Towards an effective relativistic density functional for dense matter in supernovae and compact stars

Stefan Typel
GSI Helmholtzzentrum für Schwerionenforschung, Darmstadt
Nuclear Astrophysics Virtual Institute

Astrophysics and Nuclear Structure

International Workshop XLI on Gross Properties of Nuclei and Nuclear Excitations
January 26 – February 1, 2013
Hirschegg, Kleinwalsertal, Austria

Outline

- **Introduction**
  Astrophysics and Equation of State, Nuclear and Stellar Matter,
  Constraints, Correlations, Relativistic Density Functional
- **Nuclear Correlations in Matter**
  Generalized Relativistic Density Functional, Light and Heavy Clusters,
  Low-Density Limit, Scattering Correlations, Neutron Matter
- **Coulomb Correlations in Matter**
  Coulomb Interaction in Matter, One-Component Plasma,
  Gas/Liquid Phase, Solid Phase
- **Summary**

Introduction

Astrophysics and Equation of State

- essential ingredient in astrophysical model calculations:

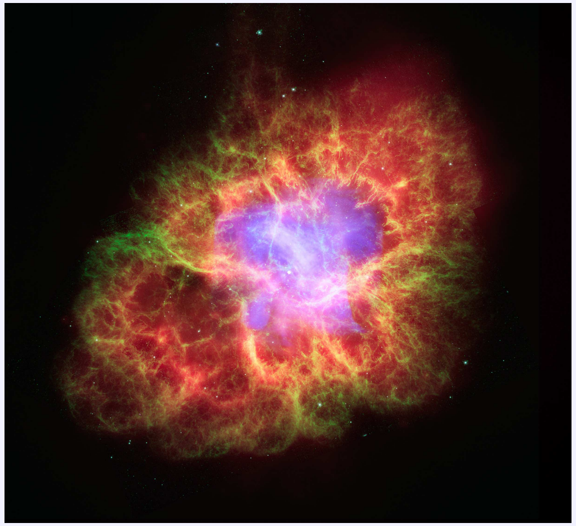
  Equation(s) of State (EoS) of dense matter

  ⇒ dynamical evolution of supernovae

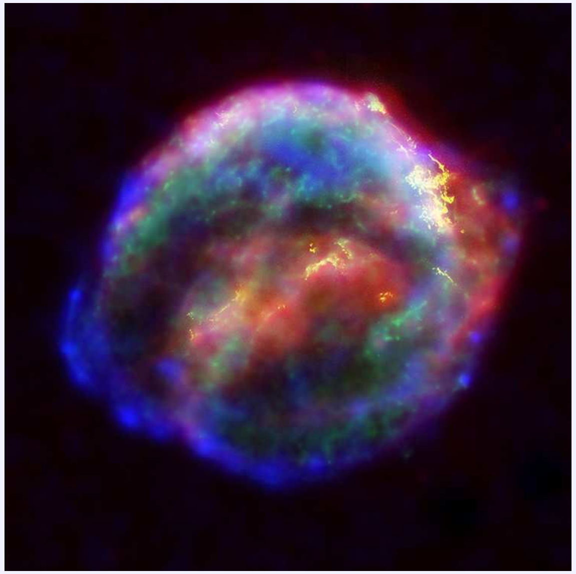
  ⇒ static properties of neutron stars

  ⇒ conditions for nucleosynthesis

  ⇒ energetics, chemical composition, transport properties, . . .



X-ray: NASA/CXC/J.Hester (ASU)
Optical: NASA/ESA/J.Hester & A.Loll (ASU)
Infrared: NASA/JPL-Caltech/R.Gehrz (Univ. Minn.)



NASA/ESA/R.Sankrit & W.Blair (Johns Hopkins Univ.)

Astrophysics and Equation of State

- essential ingredient in astrophysical model calculations:

  Equation(s) of State (EoS) of dense matter

  ⇒ dynamical evolution of supernovae

  ⇒ static properties of neutron stars

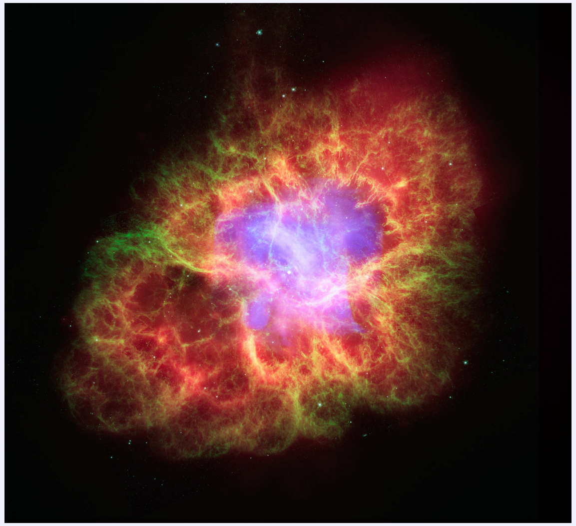
  ⇒ conditions for nucleosynthesis

  ⇒ energetics, chemical composition, transport properties, . . .

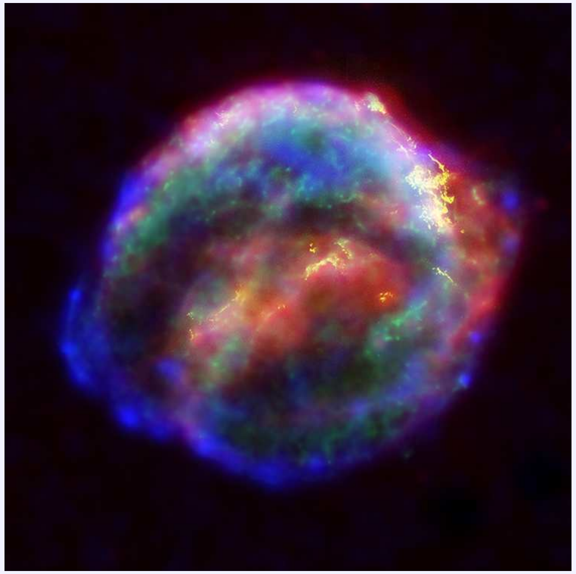
- **timescale of reactions ≪ timescale of system evolution**

  ⇒ equilibrium (thermal, chemical, . . .)

  ⇒ application of **EoS** reasonable



X-ray: NASA/CXC/J.Hester (ASU)
Optical: NASA/ESA/J.Hester & A.Loll (ASU)
Infrared: NASA/JPL-Caltech/R.Gehrz (Univ. Minn.)



NASA/ESA/R.Sankrit & W.Blair (Johns Hopkins Univ.)

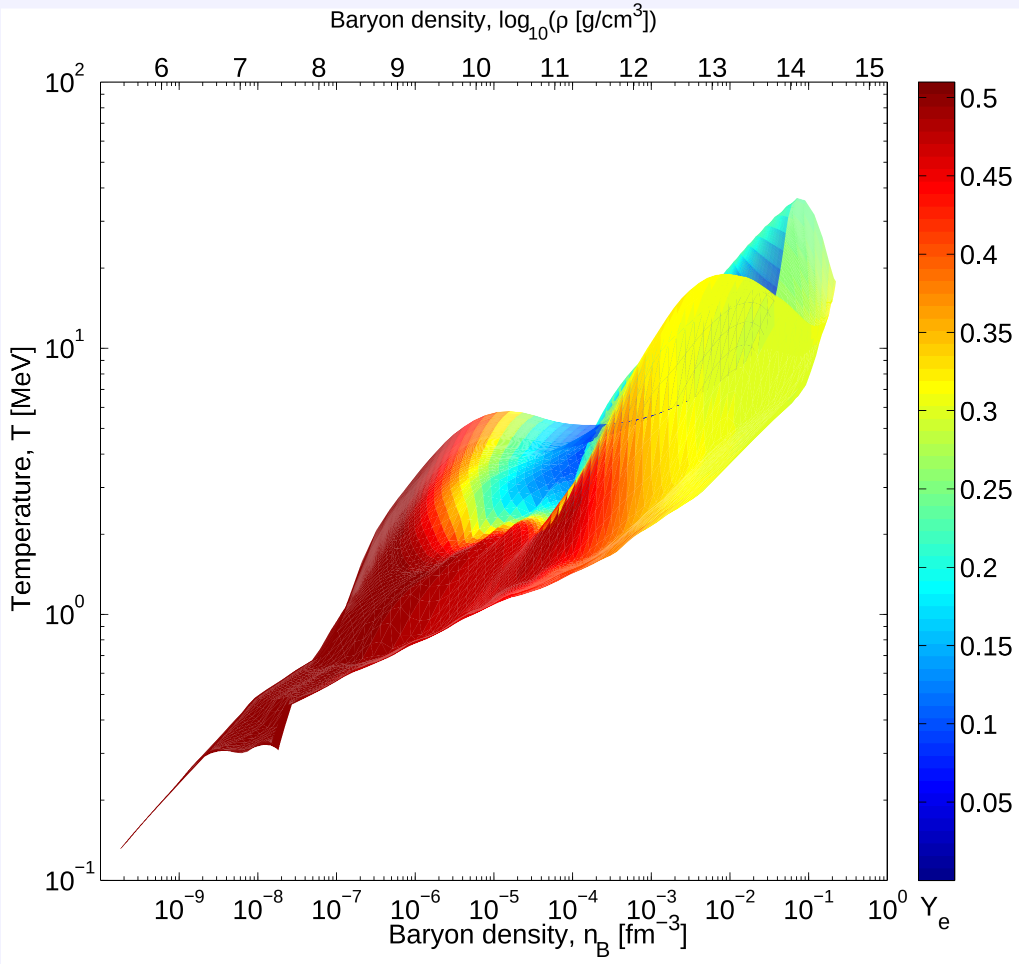
EoS Parameters

standard choice:

- **density**:
  $10^{-9} \lesssim \varrho/\varrho_{\rm sat} \lesssim 10$
  with nuclear saturation density
  $\varrho_{\rm sat} \approx 2.5 \cdot 10^{14}$ g/cm^3
  ($n_{\rm sat} = \varrho_{\rm sat}/m_n \approx 0.15$ fm^{-3})
- **temperature**:
  0 MeV $\leq k_B T \lesssim 50$ MeV
  ($\hat{=}\ 5.8 \cdot 10^{11}$ K)
- **electron fraction**:
  $0 \leq Y_e \lesssim 0.6$

sometimes other choices
more appropriate:
e.g. crust of neutron stars
(density $\longrightarrow$ pressure)

simulation of core-collapse supernova



T. Fischer, GSI/TU Darmstadt

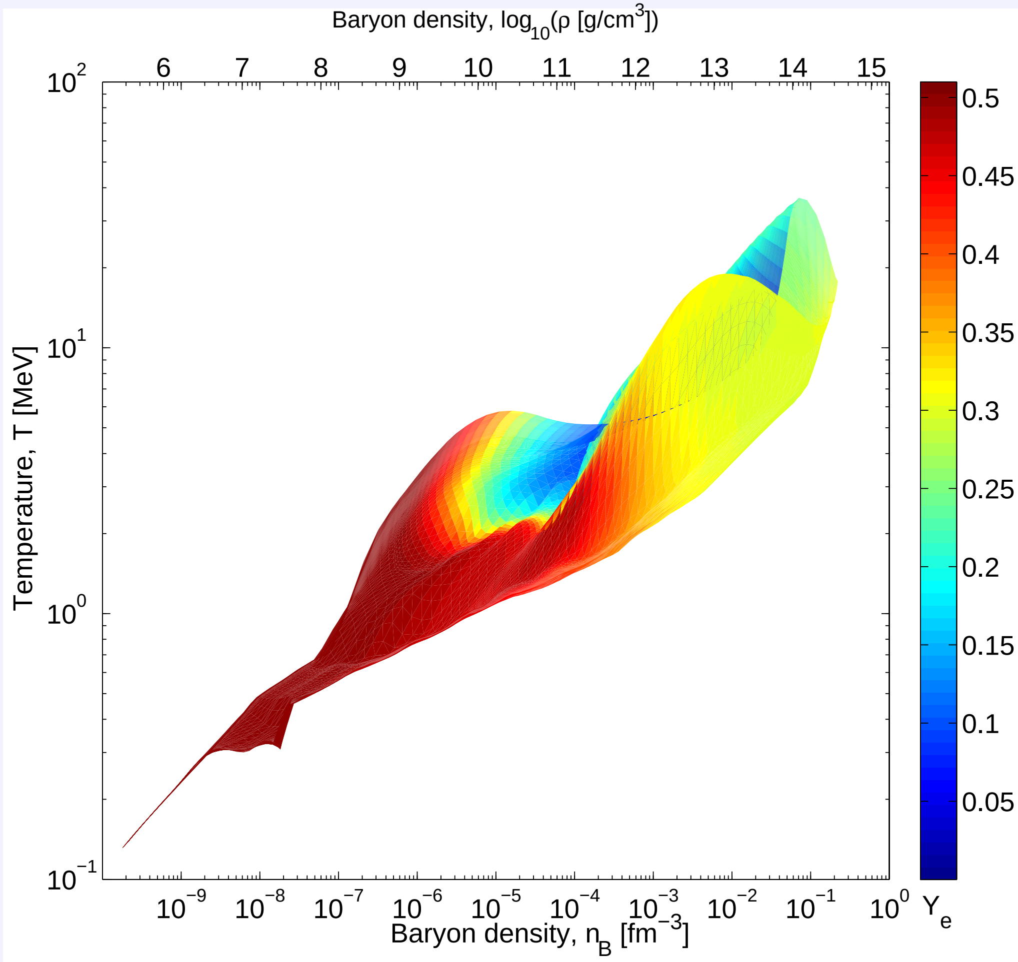
EoS Constituents

most relevant particles:
(at low temperatures and
not too high densities)

- **neutrons, protons**
- **nuclei**
- **electrons, (muons)**
  (charge neutrality!)
- **neutrinos**
  (often not in equilibrium,
  treated independently of EoS)

more particles under extreme conditions:
e.g. high densities, high temperatures
(hyperons, mesons, . . .)

simulation of core-collapse supernova



T. Fischer, GSI/TU Darmstadt

EoS for Astrophysical Applications

- many EoS developed in the past:
  from simple parametizations to sophisticated models
- many investigations of detailed aspects:
  often restricted to particular conditions

$\Rightarrow$ only few realistic global EoS used in astrophysical simulations

EoS for Astrophysical Applications

- many EoS developed in the past:
  from simple parametizations to sophisticated models
- many investigations of detailed aspects:
  often restricted to particular conditions

$\Rightarrow$ only few realistic global EoS used in astrophysical simulations

- **challenge:**
  covering of full parameter space in a single model
  $\Rightarrow$ combination of different features/approaches required

EoS for Astrophysical Applications

- many EoS developed in the past:
  from simple parametizations to sophisticated models
- many investigations of detailed aspects:
  often restricted to particular conditions

$\Rightarrow$ only few realistic global EoS used in astrophysical simulations

- **challenge:**
  covering of full parameter space in a single model
  $\Rightarrow$ combination of different features/approaches required
- **here:**
  - effect of correlations
    $\Rightarrow$ formation and dissolution of clusters
    $\Rightarrow$ phase transition: gas/liquid $\leftrightarrow$ solid

EoS for Astrophysical Applications

- many EoS developed in the past:
  from simple parametizations to sophisticated models
- many investigations of detailed aspects:
  often restricted to particular conditions

$\Rightarrow$ only few realistic global EoS used in astrophysical simulations

- **challenge:**
  covering of full parameter space in a single model
  $\Rightarrow$ combination of different features/approaches required
- **here:**
  - effect of correlations
    $\Rightarrow$ formation and dissolution of clusters
    $\Rightarrow$ phase transition: gas/liquid $\leftrightarrow$ solid
- important distinction:
  nuclear matter $\leftrightarrow$ stellar matter
  $\Rightarrow$ very different systems

