Transport Coefficients of NJL Model: Chiral Symmetry Broken Phase

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Introduction: Hydrodynamics

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- A "hierarchy" of dynamical description

Micro: QCD $l_{\rm micro} \sim d_{\rm inter}$ $\mathcal{L} = \bar{\psi}_f (iD_\mu \gamma^\mu - m_f) \psi_f - \frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu}$

Meso: kinetic $l_{\rm meso} \sim l_{\rm mfp}$ $\frac{\partial f_p}{\partial t} + \mathbf{v} \cdot \frac{\partial f_p}{\partial \mathbf{x}} + \mathbf{F} \cdot \frac{\partial f_p}{\partial \mathbf{p}} = \mathcal{C}[f_p]$

Macro: hydro $l_{\rm macro} \sim \rho / \partial_x \rho$ $\frac{\partial}{\partial t}(\rho v_i) + \frac{\partial}{\partial x_j} \Pi_{ij} = 0$

- Top to bottom "*coarse graining* ": short-distance information loses, universality increases. Hydro equations are general.
- Only those long wavelength low frequency modes are included in hydrodynamics: conserved densities, Goldstone modes. Short-distance physics is only in transport coefficients + EOS.

Introduction: Hydrodynamics

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- **Relativistic hydrodynamic equations (conservation laws):**

$$\partial_\mu T^{\mu\nu} = 0 \qquad \partial_\mu N^\mu = 0$$

- **Constructive relations (Landau-Lifshitz frame)**

$$T^{\mu\nu} = (\varepsilon + P)\, u^\mu u^\nu - P g^{\mu\nu} + \tau^{\mu\nu}$$

$$N^\mu = n u^\mu + j^\mu$$

Ideal +Dissipative

$$\tau^{\mu\nu} = \eta \left(\nabla^\mu u^\nu + \nabla^\nu u^\mu - \frac{2}{3} \Delta^{\mu\nu} \partial_\mu u^\mu \right) + \zeta \Delta^{\mu\nu} \partial_\sigma u^\sigma + O(\partial^2)$$

$$j^\mu = \kappa \left(\frac{nT}{\varepsilon + P} \right)^2 \nabla^\mu \left(\frac{\mu}{T} \right) + O(\partial^2)$$

- **Transport coefficients: shear viscosity η , bulk viscosity ζ , heat conductivity κ .**

Introduction: Transport coefficients

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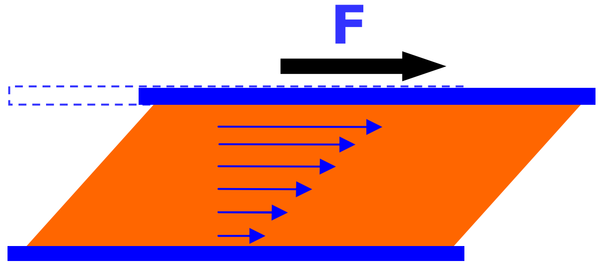
- Phenomenological meaning of transport coefficients.

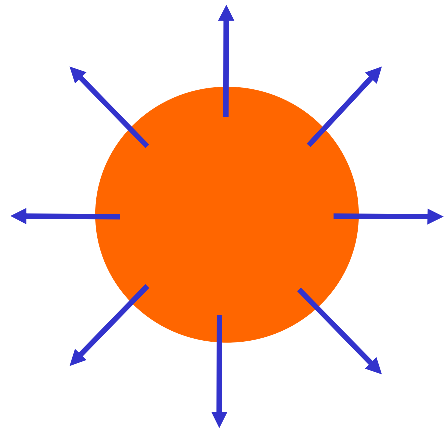
shear force

$$F \sim \eta \frac{\partial v_x}{\partial y} \qquad \dot{s} \sim \frac{\eta}{2T}\left(\frac{\partial v_x}{\partial y}\right)^2$$



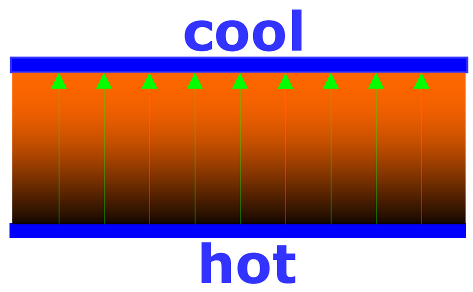
pressure shift

$$\Delta P \sim -\zeta \nabla \cdot \mathbf{v} \qquad \dot{s} \sim \frac{\zeta}{T}(\nabla \cdot \mathbf{v})^2$$



heat flow

$$Q_y \sim -\kappa \frac{\partial T}{\partial y} \qquad \dot{s} \sim \frac{\kappa}{T^2}\left(\frac{\partial T}{\partial y}\right)^2$$

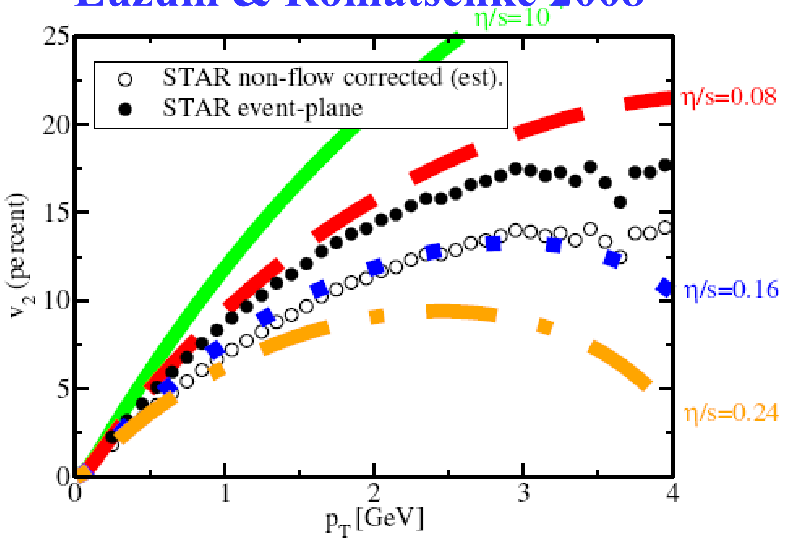


Introduction: Transport coefficients

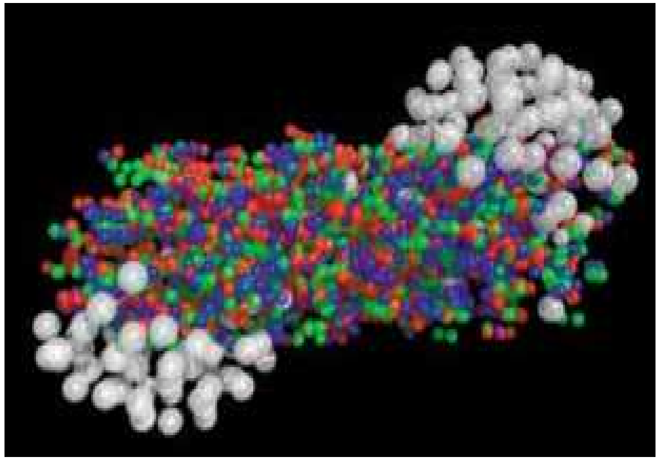
- Transport coefficients connect micro to macro physics.

