Functional renormalisation group studies of hot an dense matter: an overview

Jan M. Pawlowski

Universität Heidelberg & ExtreMe Matter Institute

Hirschegg, January 20th 2016

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Bundesministerium für Bildung und Forschung

European Research Council

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EMMI

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an overview

- **talks at the workshop**

C. Almasi	**'Modeling chiral criticality and its consequences for heavy ion collisions'**
C. Fischer	**'Locating QCD's critical end point'**
M. Mitter	**'Chiral symmetry breaking in continuum QCD'**
R.-A. Tripolt	**'Spectral functions and transport coefficients with the FRG'**
J. Eser	**'FRG-study of the chiral phase transition in a QM model with (axial) vector mesons'**
N. Christiansen	**'Transport coefficients in Yang-Mills theory and QCD'**

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- talk to Bernd-Jochen

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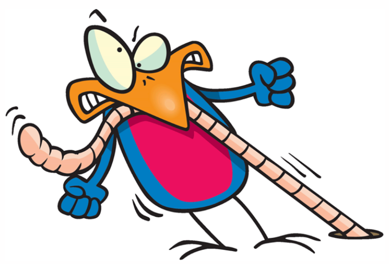
the early bird catches the worm



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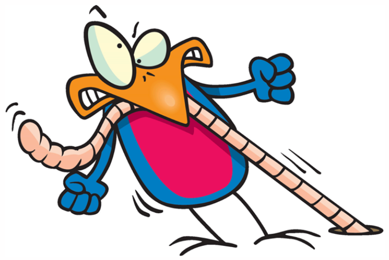

we do not have the worm yet but we see it already!

an overview

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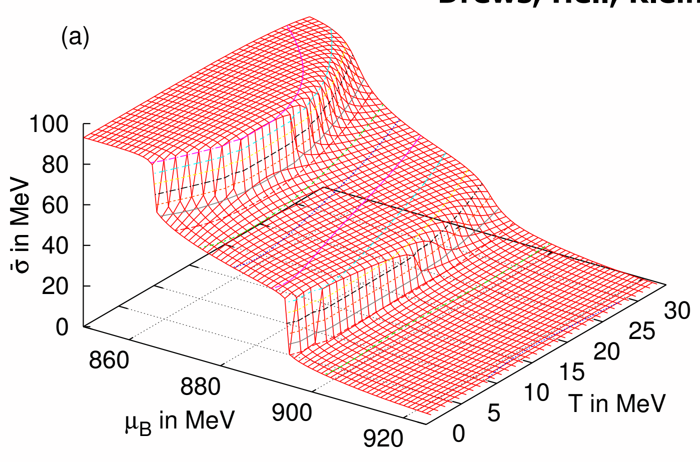
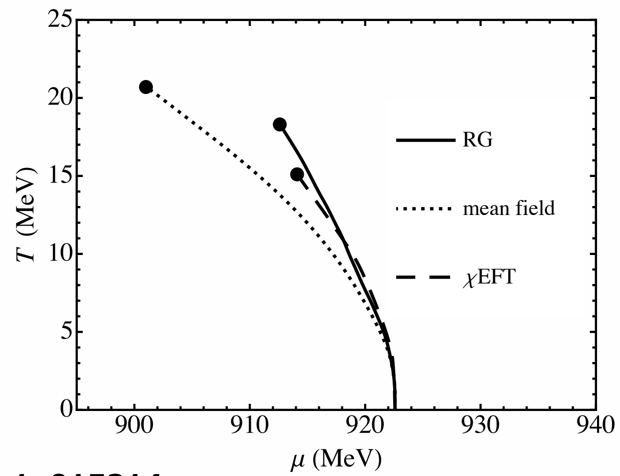
- **topics not treated (selection)**

liquid-gas transition

Drews, Hell, Klein, Weise, Phys.Rev. D88 (2013) 9, 096011





Weyrich, Strodthoff, Smekal, Phys.Rev. C92 (2015) 1, 015214

an overview

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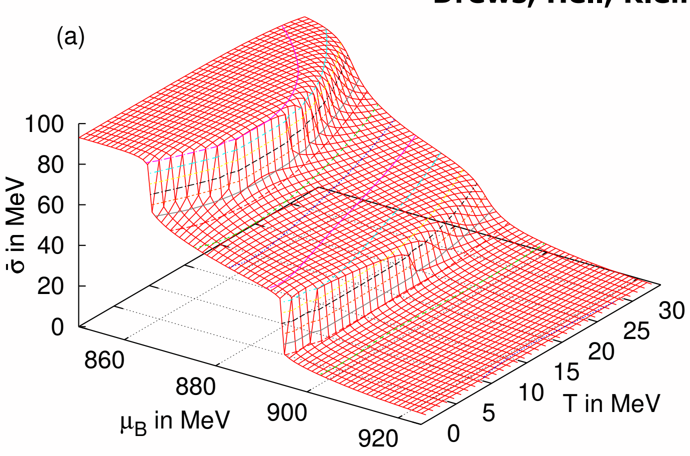
- topics not treated (selection)

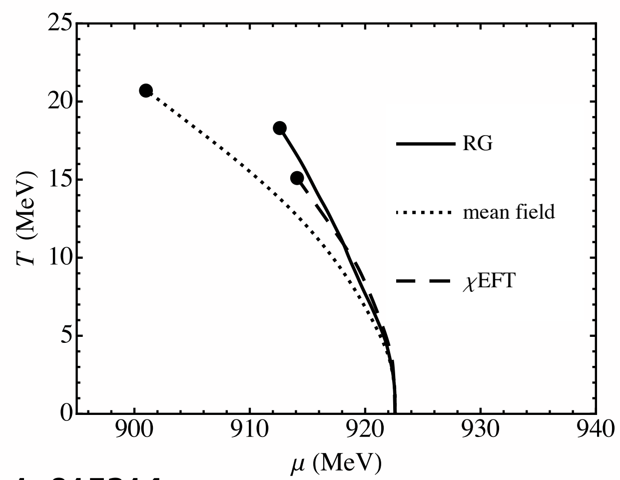
liquid-gas transition

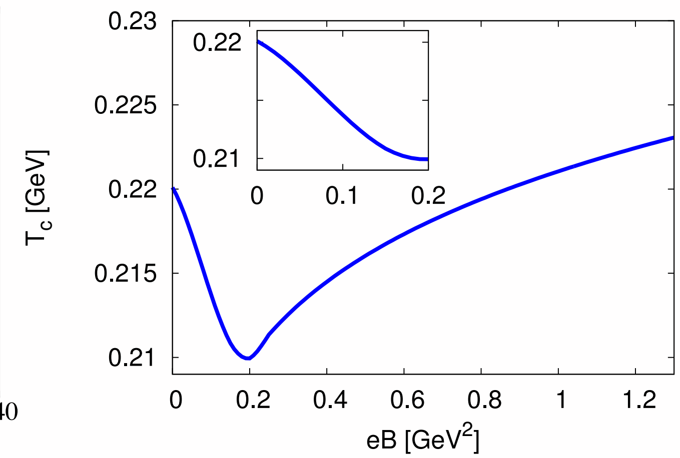
(inverse) magnetic catalysis

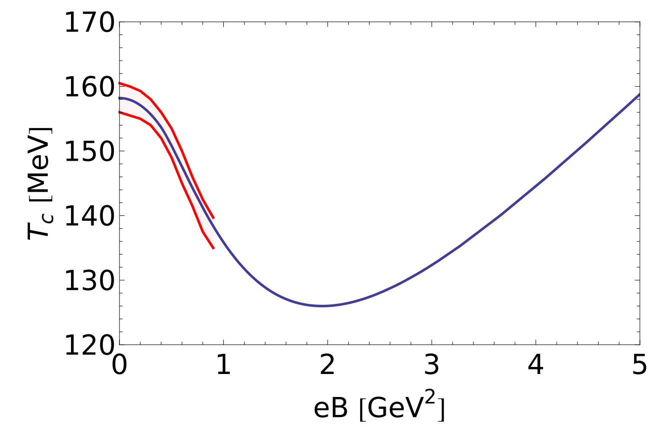
Drews, Hell, Klein, Weise, Phys.Rev. D88 (2013) 9, 096011

Mueller, Pawlowski, Phys.Rev. D91 (2015) 11, 116010





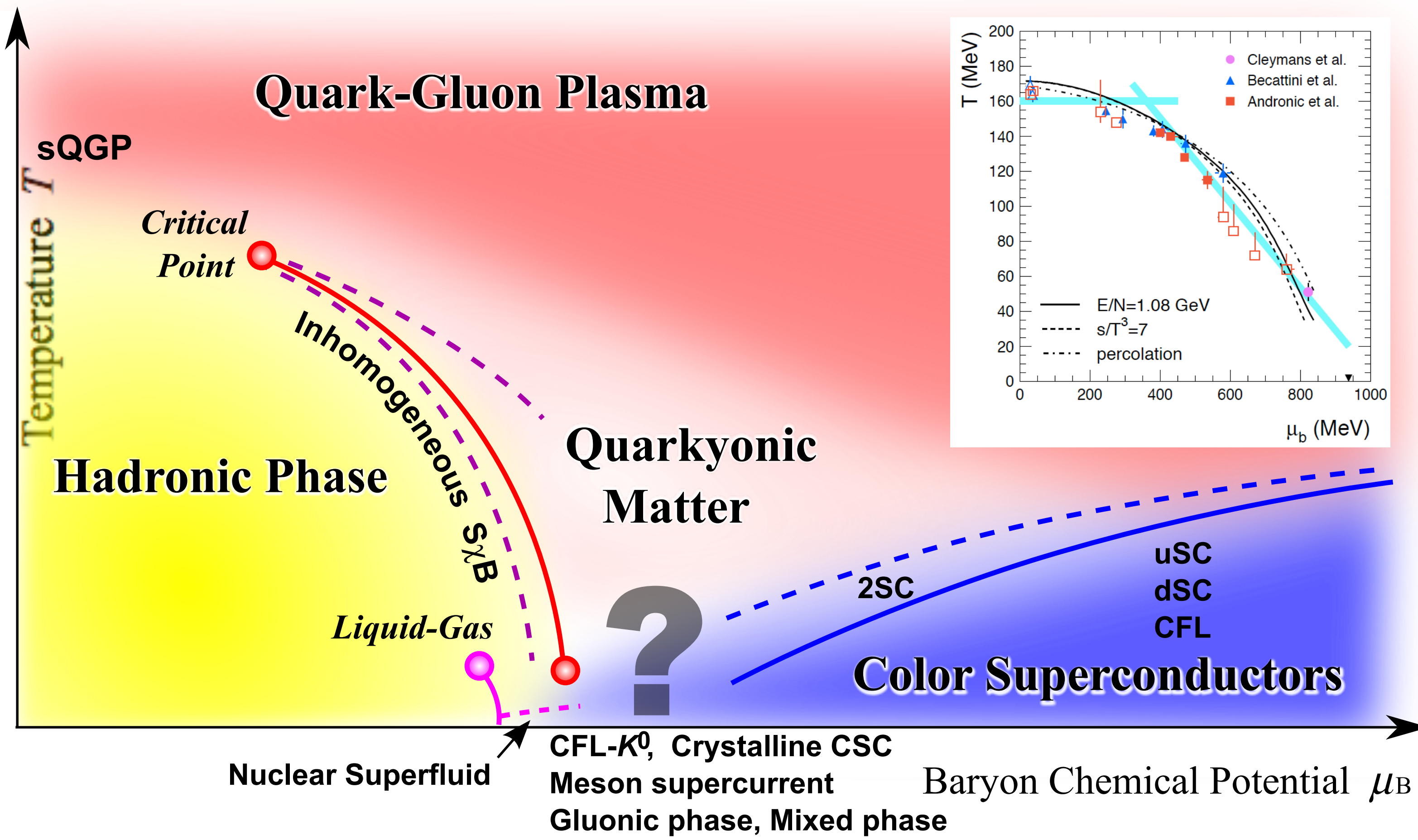


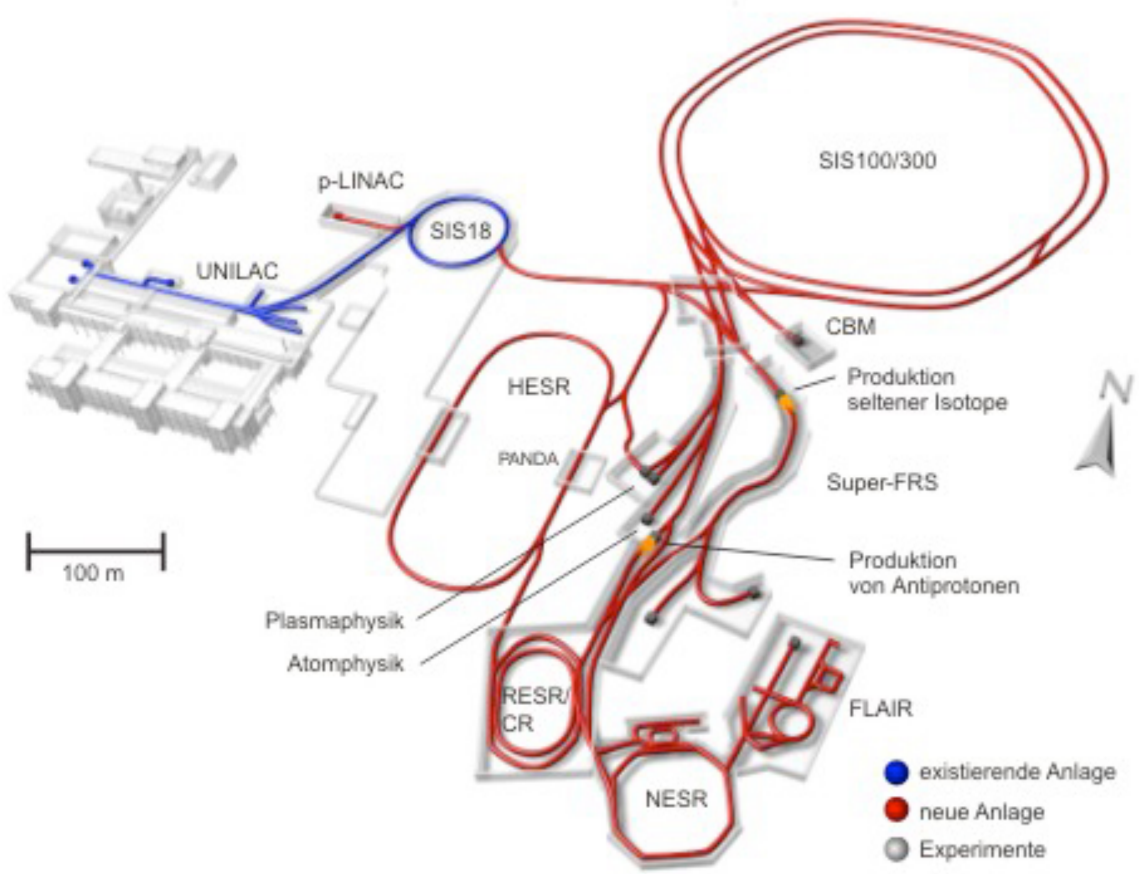


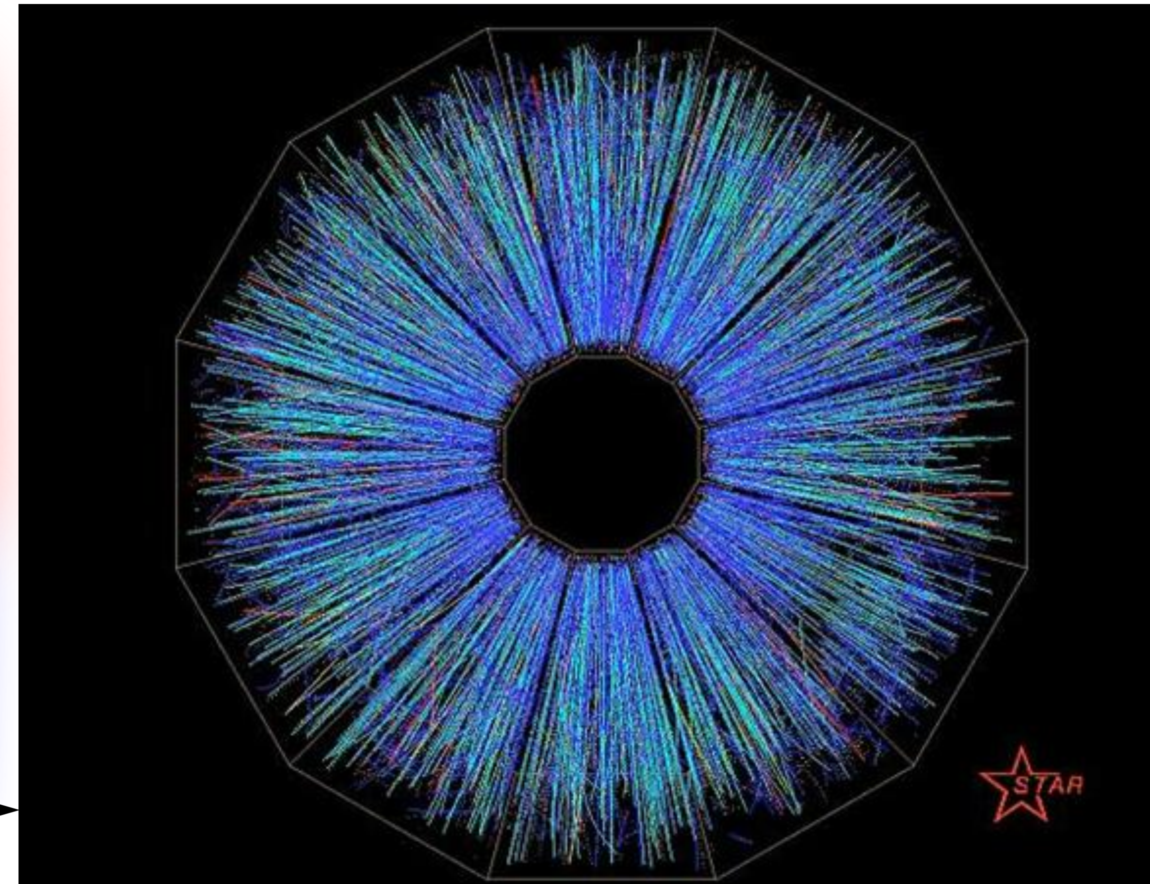
Weyrich, Strodthoff, Smekal, Phys.Rev. C92 (2015) 1, 015214

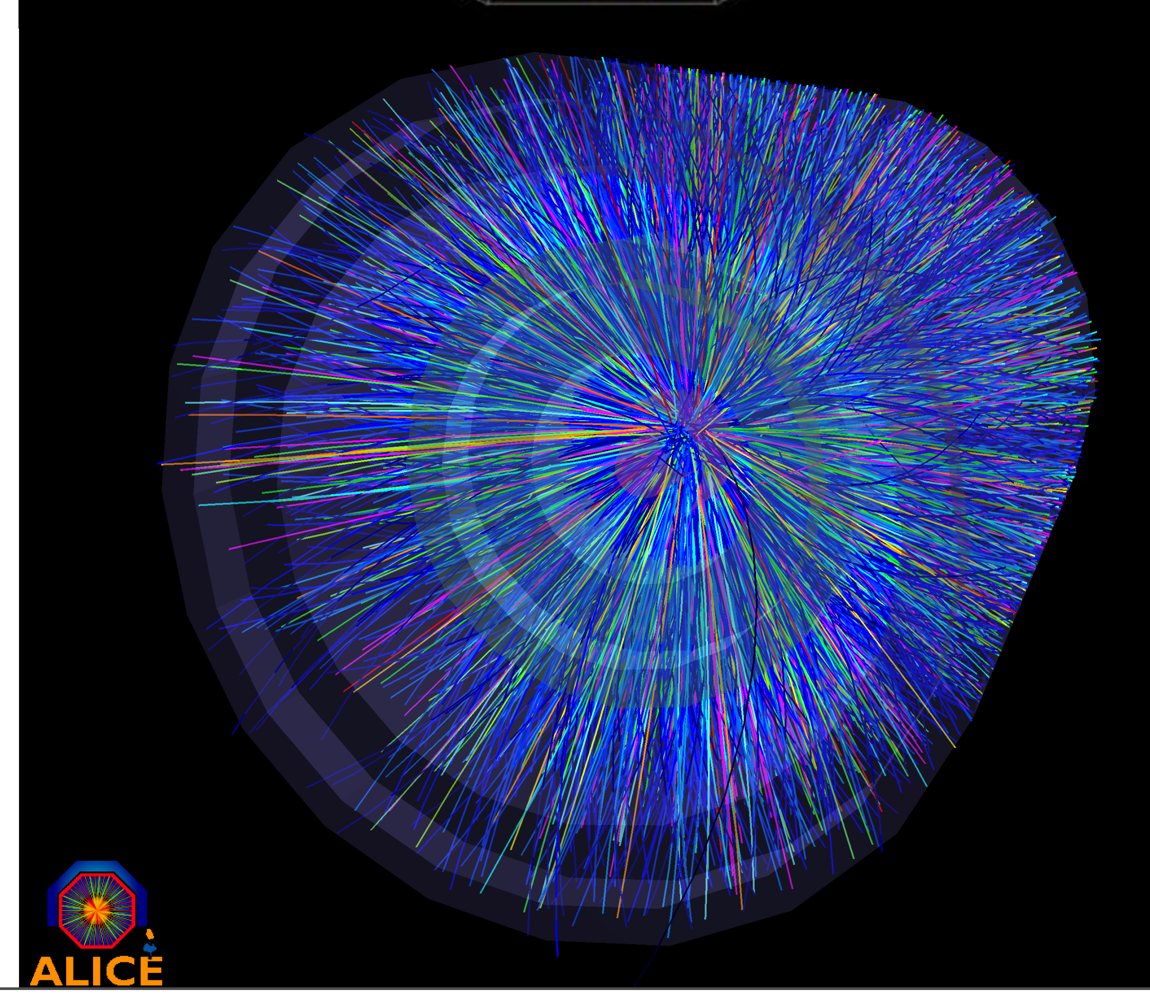
Braun, Mian, Rechenberger, arXiv:1412.6025

Heavy ion collisions





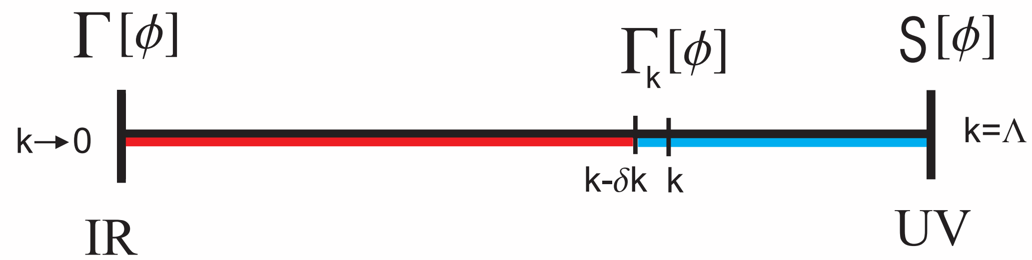




Functional RG for QCD

JMP, AIP Conf.Proc. 1343 (2011)
Nucl.Phys. A931 (2014) 113

free energy at momentum scale k



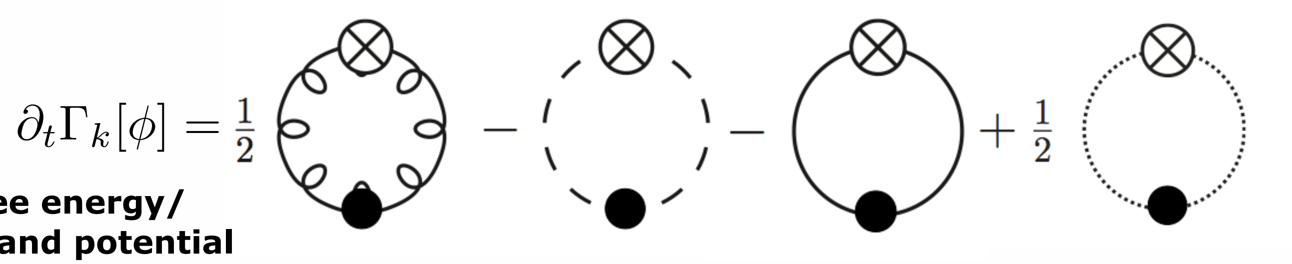
Phase diagram survey

JMP, Schladming '13

ab initio

glue quantum fluctuations

hadronic quantum fluctuations



free energy/ grand potential

quark quantum fluctuations

RG-scale k: $t = \ln k$

Functional RG for QCD

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Nucl.Phys. A931 (2014) 113

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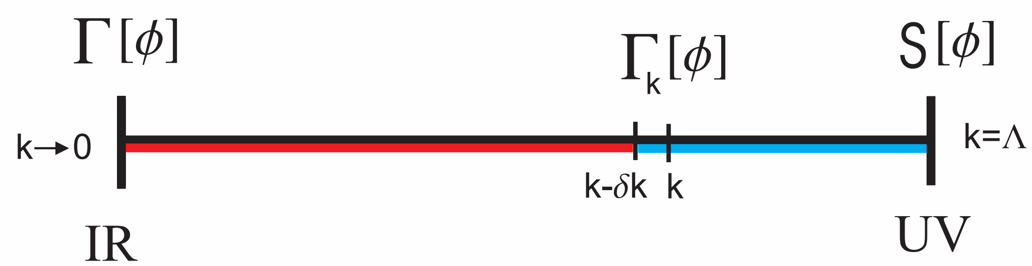

Phase diagram survey

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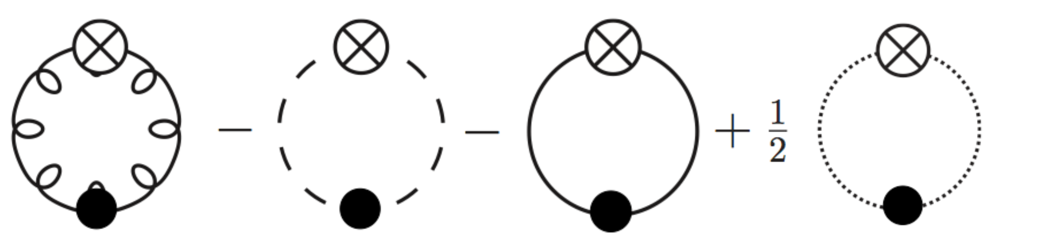
$\partial_t \Gamma_k[\phi] = \frac{1}{2}$

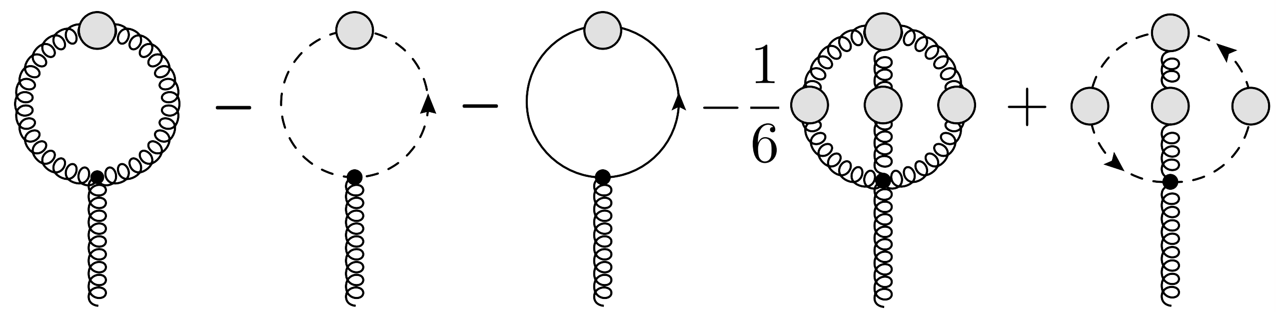


free energy/ grand potential

quark quantum fluctuations

$\text{RG-scale k:}\ t = \ln k$

functional DSE : $\dfrac{\delta\,(\Gamma - S)}{\delta A_0} = \dfrac{1}{2}$

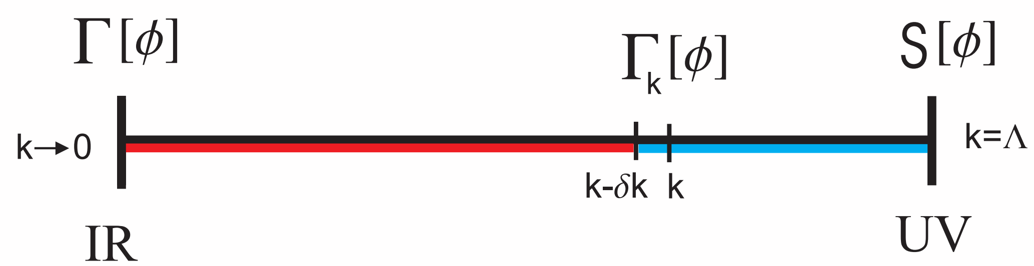


A_0 **: background field**

Functional RG for QCD

JMP, AIP Conf.Proc. 1343 (2011)
Nucl.Phys. A931 (2014) 113

free energy at momentum scale k



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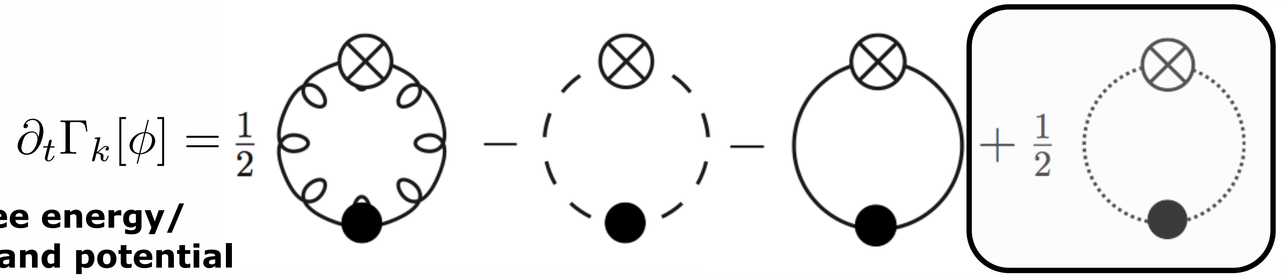

