

Toward Realistic Description of Low energy Fusion for Astrophysics: Application to N-⁴He and d-⁴He Scattering

International Workshop XLI on Gross Properties of Nuclei and Nuclear Excitations:
“Astrophysics and Nuclear Structure”

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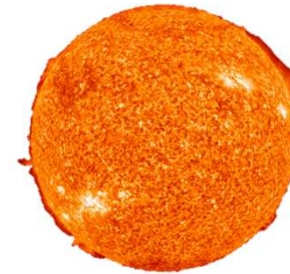


From Stars to Nuclei



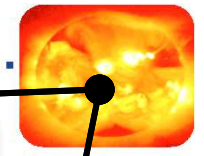
Evolution of stars: birth, main sequence and death

The sun ☉



H	73.46%
He	24.85%
O	0.77%
C	0.29%
Fe	0.16%

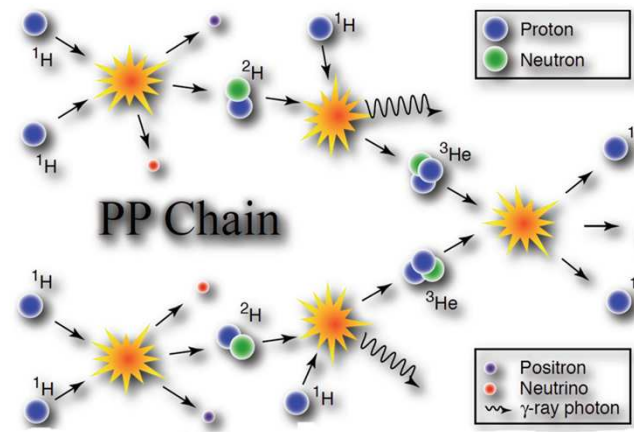
Fusion processes play an important role in determining the evolution of our universe: nucleosynthesis, stellar evolution ...



Sun



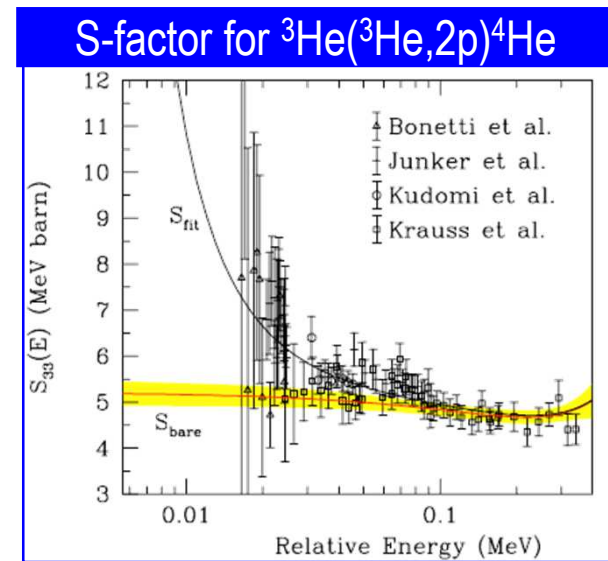
- What powers stars ?
- How long does a star live ?
- ...



Nuclear astrophysics community relies on accurate fusion reactions observables among others.

Challenging for both experiment and theory:

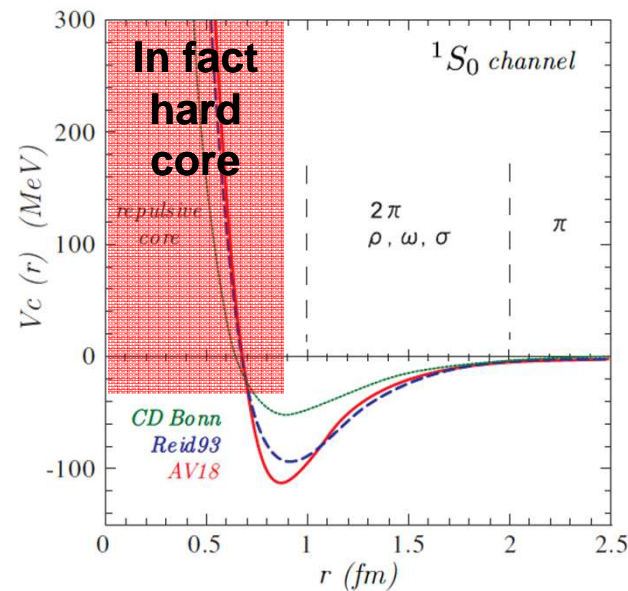
- Low rates: Coulomb repulsion between target and projectile + low energy (quantum tunneling effects).
- Projectile and target are not fully ionized in a lab. This leads to laboratory electron screening
- Fundamental theory is still missing



Some words about the ingredients of an *ab initio* calculation

A high precision nuclear Hamiltonian

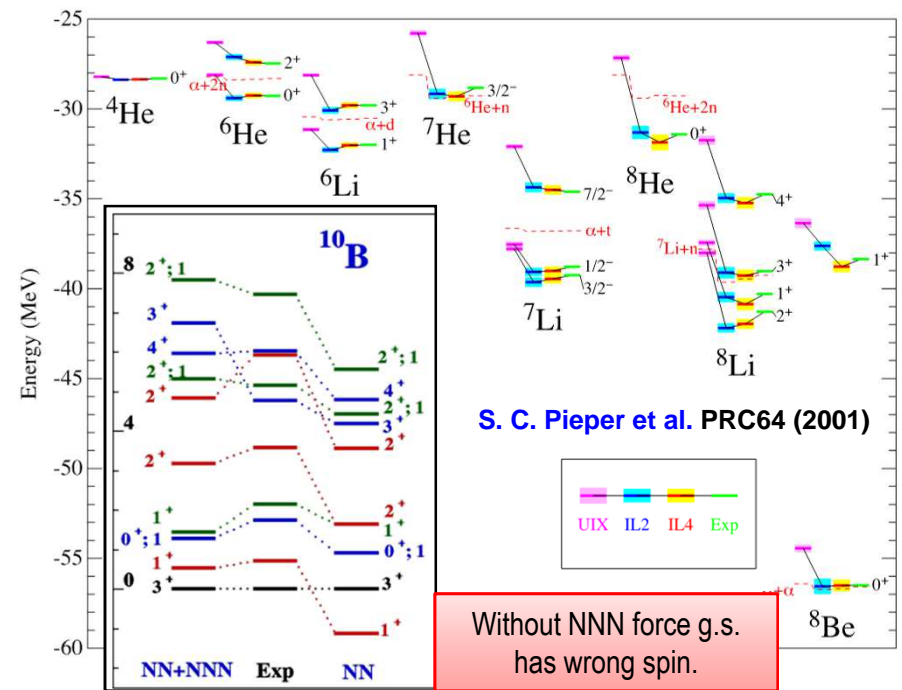
We have a NN interaction...



N. Ishii et al. PRL99 (2007)

The nuclear interaction has a strong repulsive core. This makes nuclear structure calculation converge slowly.

...and also NNN interaction



P. Navrátil et al. PRL 99 (2007)

We need a NNN interaction to achieve a high-precision. This is ~100 times numerically costlier.

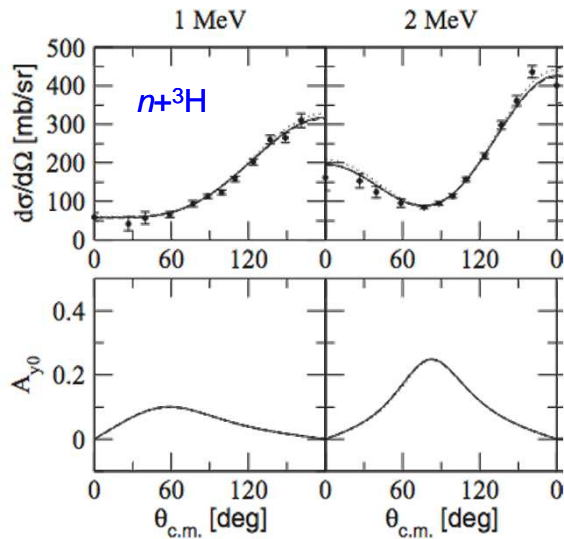
Fig. 3 (Pieper, et al.)

Status of nuclear reaction models

- *Ab initio* nuclear reactions lagging behind structure calculations

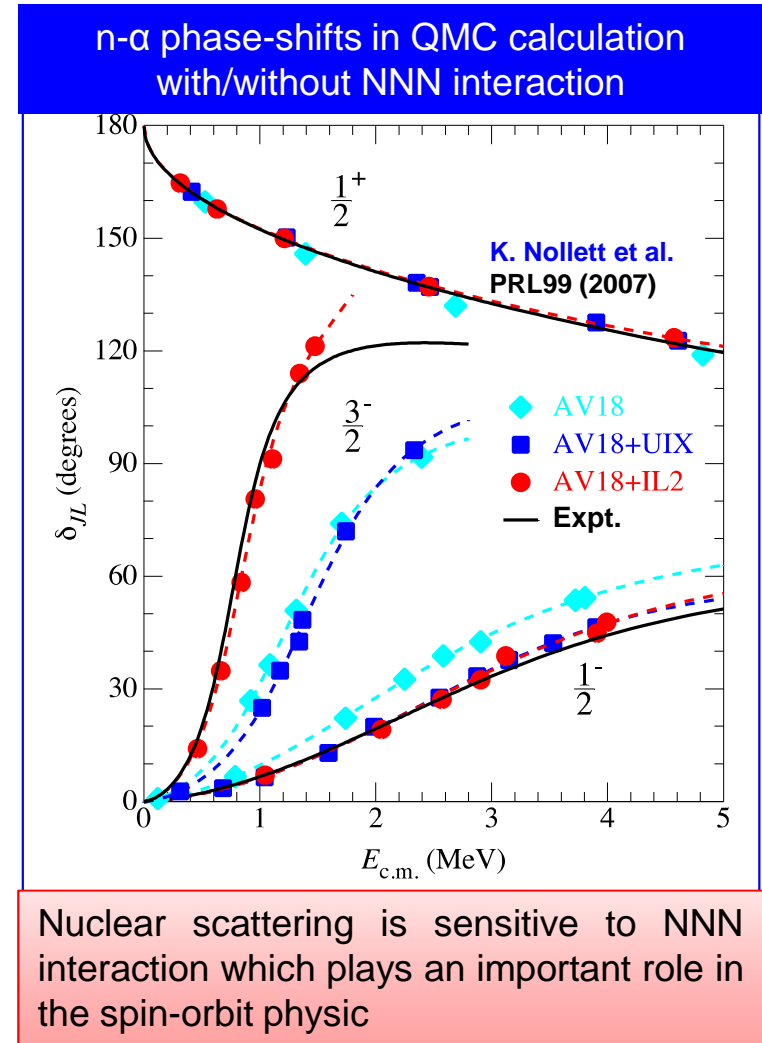
- Exact reaction calculations for very light systems **A=3,4**

- Faddeev / Faddeev-Yacubovsky
- Alt-Grassberger-Sandhas
- Hyperspherical Harmonics, ...



