

Chiral three-nucleon forces: From neutron matter to neutron stars

Kai Hebeler (OSU)

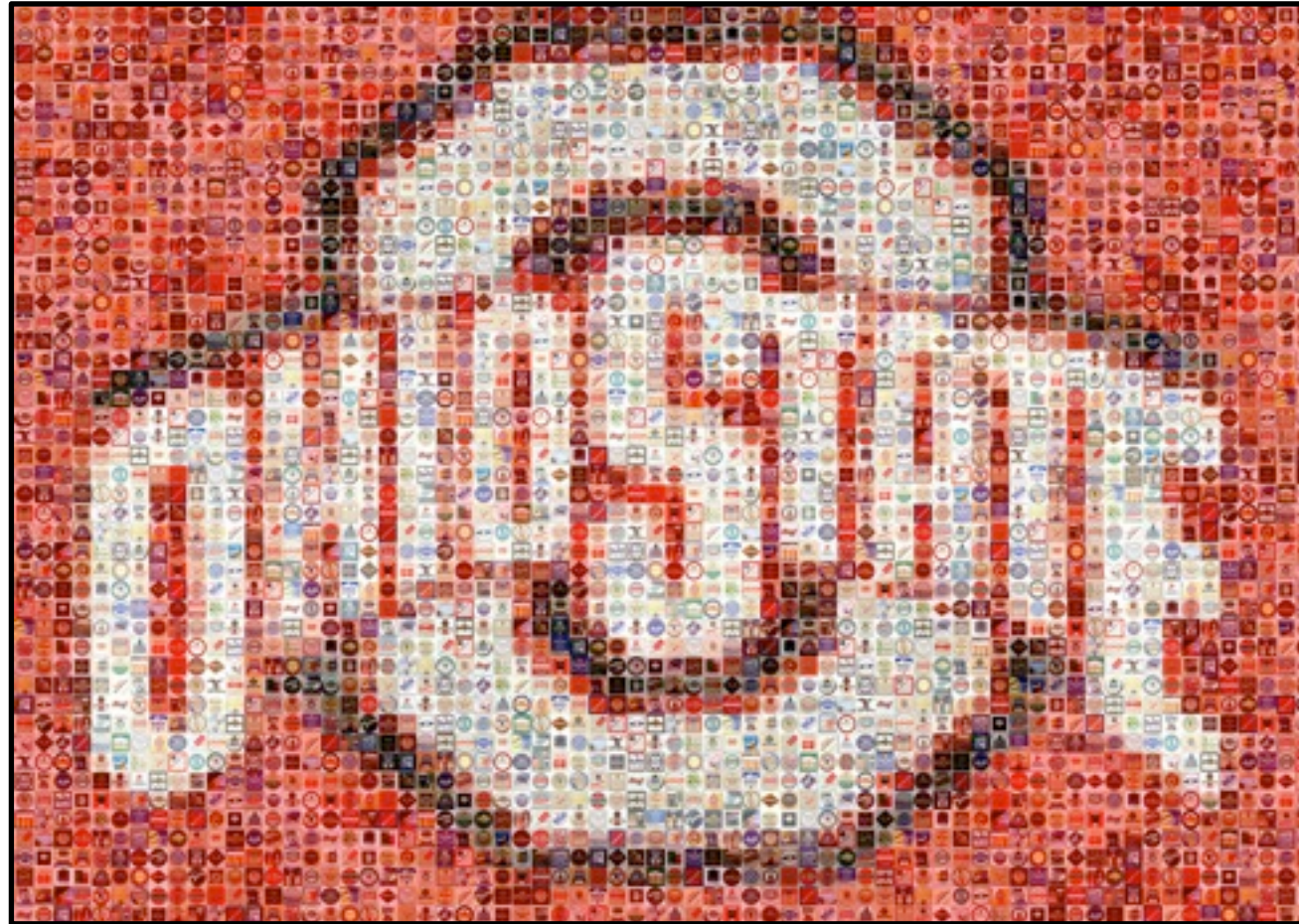
**Hirscheegg 2013:
Astrophysics and Nuclear Structure**

Hirscheegg, January 29, 2013



Resolution: The higher the better?

in the nuclear physics here we are interested in low-energy observables
(long-wavelength information)



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- resolution of very small (irrelevant) structures can obscure this information
- small details have nothing to do with long-wavelength information

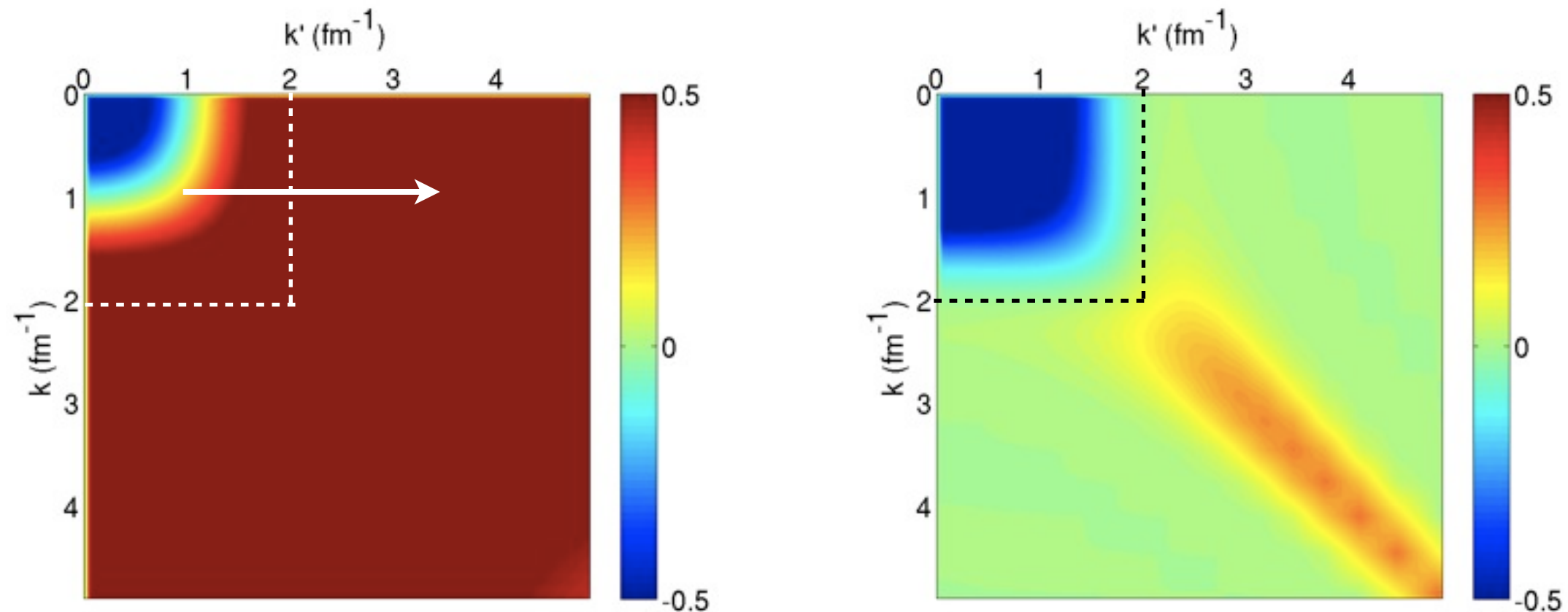
Strategy: Use a low-resolution version



- long-wavelength information is preserved
- distortion at small distance significantly reduced
- much less information necessary

In nuclear physics:
Use **renormalization group (RG)** to change resolution.

Changing the resolution: The (Similarity) Renormalization Group

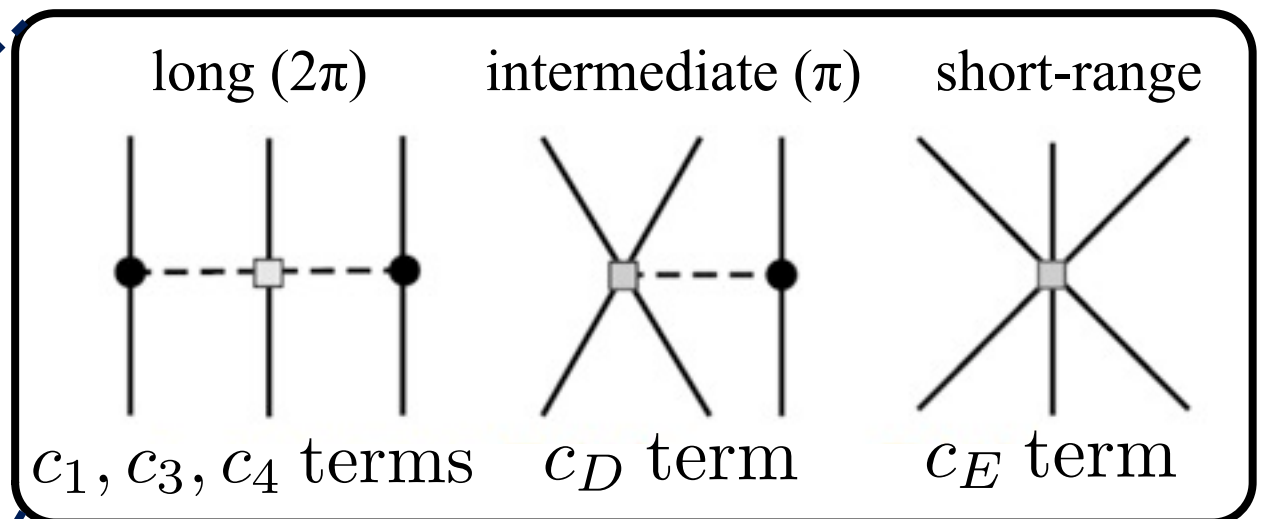
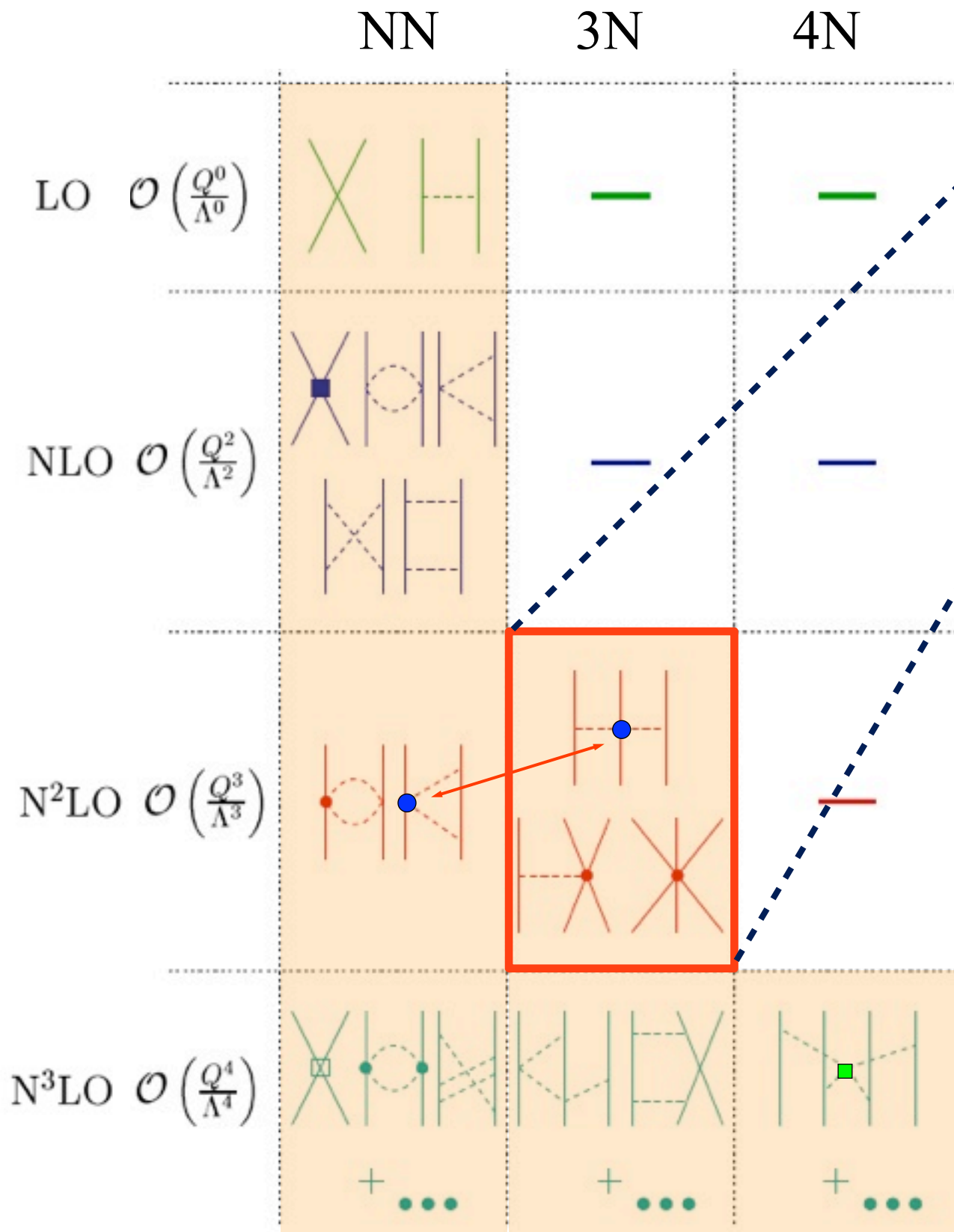


- elimination of coupling between low- and high momentum components, many-body problem more perturbative and tractable
- observables unaffected by resolution change (for exact calculations)
- residual resolution dependences can be used as tool to test calculations

Not the full story:

RG transformation also changes **three-body** (and higher-body) interactions!

Chiral EFT for nuclear forces, leading order 3N forces



large uncertainties in coupling constants at present:

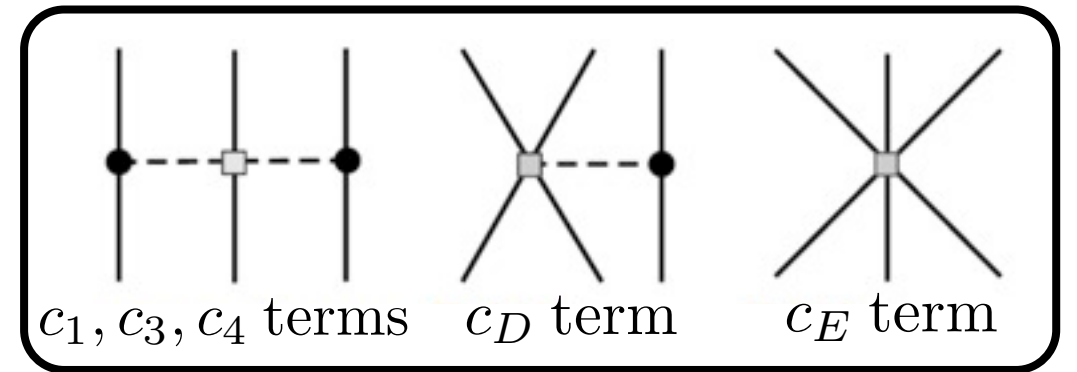
$$c_1 = -0.9^{+0.2}_{-0.5}, \quad c_3 = -4.7^{+1.5}_{-1.0}, \quad c_4 = 3.5^{+0.5}_{-0.2}$$

lead to theoretical uncertainties in many-body observables

use chiral interactions as input for RG evolution

RG evolution of 3N interactions

- So far (in momentum basis):
intermediate (c_D) and short-range (c_E) 3NF couplings fitted to few-body systems at different resolution scales:



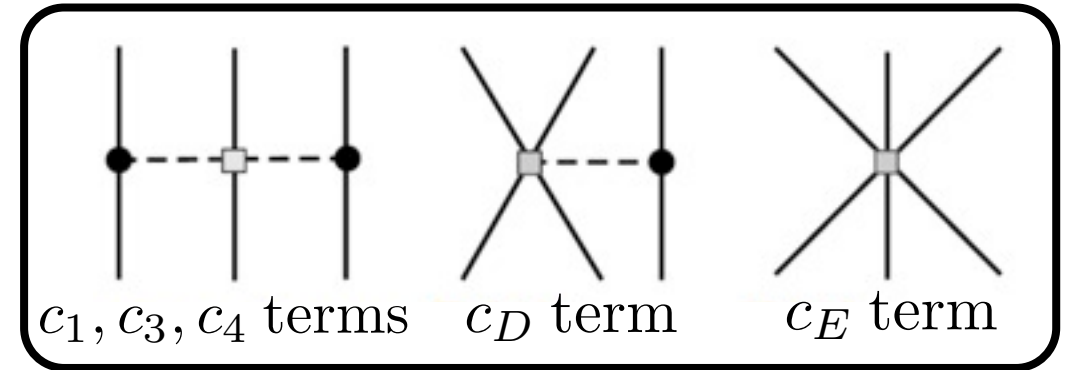
$$E_{3\text{H}} = -8.482 \text{ MeV} \quad \text{and} \quad r_{4\text{He}} = 1.464 \text{ fm}$$

→ coupling constants of natural size

- in neutron matter contributions from c_D , c_E and c_4 terms vanish
- long-range 2π contributions assumed to be invariant under RG evolution

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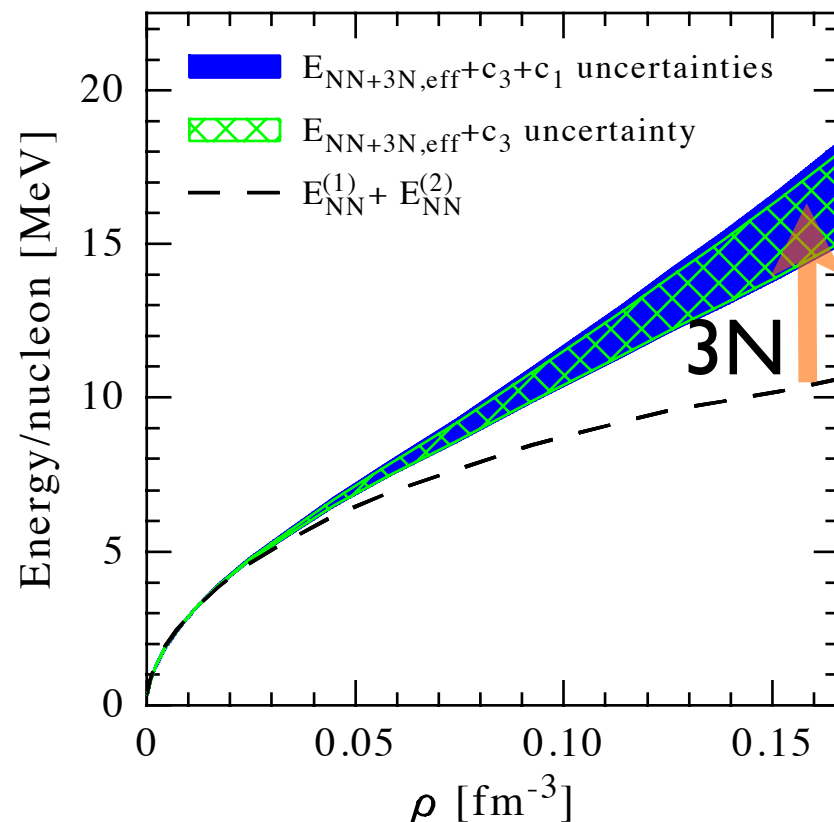
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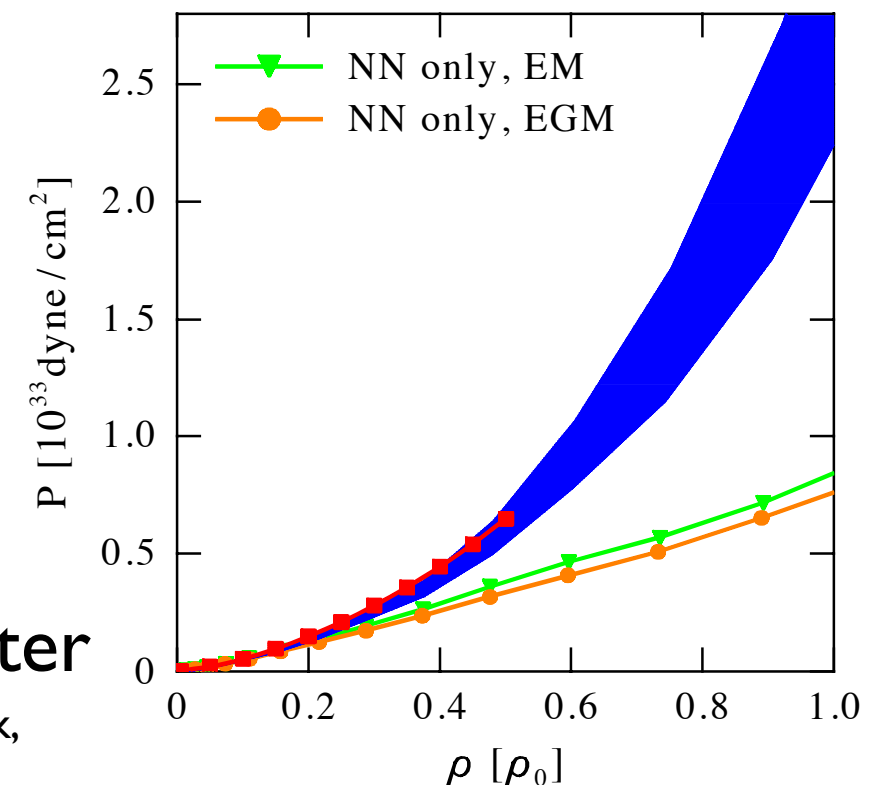


pure neutron matter

KH and Schwenk PRC 82, 014314 (2010)

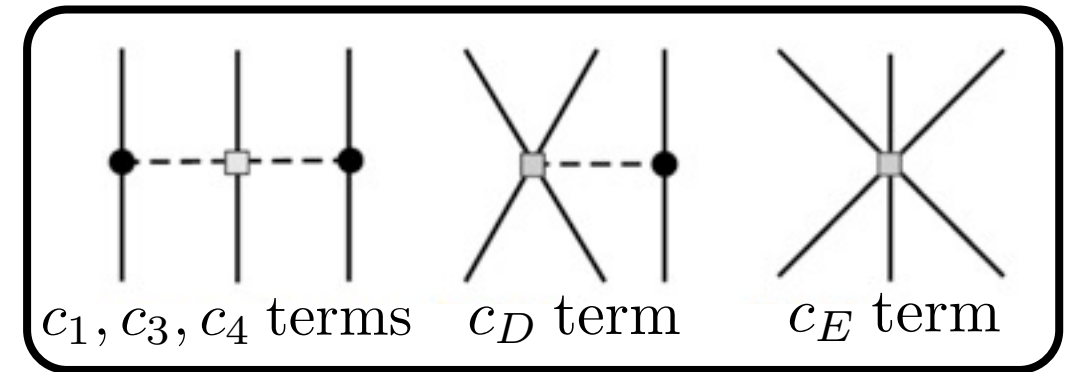
neutron star matter

KH, Lattimer, Pethick, Schwenk,
PRL 105, 161102 (2010)