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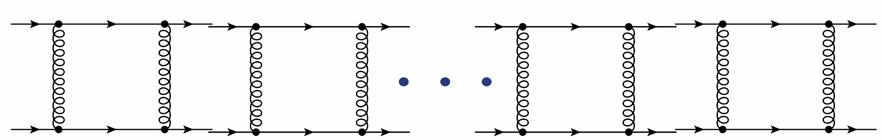
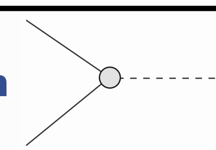


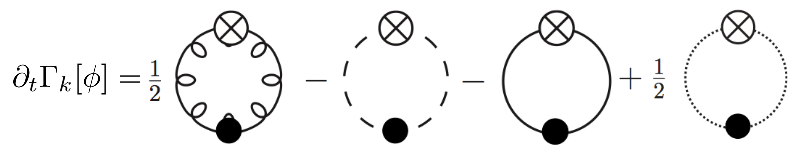
Dynamical hadronisation



dynamical

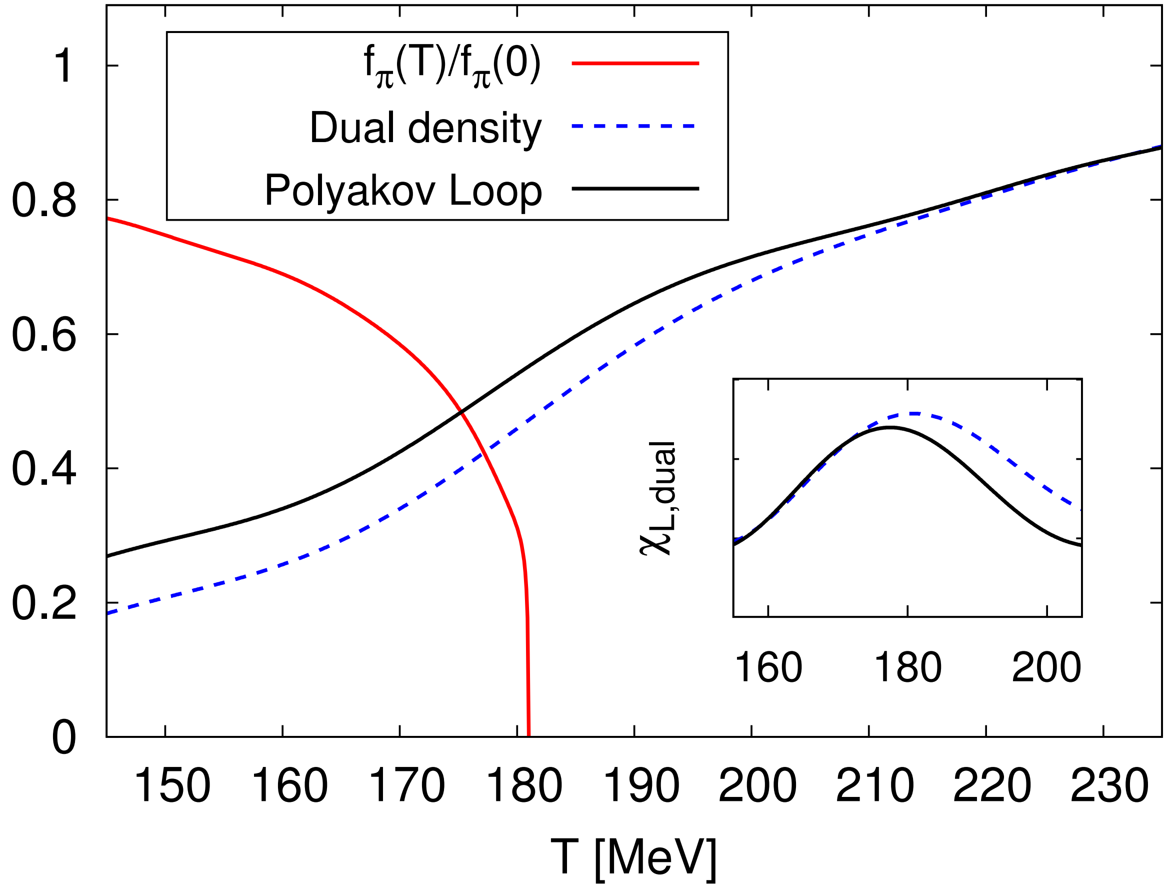
Gies, Wetterich '01
JMP '05
Flörchinger, Wetterich '09

QCD

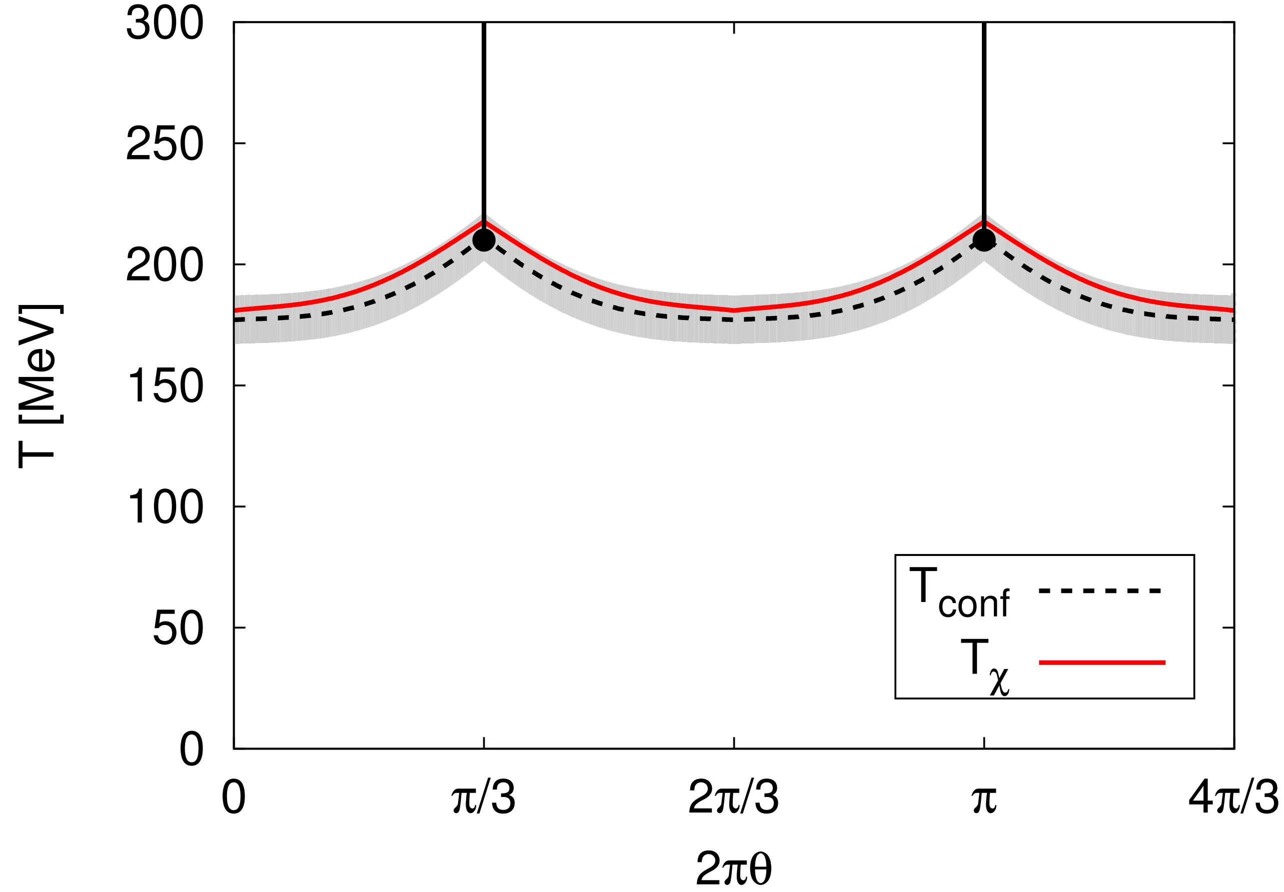


2 flavors & chiral limit

vanishing density



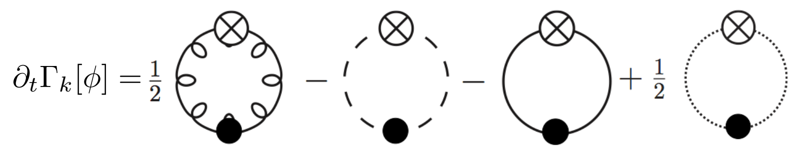
imaginary chemical potential



Braun, Haas, Marhauser, JMP, PRL 106 (2011) 022002

Flows towards the fluctuating PQM model for low energies

QCD



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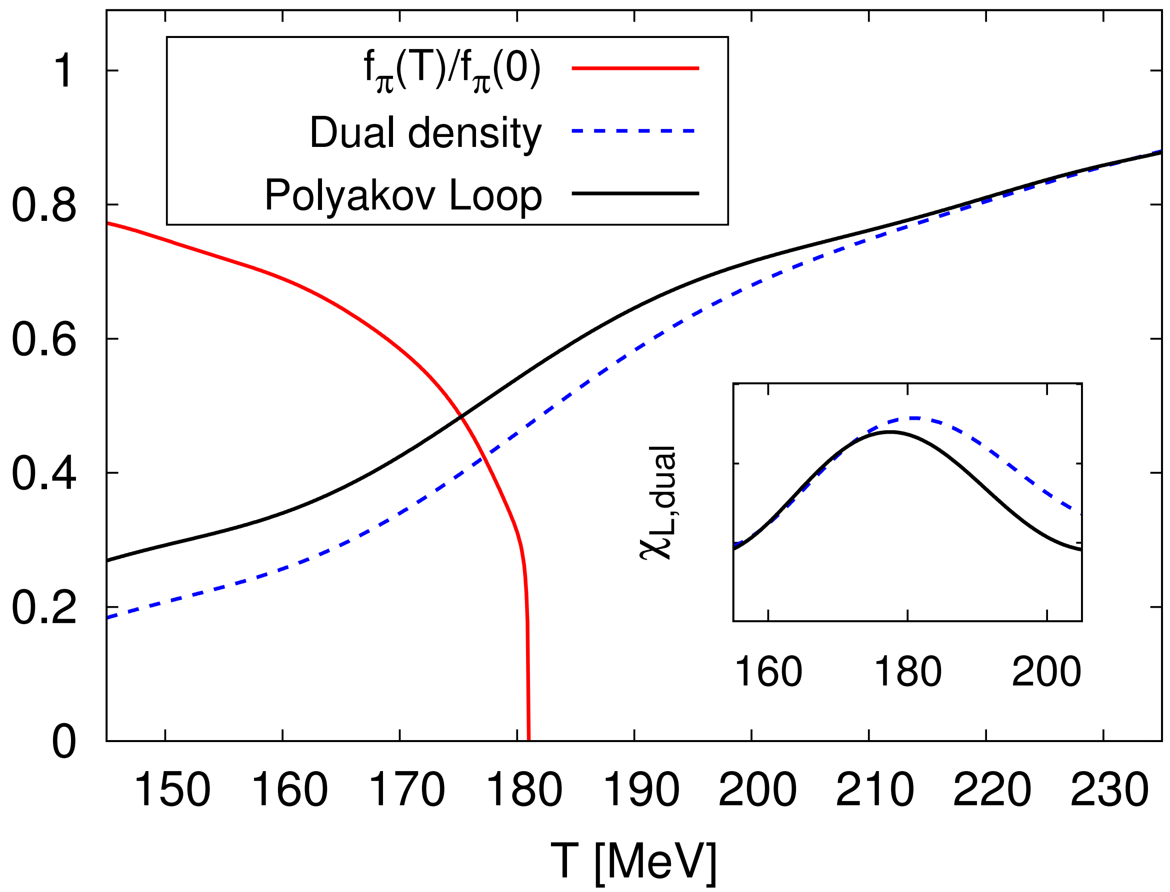

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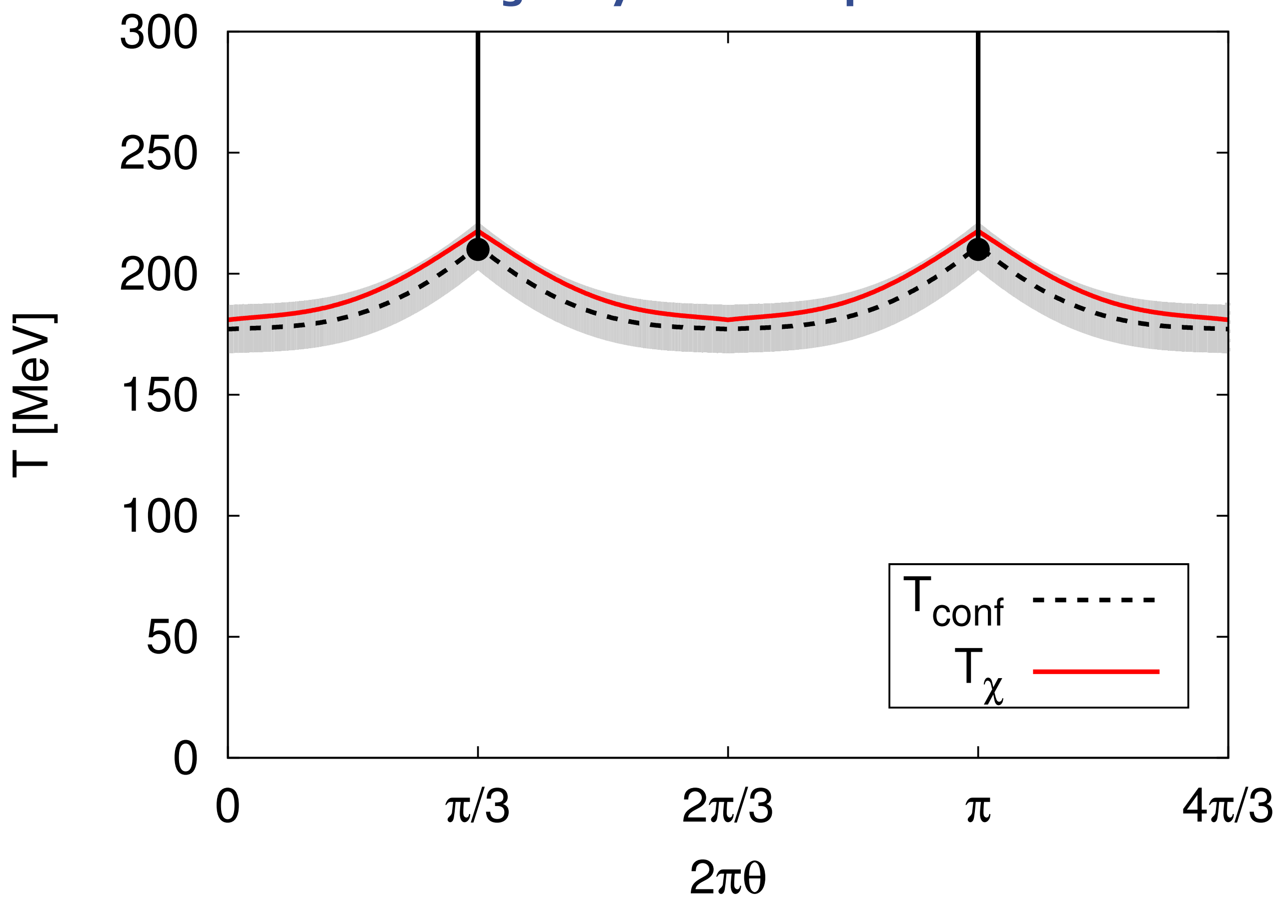
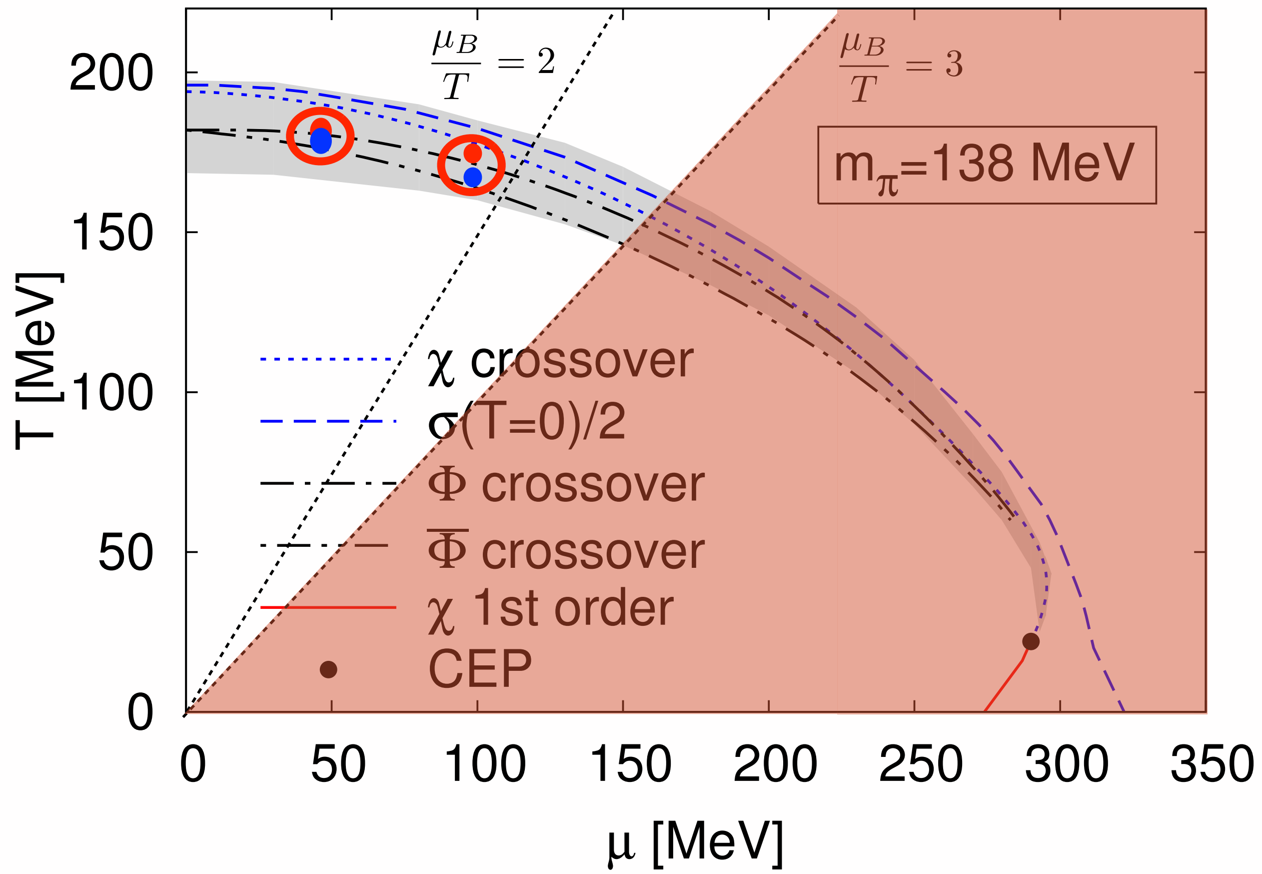


Braun, Haas, Marhauser, JMP, PRL 106 (2011) 022002

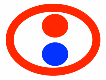
Flows towards the fluctuating PQM model for low energies

fQCD: motivation

Phase diagram of quantised 2-flavor PQM-model



Herbst, JMP, Schaefer, PLB 696 (2011) 58-67
PRD 88 (2013) 1, 014007



FRG QCD results at finite density

Haas, Braun, JMP '09, unpublished

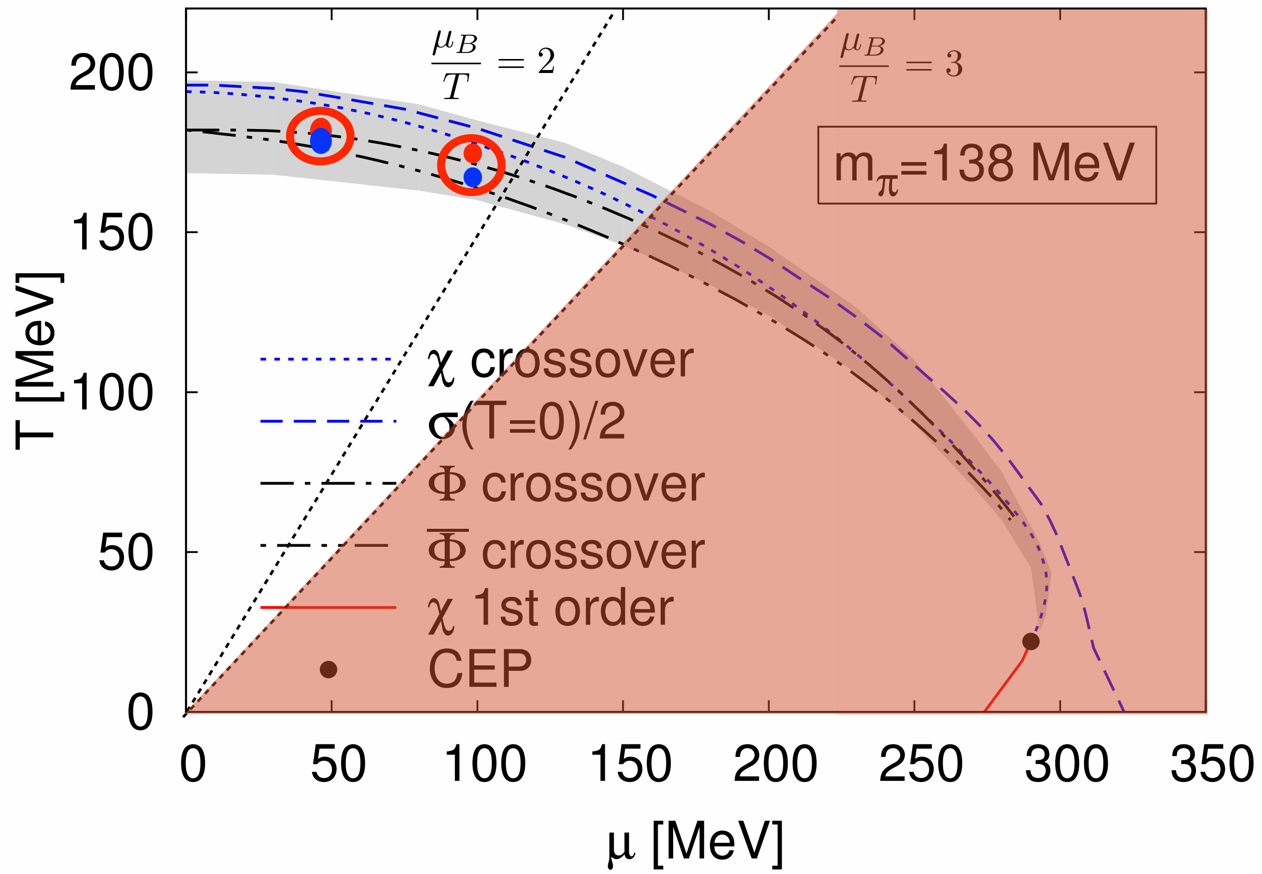
Extension of FRG QCD results at imaginary chemical potential

Braun, Haas, Marhauser, JMP, PRL 106 (2011) 022002

fQCD: motivation

see talk of Mario Mitter

Phase diagram of quantised 2-flavor PQM-model



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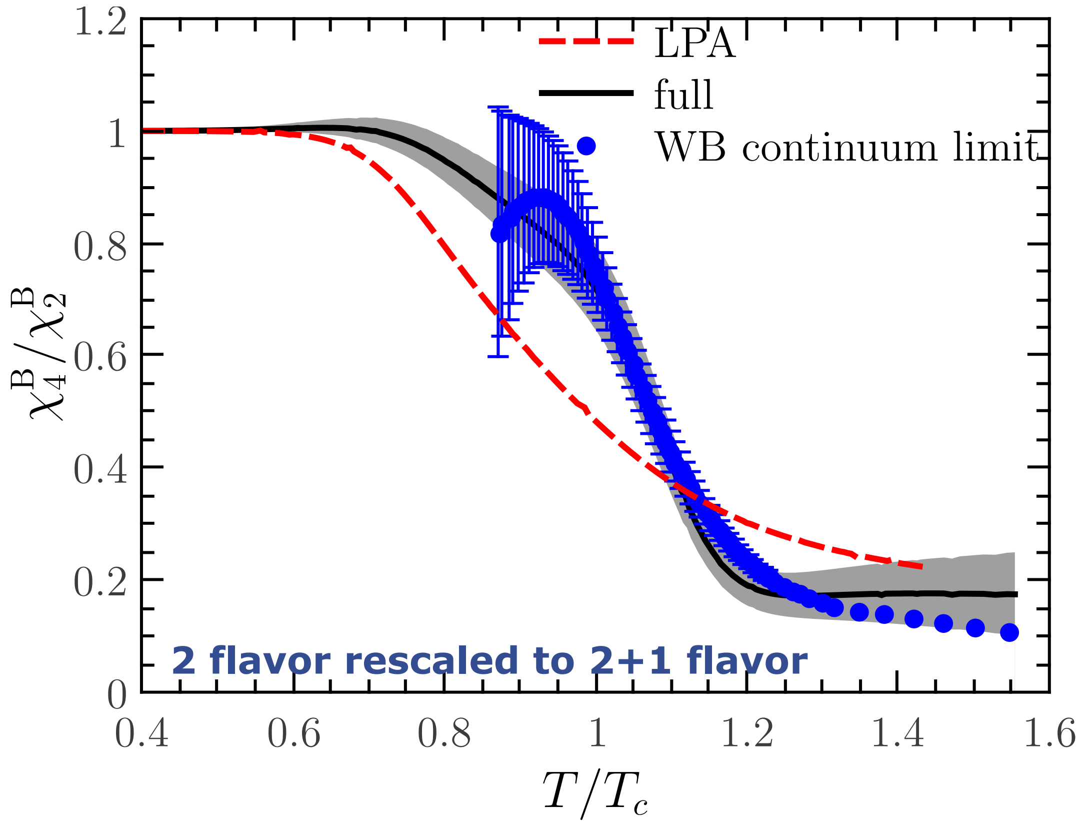
Thermodynamics