Macro

Luzum & Romatschke 2008



Micro

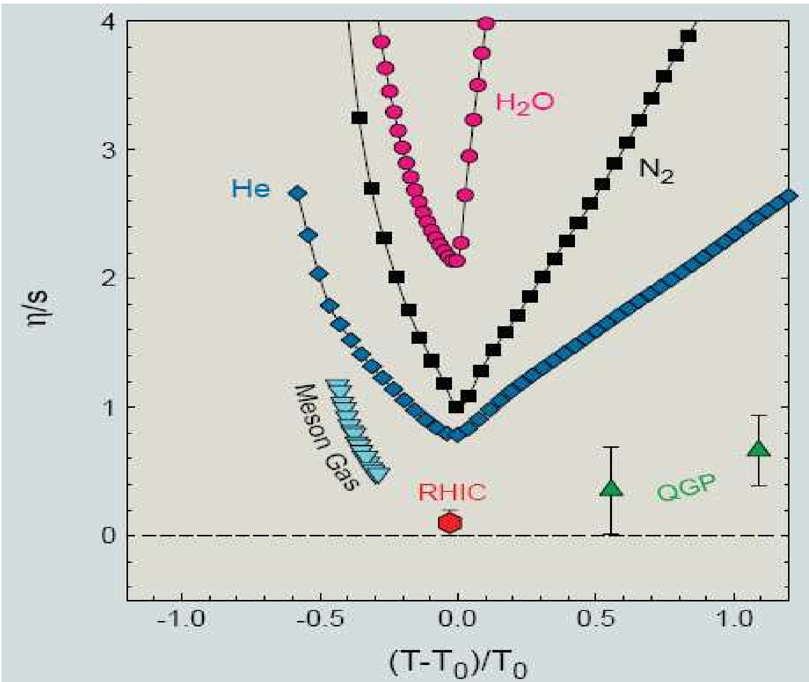


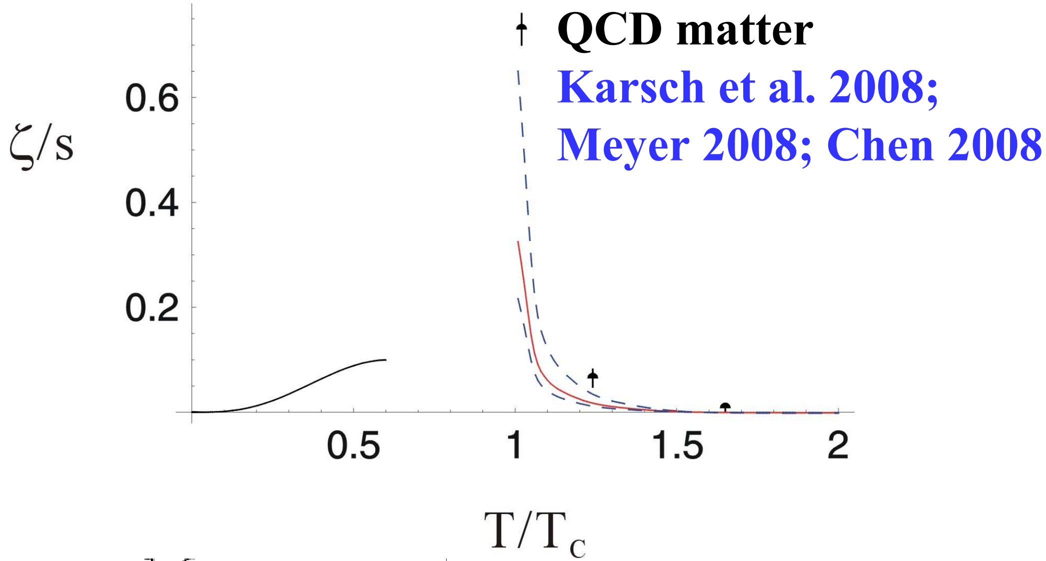
Large v2 → **Small shear viscosity** → **Strongly coupled matter**

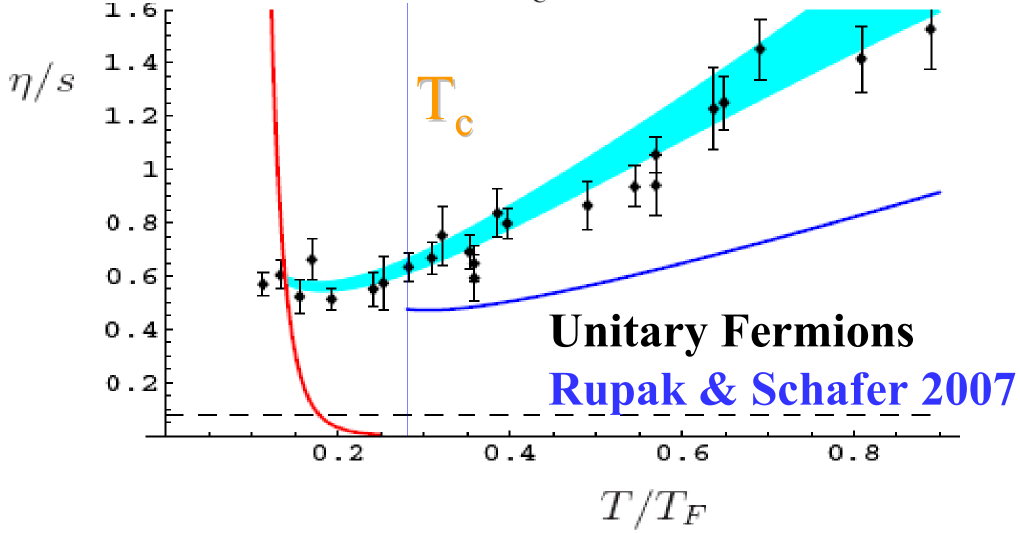
Introduction: Transport coefficients

- Transport coefficients are related to phase transition.

Lacey et al.2007

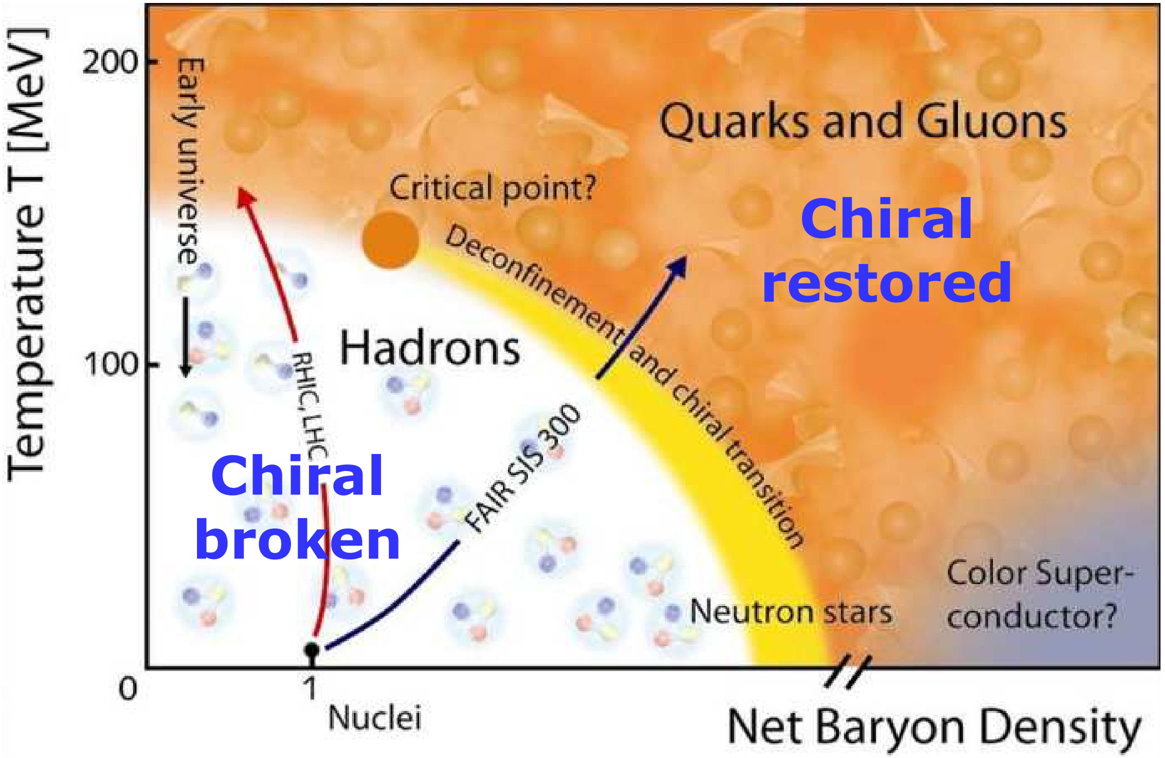






Dynamical critical behaviors: talks of Skokov, Nakano, Llanes-Estrada.