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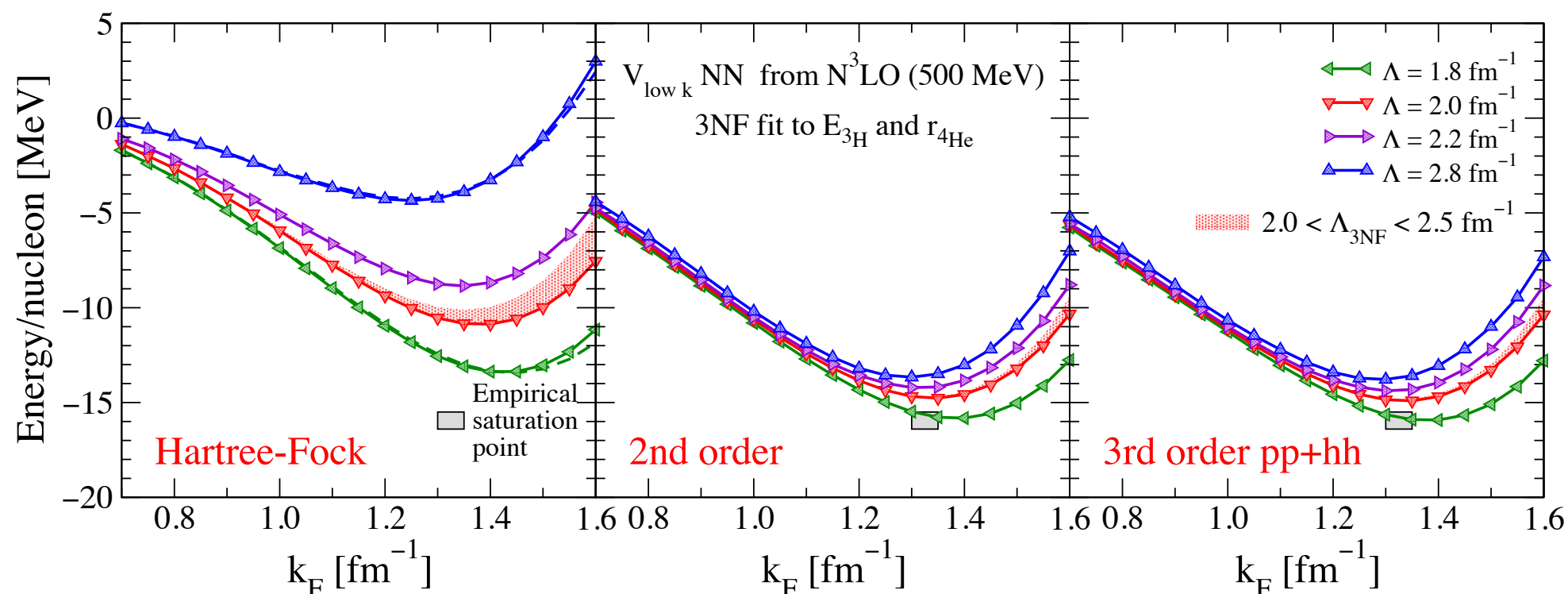
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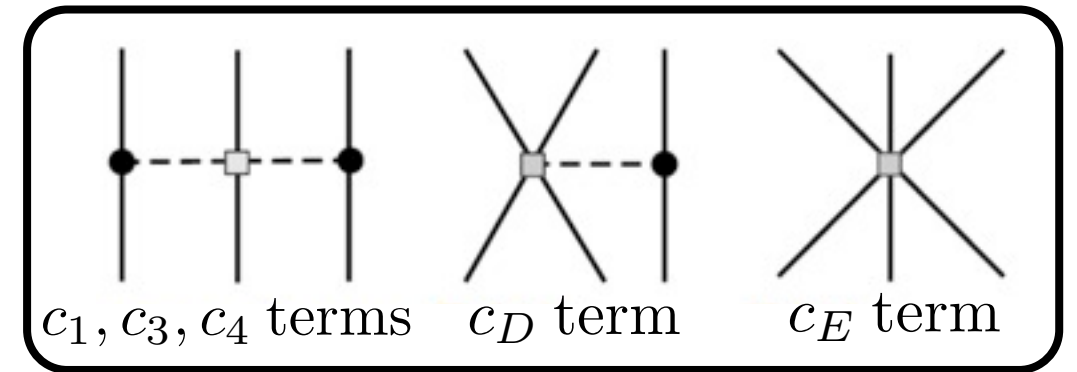


symmetric
nuclear matter

KH, Bogner, Furnstahl, Nogga,
PRC(R) 83, 031301 (2011)

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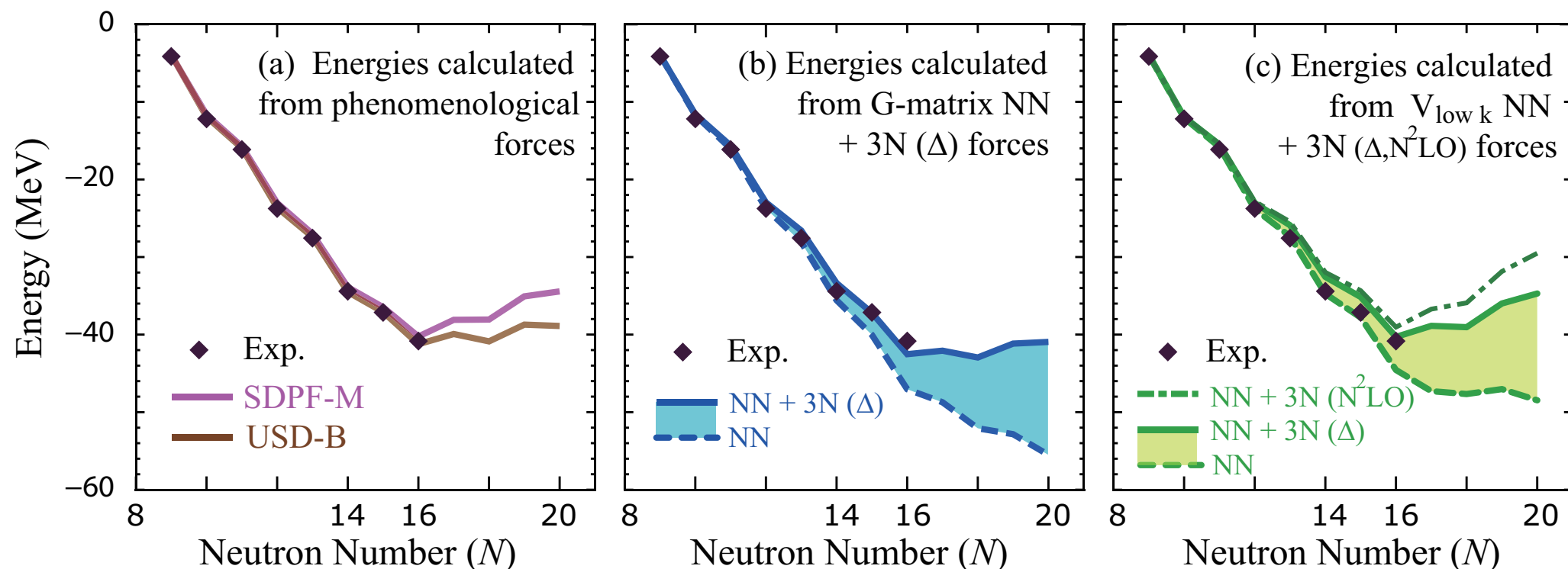
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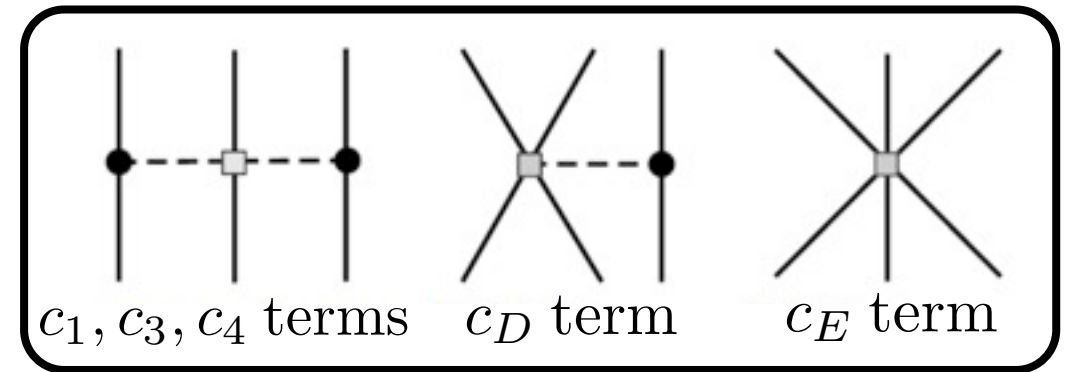


nuclear
shell model

Otsuka et al.,
PRL 105, 032501 (2010)

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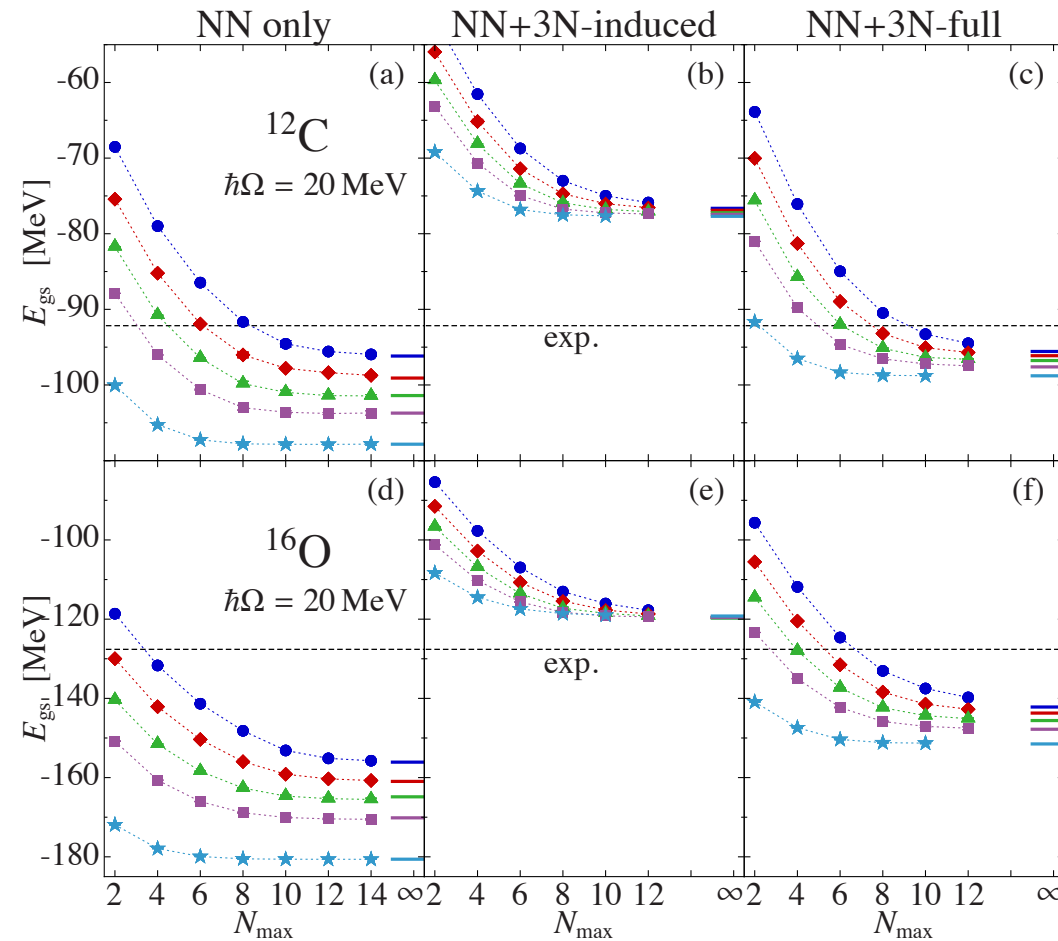
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 - has been achieved in oscillator basis (Jurgenson, Roth)
 - promising results in very light nuclei
 - puzzling effects in heavier nuclei (higher-body forces?)
 - not immediately applicable to infinite systems
 - limitations on $\hbar\Omega$

RG evolution of 3N interactions



Roth et al. PRL 107, 072501 (2011)

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Consistent 3NF evolution in momentum basis:

Current/future developments

- application to infinite systems
 - ▶ equation of state
 - ▶ systematic study of induced many-body contributions, scaling behavior
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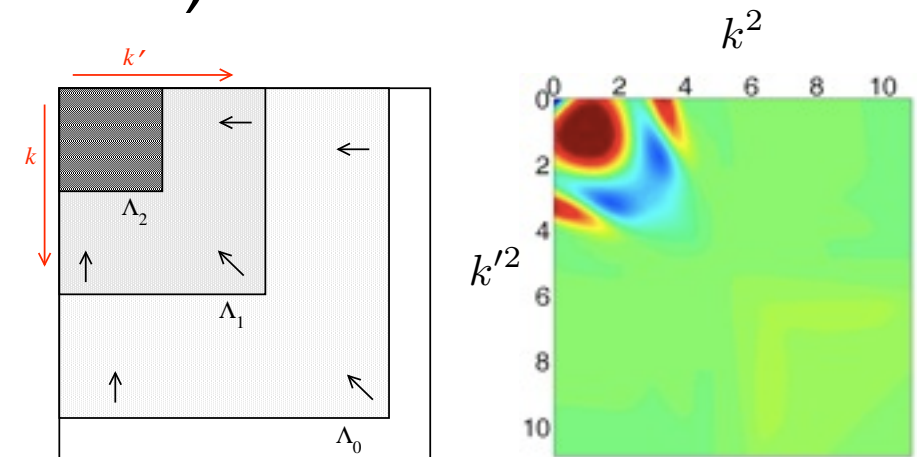
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 - ▶ improved efficiency of evolution
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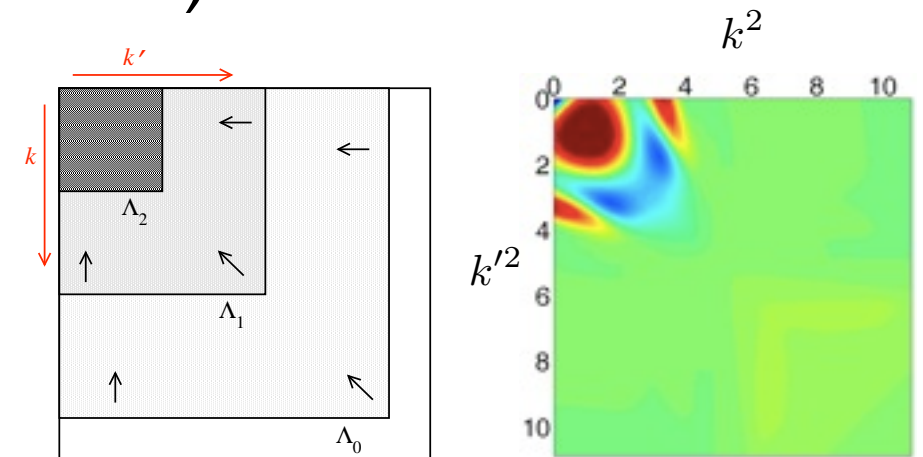


Anderson et al., PRC 77, 037001 (2008)

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- explicit calculation of unitary 3N transformation
 - ▶ RG evolution of operators
 - ▶ study of correlations in nuclear systems \longrightarrow factorization

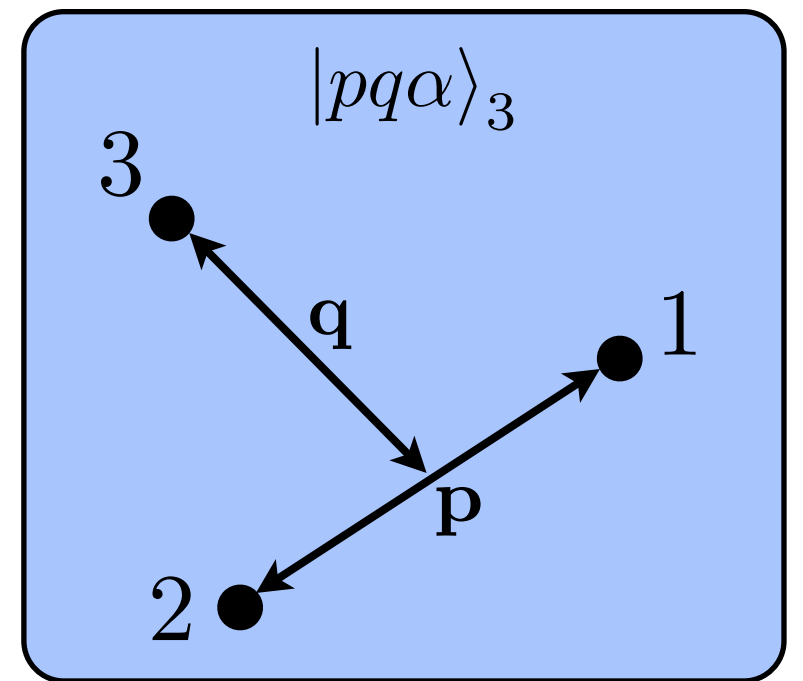
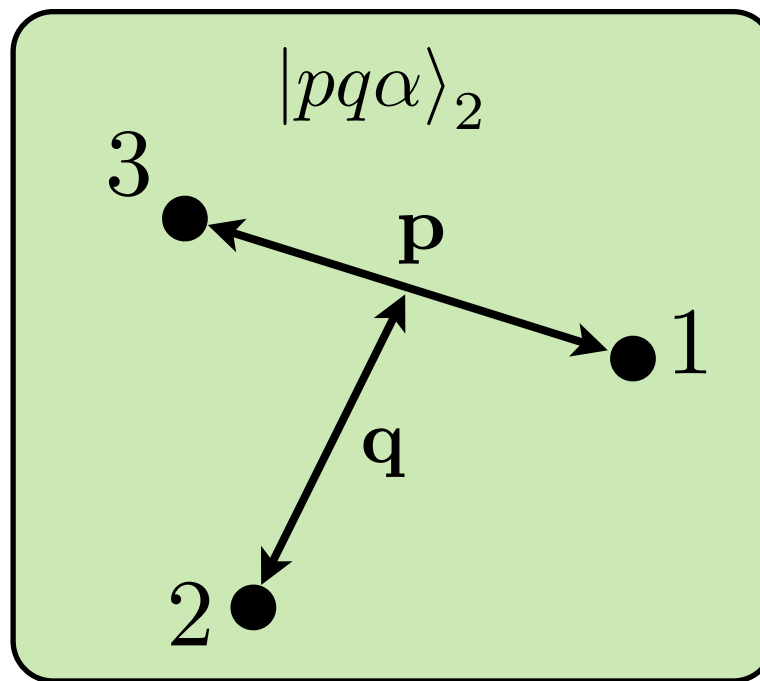
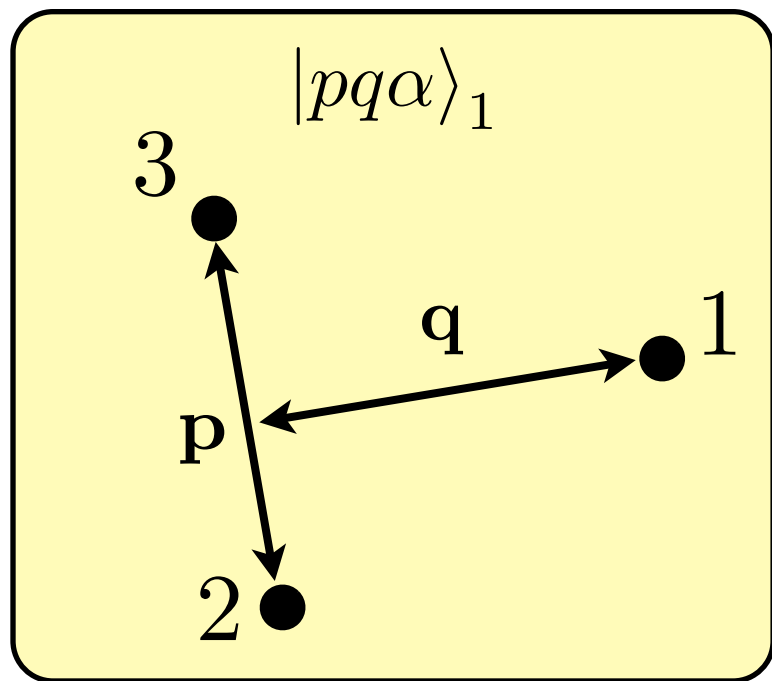


Anderson et al., PRC 77, 037001 (2008)

RG evolution of 3N interactions in momentum space

Three-body Faddeev basis:

$$|pq\alpha\rangle_i \equiv |p_i q_i; [(LS)J(l s_i)j] \mathcal{J} \mathcal{J}_z (T t_i) \mathcal{T} \mathcal{T}_z\rangle$$



$${}_i \langle pq\alpha | P | p' q' \alpha' \rangle_i = {}_i \langle pq\alpha | p' q' \alpha' \rangle_j$$

Faddeev bound-state equation:

$$|\psi_i\rangle = G_0 [2t_i P + (1 + t_i G_0) V_{3N}^i (1 + 2P)] |\psi_i\rangle$$

SRG flow equations of NN and 3N forces in momentum basis

$$\frac{dH_s}{ds} = [\eta_s, H_s] \quad \eta_s = [T_{\text{rel}}, H_s]$$

$$H = T + V_{12} + V_{13} + V_{23} + V_{123}$$

- spectators correspond to delta functions, matrix representation of H_s ill-defined
- **solution**: explicit separation of NN and 3N flow equations

$$\begin{aligned} \frac{dV_{ij}}{ds} &= [[T_{ij}, V_{ij}], T_{ij} + V_{ij}], \\ \frac{dV_{123}}{ds} &= [[T_{12}, V_{12}], V_{13} + V_{23} + V_{123}] \\ &\quad + [[T_{13}, V_{13}], V_{12} + V_{23} + V_{123}] \\ &\quad + [[T_{23}, V_{23}], V_{12} + V_{13} + V_{123}] \\ &\quad + [[T_{\text{rel}}, V_{123}], H_s] \end{aligned}$$

- only connected terms remain in $\frac{dV_{123}}{ds}$, ‘dangerous’ delta functions cancel

SRG evolution in momentum space

- evolve the antisymmetrized 3N interaction

$$\overline{V}_{123} = {}_i \langle pq\alpha | (1 + P_{123} + P_{132}) V_{123}^{(i)} (1 + P_{123} + P_{132}) | p'q'\alpha' \rangle_i$$

- embed NN interaction in 3N basis:

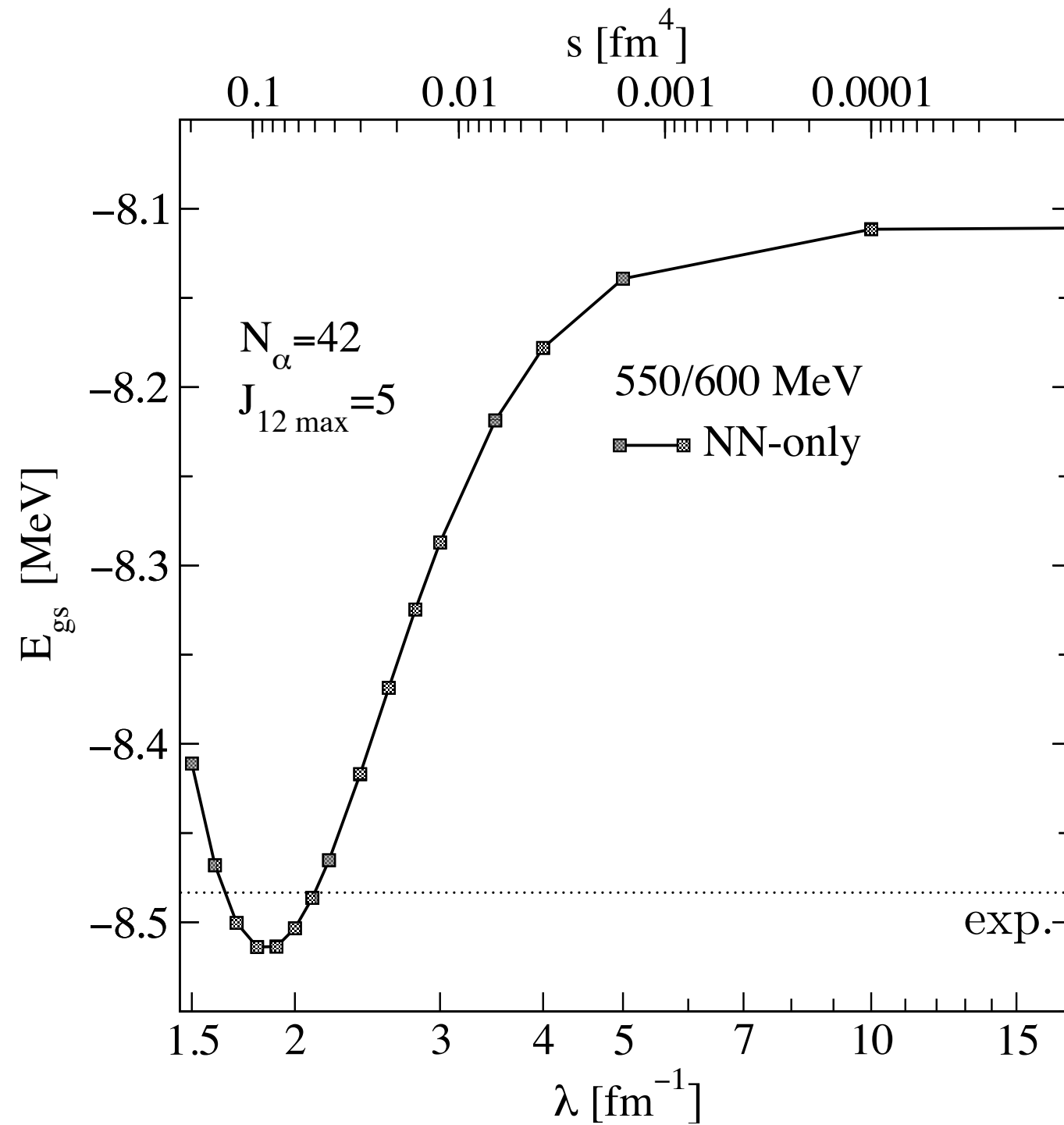
$$V_{13} = P_{123} V_{12} P_{132}, \quad V_{23} = P_{132} V_{12} P_{123}$$

$$\text{with } {}_3 \langle pq\alpha | V_{12} | p'q'\alpha' \rangle_3 = \langle p\tilde{\alpha} | V_{\text{NN}} | p'\tilde{\alpha}' \rangle \delta(q - q') / q^2$$