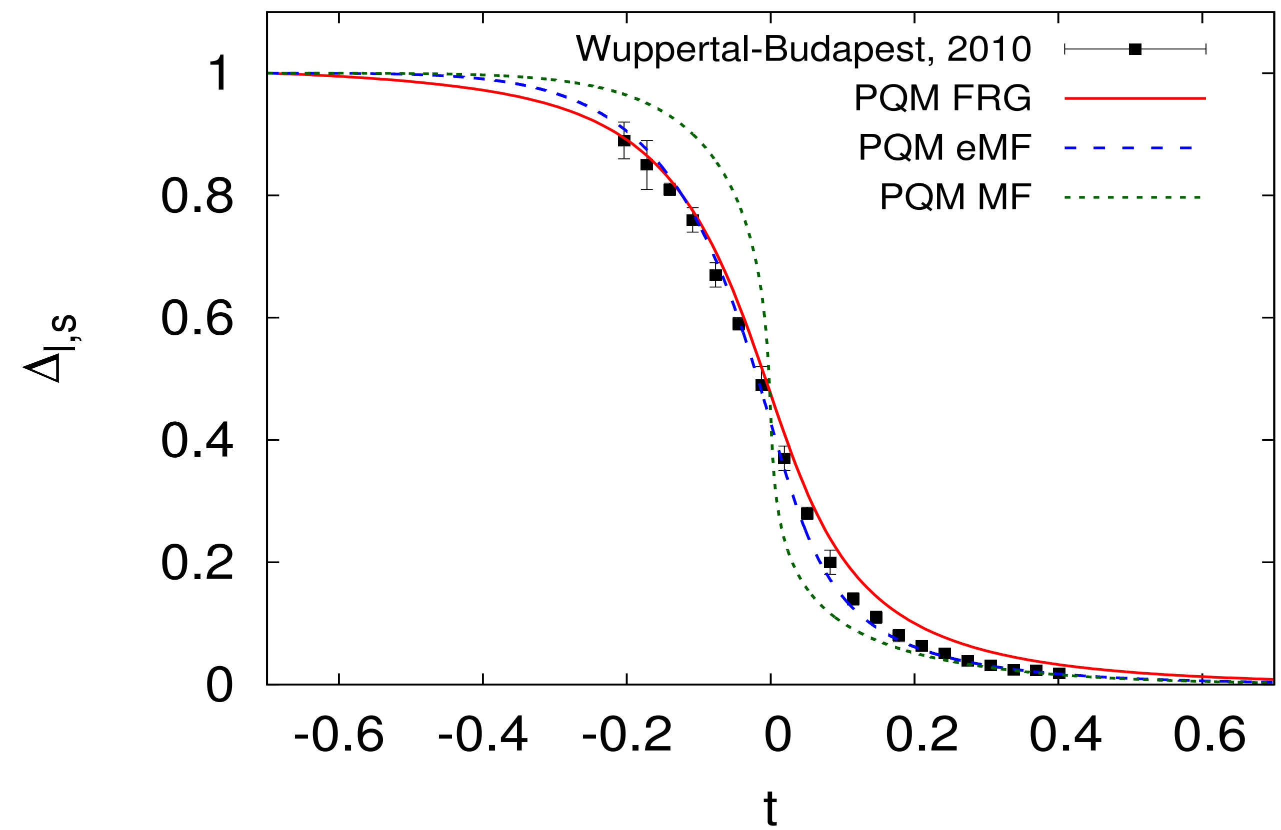
2+1 flavor QCD - enhanced PQM-model

Kurtosis

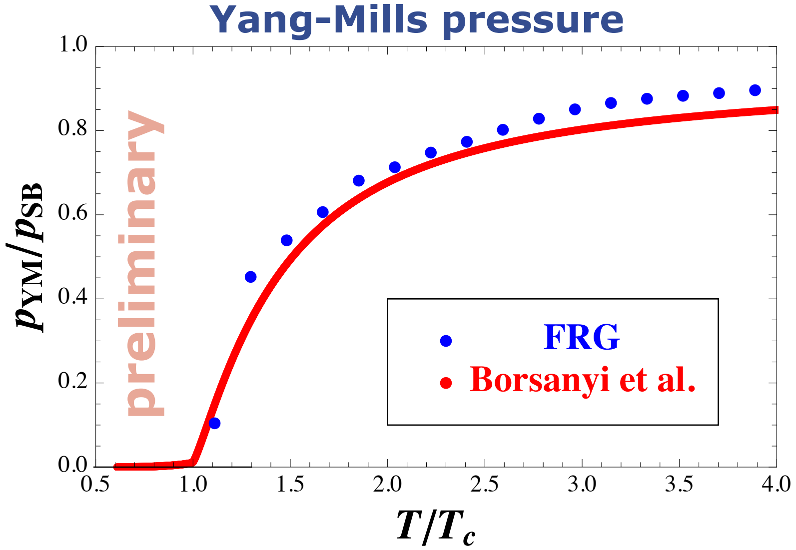


Fu, JMP, PRD 92 (2015) 11, 116006

reduced chiral condensate



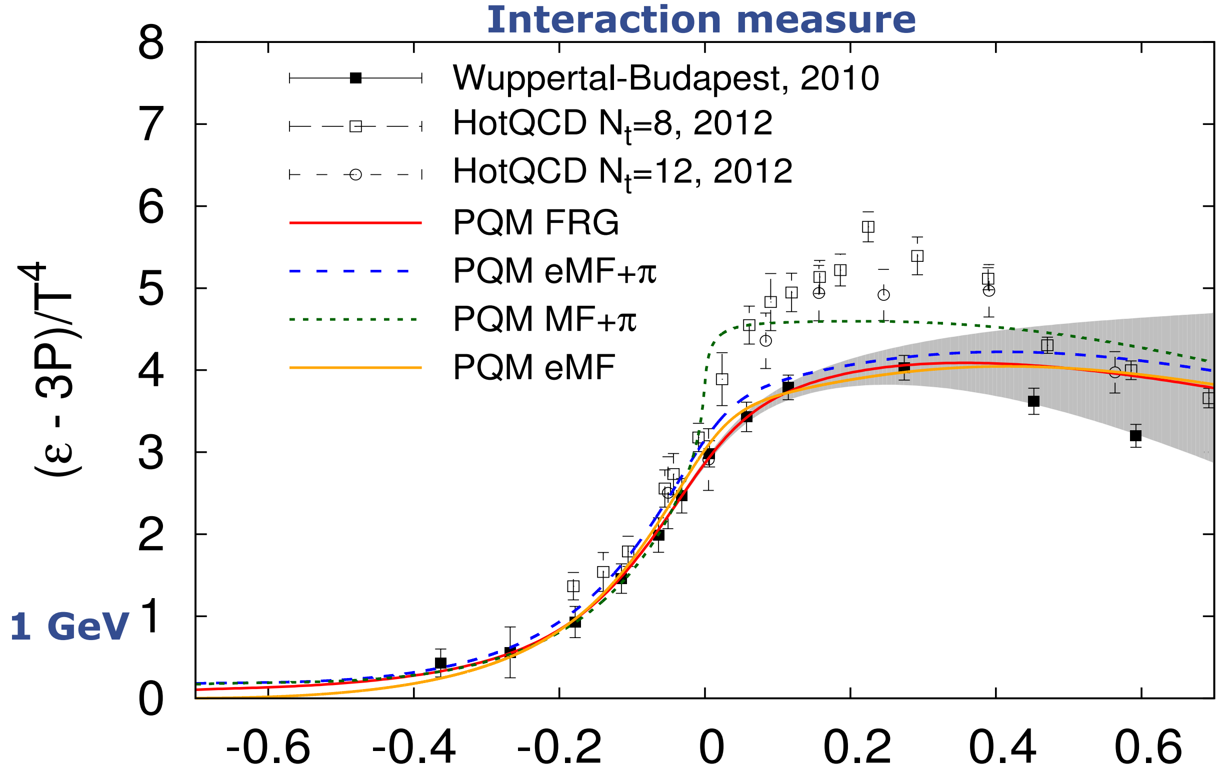
Yang-Mills pressure



Fister, JMP '11

Shaded area:
systematic error estimate
due to low initial UV scale 1 GeV

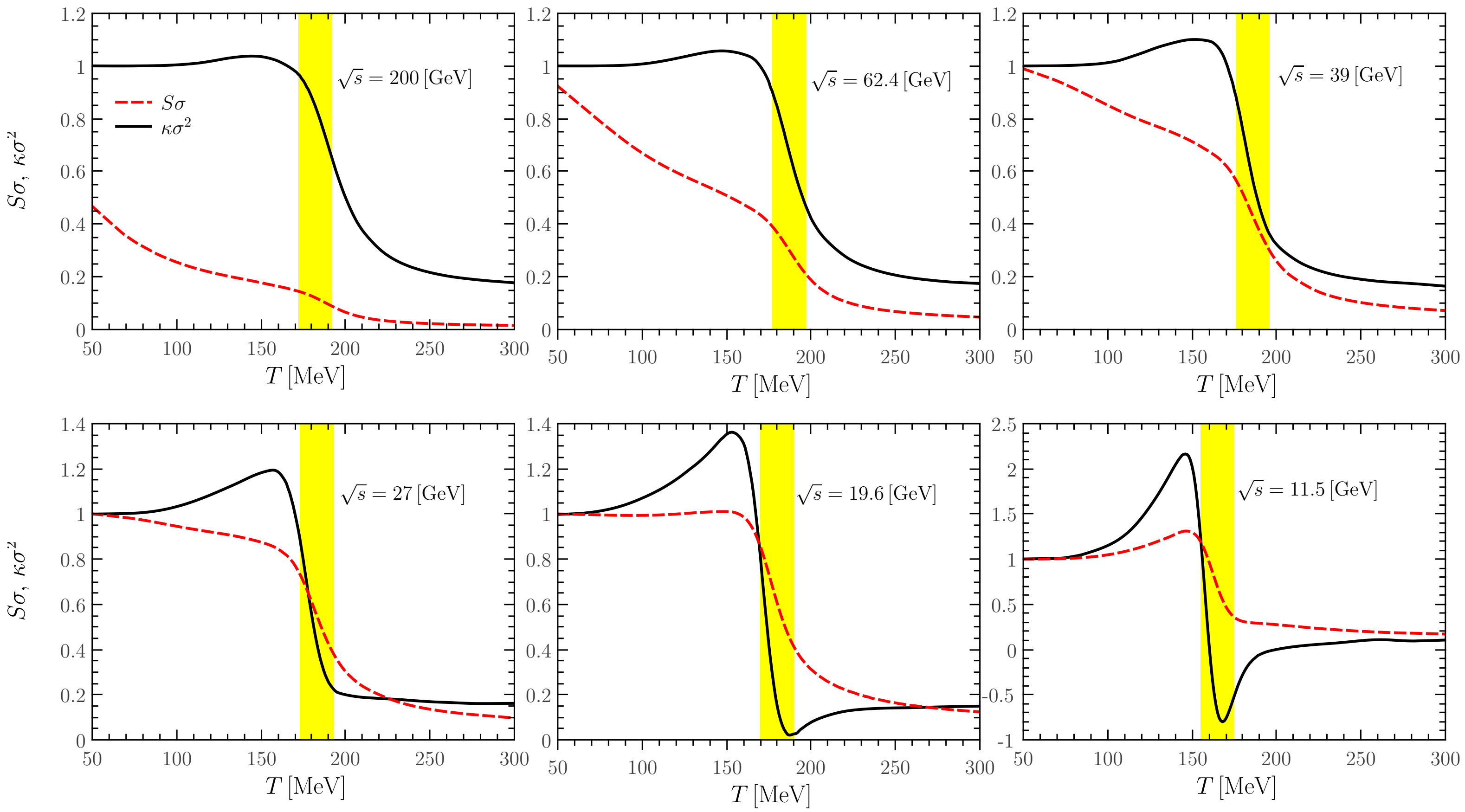
Interaction measure

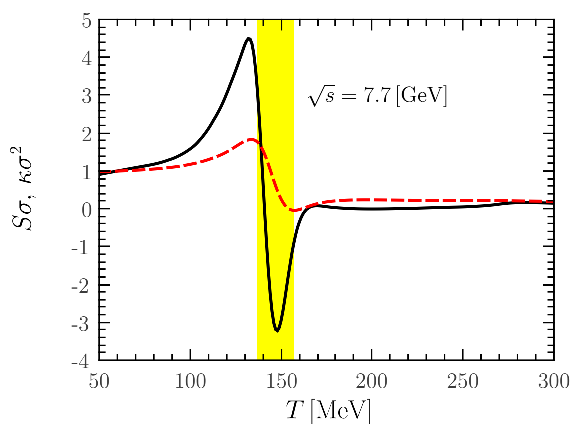


Herbst, Mitter et al, PLB 731 (2014) 248-256

Fluctuations

Skewness, Kurtosis

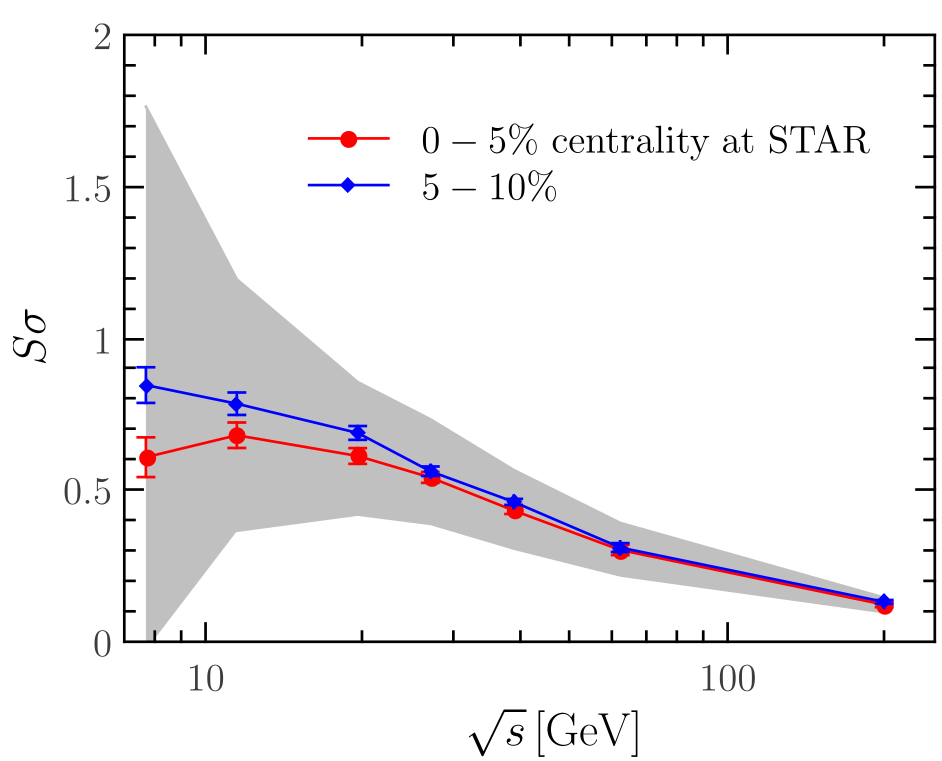


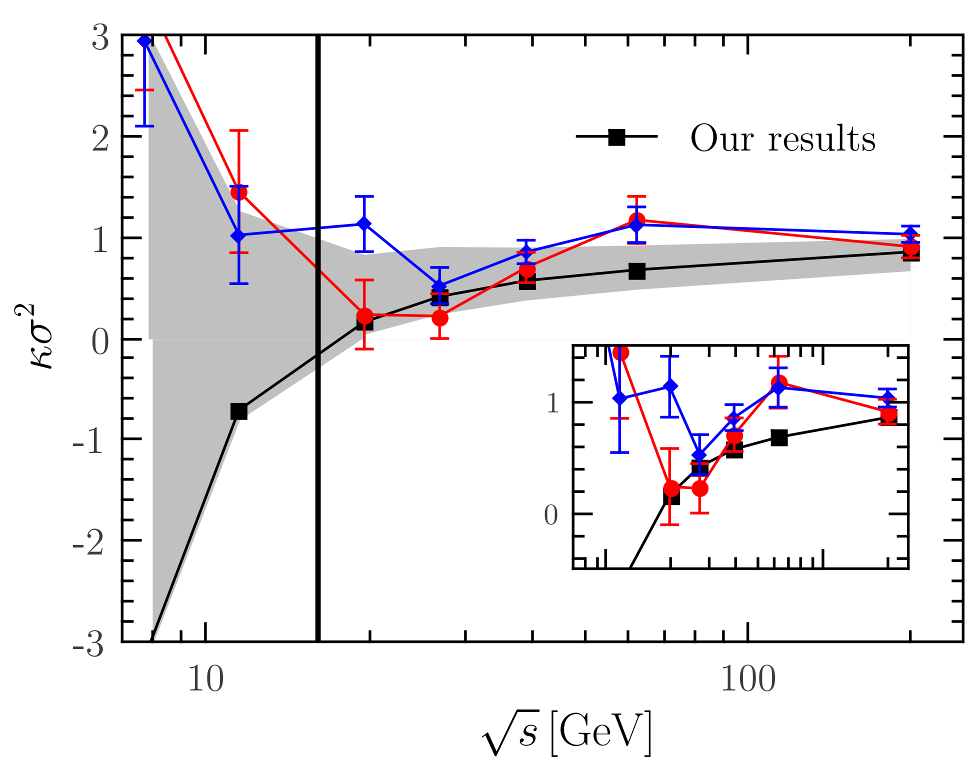


$$\chi_n^{\mathrm{B}} = \frac{\partial^n}{\partial(\mu_{\mathrm{B}}/T)^n}\frac{p}{T^4}$$

$$\sigma^2 = VT^3\chi_2^{\mathrm{B}}$$

$$S = \frac{\chi_3^{\mathrm{B}}}{\chi_2^{\mathrm{B}}\sigma} \qquad \kappa = \frac{\chi_4^{\mathrm{B}}}{\chi_2^{\mathrm{B}}\sigma^2}$$

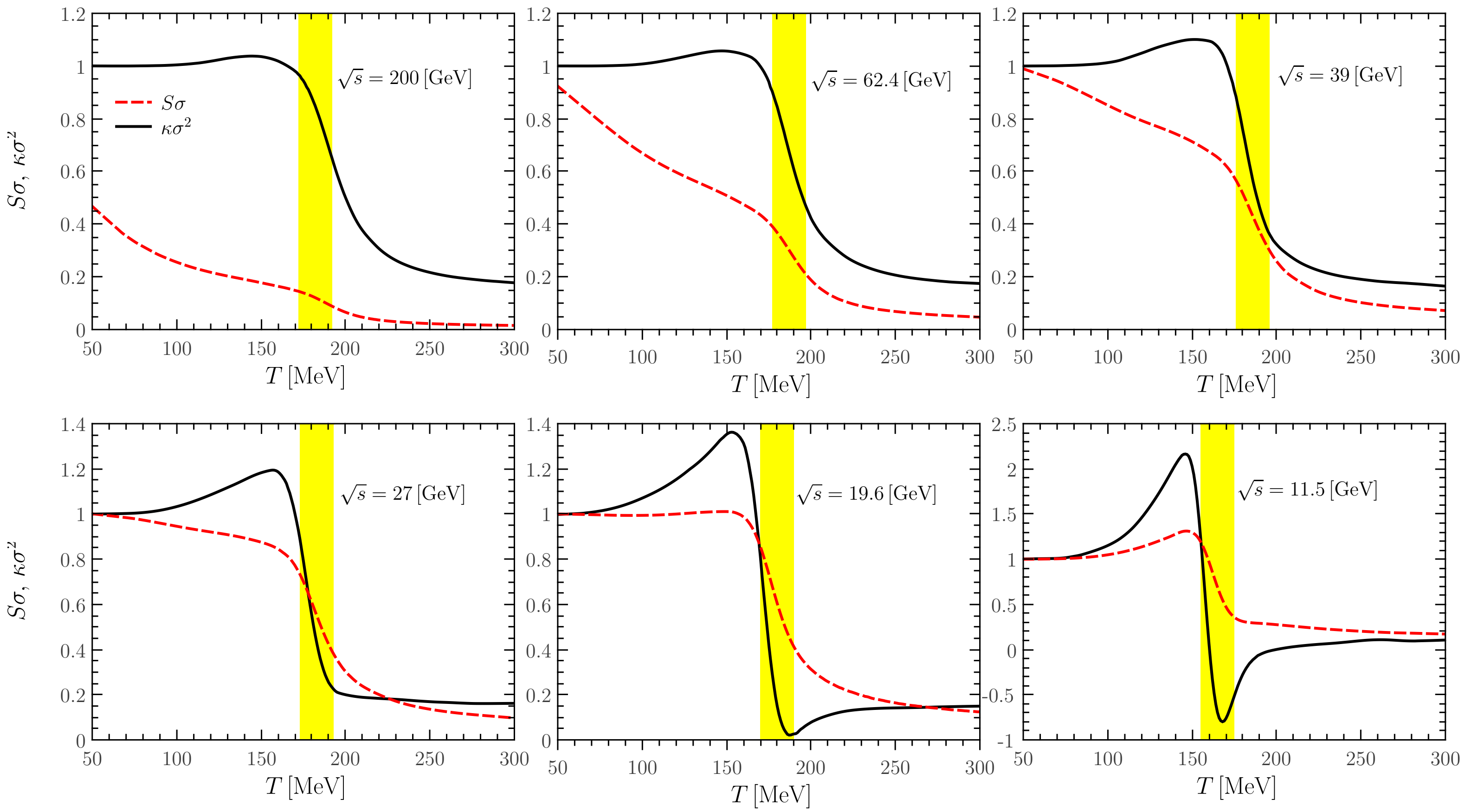


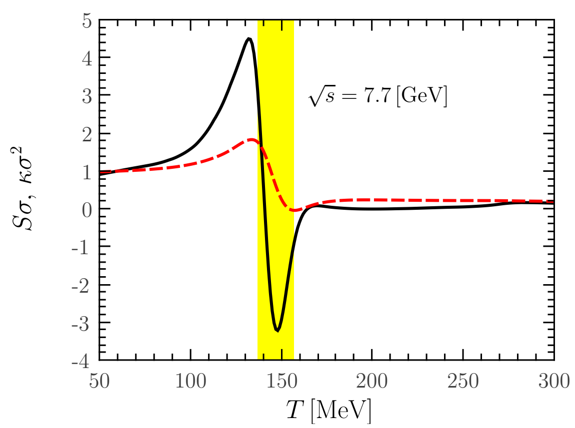


Fu, JMP, arXiv:1512.08461

Fluctuations

Skewness, Kurtosis





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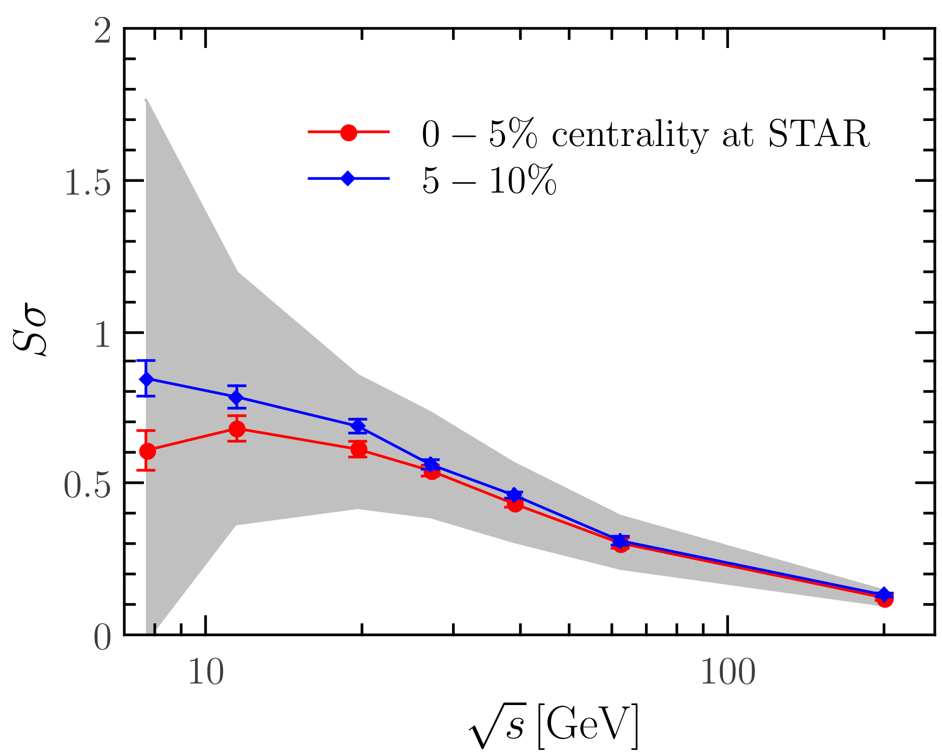
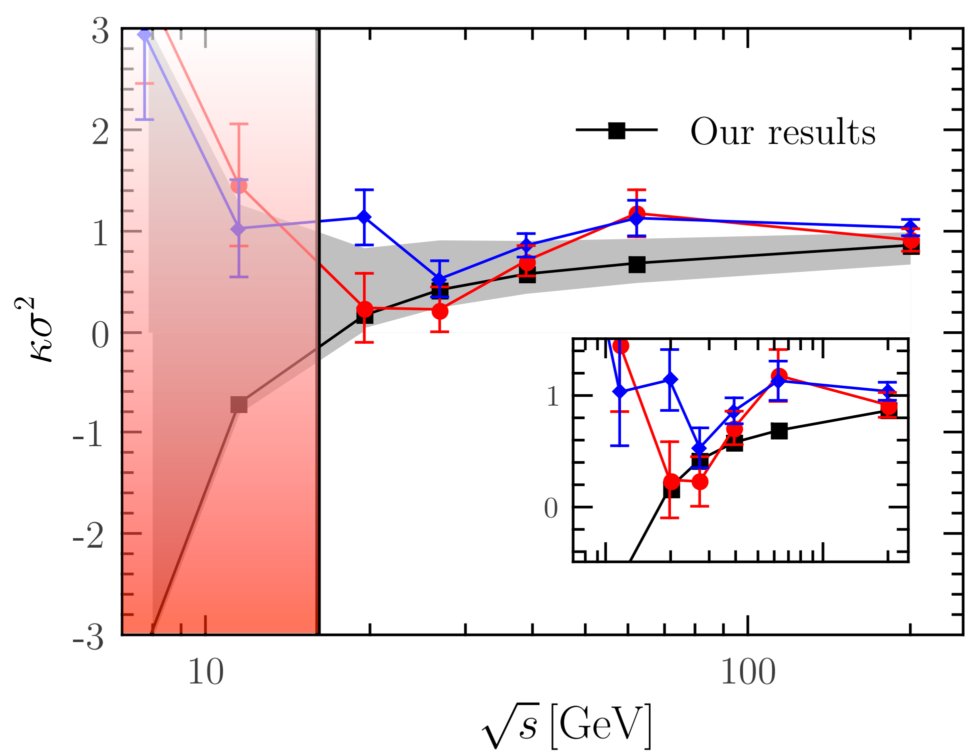
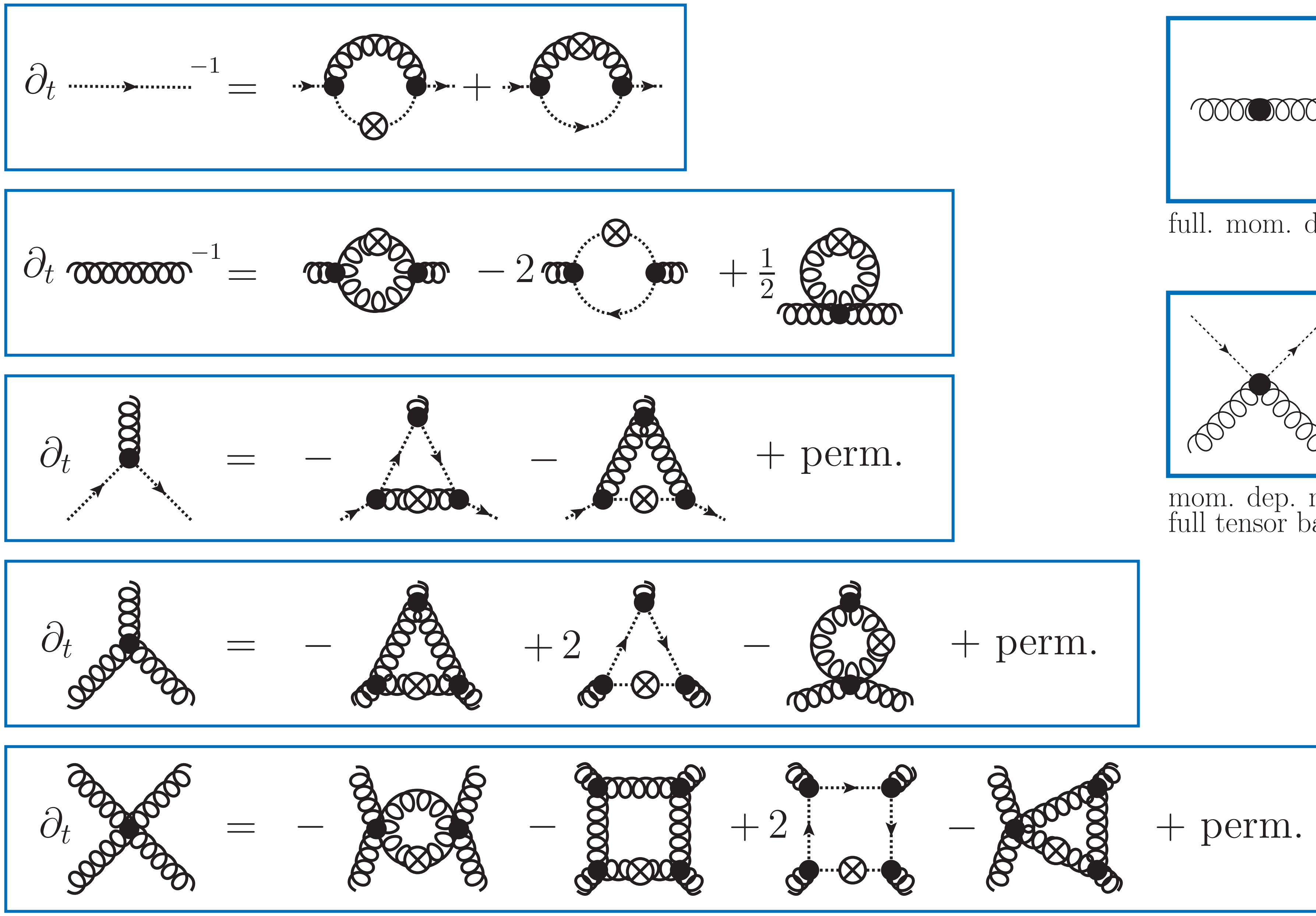
Fu, JMP, Schaefer, Rennecke, work in progress

