Exploratory calculations of baryon resonances and exotic charmonia from Lattice QCD using Lüscher's method

Daniel Mohler

Hirschegg, January 15th, 2018



Outline

Introduction and motivation

- Recent progress in Lattice QCD simulations
- Coordinated Lattice Simulations (CLS)
- Illustrative example: ρ resonance and relevance to $(g-2)_{\mu}$

Exploratory calculations for charmonium resonances and bound states • Previous results for the χ'_{c0} / X(3915)

- Towards charmonium resonances from coupled-channel simulations
- Exploratory calculations for baryon resonances
 - Previous results
 - Challenges
 - Towards coupled-channel results for the $\Lambda(1405)$

Conclusions and Outlook

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Lattice Quantum Chromodynamics: What do we calculate?

Regularization of QCD by a 4-d Euclidean space-time lattice. (Kenneth Wilson 1974) Provides a calculational method for QCD



Euclidean correlator of two Hilbert-space operators \hat{O}_1 and \hat{O}_2 .

$$ig\langle \hat{O}_2(t)\hat{O}_1(0)ig
angle = \sum_n e^{-t\Delta E_n} \langle 0|\hat{O}_2|n
angle \langle n|\hat{O}_1|0
angle \ = rac{1}{Z}\int \mathcal{D}[\psi,\bar{\psi},U]e^{-S_E}O_2[\psi,\bar{\psi},U]O_1[\psi,\bar{\psi},U]$$

- Path integral over the Euclidean action S_{E,QCD}[ψ, ψ̄, U];
 (a sum over quantum fluctuations)
- Can be evaluated with *Markov Chain Monte Carlo* (using methods well established in statistical physics)

Recent progress in Lattice QCD

- Dynamical simulations with 2+1(+1) flavors of sea quarks
- Simulations at physical pion (light-quark) masses
- Isospin splitting and QCD+QED simulations
- Improved heavy quark actions for charm



BMW Collaboration, Borsanyi et al. Science 347 1452 (2015)

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Progress from an old idea: Lüscher's finite-volume method

M. Lüscher Commun. Math. Phys. 105 (1986) 153; Nucl. Phys. B 354 (1991) 531; Nucl. Phys. B 364 (1991) 237.

Basic observation: Finite volume, multi-particle energies are shifted with regard to the free energy levels due to the interaction

$$E = E(p_1) + E(p_2) + \Delta_E$$

- Energy shifts encode scattering amplitude(s)
- Original method: Elastic scattering in the rest-frame in multiple spatial volumes *L*³
- Coupled 2-hadron channels well understood
- 2 ↔ 1 and 2 ↔ 2 transitions well understood (example ππ → πγ^{*})
- significant progress for 3-particle scattering
 → see talk by Akaki Rusetsky



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Fully systematic calculation vs. exploratory study

Important lattice systematics from

- Taking the *continuum limit*: $a(g,m) \rightarrow 0$
- Taking the *infinite volume limit*: $L \to \infty$
- Calculation at (or extrapolation to) the physical pion mass

I cover many exploratory results

- Should be compared only qualitatively to experiment
- Provide an outlook on future Lattice QCD results

Example for fully systematic results:

• Flavor physics results listed in the FLAG review http://itpwiki.unibe.ch/flag/



Baryon resonances and charmonia from LQCD

CLS 2+1 flavor ensembles: Overview

Bruno et al. JHEP 1502 043 (2015); Bali et al. PRD 94 074501 (2016)



plots by Jakob Simeth, RQCD

- Letters in the name denote the aspect ratio T/L; First digit encodes β
- Ensembles at 5 lattice spacings and with a range of $M_{\pi} \leq 420 \text{MeV}$
- Ensembles to control (or exploit) finite volume effects

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CLS 2+1 flavor ensembles: Statistics – area \propto MDU

Bruno et al. JHEP 1502 043 (2015); Bali et al. PRD 94 074501 (2016)





plots by Jakob Simeth, RQCD

- > 4000 MDU for many ensembles Typically save 1 configuration every 4 MDU
 - target statistics chosen considering largest τ_{int} (often YM action density)