M. Viviani et al.
PRC84 (2011)

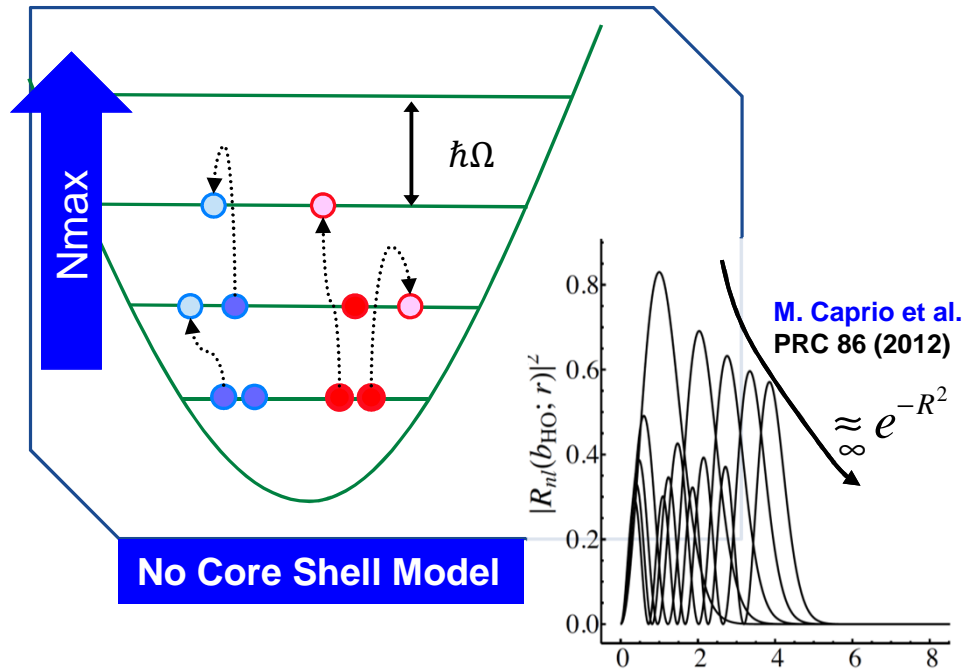
- Now trying to incorporate continuum effects in methods for light nuclei to describe reactions



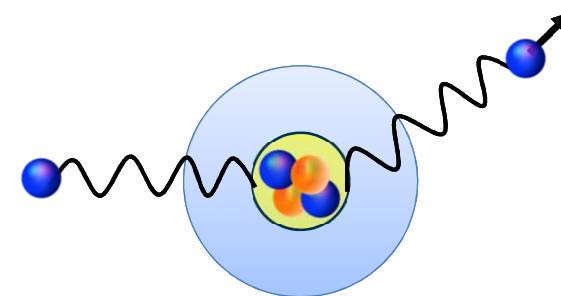
Nuclear scattering is sensitive to NNN interaction which plays an important role in the spin-orbit physic

Why is it *hard* to model nuclear reactions?

If we used Harmonic oscillator states...



... inbound and outbound waves cannot be described by finite number of basis states



For more information on boundary conditions and R-matrix see
[P. Descouvemont, D. Baye Rep.Prog.Phys.73 \(2010\)](#)

From nucleons to nuclear reactions



- **Objective:**

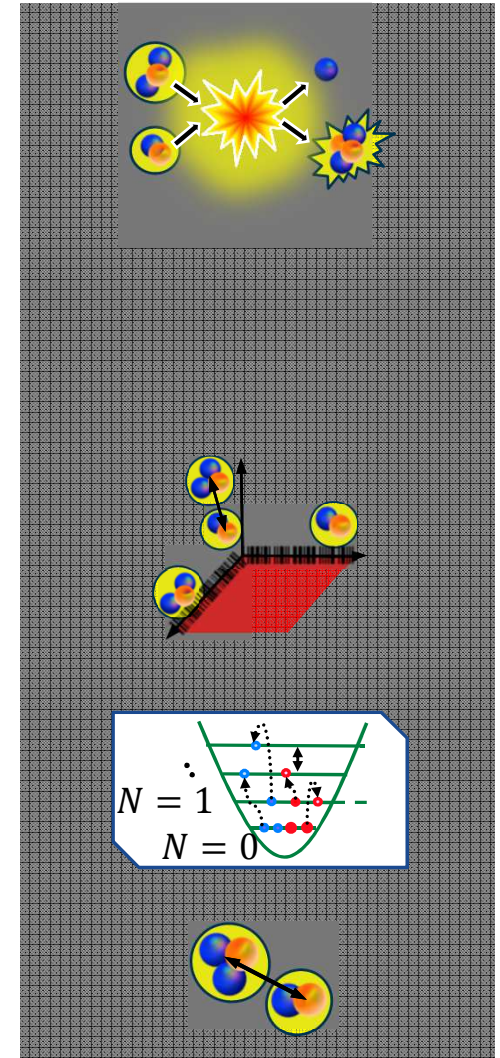
Address static and dynamical properties of light ions and describe fusion reactions.

- **Ingredients**

- High-precision nuclear interaction, two- plus three-nucleon, derived within the Chiral Effective Field Theory (EFT).

- **Recipe**

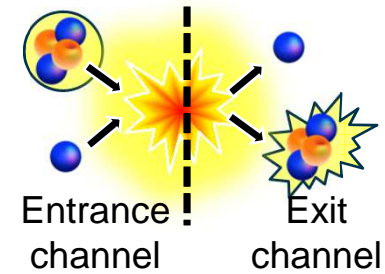
- Solve the many-body Schrödinger equation.
- Address structural properties. (bound states, narrow resonances)
 - *Ab initio* many-body approaches ($A \leq \sim 16$); No-Core Shell Model (NCSM)
- Address dynamical properties. (scattering, reactions)
 - Extend No-Core Shell-Model with the Resonating Group Method (RGM)



Ab initio NCSM/RGM: formalism for binary clusters

S. Quaglioni and P. Navrátil, Phys. Rev. Lett. 101, 092501 (2008); Phys. Rev. C 79, 044606 (2009)

Ex: n-⁴He scattering



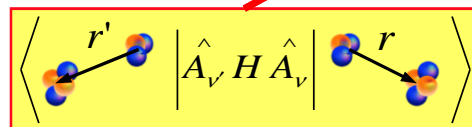
- Starts from:

$$\Psi_{RGM}^{(A)} = \sum_{\nu} \int d\vec{r} g_{\nu}(\vec{r}) \hat{A}_{\nu} \left| \Phi_{\nu r}^{(A-a,a)} \right\rangle$$

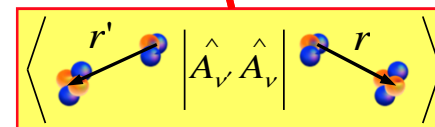
Relative wave function (unknown)
Antisymmetrizer
Channel basis

- Schrödinger equation on channel basis:

$$H \Psi_{RGM}^{(A)} = E \Psi_{RGM}^{(A)} \Rightarrow \sum_{\nu} \int d\vec{r} \left[H_{\nu\nu}(\vec{r}', \vec{r}) - E N_{\nu\nu}(\vec{r}', \vec{r}) \right] g_{\nu}(\vec{r}) = 0$$



Hamiltonian kernel



Norm (overlap) kernel

∞ NCSM densities

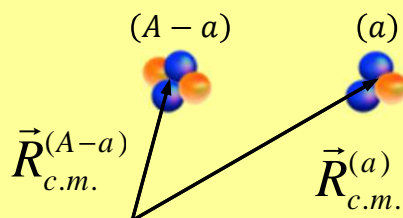
RGM accounts for: 1) interaction (Hamiltonian kernel), 2) Pauli principle (Norm kernel) between clusters and NCSM accounts for: internal structure of clusters

Matrix elements of translationally invariant operators

- Translational invariance is preserved (exactly!) also with SD cluster basis

$${}_{SD} \left\langle \Phi_{f_{SD}}^{(A-a',a')} \left| \hat{O}_{t.i.} \right| \Phi_{i_{SD}}^{(A-a,a)} \right\rangle_{SD} = \sum_{i_R f_R} M_{i_{SD} f_{SD}, i_R f_R} \left\langle \Phi_{f_R}^{(A-a',a')} \left| \hat{O}_{t.i.} \right| \Phi_{i_R}^{(A-a,a)} \right\rangle$$

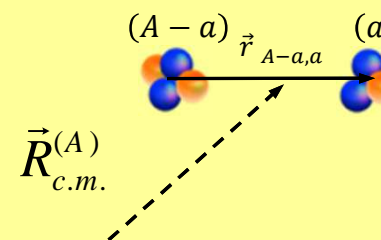
What we calculate in the “SD” channel basis



$$\left| \psi_{\alpha_1}^{(A-a)} \right\rangle_{SD} \left| \psi_{\alpha_2}^{(a)} \right\rangle \varphi_{nl}(\vec{R}_{c.m.}^{(a)})$$

Matrix inversion

Observables calculated in the translationally invariant basis



$$\left| \psi_{\alpha_1}^{(A-a)} \right\rangle \left| \psi_{\alpha_2}^{(a)} \right\rangle \varphi_{nl}(\vec{r}_{A-a,a})$$

- Advantage: can use powerful second quantization techniques

$${}_{SD} \left\langle \Phi_{v'n'}^{(A-a',a')} \left| \hat{O}_{t.i.} \right| \Phi_{vm}^{(A-a,a)} \right\rangle_{SD} \propto {}_{SD} \left\langle \psi_{\alpha_1}^{(A-a')} \left| a^+ a \right| \psi_{\alpha_1}^{(A-a)} \right\rangle_{SD}, {}_{SD} \left\langle \psi_{\alpha_1}^{(A-a')} \left| a^+ a^+ a a \right| \psi_{\alpha_1}^{(A-a)} \right\rangle_{SD}, \dots$$

Going around the hard core problem

E. Jurgenson, Navrátil, R. J. Furnstahl Phys. Rev. Lett. 103 (2009)

In configuration interaction methods we need to soften interaction to address the hard core
 We use the Similarity-Renormalization-Group (SRG) method

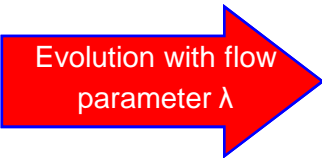
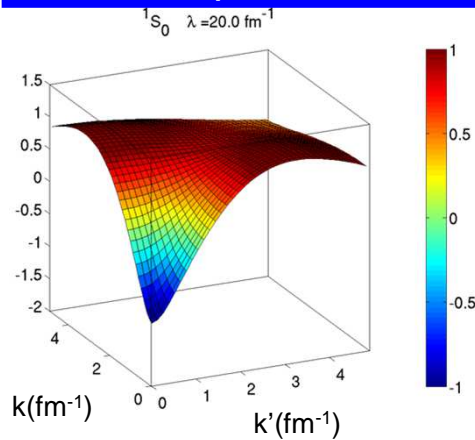
$$H_\lambda = U_\lambda H U_\lambda^\dagger$$

Unitary transformations

$$\left\{ \begin{aligned} \frac{dH_\lambda}{d\lambda} &= [\eta(\lambda), H_\lambda] \\ \eta(\lambda) &= \frac{dU_\lambda}{d\lambda} U_\lambda^\dagger \end{aligned} \right.$$

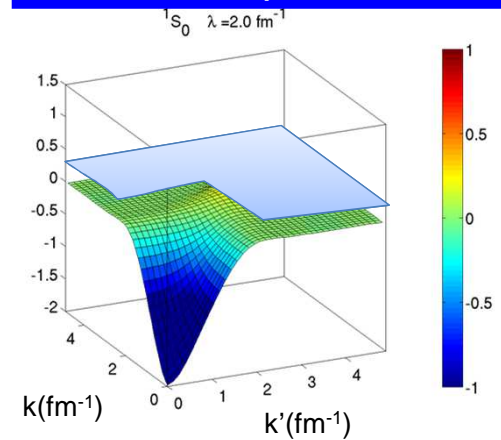
Flow parameter

Bare potential



- Preserves the physics
- Decouples high and low momentum
- Induces many-body forces

