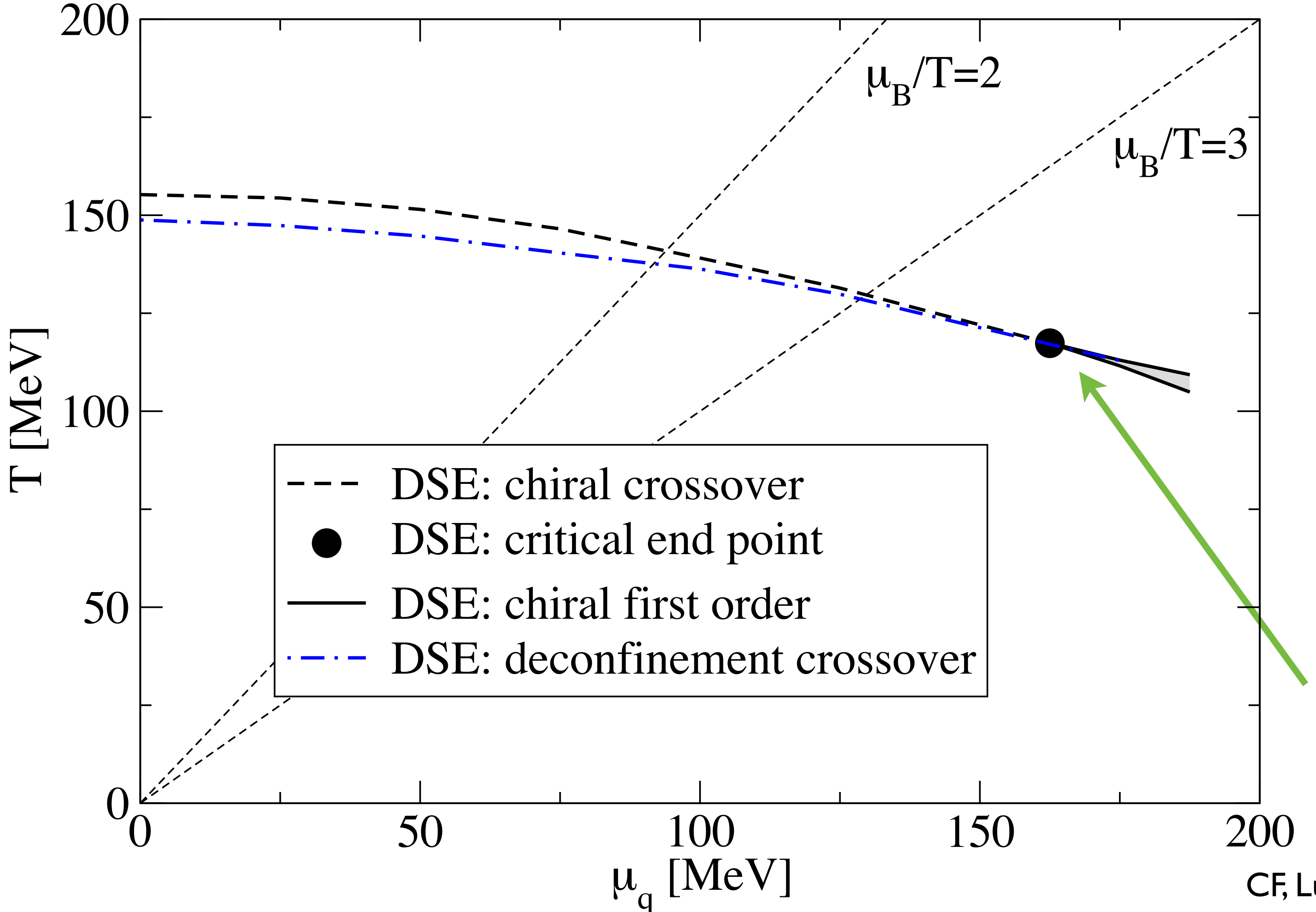
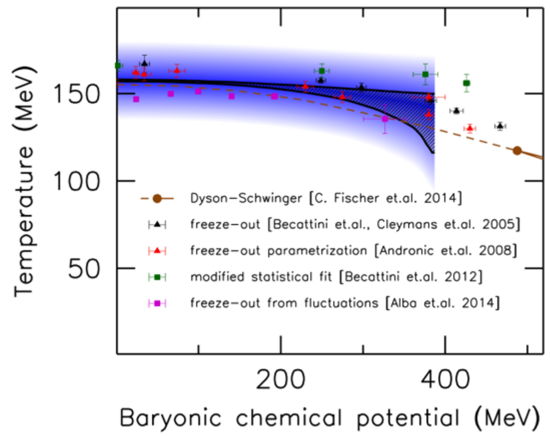


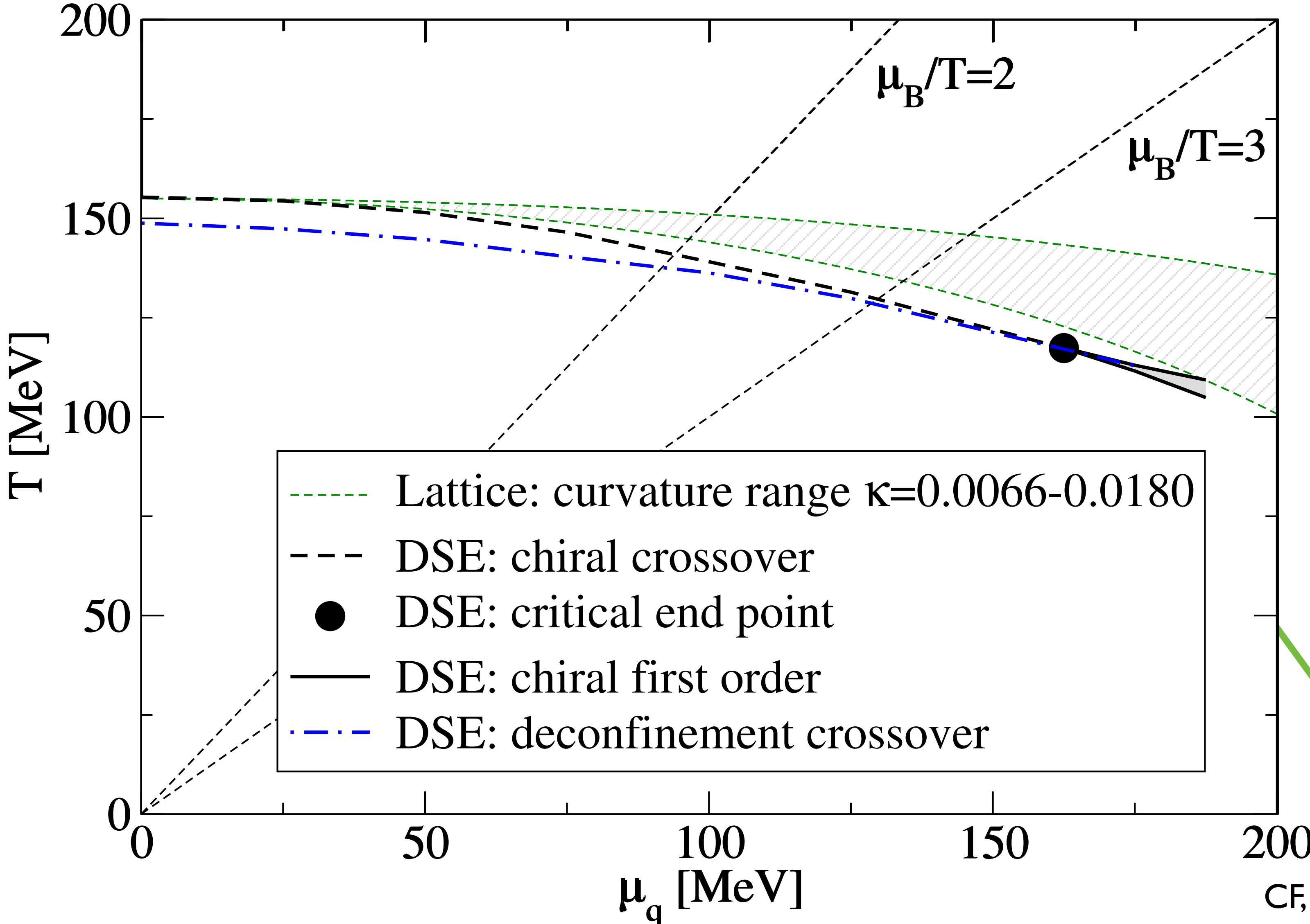


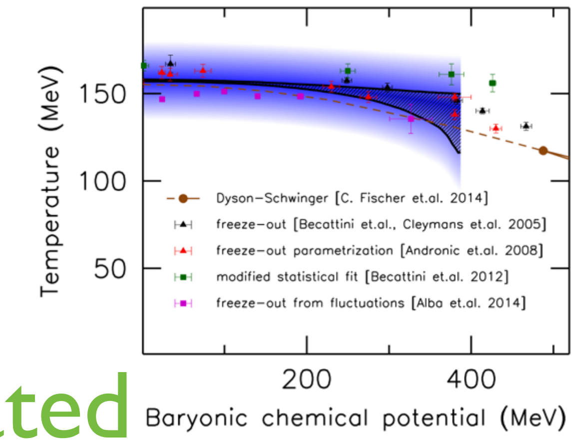
CEP at large μ

CF, Luecker, PLB 718 (2013) 1036,
CF, Fister, Luecker, Pawlowski, PLB 732 (2014) 273
CF, Luecker, Welzbacher, PRD 90 (2014) 034022

- combined evidence of FRG and DSE: no CEP at $\mu_B/T<2$

N_f=2+1: Polyakov loop and phase diagram





Extrapolated curvature from lattice

Kaczmarek at al. PRD 83 (2011) 014504,
Endrodi, Fodor, Katz, Szabo, JHEP 1104 (2011) 001
Cea, Cosmai, Papa, PRD 89 (2014) 074512