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CLS 2+1 flavor ensembles: Volumes used

Bruno et al. JHEP 1502 043 (2015); Bali et al. PRD 94 074501 (2016)



plots by Jakob Simeth, RQCD

- red: $m_{\pi}L \leq 4$; yellow: $4 \leq m_{\pi}L \leq 5$; green $5 \leq m_{\pi}L$
- Most ensembles with $m_{\pi}L \geq 4$
- Some smaller volumes to check finite size effects

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Correlation matrix built from both quark-antiquark ρ and $\pi\pi$ interpolators:

$$C(t) = \begin{pmatrix} \langle \rho(t)\rho(0)^{\dagger} \rangle & \langle \rho(t)(\pi\pi)(0)^{\dagger} \rangle \\ \langle (\pi\pi)(t)\rho(0)^{\dagger} \rangle & \langle (\pi\pi)(t)(\pi\pi)(0)^{\dagger} \rangle \end{pmatrix}$$

Where we use ρ^0 and $\pi^+\pi^-$ type interpolators:

$$\rho^{0}(P,t) \propto \sum_{\mathbf{x}} e^{-i\mathbf{P}\cdot\mathbf{x}} \left(\bar{u}\mathbf{a}\cdot\gamma u - \bar{d}\mathbf{a}\cdot\gamma d\right)(\mathbf{x},t)$$
$$(\pi\pi)(t) = \pi^{+}(\mathbf{p}_{1},t)\pi^{-}(\mathbf{p}_{2},t) - \pi^{-}(\mathbf{p}_{1},t)\pi^{+}(\mathbf{p}_{2},t)$$
$$\pi^{+}(p,t) \propto \sum_{\mathbf{x}} e^{-i\mathbf{p}\cdot\mathbf{x}}\bar{d}\gamma_{5}u(\mathbf{x},t)$$

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• Lüscher quantization condition

$$\delta_1(k) + \phi\left(\frac{L}{2\pi}k\right) = n\pi$$
 with $E_{cm}(k) = 2\sqrt{k^2 + m_\pi^2}$



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• In this simple case of elastic scattering: one phase-shift point for each energy level



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 with $E_{cm}(k) = 2\sqrt{k^2 + m_\pi^2}$



Similar results with $m_{\pi} \approx 265 \text{ MeV}$



Data from Felix Erben, Mainz

Treats all correlations correctly, see arXiv:1710.03529
Different fits for final publications: Breit-Wigner, Gounaris-Sakurai,

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Baryon resonances and charmonia from LQCD

$(g-2)_{\mu}$ and the large x_0 behavior of the vector correlator



Previous determination from arXiv:1705.01775 New analysis from Felix Erben.

- Data from Lüscher analysis \rightarrow more accurate determination of the large x_0 behavior
- Reduces uncertainty of lattice determination of the HVP contribution
- Lellouch-Lüscher yields timelike pion form factor

H.B. Meyer, PRL 107, 072002 (2011)

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 χ'_{c0} and X/Y(3915)

$$I^{G}(J^{PC}) = 0^{+}(0 \text{ or } 2^{++})$$

PDG interpreted X(3915) as a regular charmonium (χ'_{c0})

 $\begin{array}{l} {\sf Mass} \ m=3918.4\pm 1.9 \ {\sf MeV} \\ {\sf Full \ width} \ \Gamma=20\pm 5 \ {\sf MeV} \quad ({\sf S}=1.1) \end{array}$

• Some of the reasons to doubt this assignment:

Guo, Meissner Phys. Rev. **D**86, 091501 (2012) Olsen, PRD 91 057501 (2015)