Introduction: Chiral phase transition



Chiral phase transition

$$E \sim \Lambda_{QCD}$$

Nonperturbative

Effective chiral model: NJL

Transport coefficients in NJL model:

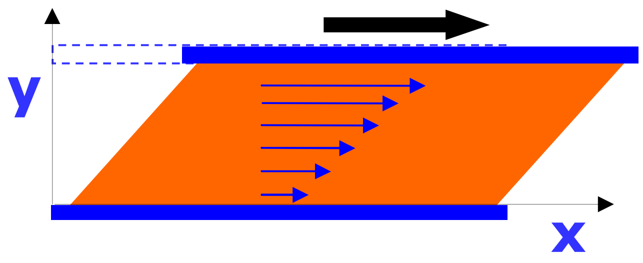
How fluid?

Critical behaviors?

NJL model: Kubo's formulae

- **Kubo's formulae give how the system near equilibrium linear responses to external weak (thermodynamic) forces.**
- **Let v_x has a small gradient in y direction.**
  The perturbation Hamiltonian reads (Zubarev 1974),

$$\hat{H}_{pert}(t) = \int_{-\infty}^{t} dt' \int d^3\mathbf{x} \hat{T}_{xy}(\mathbf{x},t')\partial_y v_x(\mathbf{x},t')$$



$$\delta\langle\hat{T}_{xy}\rangle(\mathbf{x},t) = i\int_{-\infty}^{t} dt' \langle[\hat{H}_{pert}(t'),\hat{T}_{xy}(\mathbf{x},t)]\rangle$$

$$= \boxed{\int_{-\infty}^{t} dt' \int_{-\infty}^{t'} dt'' \int d^3\mathbf{x}'' G_R^{\eta}(\mathbf{x},t;\mathbf{x}'',t'')}\partial_y v_x(\mathbf{x}'',t'')$$

$$G_R^{\eta}(\mathbf{x},t;\mathbf{x}'',t'') \equiv -i\theta(t-t'')\langle[\hat{T}^{xy}(\mathbf{x},t),\hat{T}^{xy}(\mathbf{x}'',t'')]\rangle$$

NJL model: Kubo's formulae

- After Fourier transformation, shear viscosity:

$$\eta = \frac{i}{\omega} \lim_{\omega \to 0} \lim_{\mathbf{k} \to \mathbf{0}} G_R^{\eta}(\omega, \mathbf{k}) = -\frac{\partial}{\partial \omega} \mathrm{Im} G_R^{\eta}(\omega, \mathbf{0})\Big|_{\omega \to 0}$$

- Similarly, bulk viscosity and heat conductivity:

$$\zeta = -\frac{1}{9} \frac{\partial}{\partial \omega} \mathrm{Im} G_R^{\zeta}(\omega, 0)\Big|_{\omega \to 0}$$

$$\kappa = \frac{T}{3} \left(\frac{\varepsilon + P}{nT} \right)^2 \frac{\partial}{\partial \omega} \mathrm{Im} G_R^{\kappa}(\omega, 0)\Big|_{\omega \to 0}$$

$$G_R^{\zeta}(\mathbf{x}, t) = -i\theta(t) \langle [\hat{T}_{\mu}^{\mu}(\mathbf{x}, t), \hat{T}_{\nu}^{\nu}(0)] \rangle$$

$$G_R^{\kappa}(\mathbf{x}, t) = -i\theta(t) \langle [\hat{N}^{\mu}(\mathbf{x}, t), \hat{N}_{\mu}(0)] \rangle$$

NJL model: Kubo's formulae

- **2-flavor NJL model: symmetry** $SU_C(3) \otimes SU(2)_V \otimes SU_A(2) \otimes U_B(1)$

$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ + \mu\gamma^0)\psi + g[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\boldsymbol{\tau}\psi)^2]$$

$$\hat{T}^{xy} = i\bar{\psi}\gamma^y\partial^x\psi$$

$$G^{\eta}_{\beta}(i\omega_n,\mathbf{k}) = -\int\frac{d\epsilon_1}{2\pi}\frac{d\epsilon_2}{2\pi}\int\frac{d^3\mathbf{p}}{(2\pi)^3}\frac{p_x^2[n_F(\epsilon_2-\mu)-n_F(\epsilon_1-\mu)]}{i\omega_n-\epsilon_1+\epsilon_2}\mathrm{Tr}[\gamma^2\rho(\epsilon_1,\mathbf{p}+\mathbf{k})\gamma^2\rho(\epsilon_2,\mathbf{p})]$$

$$G^{\eta}_{R}(w,\mathbf{k}) = G^{\eta}_{\beta}(i\omega_n\to\omega+i0^+,\mathbf{k})$$

$$\mathrm{Im}G^{\eta}_{R}(\omega,\mathbf{k}) = \frac{1}{2}\int\frac{d\epsilon}{2\pi}\int\frac{d^3\mathbf{p}}{(2\pi)^3}p_x^2[n_F(\epsilon-\mu)-n_F(\epsilon+\omega-\mu)]\mathrm{Tr}[\gamma^2\rho(\epsilon+\omega,\mathbf{p}+\mathbf{k})\gamma^2\rho(\epsilon,\mathbf{p})]$$

$$\eta = -\frac{1}{2}\int\frac{d^4p}{(2\pi)^4}p_x^2 n_F'(p_0-\mu)\mathrm{Tr}[\gamma^2\rho(p_0,\mathbf{p})\gamma^2\rho(p_0,\mathbf{p})]$$

Quark spectral function $\rho(p) = \rho_s + \rho_\mu\gamma^\mu$

Quark propagator $S(p_0,\mathbf{p}) = \int_{-\infty}^{\infty}\frac{d\epsilon}{2\pi}\frac{\rho(\epsilon,\mathbf{p})}{p_0-\epsilon}$

NJL model: Kubo's formulae

- Similarly, for bulk viscosity we have (EOM imposed)

$$\hat{T}^{\mu}_{\mu} = -\bar{\psi} i \gamma^{\rho} \partial_{\rho} \psi$$

- For heat conductivity we use

$$\hat{N}^{\mu} = \bar{\psi} \gamma^{\mu} \psi$$

- We have

$$\zeta = -\frac{1}{18} \int \frac{d^4 p}{(2\pi)^4} n_F'(p_0 - \mu) \mathrm{Tr} \left[\not{p} \rho(p) \right]^2$$

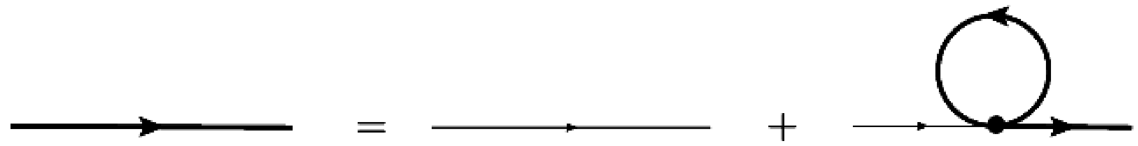
$$\kappa = -\frac{T}{6} \left(\frac{\varepsilon + P}{nT} \right)^2 \int \frac{d^4 p}{(2\pi)^4} n_F'(p_0 - \mu) \mathrm{Tr} \left[\gamma^{\mu} \rho(p) \gamma_{\mu} \rho(p) \right]$$

NJL model: mean-field approximation

- 2-flavor NJL model: symmetry $SU_C(3) \otimes SU(2)_V \otimes SU_A(2) \otimes U_B(1)$

$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ + \mu\gamma^0)\psi + g[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\boldsymbol{\tau}\psi)^2]$$

- Non-renormalizable: introduce a cutoff $\Lambda \simeq 653\mathrm{MeV}$
- Coupling constant $g\Lambda^2 \simeq 2.14 > 1$: perturbation theory fails
- Approximation scheme: 1/Nc expansion (g ~ 1/Nc).
- Leading order: O(1) to gap equation. Mean-field (Hartree) approx.