- use $P_{123} \overline{V}_{123} = P_{132} \overline{V}_{123} = \overline{V}_{123}$

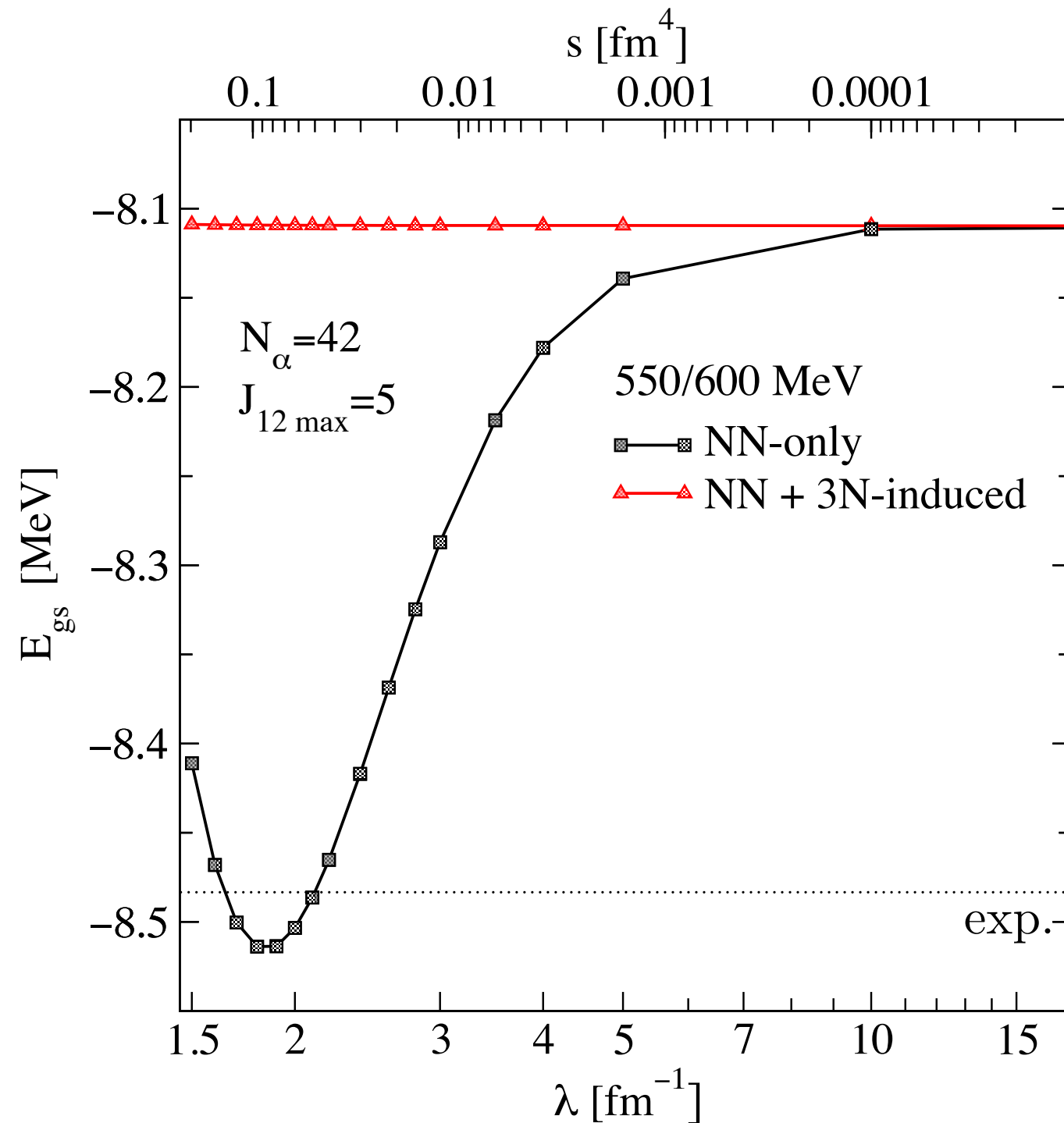
$$\begin{aligned} \Rightarrow \quad d\overline{V}_{123}/ds &= C_1(s, T, V_{\text{NN}}, P) \\ &\quad + C_2(s, T, V_{\text{NN}}, \overline{V}_{123}, P) \\ &\quad + C_3(s, T, \overline{V}_{123}) \end{aligned}$$

SRG evolution of 3N interactions in momentum space: Results for the Triton



Hebeler PRC(R) 85, 021002 (2012)

SRG evolution of 3N interactions in momentum space: Results for the Triton

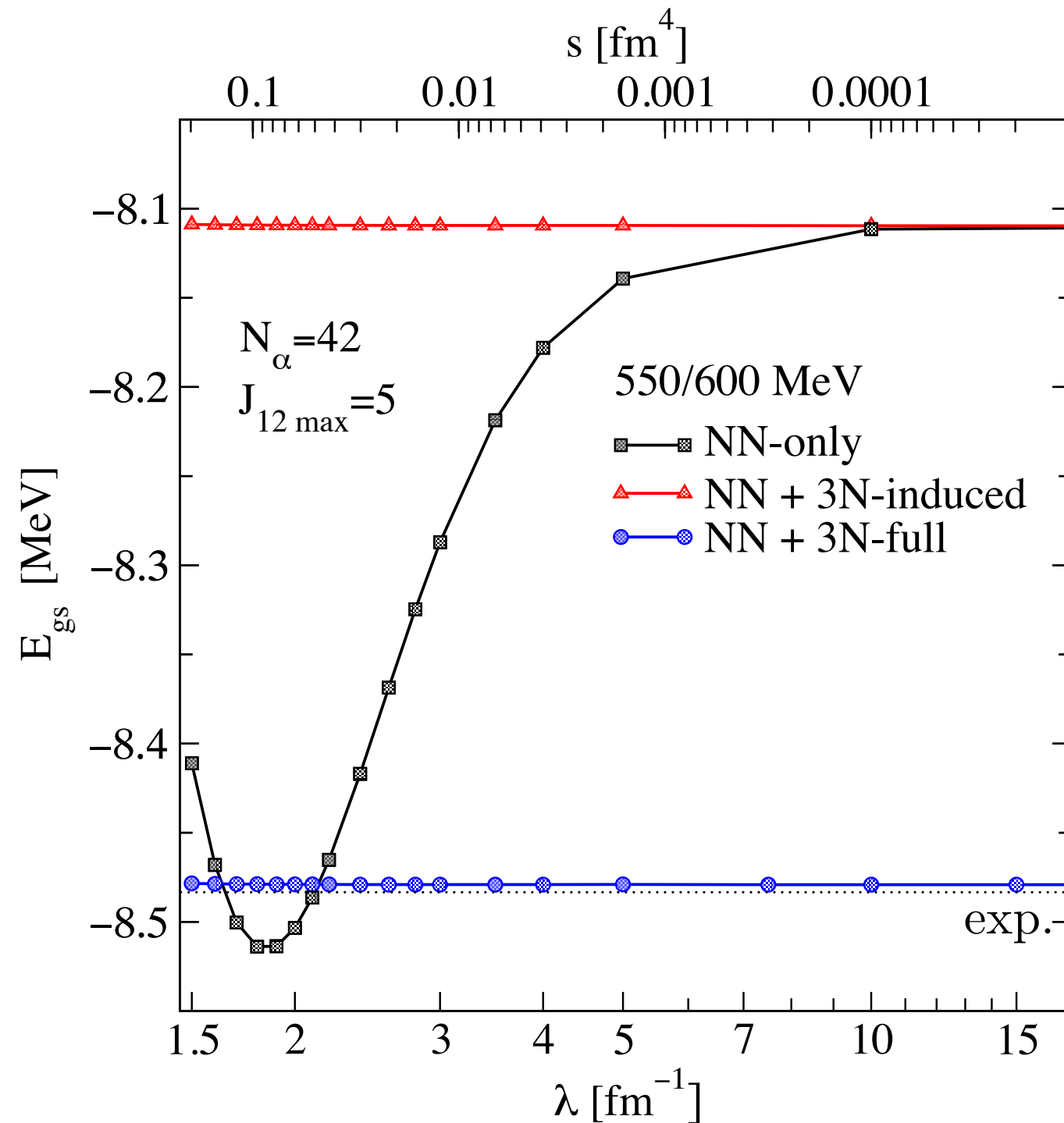


Hebeler PRC(R) 85, 021002 (2012)

It works:

Invariance of E_{gs}^{3H} within ≤ 1 eV for consistent chiral interactions at $N^2\text{LO}$

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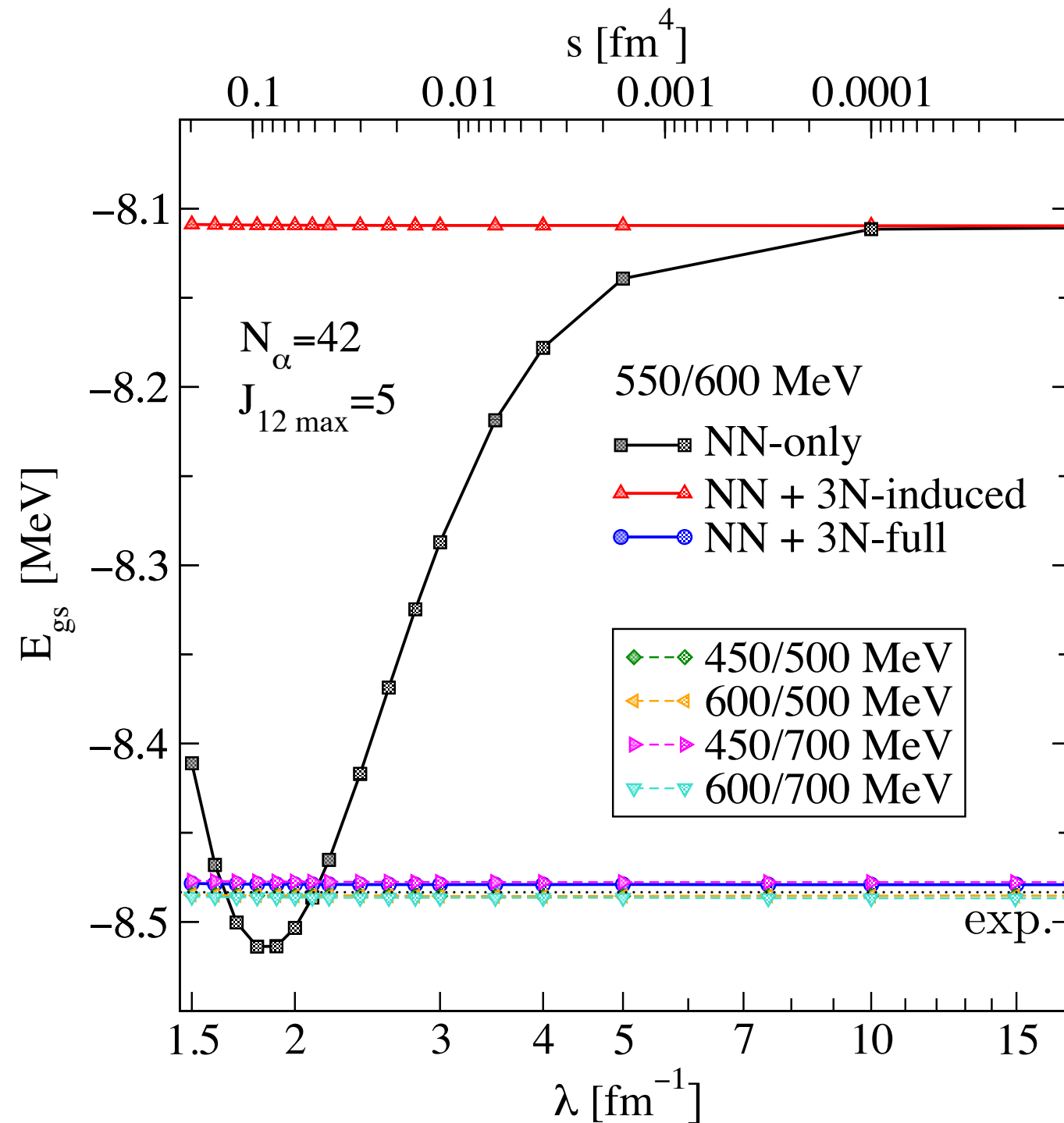


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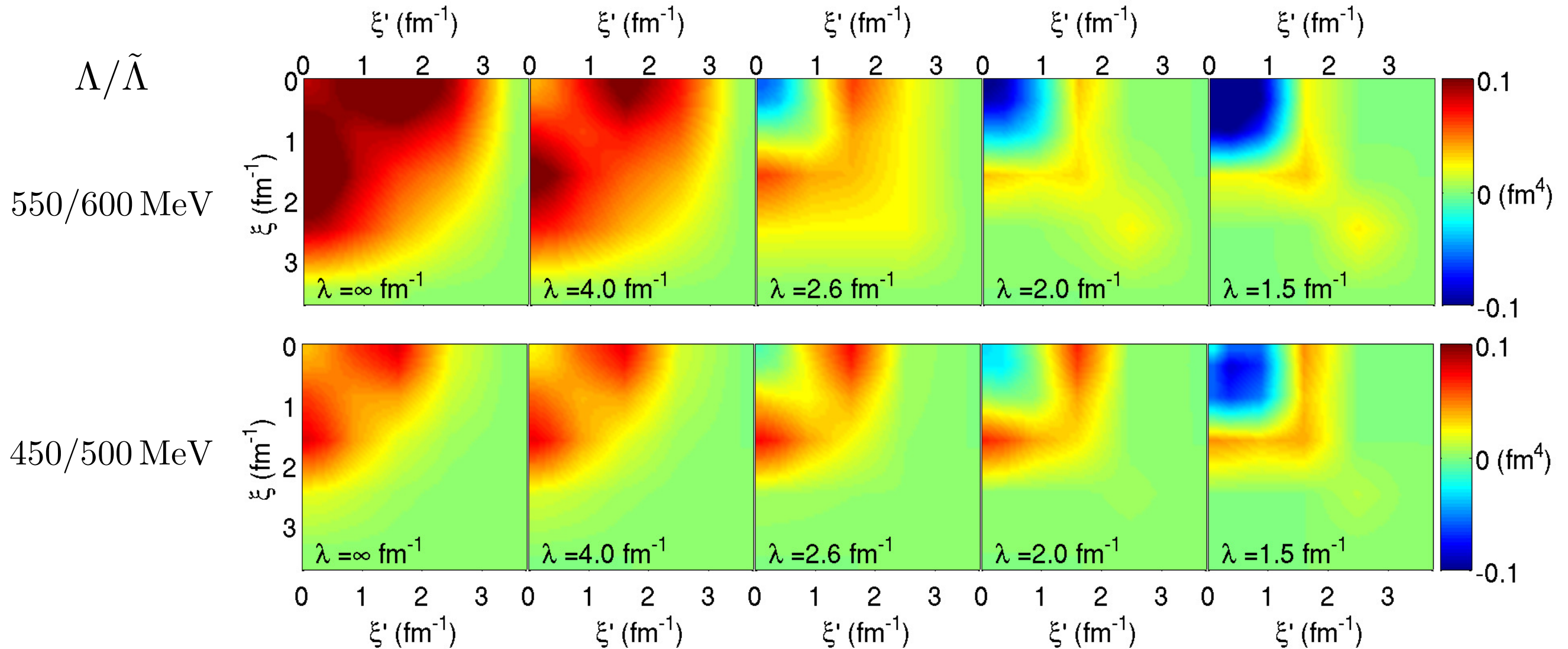
Hebeler PRC(R) 85, 021002 (2012)

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Decoupling of matrix elements

$$\theta = \frac{\pi}{12} \qquad \mathcal{T} = \mathcal{J} = \frac{1}{2}$$



KH, PRC(R) 85, 021002 (2012)

hyperradius: $\xi^2 = p^2 + \frac{3}{4}q^2$

hyperangle: $\tan \theta = \frac{2p}{\sqrt{3}q}$

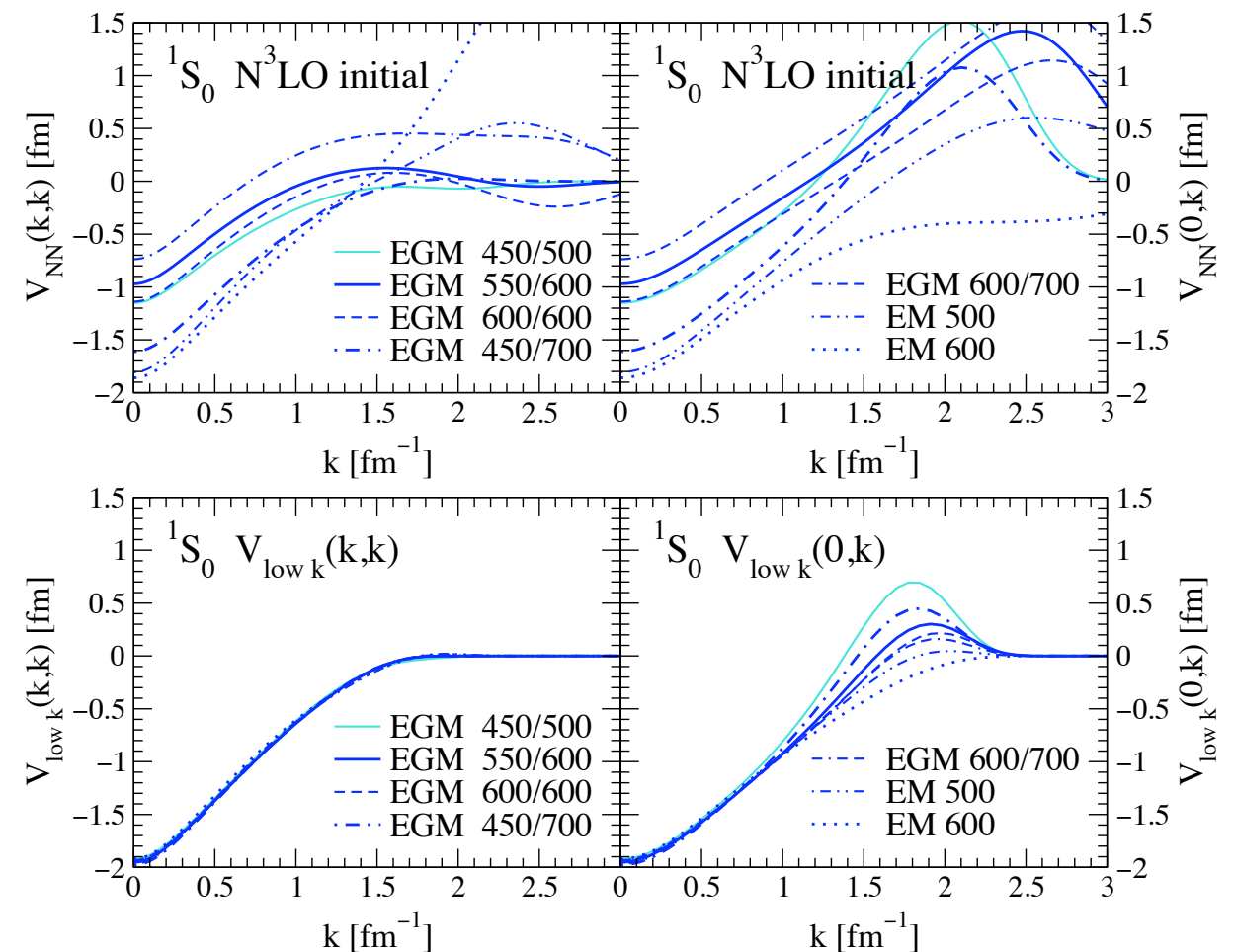
same decoupling patterns like in NN interactions

Universality in 3N interactions at low resolution

phase-shift
equivalence

common long-
range physics

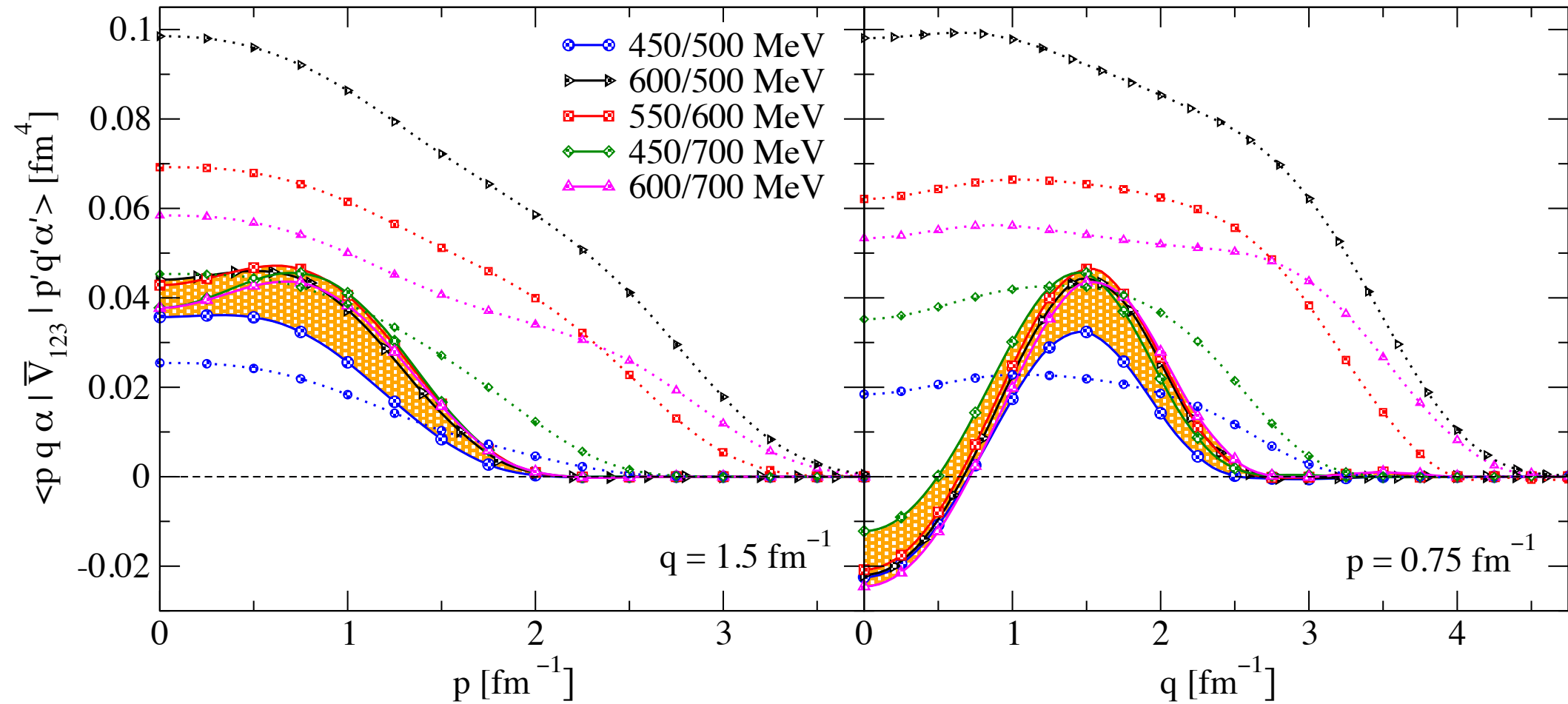
(approximate) universality of
low-resolution NN interactions



To what extent are 3N interactions constrained at low resolution?