Nuclear Matter

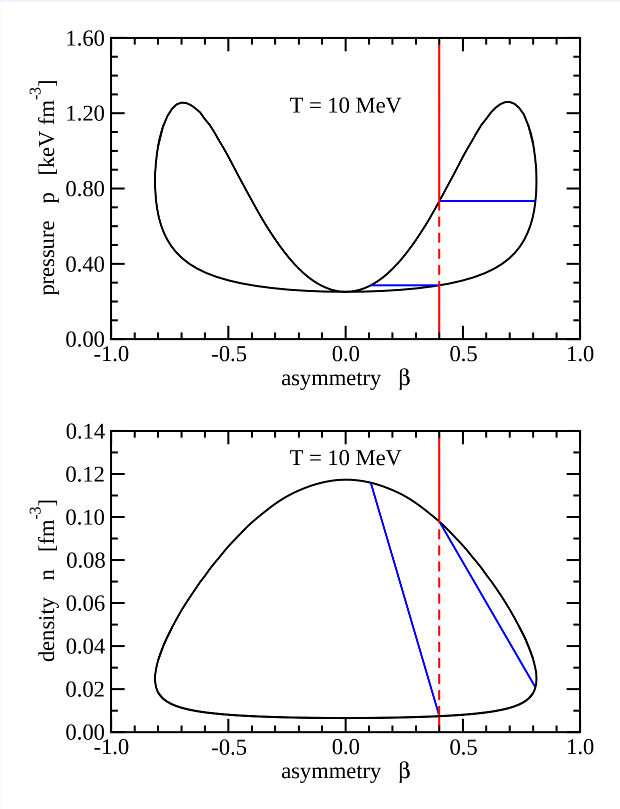
- only strongly interacting particles
- no electromagnetic interaction, no charge neutrality

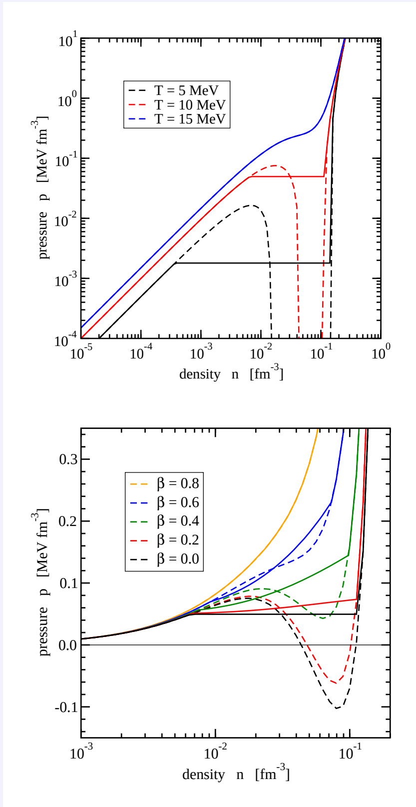
Nuclear Matter

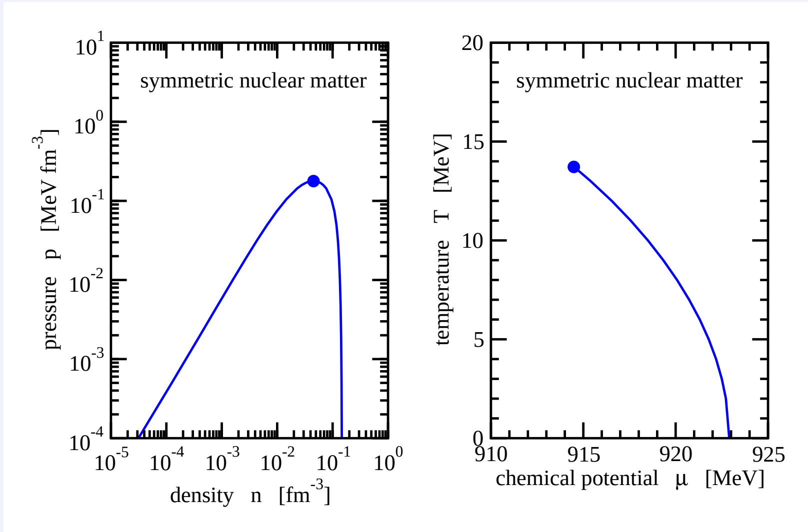
- only strongly interacting particles
- no electromagnetic interaction, no charge neutrality
- many-body correlations due to short-range nuclear interaction
  $\Rightarrow$ clustering $\Rightarrow$ liquid-gas phase transition in thermodynamic limit
  $\Rightarrow$ balance attraction $\leftrightarrow$ repulsion $\Rightarrow$ feature of saturation
- characteristic nuclear matter parameters ϱ_{sat}, E_{sat}/A, K, J, L, . . .

Nuclear Matter

- only strongly interacting particles
- no electromagnetic interaction, no charge neutrality
- many-body correlations due to short-range nuclear interaction
  $\Rightarrow$ clustering $\Rightarrow$ liquid-gas phase transition in thermodynamic limit
  $\Rightarrow$ balance attraction $\leftrightarrow$ repulsion $\Rightarrow$ feature of saturation
- characteristic nuclear matter parameters ϱ_{sat}, E_{sat}/A, K, J, L, ...







- “non-congruent” phase transition

Stellar Matter

- both hadrons and leptons
- strong and electromagnetic interaction
- condition: charge neutrality

Stellar Matter

- both hadrons and leptons
- strong and electromagnetic interaction
- condition: charge neutrality
- many-body correlations due to short-range and long-range interactions $\Rightarrow$
  - formation of inhomogeneous matter and finite-size structures
  - clustering $\Rightarrow$ new particle species (nuclei) $\Rightarrow$ change of chemical composition
  - lattice formation $\Rightarrow$ phase transition: liquid/gas $\leftrightarrow$ solid
  - "pasta phases"
  - modification of thermodynamic properties

Stellar Matter

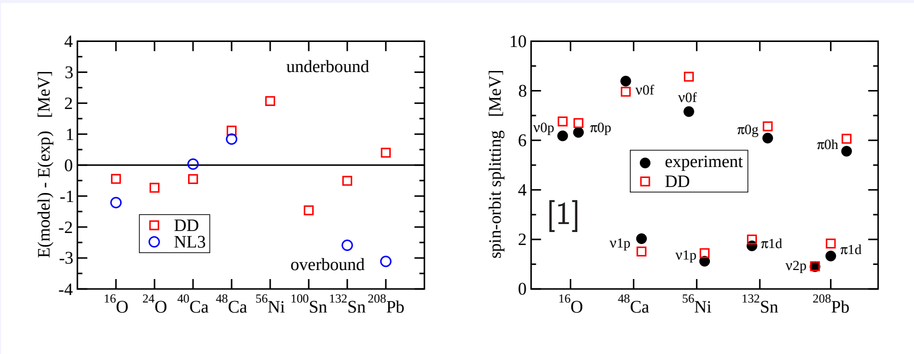
- both hadrons and leptons
- strong and electromagnetic interaction
- condition: charge neutrality
- many-body correlations due to short-range and long-range interactions ⇒
  - formation of inhomogeneous matter and finite-size structures
  - clustering ⇒ new particle species (nuclei) ⇒ change of chemical composition
  - lattice formation ⇒ phase transition: liquid/gas ↔ solid
  - “pasta phases”
  - modification of thermodynamic properties

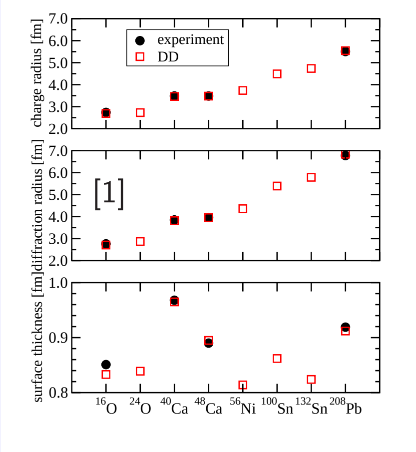
aim:

- consider these (and more) features by extending relativistic mean-field (RMF) model for nuclei
- theoretical formulation as “density functional” with well-constrained parameters

Constraints

- **nuclear physics**
  - nuclei (binding energy, radii, charge formfactor, spin-orbit splittings, . . .)

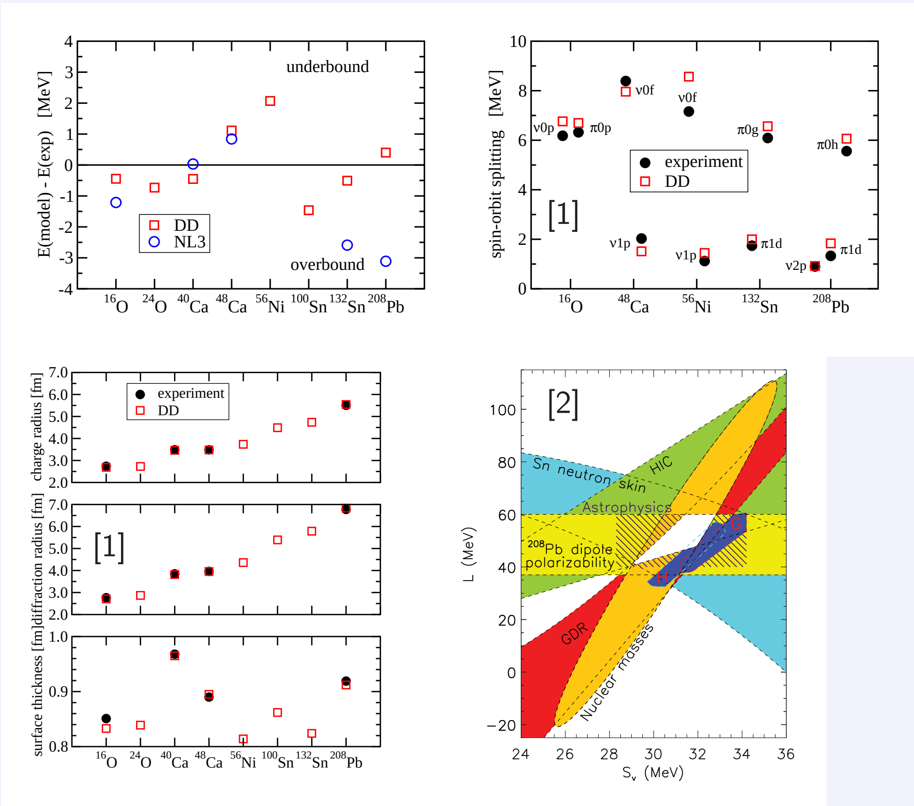




[1] S. Typel, Phys. Rev. C 71 (2005) 064301

Constraints

- **nuclear physics**
  - nuclei (binding energy, radii, charge formfactor, spin-orbit splittings, . . .)
  - nuclear matter (saturation properties, characteristic parameters, . . .)

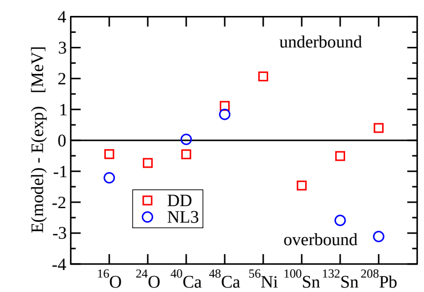


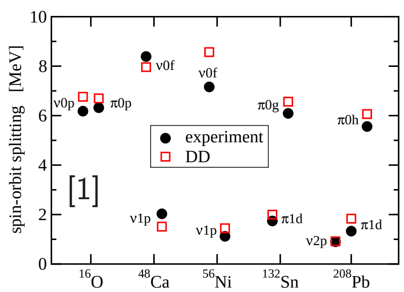
[1] S. Typel, Phys. Rev. C 71 (2005) 064301

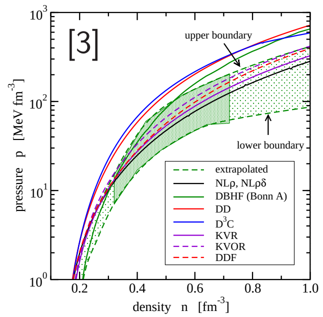
[2] J.M. Lattimer, Y. Lim, arXiv:1203.4286

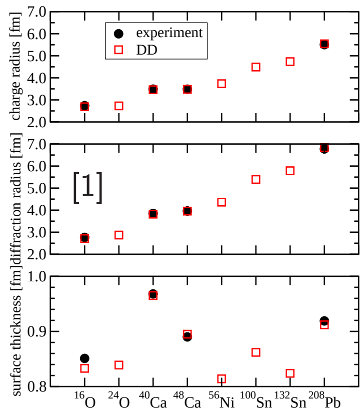
Constraints

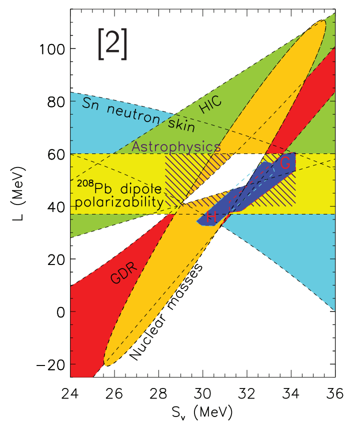
- **nuclear physics**
  - nuclei (binding energy, radii, charge formfactor, spin-orbit splittings, . . .)
  - nuclear matter (saturation properties, characteristic parameters, . . .)
  - heavy-ion collisions (flow, particle production, fragment yields, . . .)











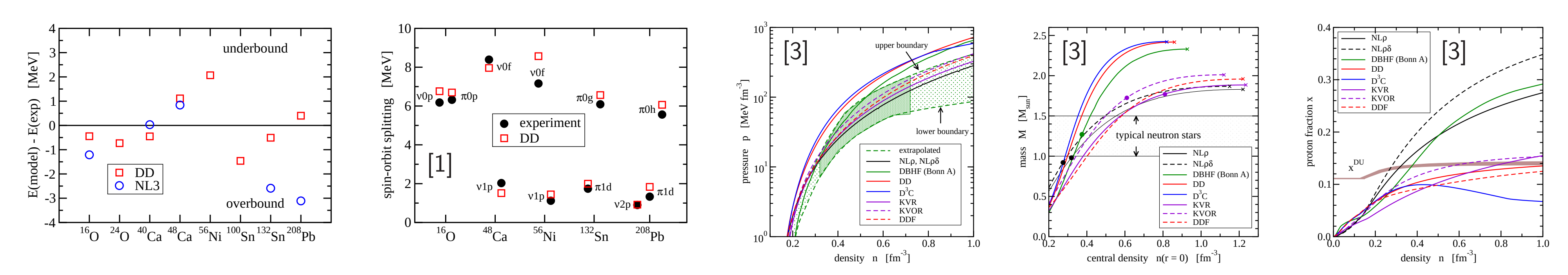
[1] S. Typel, Phys. Rev. C 71 (2005) 064301

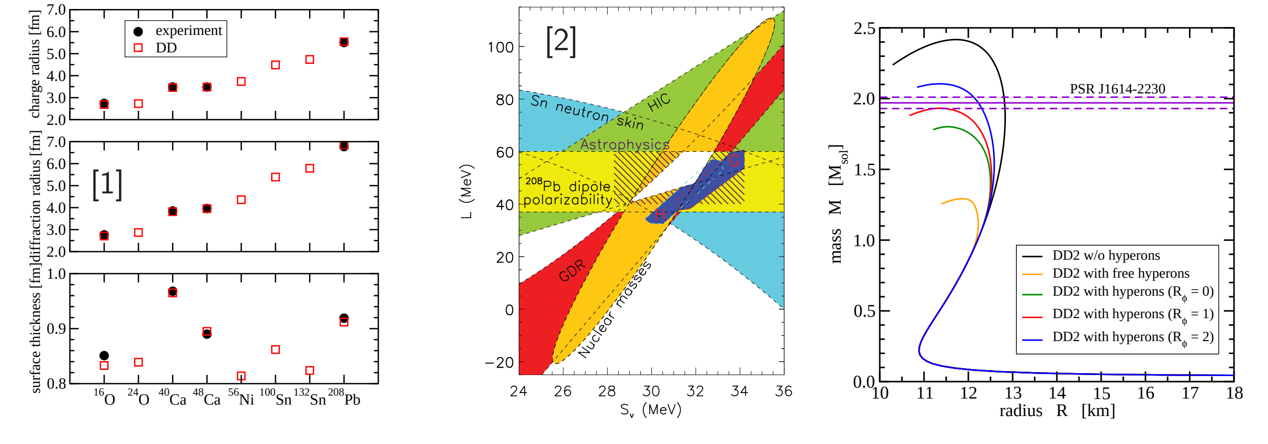
[2] J.M. Lattimer, Y. Lim, arXiv:1203.4286

[3] T. Klähn et al., Phys. Rev. C 74 (2006) 035802

Constraints

- **nuclear physics**
  - nuclei (binding energy, radii, charge formfactor, spin-orbit splittings, . . .)
  - nuclear matter (saturation properties, characteristic parameters, . . .)
  - heavy-ion collisions (flow, particle production, fragment yields, . . .)
- **astrophysics**
  - compact stars (static properties, cooling, . . .)