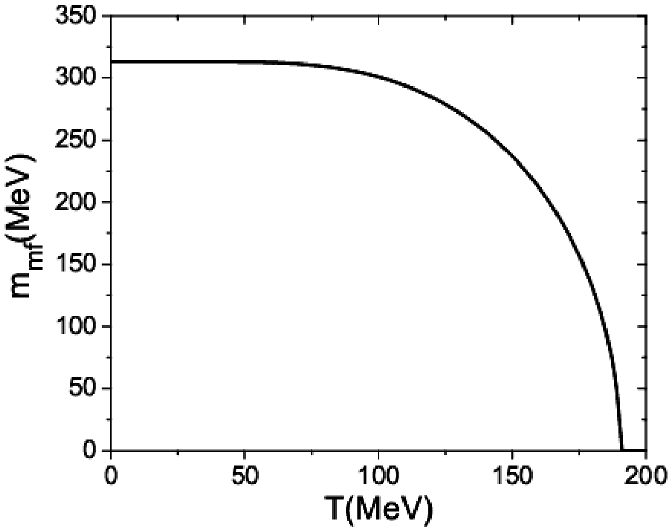


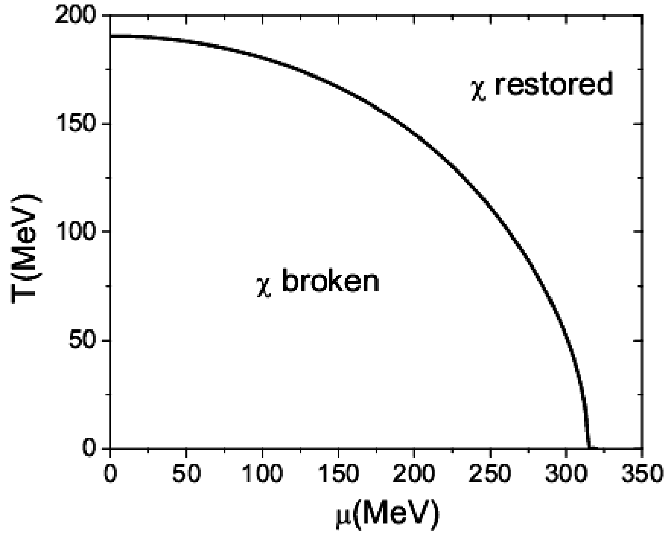
$$S_{\mathrm{mf}}^{-1}(p) = S_0^{-1}(p) - \Sigma_{\mathrm{mf}}(p)$$

$$m_{\mathrm{mf}} = \Sigma_{\mathrm{mf}} = 2m_{\mathrm{mf}}N_cN_fg\int\frac{d^3\mathbf{p}}{(2\pi)^3}\frac{1}{E_\mathbf{p}}\left[\tanh\frac{E_\mathbf{p}+\mu}{2T} + \tanh\frac{E_\mathbf{p}-\mu}{2T}\right]$$

NJL model: mean-field approximation

- Mean-field gives 2nd phase transition at Tc, 1st at μ_c.





- Mean-field mass is always real.

$$\rho_{\mathrm{mf}}(\epsilon) \propto \delta(\epsilon \pm E_{\mathbf{p}}) \quad \longrightarrow \quad \rho_{\mathrm{mf}}(\epsilon)\rho_{\mathrm{mf}}(\epsilon+\omega) = 0$$

Mean-field does not contribute to transport coefficients

- We need to go beyond mean-field approx.. How to do?

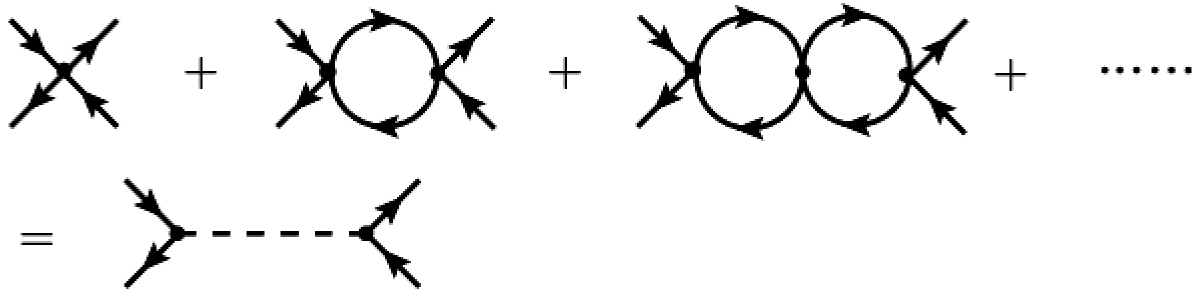
NJL model: O(1/Nc) correction

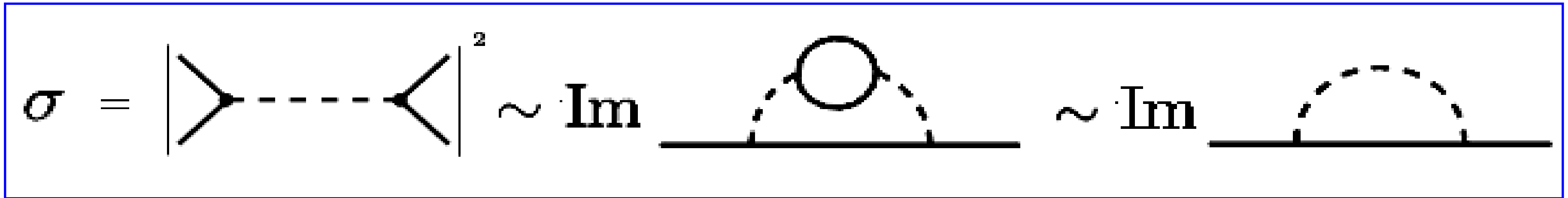
- **Kinetic argument gives**

$$\eta \sim \frac{1}{3} n \bar{p} l_{\mathrm{mfp}} \sim \frac{T}{\sigma} \qquad\qquad \zeta \sim \eta \left(\frac{1}{3} - c_s^2 \right)^2$$

- **O(1/Nc) contribution to quark cross section: RPA bubbles**

e. g. :





Quark self-energy should contain this sunrise diagram

NJL model: RPA for mesons

- RPA bubble summation = Bethe-Salpeter equation for meson

Meson propagator:

$$D_M^{-1}(q) = (-2g)^{-1} + \Pi_M(q)$$

Meson self-energy:

$$\Pi_M(q) = i \int \frac{d^4p}{(2\pi)^4} \mathrm{Tr}\left[V_M^* S_{\mathrm{mf}}(p+q) V_M S_{\mathrm{mf}}(p)\right]$$