- only two low-energy constants c_D and c_E
- 3N interactions give only subleading contributions to observables

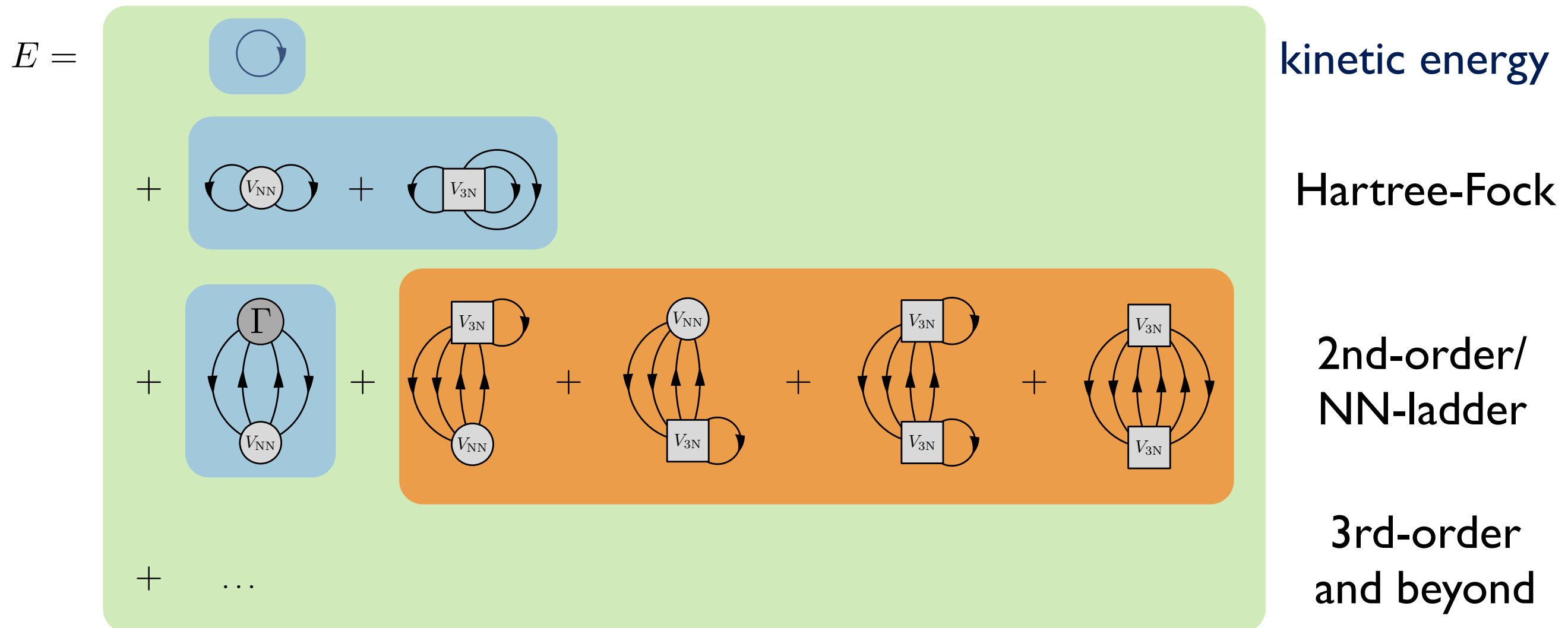
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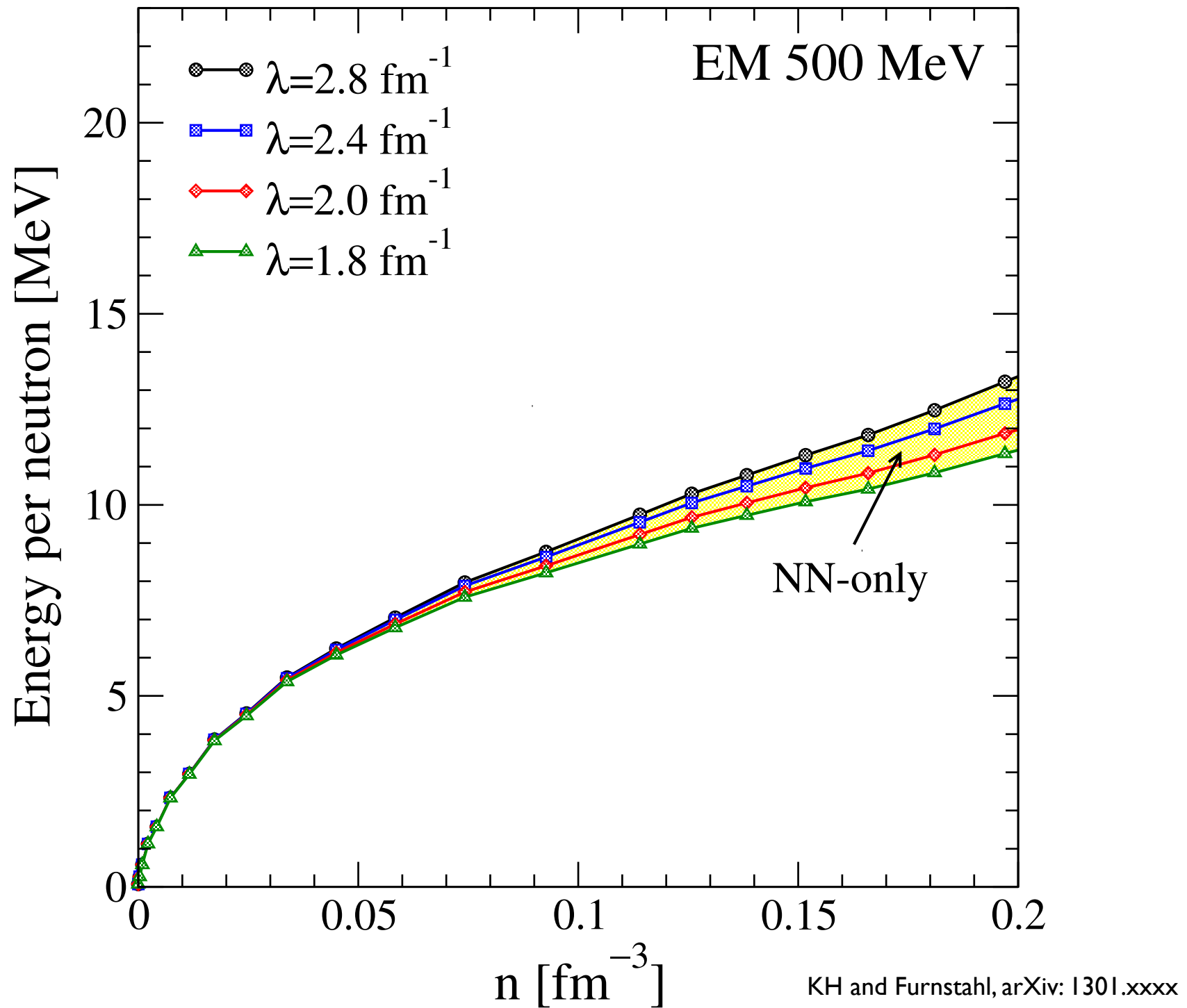
- remarkably reduced scheme dependence for typical momenta $\sim 1 \text{ fm}^{-1}$, matrix elements with significant phase space well constrained at low resolution
- new momentum structures induced at low resolution
- study based on $N^2\text{LO}$ chiral interactions, improved universality at $N^3\text{LO}$?

First application to neutron matter: Equation of state



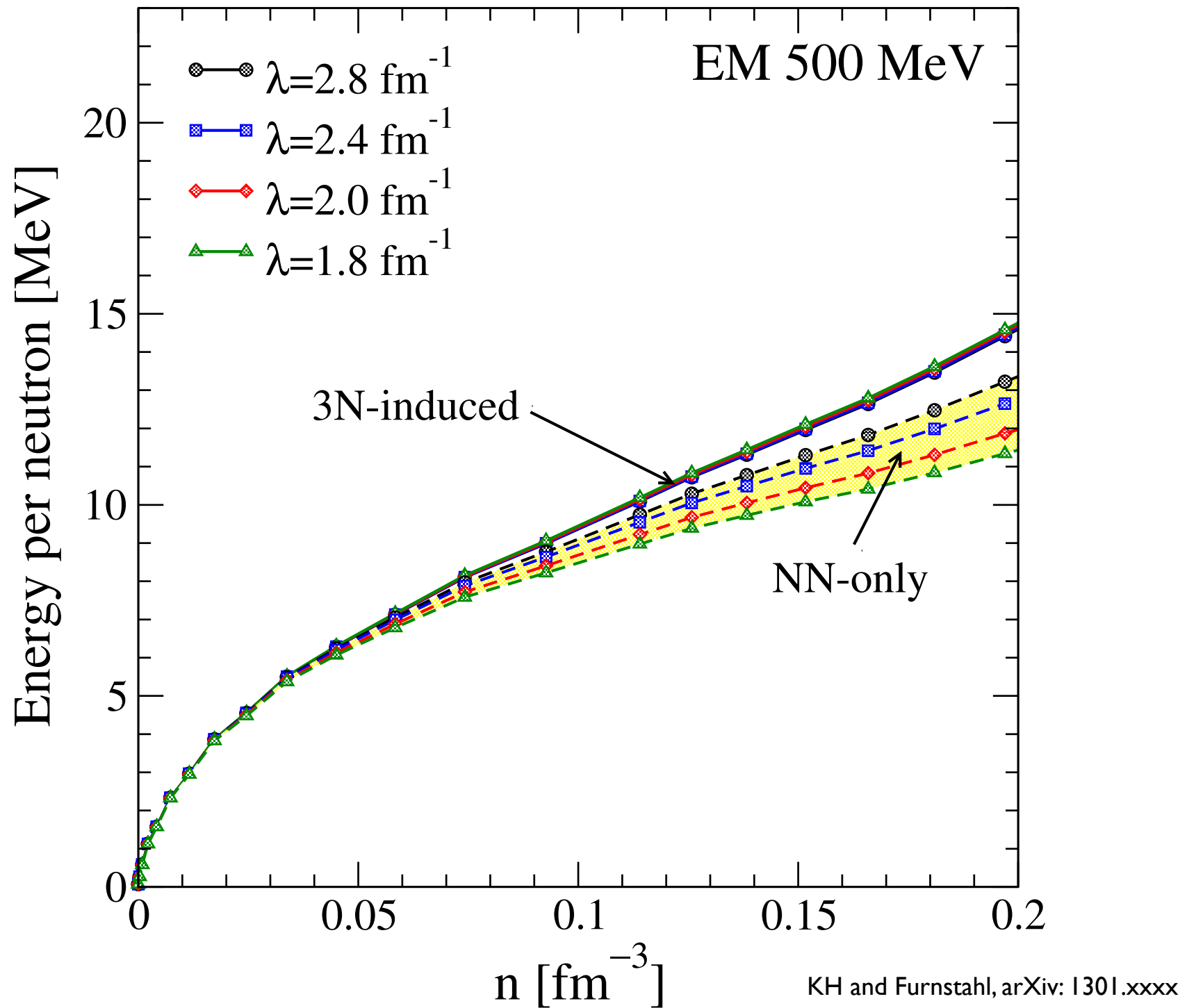
- evolve consistently NN + 3NF in the isospin $\mathcal{T} = 3/2$ channel
- calculate EOS by taking all blue-boxed contributions into account
- in this approximation NN and 3NF contributions factorize

First results for neutron matter



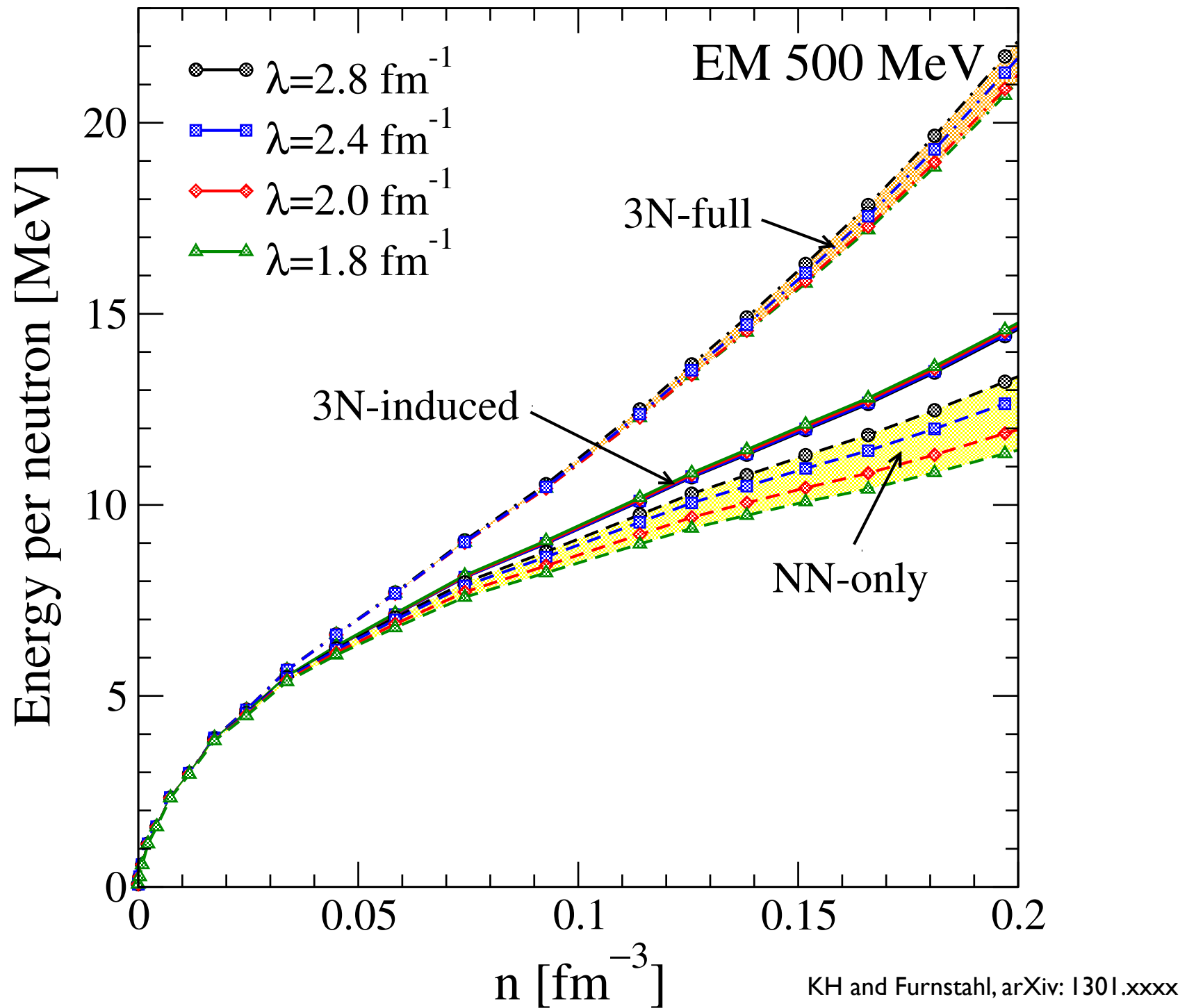
- all partial waves included up to $\mathcal{J} = 9/2$ in SRG evolution and EOS calculation
- consistent 3NF with $c_1 = -0.81 \text{ GeV}^{-1}$ and $c_3 = -3.2 \text{ GeV}^{-1}$