CEP at large μ

CF, Luecker, PLB 718 (2013) 1036,
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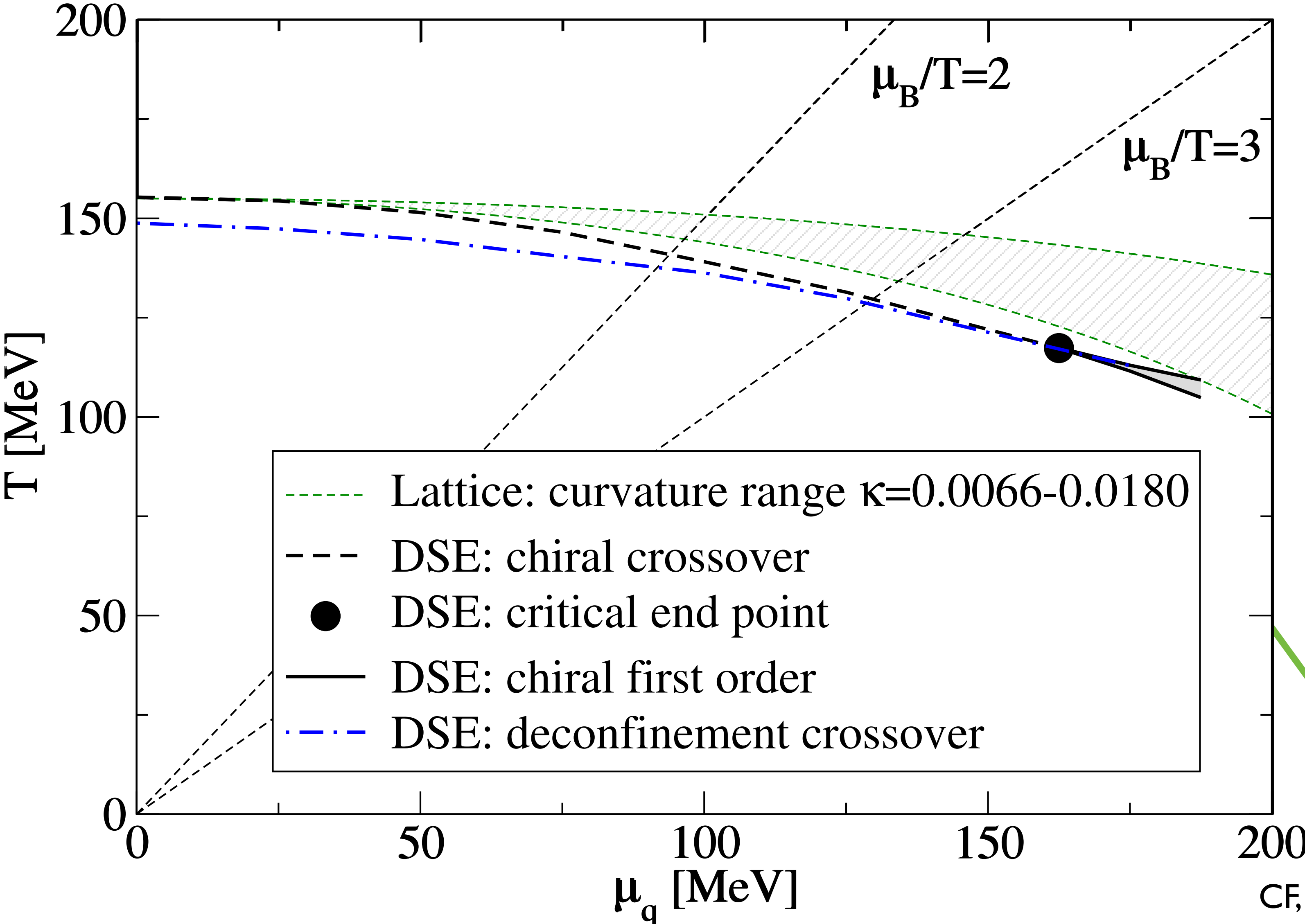

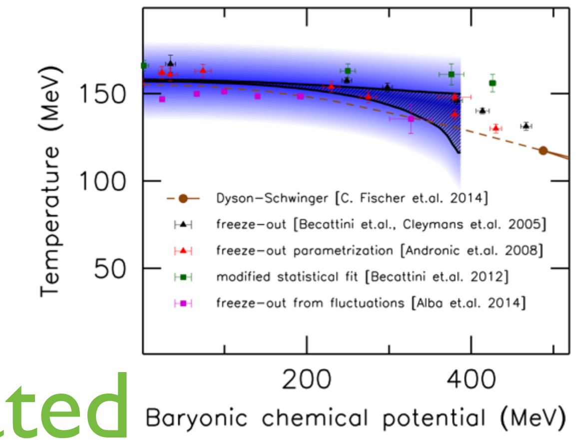

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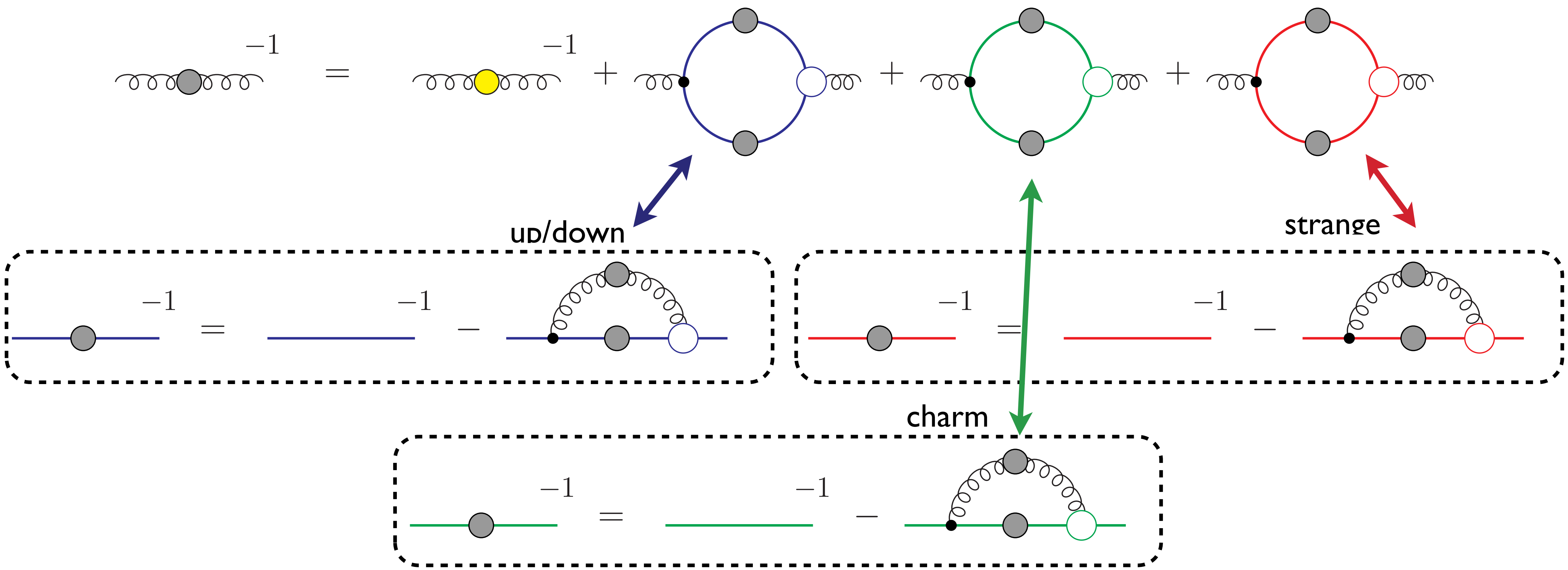
- combined evidence of FRG and DSE: no CEP at $\mu_B/T<2$

Caveat: baryon effects missing...

Nc=2: Brauner, Fukushima and Hidaka, PRD 80 (2009) 74035
Strodthoff, Schaefer and Smekal, PRD 85 (2012) 074007

Nf=2+1+1-QCD with DSEs



- Physical up/down, strange and charm quark masses
- Transition controlled by chiral dynamics
- *no lattice or model results available yet*

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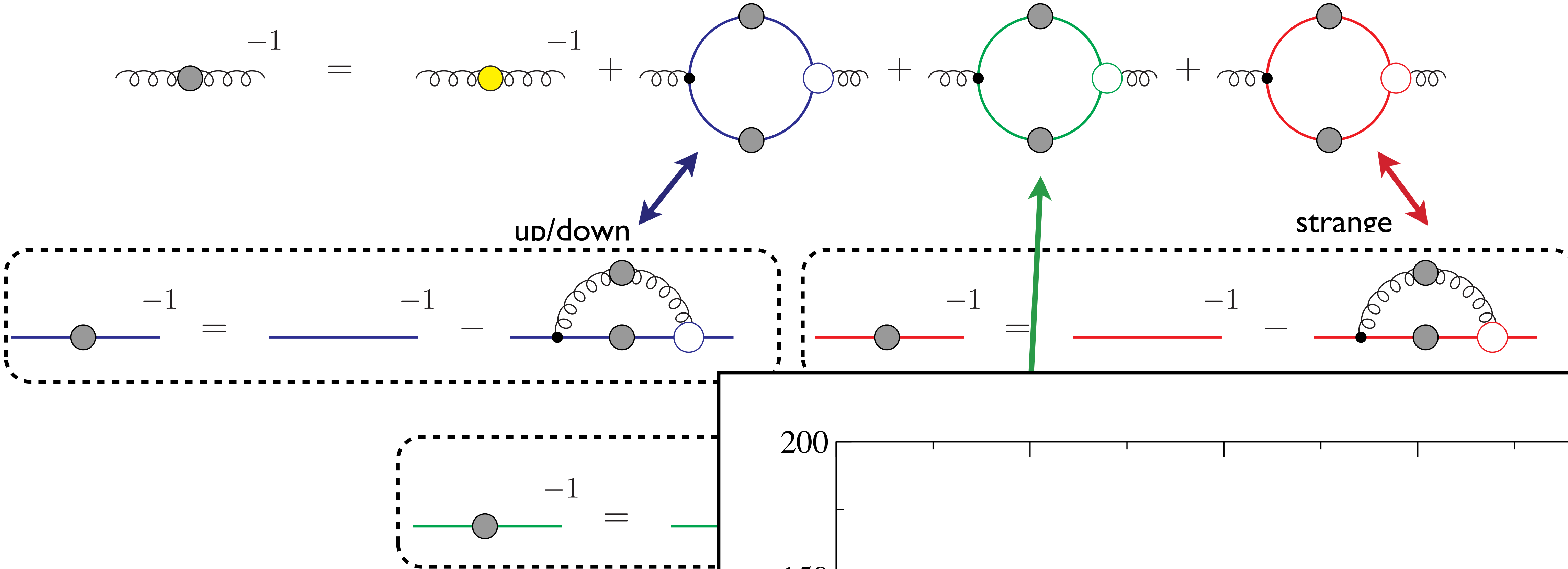