Quark-Meson vertices:

$$V_M = \begin{cases} 1 & M = \sigma \\ i\tau_+\gamma_5 & M = \pi_+ \\ i\tau_-\gamma_5 & M = \pi_- \\ i\tau_3\gamma_5 & M = \pi_0 \end{cases}, \qquad V_M^* = \begin{cases} 1 & M = \sigma \\ i\tau_-\gamma_5 & M = \pi_+ \\ i\tau_+\gamma_5 & M = \pi_- \\ i\tau_3\gamma_5 & M = \pi_0 \end{cases}$$

NJL model: RPA for mesons

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- Direct calculation shows (below Tc)

$$\mathrm{Re}\Pi_M(\omega,\mathbf{0}) = N_c N_f \mathcal{P}\int \frac{d^3\mathbf{p}}{(2\pi)^3}\frac{1}{E_p}\frac{E_p^2-\epsilon_M^2/4}{E_p^2-\omega^2/4}\left[\tanh\frac{E_p+\mu}{2T}+\tanh\frac{E_p-\mu}{2T}\right]$$

$$\begin{aligned}\mathrm{Im}\Pi_M(\omega,\mathbf{0}) &= N_c N_f\frac{\omega^2-\epsilon_M^2}{8\pi\omega}\sqrt{\omega^2-4m^2}\left[\tanh\frac{\omega+2\mu}{4T}+\tanh\frac{\omega-2\mu}{4T}\right]\theta(\omega-2m)\\ &- N_c N_f\frac{\omega^2-\epsilon_M^2}{8\pi\omega}\sqrt{\omega^2-4m^2}\left[\tanh\frac{\omega+2\mu}{4T}+\tanh\frac{\omega-2\mu}{4T}\right]\theta(-\omega-2m)\end{aligned}$$

- Position of pole of meson propagator gives mass and width.

$$1-2g\Pi_M(\mathcal{M}_M,\mathbf{0})=0 \qquad \mathcal{M}_M = m_M - i\Gamma_M/2$$

- The pole mass is zero for pion, and 2m for sigma. Pole width is zero. Pion is Goldstone boson for chiral symmetry breaking.

NJL model: RPA for mesons

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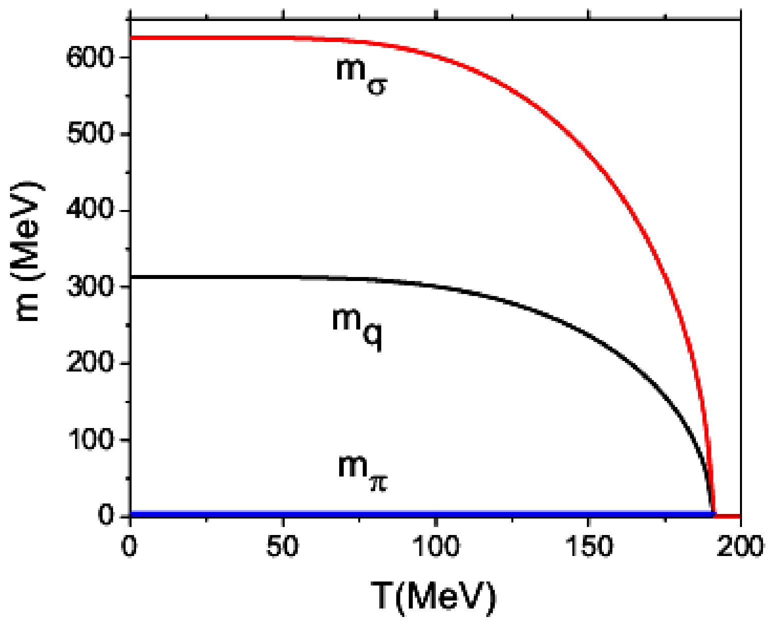
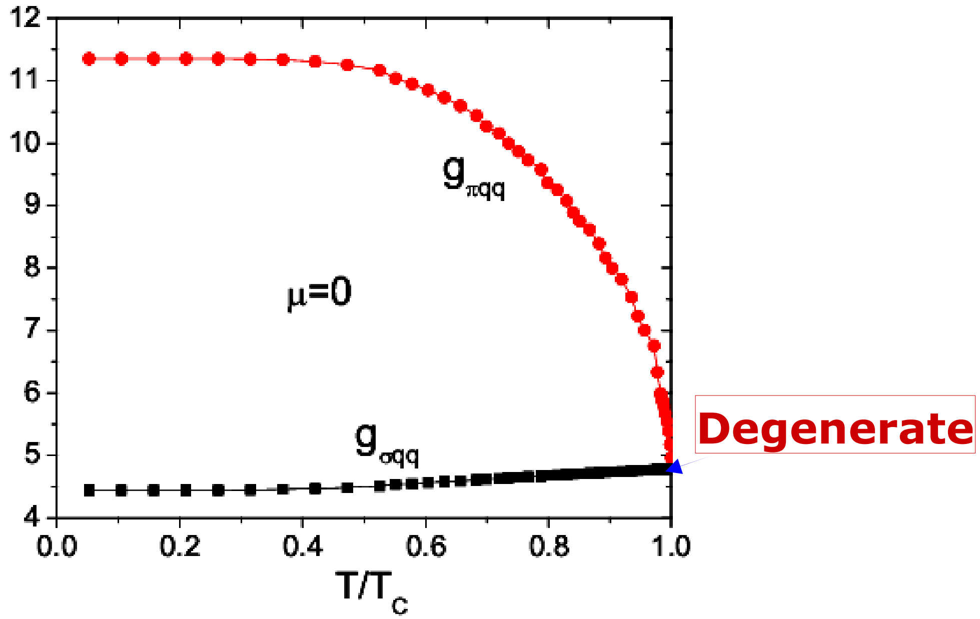
- **Use pole approximation (valid when pole is well separated from continuum states)**

$$D_M(q) \approx \frac{-g_{Mqq}^2}{q^2 - M_M^2} \qquad g_{Mqq}^2 = \left(\frac{\partial \mathrm{Re}\Pi_M}{\partial q_0^2} \right)^{-1} \Bigg|_{q_0 = m_M, \mathbf{q}=0}$$





NJL model: RPA for mesons

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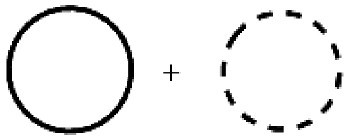
- **Thermodynamic potential**

$\Omega =$ 

$$\approx \Omega_{\rm mf} + \sum_M \int \frac{d^3\mathbf{q}}{(2\pi)^3} \left[\frac{E_M}{2} + T \ln\left(1 - e^{-\beta E_M}\right) \right]$$

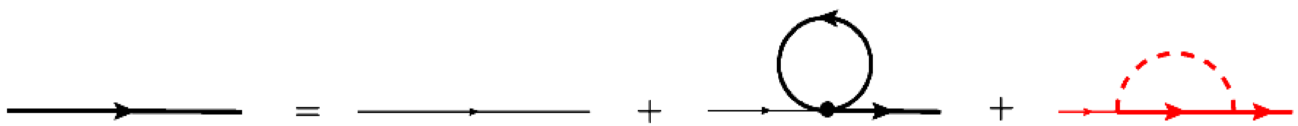
- **Entropy density**

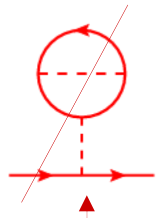
$$s = 2N_c N_f \int \frac{d^3\mathbf{p}}{(2\pi)^3} \left[\ln(1 + e^{-\beta(E_\mathbf{p} - \mu)}) + n_F(E_\mathbf{p} - \mu) \frac{E_\mathbf{p} - \mu}{T} \right] + (\mu \to -\mu)$$

$$- \sum_M \int \frac{d^3\mathbf{q}}{(2\pi)^3} \left[\ln\left(1 - e^{-\beta E_M}\right) - n_B(E_M) \frac{E_M}{T} \right].$$