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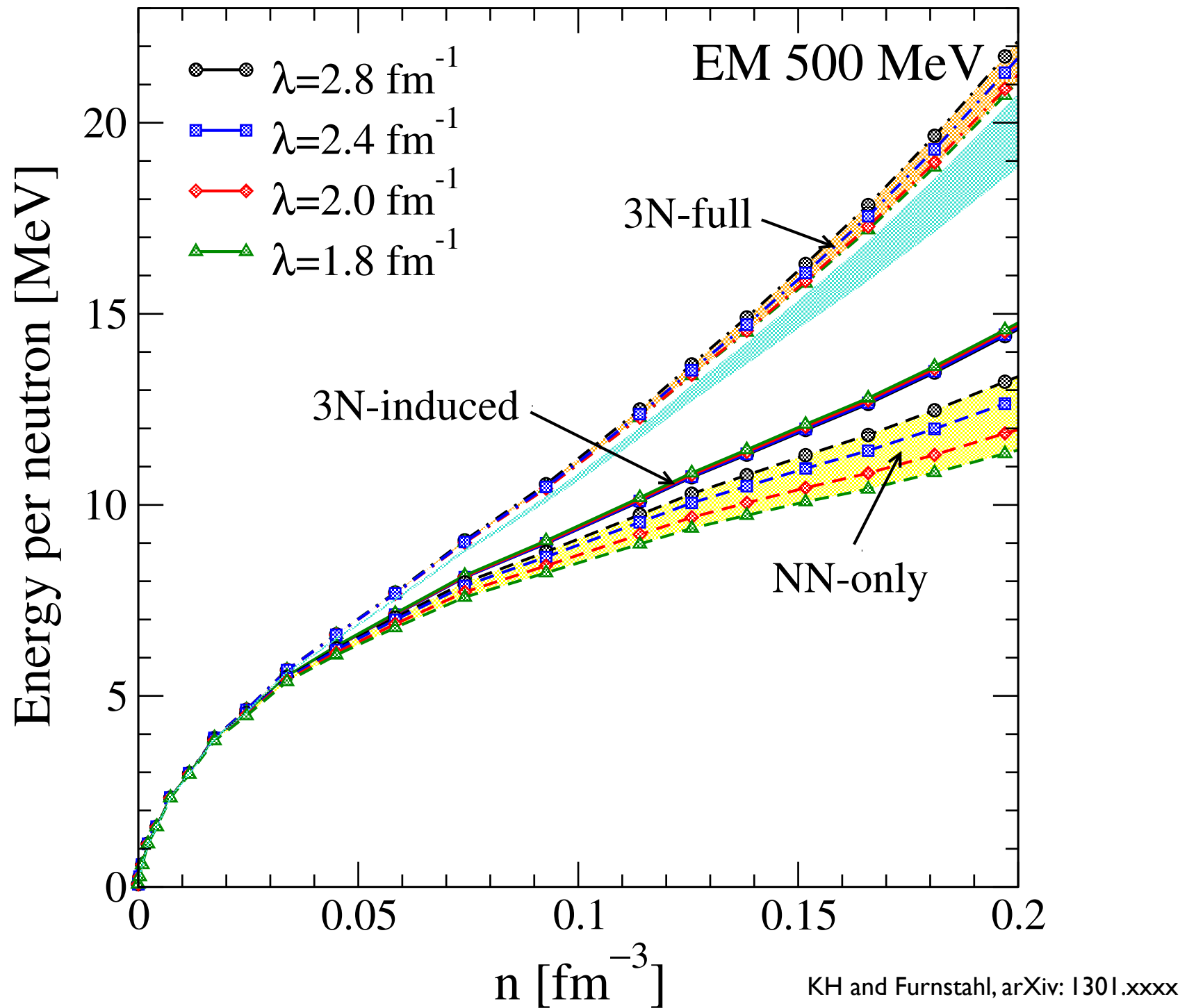
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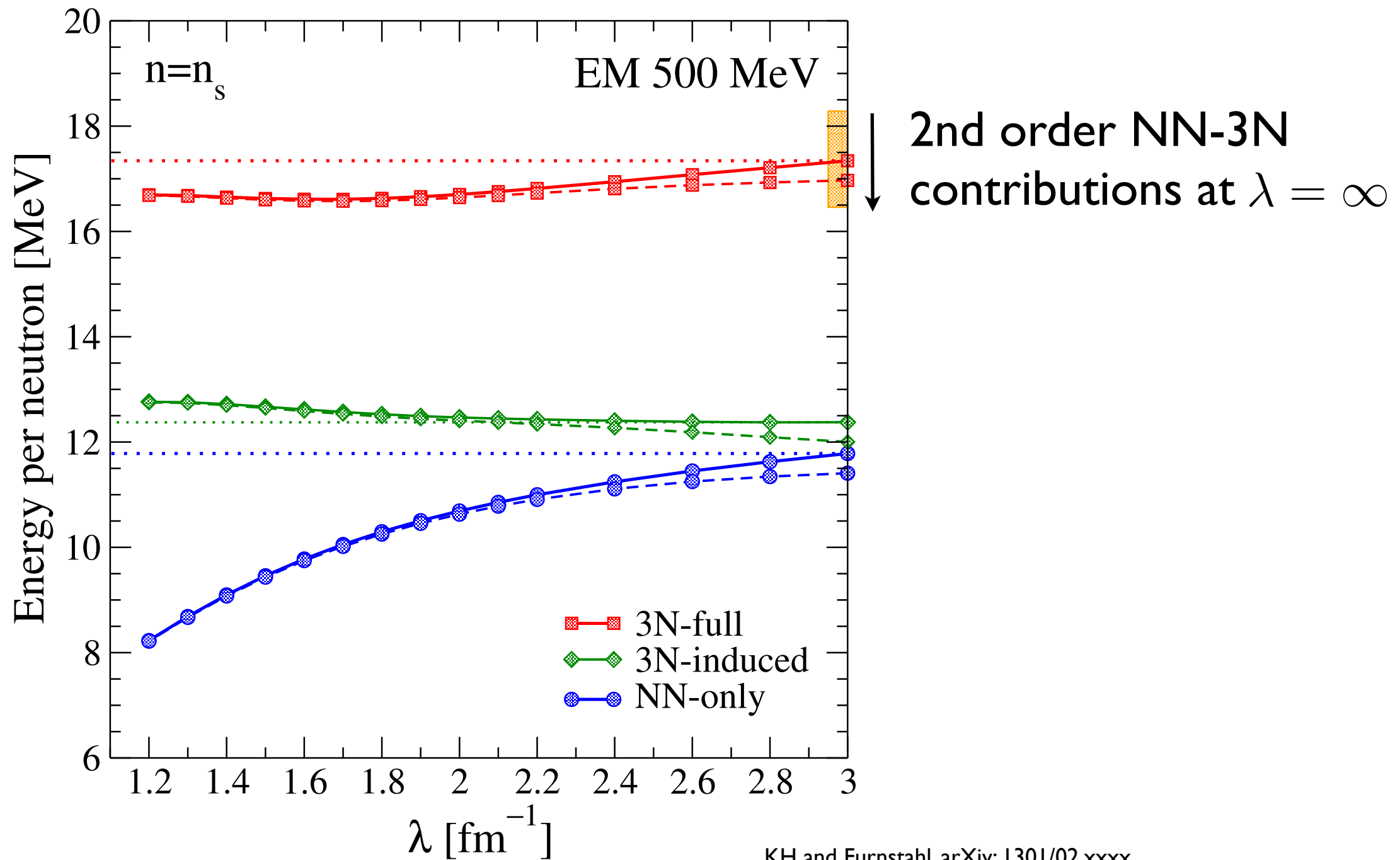
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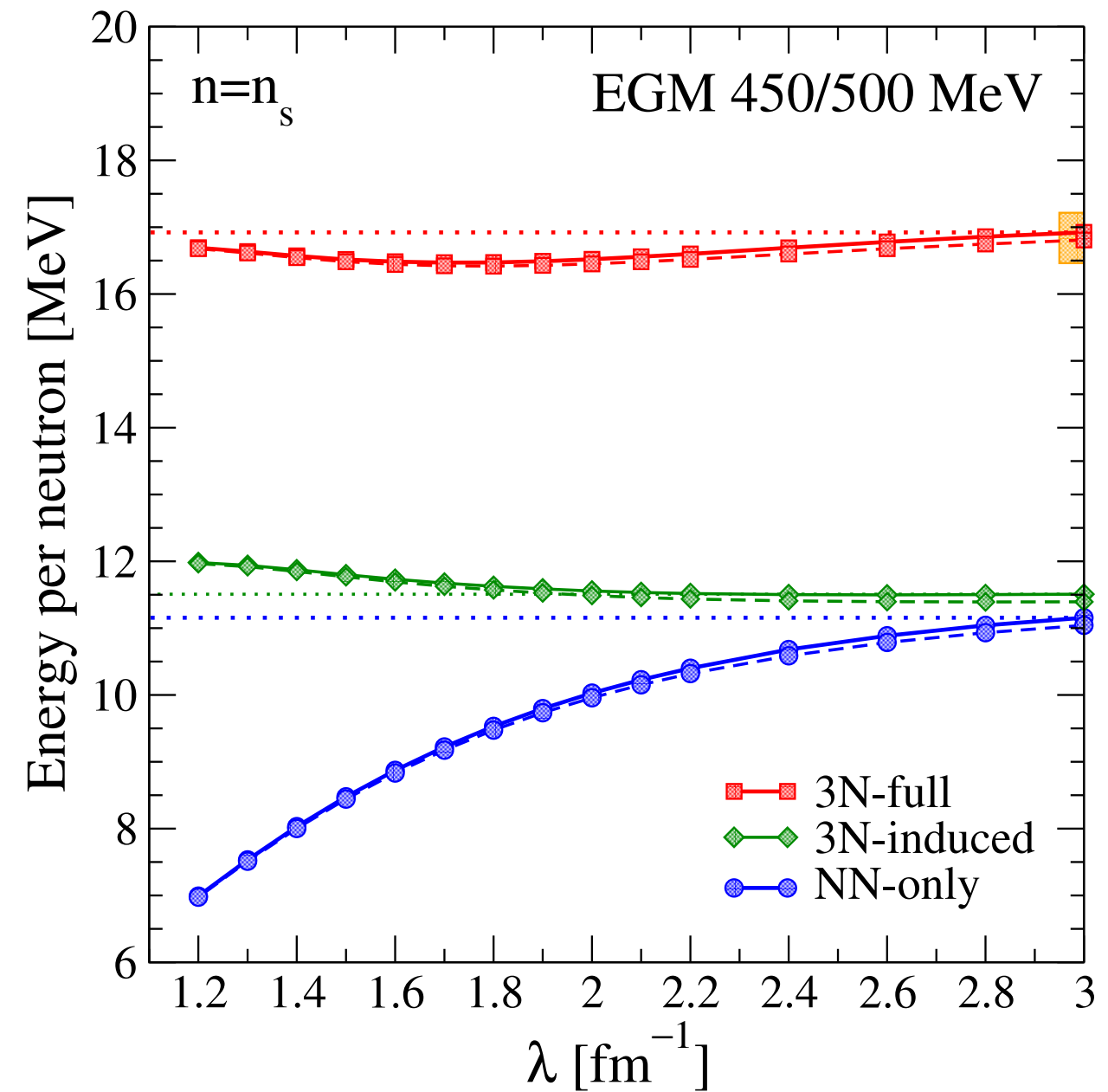
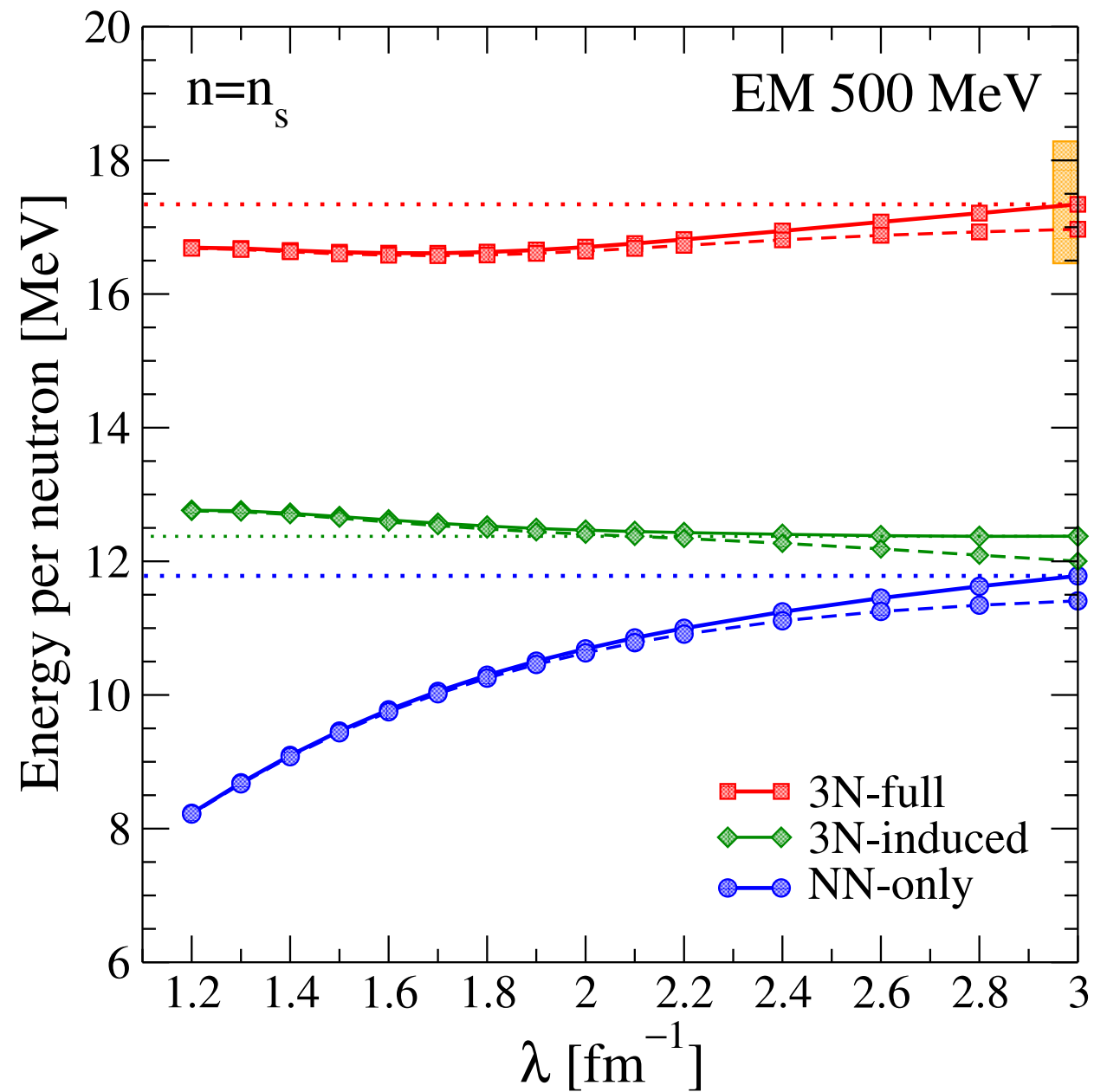
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Resolution-scale dependence at saturation density



- solid lines: NN resummed, dashed lines: 2nd order
- variations: NN-only 3.6 MeV, 3N-induced: 390 keV, 3N-full: 650 keV
- indications for 4N forces at small λ ?

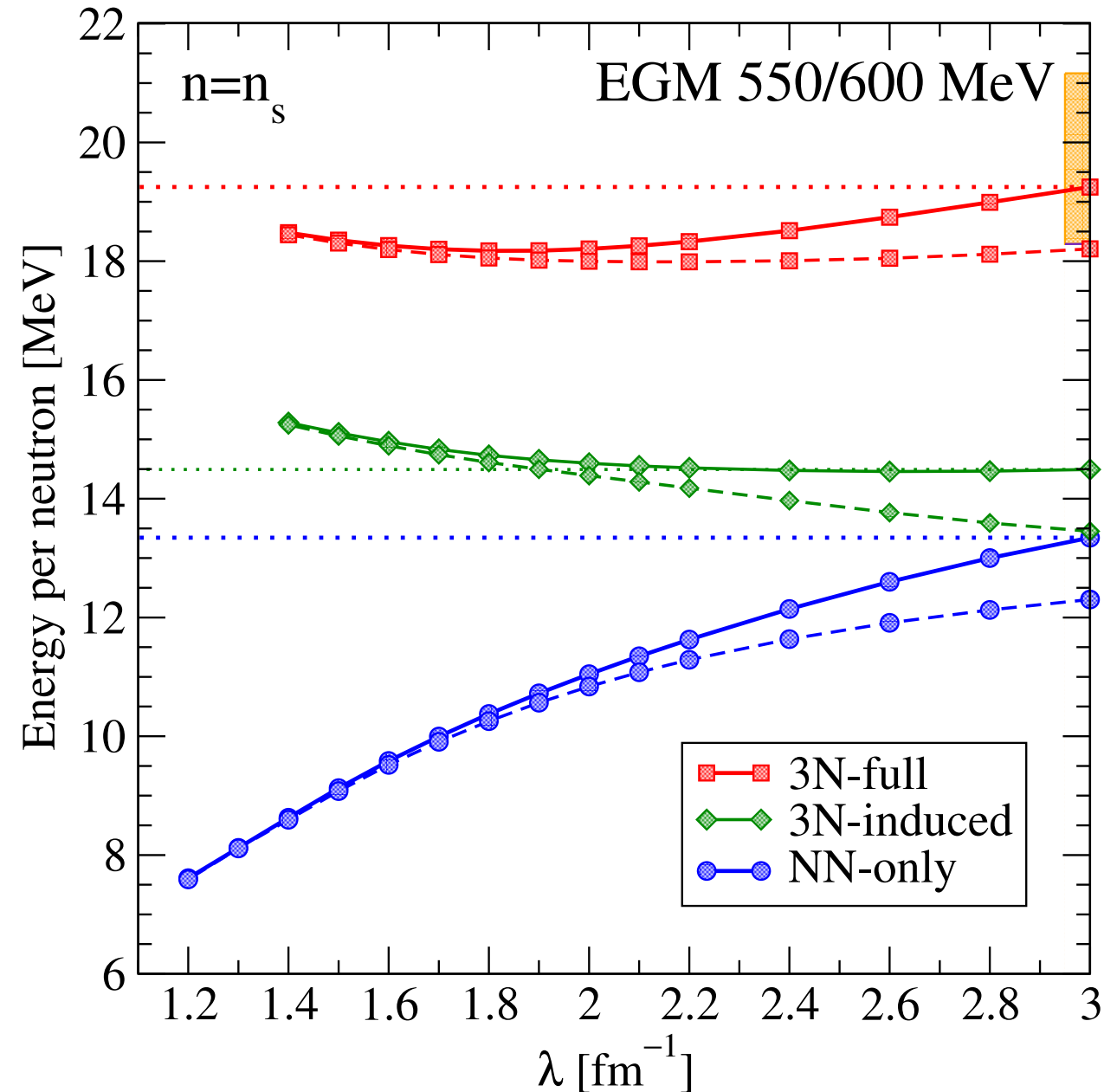
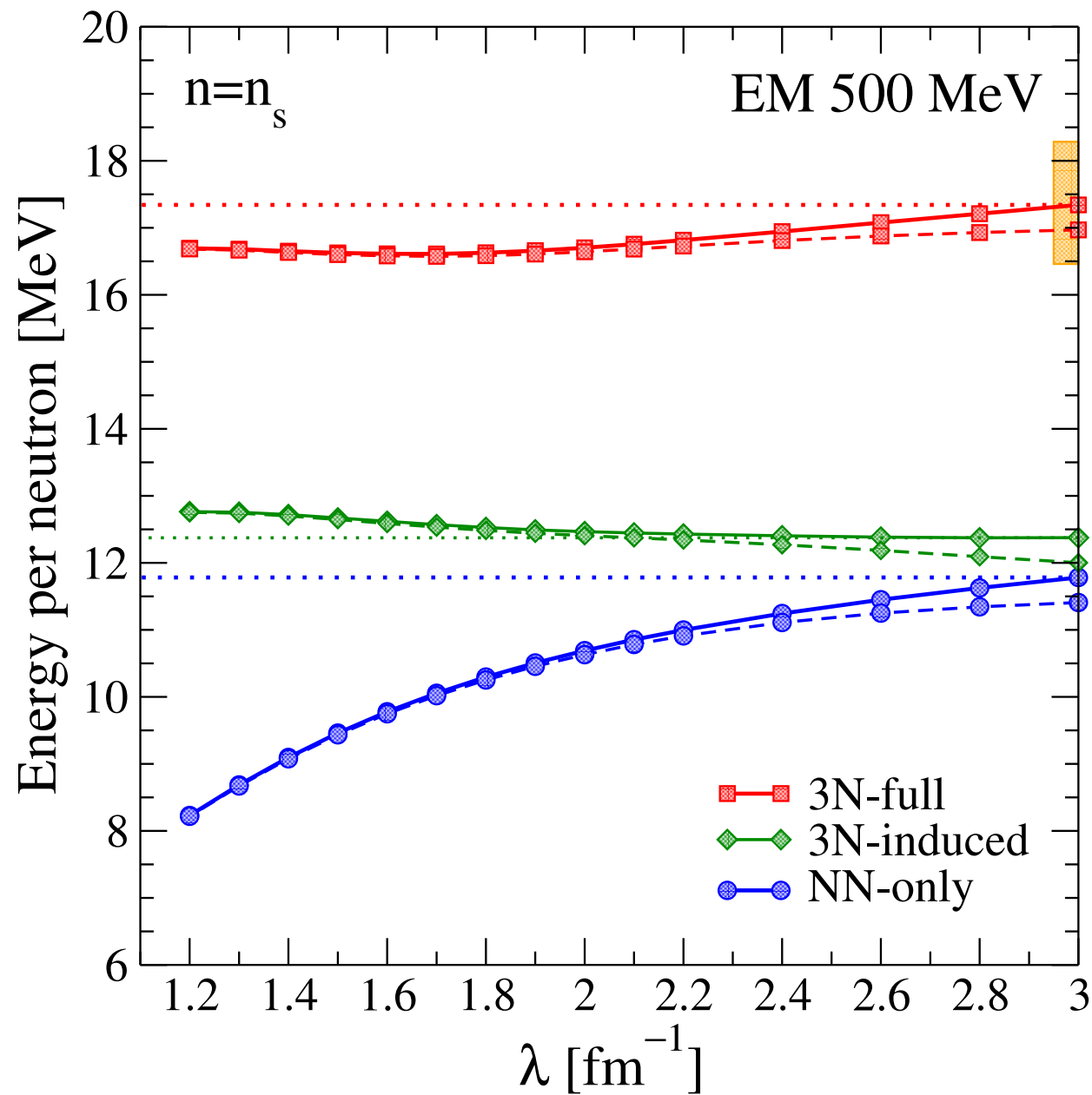
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KH and Furnstahl, arXiv: 1301/02.xxxx

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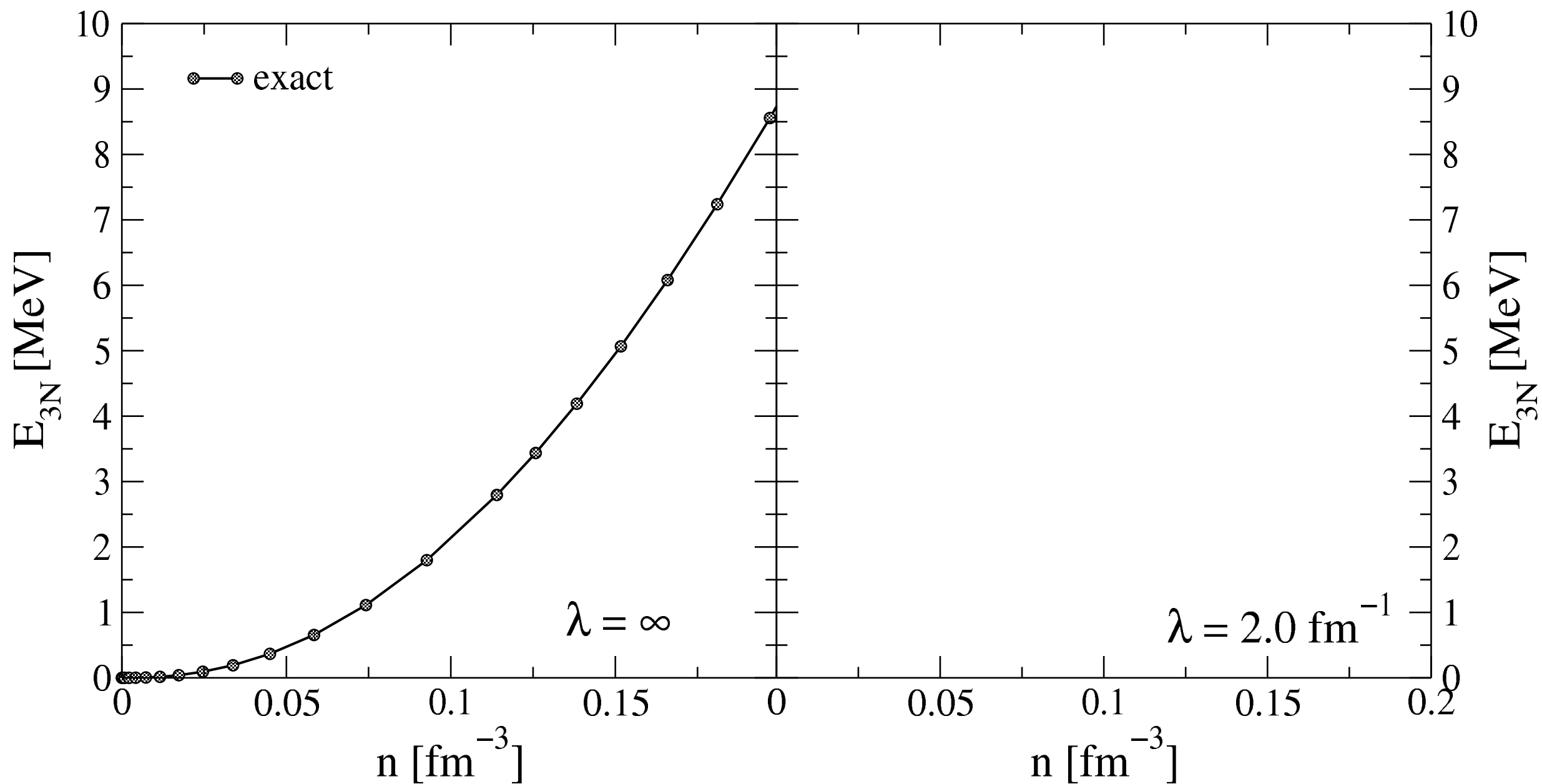
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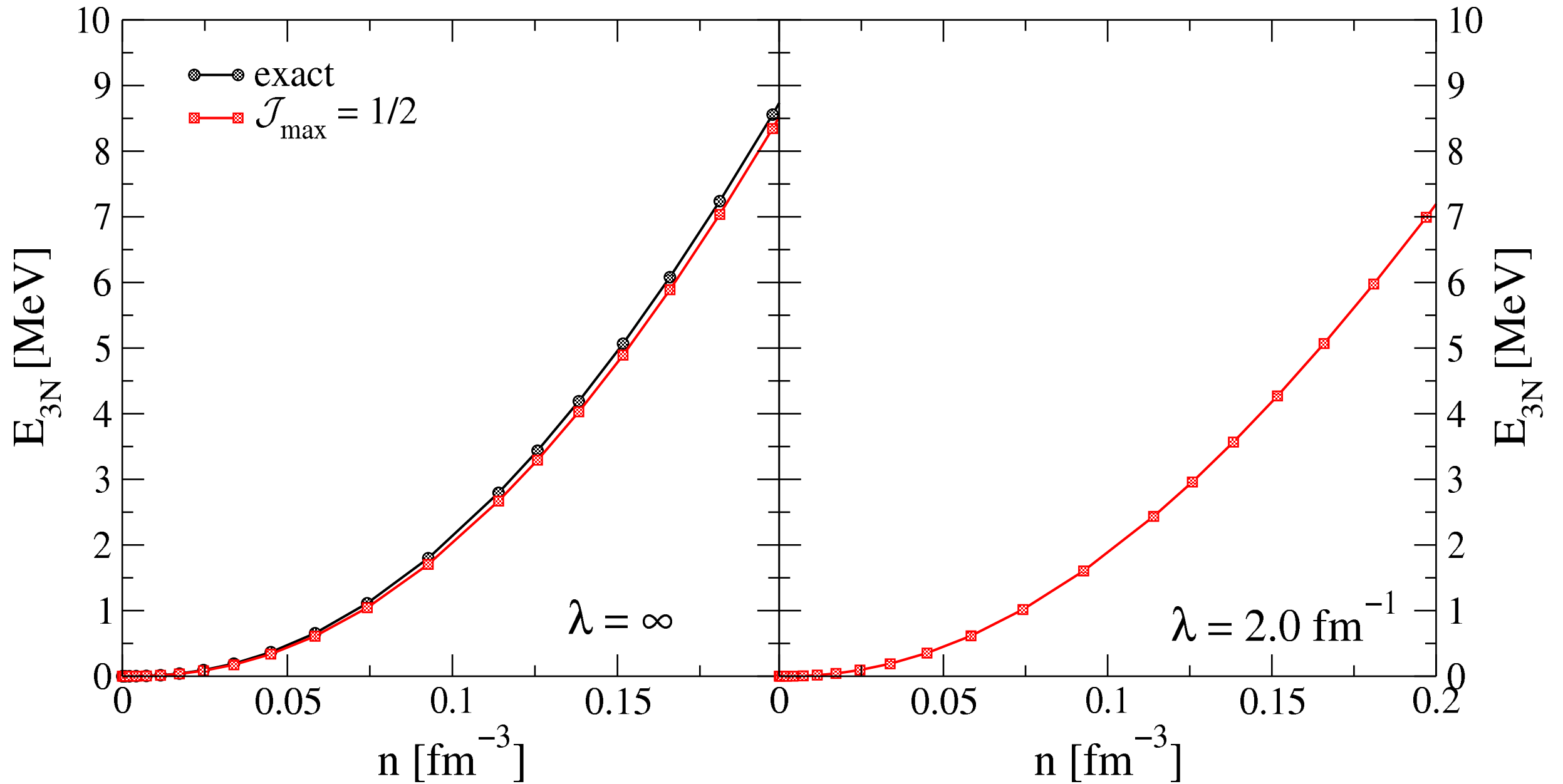
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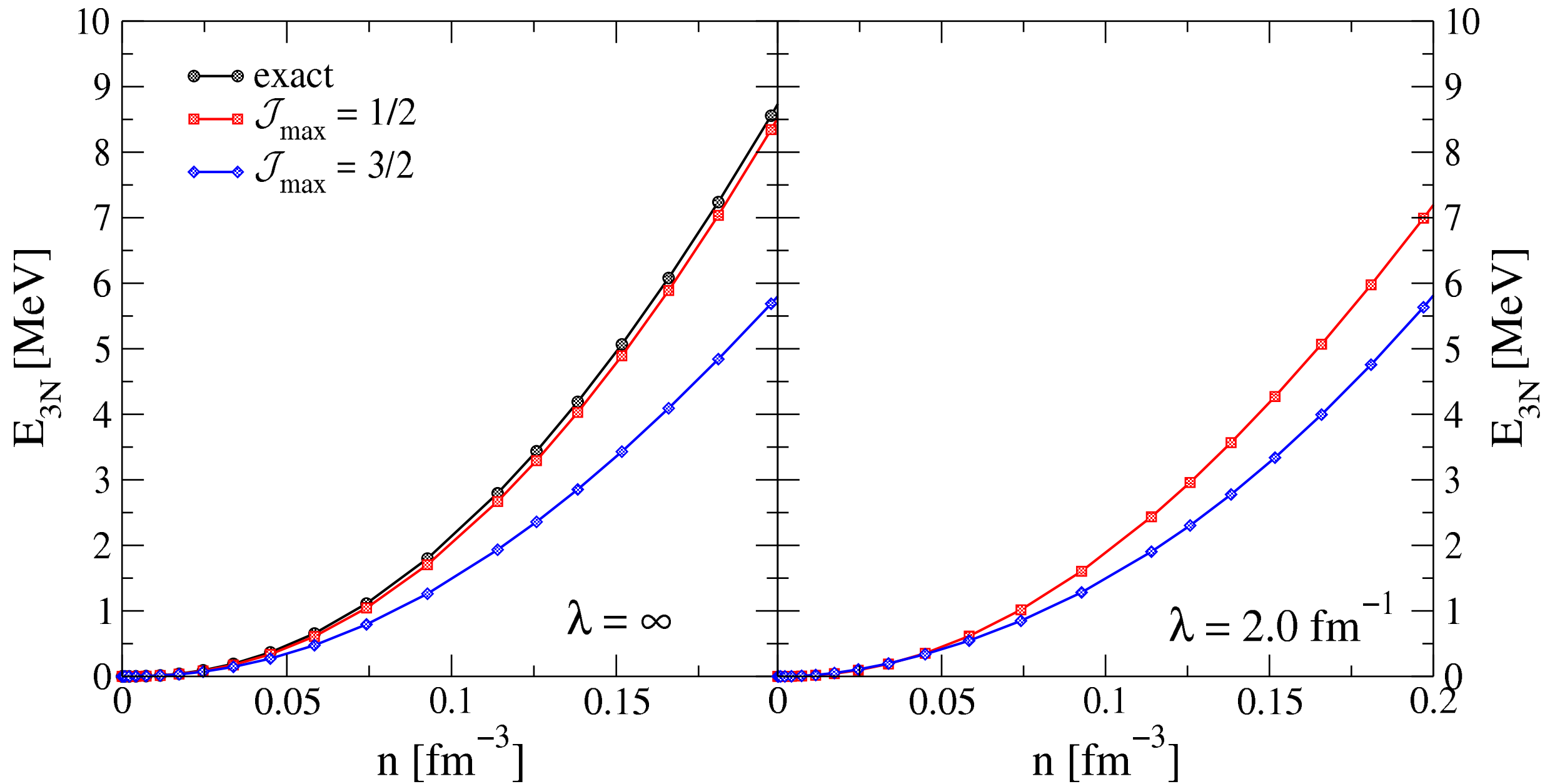
Partial-wave convergence of 3NF contributions