[1] S. Typel, Phys. Rev. C 71 (2005) 064301

[2] J.M. Lattimer, Y. Lim, arXiv:1203.4286

[3] T. Klähn et al., Phys. Rev. C 74 (2006) 035802

Correlations

- interacting many-body system
  $\Rightarrow$ information on correlations in spectral functions

Correlations

- interacting many-body system
  $\Rightarrow$ information on correlations in spectral functions
- approximation: quasiparticles with self-energies
  - change of particle properties
  - reduction of residual correlations
  - definition of chemical composition?
  - extreme case: uncorrelated quasiparticles

Correlations

- interacting many-body system
  ⇒ information on correlations in spectral functions
- approximation: quasiparticles with self-energies
  - change of particle properties
  - reduction of residual correlations
  - definition of chemical composition?
  - extreme case: uncorrelated quasiparticles
- quasiparticle concept very successful in nuclear physics
  ⇒ phenomenological mean-field models (e.g. Skyrme, Gogny, relativistic) with only nucleons as degrees of freedom

Correlations

- interacting many-body system
  ⇒ information on correlations in spectral functions
- approximation: quasiparticles with self-energies
  - change of particle properties
  - reduction of residual correlations
  - definition of chemical composition?
  - extreme case: uncorrelated quasiparticles
- quasiparticle concept very successful in nuclear physics
  ⇒ phenomenological mean-field models (e.g. Skyrme, Gogny, relativistic)
  with only nucleons as degrees of freedom
- low densities: clusters as new degrees of freedom
  ⇒ benchmark: virial equation of state
  (see e.g. C. J. Horowitz, A. Schwenk, Nucl. Phys. A 776 (2006) 55)

Correlations

- interacting many-body system
  ⇒ information on correlations in spectral functions
- approximation: quasiparticles with self-energies
  - change of particle properties
  - reduction of residual correlations
  - definition of chemical composition?
  - extreme case: uncorrelated quasiparticles
- quasiparticle concept very successful in nuclear physics
  ⇒ phenomenological mean-field models (e.g. Skyrme, Gogny, relativistic)
  with only nucleons as degrees of freedom
- low densities: clusters as new degrees of freedom
  ⇒ benchmark: virial equation of state
  (see e.g. C. J. Horowitz, A. Schwenk, Nucl. Phys. A 776 (2006) 55)

⇒ transition in unified model?

Relativistic Density Functional

- constituents: nucleons $\Rightarrow \psi_i$ $(i = n, p)$ Dirac spinors

Relativistic Density Functional

- constituents: nucleons $\Rightarrow \psi_i$ $(i = n, p)$ Dirac spinors
- interaction: ◦ strong $\Rightarrow$ meson fields A_m $(m = \sigma, \omega, \rho$, convenient auxiliary fields)
  ◦ electromagnetic $\Rightarrow A_\gamma$

Relativistic Density Functional

- constituents: nucleons $\Rightarrow \psi_i$ $(i = n, p)$ Dirac spinors
- interaction: ○ strong $\Rightarrow$ meson fields A_m ($m = \sigma, \omega, \rho$, convenient auxiliary fields)
  ○ electromagnetic $\Rightarrow A_\gamma$
- energy of nucleus $E = \int d^3r \; \varepsilon(\vec{r}) + E_{\text{cm}} + E_{\text{pair}} + \ldots$

  with energy density functional

$$\varepsilon = \sum_i w_i \left[t_i + (m_i - \Gamma_{i\sigma} A_\sigma) n_i^{(s)} + (\Gamma_{i\omega} A_\omega + \Gamma_{i\rho} A_\rho + \Gamma_{i\gamma} A_\gamma) n_i \right]$$

$$+ \frac{1}{2} \left(m_\sigma^2 A_\sigma^2 + \vec{\nabla} A_\sigma \cdot \vec{\nabla} A_\sigma - m_\omega^2 A_\omega^2 - \vec{\nabla} A_\omega \cdot \vec{\nabla} A_\omega - m_\rho^2 A_\rho^2 - \vec{\nabla} A_\rho \cdot \vec{\nabla} A_\rho - \vec{\nabla} A_\gamma \cdot \vec{\nabla} A_\gamma \right)$$

  ○ single-particle densities $t_i = \bar{\psi}_i \vec{\gamma} \cdot \hat{\vec{p}} \psi_i \qquad n_i^{(s)} = \bar{\psi}_i \psi_i \qquad n_i = \bar{\psi}_i \gamma_0 \psi_i$

  ○ occupation numbers w_i

Relativistic Density Functional

- constituents: nucleons $\Rightarrow \psi_i$ $(i = n, p)$ Dirac spinors
- interaction: ○ strong $\Rightarrow$ meson fields A_m $(m = \sigma, \omega, \rho$, convenient auxiliary fields)
  ○ electromagnetic $\Rightarrow A_\gamma$
- energy of nucleus $E = \int d^3r\, \varepsilon(\vec{r}) + E_{\text{cm}} + E_{\text{pair}} + \ldots$

  with energy density functional

$$\varepsilon = \sum_i w_i \left[t_i + (m_i - \Gamma_{i\sigma} A_\sigma) n_i^{(s)} + (\Gamma_{i\omega} A_\omega + \Gamma_{i\rho} A_\rho + \Gamma_{i\gamma} A_\gamma) n_i \right]$$

$$+ \frac{1}{2} \left(m_\sigma^2 A_\sigma^2 + \vec{\nabla} A_\sigma \cdot \vec{\nabla} A_\sigma - m_\omega^2 A_\omega^2 - \vec{\nabla} A_\omega \cdot \vec{\nabla} A_\omega - m_\rho^2 A_\rho^2 - \vec{\nabla} A_\rho \cdot \vec{\nabla} A_\rho - \vec{\nabla} A_\gamma \cdot \vec{\nabla} A_\gamma \right)$$

  ○ single-particle densities $t_i = \bar{\psi}_i \vec{\gamma} \cdot \hat{\vec{p}} \psi_i \qquad n_i^{(s)} = \bar{\psi}_i \psi_i \qquad n_i = \bar{\psi}_i \gamma_0 \psi_i$

  ○ occupation numbers w_i

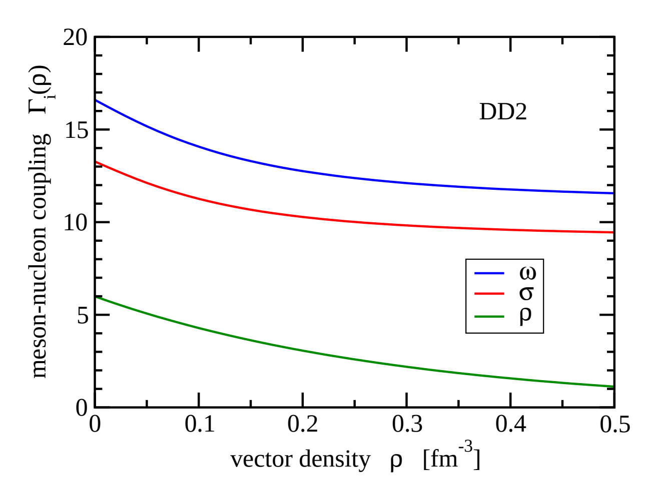
  ○ density dependent meson-nucleon couplings

  $\Gamma_{im} = g_{im} \Gamma_m(\varrho) \qquad \varrho = n_n + n_p$

  $\Rightarrow$ medium dependent interaction

  $\Rightarrow$ rearrangement contributions to self-energies

  ○ $\Gamma_{i\gamma} = Q_i \Gamma_\gamma$ with charge number Q_i



Nuclear Correlations in Matter

Theoretical Approaches

- ideal mixture of independent particles, no interaction
  $\Rightarrow$ **Nuclear Statistical Equilibrium/Law of Mass Action**
  most simple approach, suppression of nuclei $\Rightarrow$ excluded volume mechanism

Theoretical Approaches

- ideal mixture of independent particles, no interaction
  $\Rightarrow$ **Nuclear Statistical Equilibrium/Law of Mass Action**
  most simple approach, suppression of nuclei $\Rightarrow$ excluded volume mechanism

- mixture of interacting particles/correlations
  $\Rightarrow$ **Virial Equation of State**
  model-independent low-density benchmark

Theoretical Approaches

- ideal mixture of independent particles, no interaction
  $\Rightarrow$ **Nuclear Statistical Equilibrium/Law of Mass Action**
  most simple approach, suppression of nuclei $\Rightarrow$ excluded volume mechanism

- mixture of interacting particles/correlations
  $\Rightarrow$ **Virial Equation of State**
  model-independent low-density benchmark

- considering medium effects with increasing density
  $\Rightarrow$ **Quantum Statistical/Generalized Beth-Uhlenbeck Approach**
  correlations of quasiparticles with medium-dependent properties,
  microscopic origin of cluster dissolution/Mott effect (action of Pauli principle)

Theoretical Approaches

- ideal mixture of independent particles, no interaction
  ⇒ **Nuclear Statistical Equilibrium/Law of Mass Action**
  most simple approach, suppression of nuclei ⇒ excluded volume mechanism

- mixture of interacting particles/correlations
  ⇒ **Virial Equation of State**
  model-independent low-density benchmark

- considering medium effects with increasing density
  ⇒ **Quantum Statistical/Generalized Beth-Uhlenbeck Approach**
  correlations of quasiparticles with medium-dependent properties,
  microscopic origin of cluster dissolution/Mott effect (action of Pauli principle)

- interpolation from low to high densities around nuclear saturation
  ⇒ **Generalized Relativistic Density Functional**
  correct limits, formation and dissolution of nuclei

Generalized Relativistic Density Functional

- include **new degrees of freedom** with medium-dependent properties:
  - light nuclei (deuteron, triton, helion, α-particle)
  - nucleon-nucleon scattering correlations (nn, pp, np channels)
  - heavy nuclei ($A > 4$)

  $\Rightarrow$ interaction via minimal coupling to mesons/photon with scaled strengths

Generalized Relativistic Density Functional

- include **new degrees of freedom** with medium-dependent properties:
  - ◦ light nuclei (deuteron, triton, helion, α-particle)
  - ◦ nucleon-nucleon scattering correlations (nn, pp, np channels)
  - ◦ heavy nuclei ($A > 4$)

  $\Rightarrow$ interaction via minimal coupling to mesons/photon with scaled strengths
- **model parameters**
  - ◦ vacuum masses of nucleons, electrons, nuclei
  - ◦ effective resonance energies and degeneracy factors
  - ◦ density-dependent meson-nucleon/nucleus couplings, fitted to properties of atomic nuclei
  - ◦ medium-dependent mass shifts of clusters (bound and continuum states)

Details:

S. Typel, G. Röpke, T. Klähn, D. Blaschke, H.H. Wolter, Phys. Rev. C 81 (2010) 015803

M. D. Voskresenskaya, S. Typel, Nucl. Phys. A 887 (2012) 42

G. Röpke, N.-U. Bastian, D. Blaschke, T. Klähn, S. Typel, H.H. Wolter, Nucl. Phys. A 897 (2013) 70

Light Nuclei

shift of binding energies/masses

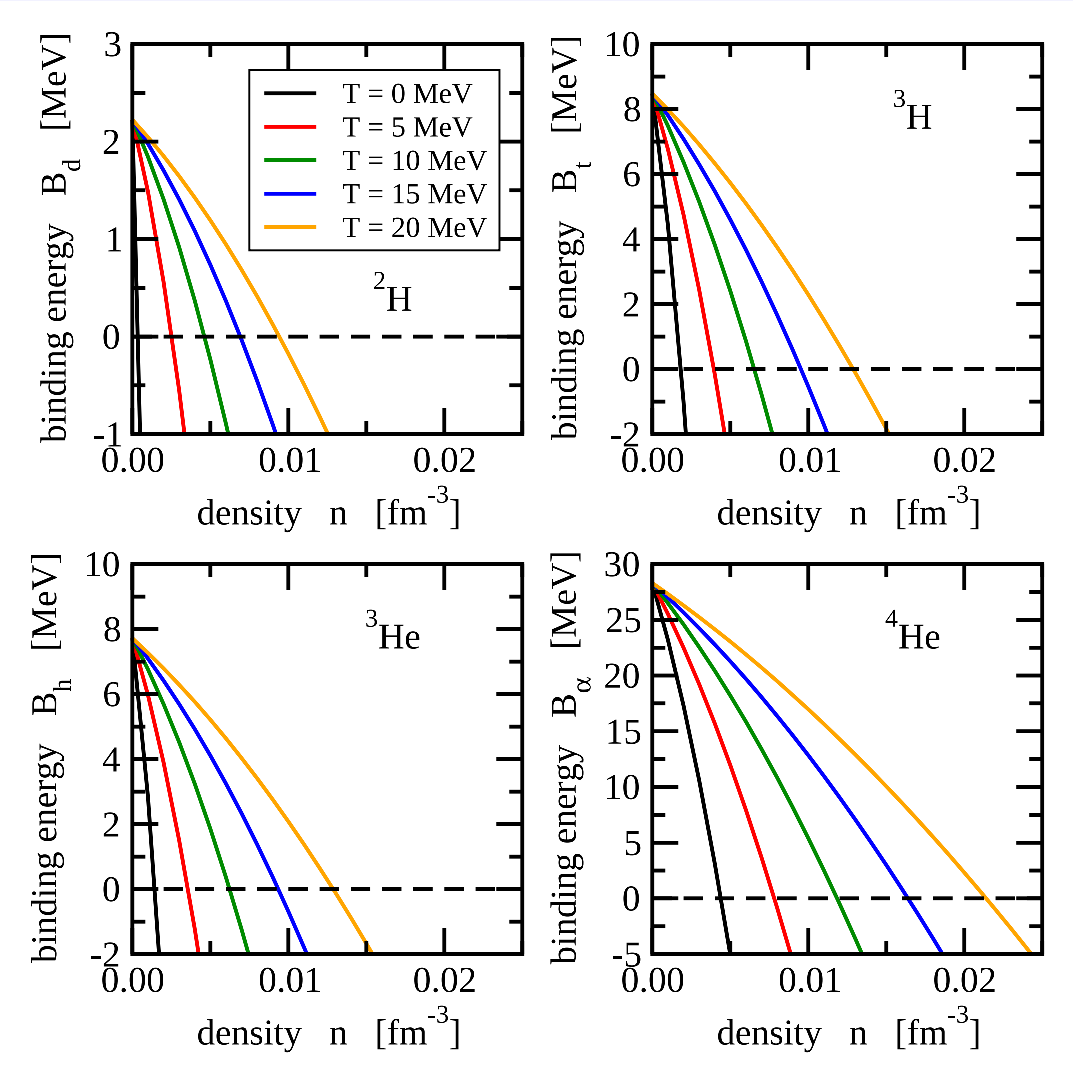
- solve in-medium Schrödinger equation with realistic nucleon-nucleon potentials
- parametrization of shifts Δm_i
- main effect: Pauli principle
  $\Rightarrow$ blocking of states in the medium!