Evolved potential

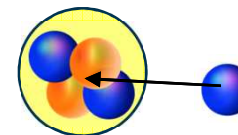
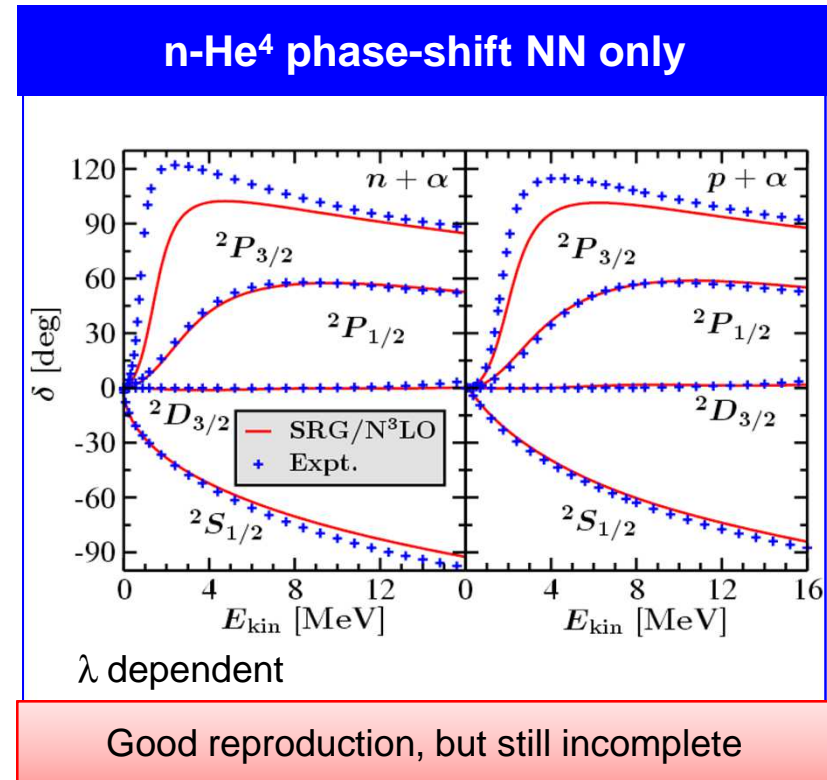


Demonstrated capability to describe binary-cluster reactions starting from NN interactions

- ☑ Nucleon-nucleus collisions
 - ✓ n - ^3H , p - ^3He , N - ^4He , n - ^{10}Be scattering with $N^3\text{LO}$ NN (mod. Lee-Suzuki eff. Int.)
 - ✓ Nucleon scattering on ^3H , $^3,4\text{He}$, ^7Li , ^7Be , ^{12}C , ^{16}O with SRG- $N^3\text{LO}$
 - ✓ $^7\text{Be}(p,\gamma)^8\text{B}$ radiative capture with SRG- $N^3\text{LO}$

- ☑ Deuterium-nucleus collisions
 - ✓ d - ^4He scattering and ^6Li structure with SRG- $N^3\text{LO}$

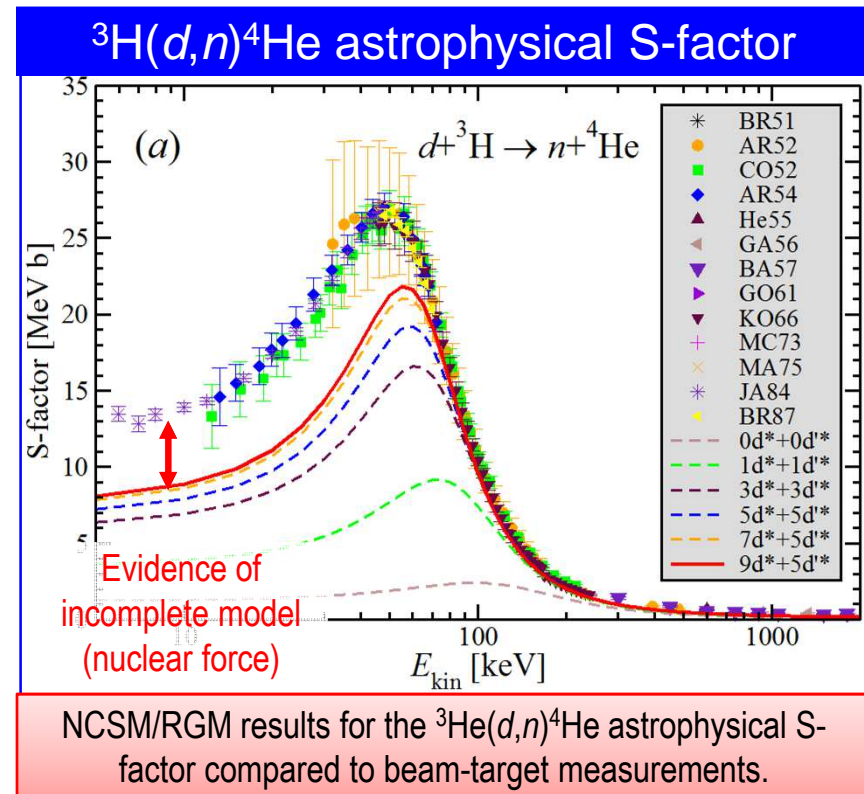
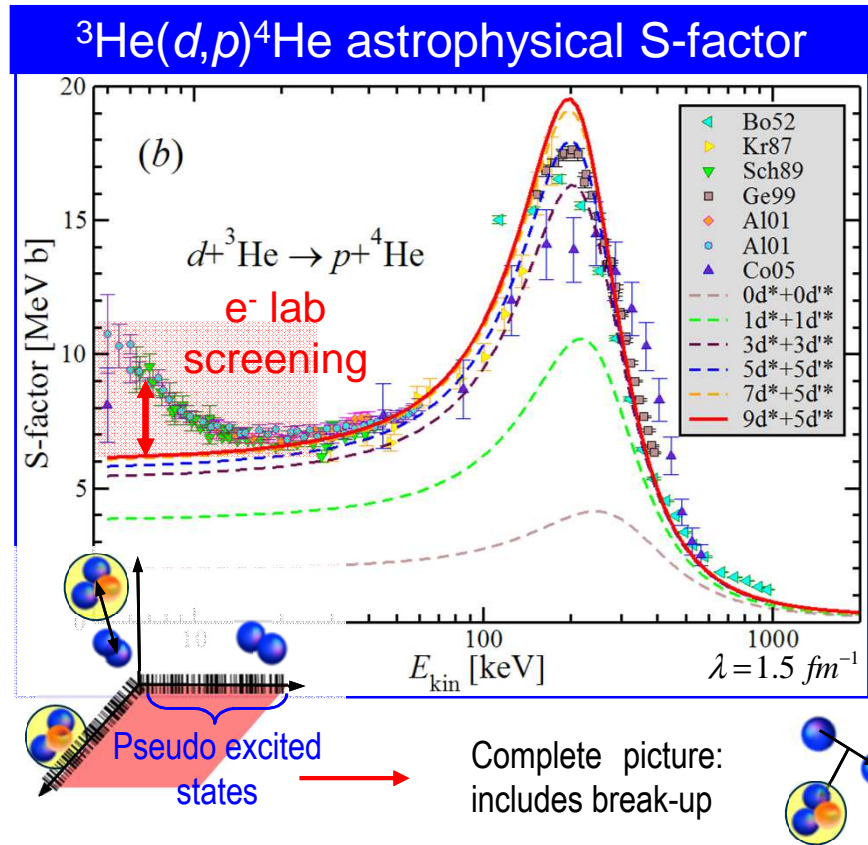
- ☑ (d,N) transfer reactions
 - ✓ $^3\text{H}(d,n)^4\text{He}$ and $^3\text{He}(d,p)^4\text{He}$ reactions with SRG- $N^3\text{LO}$



n on ^4He scattering

Ab initio many-body calculations of the ${}^3\text{H}(d,n){}^4\text{He}$ and ${}^3\text{He}(d,p){}^4\text{He}$ fusion

P. Navrátil, S. Quaglioni, PRL 108, 042503 (2012)



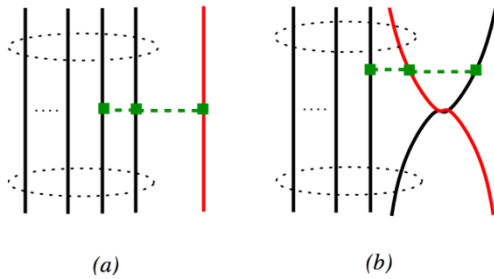
Calculated S-factors converge with the inclusion of the virtual breakup of the deuterium, obtained by means of excited 3S_1 - 3D_1 (d^*) and 3D_2 (d^{**}) pseudo-states.

Incomplete nuclear interaction: requires NNN force (SRG-induced + “real”)

Including the NNN force into the NCSM/RGM approach nucleon-nucleus formalism

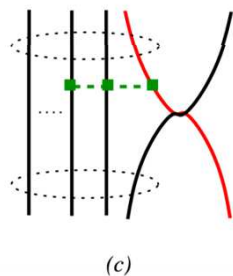
$$\left\langle \Phi_{\nu r'}^{J\pi T} \left| \hat{A}_{\nu'} V^{NNN} \hat{A}_{\nu} \right| \Phi_{\nu r}^{J\pi T} \right\rangle = \left\langle \begin{array}{c} (A-1) \\ \text{---} \\ r' \\ (a'=1) \end{array} \left| V^{NNN} \left(1 - \sum_{i=1}^{A-1} \hat{P}_{iA} \right) \right| \begin{array}{c} (A-1) \\ \text{---} \\ r \\ (a=1) \end{array} \right\rangle$$

$$\mathcal{V}_{\nu'\nu}^{NNN}(r, r') = \sum R_{n'l'}(r') R_{nl}(r) \left[\frac{(A-1)(A-2)}{2} \langle \Phi_{\nu'n'}^{J\pi T} | V_{A-2A-1A} (1 - 2P_{A-1A}) | \Phi_{\nu n}^{J\pi T} \rangle \right.$$



Direct potential:

$$\propto_{SD} \left\langle \psi_{\alpha_1}^{(A-1)} \left| a_i^+ a_j^+ a_l a_k \right| \psi_{\alpha_1}^{(A-1)} \right\rangle_{SD}$$

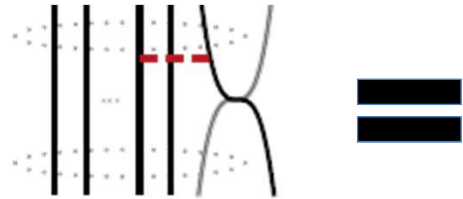


Exchange potential:

$$\propto_{SD} \left\langle \psi_{\alpha_1}^{(A-1)} \left| a_h^+ a_i^+ a_j^+ a_m a_l a_k \right| \psi_{\alpha_1}^{(A-1)} \right\rangle_{SD}$$

$$- \frac{(A-1)(A-2)(A-3)}{2} \left\langle \Phi_{\nu'n'}^{J\pi T} \left| P_{A-1A} V_{A-3A-2A-1} \right| \Phi_{\nu n}^{J\pi T} \right\rangle \cdot$$