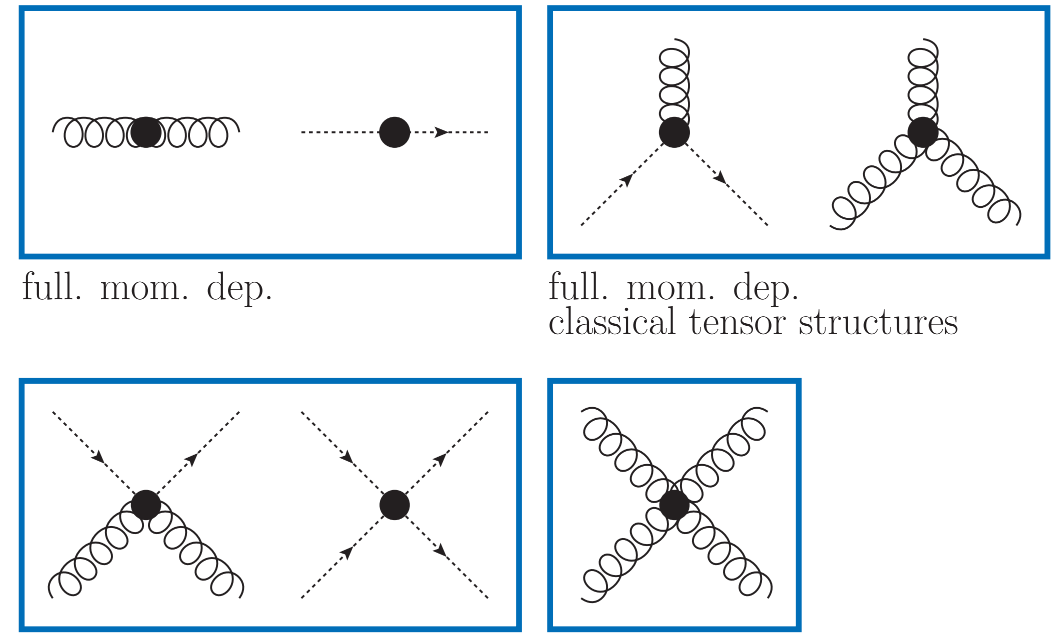




Fu, JMP, arXiv:1512.08461

Glue sector





full. mom. dep.

full. mom. dep.
classical tensor structures

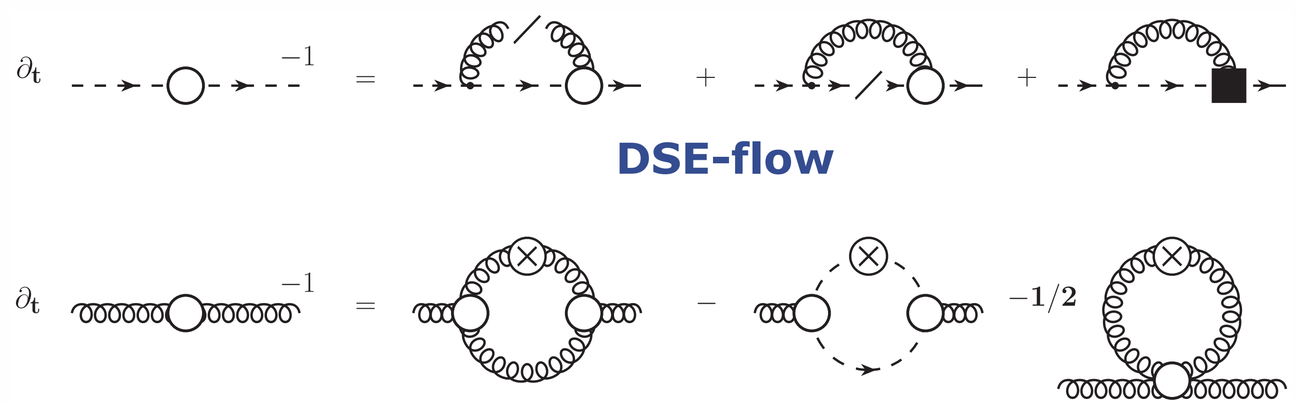
mom. dep. needed by tadpoles
full tensor basis

sym. point mom. dep. and
mom. dep. needed by tadpole
classical tensor structure

Aiming at apparent convergence

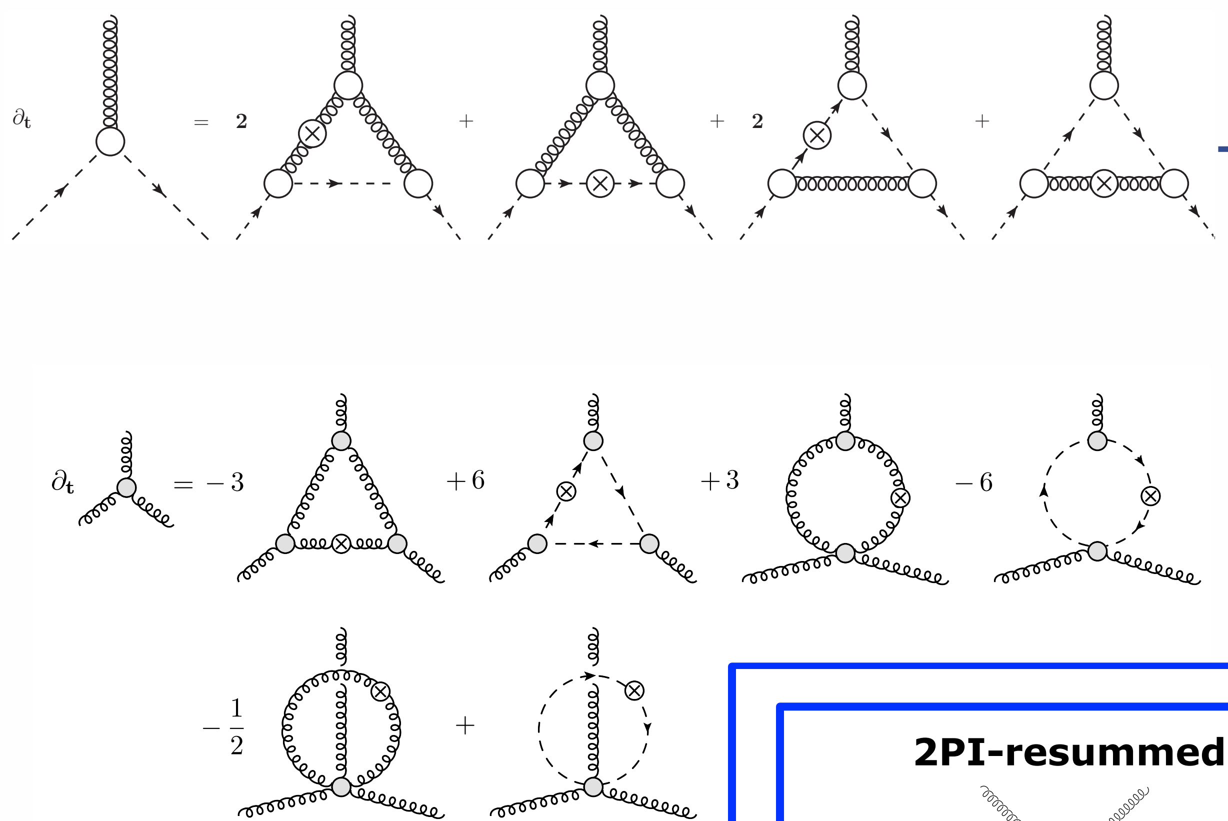
Cyrol, Fister, Mitter, JMP, Strodthoff, in prep.

Glue sector



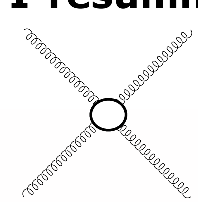
DSE-flow

2PI-resummation

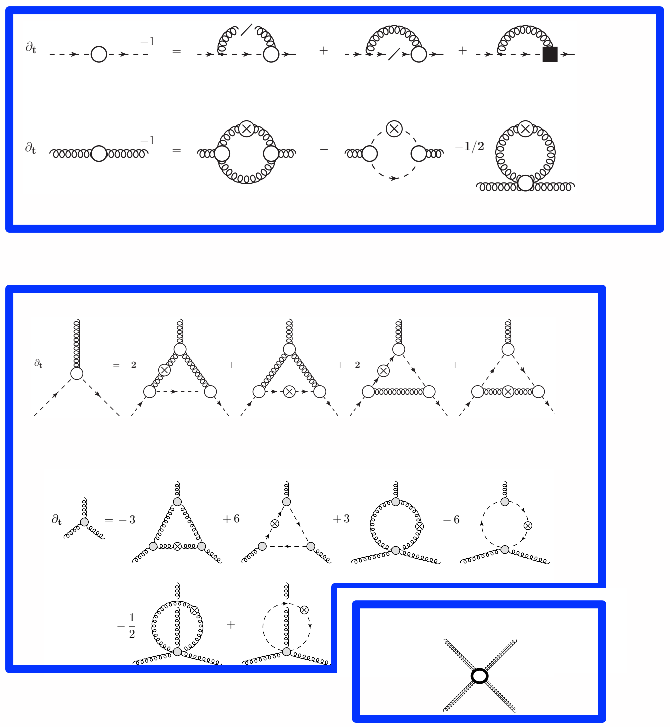


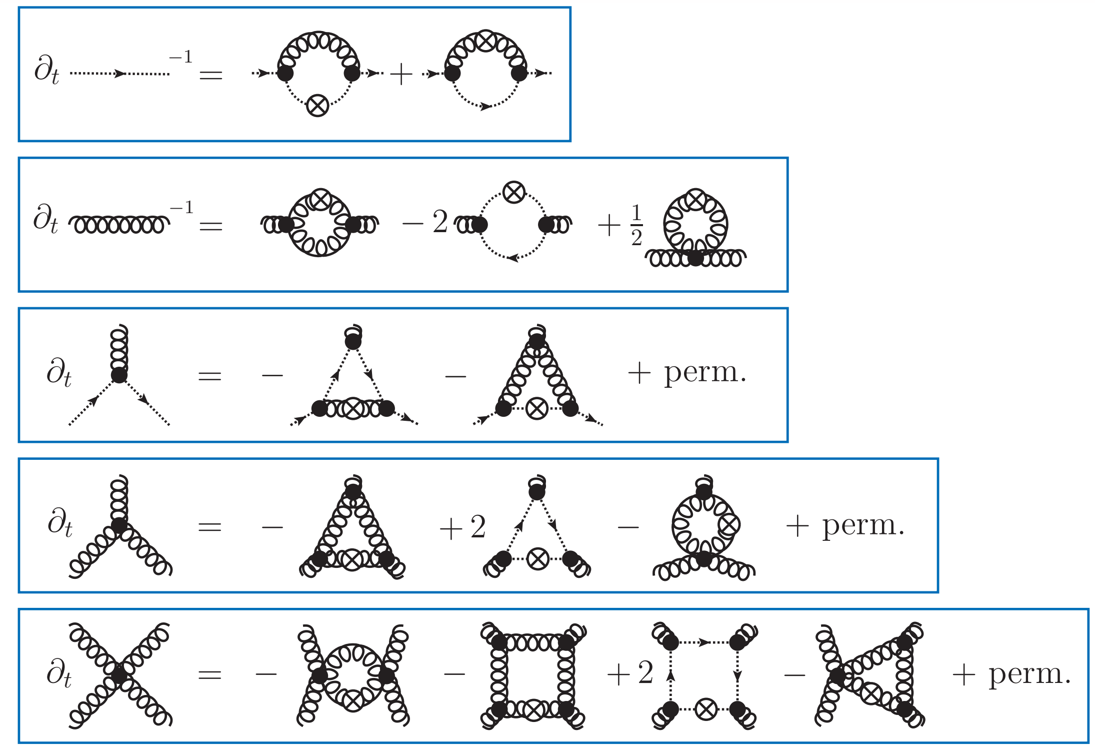
+ ...

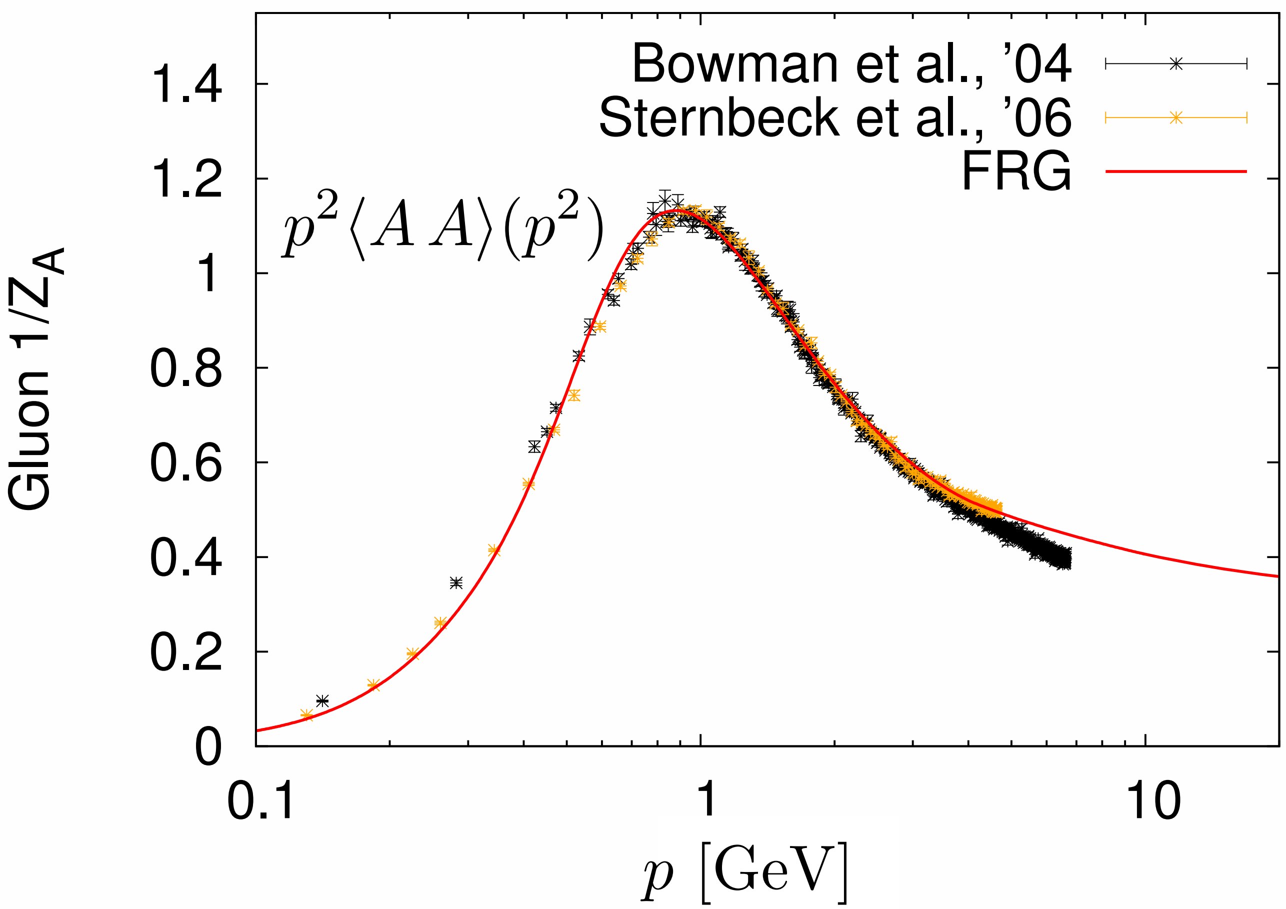
2PI-resummed



Glue sector

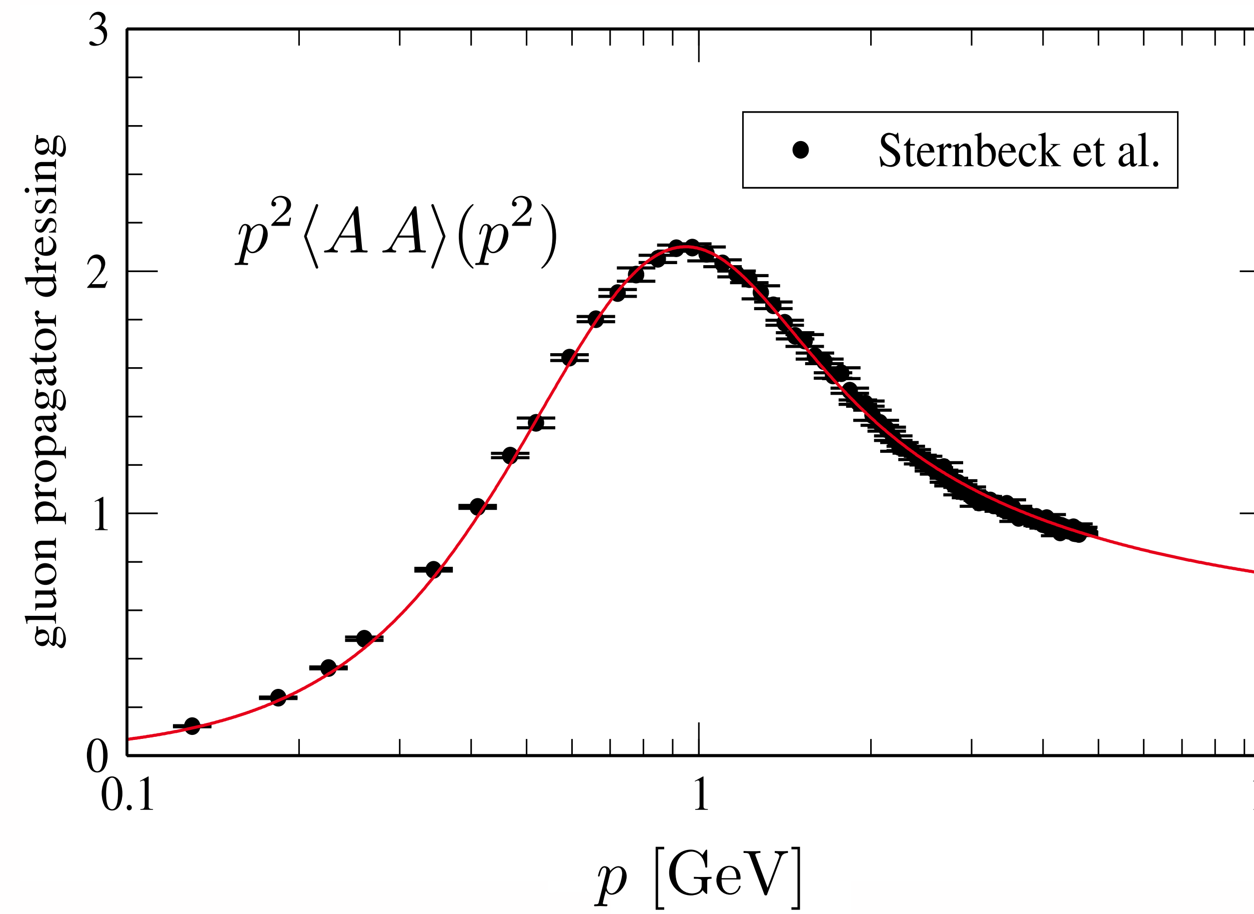






Fischer, Maas, JMP, Annals Phys. 324 (2009) 2408

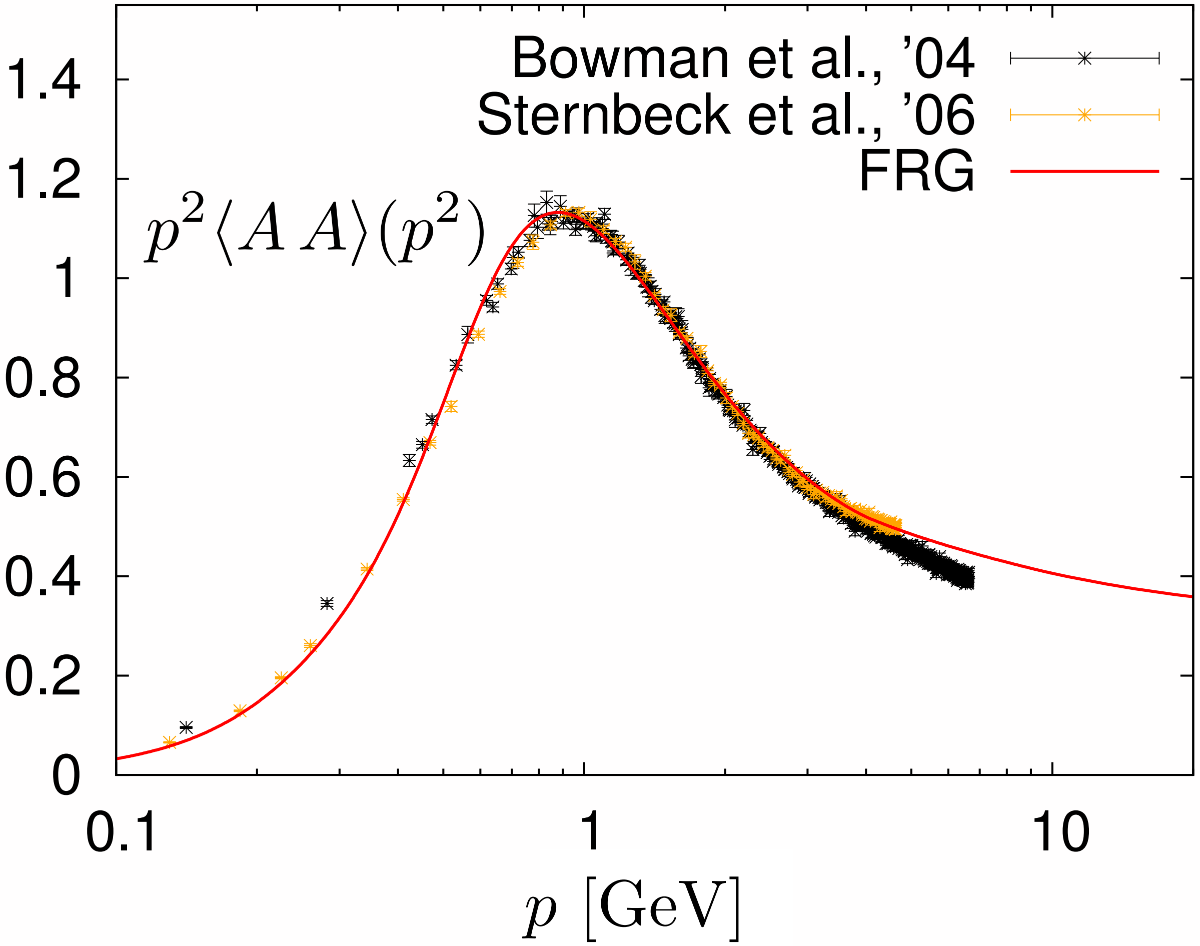
Fister, JMP '14



Cyrol, Fister, Mitter, JMP, Strodthoff, in prep.

Glue sector

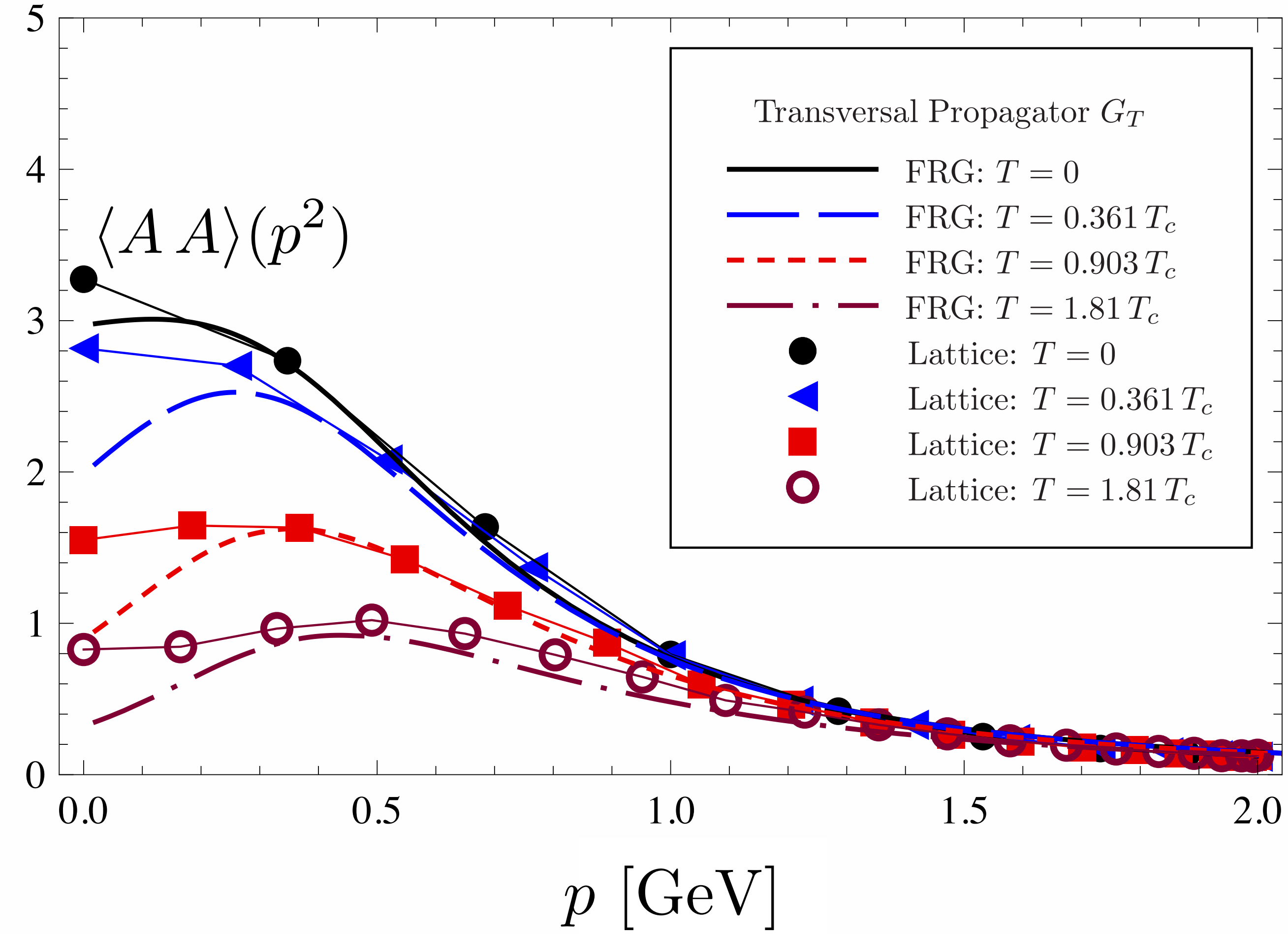
Yang-Mills propagators, T=0



Fischer, Maas, JMP, Annals Phys. 324 (2009) 2408

Fister, JMP '14

Yang-Mills propagators, finite T

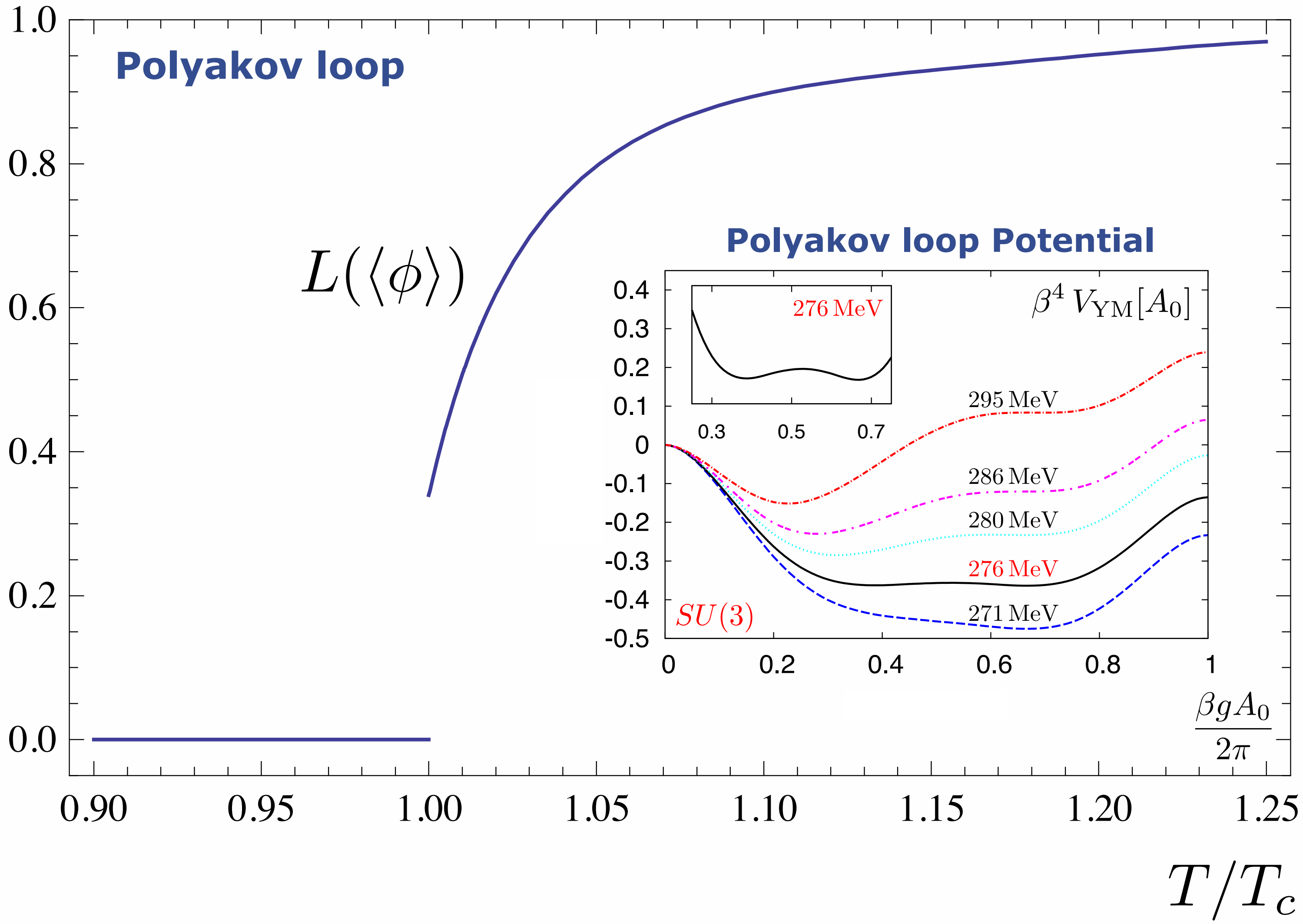


Fister, JMP, arXiv:1112.5440

Confinement

FRG: Braun, Gies, JMP, PLB 684 (2010) 262

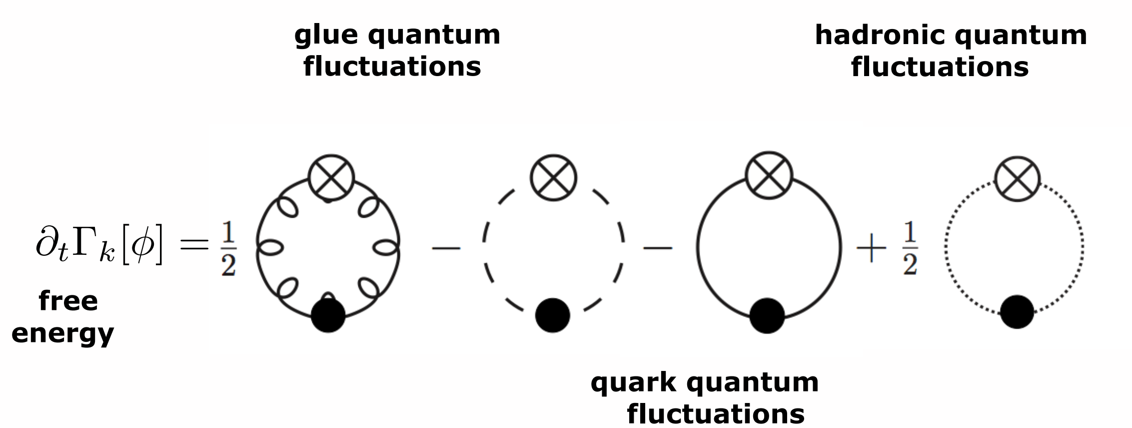
FRG, DSE, 2PI: Fister, JMP, PRD 88 (2013) 045010



$$T_c/\sqrt{\sigma} = 0.658 \pm 0.023$$

$$\text{lattice}: T_c/\sqrt{\sigma} = 0.646$$

$$\boxed{L[A_0] = \frac{1}{N_c} \mathrm{tr}\, \mathcal{P} e^{i g \int_0^\beta A_0(x)}}$$