Light Nuclei

shift of binding energies/masses

- solve in-medium Schrödinger equation with realistic nucleon-nucleon potentials
- parametrization of shifts Δm_i
- main effect: Pauli principle
  $\Rightarrow$ blocking of states in the medium!

- example: symmetric nuclear matter, nuclei at rest in medium
- in vacuum:
  experimental binding energies
- nuclei become unbound $(B_i < 0)$ with increasing density of medium
- dissolution of clusters at high densities $\Rightarrow$ Mott effect



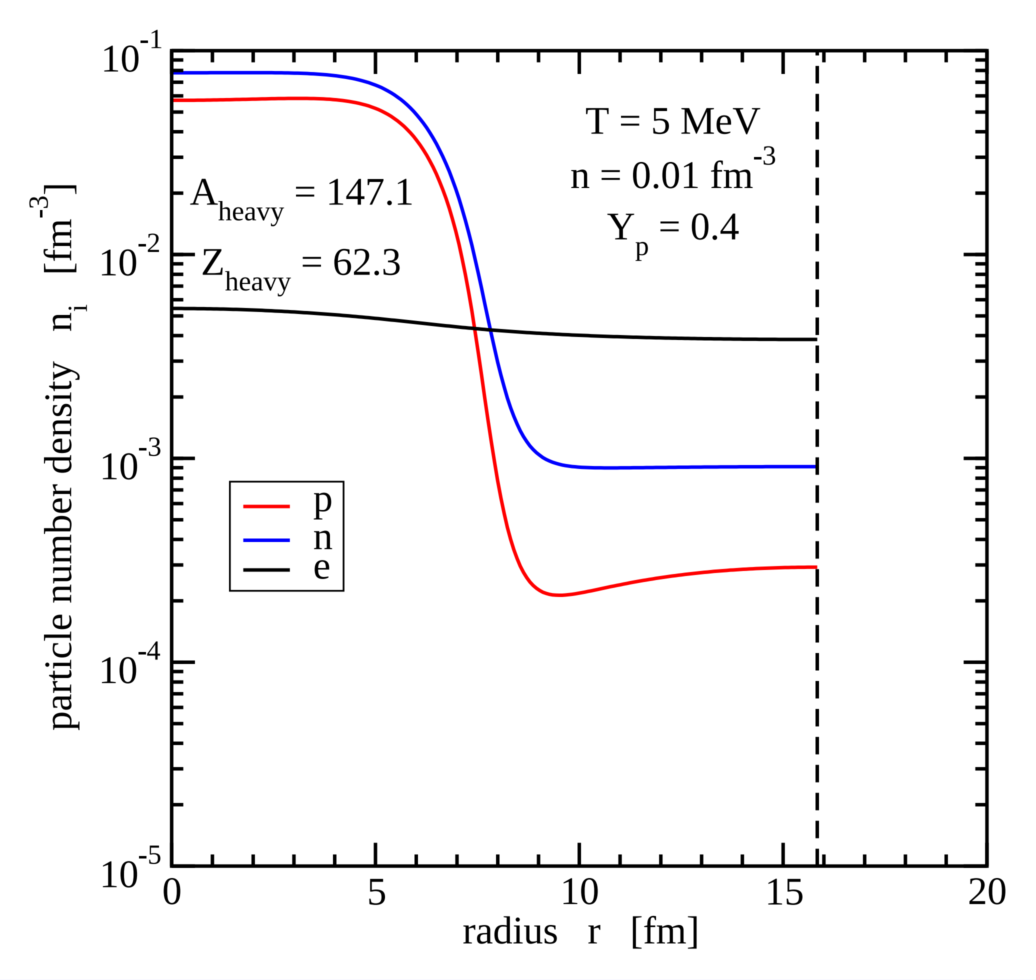
Heavy Nuclei I

inhomogeneous matter at low densities

- comparison with uniform matter
  $\Rightarrow$ increase in binding energy

inhomogeneous matter at low densities

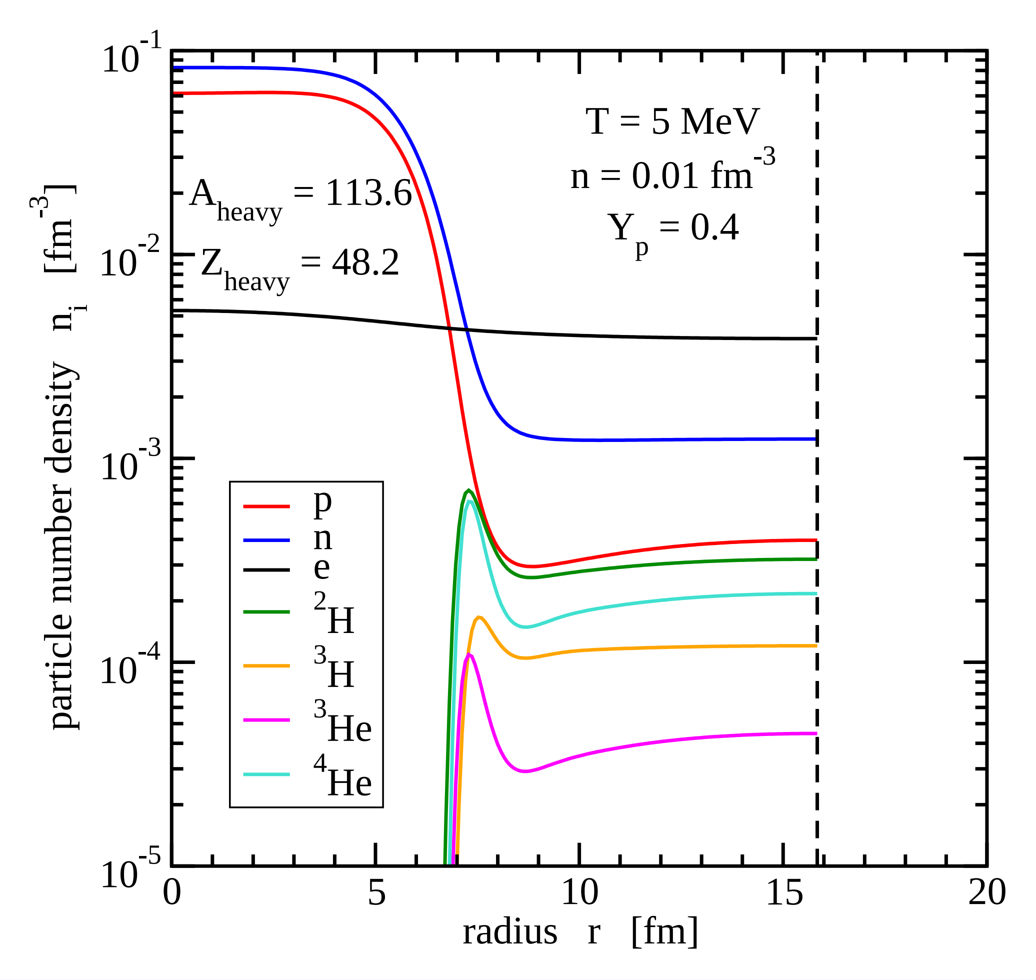
- comparison with uniform matter
  $\Rightarrow$ increase in binding energy
- spherical Wigner-Seitz cell calculation
  - generalized rel. density functional
  - extended Thomas-Fermi approximation
  - electrons for charge compensation
  - heavy nucleus surrounded by gas of nucleons
- self-consistent calculation with interacting nucleons, electrons



Heavy Nuclei I

inhomogeneous matter at low densities

- comparison with uniform matter
  ⇒ increase in binding energy
- spherical Wigner-Seitz cell calculation
  - generalized rel. density functional
  - extended Thomas-Fermi approximation
  - electrons for charge compensation
  - heavy nucleus surrounded by gas of nucleons and light clusters
- self-consistent calculation with interacting nucleons, electrons and light nuclei
- increased probability of finding light clusters at surface of heavy nucleus



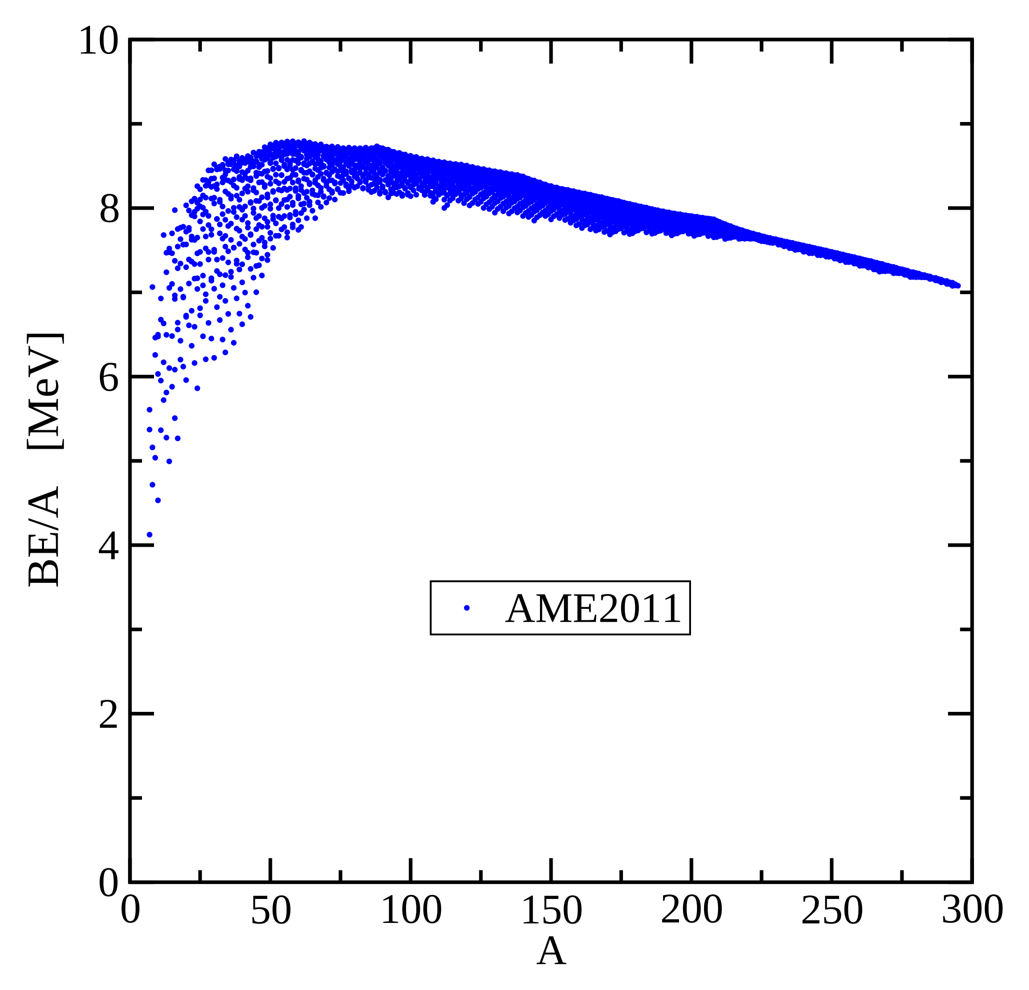
Heavy Nuclei II

- traditional approach in EoS tables:
  - single-nucleus approximation (SNA)
    (one representative heavy nucleus)
  - no distribution of nuclei

Heavy Nuclei II

- traditional approach in EoS tables:
  - single-nucleus approximation (SNA) (one representative heavy nucleus)
  - no distribution of nuclei
- extended approach:
  - full table of nuclei included (c.f. NSE calculations)
  - vacuum binding energies needed
  - medium-dependent shift of binding energies from SNA

binding energy per nucleon

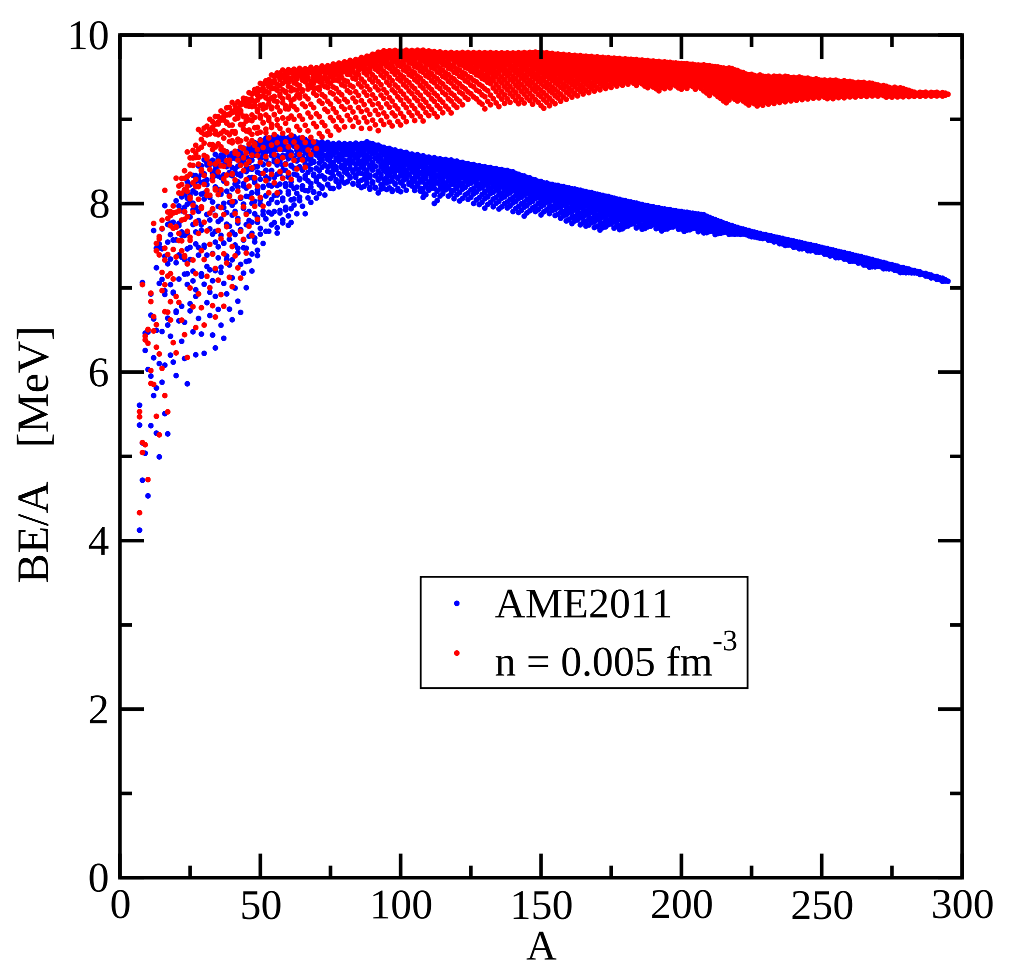


AME2011: G. Audi, W. Meng (private communication)