We are following two ways of handling the three-body density



+ efficient for addressing different projectile
 + coupled scheme (makes a good use of Racah algebra)

$$\sum_{j_0 t_0} \sum_{j'_0 t'_0} \sum_{n_\alpha l_\alpha j_\alpha} \sum_{n'_\alpha l'_\alpha j'_\alpha} \sum_{n_b l_b j_b} \sum_{n'_b l'_b j'_b} \sum_{g' t'_g} \frac{1}{12} \hat{j}_0 \hat{T}_0 \hat{g}' \hat{t}'_g (-$$

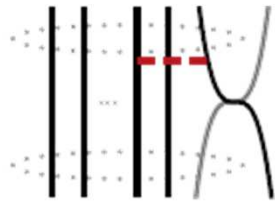
$$\left\{ \begin{matrix} I_\beta & g' & I'_1 \\ J_0 & j'_0 & j' \\ J_1 & j & J \end{matrix} \right\} \left\{ \begin{matrix} T_\beta & t'_g & T'_1 \\ T_0 & t'_0 & \frac{1}{2} \\ T_1 & \frac{1}{2} & T \end{matrix} \right\}$$

$$\langle \phi^{(A-1)} I'_1 T'_1 \| (a_{n_j}^\dagger (a_{n'_b l'_b j'_b}^\dagger a_{n'_\alpha l'_\alpha j'_\alpha}^\dagger)_{j'_0 t'_0}^{g' t'_g} \| \phi^{(A-4)} I_\beta T_\beta \rangle$$

$$\langle \phi^{(A-4)} I_\beta T_\beta \| ((\tilde{a}_{n_\alpha l_\alpha j_\alpha} \tilde{a}_{n_\alpha l_\alpha j_\alpha})_{j_0 t_0} \tilde{a}_{n_b l_b j_b})_{j_0 T_0} \| \phi^{(A-1)} I_1 T_1 \rangle$$

$${}_a \langle ((n'_\alpha l'_\alpha j'_\alpha, n'_b l'_b j'_b)_{j'_0 t'_0}, n' l' j')_{J_0 T_0} | V_{3N} | ((n_\alpha l_\alpha j_\alpha, n_\alpha l_\alpha j_\alpha)_{j_0 t_0}, n_b l_b j_b)_{J_0 T_0} \rangle_a$$

We are following two ways of handling the three-body density



=

- + acces to heavier targets
- + no averaged isospin
- + perfectly parallel

$$\sum_{jj'} \sum_{M_1 m_j} \sum_{M_{T_1} m_t} \sum_{M'_1 m'_j} \sum_{M'_{T_1} m'_t} \frac{1}{12} (-1)^{I_1 + I'_1 +}$$

$$\begin{pmatrix} I_1 & j & J \\ M_1 & m_j & M_J \end{pmatrix} \begin{pmatrix} T_1 & \frac{1}{2} & T \\ M_{T_1} & m_t & M_T \end{pmatrix}$$

treatment of M_J & M_T q.n.
new computational scheme
& infrastructure

$$\sum_{\beta_{A-3}} \sum_{\beta_{A-2}} \sum_{\beta'_{A-3}} \sum_{\beta'_{A-2}} \sum_{\beta'_{A-1}}$$

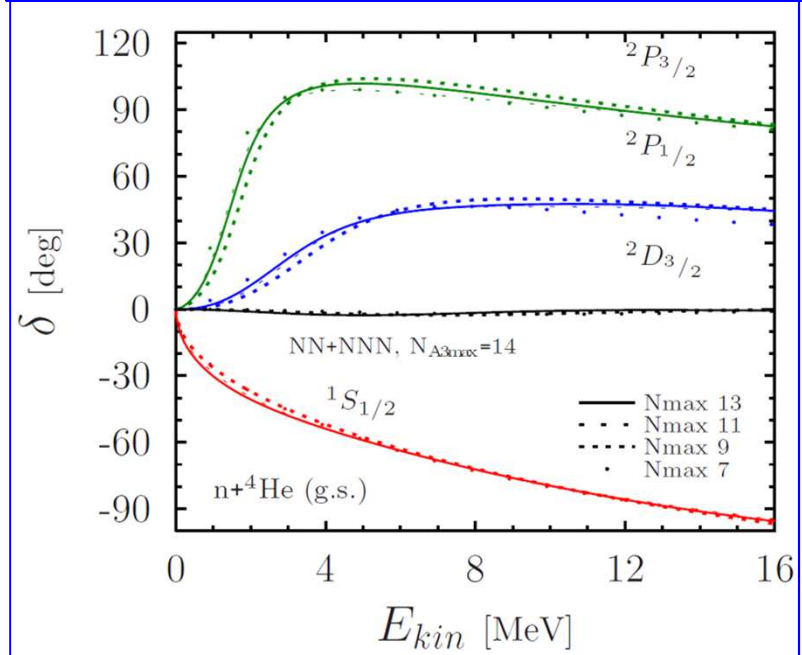
$$\langle \phi^{(A-1)}_{I'_1 M'_1 T'_1 M'_{T_1}} | \alpha_{n l j m_j \frac{1}{2} m_t}^\dagger \alpha_{\beta_{A-2}}^\dagger \alpha_{\beta_{A-3}}^\dagger \alpha_{\beta'_{A-3}} \alpha_{\beta'_{A-2}} \alpha_{\beta'_{A-1}} | \phi^{(A-1)}_{I_1 M_1 T_1 M_{T_1}} \rangle$$

$$\alpha \langle \beta_{A-3} \beta_{A-2} n' l' j' m'_j \frac{1}{2} m'_t | V_{3N} | \beta'_{A-3} \beta'_{A-2} \beta'_{A-1} \rangle \alpha$$

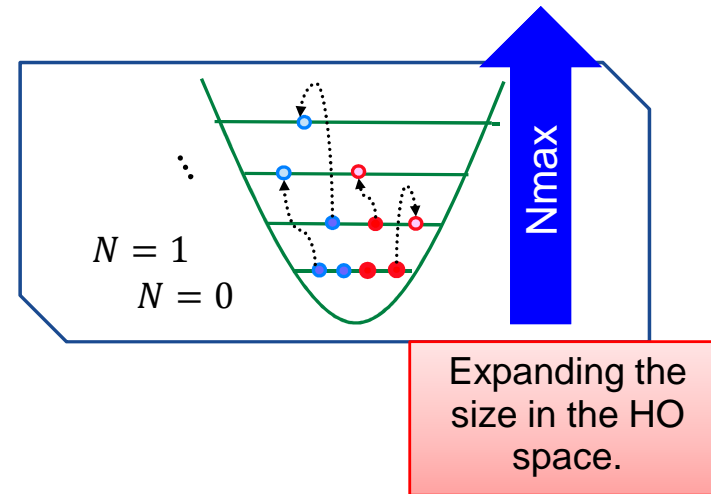
n-⁴He scattering: study of the model space convergence

G. Hupin, J. Langhammer, S. Quaglioni, P. Navrátil, R. Roth, work in progress

Convergence with respect to the number of HO major shells



Convergence of the phase-shifts as a function of the HO basis size.

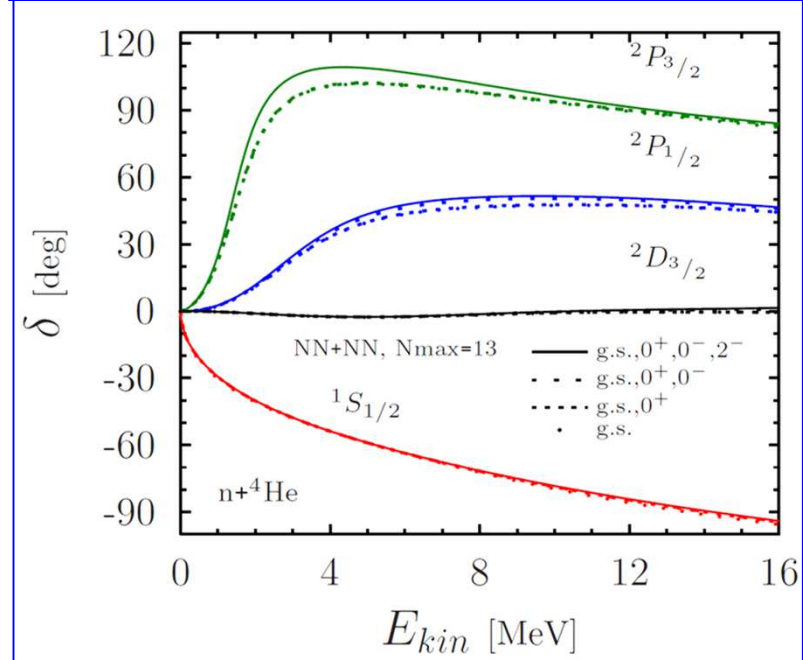


- Calculations are done using the chiral EFT interaction at N³LO for the NN and N²LO for the NNN.
- For this study, we take $\hbar\omega=20$
- The SRG flow parameter is $\lambda=2.0$
- Convergence pattern is good.

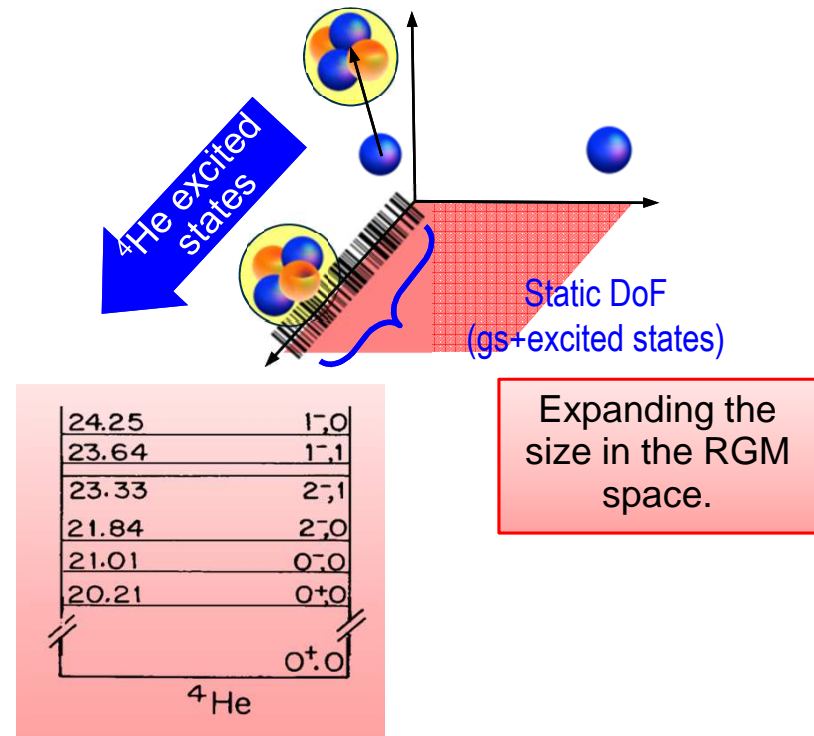
n-⁴He scattering: study of the RGM convergence in the NNN case