NJL model: Meson dressed quark

- **Meson-dressed quark propagator: (partly) O(1/Nc) correction.**





$$S^{-1}(p) = S_{\mathrm{mf}}^{-1}(p) - \sum_M \Sigma_M(p)$$

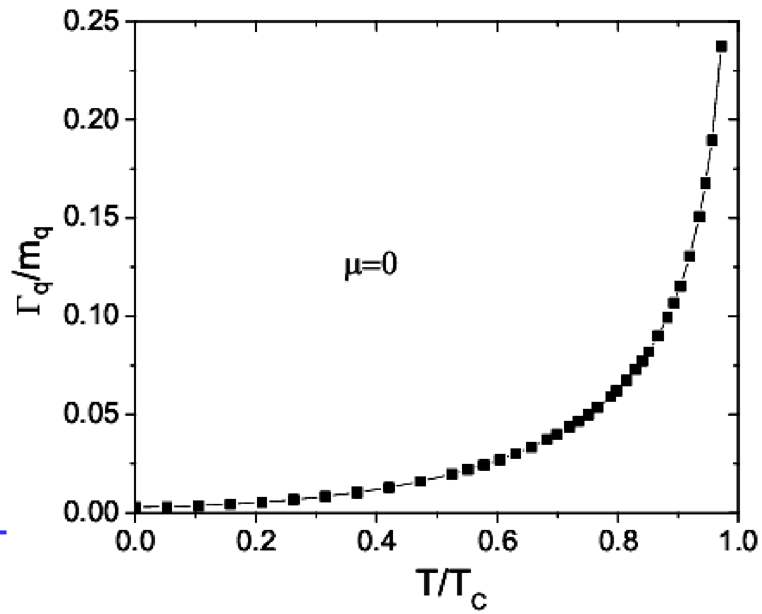
$$\Sigma_M(p) = i\int \frac{d^4q}{(2\pi)^4} V_M i S_{mf}(p+q) V_M^* i D_M(q)$$

Does not generate width

$$S(p) \approx \frac{Z + iW}{\not{p} - M + i\Gamma_q/2}$$

Width/mass<<1:

$$\Gamma_q = \frac{2\mathrm{Im}(\Sigma_0(M,\mathbf{0}) + \Sigma_s(M,\mathbf{0}))}{\partial\mathrm{Re}(\Sigma_0(M,\mathbf{0}) + \Sigma_s(M,\mathbf{0}))/\partial M - 1}$$



Transport coefficients

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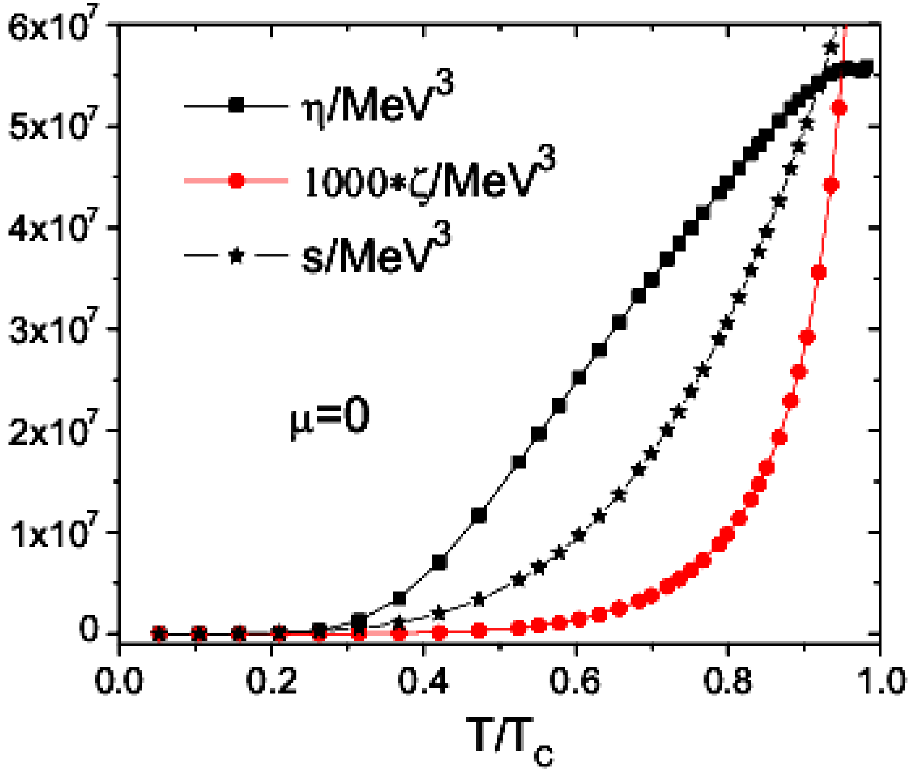
- Shear and bulk viscosities at zero chemical potential.



* Shear and bulk viscosities increase along T.

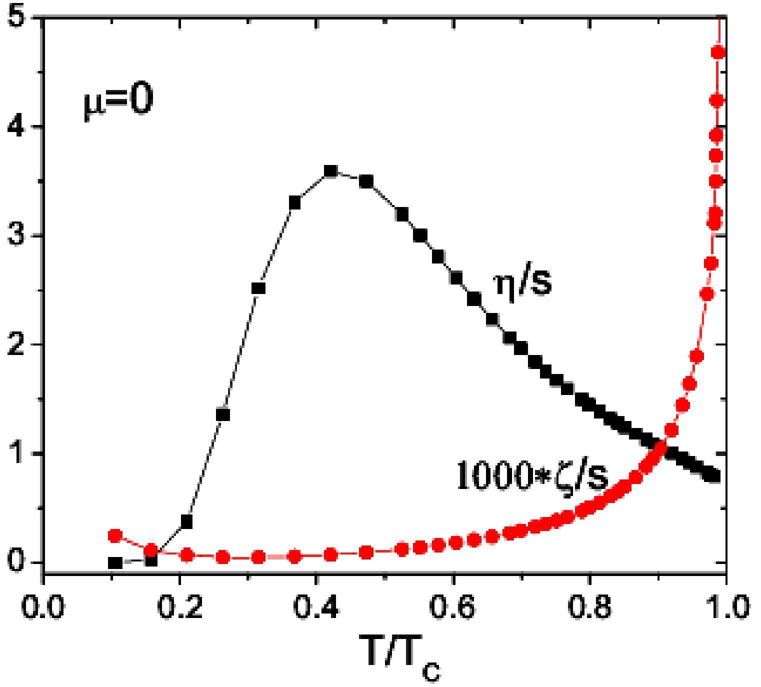
* Bulk viscosity is much smaller than shear.

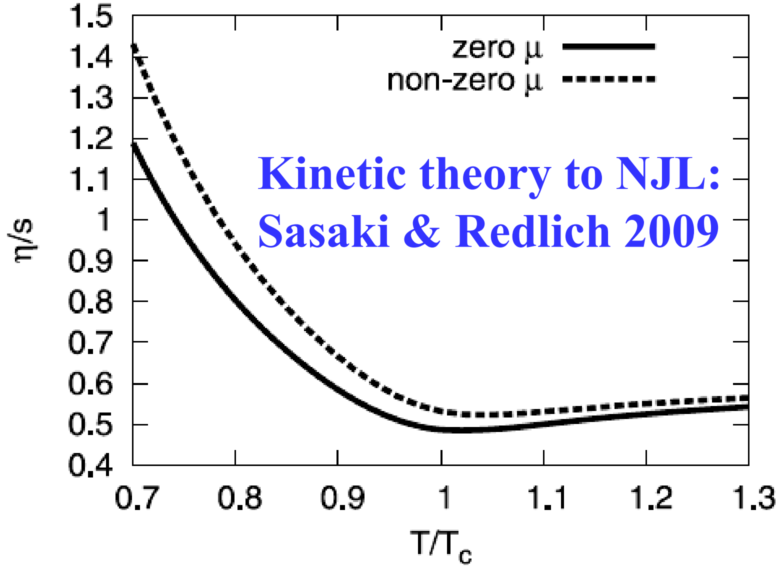
*Viscosities are small at low temperature.