Heavy Nuclei II

- traditional approach in EoS tables:
  - single-nucleus approximation (SNA) (one representative heavy nucleus)
  - no distribution of nuclei
- extended approach:
  - full table of nuclei included (c.f. NSE calculations)
  - vacuum binding energies needed
  - medium-dependent shift of binding energies from SNA

binding energy per nucleon (T=0 MeV, np symmetric matter)

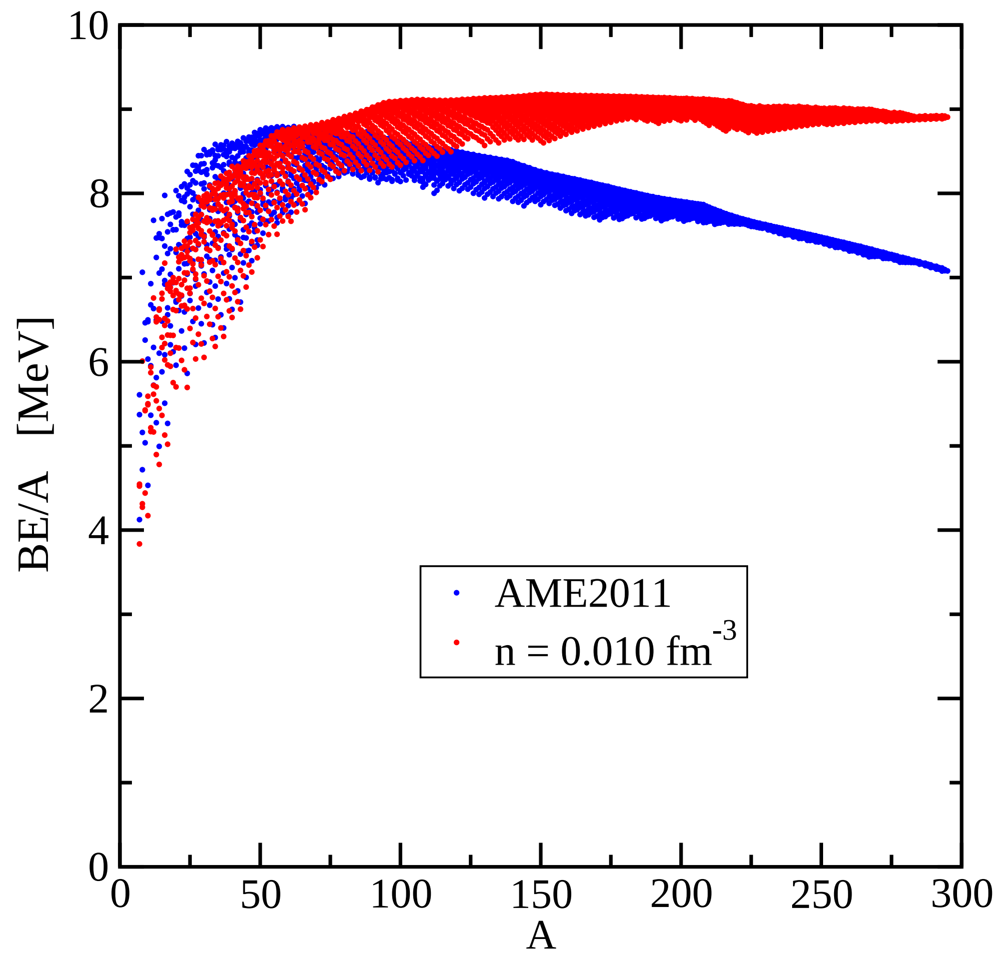


AME2011: G. Audi, W. Meng (private communication)

Heavy Nuclei II

- traditional approach in EoS tables:
  - single-nucleus approximation (SNA) (one representative heavy nucleus)
  - no distribution of nuclei
- extended approach:
  - full table of nuclei included (c.f. NSE calculations)
  - vacuum binding energies needed
  - medium-dependent shift of binding energies from SNA

binding energy per nucleon (T=0 MeV, np symmetric matter)

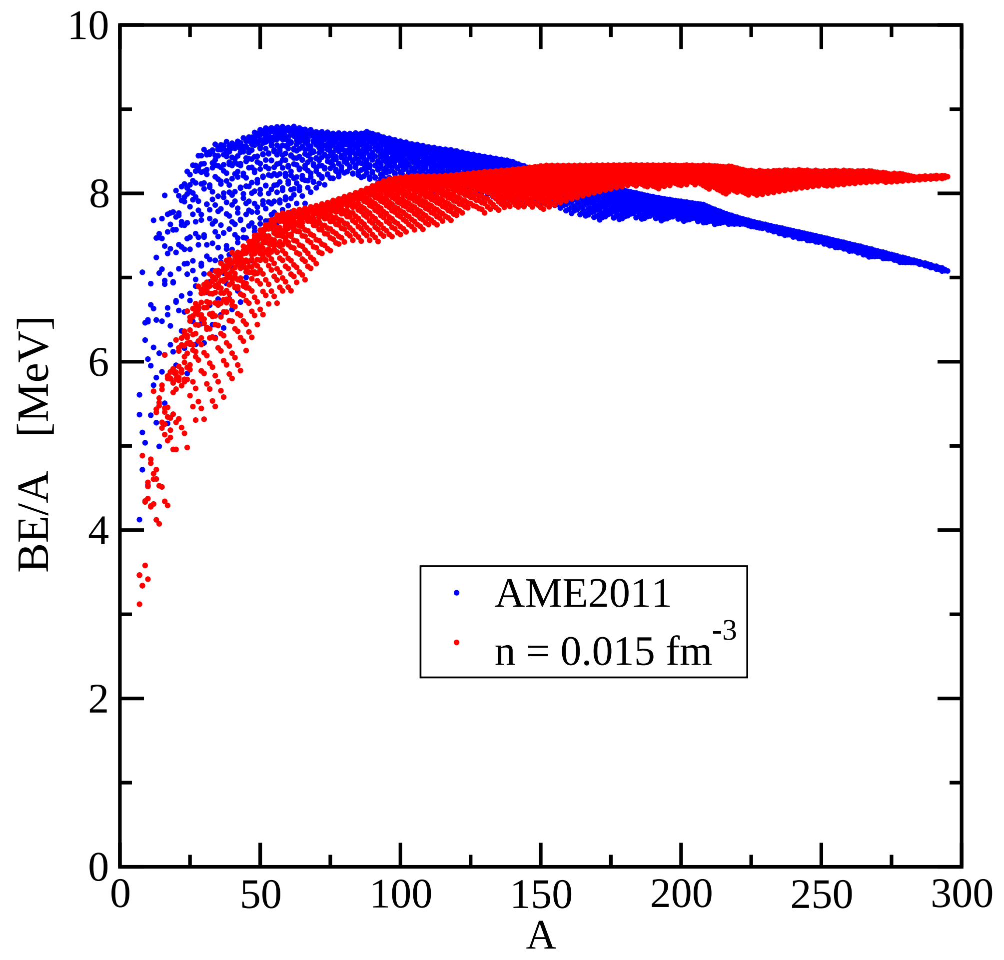


AME2011: G. Audi, W. Meng (private communication)

Heavy Nuclei II

- traditional approach in EoS tables:
  - single-nucleus approximation (SNA) (one representative heavy nucleus)
  - no distribution of nuclei
- extended approach:
  - full table of nuclei included (c.f. NSE calculations)
  - vacuum binding energies needed
  - medium-dependent shift of binding energies from SNA

binding energy per nucleon (T=0 MeV, np symmetric matter)

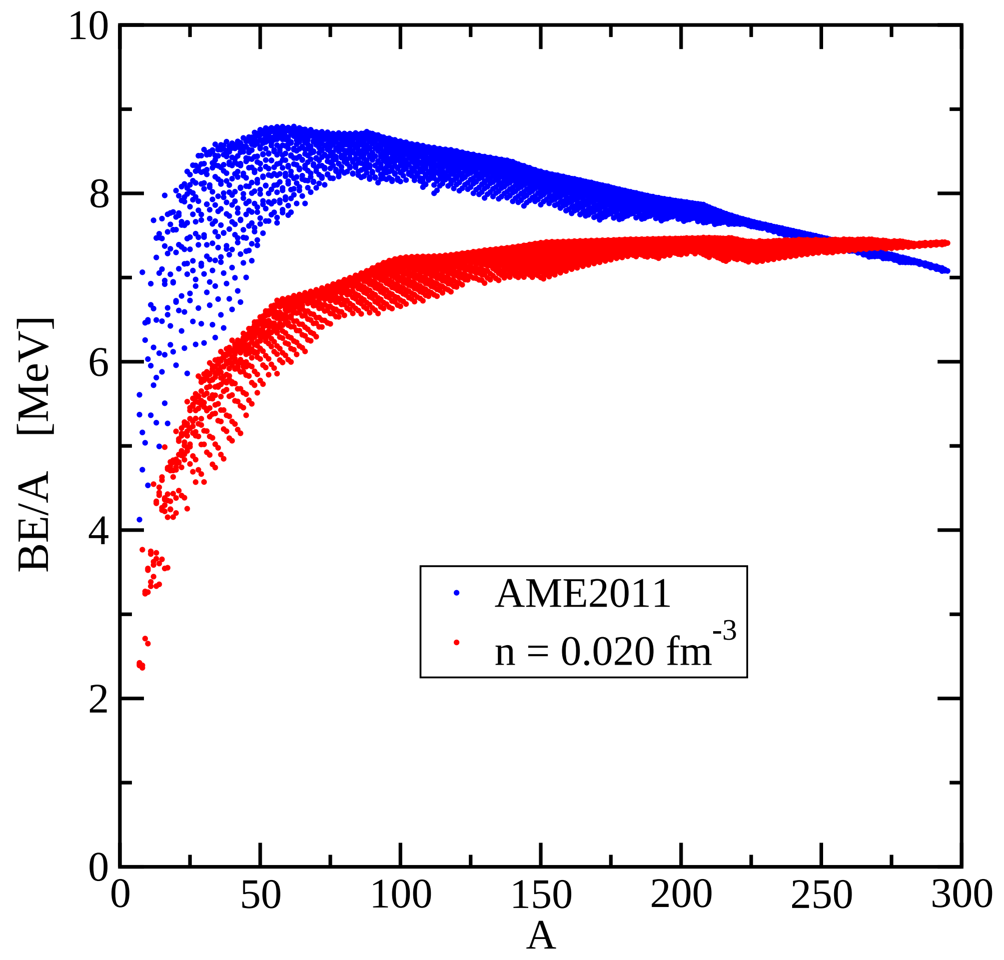


AME2011: G. Audi, W. Meng (private communication)

Heavy Nuclei II

- traditional approach in EoS tables:
  - single-nucleus approximation (SNA) (one representative heavy nucleus)
  - no distribution of nuclei
- extended approach:
  - full table of nuclei included (c.f. NSE calculations)
  - vacuum binding energies needed
  - medium-dependent shift of binding energies from SNA

binding energy per nucleon (T=0 MeV, np symmetric matter)

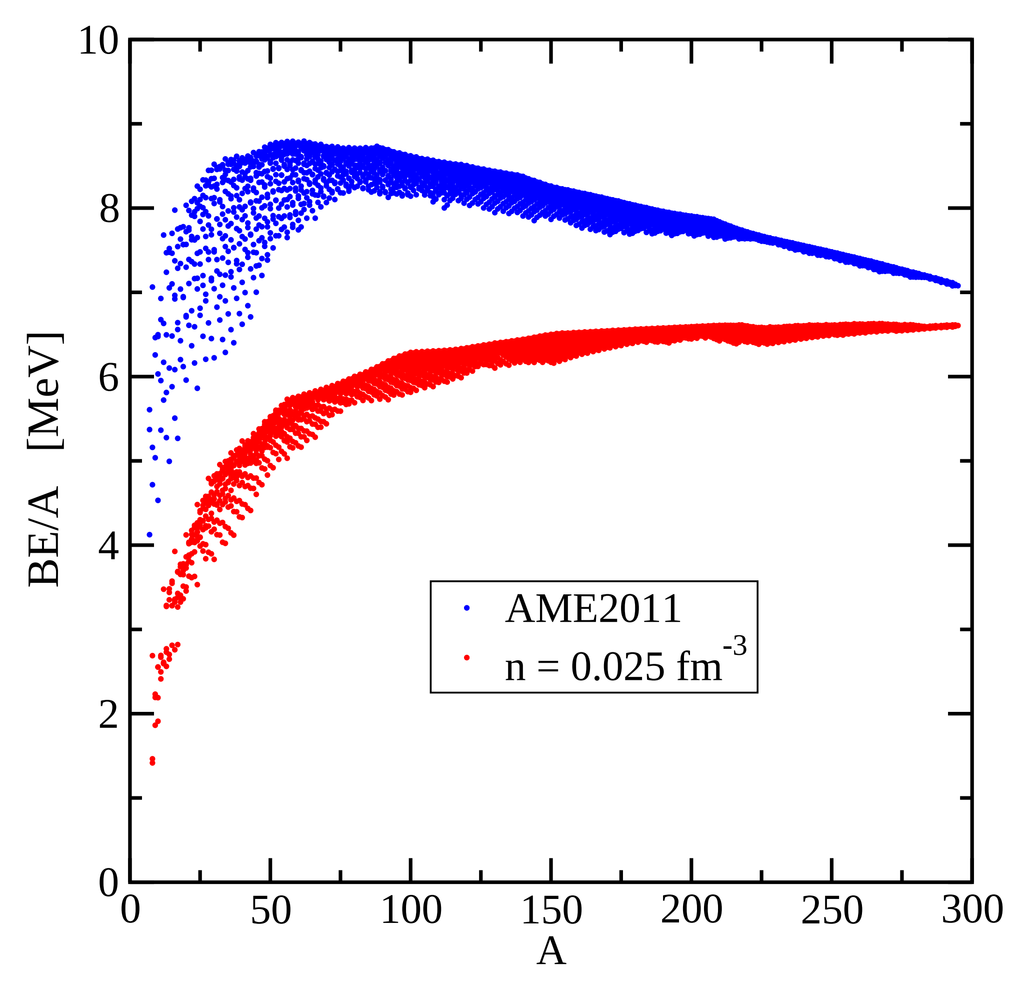


AME2011: G. Audi, W. Meng (private communication)

Heavy Nuclei II

- traditional approach in EoS tables:
  - single-nucleus approximation (SNA) (one representative heavy nucleus)
  - no distribution of nuclei
- extended approach:
  - full table of nuclei included (c.f. NSE calculations)
  - vacuum binding energies needed
  - medium-dependent shift of binding energies from SNA

binding energy per nucleon (T=0 MeV, np symmetric matter)