- No evidence for fall-apart mode $X(3915) \rightarrow \overline{D}D$
- Spin splitting $m_{\chi_{c2}(2P)} m_{\chi_{c0}(2P)}$ too small
- Large OZI suppressed $X(3915) \rightarrow \omega J/\psi$
- Width should be significantly larger than $\Gamma_{\chi_{c2}(2P)}$
- Zhou *et al.* (PRL 115 2, 022001 (2015)) argue that what is dubbed X(3915) is the spin 2 state already known and suggests that a broader state is hiding in the experiment data.
- Observation of an alternative χ_{c0}(2P) by Belle: Chilikin *et al.* PRD 95 112003 (2017)

$$M = 3862^{+26+40}_{-32-13} \text{ MeV} \qquad \Gamma = 201^{+154+88}_{-067-82} \text{ MeV}$$

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χ'_{c0} : Exploratory lattice calculation



Lang, Leskovec, DM, Prelovsek, JHEP 1509 089 (2015)

- Assumes only $\overline{D}D$ is relevant
- Lattice data suggests a fairly narrow resonance with 3.9 GeV < M < 4.0 GeV and $\Gamma < 100 \text{MeV}$
- Future experiment and lattice QCD results needed to clarify the situation

χ_{c0}^{\prime} : Improvements and challenges

with G. Bali, S. Collins, M. Padmanath, S. Piemonte, S. Prelovsek

Improvements:

- High-precision determinations of the energy splittings needed
 → significantly improve statistics by using CLS ensembles
- Bigger density of energy level needed
 - \rightarrow Calculation in multiple volumes: CLS ensembles U101, H105, N101
 - \rightarrow Add information from moving frames
- Treatment as a single-channel problem only sensible if X(3915) is indeed a spin-2 state
 - \rightarrow consider coupled channel $D\bar{D}$, $J/\psi\omega$ and $D_s\bar{D}_s$

Challenges:

- Need strategy for dealing with (largish) discretization effects
- $Tr(M) = \text{const. trajectory means } D_s \overline{D}_s$ threshold lower

Charm-quark mass tuning and expected energy levels

D-meson mass as a function of the Expected energy levels: CLS-ensembles charm-quark hopping parameter κ_c U101, H105, N101 2000sqrt(s) [GeV] 3.80 $\kappa_c = 0.123147$ 19503.75 1900 $m_D \; [{\rm MeV}]$ 3.70 0.124056 3.65 18503.60 1800 3.55 0.12522 3.50 17507.95 8.00 8.05 8.10 8.15 2.0 2.5 3.0 3.5 4.0 4.5

- Currently: Low statistics results on U101
- For these results: Charm quark tuning by RQCD
- Will add further charm-quark masses if needed

Interpolator basis

$$A_1^{++}$$
 $(J^{PC} = 0^{++}, 4^{++}, \ldots)$

Label n	Operator		
0	$\bar{q} q$		
1	$ar{q} \ \gamma_i \overline{ abla}_i \ q$		
2	$\bar{q} \gamma_i \gamma_t \overrightarrow{\nabla}_i q$		
3	$\bar{q} \overleftarrow{\nabla}_i \overrightarrow{\nabla}_i q$		
4	$\bar{q} \Delta \Delta q$		
5	$\bar{q}_i \Delta \gamma_i \nabla_i q$		
6	$\bar{q} \overleftarrow{\Delta} \gamma_i \gamma_t \overrightarrow{\nabla}_i q$		
7	$O^{\bar{D}(0)D(0)} \sim \bar{c}\gamma_5 l \bar{l}\gamma_5 c$		
8	$O^{\bar{D}(0)D(0)} \sim \bar{c}\gamma_5\gamma_t l\bar{l}\gamma_5\gamma_t c$		
9	$O^{\bar{D}(p)D(-p)} \sim \bar{c}\gamma_5 l \bar{l}\gamma_5 c$		
10	$O^{\bar{D}^*(0)D^*(0)} \sim \bar{c}\gamma_i l \bar{l}\gamma_i c$		
11	$O^{\bar{D}^*(0)D^*(0)} \sim \bar{c}\gamma_i\gamma_t l\bar{l}\gamma_i\gamma_t c$		
12	$O^{J/\psi(0)\omega(0)} \sim \bar{c}\gamma_i c \bar{l}\gamma_i l$		
13	$O^{J/\psi(0)\omega(0)} \sim \bar{c}\gamma_i\gamma_t c \bar{l}\gamma_i\gamma_t l$		
14	$O^{ar{D}_s(0)D_s(0)}\simar{c}\gamma_5sar{s}\gamma_5c$		
15	$O^{\bar{D}_s(0)D_s(0)} \sim \bar{c}\gamma_5\gamma_t s \bar{s}\gamma_5\gamma_t c$		