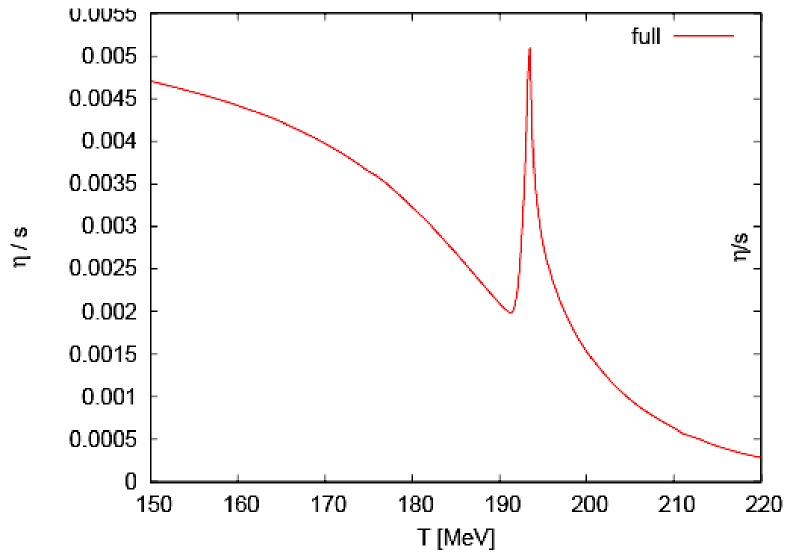
* Bulk viscosity shows up a "divergence" near Tc.

Transport coefficients

- Shear and bulk viscosities over entropy density at zero chemical potential



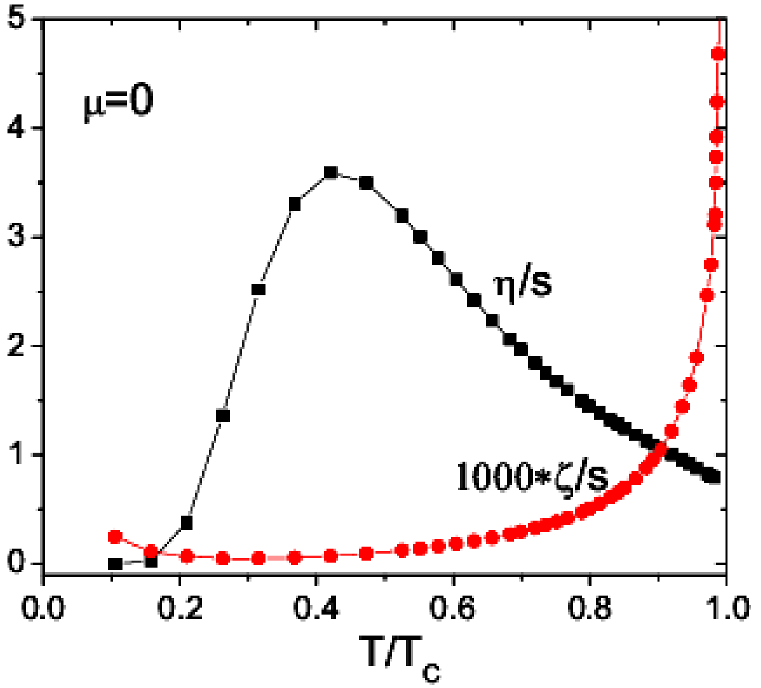


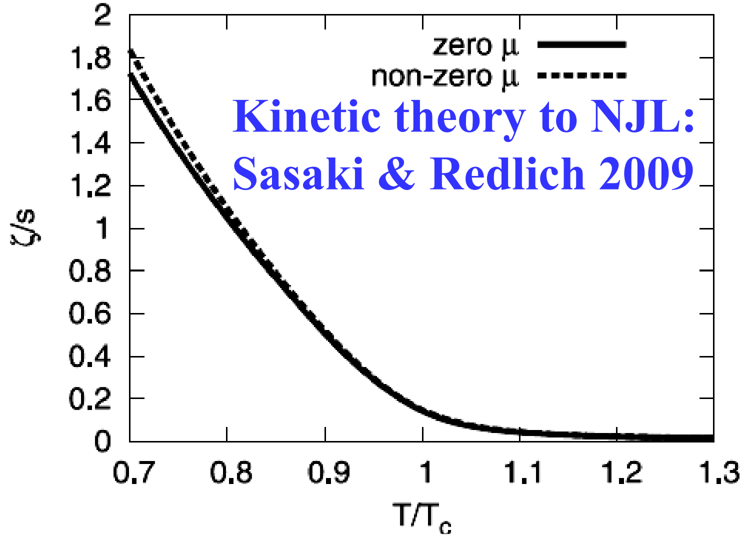


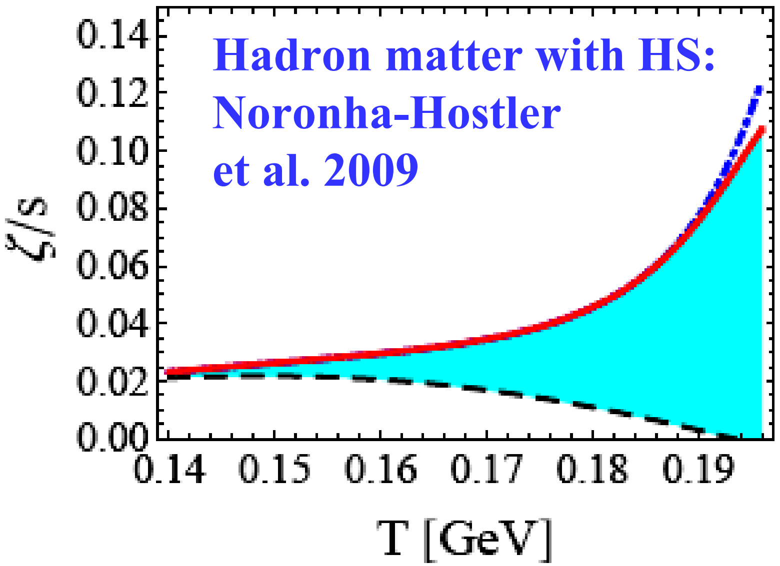
1/Nc analysis to shear viscosity in NJL: Buballa, Heckmann & Wambach 2008