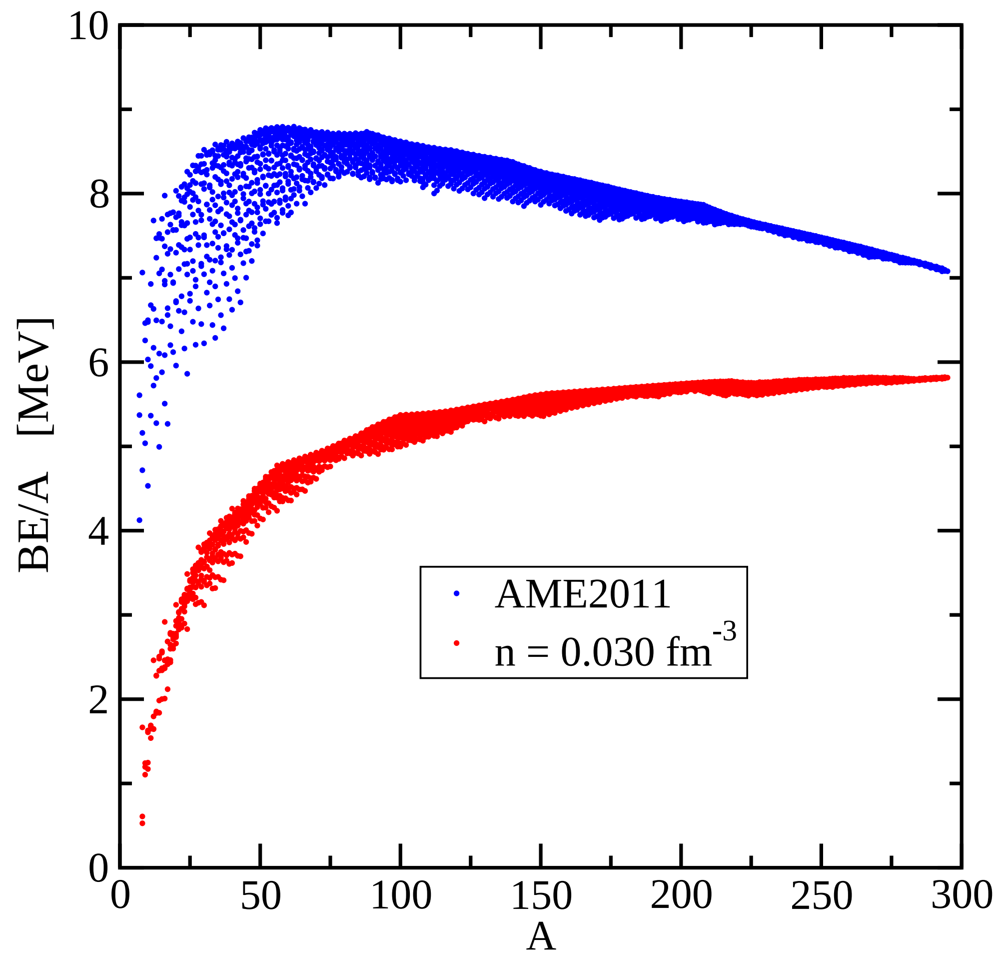


AME2011: G. Audi, W. Meng (private communication)

Heavy Nuclei II

- traditional approach in EoS tables:
  - single-nucleus approximation (SNA) (one representative heavy nucleus)
  - no distribution of nuclei
- extended approach:
  - full table of nuclei included (c.f. NSE calculations)
  - vacuum binding energies needed
  - medium-dependent shift of binding energies from SNA

binding energy per nucleon (T=0 MeV, np symmetric matter)

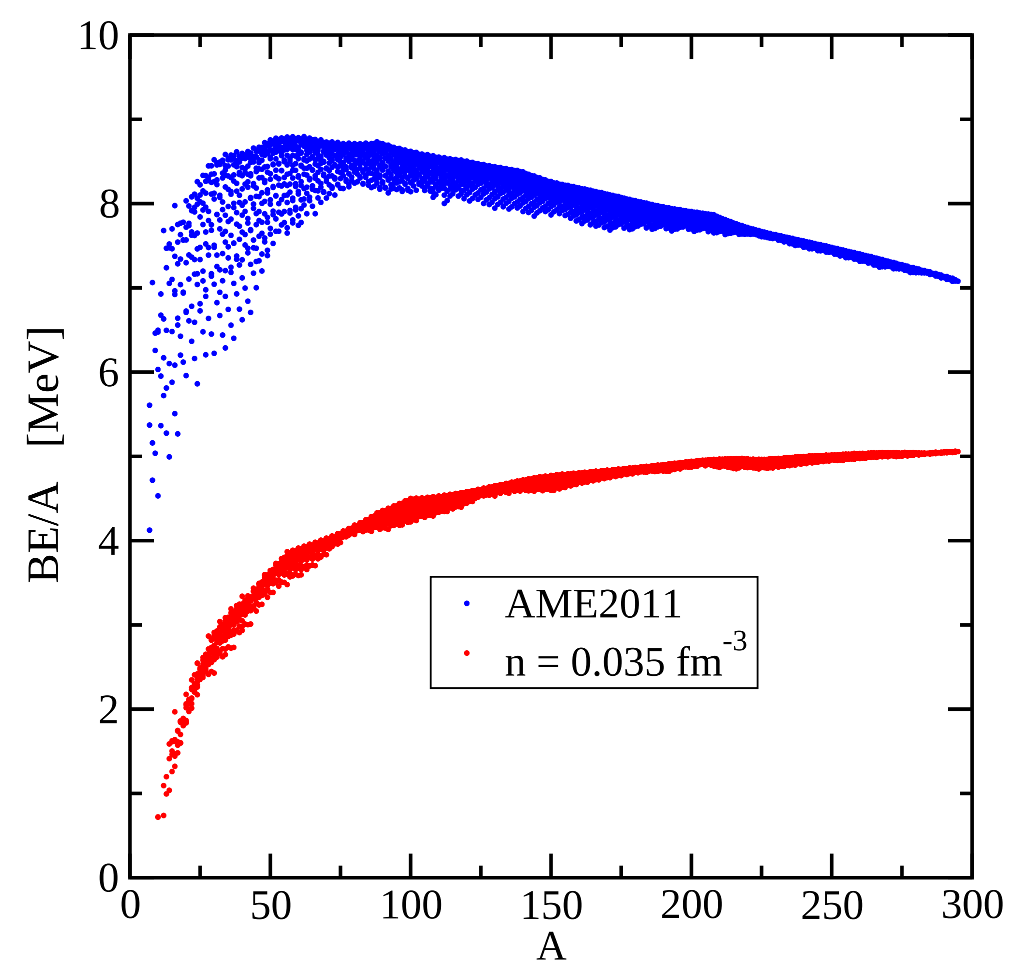


AME2011: G. Audi, W. Meng (private communication)

Heavy Nuclei II

- traditional approach in EoS tables:
  - single-nucleus approximation (SNA) (one representative heavy nucleus)
  - no distribution of nuclei
- extended approach:
  - full table of nuclei included (c.f. NSE calculations)
  - vacuum binding energies needed
  - medium-dependent shift of binding energies from SNA
- medium effects:
  - relative stabilization of heavier and exotic nuclei
  - dissolution of nuclei depending on density, temperature, np-asymmetry
- parametrization of mass shifts Δm_i, only preliminary results

binding energy per nucleon (T=0 MeV, np symmetric matter)



AME2011: G. Audi, W. Meng (private communication)

Low-Density Limit I

- only **two-body correlations** relevant
- **comparison** of generalized relativistic density functional
  with virial Equation of State (model-independent benchmark,
  depends only on experimental binding energies and phase shifts $\delta_l^{(ij)}$)

Low-Density Limit I

- only **two-body correlations** relevant
- **comparison** of generalized relativistic density functional with virial Equation of State (model-independent benchmark, depends only on experimental binding energies and phase shifts $\delta_l^{(ij)}$)
- **fugacity expansion** of thermodynamic potential Ω
  $\Rightarrow$ consistency relations with virial coefficients and zero-density meson-nucleon couplings $C_m = \Gamma_m^2/m_m^2$ $(m = \omega, \sigma, \rho, \delta)$
  $\Rightarrow$ effective resonance energies $E_{ij}(T)$ $(i, j = n, p)$ representing NN scattering correlations
  $\Rightarrow$ effective degeneracy factors $g_{ij}^{(\text{eff})}(T)$ (cf. treatment of excited states of nuclei)
  $\Rightarrow$ relativistic corrections

Low-Density Limit II

- **zero temperature limit of consistency relations without scattering correlations**
  - $$C_{\omega} - C_{\sigma} = \frac{\pi}{2m} \left[a_{nn}(^{1}S_{0}) + a_{pp}(^{1}S_{0}) + a_{np}(^{1}S_{0}) + 3a_{np}(^{3}S_{1}) \right]$$
  - $$C_{\rho} - C_{\delta} = \frac{\pi}{2m} \left[a_{nn}(^{1}S_{0}) + a_{pp}(^{1}S_{0}) - a_{np}(^{1}S_{0}) - 3a_{np}(^{3}S_{1}) \right]$$

  with scattering lengths a_{ij} and assuming $m = m_n = m_p$

Low-Density Limit II

- **zero temperature limit of consistency relations without scattering correlations**
  - $$C_{\omega} - C_{\sigma} = \frac{\pi}{2m}\left[a_{nn}(^{1}S_{0}) + a_{pp}(^{1}S_{0}) + a_{np}(^{1}S_{0}) + 3a_{np}(^{3}S_{1})\right]$$
  - $$C_{\rho} - C_{\delta} = \frac{\pi}{2m}\left[a_{nn}(^{1}S_{0}) + a_{pp}(^{1}S_{0}) - a_{np}(^{1}S_{0}) - 3a_{np}(^{3}S_{1})\right]$$

  with scattering lengths a_{ij} and assuming $m = m_n = m_p$

- **comparison of experiment with RMF parametrizations**

	exp.	DD2 [1] (ω, σ, ρ)	DD-MEδ [2] $(\omega, \sigma, \rho, \delta)$
$C_{\omega} - C_{\sigma}$ [fm^2]	-14.15	-5.39	-4.90
$C_{\rho} - C_{\delta}$ [fm^2]	-9.61	2.48	2.55

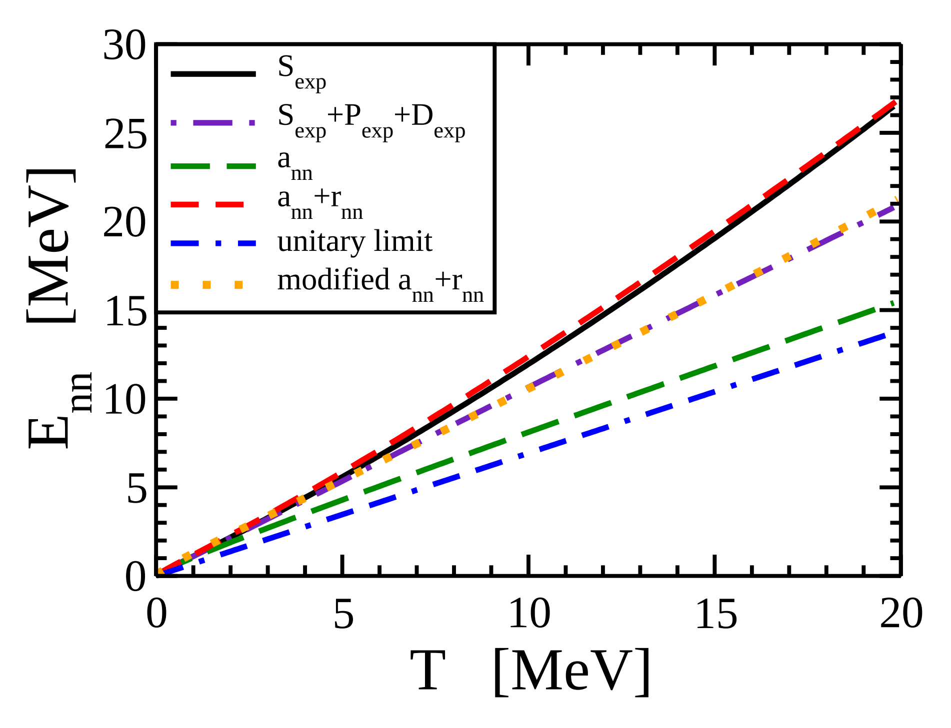
[1] S. Typel et al., Phys. Rev. C 81 (2010) 015803, [2] X. Roca-Maza et al., Phys. Rev. C 84 (2011) 054309

$\Rightarrow$ conventional mean-field models don't reproduce effect of correlations at very-low densities

$\Rightarrow$ explicit scattering correlations needed

NN Scattering Correlations

- **effective resonance energies** $$\sum_l g_l^{(ij)} \int \frac{dE}{\pi} \frac{d\delta_l^{(ij)}}{dE} \exp\left(-\frac{E}{T}\right) = \pm g_0^{(ij)} \exp\left(-\frac{E_{ij}}{T}\right)$$



effective-range expansion for s-wave phase shifts:

$$k \cot \delta_0^{(ij)} = -\frac{1}{a_{ij}} + \frac{r_{ij}}{2} k^2$$

$\Rightarrow$ analytical results

low T: $I_0^{(ij)}(T) \to -a_{ij}\sqrt{\mu_{ij} T/(2\pi)}$

unitary limit: $E_{ij}(T) = T \ln 2$

NN Scattering Correlations

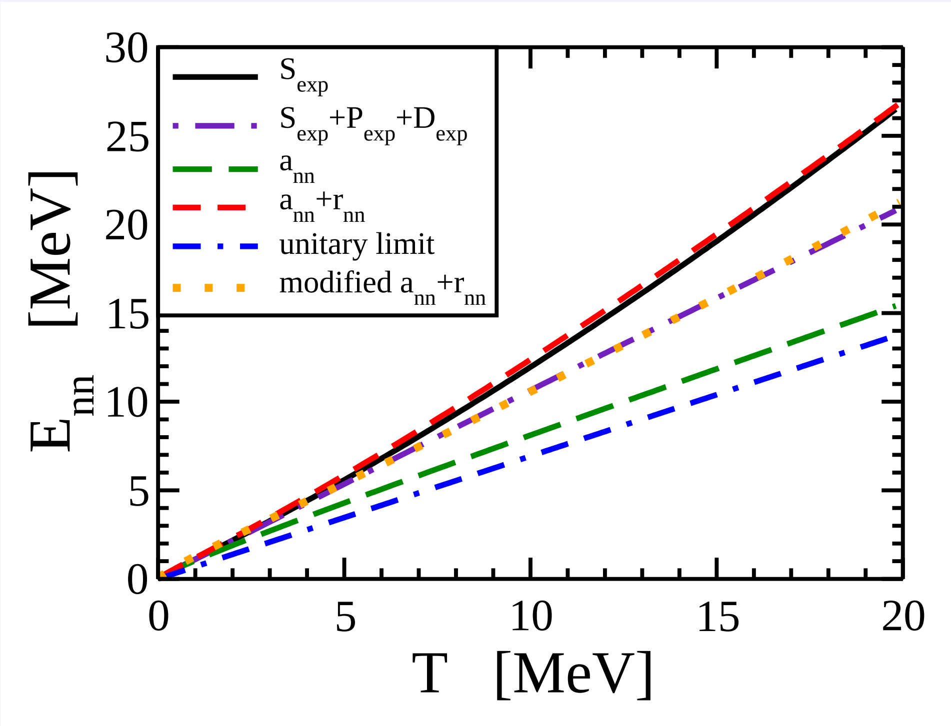
∘ **effective resonance energies**

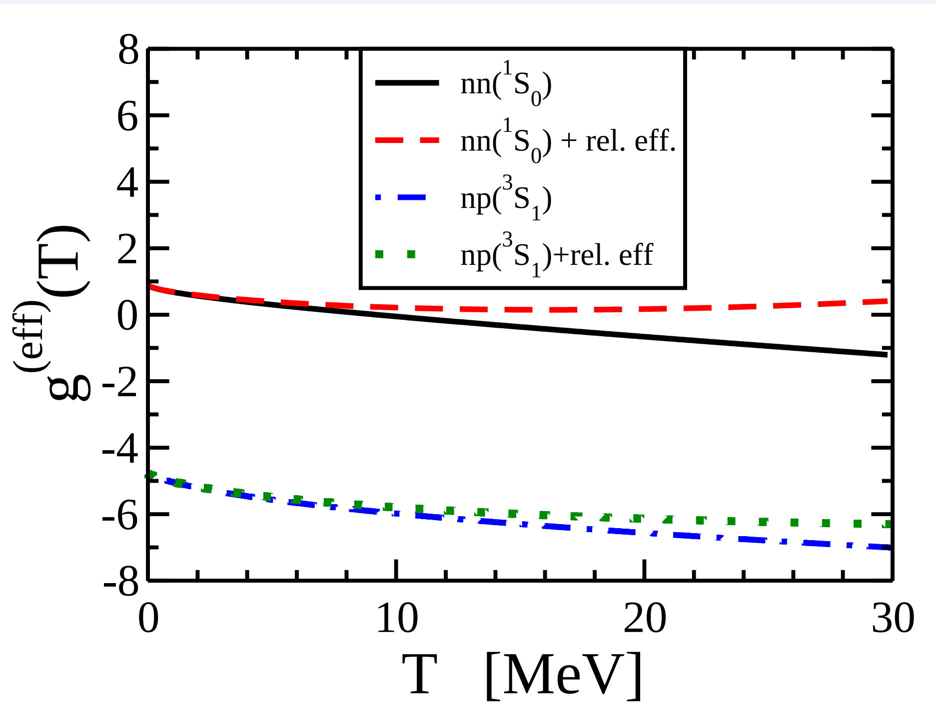
$$\sum_l g_l^{(ij)} \int \frac{dE}{\pi} \frac{d\delta_l^{(ij)}}{dE} \exp\left(-\frac{E}{T}\right) = \pm g_0^{(ij)} \exp\left(-\frac{E_{ij}}{T}\right)$$