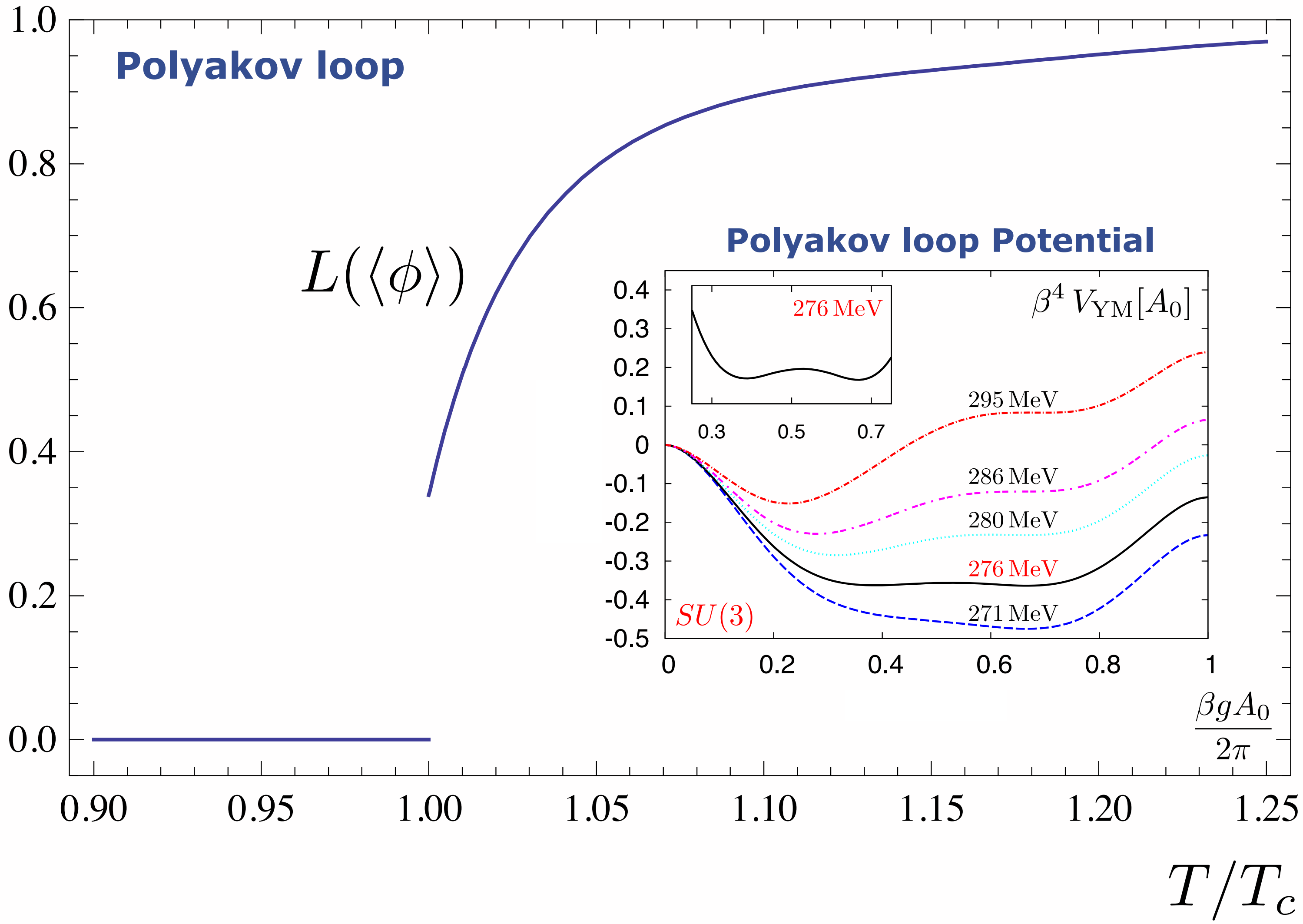


$$\mathcal{P} e^{i g \int_0^\beta A_0(x)} = e^{i\phi}$$

Confinement

FRG: Braun, Gies, JMP, PLB 684 (2010) 262

FRG, DSE, 2PI: Fister, JMP, PRD 88 (2013) 045010



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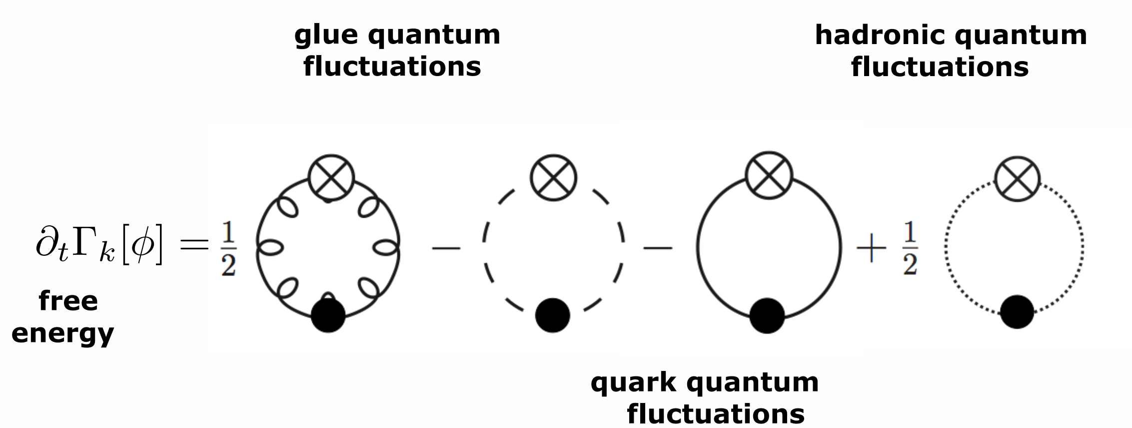


confinement

gluon propagator

gapped relative to

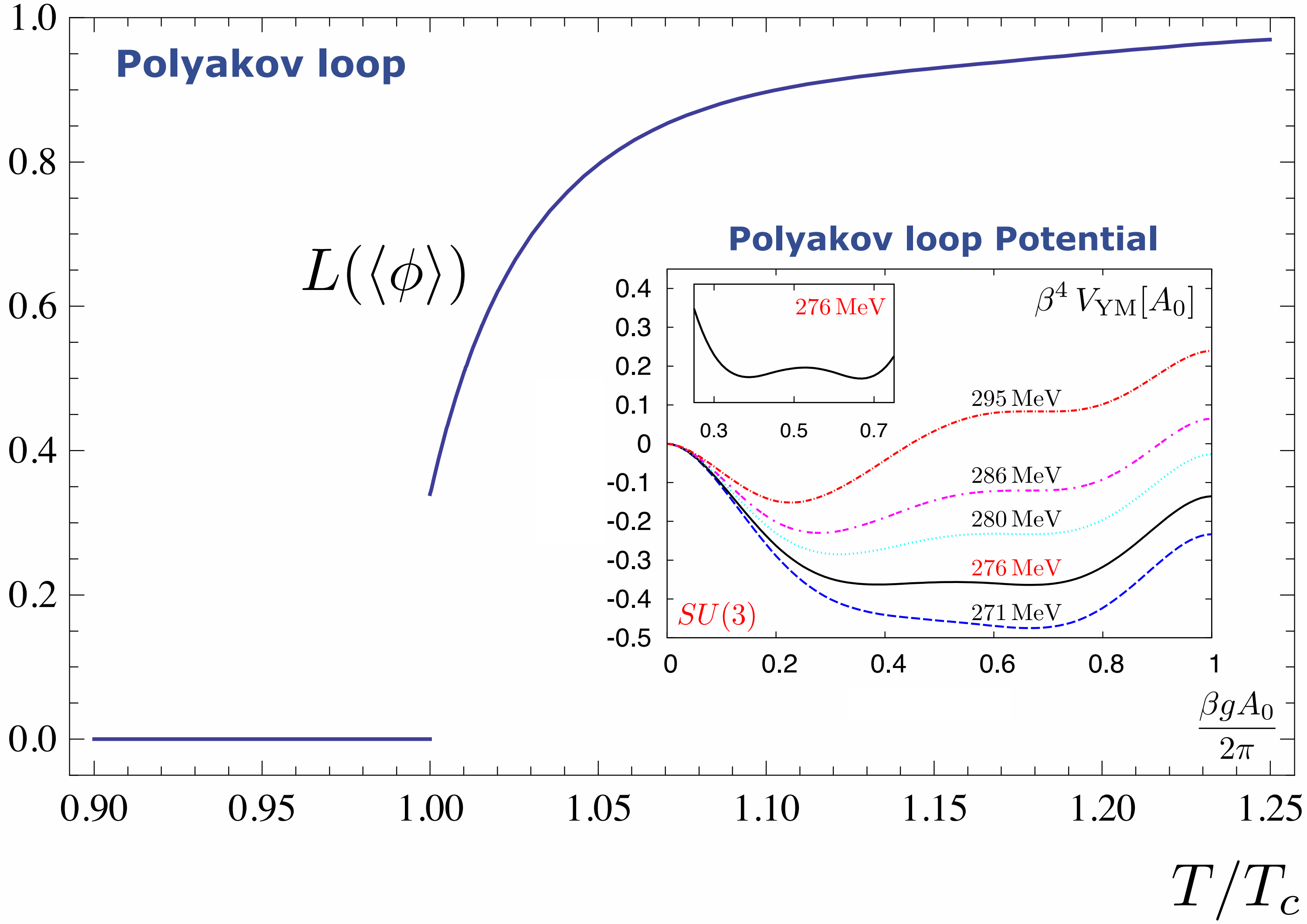
ghost propagator

Braun, Gies, JMP '07
Marhauser, JMP '08
Fister, JMP '13

Confinement

FRG: Braun, Gies, JMP, PLB 684 (2010) 262

FRG, DSE, 2PI: Fister, JMP, PRD 88 (2013) 045010



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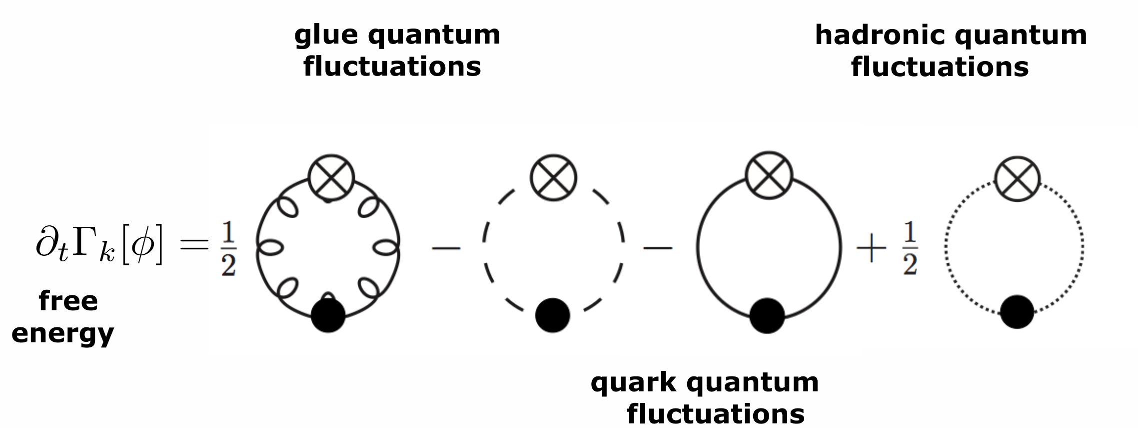
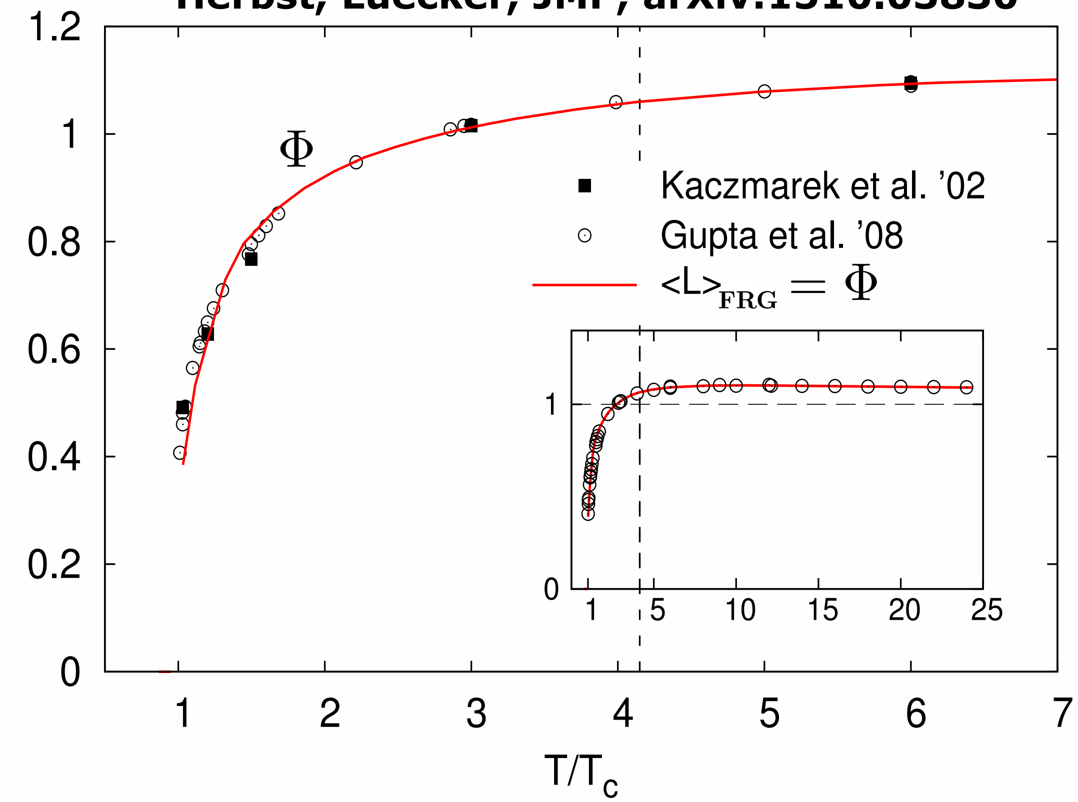
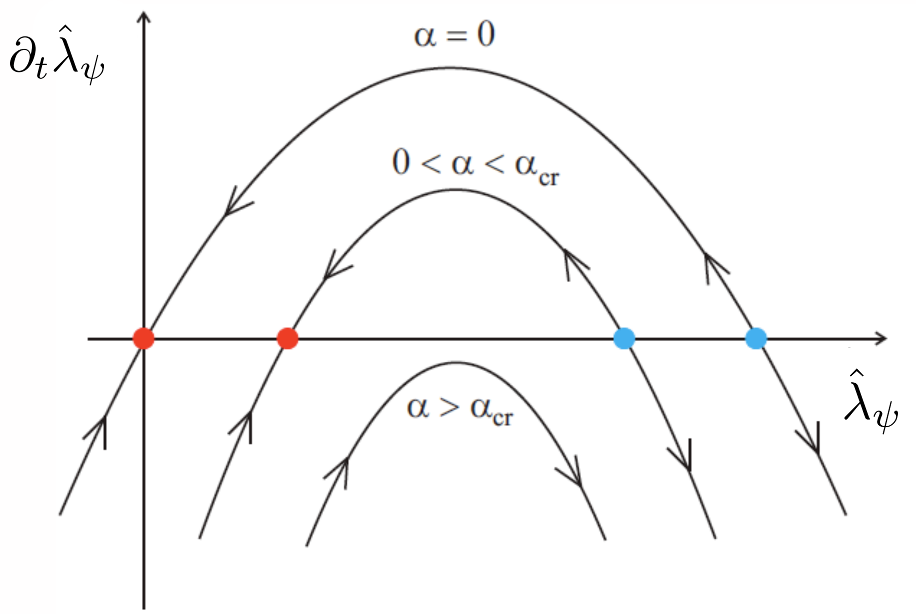
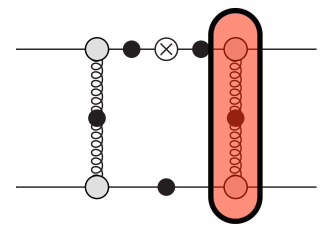


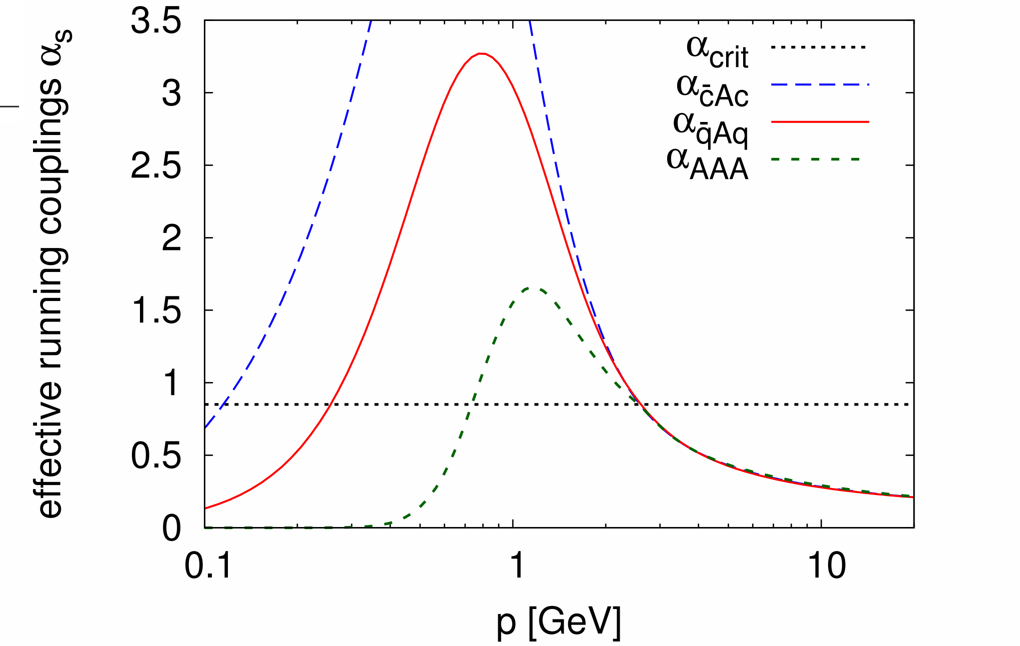
Herbst, Luecker, JMP, arXiv:1510.03830



Confinement & symmetry breaking



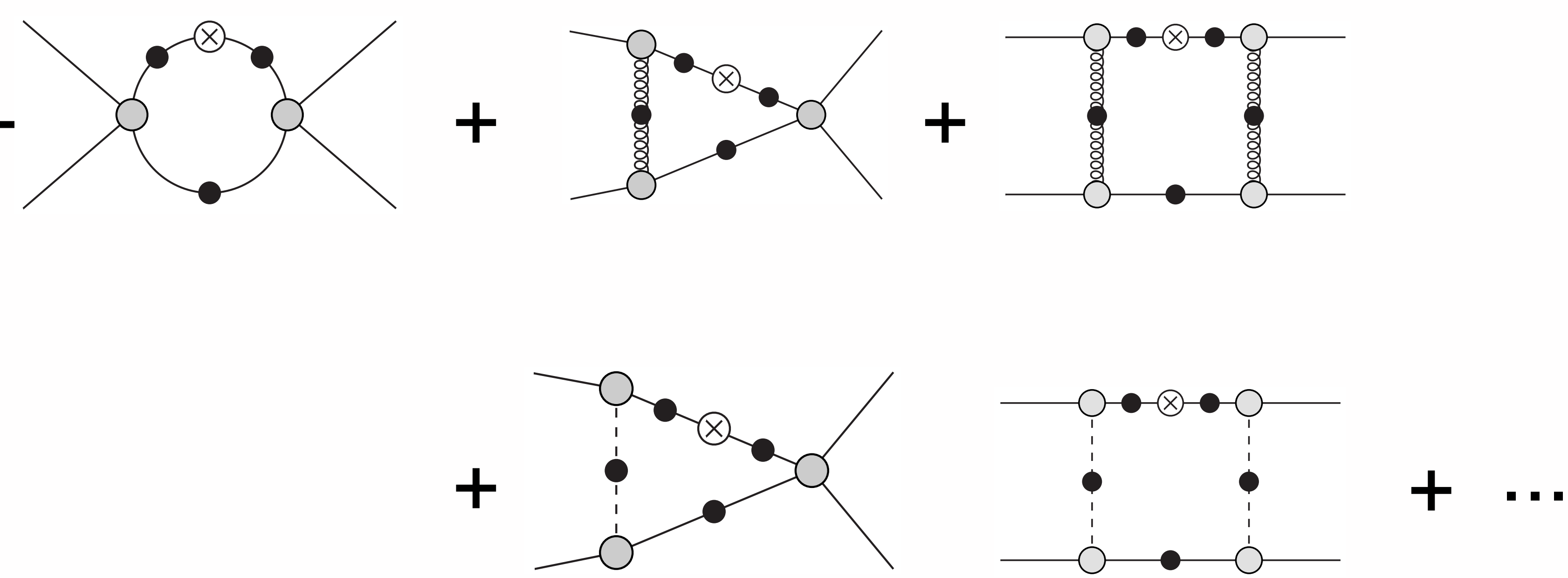




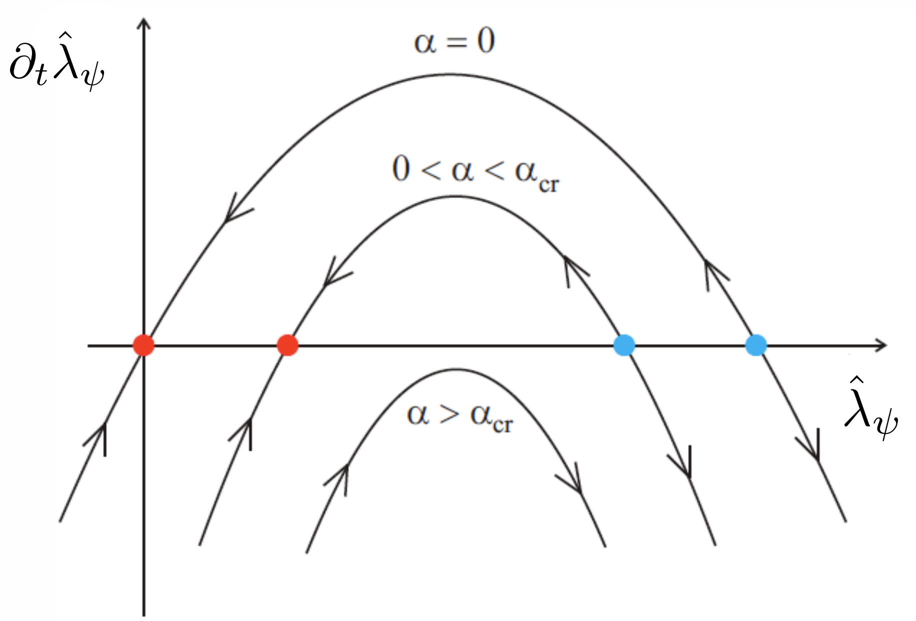
Mitter, JMP, Strodthoff '14

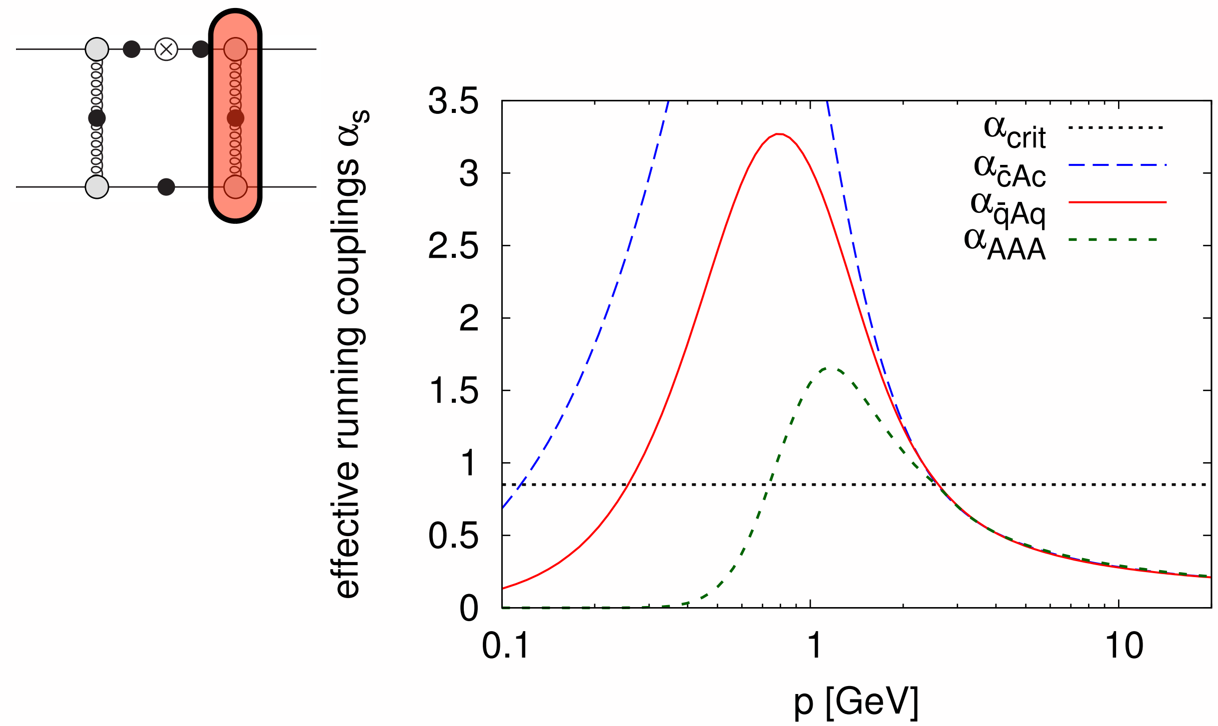
Braun, Fister, Haas, JMP, Rennecke '14

$$k\partial_k \hat\lambda_\psi = 2\hat\lambda_\psi + \dots$$



Confinement & symmetry breaking





Mitter, JMP, Strodthoff ’14

Braun, Fister, Haas, JMP, Rennecke ’14

dynamical correlation of confinement
and
chiral symmetry breaking

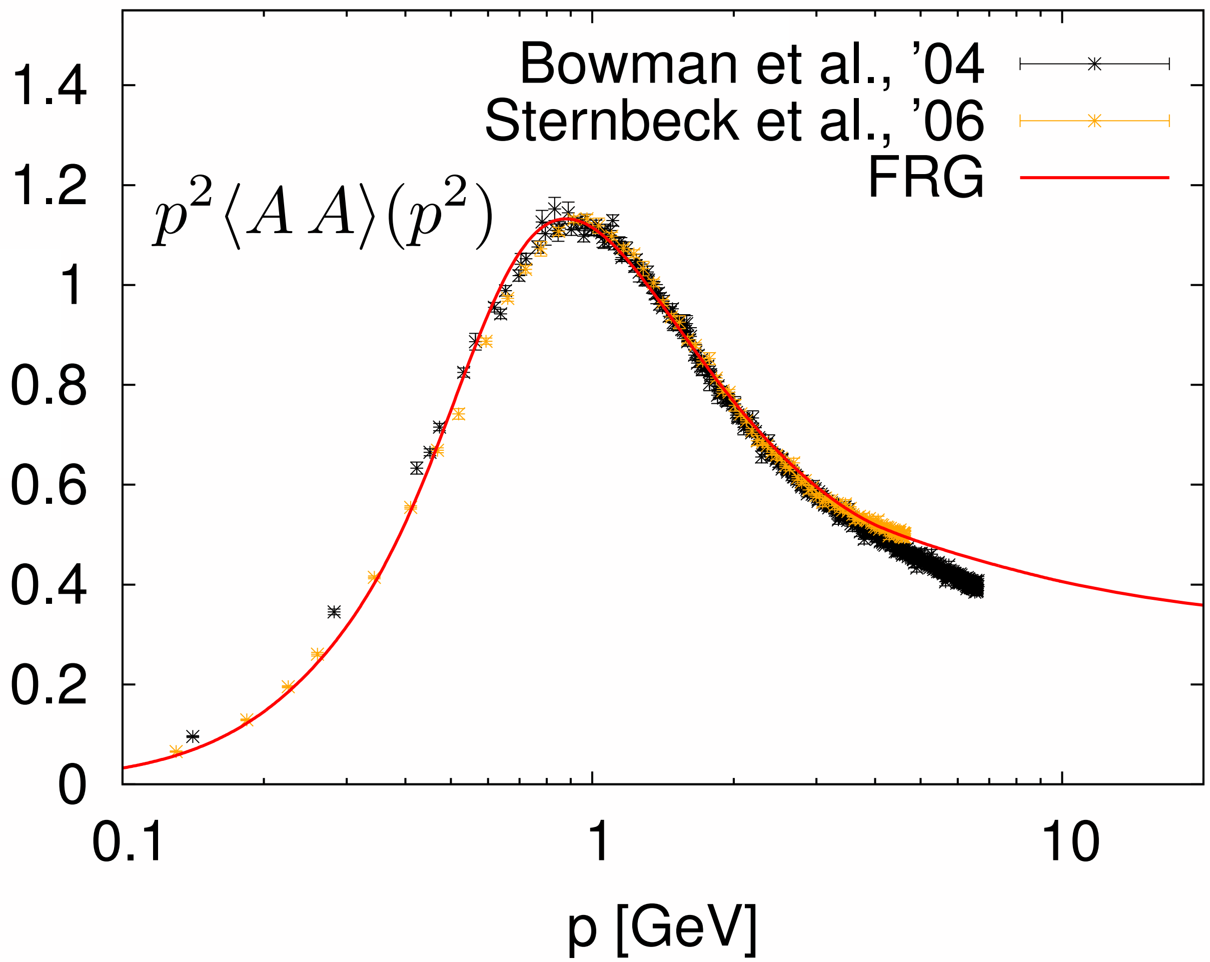
confinement	chiral symmetry breaking
gluon propagator gapped relative to ghost propagator	gluon propagator not gapped too much