G. Hupin, J. Langhammer, S. Quaglioni, P. Navrátil, R. Roth, work in progress

NNN case: convergence with respect to the many body space, first excited states



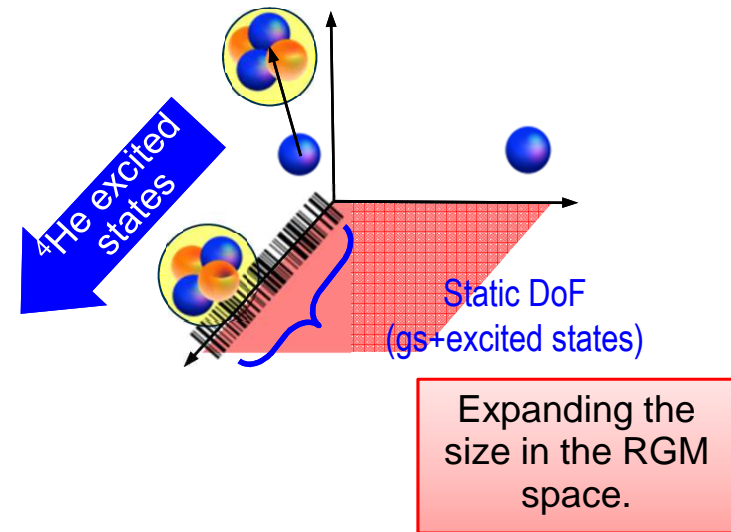
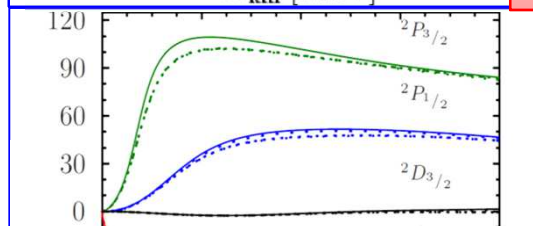
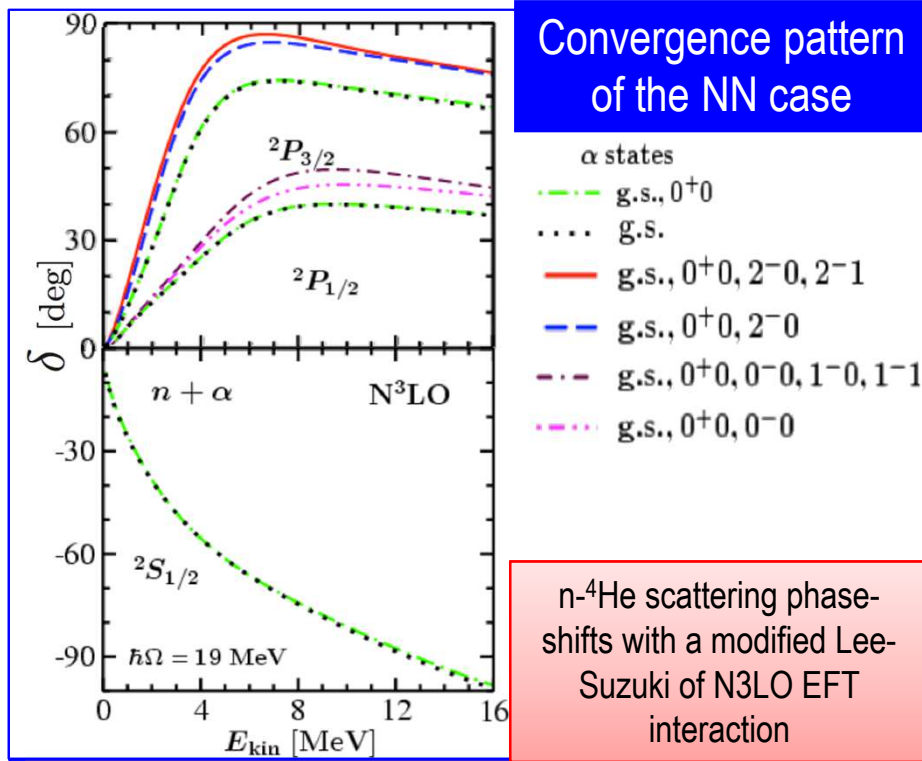
Convergence of the phase-shifts as a function of the ⁴He excited states.



• A systematic exploration of the Nmax and # of target eigenstates is ongoing.

n-⁴He scattering: study of the RGM convergence in the NN case

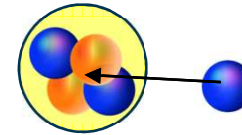
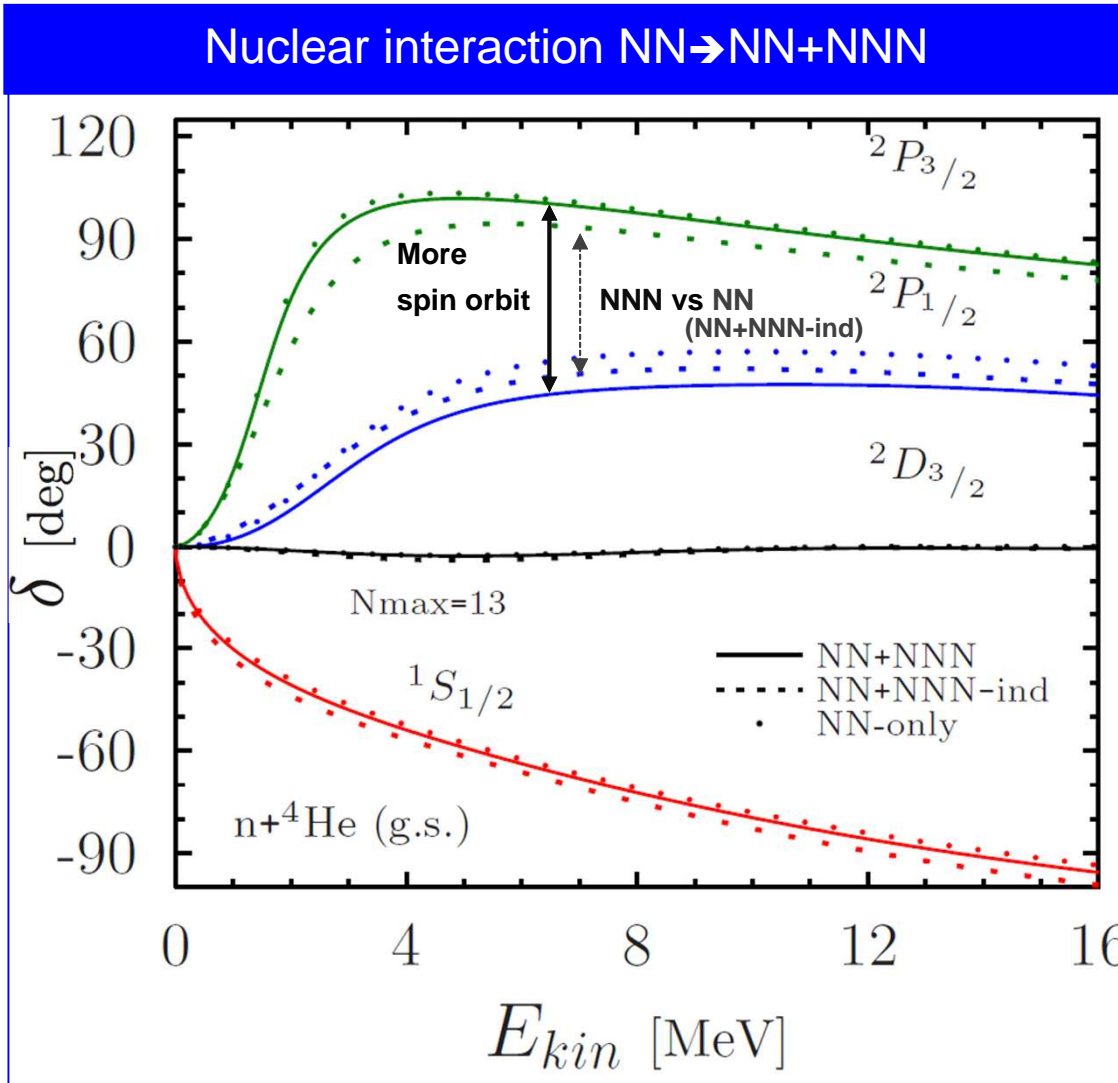
P. Navrátil and S. Quaglioni, Phys. Rev. C 83 044609, (2011)



- The phase-shifts are sensitive to the inclusion of the first six excited states of ⁴He.
- The convergence pattern of the NNN-full is similar.

n-⁴He scattering: NN versus NNN interactions, first results

G. Hupin, J. Langhammer, S. Quaglioni, P. Navrátil, R. Roth, work in progress



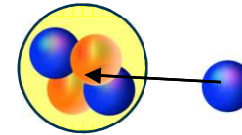
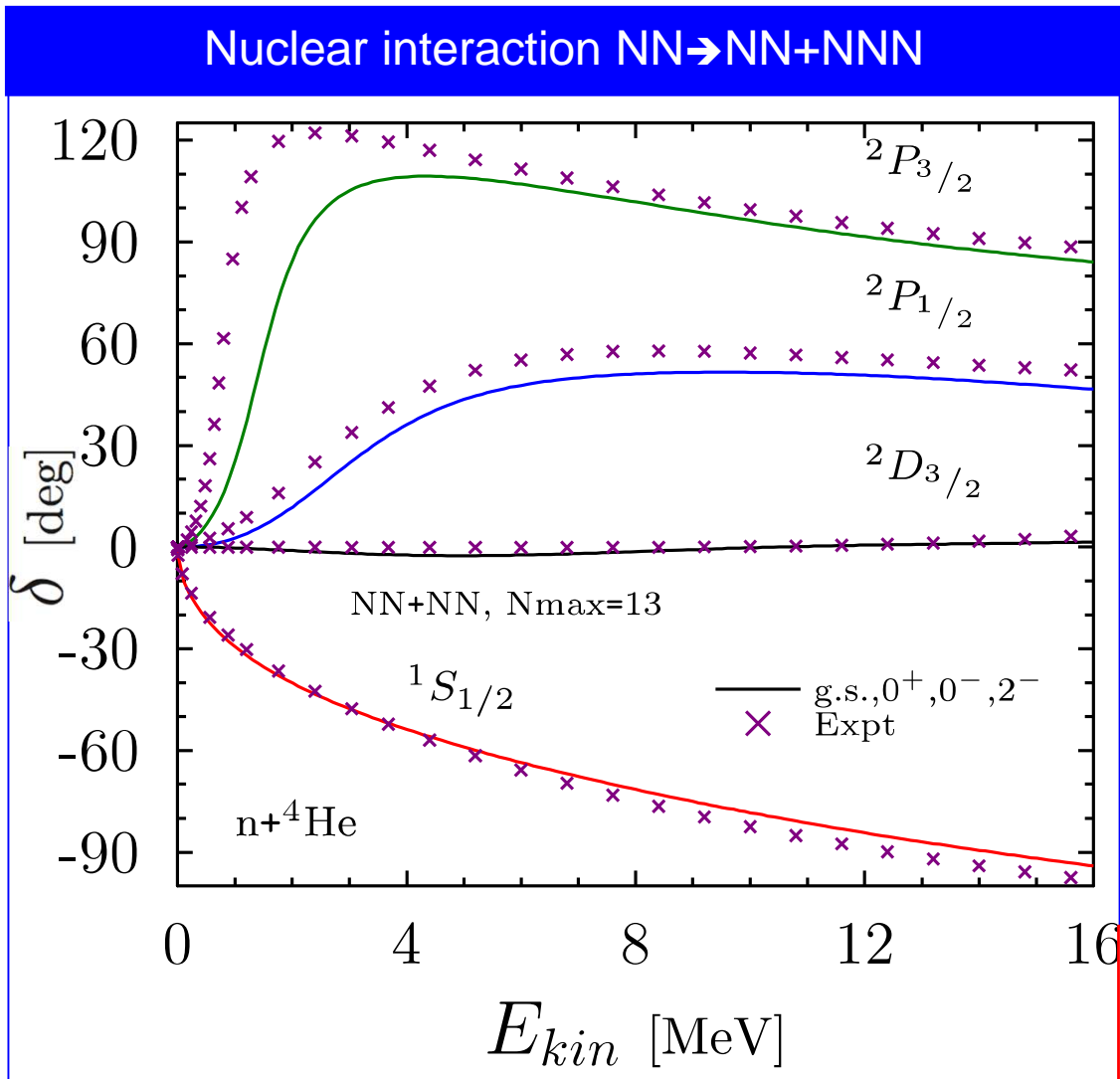
n-⁴He scattering

- The largest splitting between *P* waves is obtained with NN+NNN.