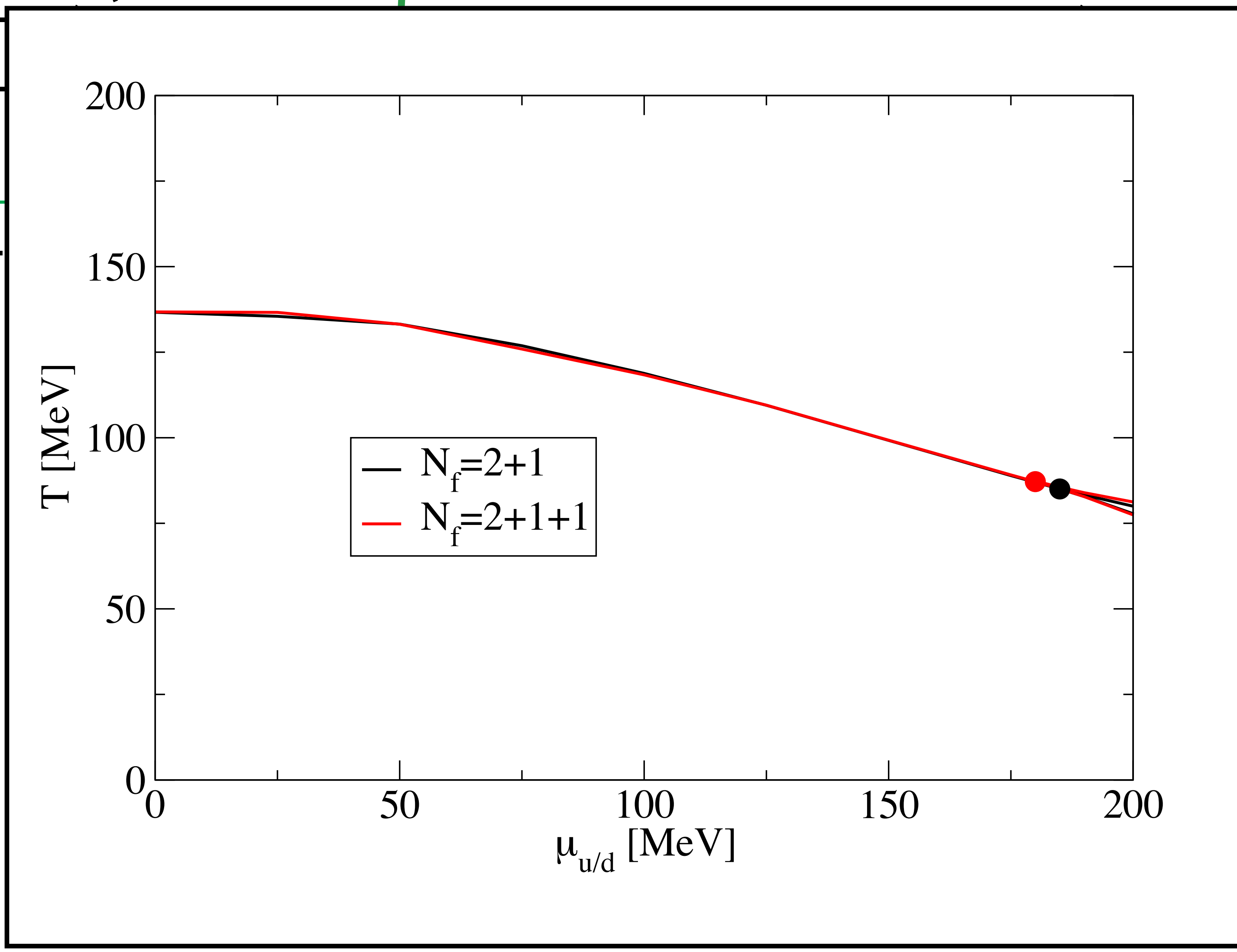

- Physical up/down, strange and charm quark masses
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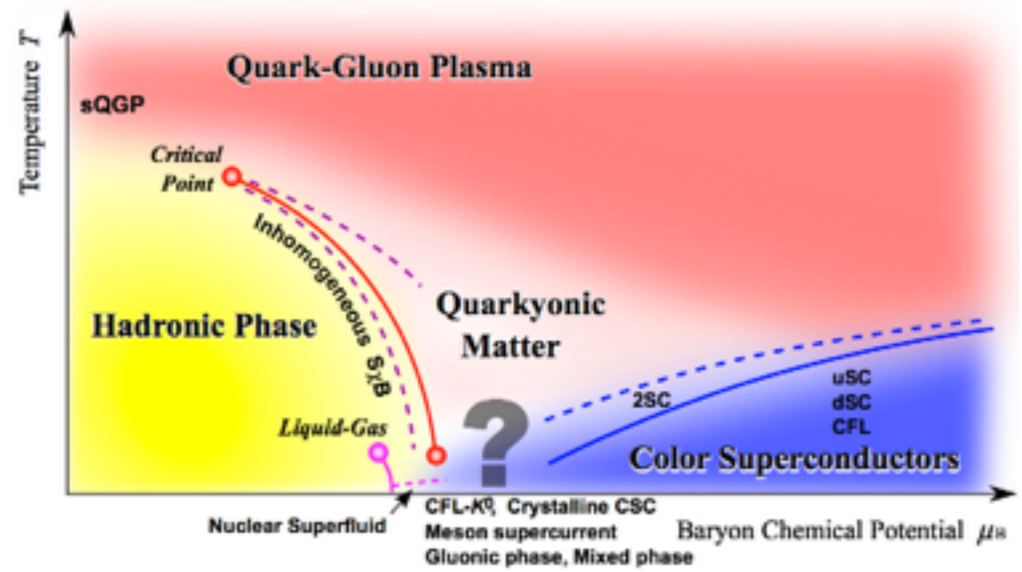
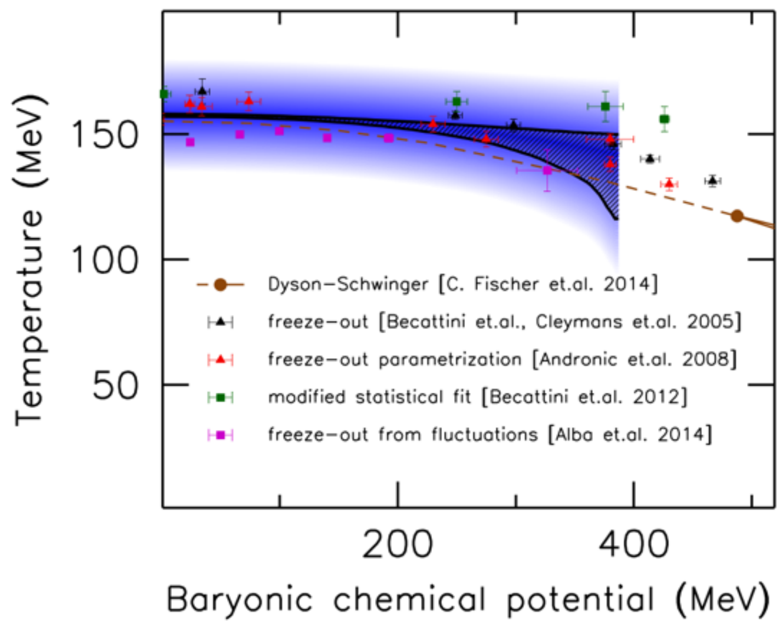
CF, Luecker, Welzbacher, PRD 90 (2014) 034022