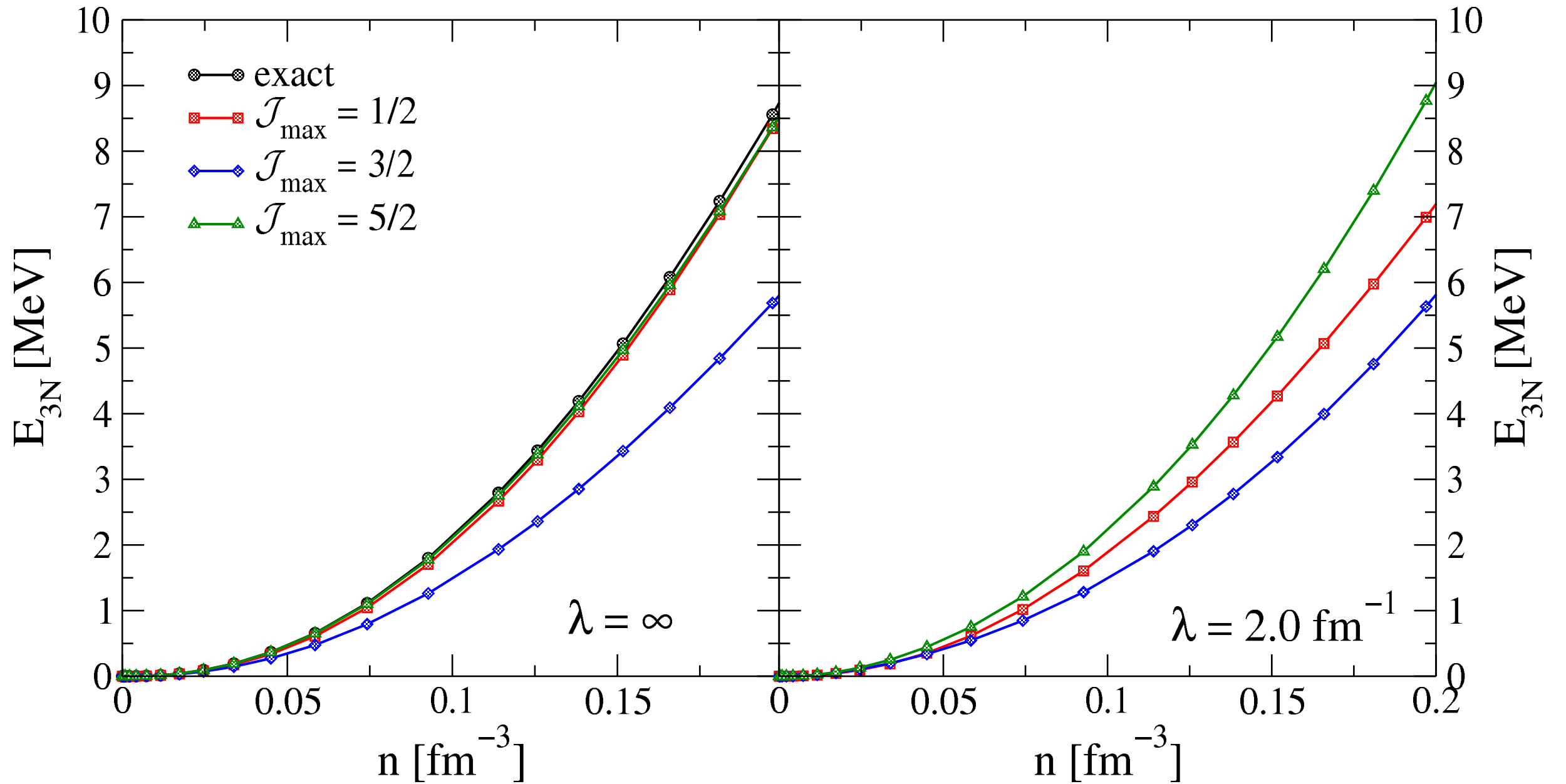
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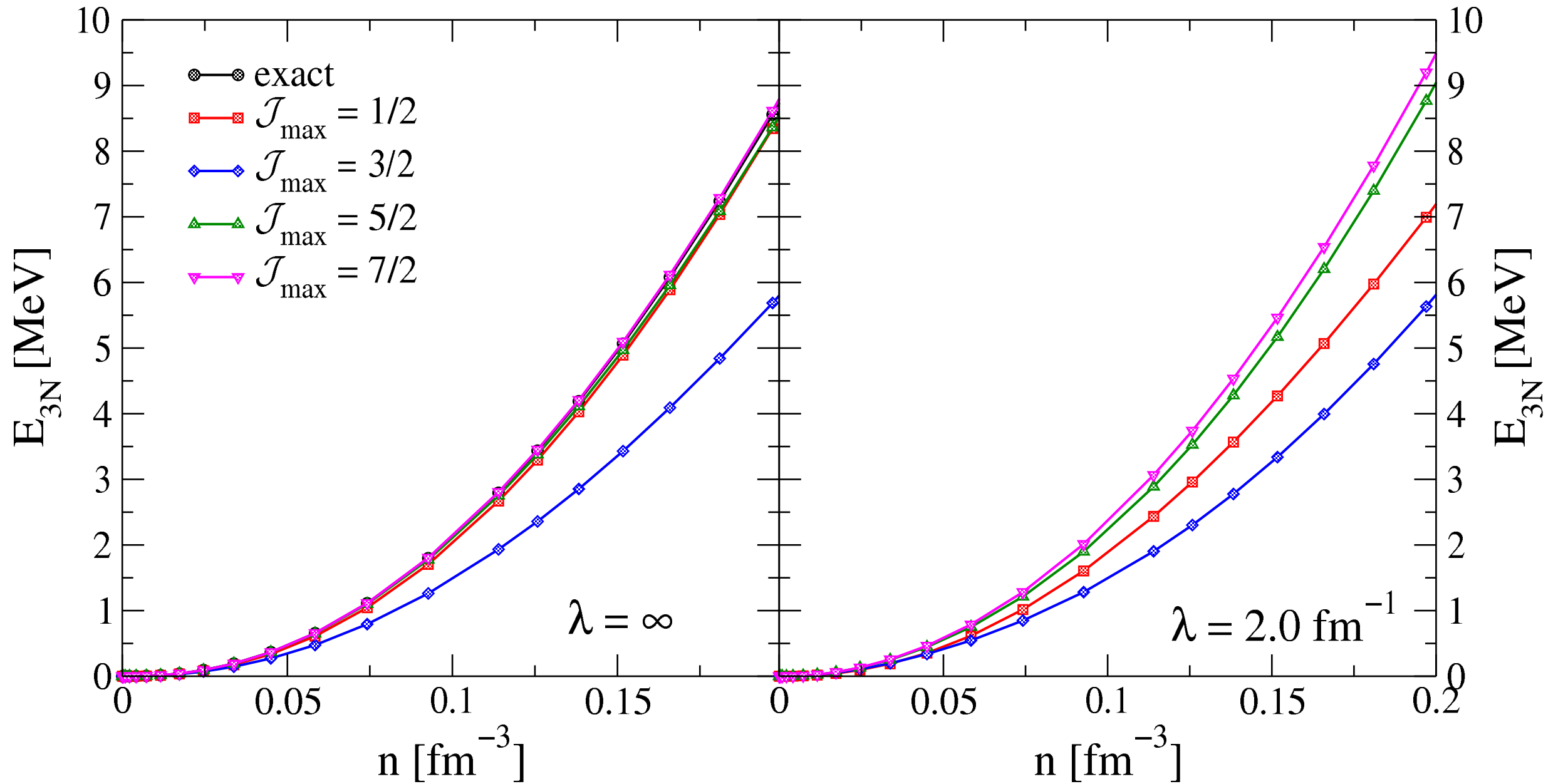
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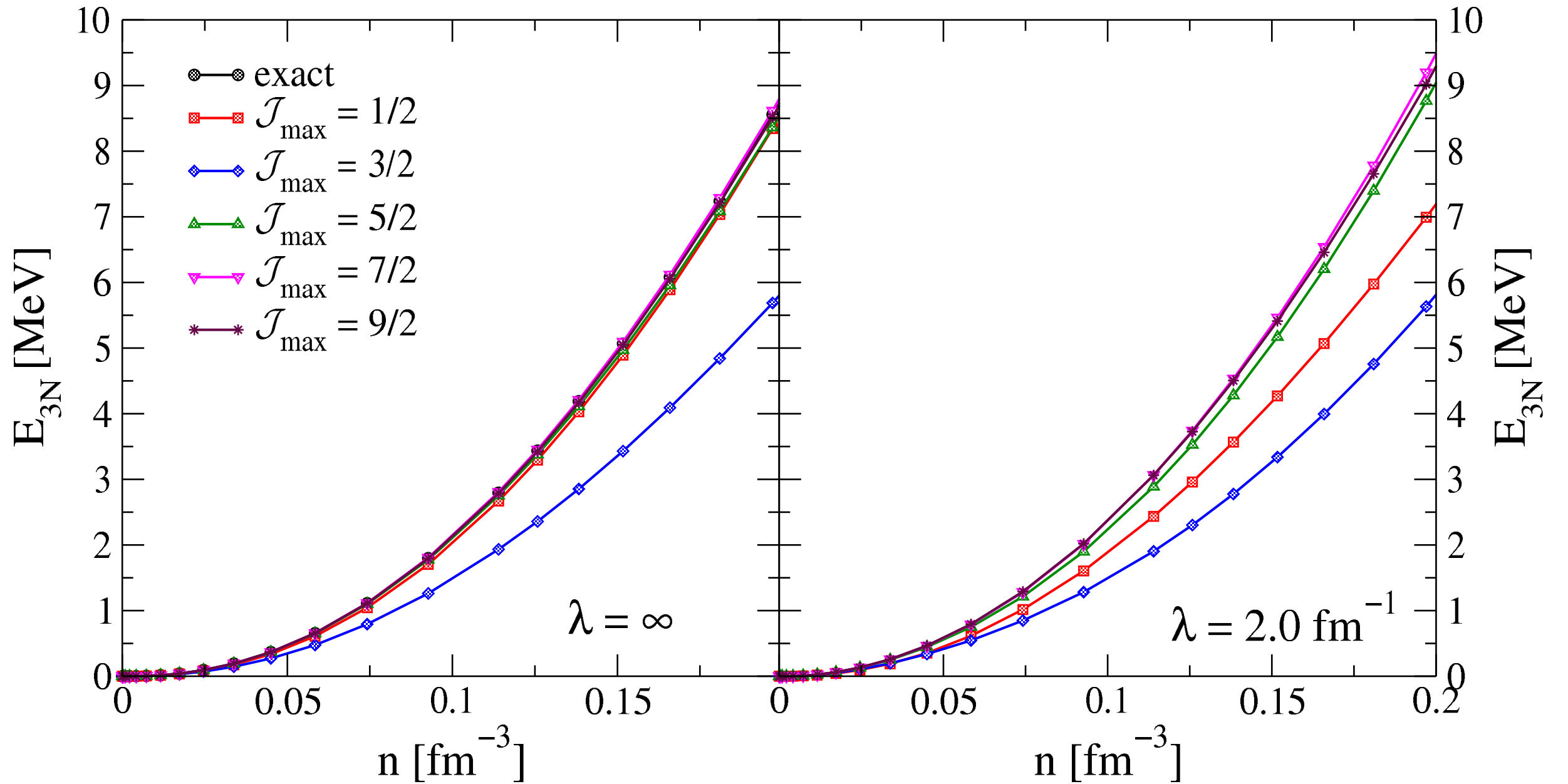
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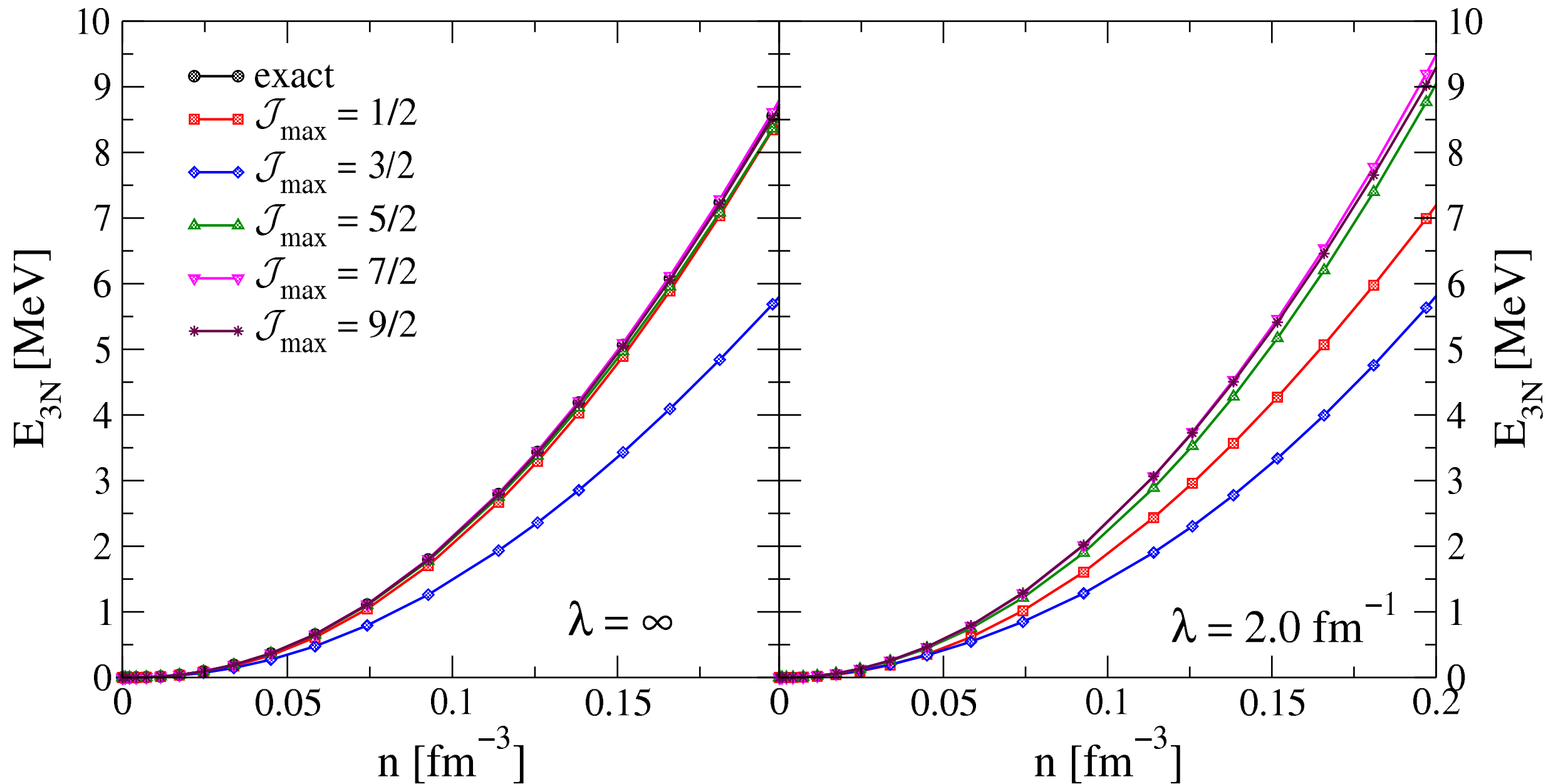
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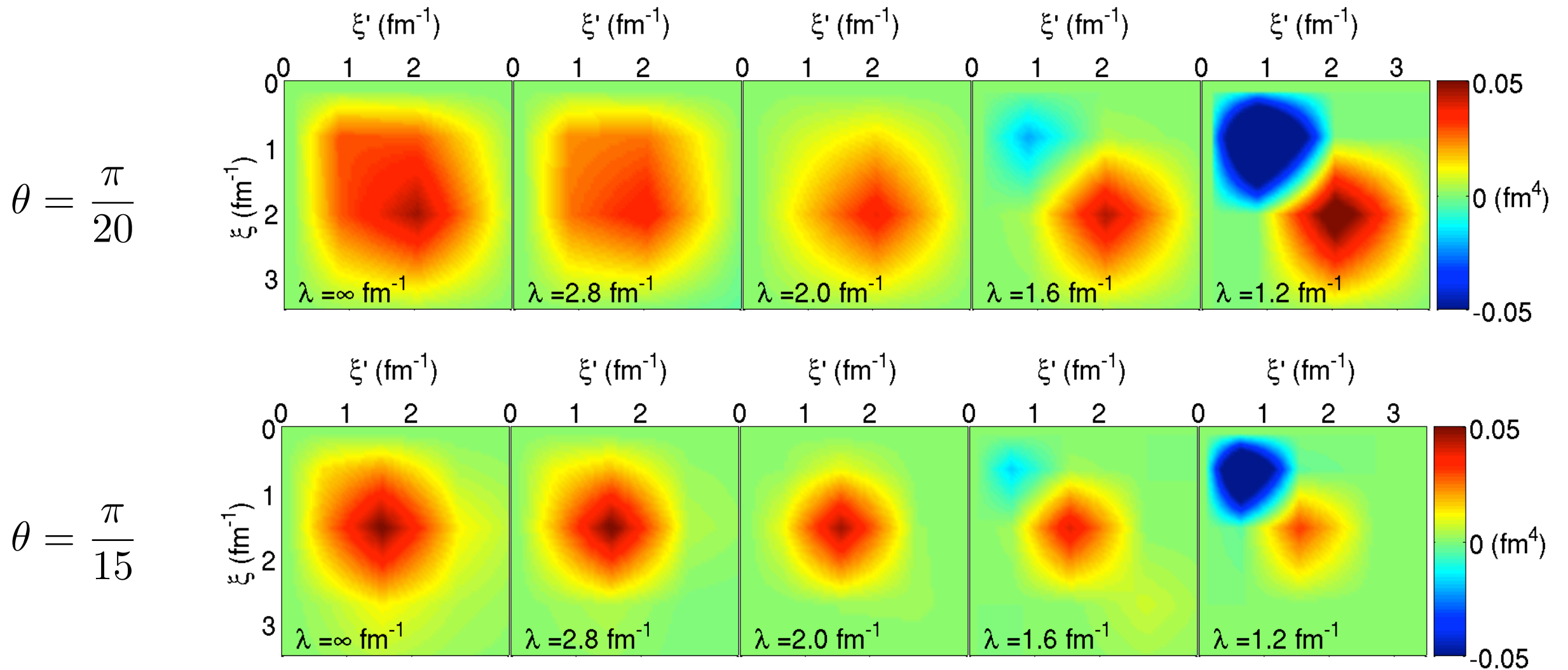
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- E_{3N} agrees within 0.4 % with the exact result at saturation density
- E_{3N} converged in partial waves at both scales, $\lambda = \infty$ and $\lambda = 2.0 \text{ fm}^{-1}$

Matrix elements of evolved 3-neutron interactions

$$\xi^2 = p^2 + \frac{3}{4}q^2 \quad \tan \theta = \frac{2p}{\sqrt{3}q}$$

show dominant channel for $\mathcal{J} = 1/2$ and positive total parity:

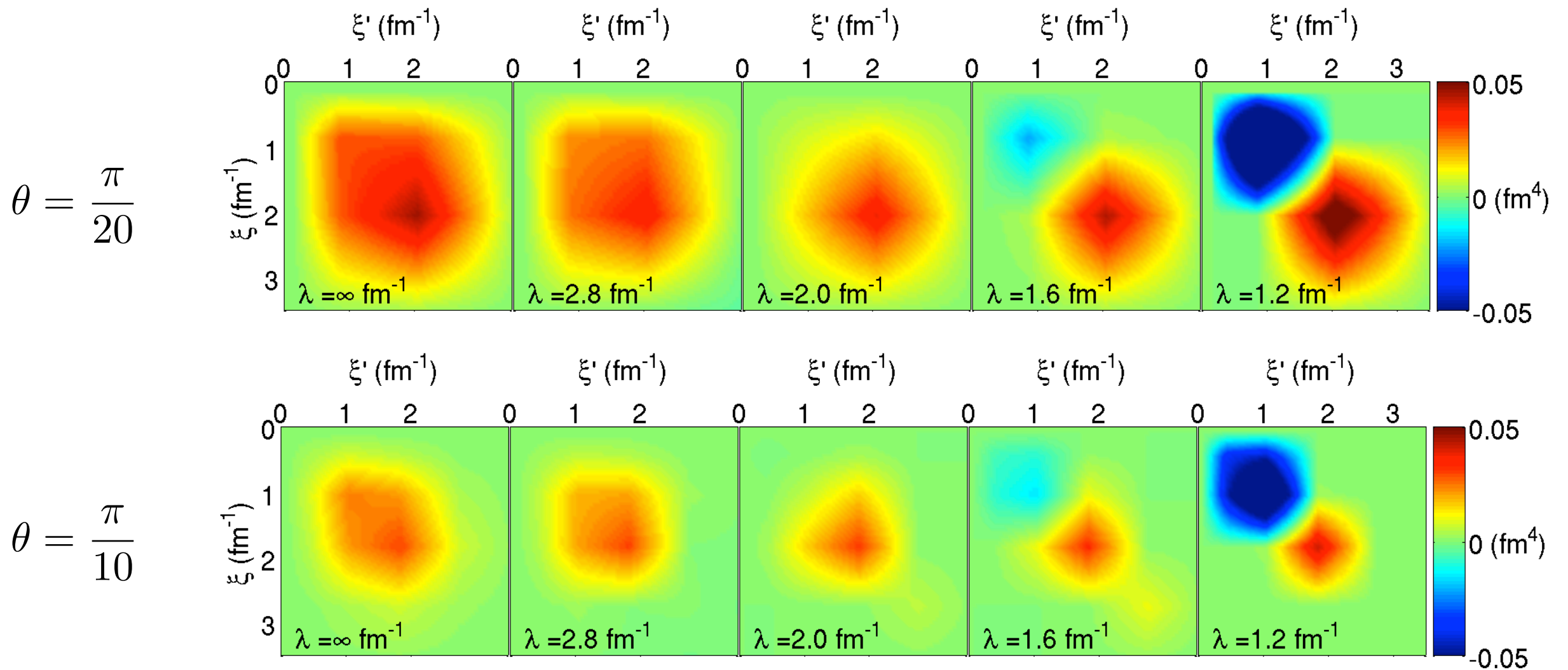


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- moderate effects in range $\lambda = \infty$ to $\lambda = 2.0 \text{ fm}^{-1}$

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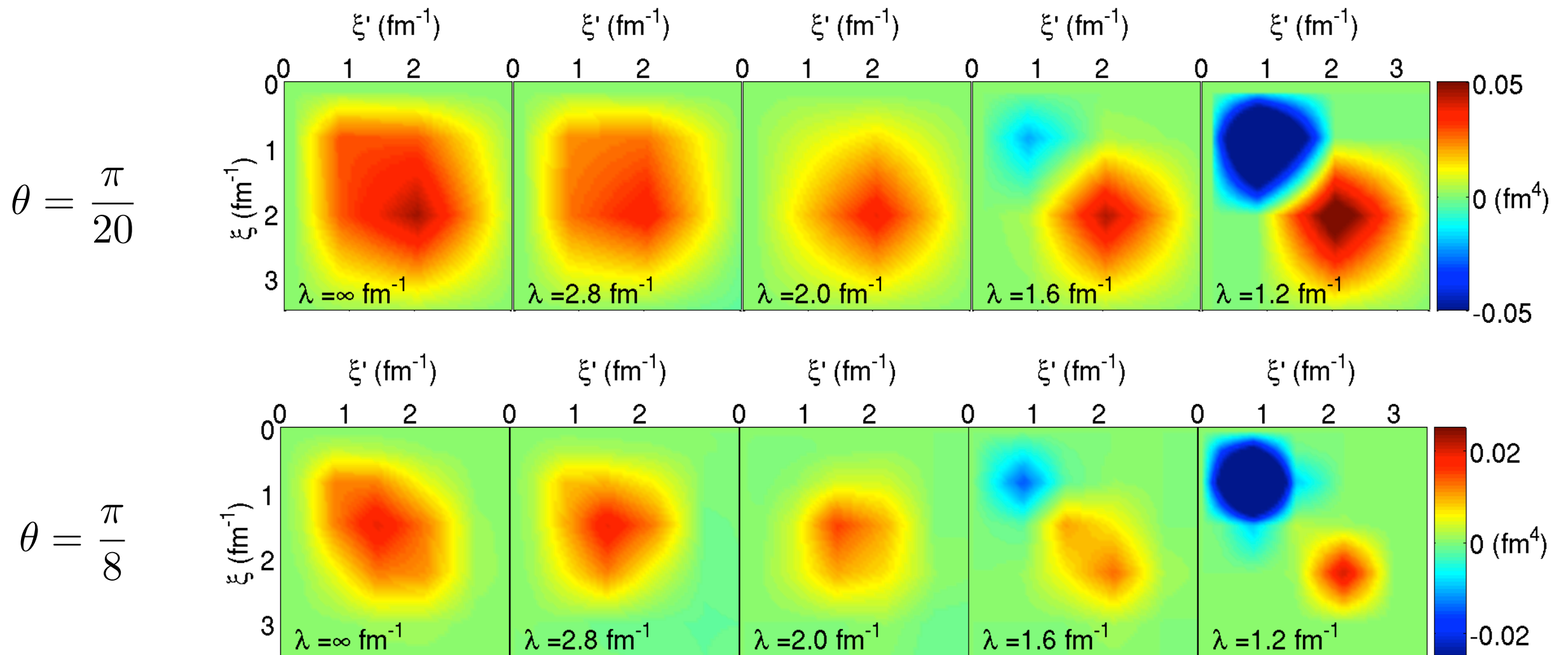


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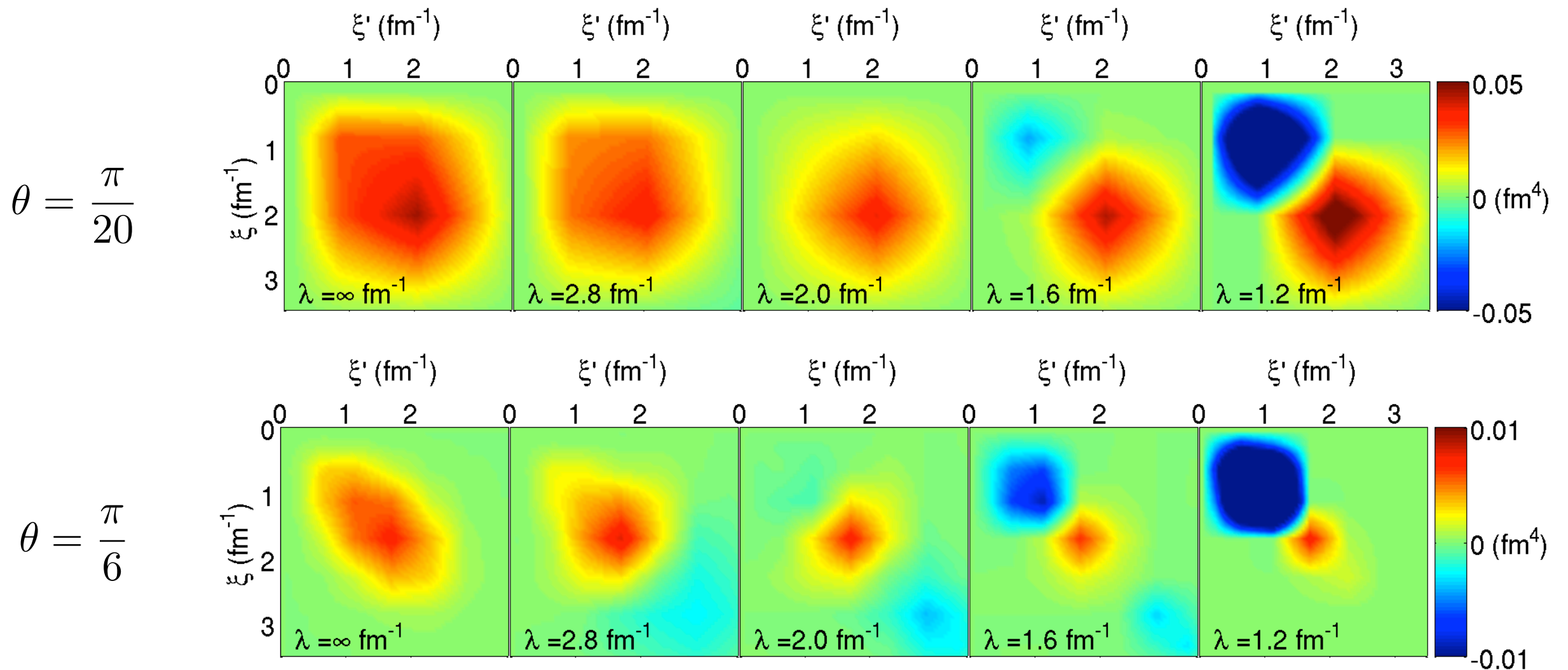


- strong renormalization effects of long-range two-pion exchange
- moderate effects in range $\lambda = \infty$ to $\lambda = 2.0 \text{ fm}^{-1}$

Matrix elements of evolved 3-neutron interactions

$$\xi^2 = p^2 + \frac{3}{4}q^2 \quad \tan \theta = \frac{2p}{\sqrt{3}q}$$

show dominant channel for $\mathcal{J} = 1/2$ and positive total parity:

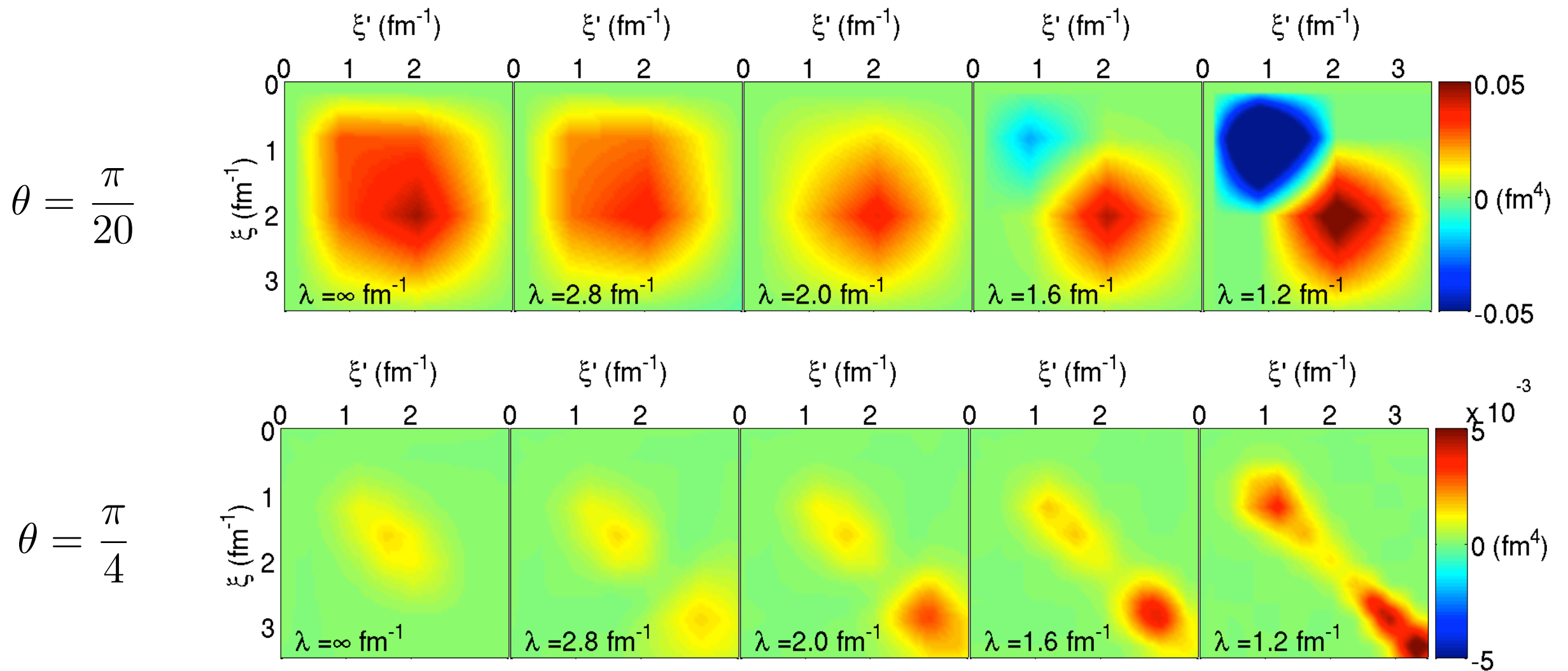


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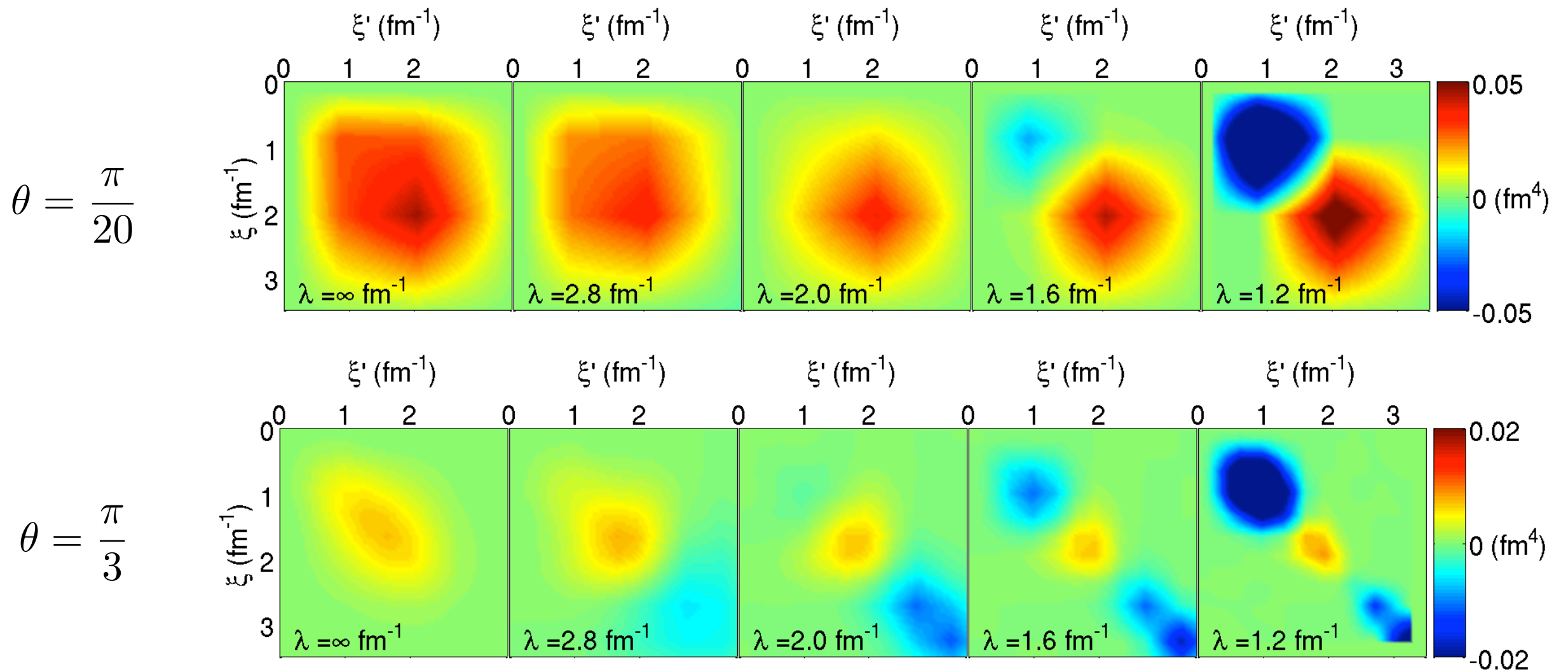


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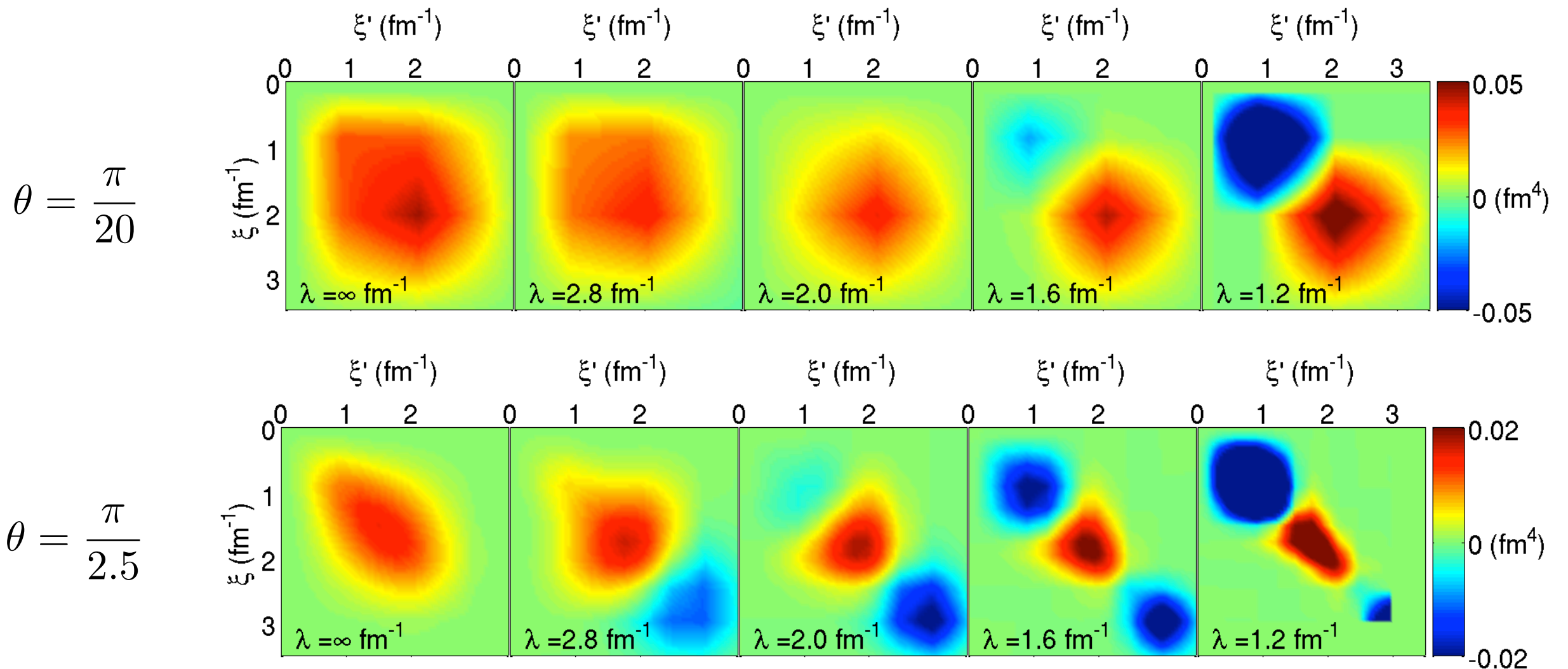


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Matrix elements of evolved 3-neutron interactions

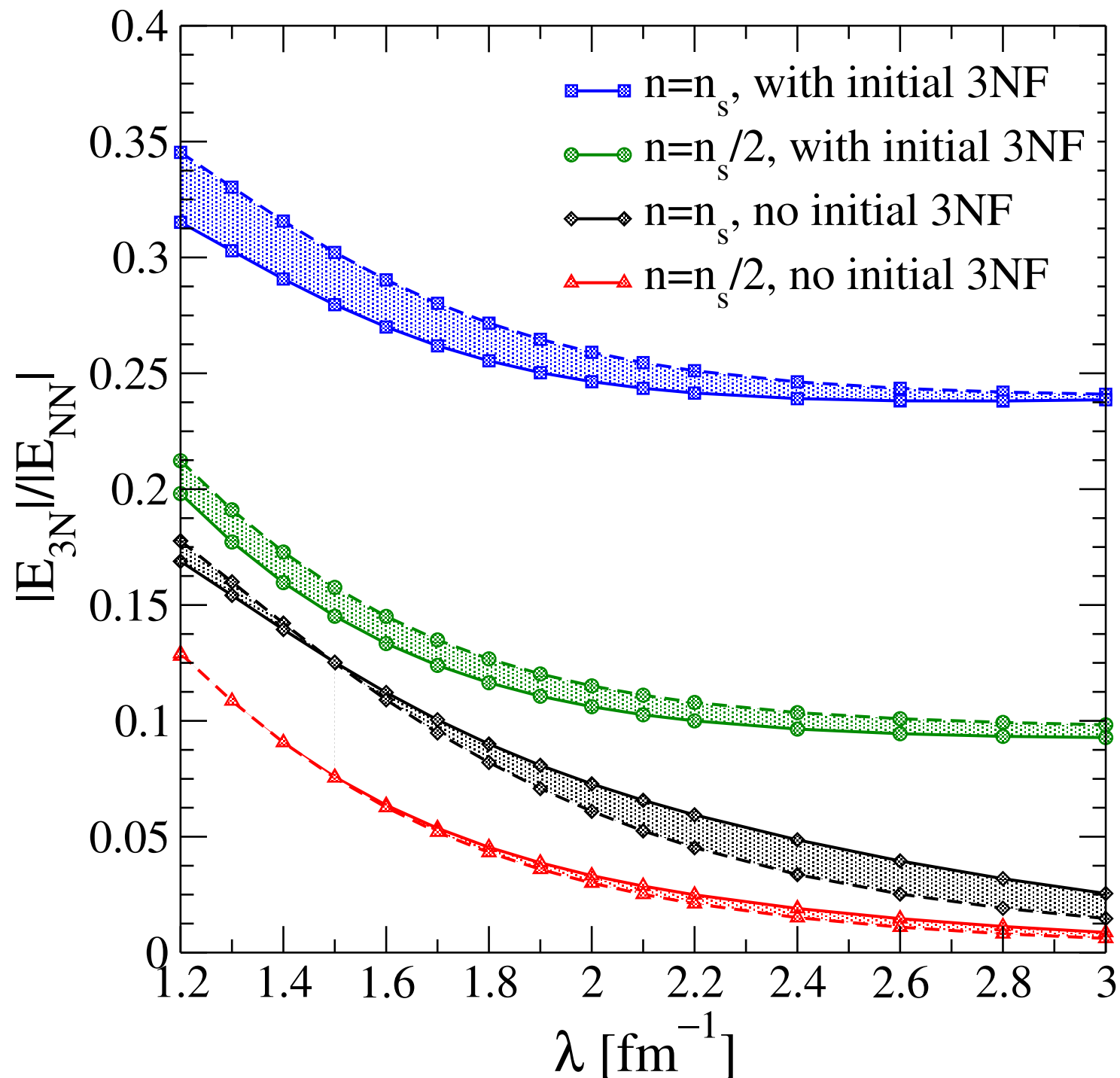
$$\xi^2 = p^2 + \frac{3}{4}q^2 \quad \tan \theta = \frac{2p}{\sqrt{3}q}$$

show dominant channel for $\mathcal{J} = 1/2$ and positive total parity:



- strong renormalization effects of long-range two-pion exchange
- moderate effects in range $\lambda = \infty$ to $\lambda = 2.0 \text{ fm}^{-1}$

Scaling of three-body contributions



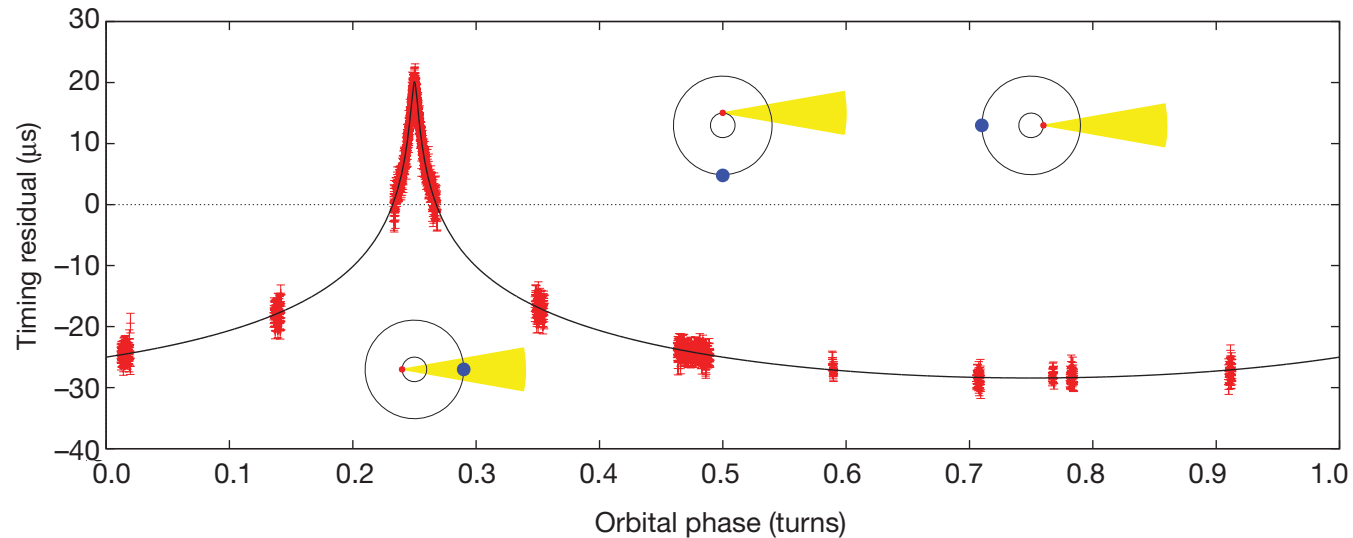
- relative size of 3N contribution grows systematically towards smaller
- no obvious trend with density (may be obscured by cancellations among contributions)

Constraints on the nuclear equation of state (EOS)

nature

A two-solar-mass neutron star measured using Shapiro delay

P. B. Demorest¹, T. Pennucci², S. M. Ransom¹, M. S. E. Roberts³ & J. W. T. Hessels^{4,5}



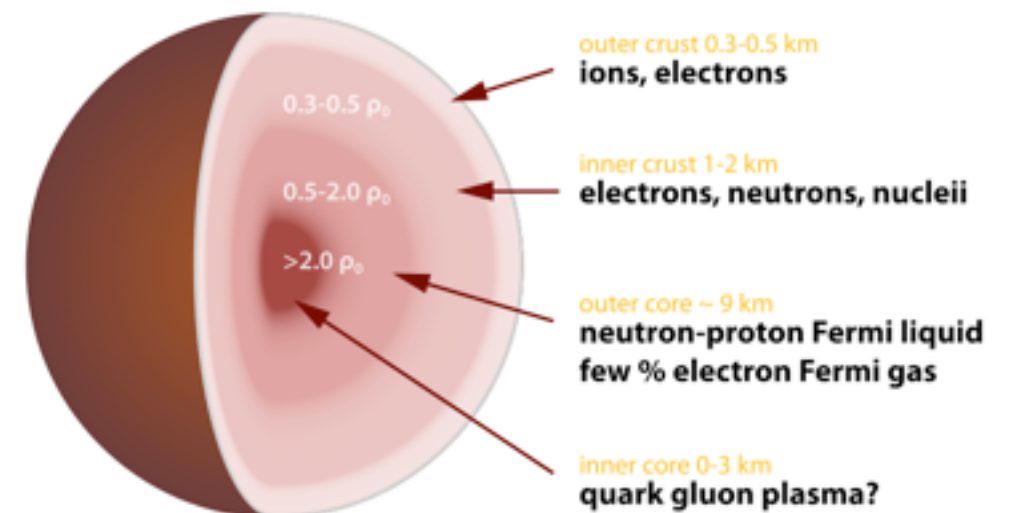
Demorest et al., Nature 467, 1081 (2010)

$$M_{\text{max}} = 1.65 M_{\odot} \rightarrow 1.97 \pm 0.04 M_{\odot}$$

Calculation of neutron star properties requires EOS up to high densities.