Comparison between NN, NN+NNN-ind and NN+NNN at Nmax=13

n-⁴He scattering: NN+NNN with the first three excited states

G. Hupin, J. Langhammer, S. Quaglioni, P. Navrátil, R. Roth, work in progress



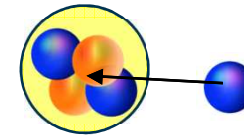
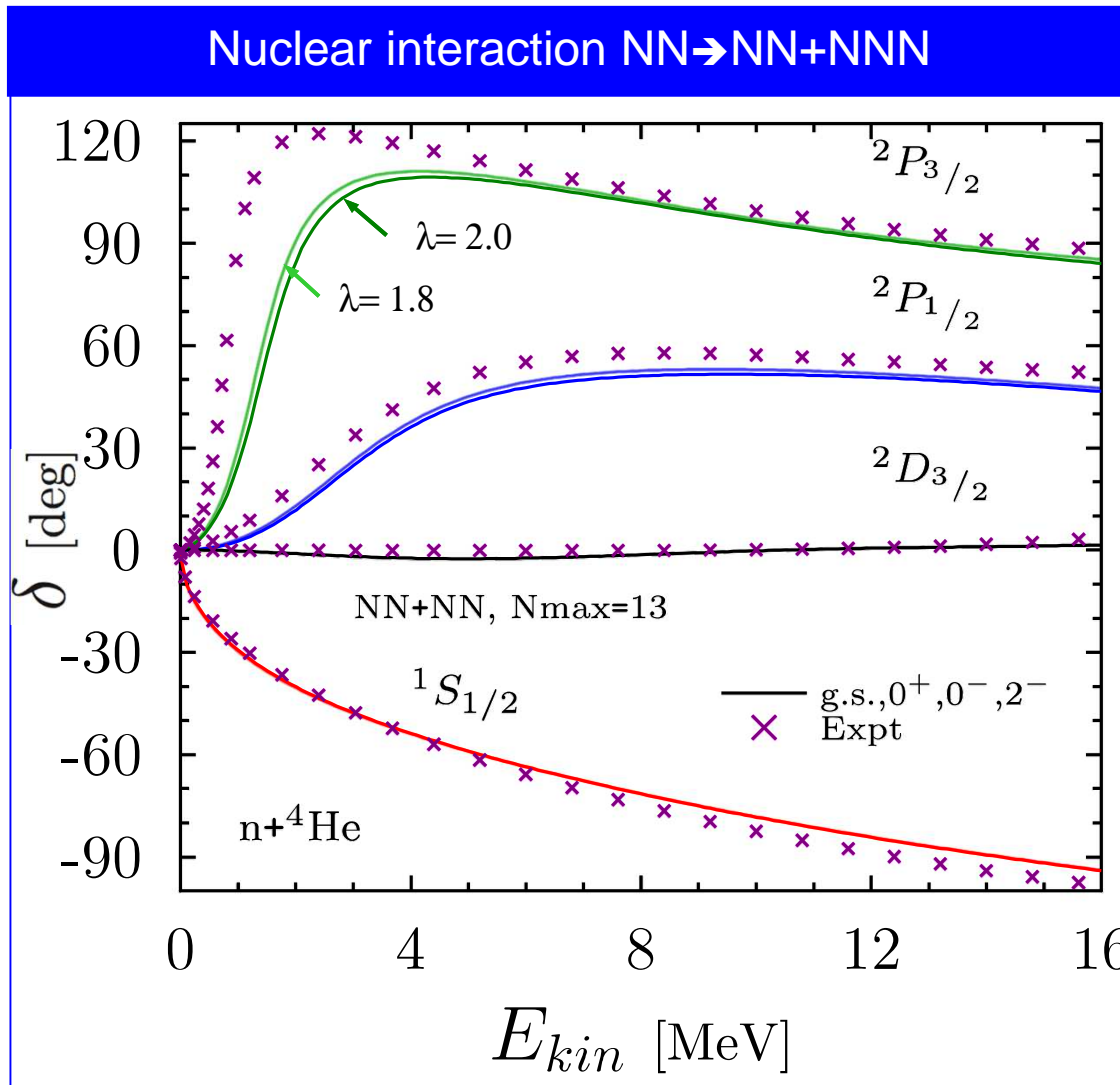
n on ⁴He scattering

- The largest splitting between *P* waves is obtained with NN+NNN.
- We expect a better reproduction of data when all the first six excited state of ⁴He included (ongoing study).
- The present NNN force is incomplete (N^2LO only).

Comparison between NN+NNN and experiment at $N_{max}=13$ and $\lambda=2.0$

Preliminary results for the lambda dependence

G. Hupin, J. Langhammer, S. Quaglioni, P. Navrátil, R. Roth, work in progress



n on ⁴He scattering

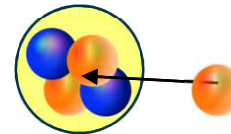
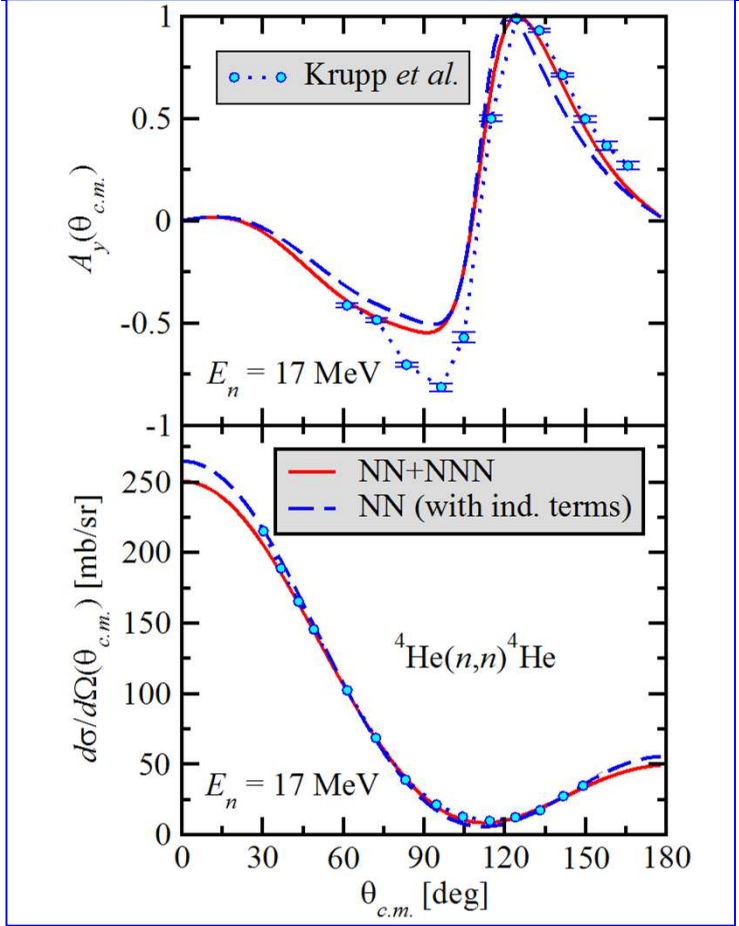
- The SRG evolution is to a good extend unitary.
- A wider range of λ value needs to be explored before a definite conclusion.
- The last sixth excited states of ⁴He should be included to compare converged results

Comparison with NN+NNN at Nmax=13 between $\lambda=2.0$ and 1.8

Cross section and p-⁴He scattering

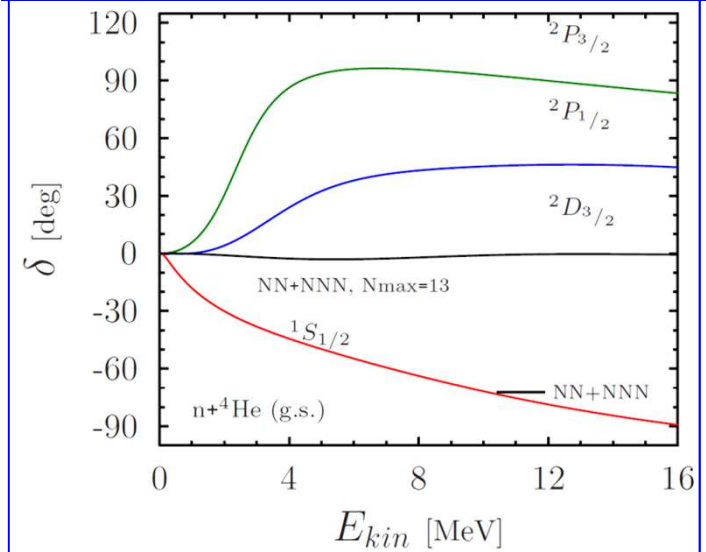
G. Hupin, J. Langhammer, S. Quaglioni, P. Navrátil, R. Roth, work in progress