Overview

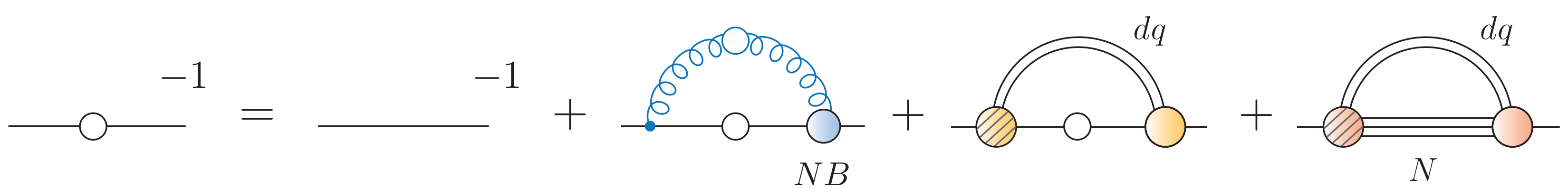
1.Introduction



2.Gluons, quarks and the CEP

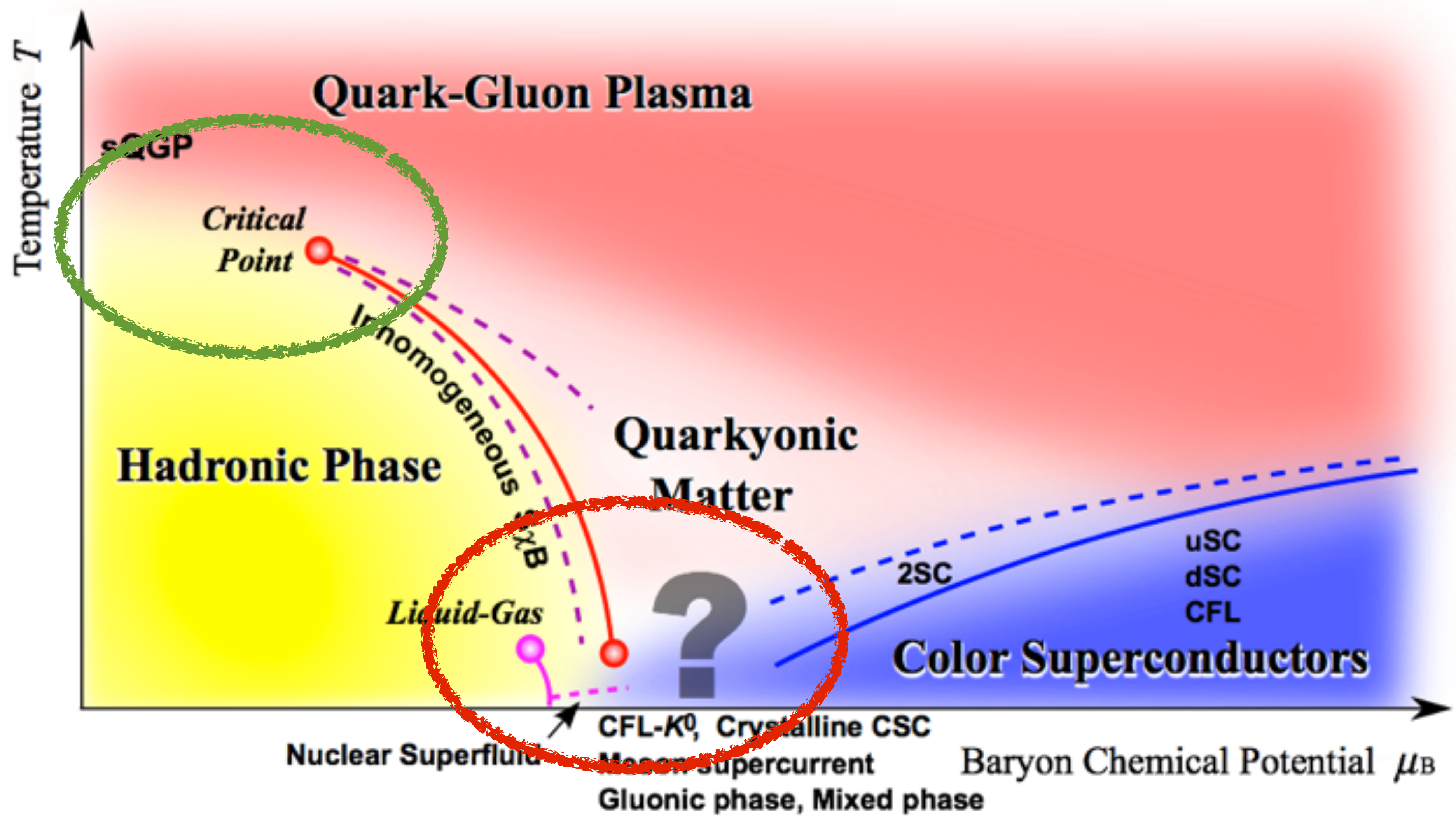


3.Baryon effects on the CEP



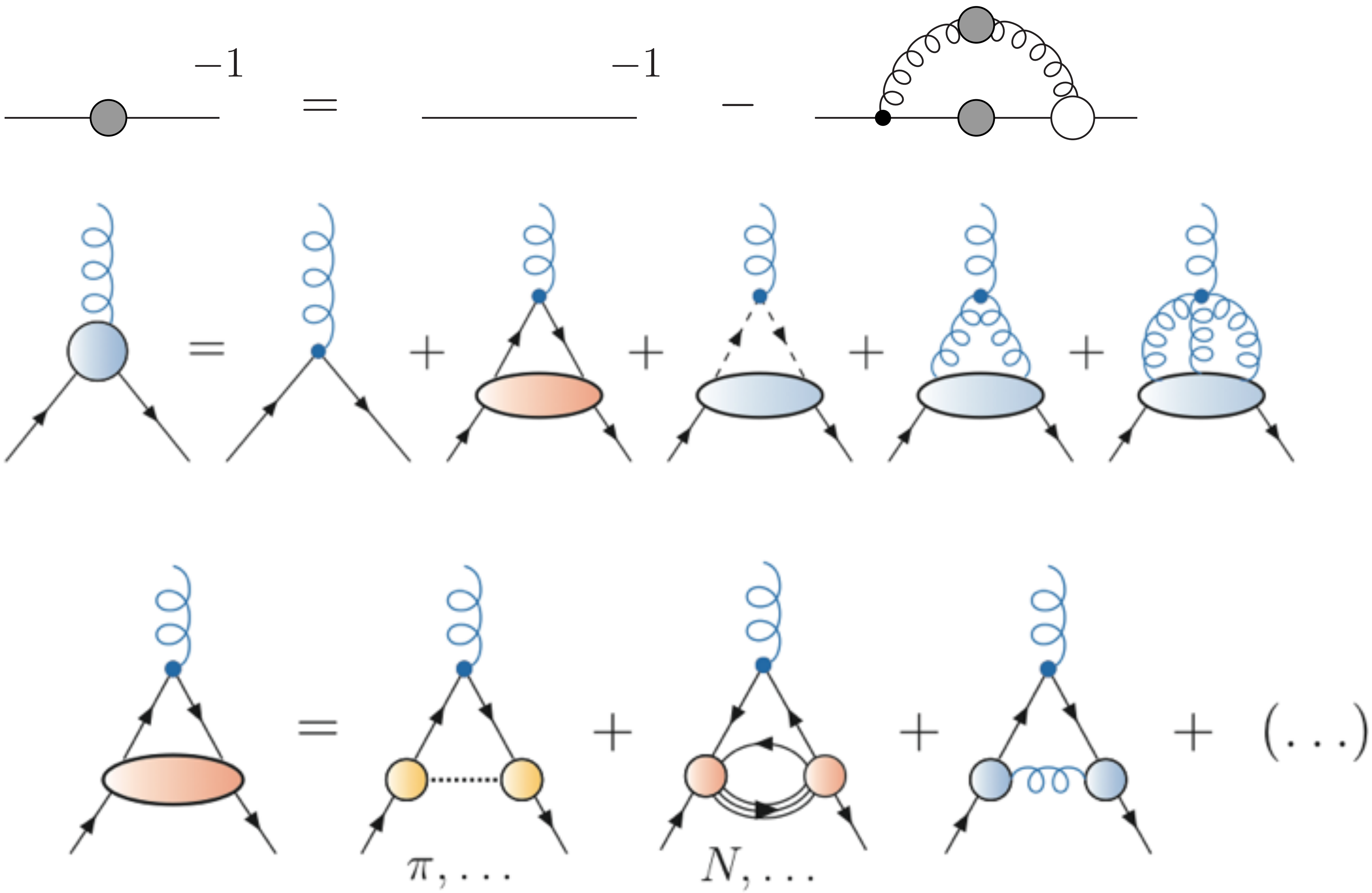
QCD phase transitions I

Fukushima, Hatsuda, Rept. Prog. Phys. 74 (2011) 014001



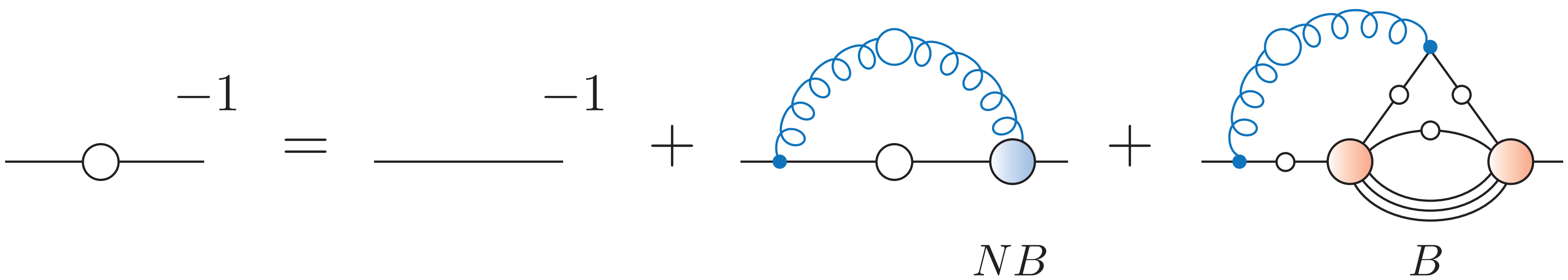
- Low temperatures, large chemical potential:
  baryons are important degrees of freedom
- How do baryons affect the quark condensate ??

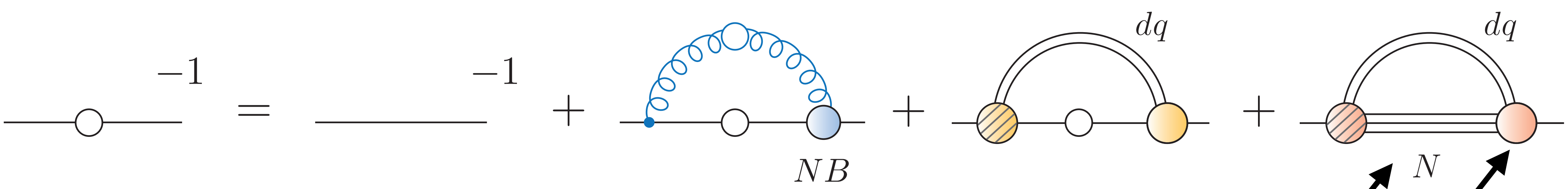
Baryon effects onto quark I



- ‘Off-shell baryons’ do affect quark conde

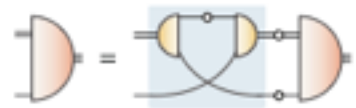
Baryon effects onto quark II





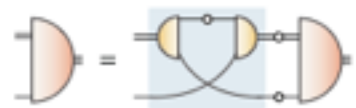
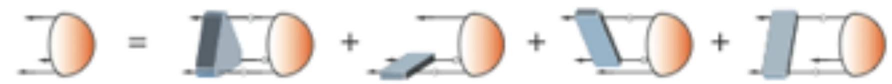
- Dependence on T and μ via -propagators
  -wave functions
- Exploratory calculation: use wave functions from T=μ=0

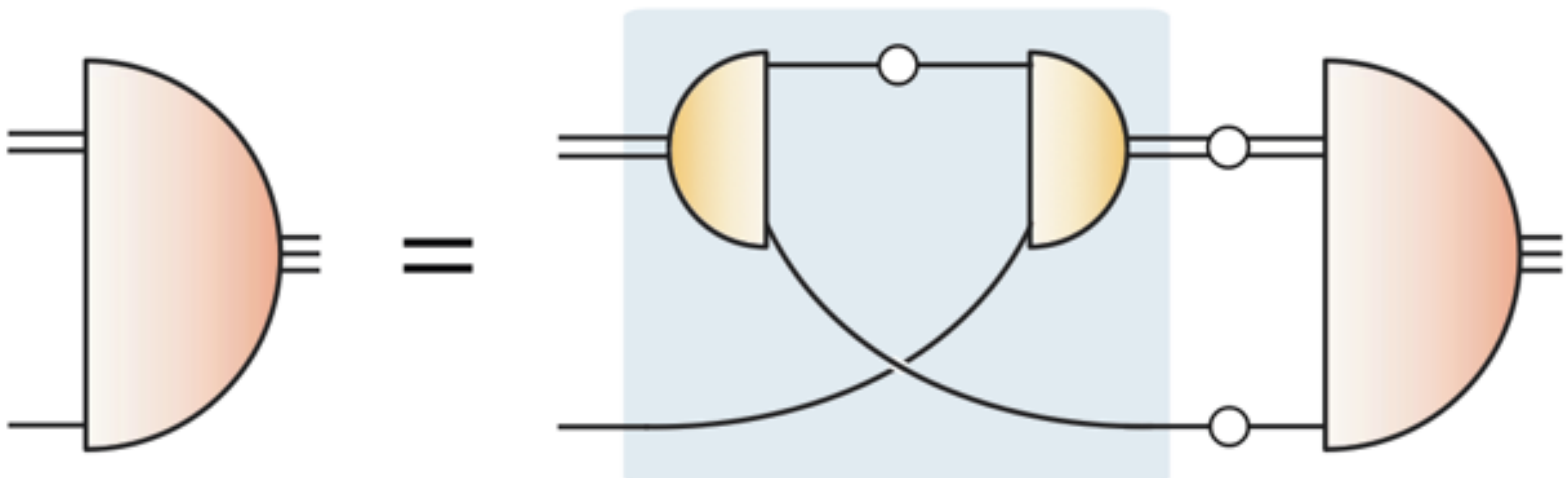
DSE/Faddeev landscape (T=μ=0)

	Quark-diquark 			Three-quark 		
	Contact interaction	**QCD-based model**	**DSE (RL)**	**RL**	**bRL**	**bRL + 3q**
N, Δ masses	√	√	√	√	√	...
N, Δ em. FFs	√	√	√	√		
$N \to \Delta\gamma$	√	√	√	...		
Roper	√	√		...		
$N \to N^*\gamma$	√	√		...		
$N^*(1535), \ldots$	...	...		...	...	
$N \to N^*\gamma$	...	...				
	Roberts et al	Oettel, Alkofer Roberts, Bloch Segovia et al.	Eichmann, Alkofer Nicmorus, Krassnigg	Eichmann, Alkofer Sanchis-Alepuz, CF	Sanchis-Alepuz, CF Williams	

Eichmann, N*-Workshop, Trento 2015

DSE/Faddeev landscape (T=μ=0)

	Quark-diquark			Three-quark		
						
	Contact interaction	QCD-based model	DSE (RL)	RL	bRL	bRL + 3q
N, Δ					√	…
N, Δ						
$N \to$						
Roper						
$N \to$						
$N^*(1535), \ldots$	…	…		…	…	
$N \to N^*\gamma$	…	…				



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Eichmann, N*-Workshop, Trento 2015

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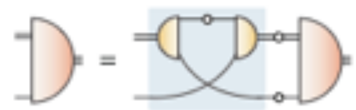
	Quark-diquark 			Three-quark 		
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N, Δ masses	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$	$\dots$
N, Δ em. FFs	$\surd$	$\surd$	$\surd$	$\surd$		
$N \to \Delta\gamma$	$\surd$	$\surd$	$\surd$	$\dots$		
Roper	$\surd$	$\surd$		$\dots$		
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Eichmann, N*-Workshop, Trento 2015

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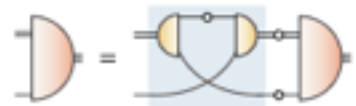
	Quark-diquark 			Three-quark 		
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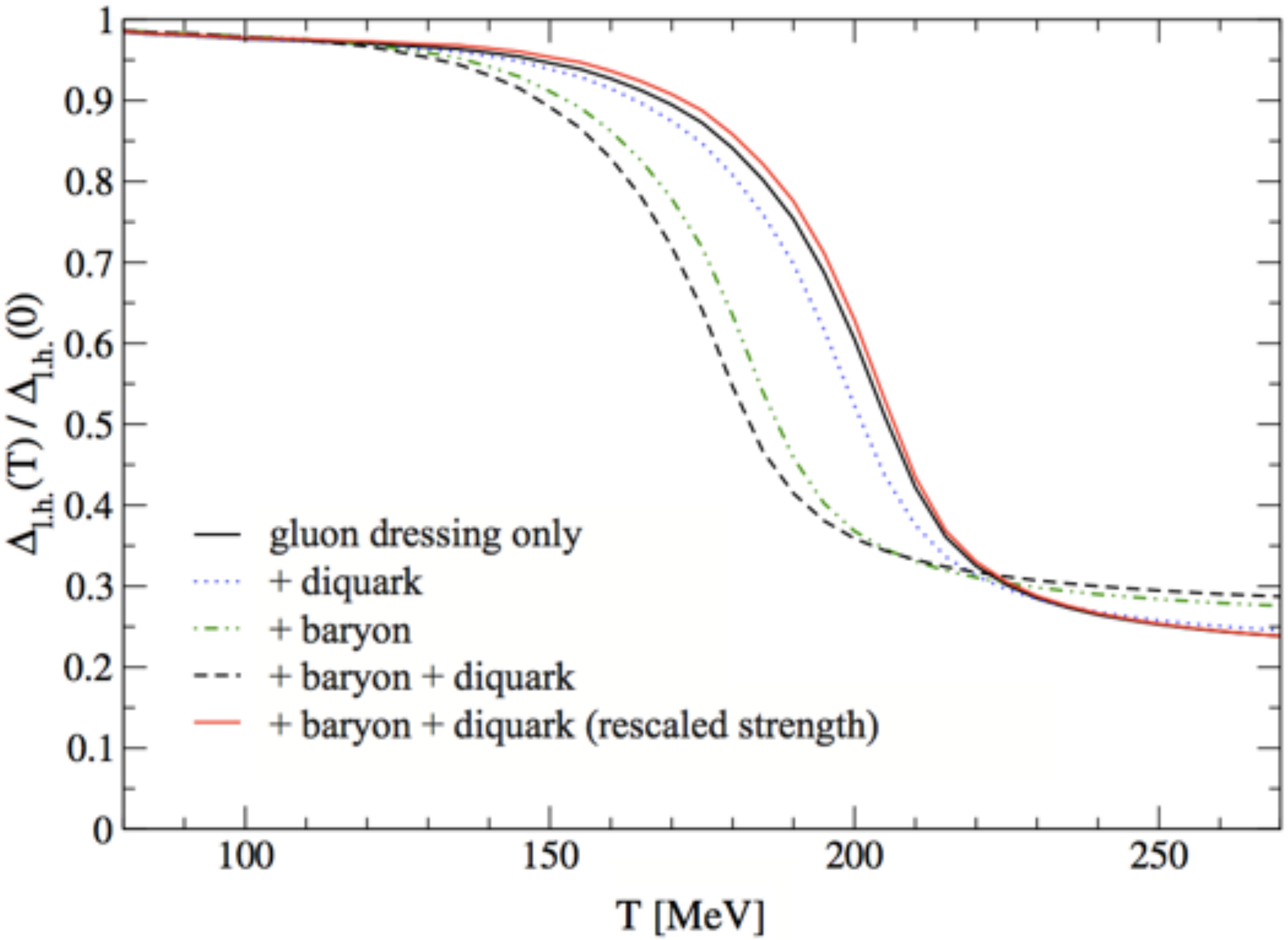
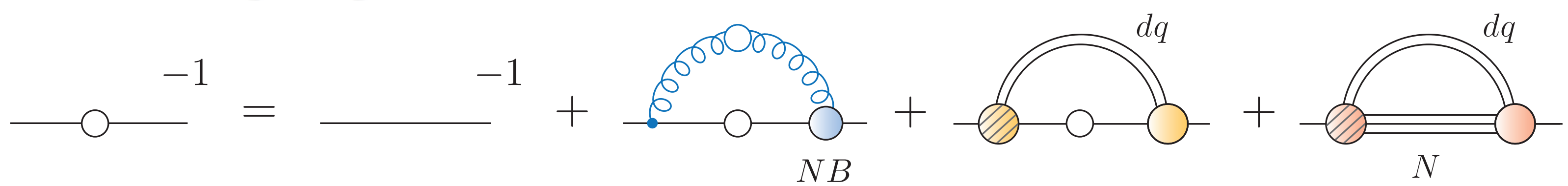
Eichmann, N*-Workshop, Trento 2015