Transport coefficients

- Shear and bulk viscosities over entropy density at zero chemical potential

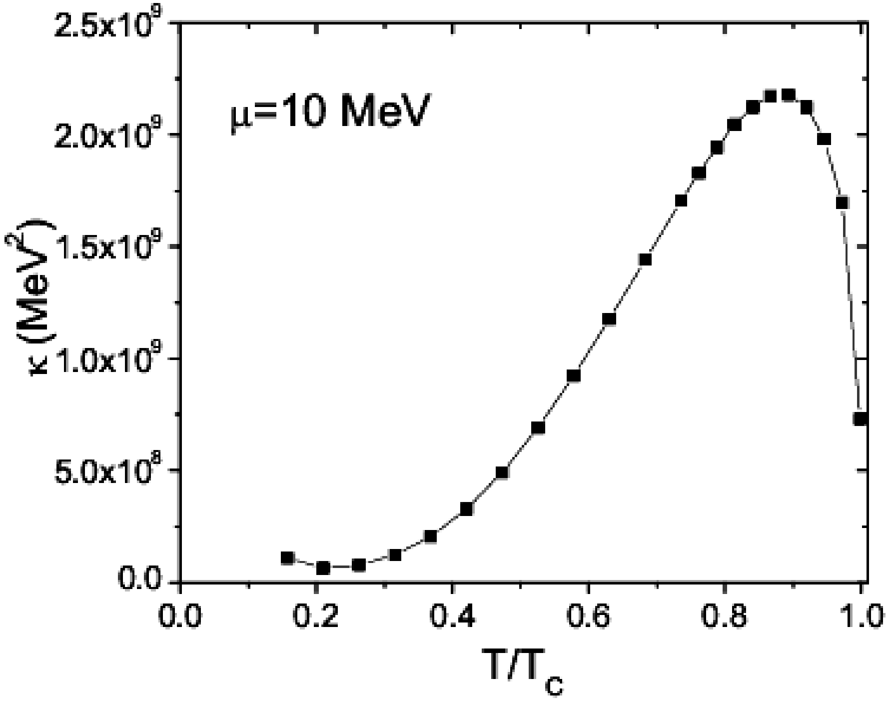






Transport coefficients

☐ Heat conductivity: mu=10 MeV.

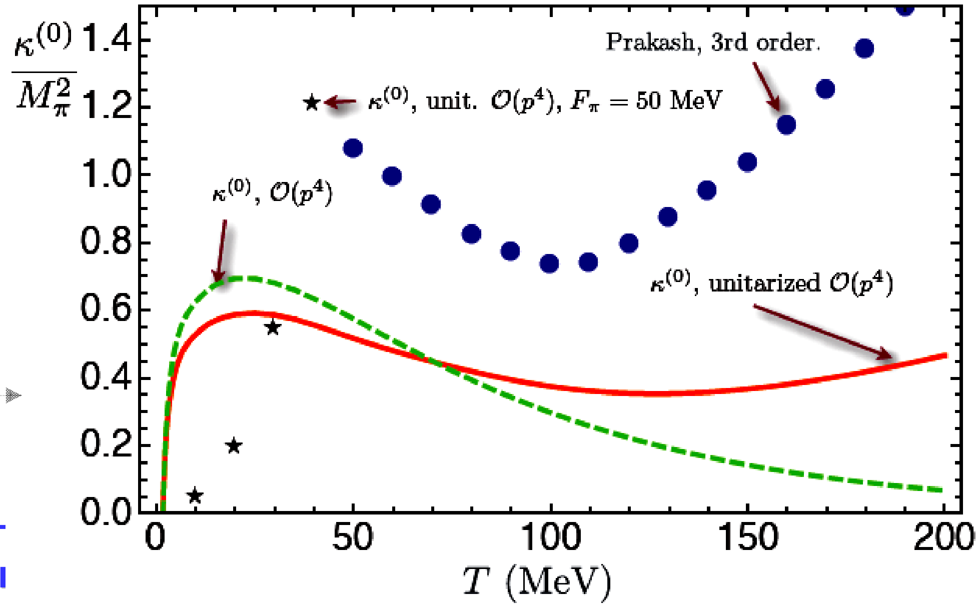


*** Very large value.**

***Non-monotonous .**

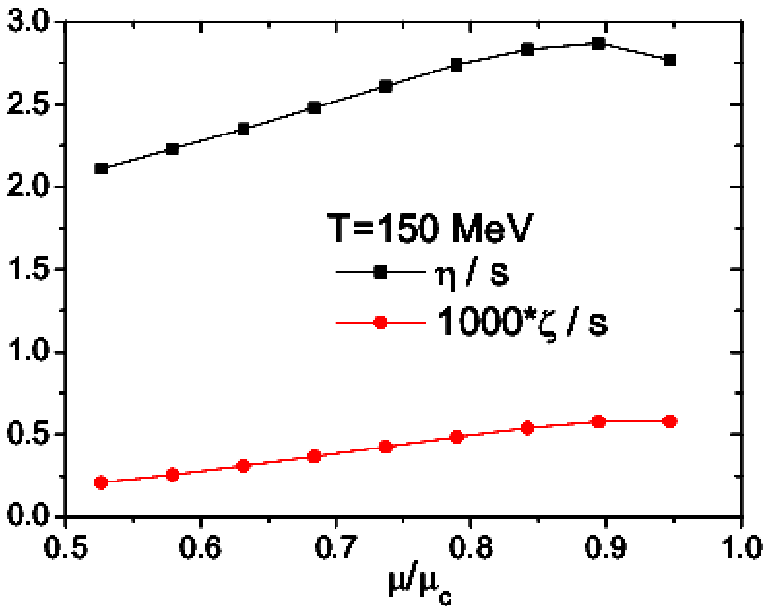
***Drops fast approaching Tc .**

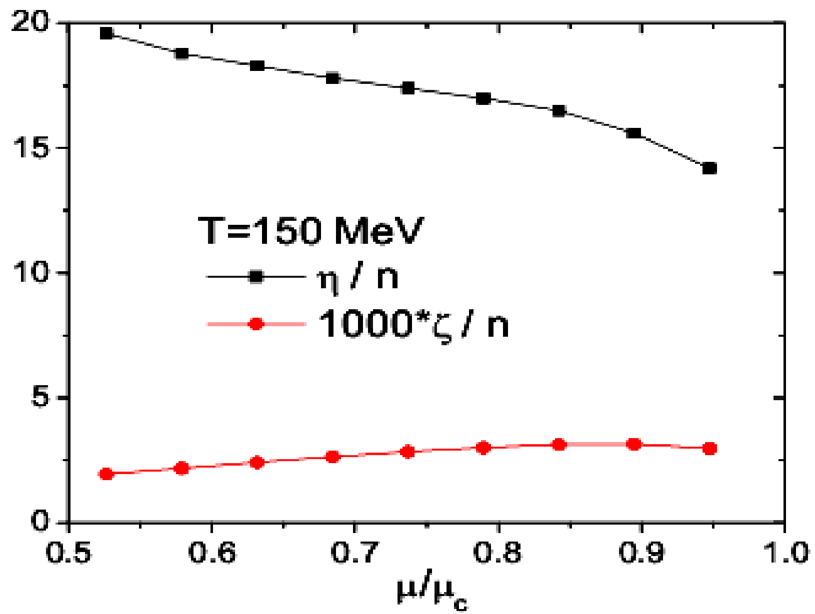
Chiral perturbation theory:
Fernandez-Fraile et al. 2008



Transport coefficients

- Shear and bulk viscosities at fixed temperature T=150 MeV.





Summary and outlooks

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- We obtain the Kubo's formulae for shear viscosity, bulk viscosity and heat conductivity in NJL model.
- We consider the RPA meson feedback effects to quark through a sunrise diagram which contribute a finite width to quark.
- At zero chemical potential, near Tc, shear viscosity decreases but bulk viscosity increases and shows a divergent behavior.
- We obtain a large heat conductivity at low mu, which drops fast when T -> Tc.
- More consistent approach: full O(1/Nc) correction to quark and O(1/Nc^2) correction to meson Bethe-Salpeter equation.
- Extention to T>Tc and mu>mu_c.
- Landscape of transport coefficients: can we use transport coefficients to fix phase transition line and tri-critical point?
- Transport coefficients in 2nd order hydrodynamics.

Thanks

谢谢

Comments are welcome

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Back up: Transport coefficients

- Shear and bulk viscosities over entropy density at zero chemical potential