Analyzing power and n-⁴He cross section



p-⁴He scattering

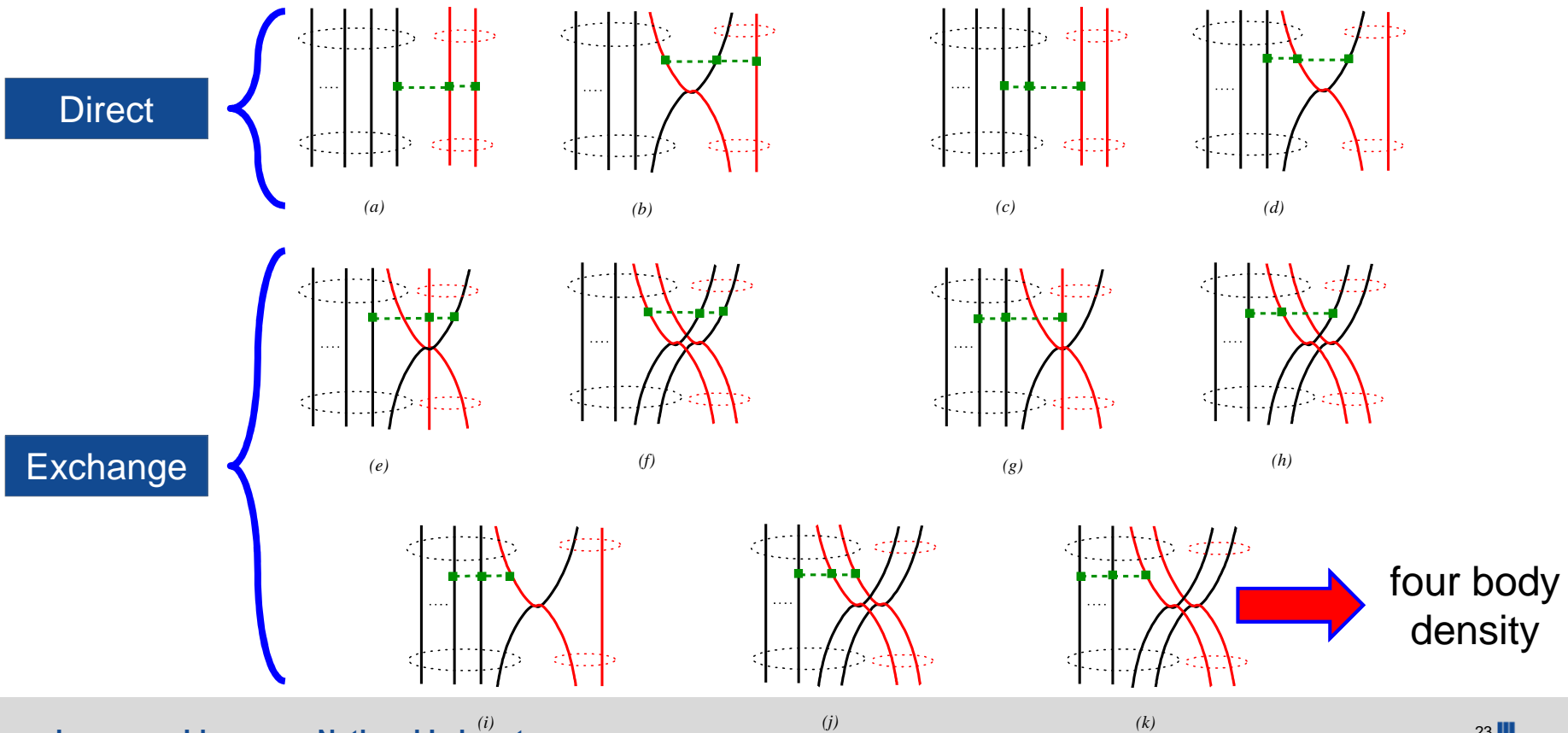
p-⁴He phase-shifts



The breaking of isospin symmetry was tested in the p-⁴He scattering

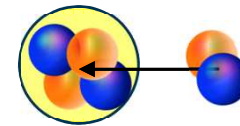
Including the NNN force into the NCSM/RGM approach deuteron-nucleus formalism

$$\left\langle \Phi_{v'r'}^{J\pi T} \left| \hat{A}_{v'} V^{NNN} \hat{A}_v \right| \Phi_{vr}^{J\pi T} \right\rangle = \left\langle \begin{array}{c} (A-2) \\ \text{diagram} \\ (a=2) \end{array} \right| V^{NNN} \left(1 - \sum_{i=1}^{A-2} \sum_{k=A-1}^A \hat{P}_{i,k} + \sum_{i < j=1}^{A-2} \hat{P}_{i,A-1} \hat{P}_{j,A} \right) \left| \begin{array}{c} (A-2) \\ \text{diagram} \\ (a=2) \end{array} \right\rangle$$

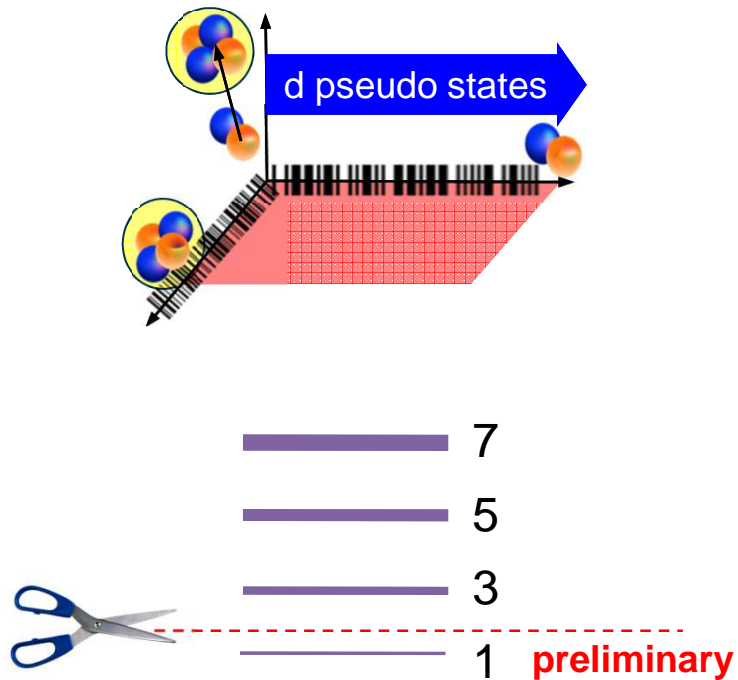


${}^4\text{He}(d,d){}^4\text{He}$ with NN+NNN interaction

G. Hupin, S. Quaglioni, P. Navratil, work in progress

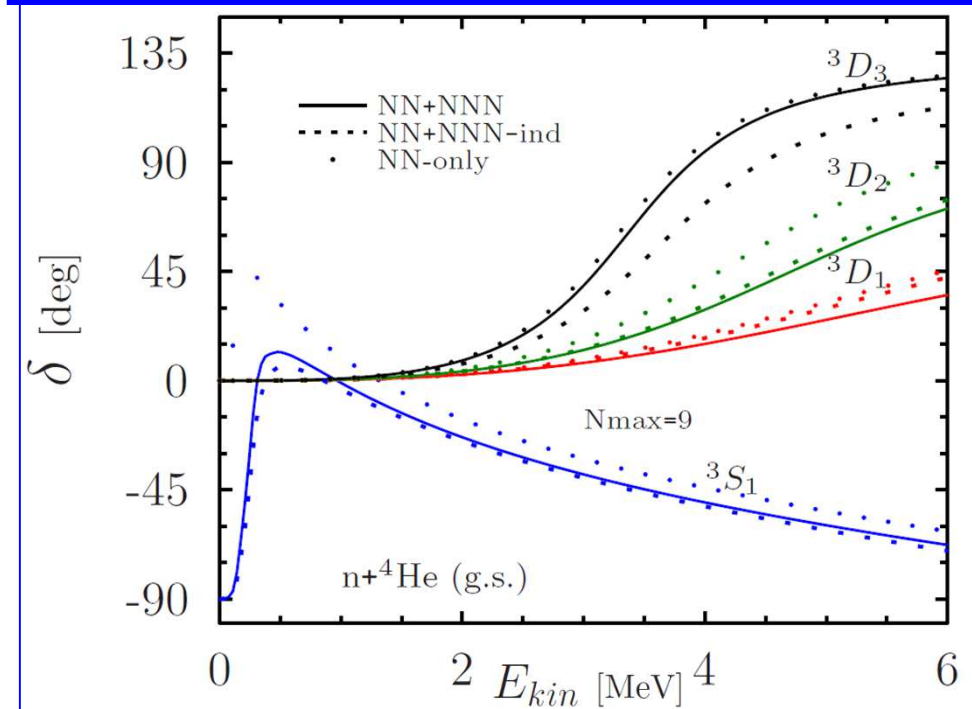


d- ${}^4\text{He}$
scattering



Pseudo-states
in each channel
 $d(\text{g.s.}), {}^3S_1, {}^3D_1, {}^3D_2, {}^3D_3, {}^3G_3$

Phase shifts with $\lambda = 2 \text{ fm}^{-1}$

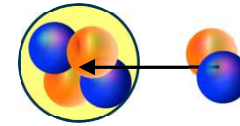


$d(\text{g.s.}) + {}^4\text{He}(\text{g.s.})$ scattering phase-shifts for NN, NN+NNN-induced and NN+NNN potential with $\lambda = 2 \text{ fm}^{-1}$.

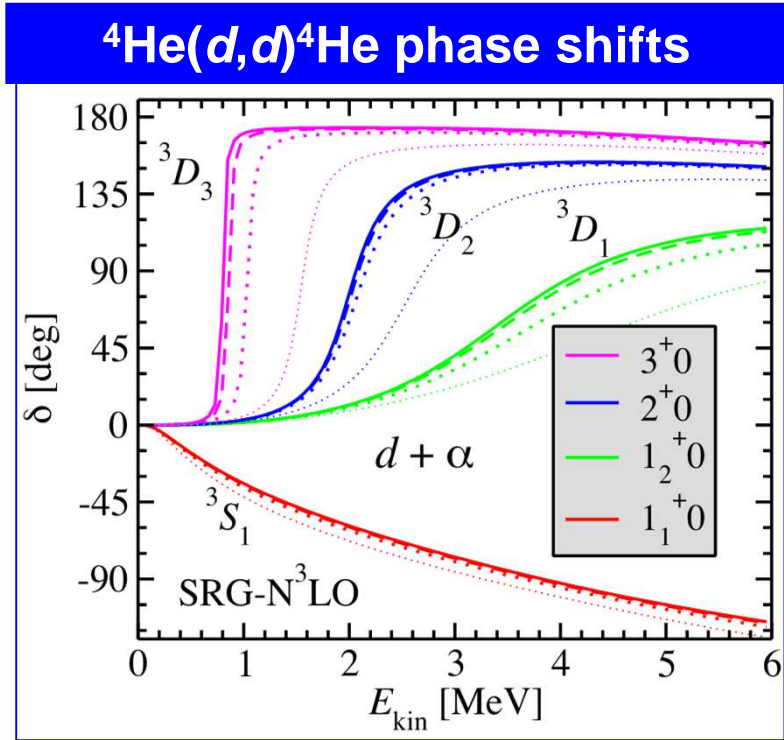
Preliminary results in a small model space and with only d and ${}^4\text{He}$ g.s., look promising