Cyrol, Fister, Mitter, JMP, Strodthoff, in prep

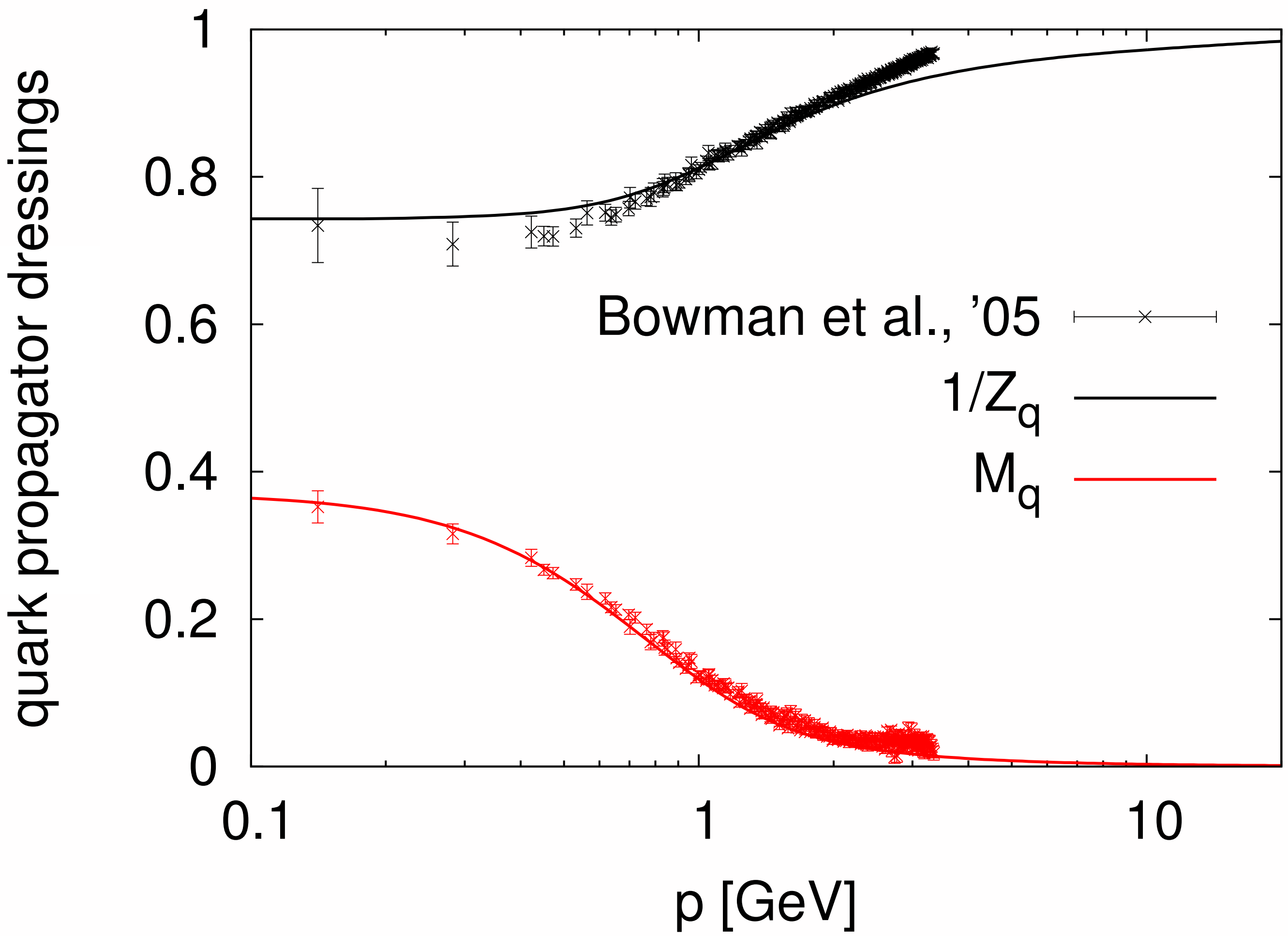
Chiral symmetry breaking

FRG-quenched QCD vs lattice-quenced QCD

unquenched gluon dressing



quark propagator



see talk of Mario Mitter

$$\mathbf{N_f = 2}$$

Mitter, JMP, Strodthoff, PRD 91 (2015) 054035

Chiral symmetry breaking

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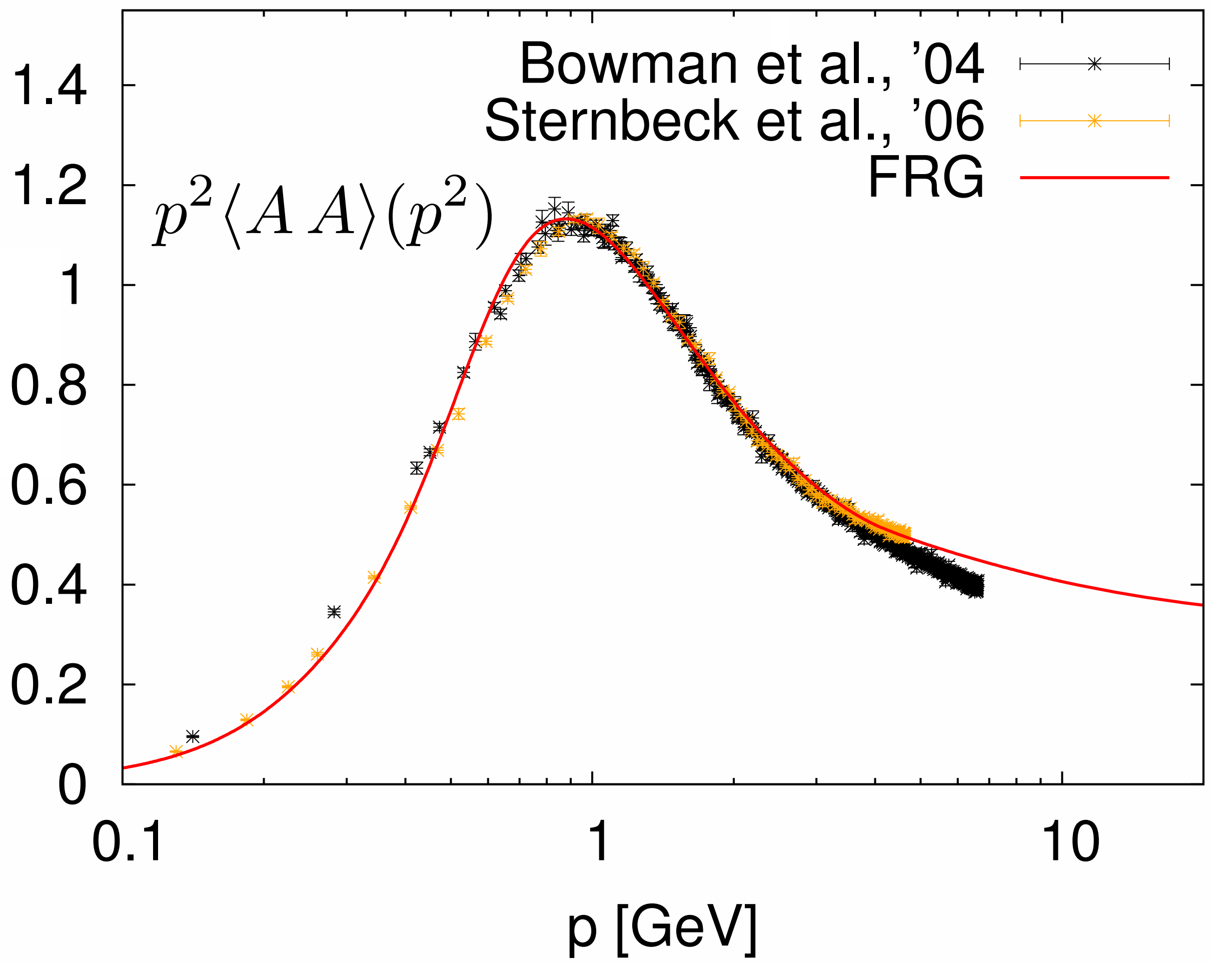
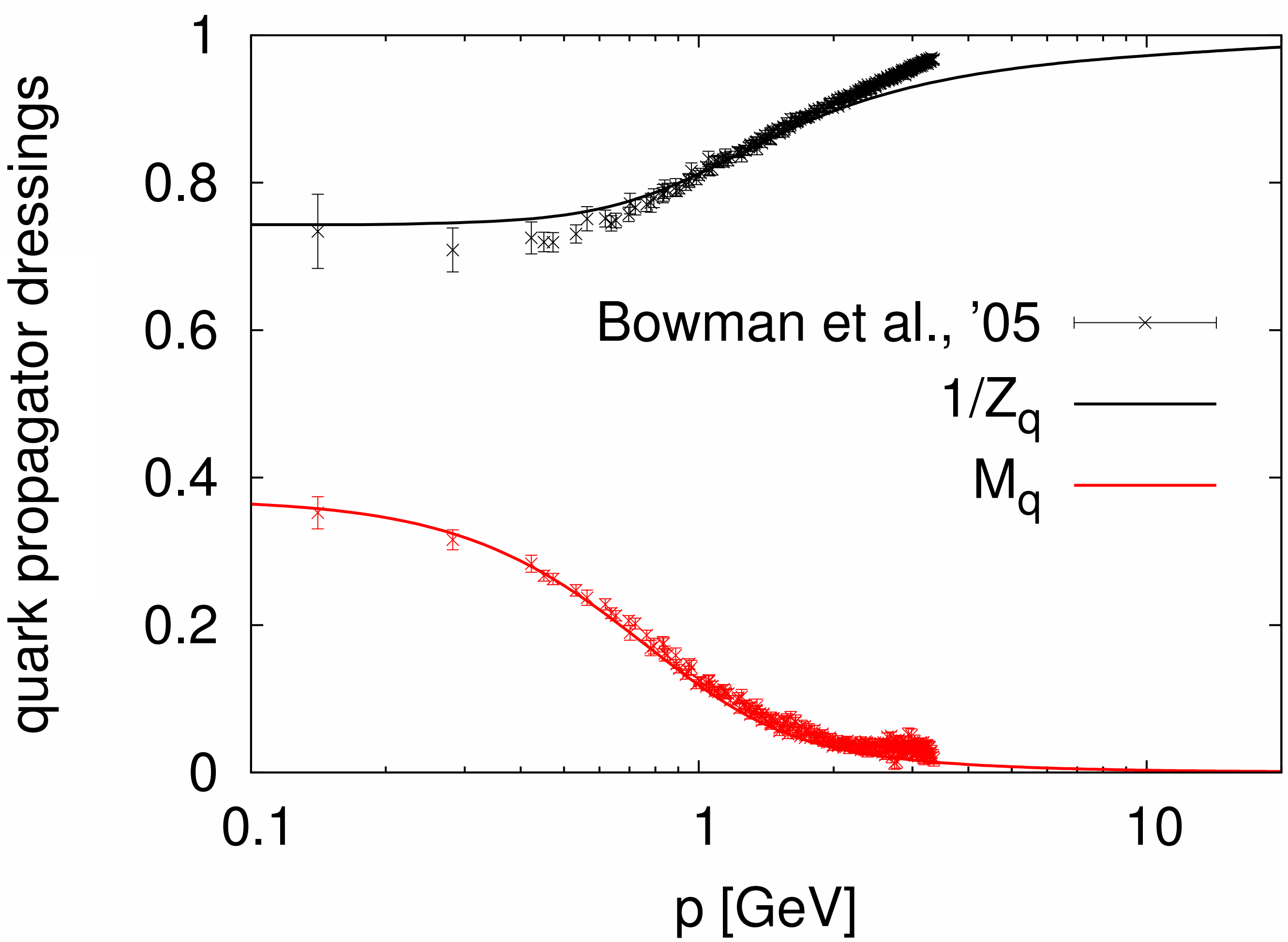


quark propagator



see talk of Mario Mitter

$$\mathbf{N_f = 2}$$

see also Williams, Fischer, Heupel, arXiv:1512.00455

Mitter, JMP, Strodthoff, PRD 91 (2015) 054035

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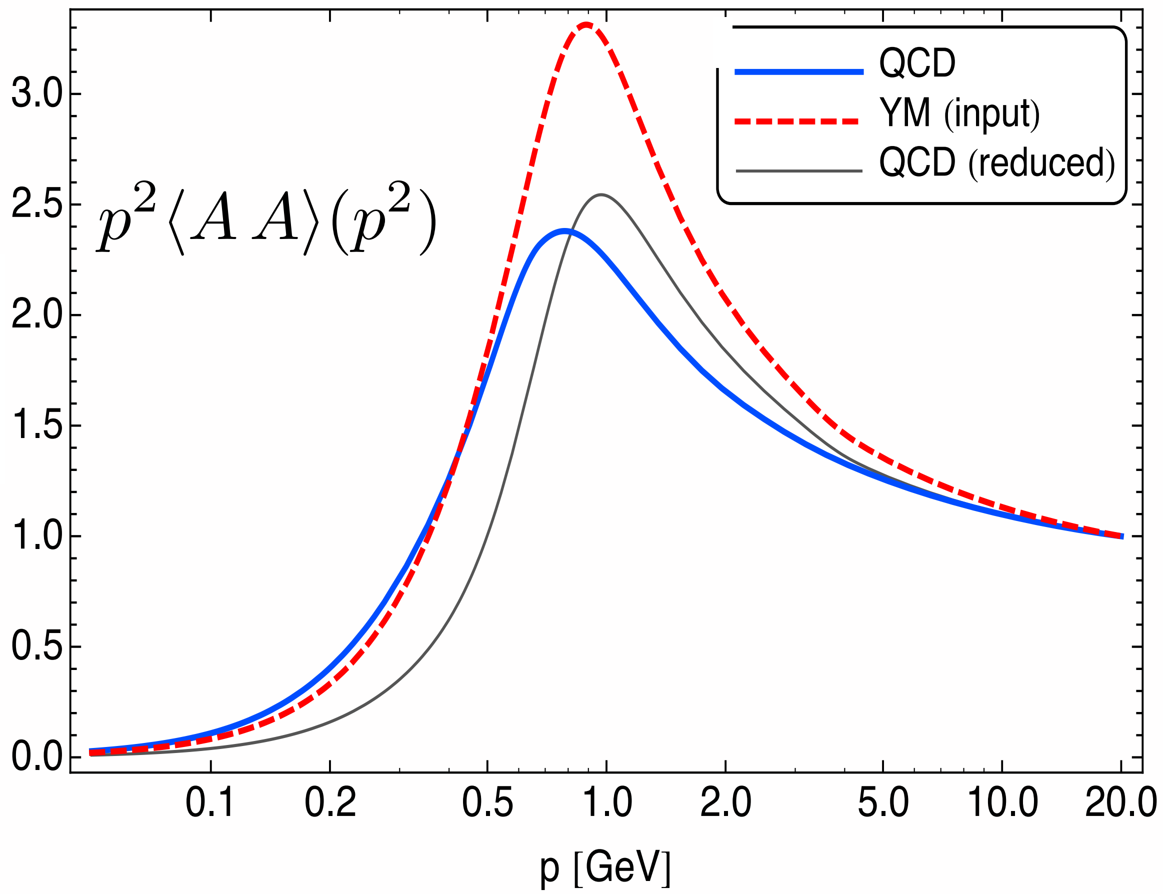
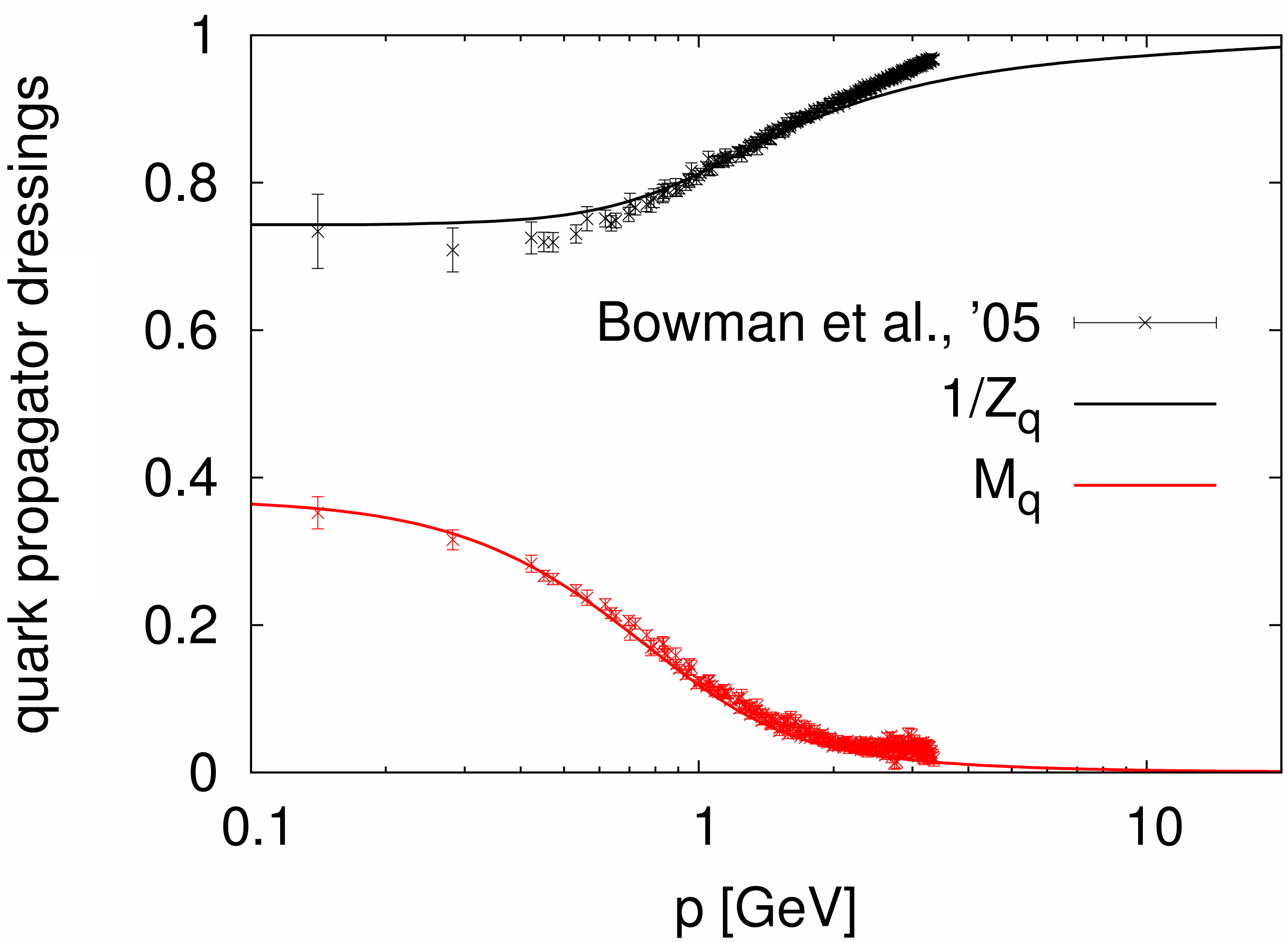


Braun, Fister, Haas, JMP, Rennecke, arXiv:1412.1045

Rennecke, Phys.Rev. D92 (2015) 7, 076012

quark propagator



Mitter, JMP, Strodthoff, PRD 91 (2015) 054035

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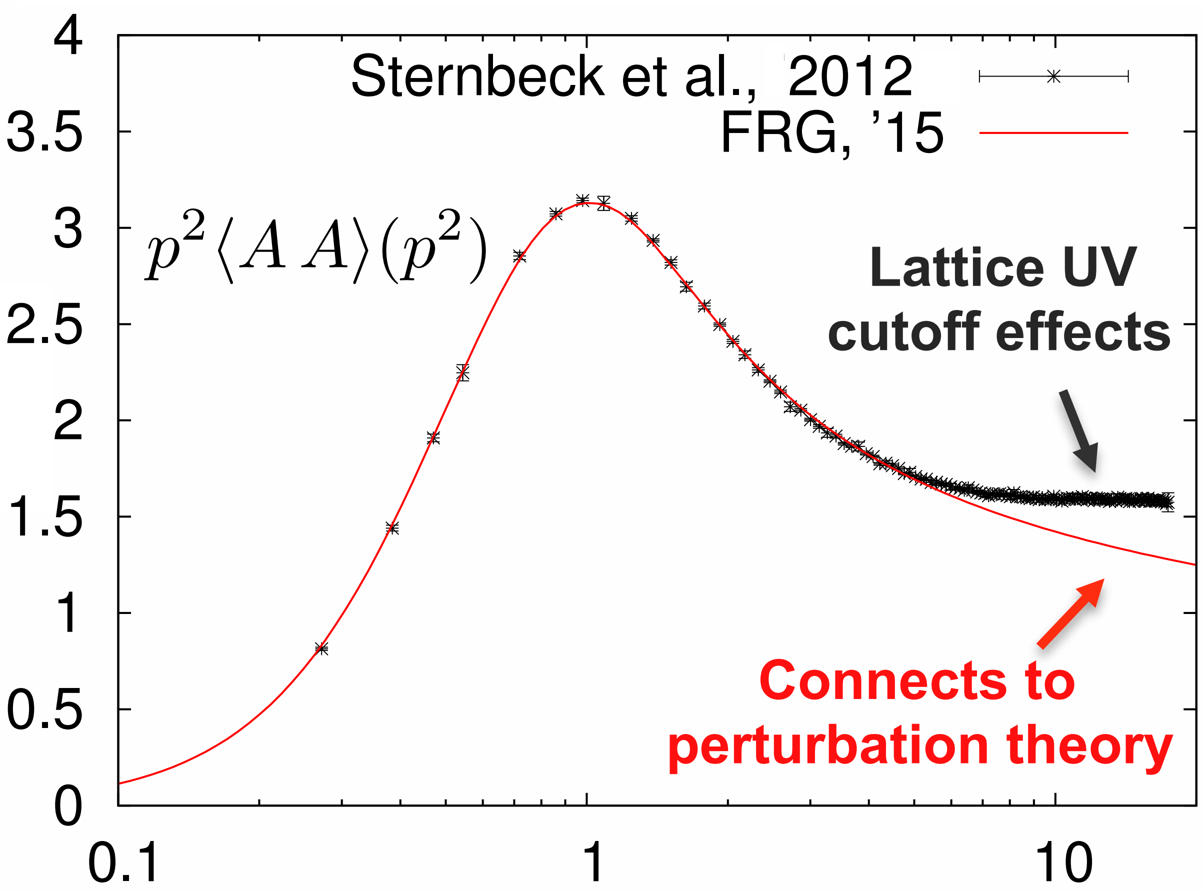
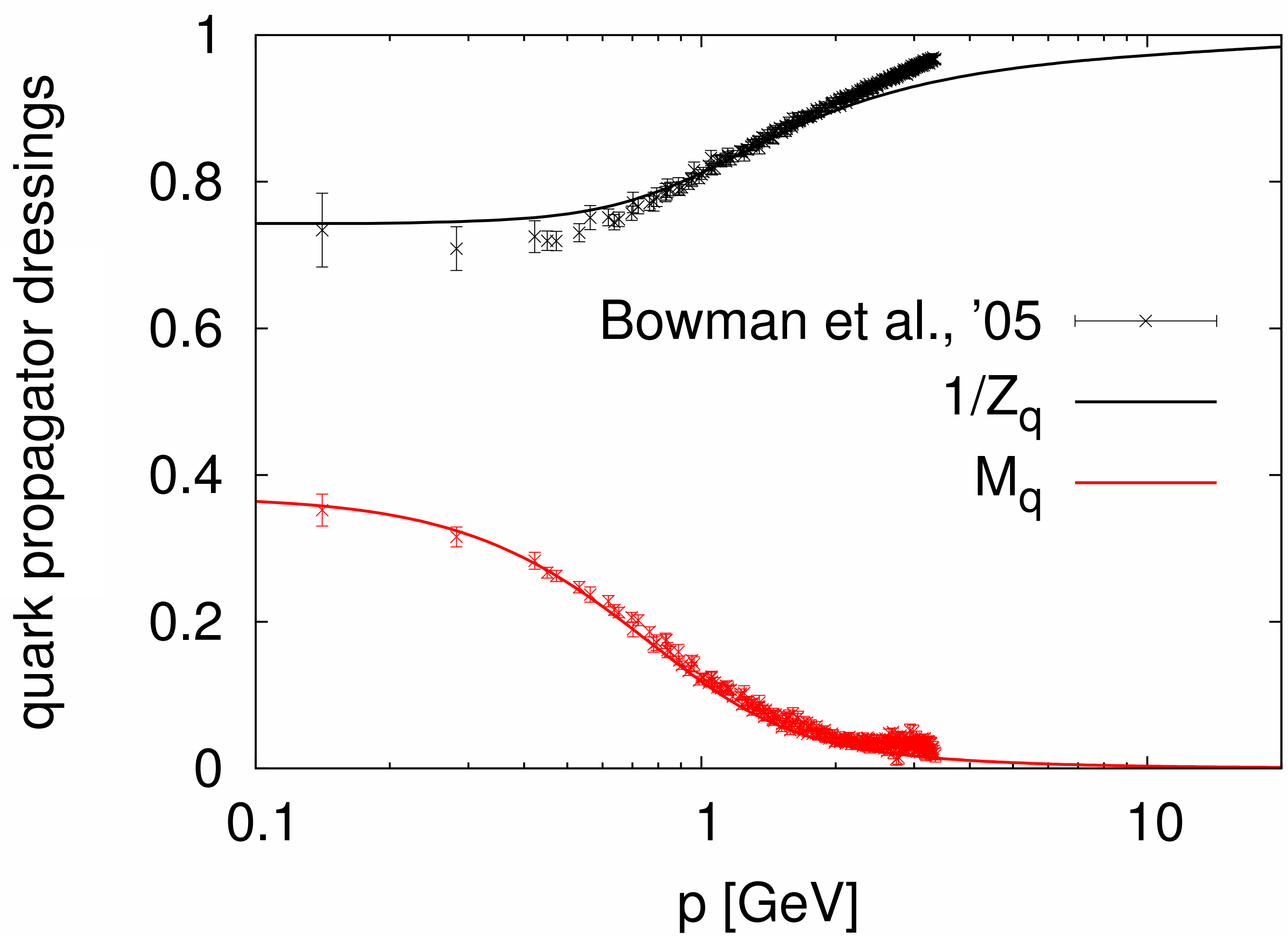