Baryon effects - results (N_f=2)

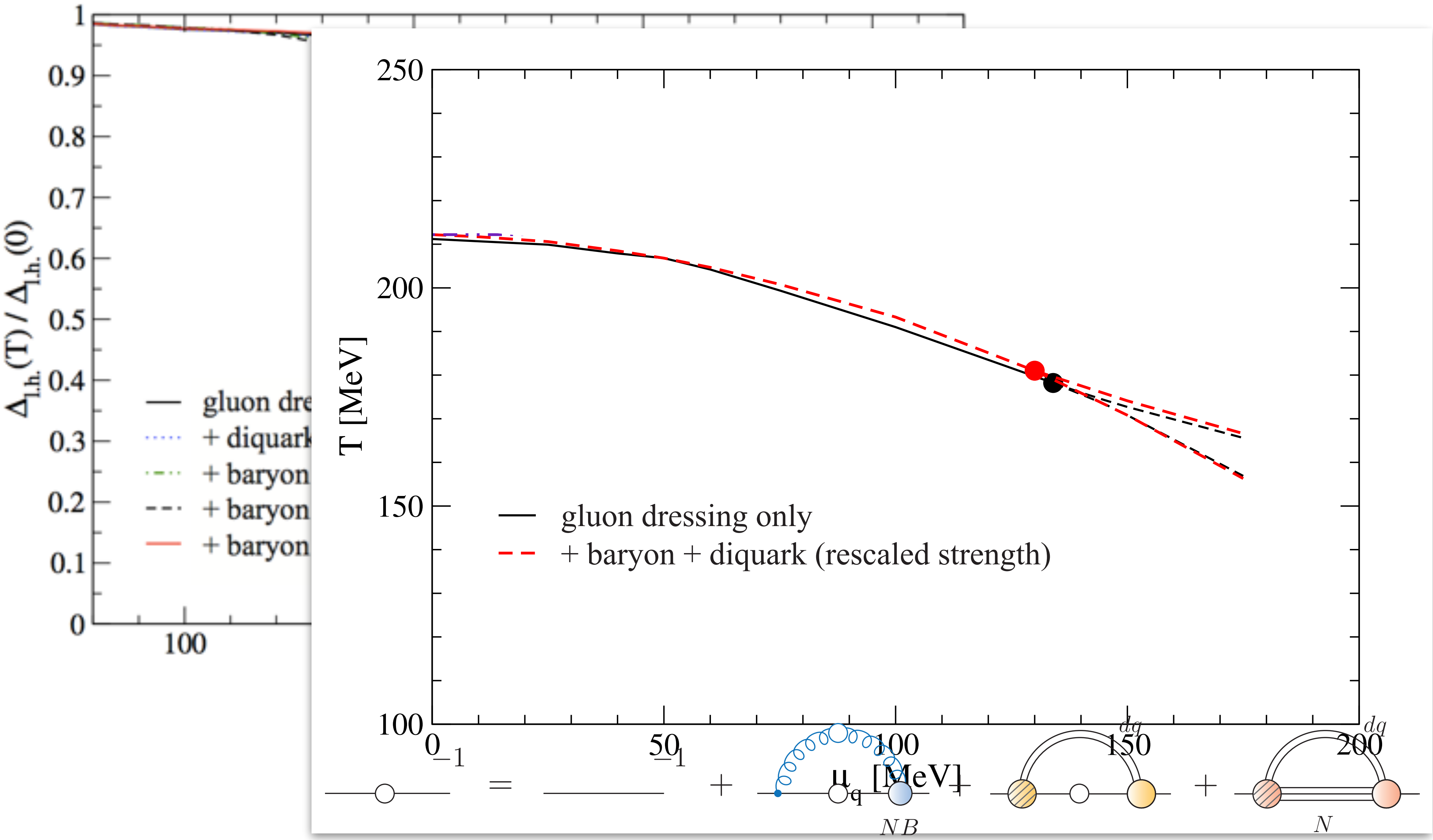




- Zero chemical potential: no effects after rescaling

Eichmann, CF, Welzbacher, PRD in press, arXiv:1509.02082

Baryon effects - results (N_f=2)



- Zero chemical potential: no effects after rescaling
- CEP: almost no effects

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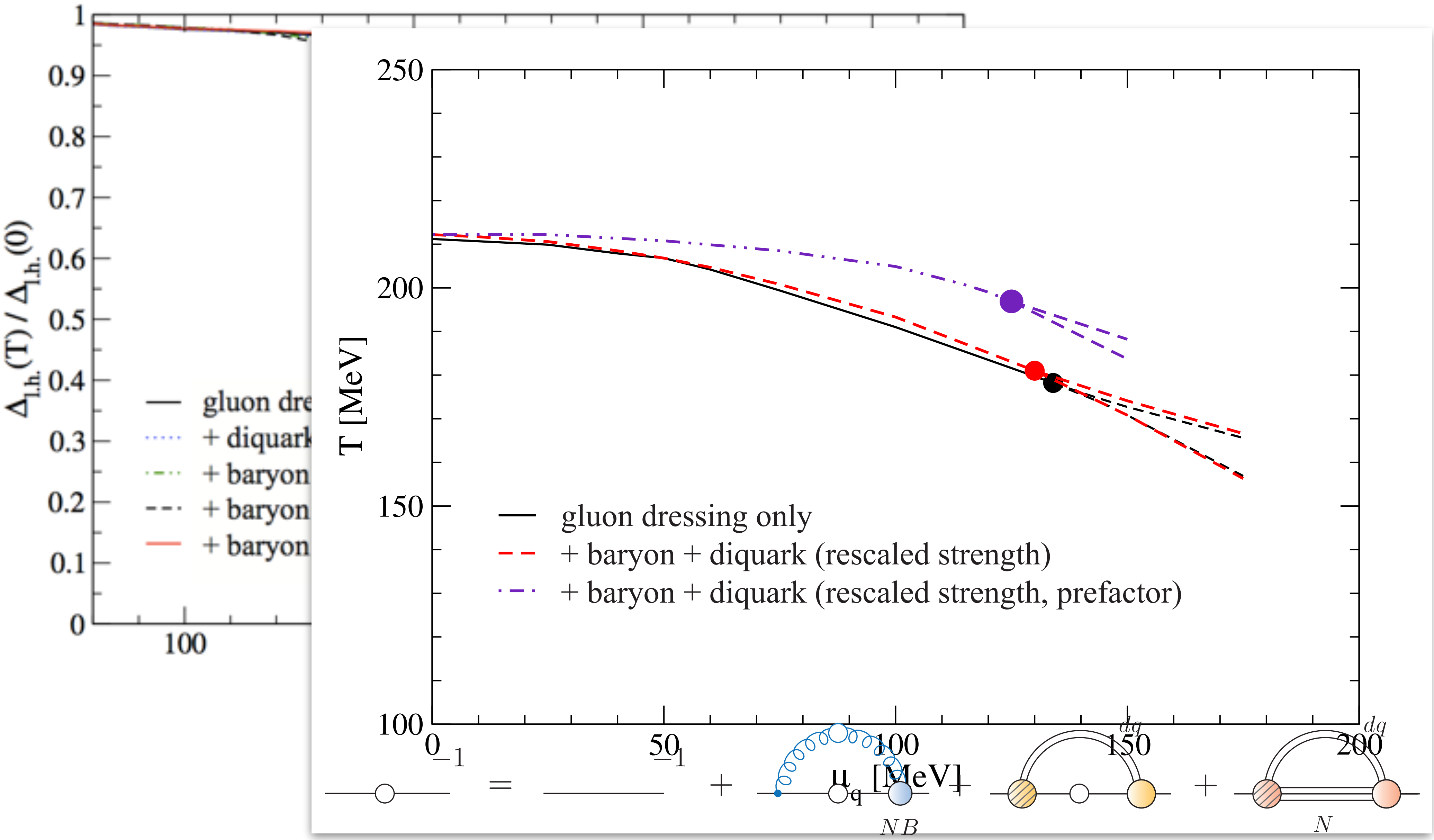
Baryon effects - results (N_f=2)



- Zero chemical potential: no effects after rescaling
- CEP: almost no effects
- But: strong μ-dependence of baryon wave function may change things…