$^4\text{He}(d,d)^4\text{He}$ with NN-only

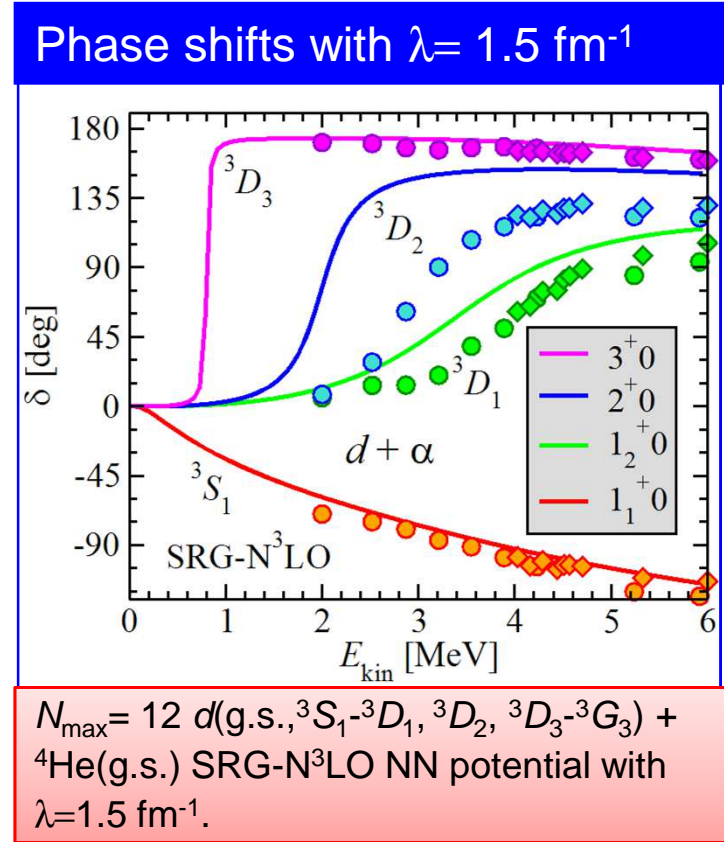
S. Quaglioni and P. Navratil



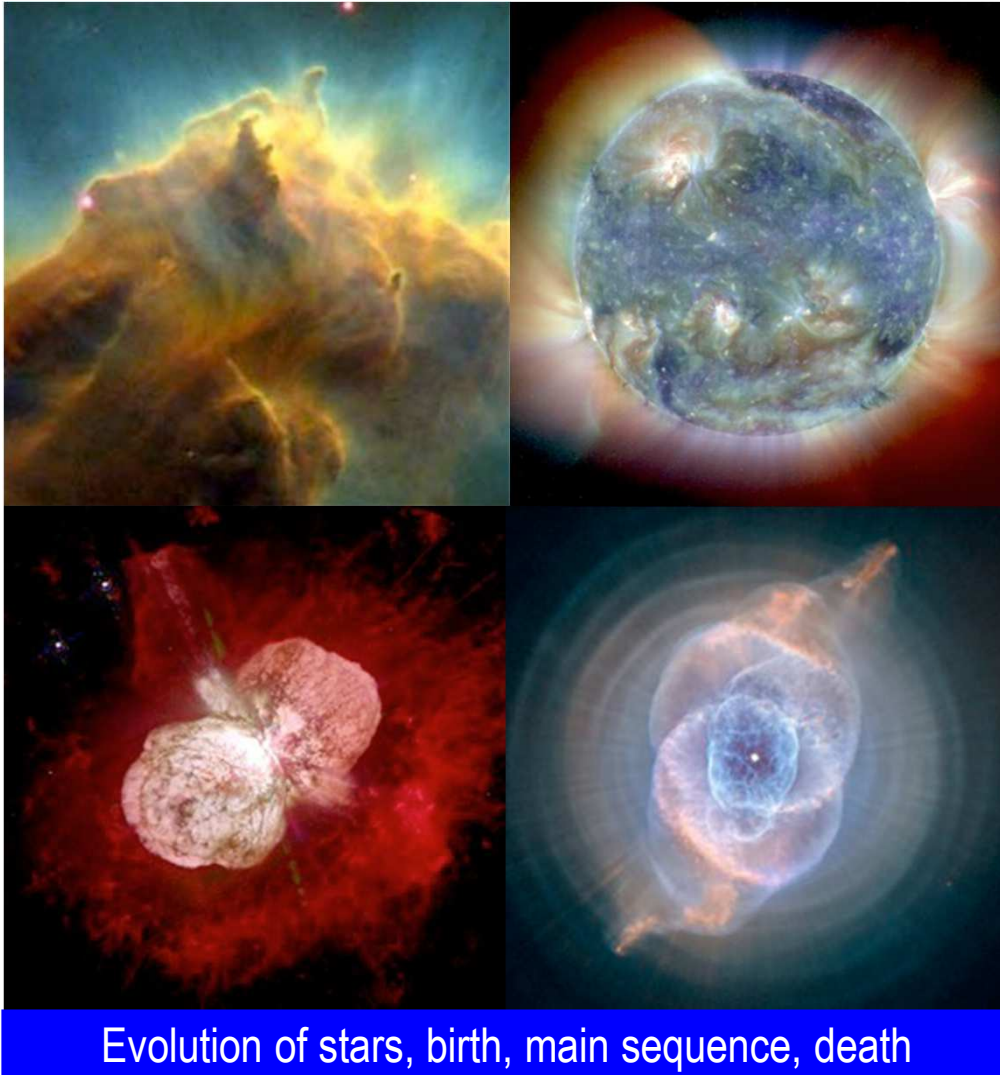
$d\text{-}^4\text{He}$
scattering



- 7
 - - - - - 5
 - 3
 - 1
- Pseudo-states
in each channel



Conclusions and Outlook



- We are extending the *ab initio* NCSM/RGM approach to describe low-energy reactions with two- and three-nucleon interactions.
- We are able to describe:
 - Nucleon-nucleus collisions with NN+NNN interaction
 - Deuterium-nucleus collisions with NN+NNN interaction
- Work in progress
 - The present NNN force is "incomplete", need to go to N³LO
 - Before definite conclusion
 - Study of λ dependence
 - Study of $\hbar\omega$ dependence
 - Scattering of heavier target

Ab initio NCSM/RGM Formalism for binary clusters

a few details

$$|\Psi^{J^{\pi T}}\rangle = \sum_{\nu} \int \frac{g_{\nu}^{J^{\pi T}}(r)}{r} \hat{A}_{\nu} \left[\left(|A-a \alpha_1 I_1^{\pi_1} T_1\rangle |a \alpha_2 I_2^{\pi_2} T_2\rangle \right)^{(sT)} Y_{\ell}(\hat{r}_{A-a,a}) \right]^{(J^{\pi T})} \frac{\delta(r-r_{A-a,a})}{rr_{A-a,a}} r^2 dr$$

Relative wave functions subject to the boundary/scattering asymptotic solution within R-matrix theory

$|\Phi_{\nu r}^{J^{\pi T}}\rangle$ (Jacobi) channel basis

We use the closure properties of HO radial wave function

$$\delta(r-r_{A-a,a}) = \sum_n R_{n\ell}(r) R_{n\ell}(r_{A-a,a})$$

We defined the RGM model space such that $n < N_{\max}$, this expansion is good for localized parts of the integration kernels.

Since we are using NCSM wave functions, it is convenient to introduce Jacobi channel states in the HO space

$$|\Phi_{\nu n}^{J^{\pi T}}\rangle = \left[\left(|A-a \alpha_1 I_1^{\pi_1} T_1\rangle |a \alpha_2 I_2^{\pi_2} T_2\rangle \right)^{(sT)} Y_{\ell}(\hat{r}_{A-a,a}) \right]^{(J^{\pi T})} R_{n\ell}(r_{A-a,a})$$

The coordinate space channel states are given by

$$|\Phi_{\nu r}^{J^{\pi T}}\rangle = \sum_n R_{n\ell}(r) |\Phi_{\nu n}^{J^{\pi T}}\rangle$$

Matrix elements of translationally invariant operators

Then the SD channel states are defined such that the eigenstates of the heaviest of the two clusters (target) are described by a SD wave function:

$$\left| \Phi_{\nu n}^{J^{\pi T}} \right\rangle_{SD} = \left[\left(\left| A-a \alpha_1 I_1^{\pi_1} T_1 \right\rangle_{SD} \left| a \alpha_2 I_2^{\pi_2} T_2 \right\rangle \right)^{(sT)} Y_{\ell} \left(\hat{R}_{c.m.}^{(a)} \right) \right]^{(J^{\pi T})} R_{n\ell} \left(R_{c.m.}^{(a)} \right)$$

$$\left| A-a \alpha_1 I_1^{\pi_1} T_1 \right\rangle \varphi_{00} \left(\vec{R}_{c.m.}^{(A-a)} \right)$$

Vector proportional to the c.m. coordinate of the $A-a$ nucleons

Vector proportional to the c.m. coordinate of the a nucleons

In the case of the nucleon-nucleus system we can applied the following basis change

$$\left| \Phi_{\nu n}^{J^{\pi T}} \right\rangle_{SD} = \sum_j \hat{s}_j^{\hat{j}} (-1)^{I_1+J+j} \left\{ \begin{matrix} I_1 & \frac{1}{2} & s \\ \ell & J & j \end{matrix} \right\}$$

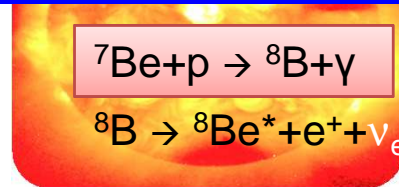
This basis is convenient to express the kernels with the help of second quantization.

$$\times \left[\left| A-1 \alpha_1 I_1^{\pi_1} T_1 \right\rangle_{SD} \varphi_{n\ell j \frac{1}{2}} \left(\vec{r}_A \sigma_A \tau_A \right) \right]^{(J^{\pi T})}$$

Ab initio many-body calculation of the ${}^7\text{Be}(p,\gamma){}^8\text{B}$ radiative capture

P. Navrátil, R. Roth, and S. Quaglioni, Phys. Lett. B704, 379 (2011)

Footprints of pp chain on earth



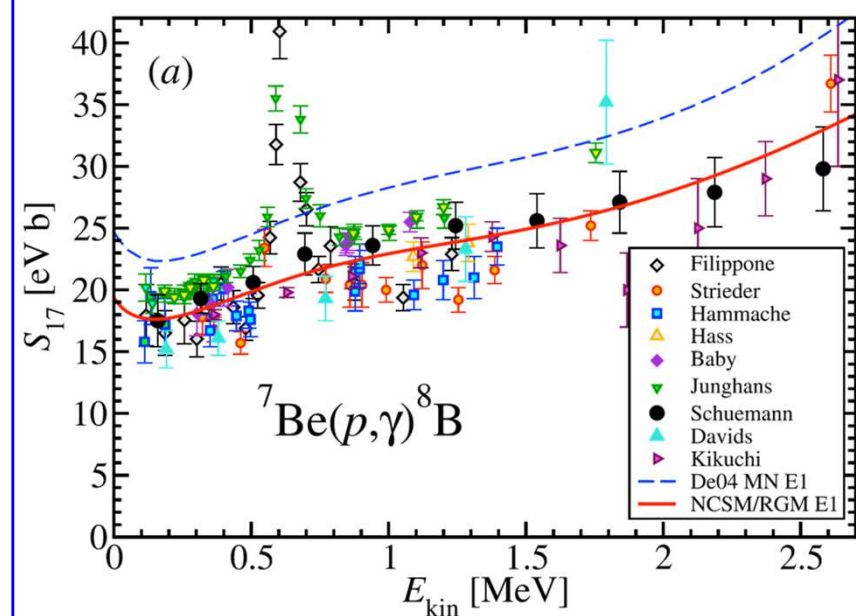
solar neutrinos
 $E_n < 15 \text{ MeV}$



The ${}^7\text{Be}(p,\gamma){}^8\text{B}$ is the final step in the nucleosynthetic chain leading to ${}^8\text{B}$ and one of the main inputs of the standard model of solar neutrinos

- ~10% error in latest $S_{17}(0)$: dominated by uncertainty in theoretical models
- NCSM/RGM results with largest realistic model space ($N_{\text{max}} = 10$):
 - $p+{}^7\text{Be}(\text{g.s.}, 1/2^-, 7/2^-, 5/2_1^-, 5/2_2^-)$
 - Siegert's E1 transition operator
- Parameter λ of SRG NN interaction chosen to reproduce separation energy: 136 keV (Expt. 137 keV)
- $S_{17}(0) = 19.4(7) \text{ eVb}$ on the lower side of, but consistent with latest evaluation

${}^7\text{Be}(p,\gamma){}^8\text{B}$ astrophysical S-factor



Ab initio theory predicts simultaneously both normalization and shape of S_{17}

Astrophysical S-factor:

$$\sigma(E) = \frac{S(E)}{E} \exp\left(-\frac{2\pi Z_1 Z_2 e^2}{\hbar \sqrt{2mE}}\right)$$