Cyrol, Mitter, JMP, Strodthoff, in prep.

quark propagator



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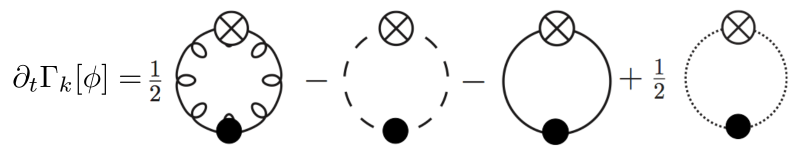
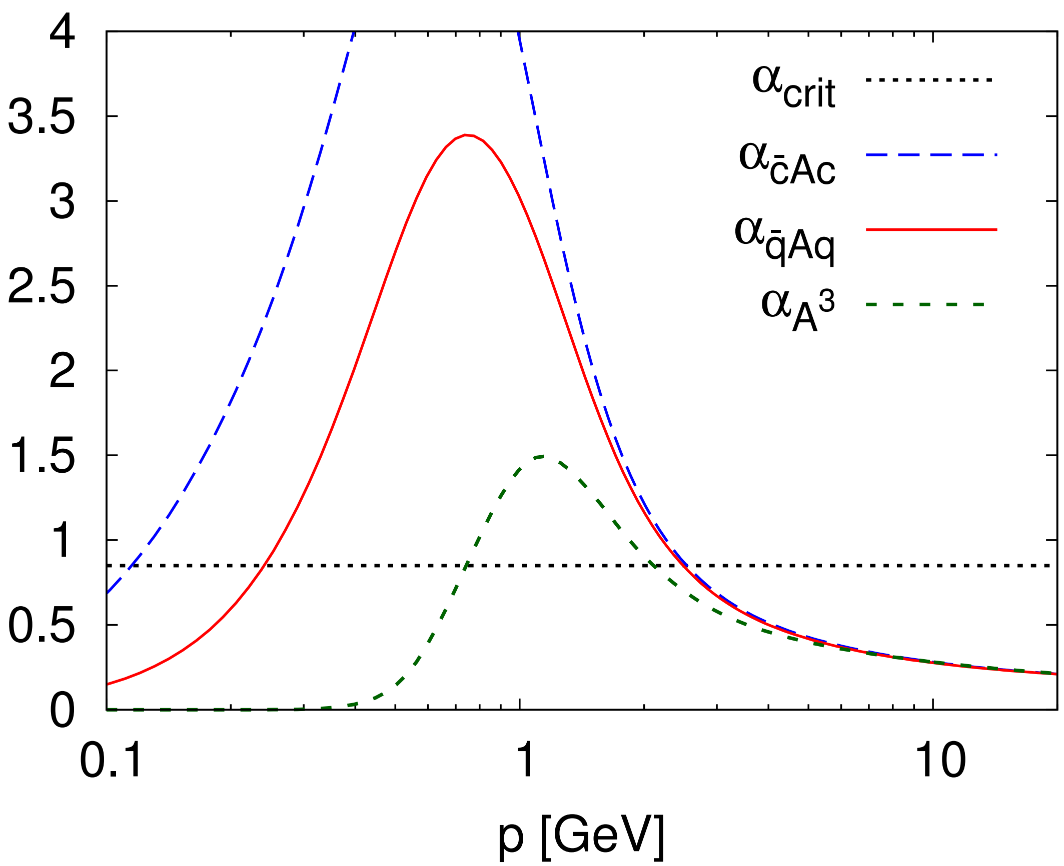
Mitter, JMP, Strodthoff, PRD 91 (2015) 054035

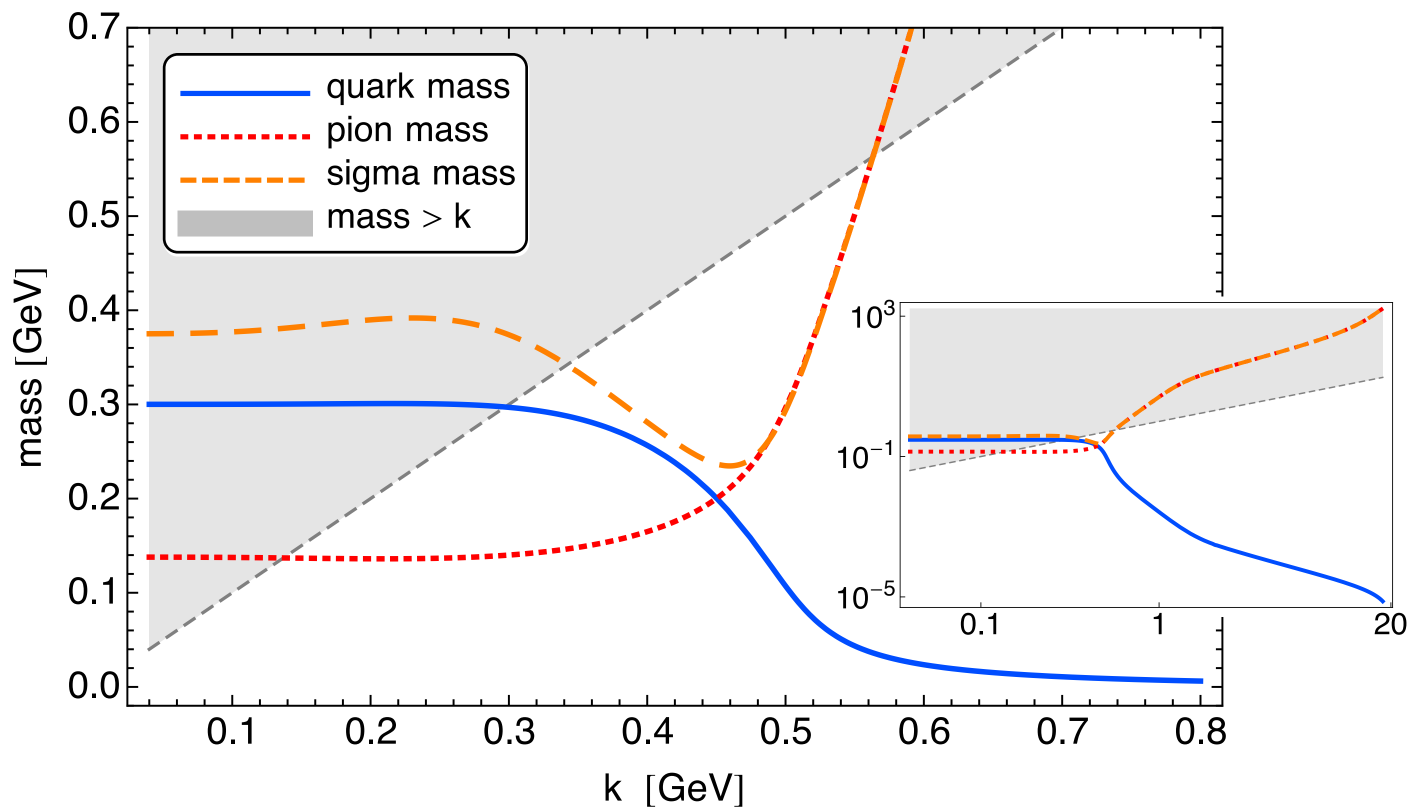
QCD



Sequential decoupling of gluon, quark, sigma, pion fluctuations



Mitter, JMP, Strodthoff, PRD 91 (2015) 054035



Braun, Fister, Haas, JMP, Rennecke, arXiv:1412.1045

Rennecke, arXiv:1504.03585

Functional Methods for QCD

Scales

- **intrinsic scale of QCD:** $\Lambda_{\rm QCD} \approx \mathbf{200}\,{\rm MeV}$
  - **glue mass gap (Landau gauge: mass gap of glue propagator)** $\boldsymbol{\Delta}\mathbf{m}_{\rm glue} \approx \boldsymbol{\Lambda}_{\rm QCD}$
  - **chiral symmetry breaking scale:** $\boldsymbol{\Delta}\mathbf{m}_\chi \approx \mathbf{m}_{\mathbf{q},{\rm constit.}} - \mathbf{m}_{\mathbf{q},{\rm current}}$ $\boldsymbol{\Delta}\mathbf{m}_\chi \approx \mathbf{300}\,{\rm MeV}$
  - **chiral/confinement critical temperatures:** $\mathbf{T}_\chi \approx \mathbf{T}_{\rm conf} \approx \mathbf{150}\,{\rm MeV}$

scales = $\mathbf{c}(\mathbf{N_f}, \mathbf{N_c})\boldsymbol{\Lambda}_{\rm QCD}$

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- **explicit mass scales of QCD:**
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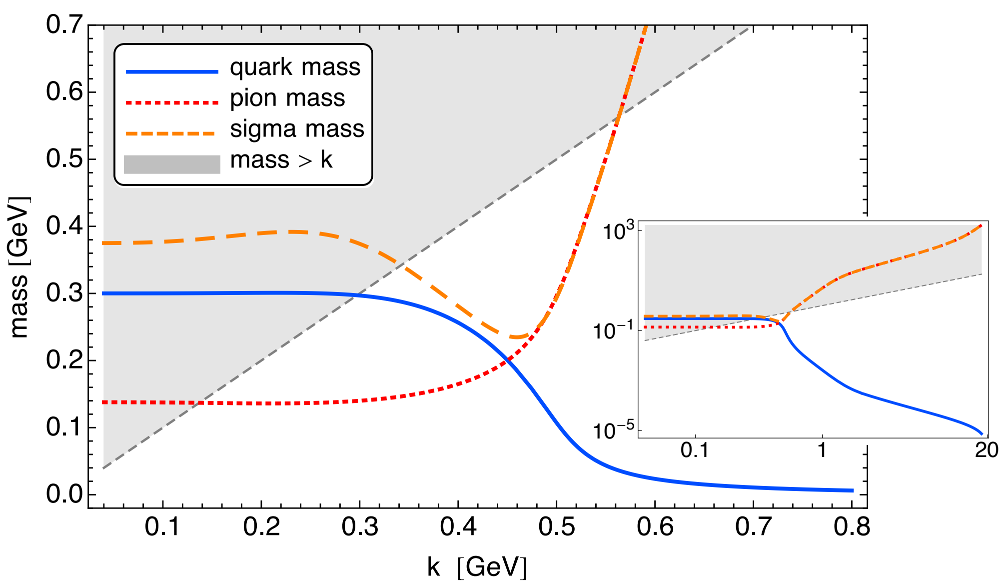
meson masses



Braun, Fister, Haas, JMP, Rennecke '14

Rennecke '15

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best done with a combination of imaginary and real time flows

see talk of Nicolai Christiansen

see talk of Ralf-Arno Tripolt

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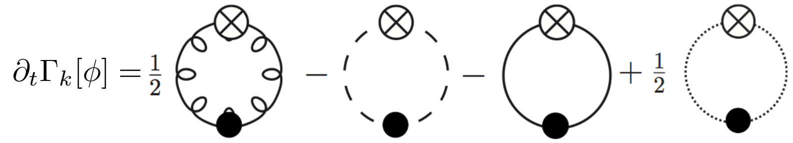
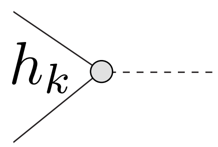
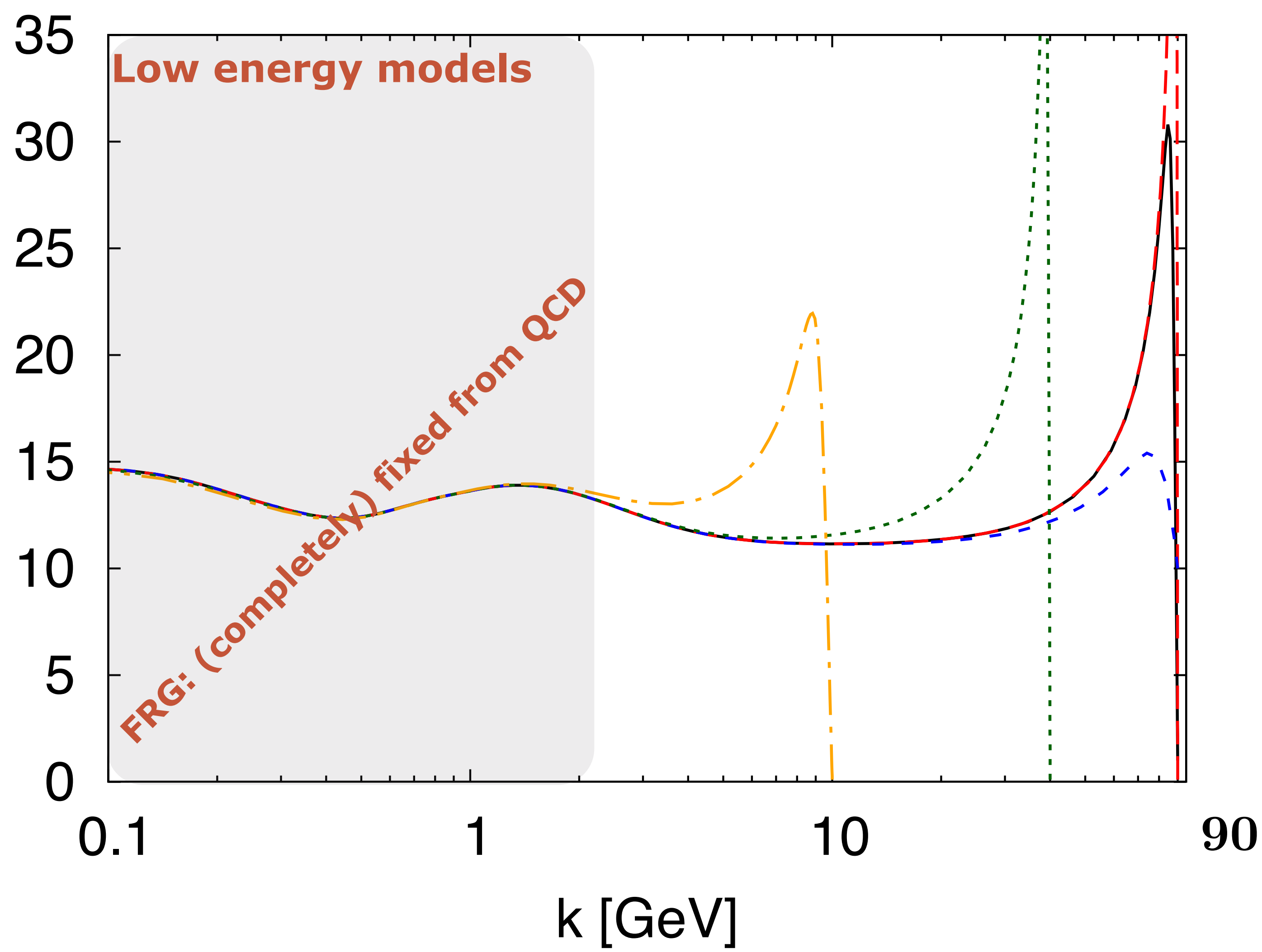
QCD



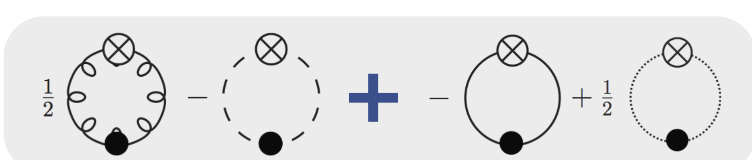


quark-meson coupling

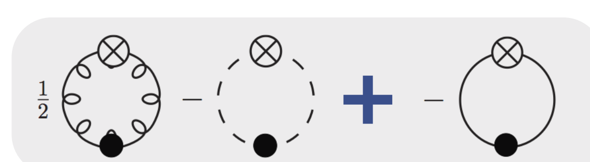


fQCD

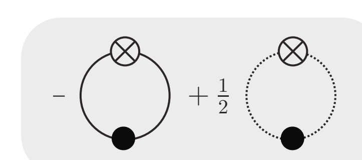
PQM-model



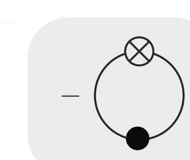
PNJL-model



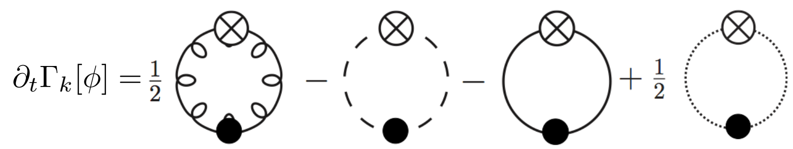
QM-model

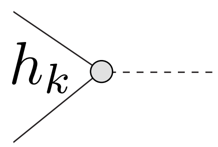


NJL-model



QCD





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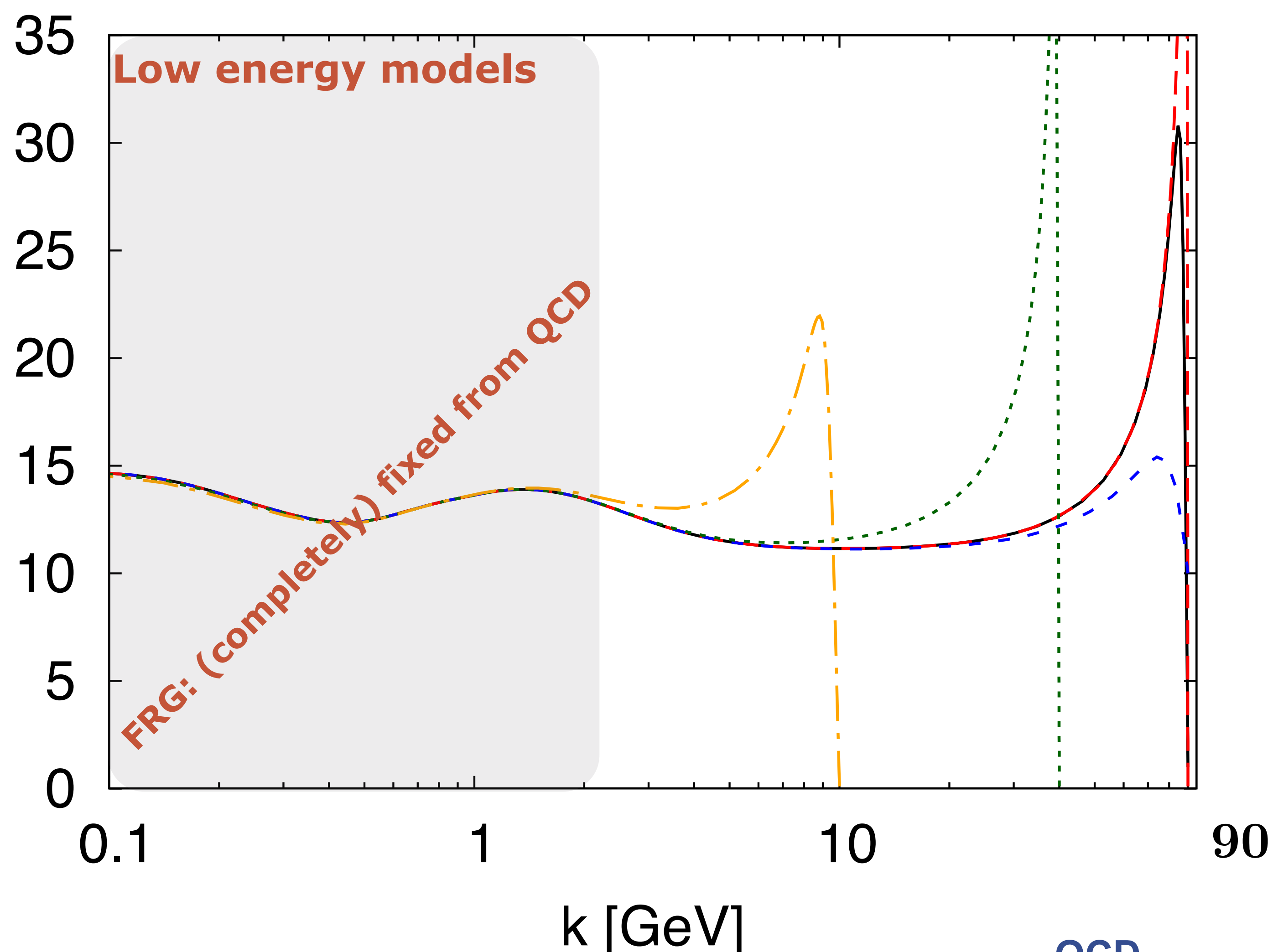
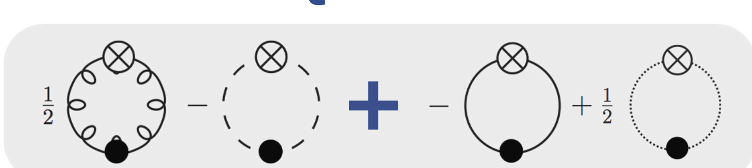


fQCD

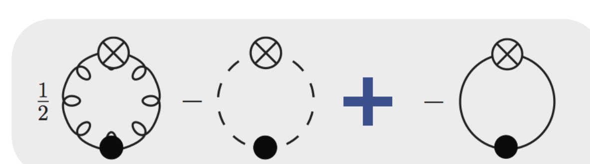
QCD

low energy models

PQM-model



PNJL-model



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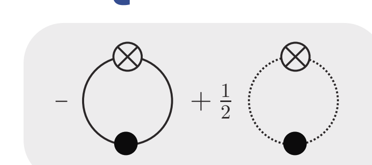

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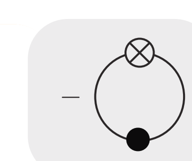
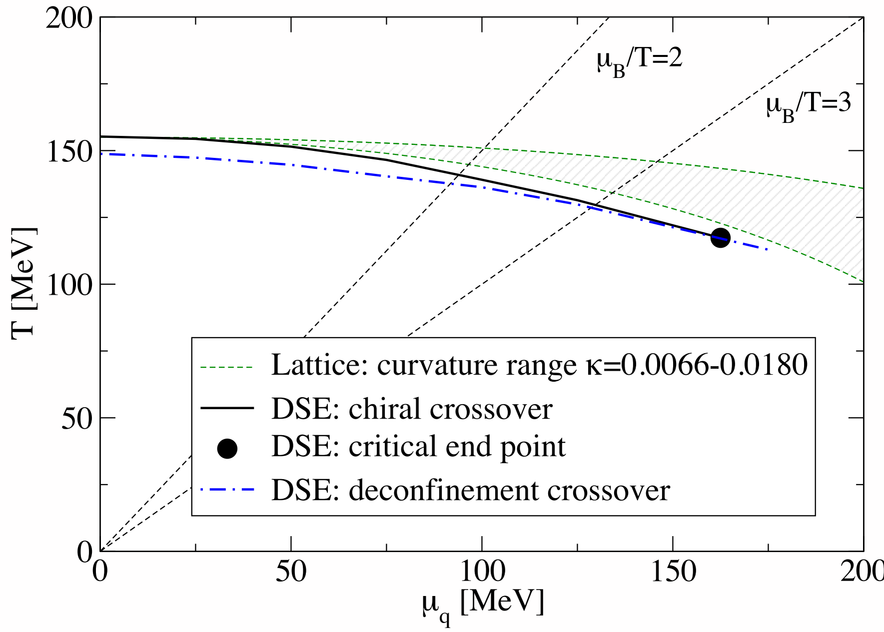


Phase structure at finite density

Phase diagram of 2+1 flavor QCD

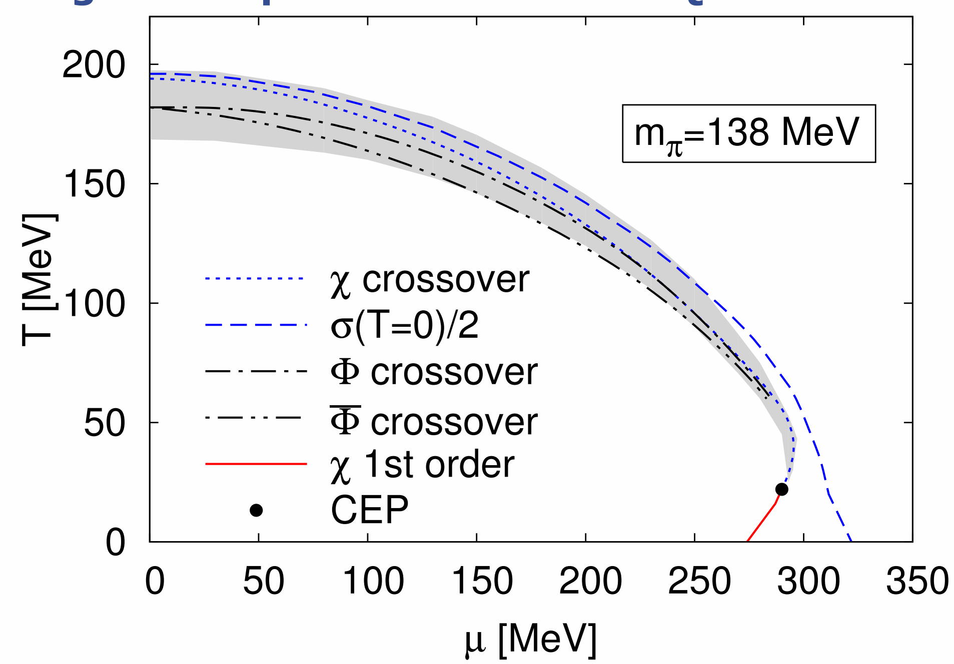


Kaczmarek at al. '11
Endrodi, Fodor, Katz, Szabo '11
Cea, Cosmai, Papa '14

Fischer, Fister, Luecker, JMP, PLB732 (2014) 248

Fischer, Luecker, Welzbacher, PRD 90 (2014) 034022

Phase diagram of quantised 2-flavor PQM-model



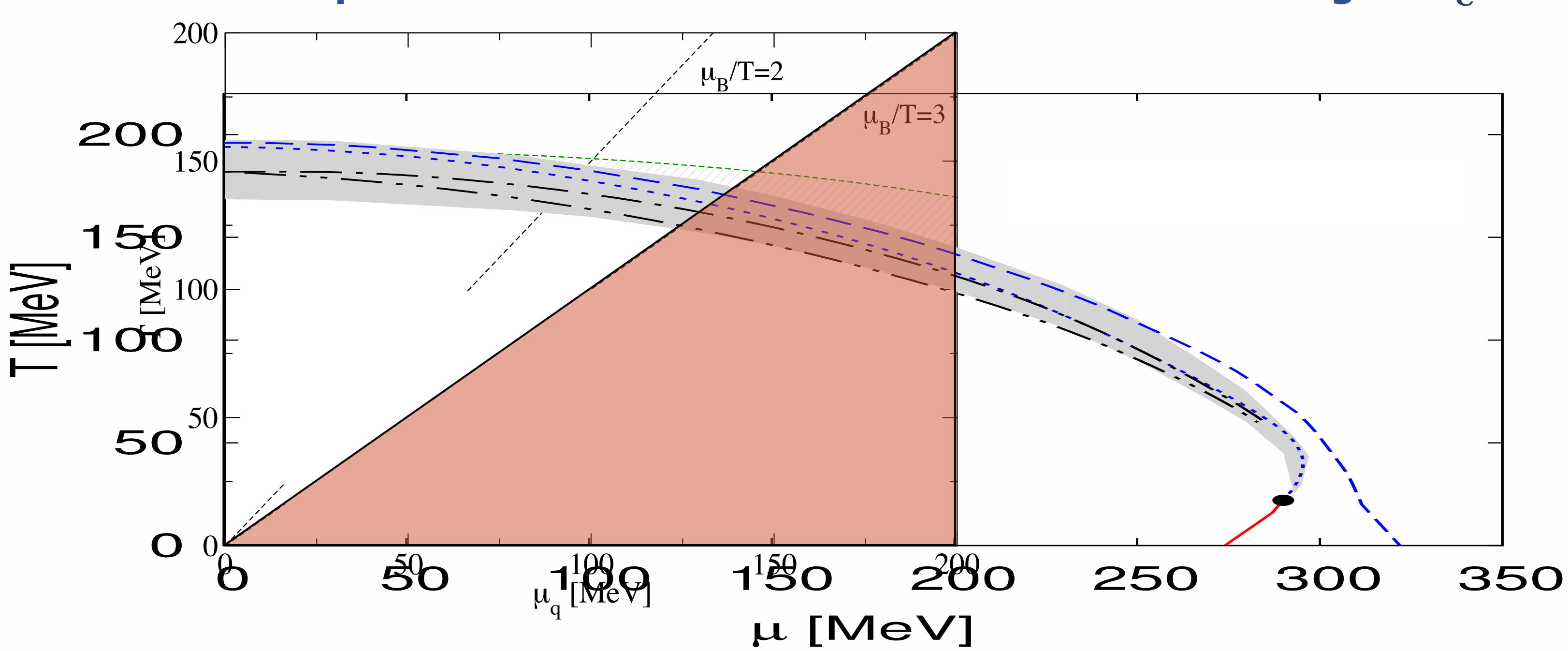
Herbst, JMP, Schaefer, PLB 696 (2011) 58-67
PRD 88 (2013) 1, 014007