Credit: NASA/Dana Berry



Strategy:

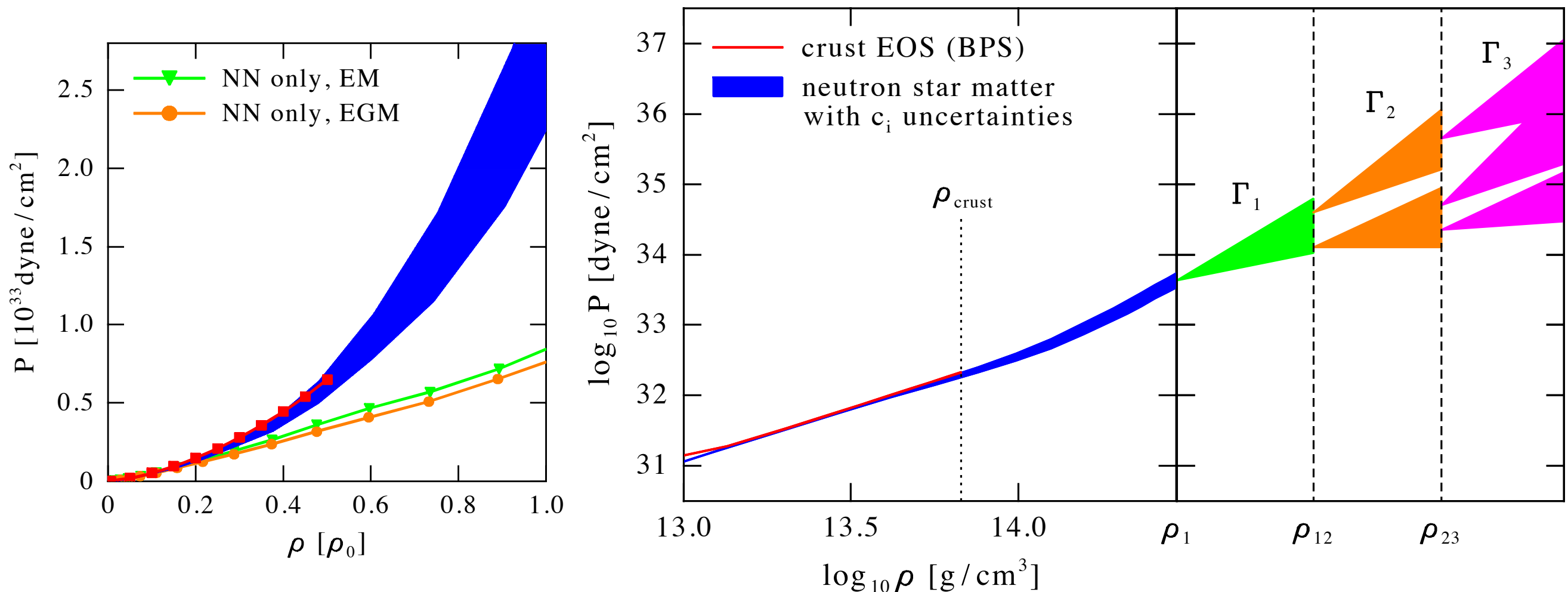
Use observations to constrain the high-density part of the nuclear EOS.

Neutron star radius constraints

incorporation of beta-equilibrium: neutron matter \longrightarrow neutron star matter

parametrize piecewise high-density extensions of EOS:

- use polytropic ansatz $p \sim \rho^\Gamma$
- range of parameters $\Gamma_1, \rho_{12}, \Gamma_2, \rho_{23}, \Gamma_3$ limited by physics!



KH, Lattimer, Pethick, Schwenk, in preparation

KH, Lattimer, Pethick, Schwenk, PRL 105, 161102 (2010)

Constraints on the nuclear equation of state

use the constraints:

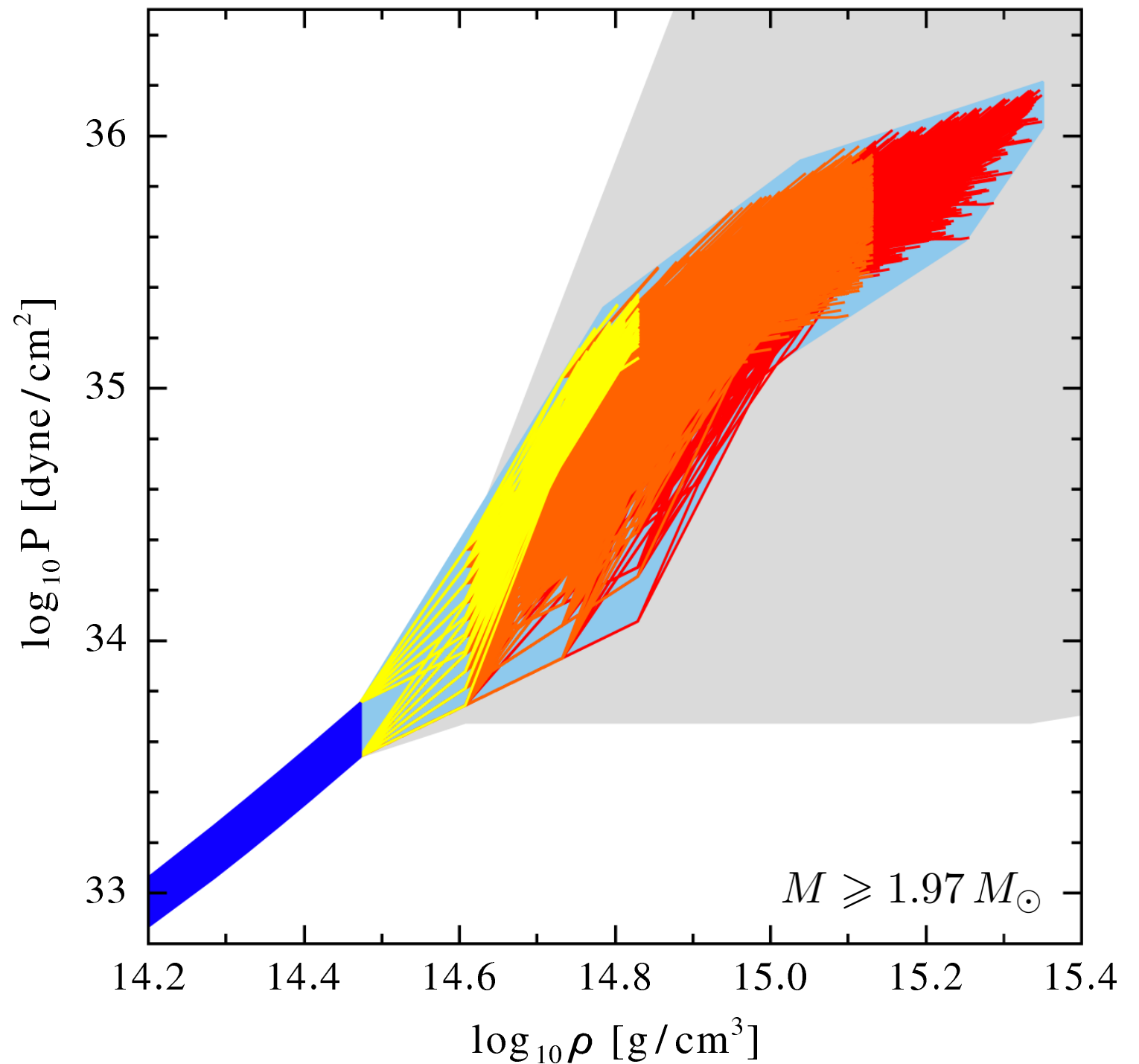
recent NS observation

$$M_{\text{max}} > 1.97 M_{\odot}$$

causality

$$v_s(\rho) = \sqrt{dP/d\varepsilon} < c$$

KH, Lattimer, Pethick, Schwenk, in preparation



significant reduction of uncertainty band

Constraints on the nuclear equation of state

use the constraints:

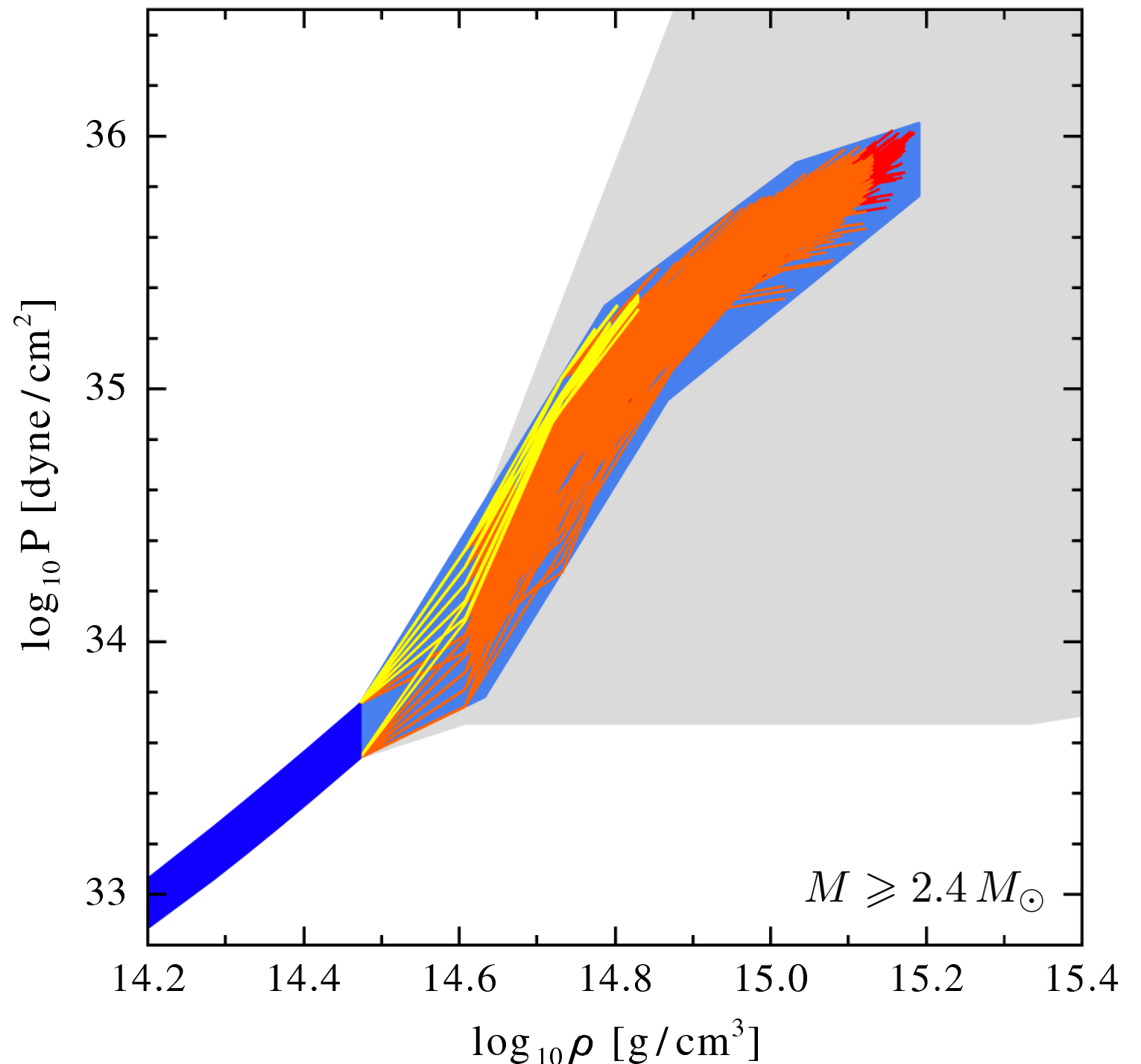
NS mass

$$M_{\text{max}} > 2.4 M_{\odot}$$

causality

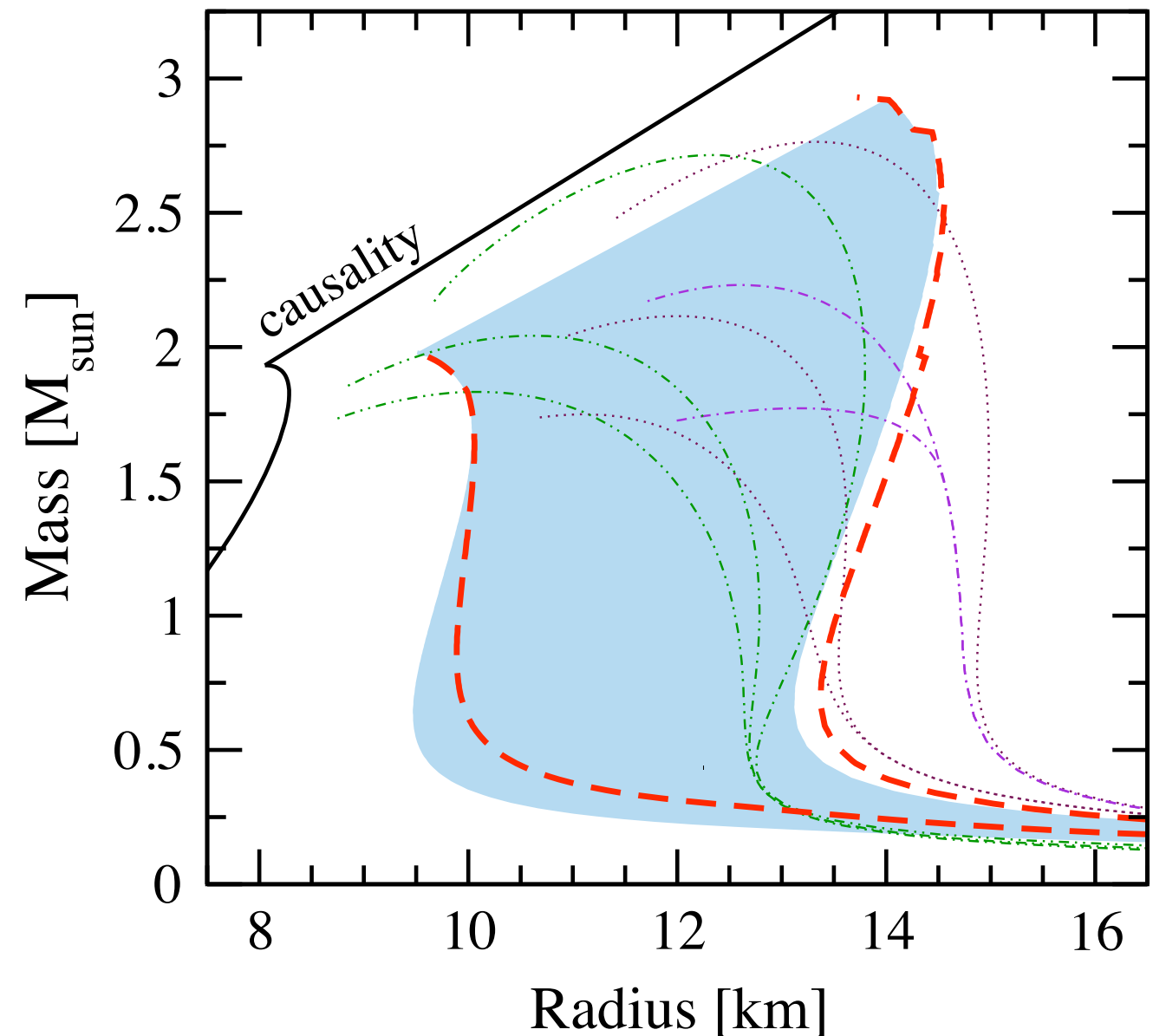
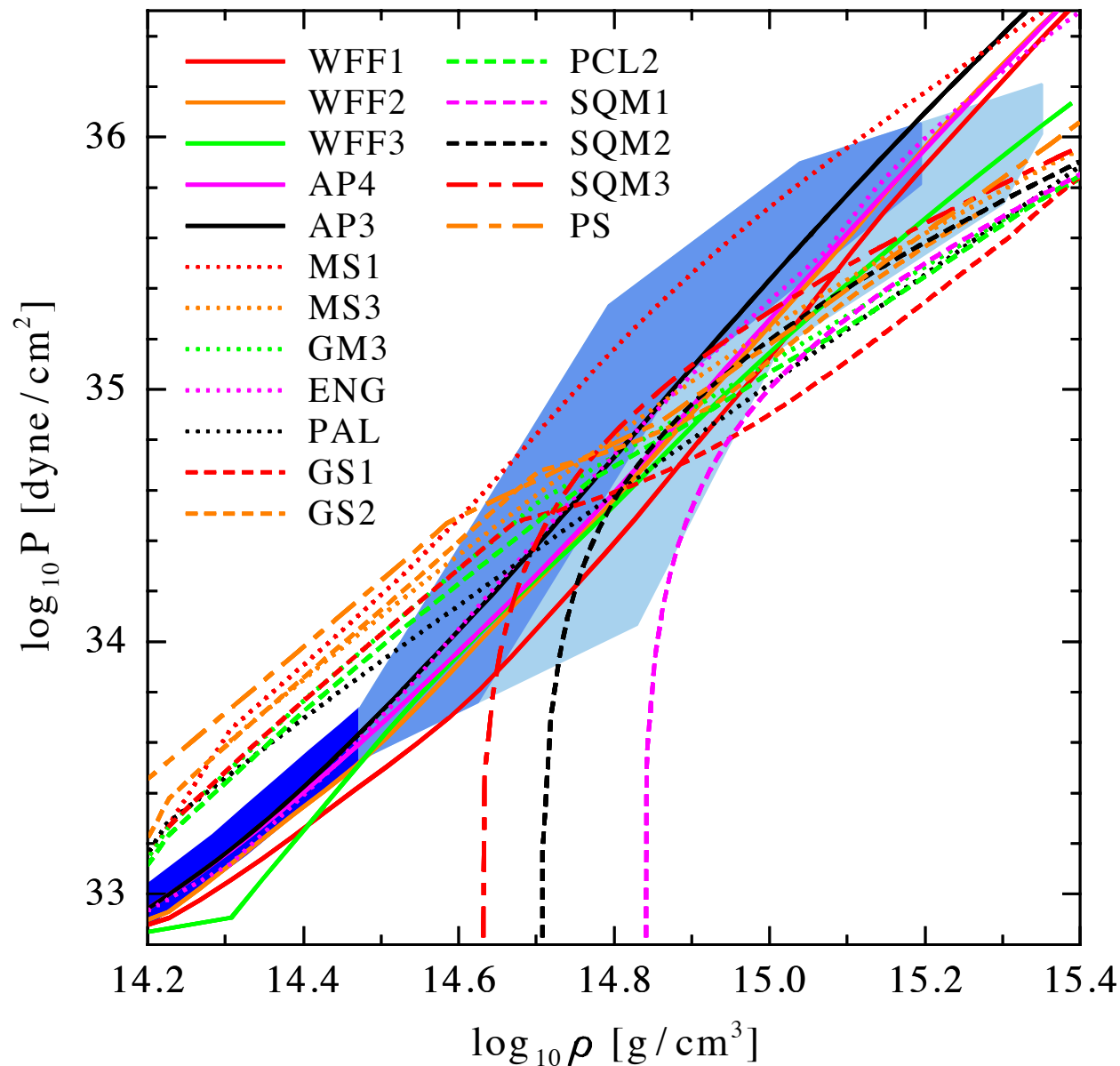
$$v_s(\rho) = \sqrt{dP/d\varepsilon} < c$$

KH, Lattimer, Pethick, Schwenk, in preparation



increased M_{max} systematically reduces width of band

Constraints on neutron star radii



KH, Lattimer, Pethick, Schwenk, in preparation
 see also KH, Lattimer, Pethick, Schwenk, PRL 105, 161102 (2010)

- low-density part of EoS sets scale for allowed high-density extensions
- radius constraint for typical $1.4 M_{\odot}$ neutron star: $9.8 - 13.4$ km

Summary

- demonstrated the feasibility of SRG evolution of NN+3NF in momentum space
- first results of neutron matter based on consistently evolved NN+3NF interactions
- strong renormalization effects of chiral two-pion exchange interaction in neutron matter
- no indications of significant contributions from 4N forces down to $\lambda = 1.2 \text{ fm}^{-1}$ in neutron matter

Outlook

- inclusion of 3NF N3LO contributions in RG evolution
- extend RG evolution to $\mathcal{T} = 1/2$ channels, application to nuclear matter
- transformation to HO basis, application to finite nuclei (CC, NCSM)
- RG evolution of operators: nuclear scaling and correlations in nuclear systems