Eichmann, CF, Welzbacher, PRD in press, arXiv:1509.02082

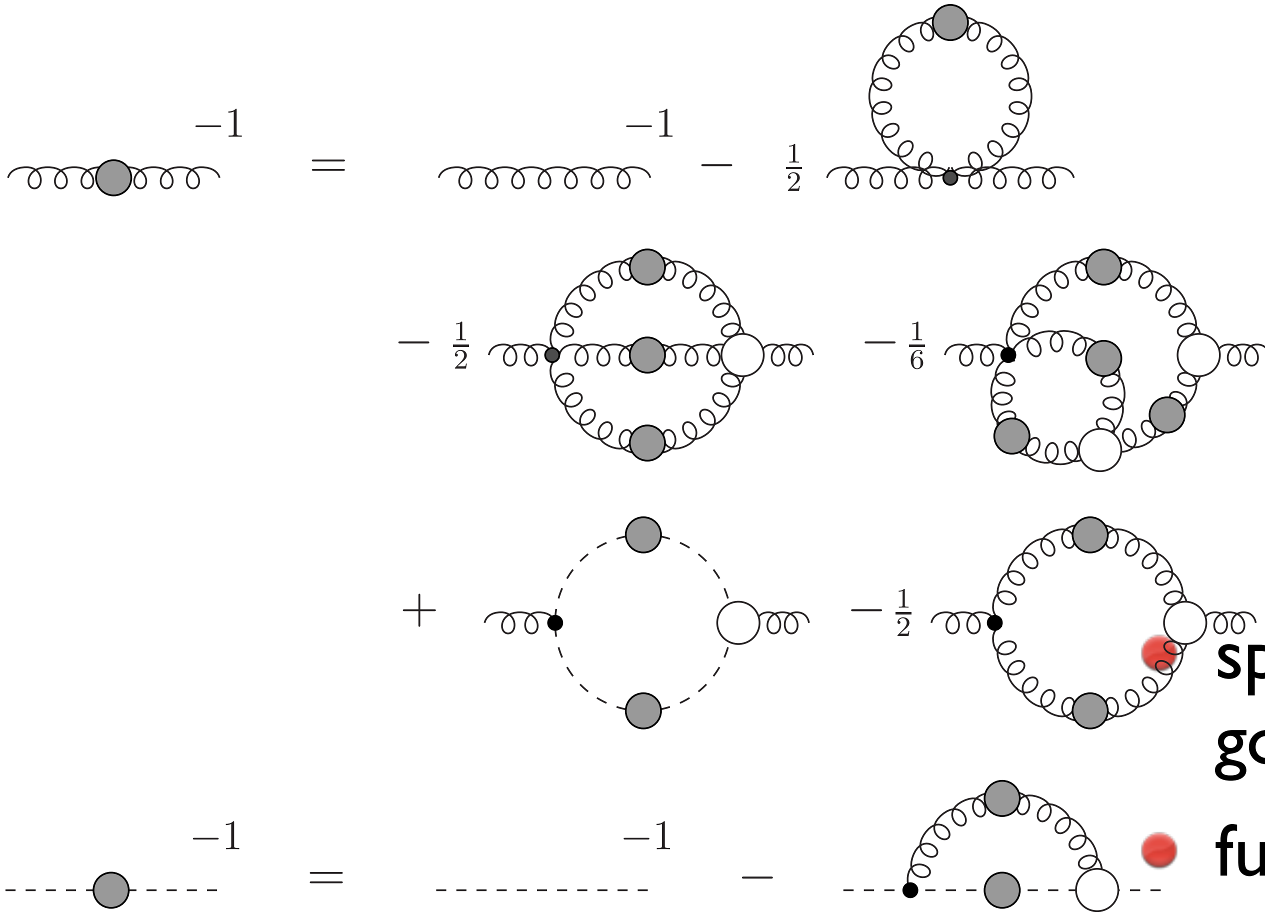
Summary

- Temperature dependent gluon propagator
  - characteristic behaviour of electric gluon
  - ‘melting’ of magnetic gluon with temperature
- Deconf. T_{pc} from dressed Polyakov-loop/Polyakov-loop potential
- QCD with finite chemical potential (beyond mean field)
  - back-reaction of quarks onto gluons important
  - N_f=2+1 and N_f=2+1+1: CEP at $\mu_c/T_c > 3$
  - charm quark does not influence CEP
  - Baryon effects may or may not be significant for CEP…

Work in progress:
- mesons and baryons at finite T and μ
- quark-gluon vertex at finite T and μ

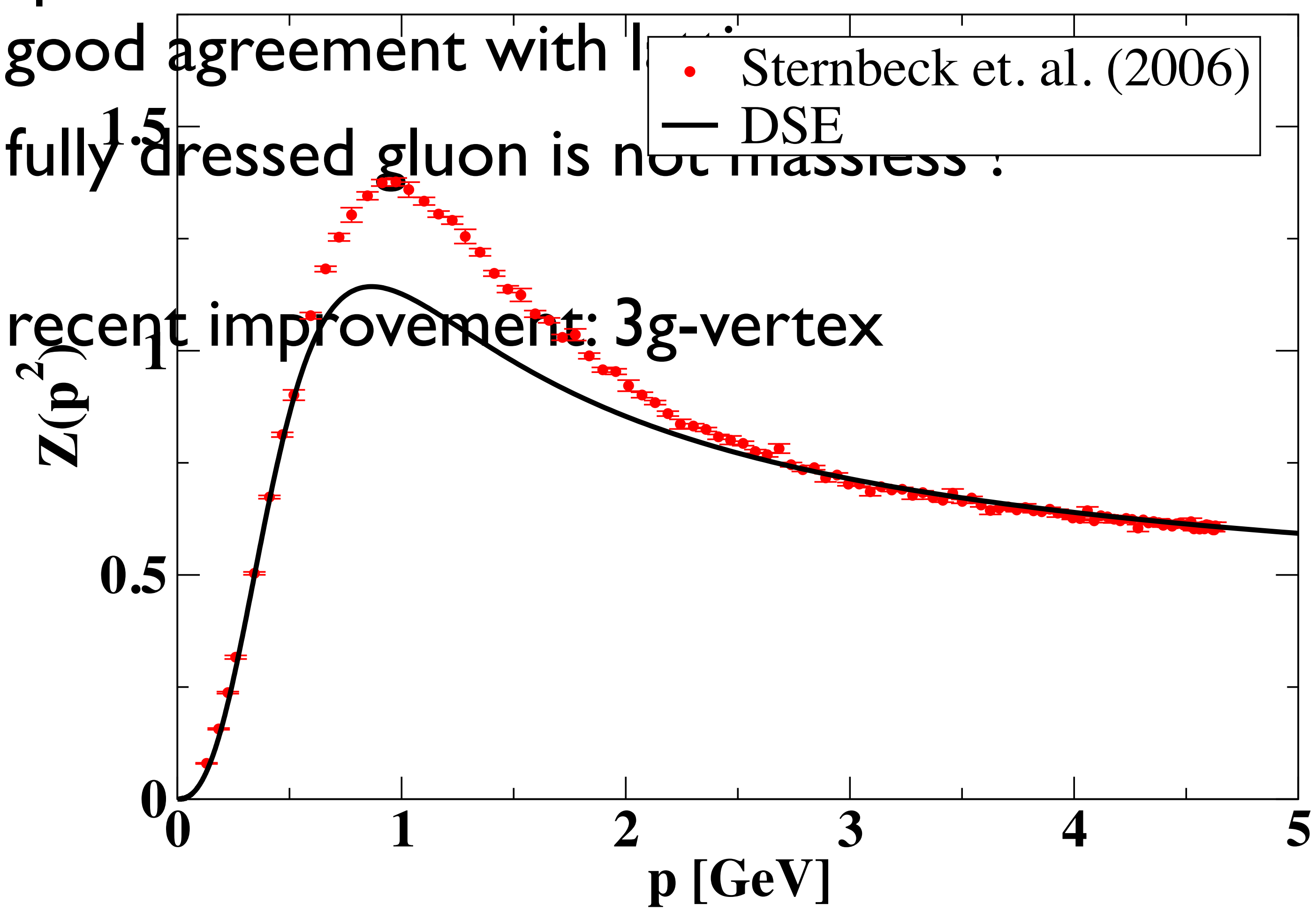
Back-up slides

Landau gauge gluon propagator



$$D_{\mu\nu}(p) = \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)\frac{Z(p^2)}{p^2}$$

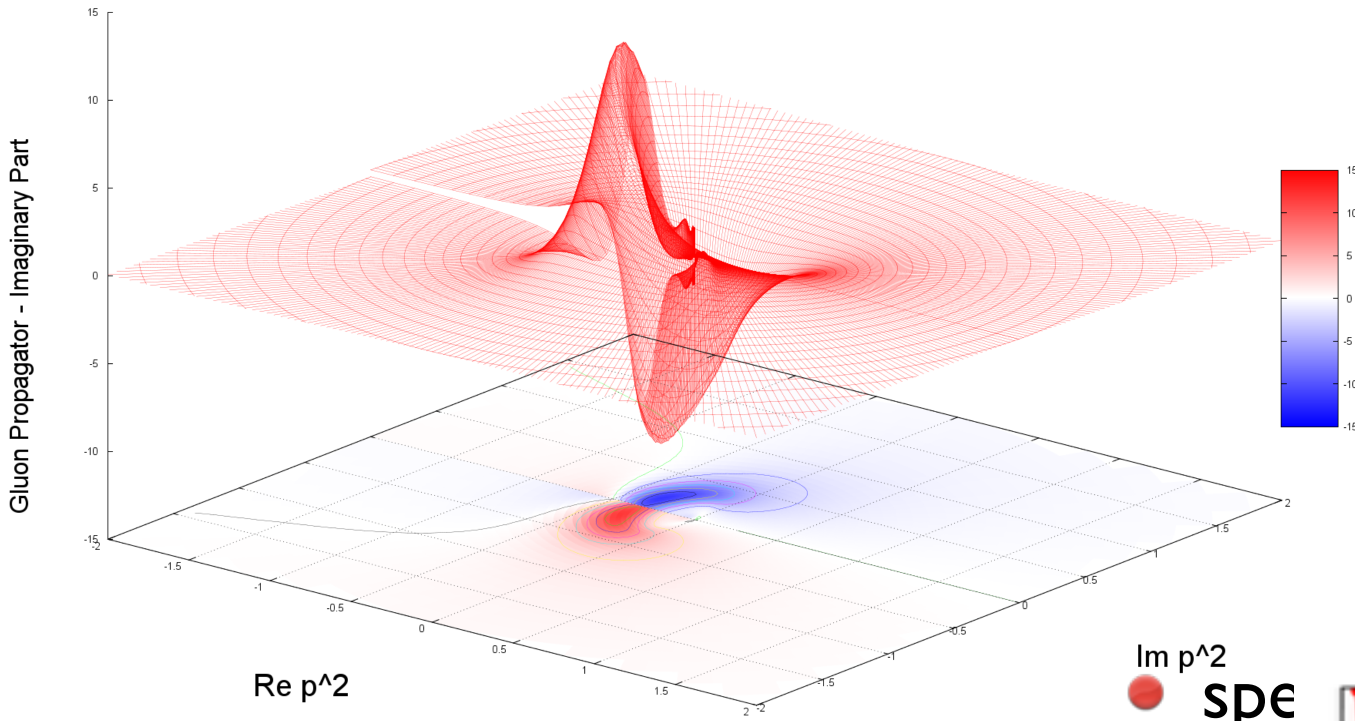
- spacelike momenta: good agreement with l...
- fully dressed gluon is not massless !
- recent improvement: 3g-vertex



Eichmann, Williams, Alkofer, Vujinovic PRD 89, (2014) 10

CF, Maas, Pawlowski, Annals Phys. 324 (2009) 2408.