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A first look at mass splittings

Preliminary results: Energy splittings from 120 configurations of U101

	$\kappa_c = 0.12522$	$\kappa_c = 0.12315$	Experiment
$m_{J/\Psi} - m_{\eta_c}$	106.9(0.6)(1.1)	98.0(0.5)(1.1)	113.2(0.7)
$m_{D_s^*}-m_{D_s}$	131.3(1.9)(1.4)	118.4(2.0)(1.3)	143.8(0.4)
$m_{D^*} - m_D$	127.8(3.9)(1.4)	115.1(4.1)(1.2)	140.66(10)
$2m_{\overline{D}} - m_{\overline{cc}}$	912.0(7.6)(9.8)	939.7(8.1)(10.1)	882.4(0.3)
$2M_{\overline{D_s}} - m_{\overline{c}\overline{c}}$	1011.7(4.2)(10.9)	1036.0(4.5)(11.1)	1084.8(0.6)
$m_{D_s} - m_D$	47.2(2.1)(0.5)	45.7(2.2)(0.5)	98.87(29)

• Unphysical $m_{D_s} - m_D$ creates a special challenge!

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Results from 3-quark interpolating fields

• Many older results from 3-quark interpolators

Example: Engel, Lang, DM, Schäfer, PRD 87 074504 (2013) $\Lambda^\prime \, {\rm s}$ in Engel et al. PRD 87 034502 (2013)



- OK at heavy quark masses where states are stable
- Typically no indications of close-by scattering thresholds
- Experience from meson sector: Spectrum not only incomplete but wrong!

Meson-baryon scattering – challenges

Nucleon noise to signal

• For the nucleon we have (argument by Parisi, Lepage)

$$N\sigma_{N,\mathbf{p}=0}^{2} = \left\langle C_{N}(\mathbf{p}=0,t;m)^{2} \right\rangle - \left\langle C_{N}(\mathbf{p}=0,t;m) \right\rangle^{2}$$

$$\propto Z_{3\pi} \mathrm{e}^{-3m_{\pi}t} + Z_{N}^{2} \mathrm{e}^{-2m_{N}t}$$

• The noise to signal ratio therefore degrades exponentially

$$rac{\sigma_N(t)}{\langle C_N(t)
angle}\simeq rac{1}{\sqrt{N}}{
m e}^{\left(m_N-rac{3}{2}m_\pi
ight)t}$$

- Contractions are more complicated
- Less cases where 3-particle scattering can be ignored (in a first step)

Nucleon noise/signal



- Slope in (most of) plateau region does not reach asymptotic value (given by $m_N \frac{3}{2}m_{\pi}$)
- Suggests that in practice noise/signal scaling is not as severe
- Exponential growth qualitatively observed

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Meson-baryon scattering - previous results

• S-wave pion-nucleon scattering with $J^P = \frac{1}{2}^{-1}$

Lang, Verduci, PRD 87 054502 (2013)

- CMF spectrum qualitatively in agreement with negative parity spectrum
- Pion-nucleon scattering in the Roper channel

Lang et al. PRD 95 014510 (2017)

- CMF spectrum from 3-quark, $N(p)\pi(-p)$, and $N\sigma$ interpolators
- No finite volume state seen for the $N^*(1440)$
- Results compatible with models with a dynamically generated Roper resonance from coupled channel $N\pi$, $N\sigma$, $\Delta\pi$
- $I = \frac{3}{2}$ p-wave nucleon pion scattering and the $\Delta(1232)$

Andersen, Bulava, Hörz, Morningstar arXiv:1710.01557

- At $m_{\pi} = 280$ MeV, observe a near threshold resonance
- Coupling $g_{\Delta N\pi}$ similar to experiment
- Should be quite feasible at lighter pion masses