see talk of Christian Fischer

FRG QCD results at finite density

Haas, Braun, JMP '09, unpublished

Comparison with 2 flavor vs 2+1 flavor scale matching of T_c

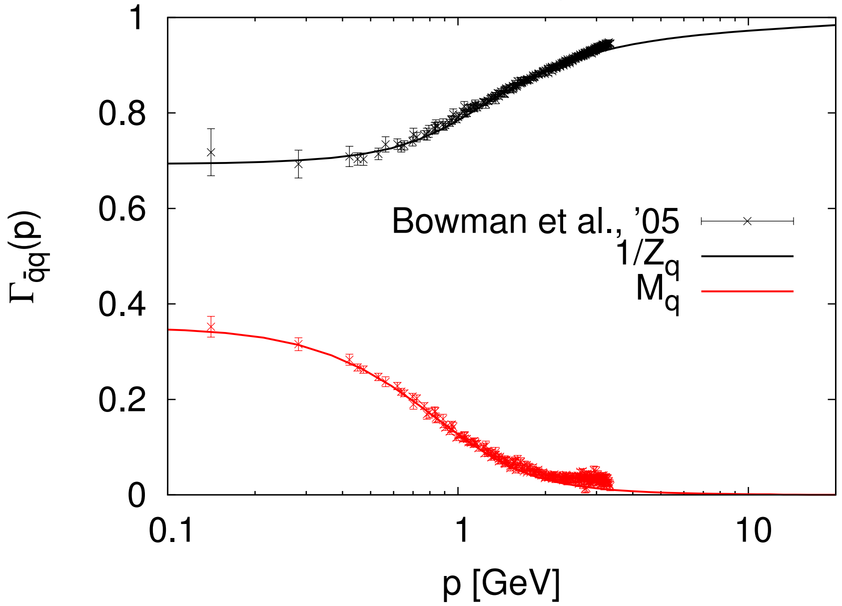


Summary & Outlook

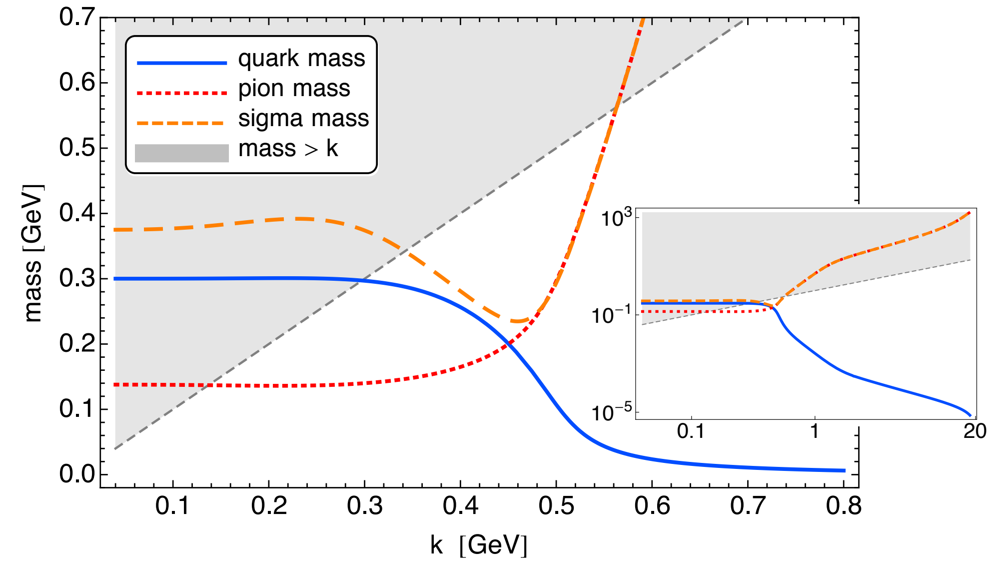
▪ Chiral Symmetry Breaking and Confinement

$$\boxed{\frac{f_{\pi,\mathrm{FRG}}}{f_{\pi,\mathrm{lattice}}} = 0.99}$$

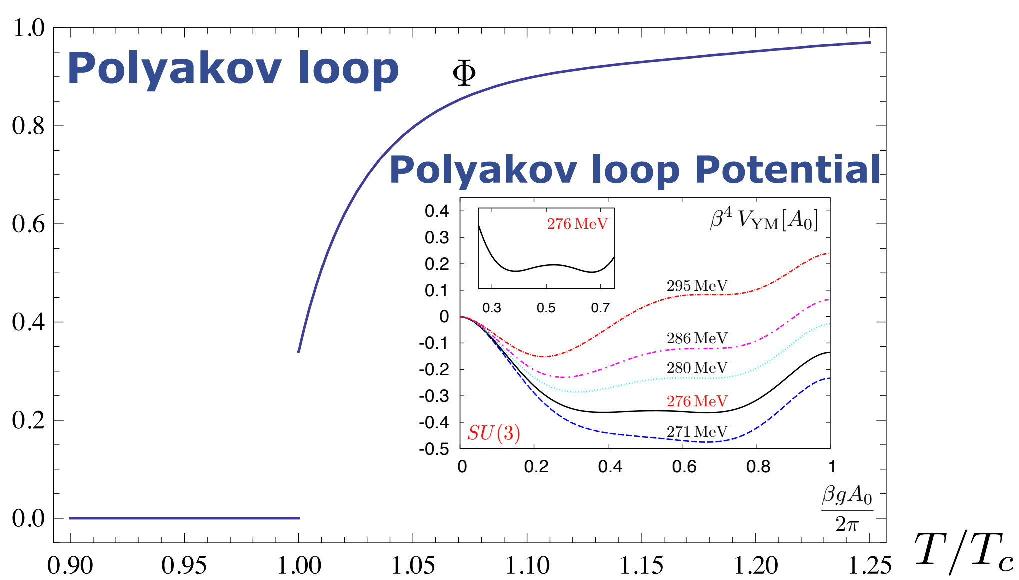
quark propagator



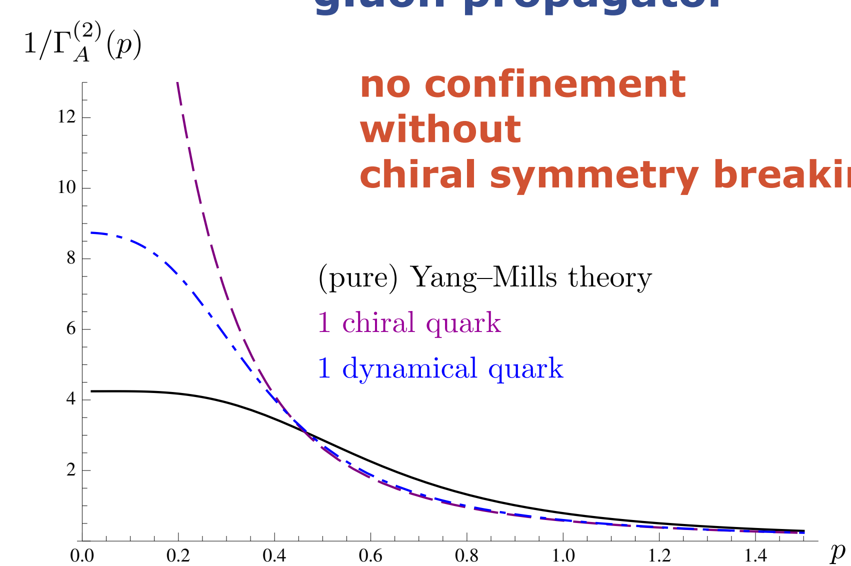
meson masses & decoupling



fQCD



gluon propagator

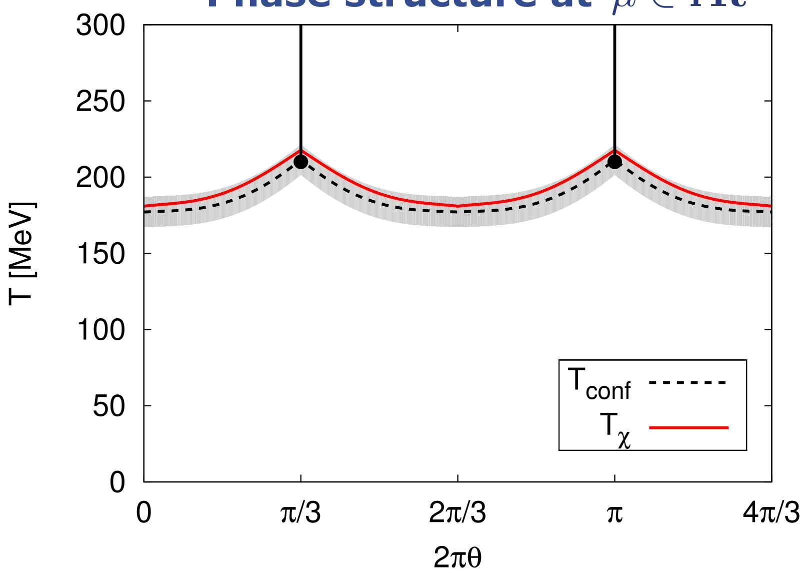


no confinement without chiral symmetry breaking

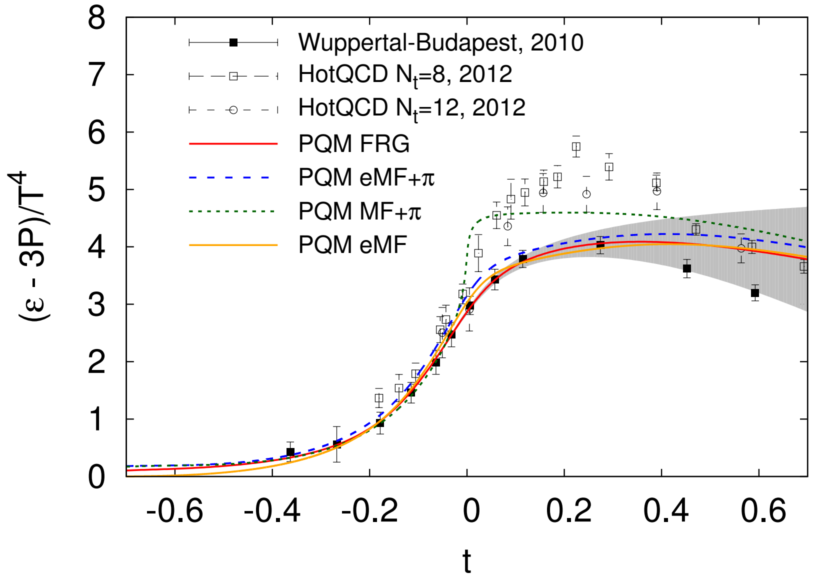
Summary & Outlook

▪ Phase structure and Transport

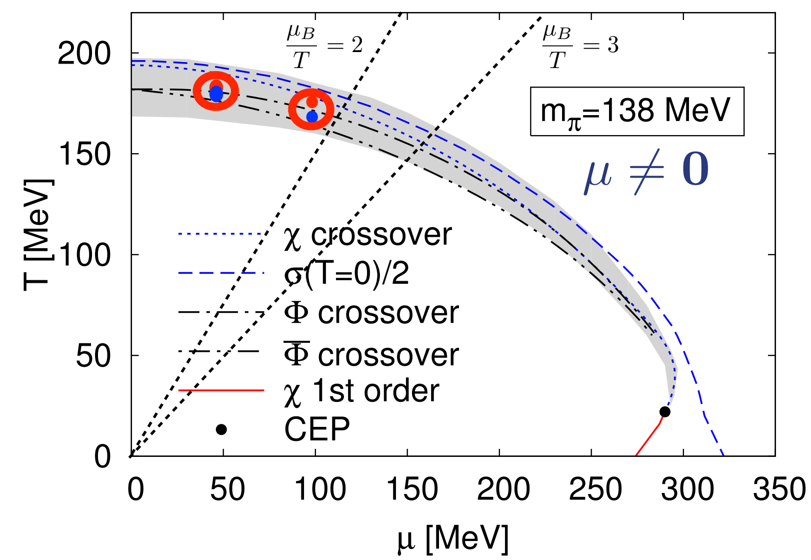
Phase structure at $\mu \in i\mathbb{R}$



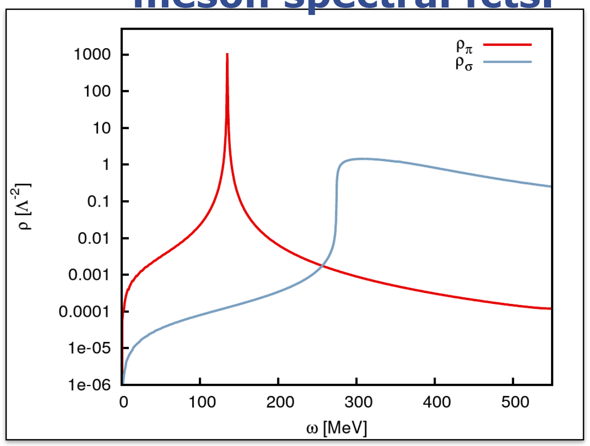
Interaction measure



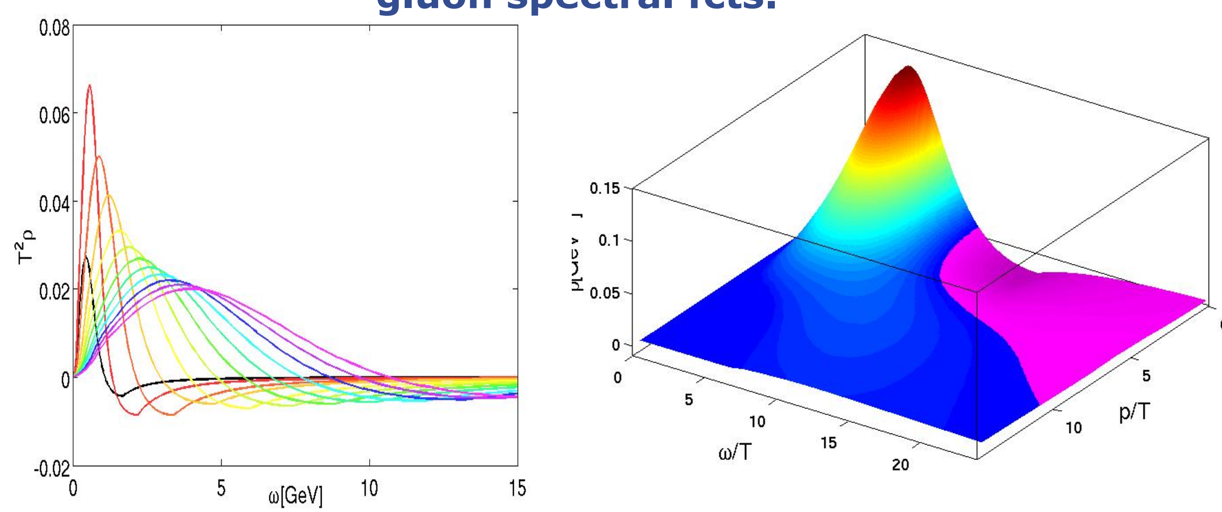
Phase structure at finite density



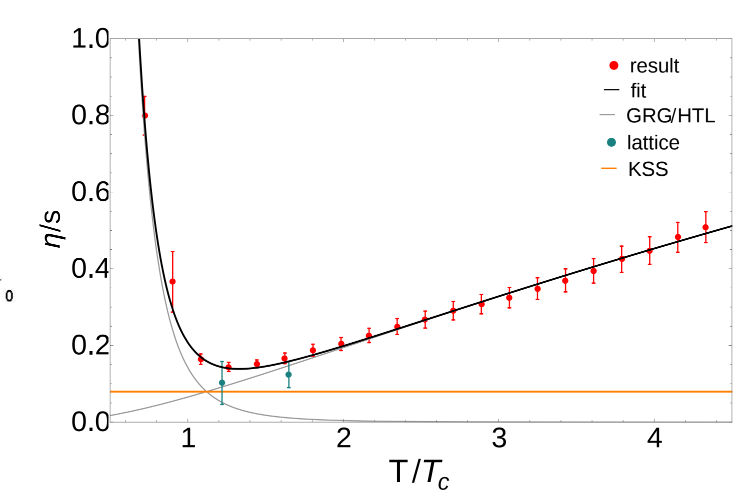
meson spectral fcts.



gluon spectral fcts.



glue viscosity/entropy



Summary & Outlook

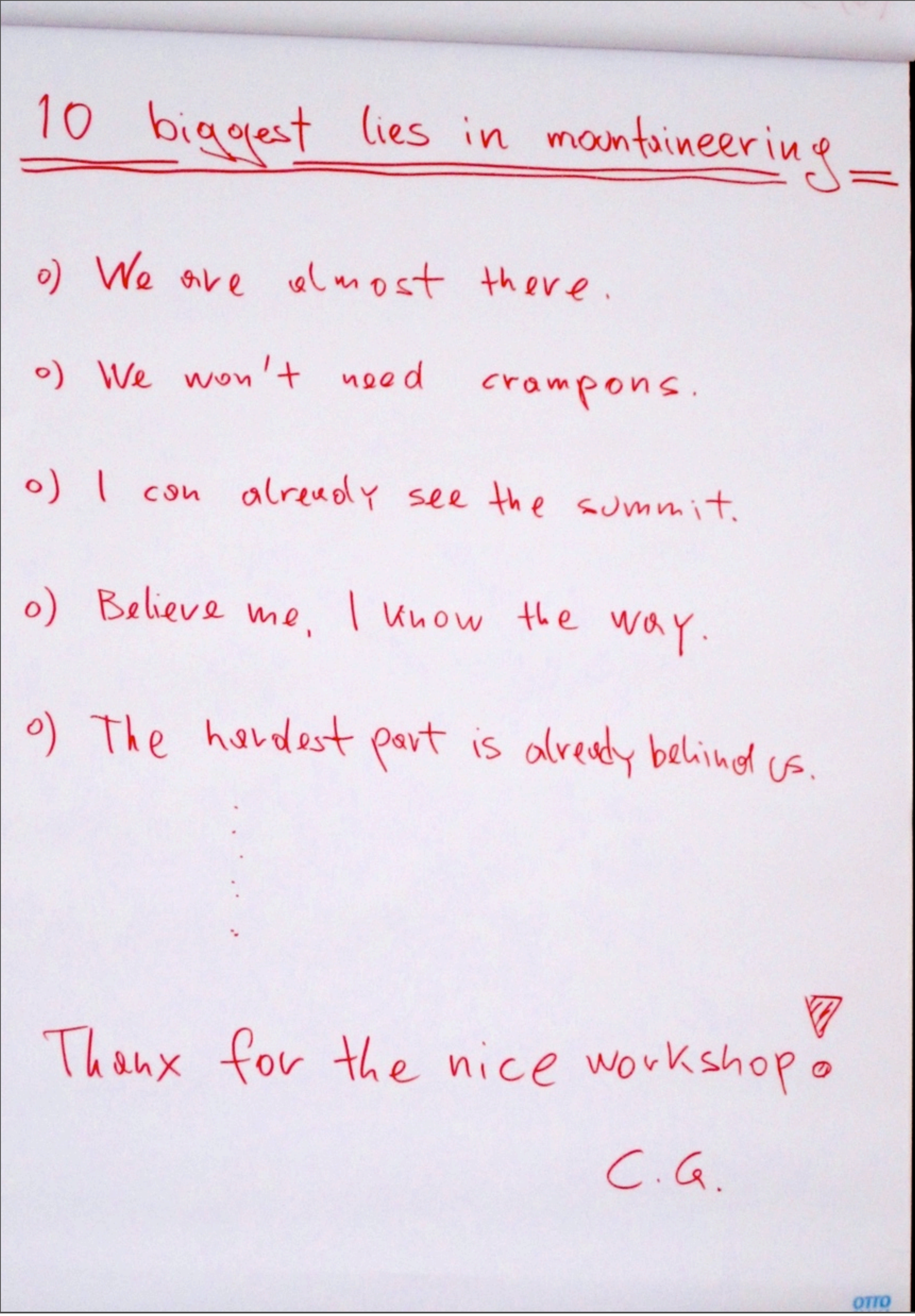
- **Chiral Symmetry Breaking and Confinement**

- **Phase Structure and Transport**

- **Towards quantitative precision**

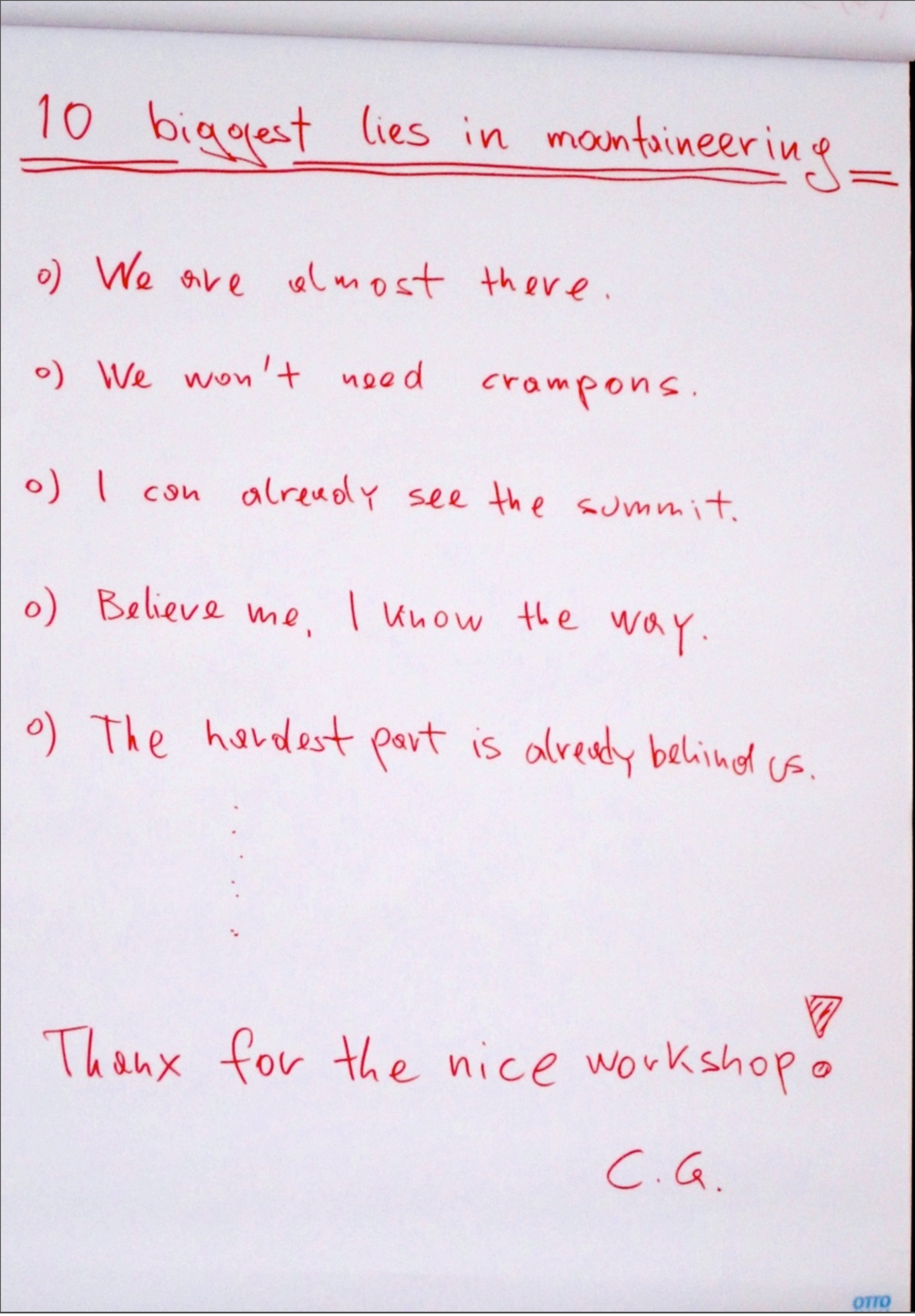
- **Baryons, high density regime & CEP, dynamics**

- **Hadronic properties**
  - **hadron spectrum & in medium modifications**
  - **low energy constants**



final word of caution

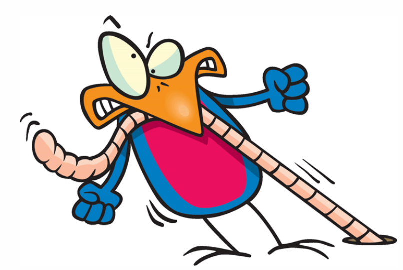
- **Original application: sign-problem**
- **General appplication: Evaluate systematic error**



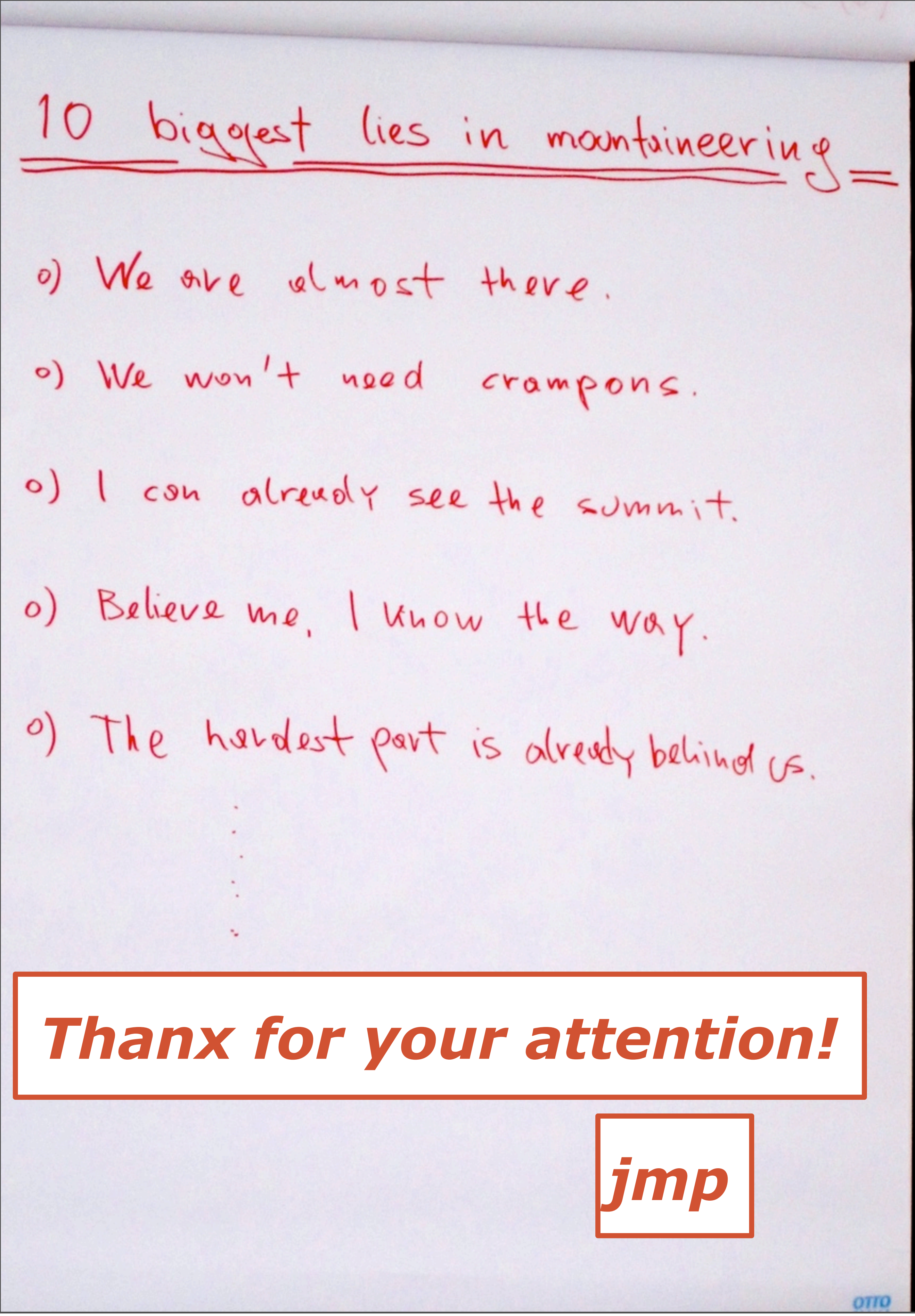
final word of caution

- Original application: sign-problem
- General appplication: Evaluate systematic error

the early bird catches the worm



we do not have the worm yet but we see it already!



Thanx for your attention!

jmp

final word of caution

- **Original application: sign-problem**

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we do not have the worm yet but we see it already!