Landau gauge gluon propagator



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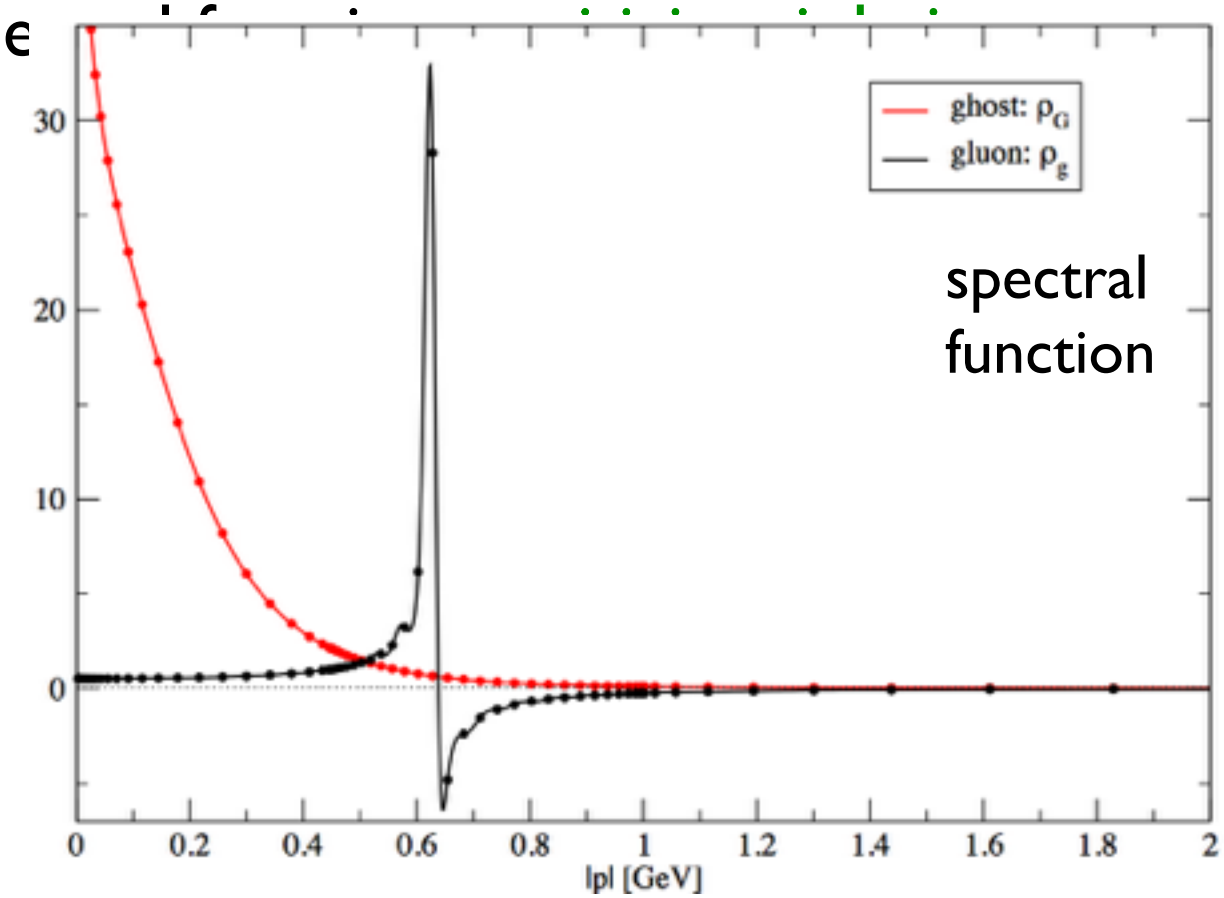


spectral function

$$600\,\text{MeV} < m_g < 700\,\text{MeV}$$

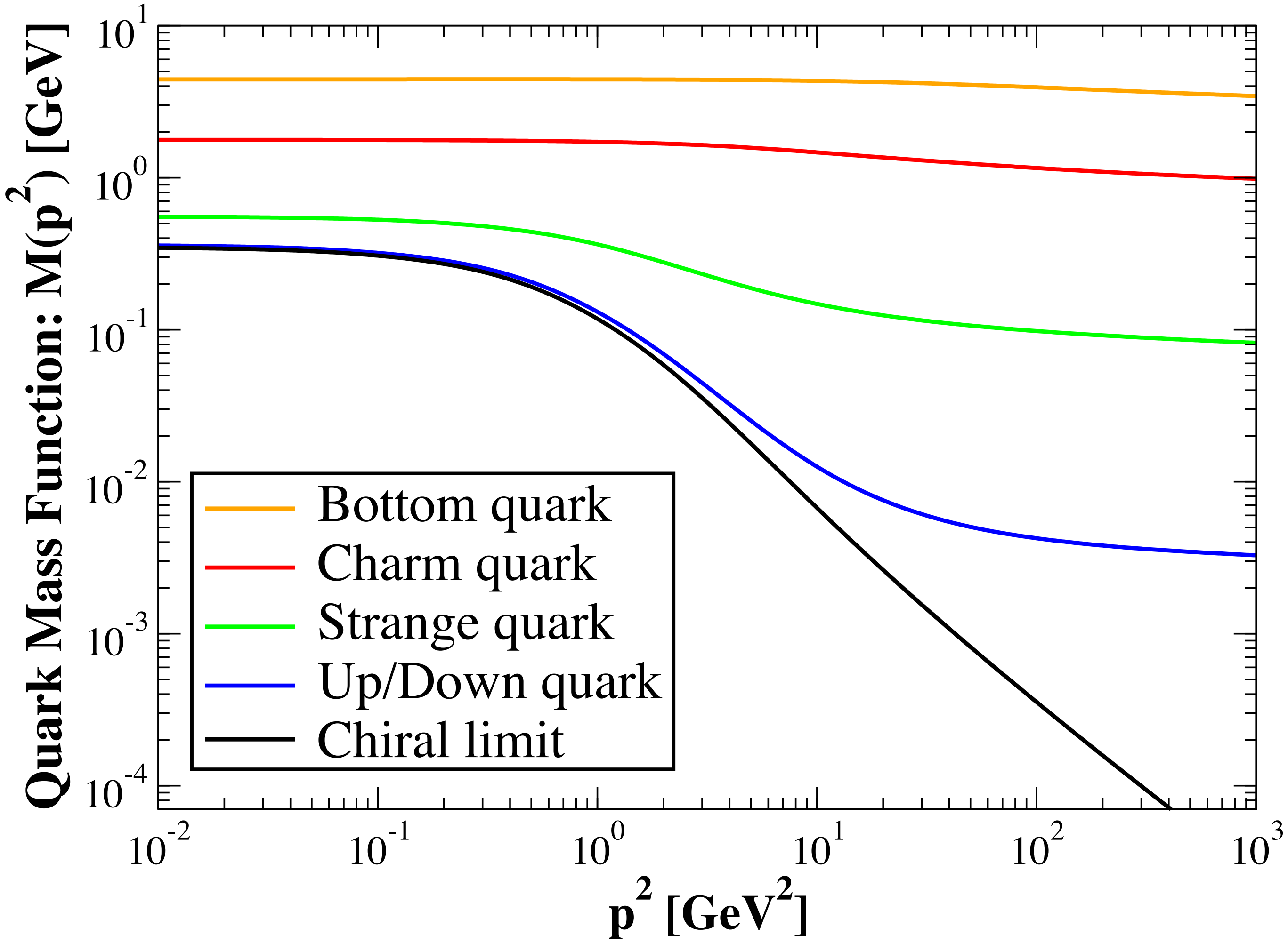
Cornwall, Papavassiliou,...

Strauss, CF, Kellermann, Phys. Rev. Lett. 109, (2012) 252001

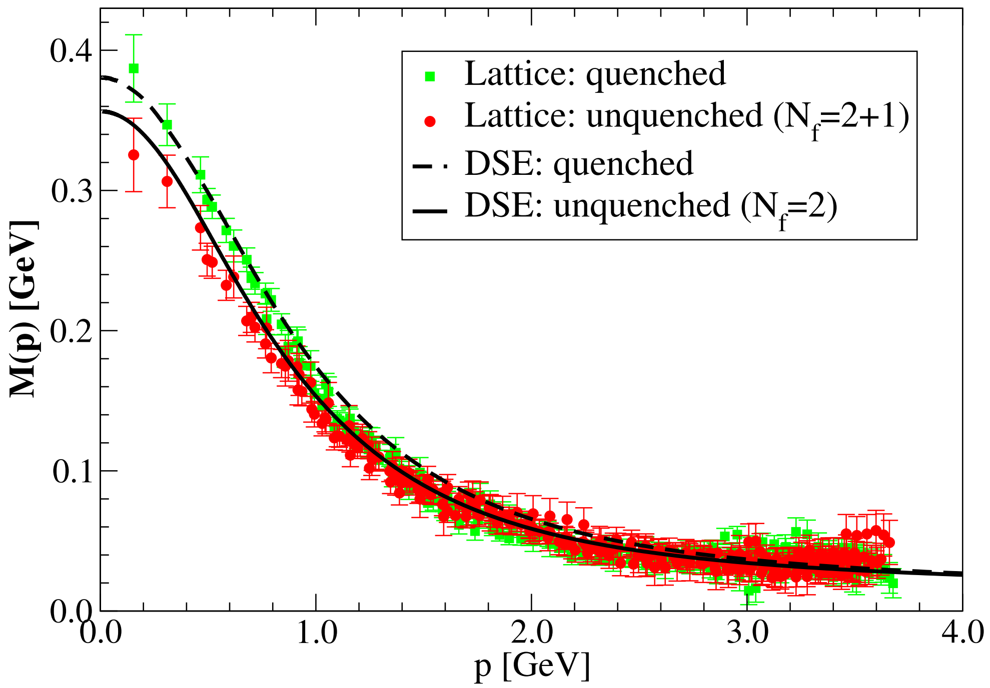
Gluon cannot appear in detector!

Quark mass: flavor dependence

Typical solution:



$$[S(p)]^{-1} = [-i\not{p} + M(p^2)]/Z_f(p^2)$$



CF, Nickel, Williams, EPJ C 60 (2009) 47

- $M(p^2)$: momentum dependent!
- Dynamical mass: $M_{strong} \approx 350$ MeV
- Flavour dependence because of m_{weak}
- Chiral condensate: $\langle \bar{\Psi}\Psi \rangle \approx (250\,\mathrm{MeV})^3$

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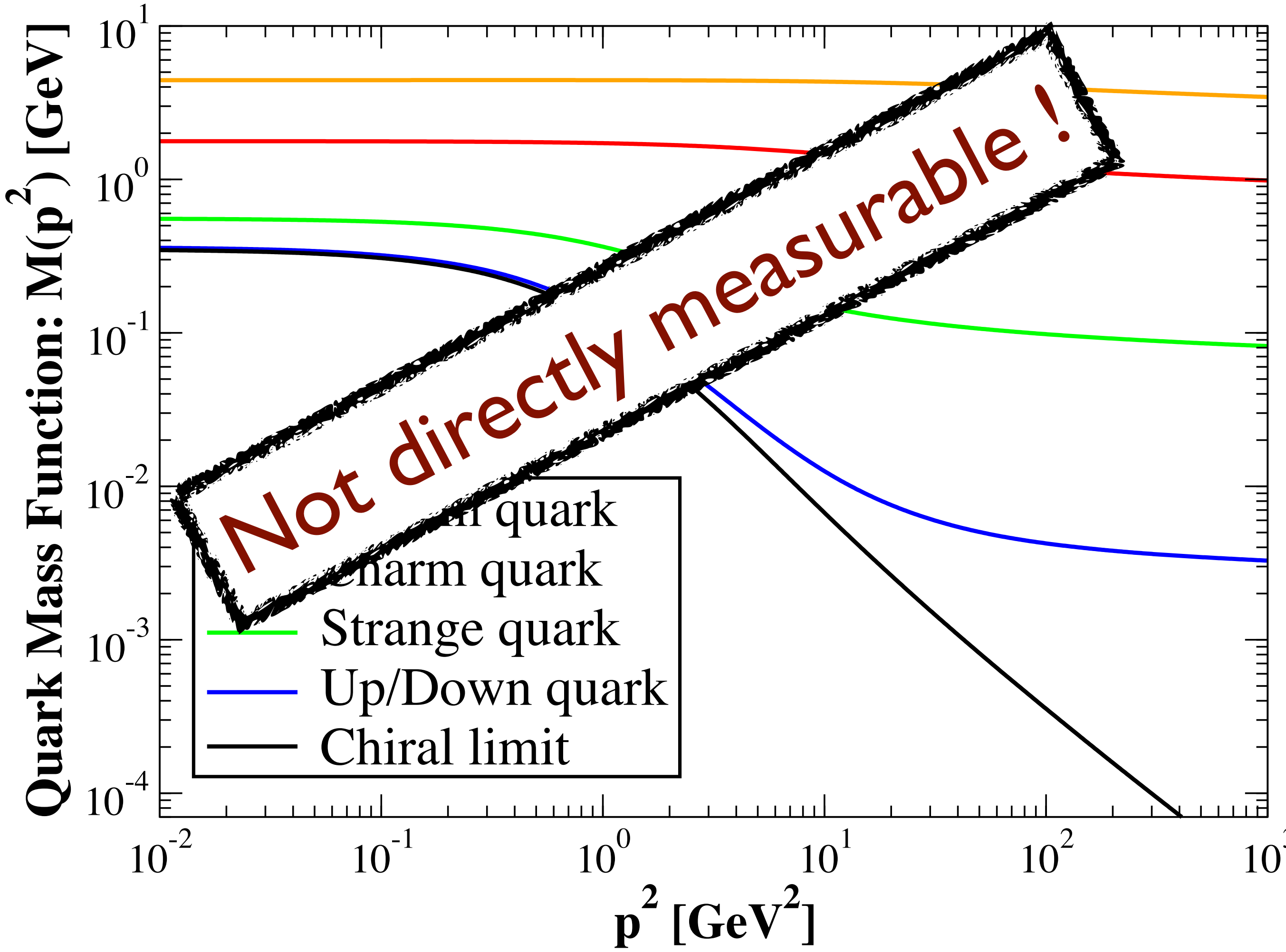