Exploratory study: $\Lambda(1405), J^P = \frac{1}{2}^{-1}$

• PDG (4 star resonance)

$$M_{\Lambda} = 1405^{+1.3}_{-1.0} MeV$$
 $\Gamma_{\Lambda} = 50.5 \pm 2.0$

- Unitarized χ PT + Model input yields 2 poles with $\Re \approx 1400$ MeV
- CLAS observes different line shapes for $\Sigma^-\pi^+$, $\Sigma^+\pi^-$ and $\Sigma^0\pi^0$ Interference between I = 0 and I = 1 amplitudes is the likely reason
- Even the $\Sigma^0 \pi^0$ is badly described by a single Breit-Wigner
- CLAS data consistent with popular 2-pole picture
- No satisfactory lattice results (although claims exist)
- Relevant channels: $\Sigma \pi$, $N\bar{K}$ (and maybe $\Lambda \eta$); simulation in isospin limit
- Goal: Explore coupled channel problem and extract scattering amplitudes from the low-lying energy spectrum

together with J. Bulava, M. Hansen, B. Hörz, C. Morningstar

Ensemble and group theory

Current data on CLS Ensemble N200

<i>a</i> [fm]	$T \times L^3$	m_{π} [MeV]	$m_K [{\rm MeV}]$	$m_{\pi}L$	Ncnfg
0.0644	$128 imes 48^3$	280	460	4.3	427

Lattice irreducible representations for a given J^P

see Morningstar et al. arXiv:1303.6816

J^P	[000]	[00n]	[0nn]	[nnn]	
$\frac{1}{2}^{+}$	G_{1g}	G_1	G	G	$\Lambda, \Lambda(1600)$
$\frac{1}{2}^{-}$	G_{1u}	G_1	G	G	$\Lambda(1405),\Lambda(1670)$
$\frac{3}{2}^{+}$	H_g	G_1, G_2	2G	F_1, F_2, G	$\Lambda(1690)$
$\frac{3}{2}^{-}$	H_u	G_1, G_2	2G	F_1, F_2, G	$\Lambda(1520),\Lambda(1690)$

A first look at some data: Truncating the basis



- Plot from an older dataset on 427 configurations
- Basis including meson-meson states starts to yield usable plateaus
- Even a diverse 3-quark basis does not yield the correct spectrum!

Rest frame calculation: Fit stability



- Results for fits from t_{min} to 20
- Non-interacting levels indicated by their central value
- Correlated differences start to become significant

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Rest frame calculation: effective range approximation



- Cannot rule out the simplest scenario of one bound state below $\Sigma\pi$
- Makes no statement about more complicated scenarios or physical m_{π}

Adding moving frames: Pattern of energy levels



- With Tr(M) = const., expect mild chiral extrapolation of *uds* states
- Our *G*₁(1) data does not have a large enough basis (caution)
- The Λ ground state is seen where expected
- No indication of levels close to $\Lambda(1600)$ in G_{1g} irrep
- No indication of levels close to $\Lambda(1520)$ in any irrep
- Apparent absence of FV states should constrain models

- Currently doubling N200 statistics ($\approx 400 \text{ configs} \rightarrow \approx 800 \text{ configs}$)
- Still enlarging the basis somewhat (include more 3-quark interpolators; include some $\Lambda \eta$)
- We plan to run more ensembles (will likely add a lighter pion mass next)
- We may add data at different lattice spacings
- Once we have bigger dataset: Check for consistency with model-inspired K-matrix parameterizations

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- Lüscher studies of meson-baryon scattering and for exotic charmonium-like states are very challenging
 - they require large statistics
 - coupled-channel formalism needed for even the simplest systems
- Making contact with experiment requires a variety of gauge ensembles
- Expect to see many new results over the next years
- Large potential to solve some of the puzzles surrounding hadrons with exotic properties (Roper, $\Lambda(1405)$, low mass charmonium-like states, ...)

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Thank you!

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Baryon resonances and charmonia from LQCD Hirschegg, Janua

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