∘ **effective degeneracy factors**

$$\sum_l g_l^{(nn)} \int \frac{dE}{\pi} \frac{d\delta_l^{(nn)}}{dE} \exp\left(-\frac{E}{T}\right) = g_{nn}^{(\mathrm{eff})}(T) \exp\left(-\frac{E_{nn}}{T}\right) - g_n^2 \frac{\lambda_{nn}^3}{\lambda_n^6} \frac{C_+}{2T}$$

$$C_+ = C\omega - C\sigma + C\rho - C\delta \,, \qquad \lambda_i = \sqrt{2\pi/(m_i T)}$$



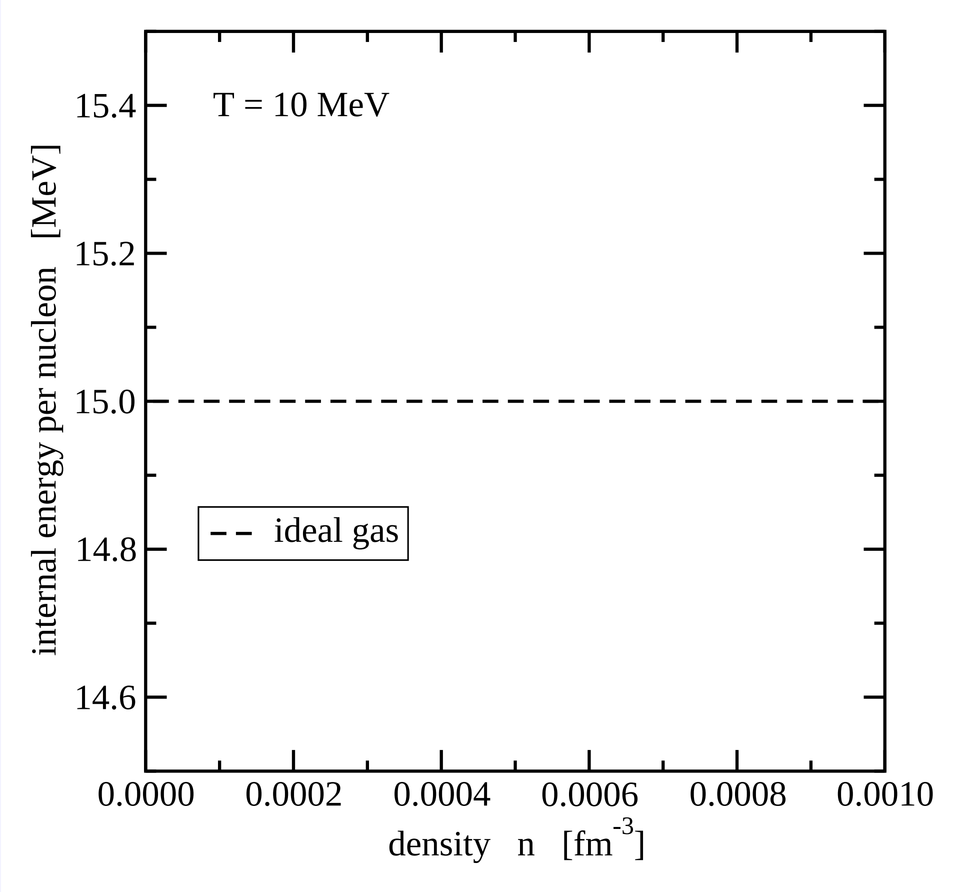


Neutron Matter at Low Densities I

comparison: different effects

- nonrelativistic ideal gas

internal energy per nucleon E/A
(ideal gas: $E/A = 3T/2$)

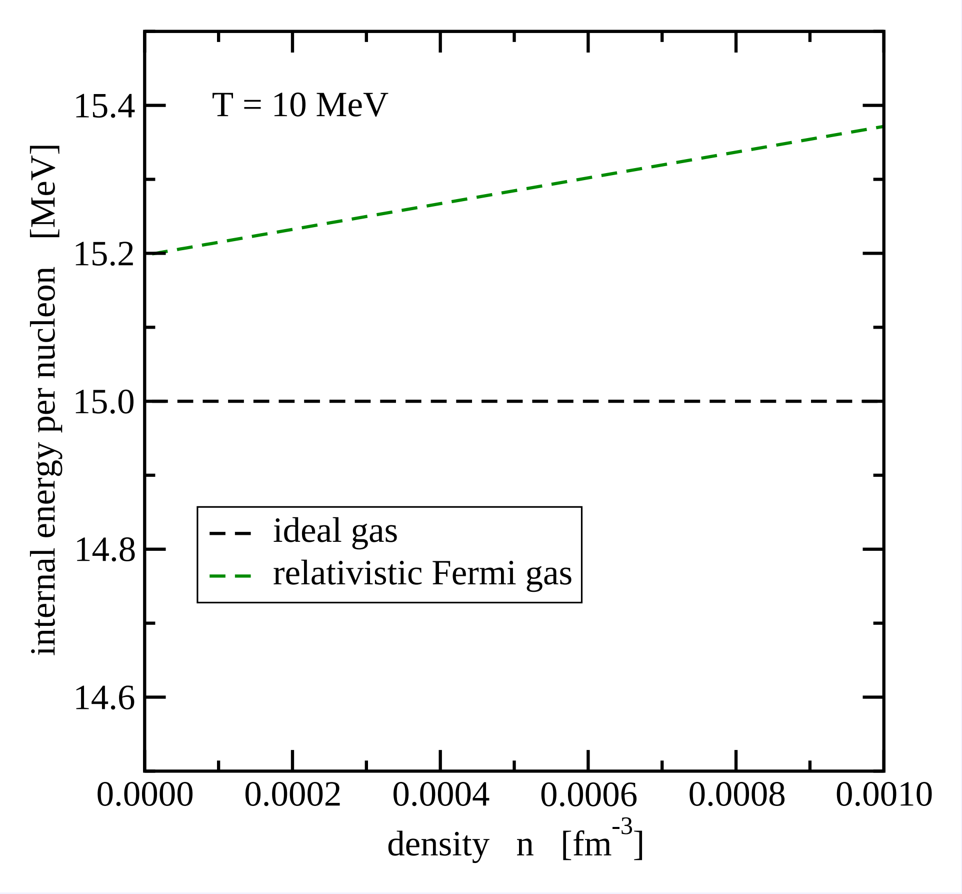


Neutron Matter at Low Densities I

comparison: different effects

- nonrelativistic ideal gas
  $\Downarrow$ rel. kinematics + quantum statistics
- relativistic Fermi gas

internal energy per nucleon E/A
(ideal gas: $E/A = 3T/2$)



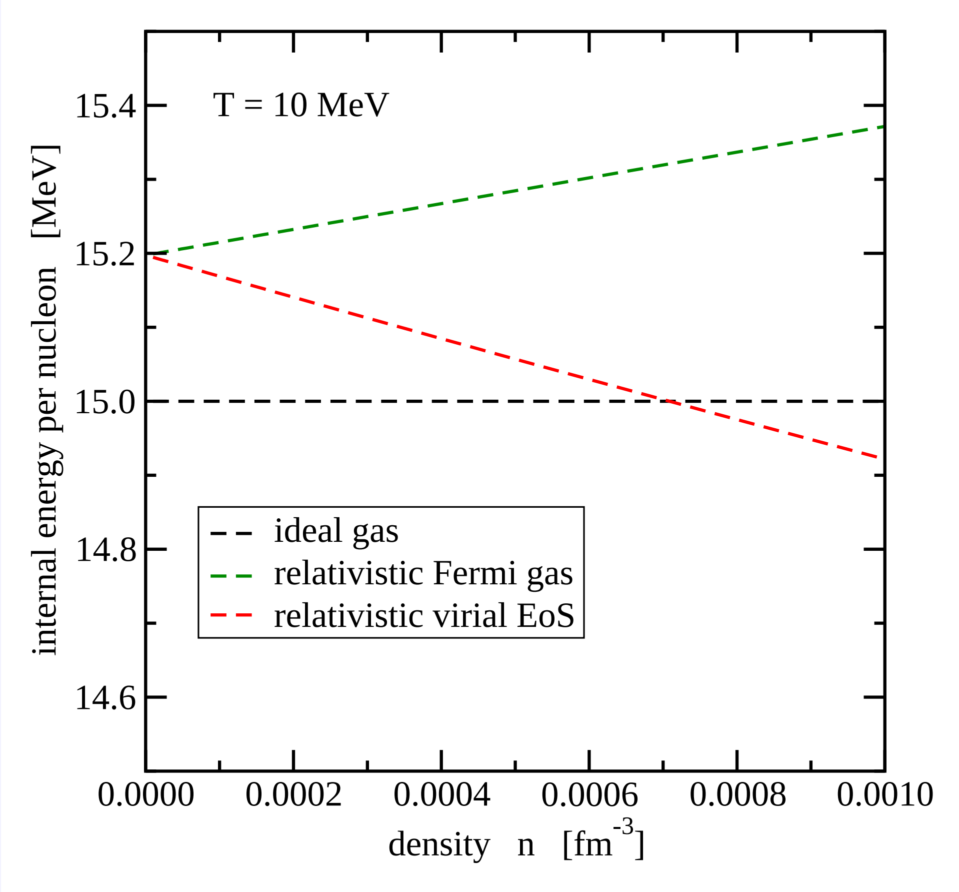
Neutron Matter at Low Densities I

comparison: different effects

- nonrelativistic ideal gas
  $\Downarrow$ rel. kinematics + quantum statistics
- relativistic Fermi gas

  $\Downarrow$ two-body correlations
- virial EoS with relativistic correction

internal energy per nucleon E/A
(ideal gas: $E/A = 3T/2$)



 Stefan Typel

Neutron Matter at Low Densities I

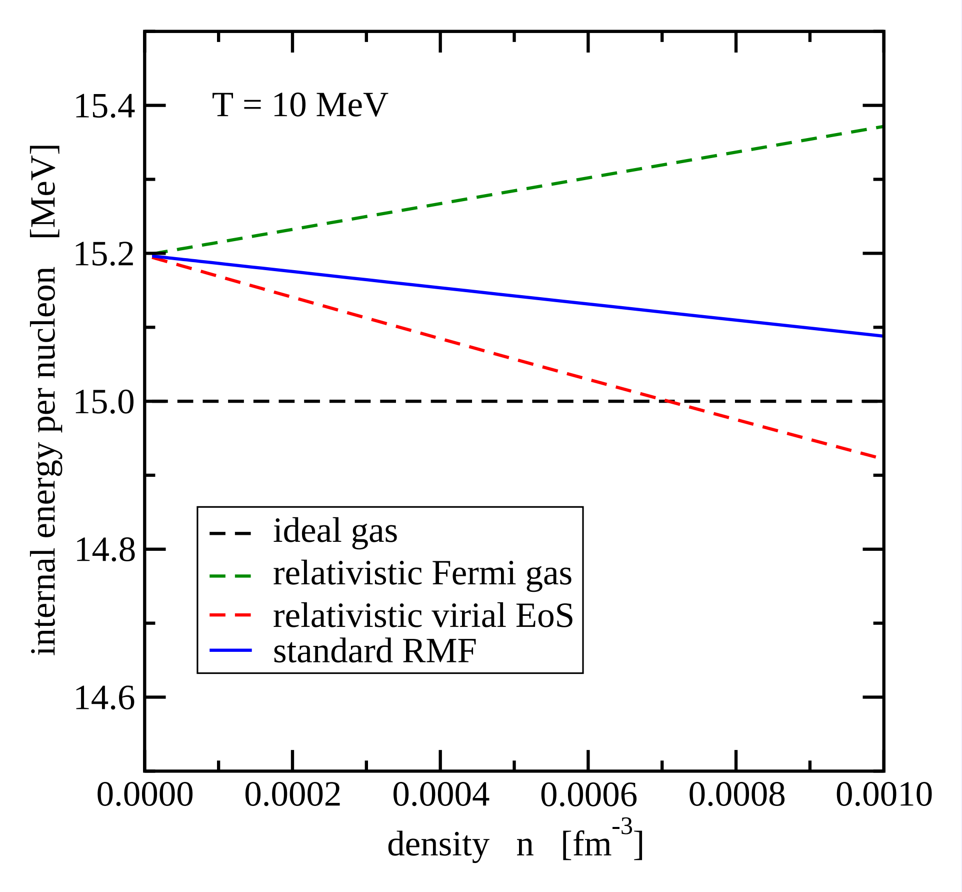
comparison: different effects

- nonrelativistic ideal gas
  ⇓ rel. kinematics + quantum statistics
- relativistic Fermi gas

  ⇓ two-body correlations
- virial EoS with relativistic correction
  (not included in standard virial EoS)

  ⇓ mean-field effects
- standard RMF model with
  density dependent couplings

internal energy per nucleon E/A
(ideal gas: $E/A = 3T/2$)