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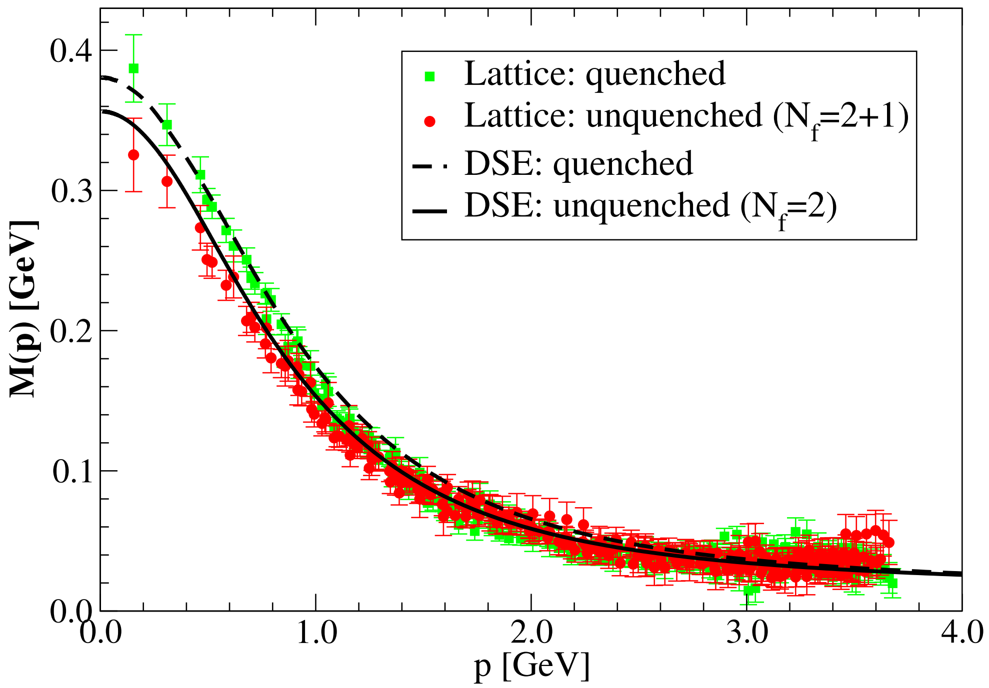

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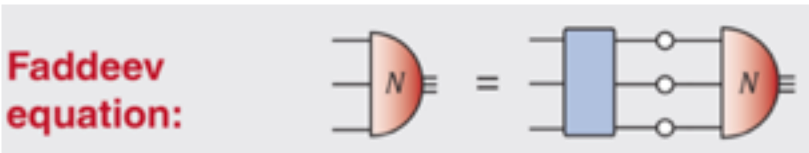
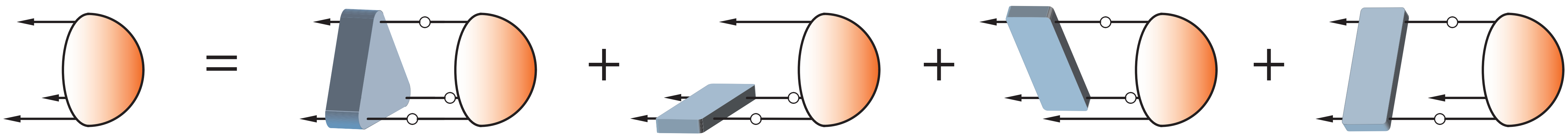
Faddeev - equation





- irreducible three-body forces

Sanchis-Alepuz, Williams, work in progress…

- two-body interactions:
  - non-perturbative gluon exchange

Eichmann, Alkofer, Krassnigg, Nicmorus, PRL 104 (2010)

  - meson exchange

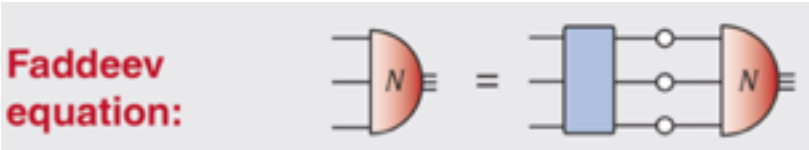
Sanchis-Alepuz, CF, Kubrak, PLB 733 (2014)

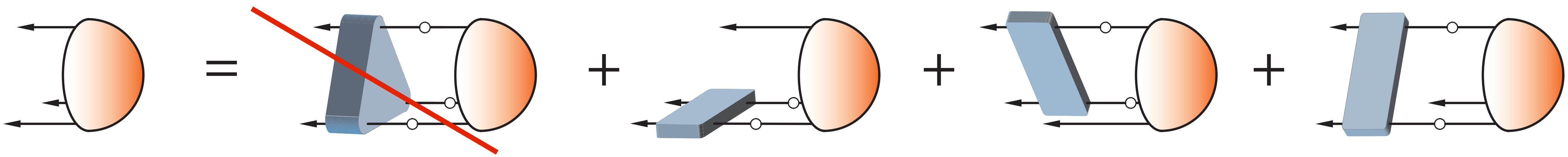
  - two-body forces beyond one-particle exchange

Sanchis-Alepuz, Williams, PLB 749 (2015) 592

- numerically expensive but manageable !

Faddeev - equation





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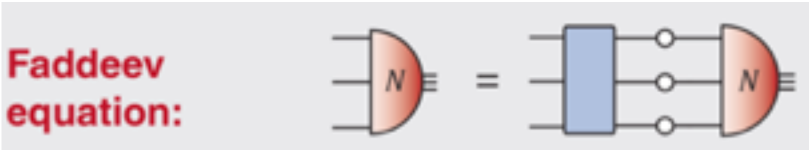

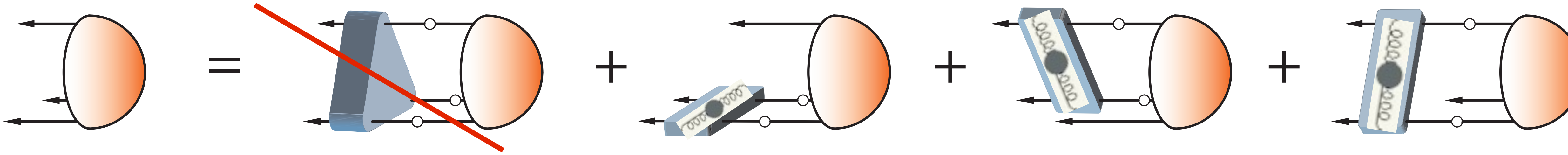

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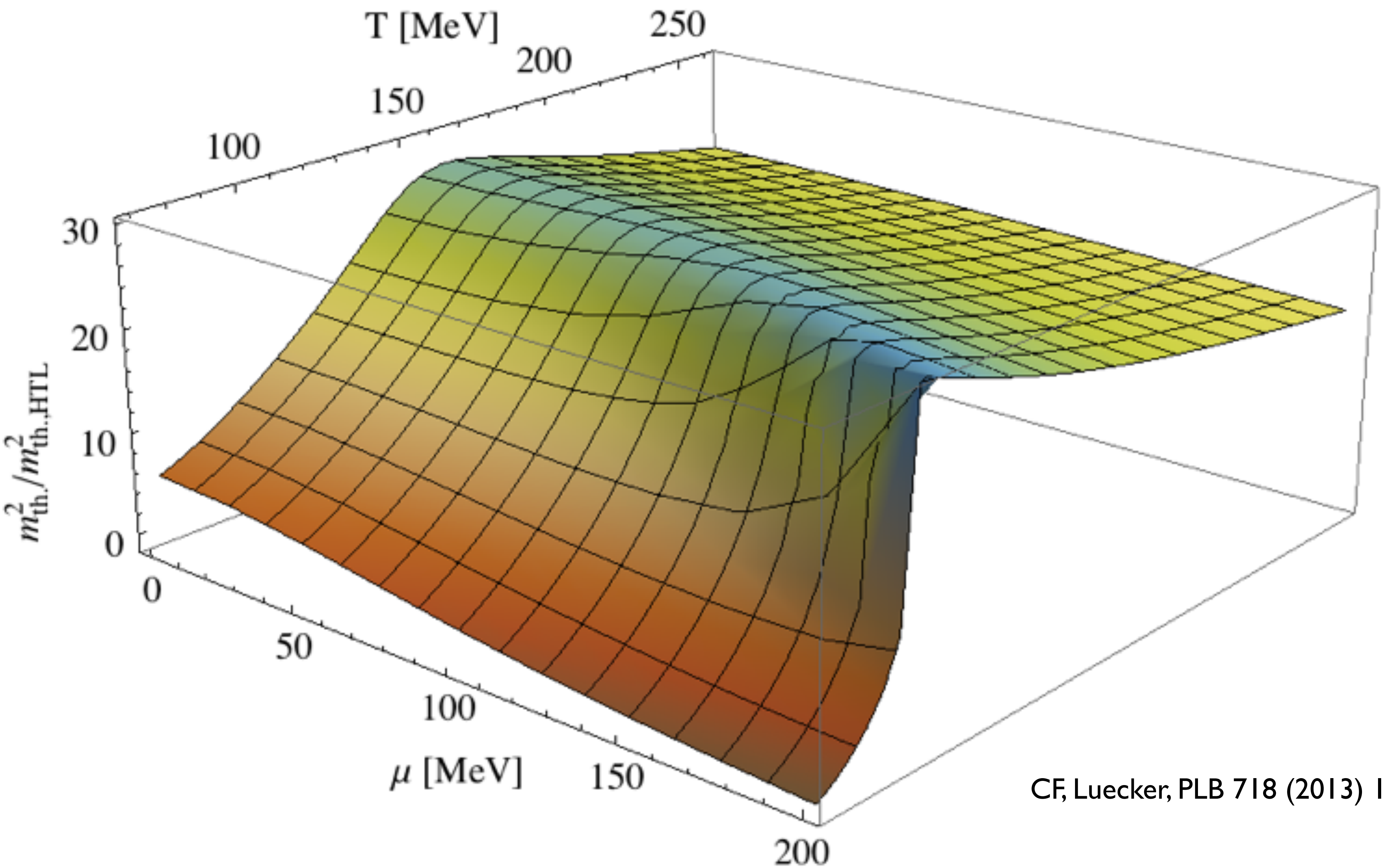
Nf=2+1: thermal electric gluon mass



CF, Luecker, PLB 718 (2013) 1036

- large temperatures: behaviour as expected from HTL
- first order transition at large chemical potential