Neutron Matter at Low Densities I

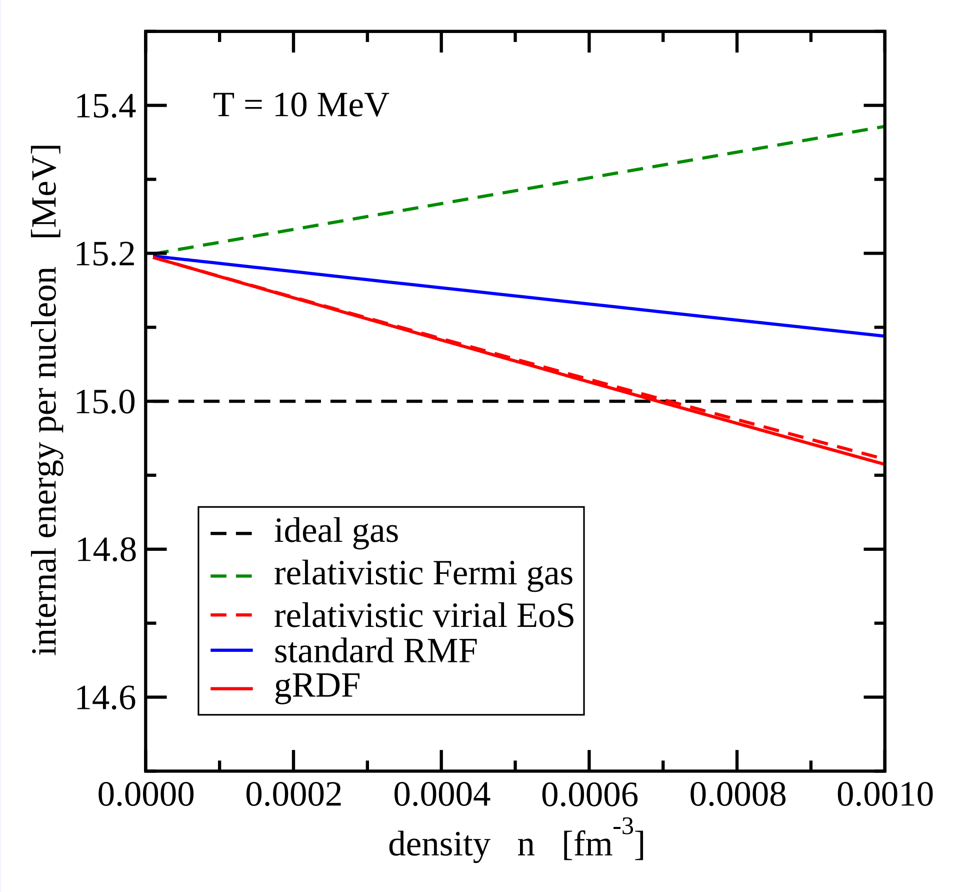
comparison: different effects

- nonrelativistic ideal gas
  $\Downarrow$ rel. kinematics + quantum statistics
- relativistic Fermi gas

  $\Downarrow$ two-body correlations
- virial EoS with relativistic correction
  (not included in standard virial EoS)

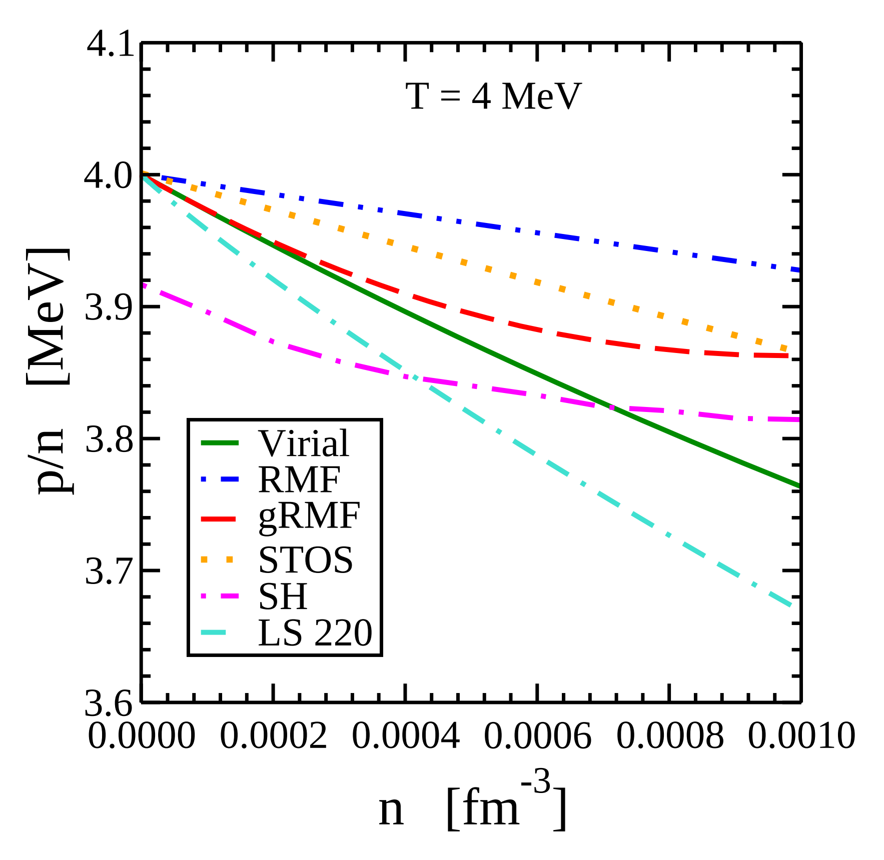
  $\Downarrow$ mean-field effects
- standard RMF model with
  density dependent couplings
  $\Downarrow$ two-body correlations
- generalized relativistic density
  functional (gRDF) with contributions
  from nn scattering

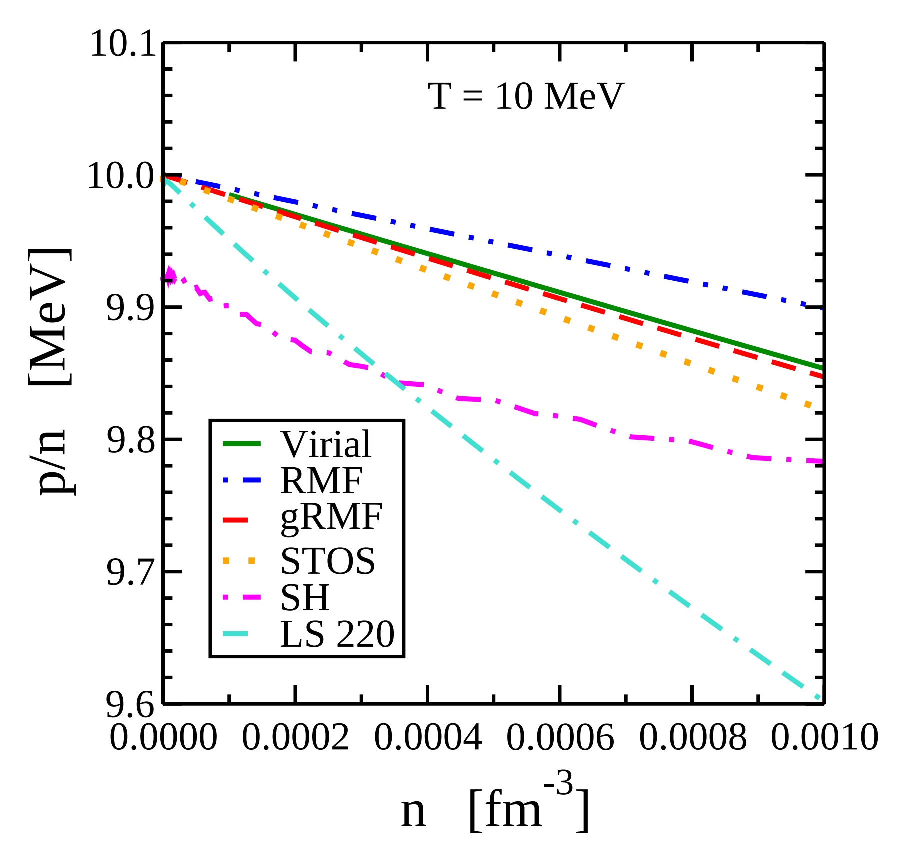
internal energy per nucleon E/A
(ideal gas: $E/A = 3T/2$)



Neutron Matter at Low Densities II

comparison: p/n in different models (ideal gas: $p/n = T$)





STOS: H. Shen et al., Nucl. Phys. A 637 (1998) 435 (TM1)

SH: G. Shen et al., Phys. Rev. C 83 (2011) 065808 (FSUGold)

LS 220: J.M. Lattimer et al., Nucl. Phys. A 535 (1991) 331 ($K = 220$ MeV)

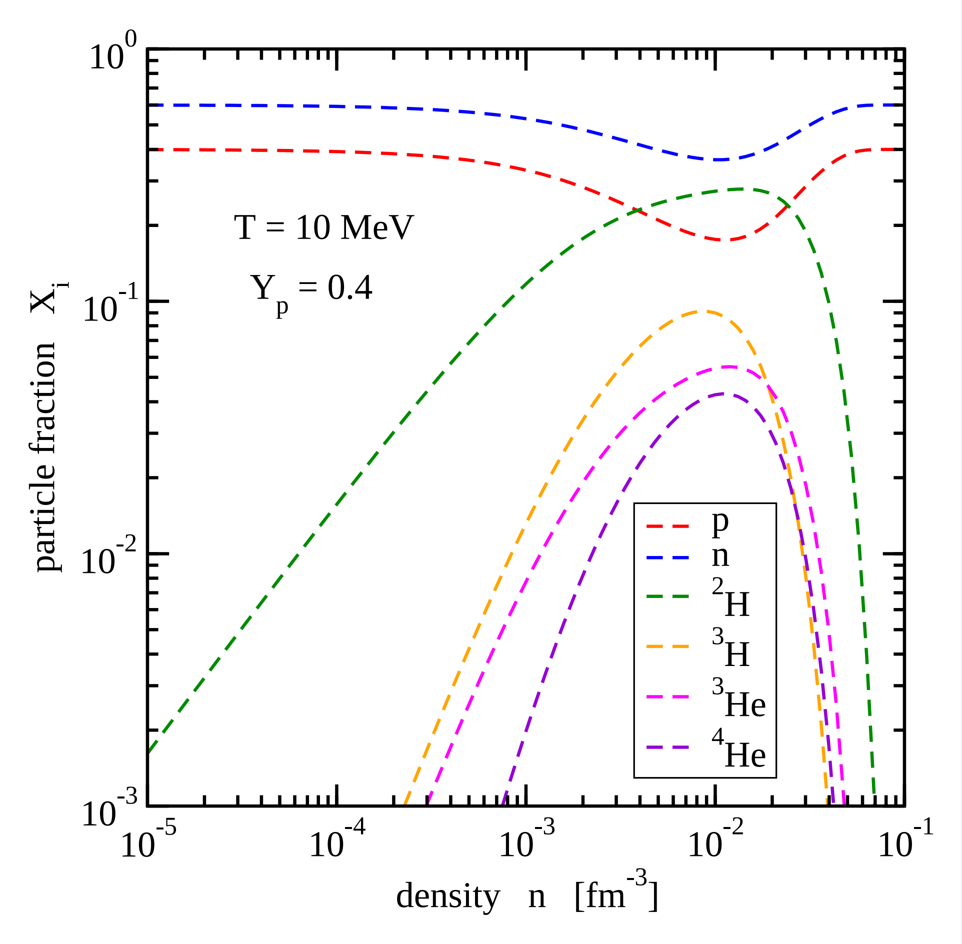
Light Clusters and Continuum Correlations

- particle fractions

$$X_i = A_i \frac{n_i}{n_b} \qquad n_b = \sum_i A_i n_i$$

- low densities:
  two-body correlations most important
- high densities:
  dissolution of clusters
  $\Rightarrow$ Mott effect

generalized relativistic density functional



(without heavy clusters)

Light Clusters and Continuum Correlations

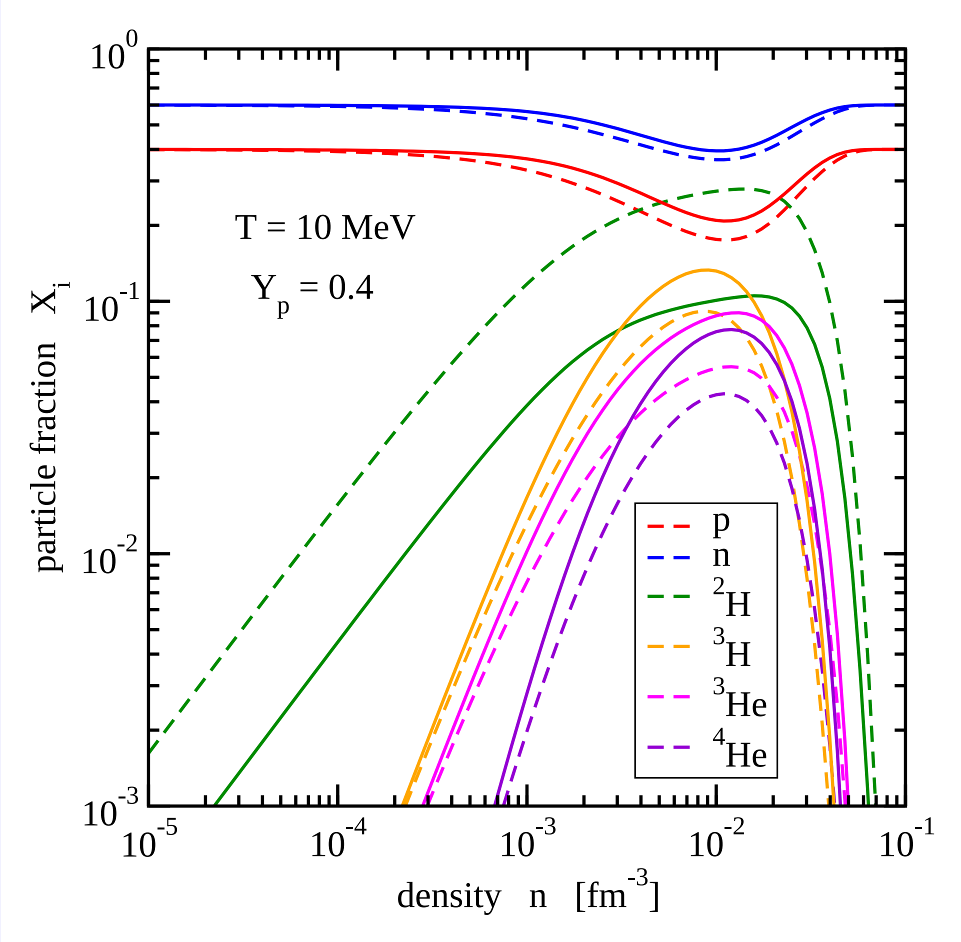
- particle fractions

$$X_i = A_i \frac{n_i}{n_b} \qquad n_b = \sum_i A_i n_i$$

- low densities:
  two-body correlations most important
- high densities:
  dissolution of clusters
  $\Rightarrow$ Mott effect
- effect of NN continuum correlations
  - ◦ dashed lines: without continuum
  - ◦ solid lines: with continuum

  $\Rightarrow$ reduction of deuteron fraction, redistribution of other particles
- correct limits with generalized relativistic density functional

generalized relativistic density functional



(without heavy clusters)

Coulomb Correlations in Matter

Coulomb Interaction in Matter

- explicit potential A_{γ} only in systems with spatially inhomogeneous charge distribution, homogeneous approaches for EoS $\Rightarrow$ effective treatment of Coulomb effects

Coulomb Interaction in Matter

- explicit potential A_{γ} only in systems with spatially inhomogeneous charge distribution, homogeneous approaches for EoS $\Rightarrow$ effective treatment of Coulomb effects
- crystal:
  lattice-periodic Coulomb potential $\longrightarrow$ potential in Wigner-Seitz approximation:
  single nucleus and electron background in spherical cell with
  size such that total charge vanishes $\Rightarrow$ screening of Coulomb potential

Coulomb Interaction in Matter

- explicit potential A_γ only in systems with spatially inhomogeneous charge distribution, homogeneous approaches for EoS $\Rightarrow$ effective treatment of Coulomb effects
- crystal:
  lattice-periodic Coulomb potential $\longrightarrow$ potential in Wigner-Seitz approximation:
  single nucleus and electron background in spherical cell with
  size such that total charge vanishes $\Rightarrow$ screening of Coulomb potential
- analytical solution for homogeneously charged sphere (ion, radius R, charge Qe) and constant electron density $n_e = 3/(4\pi R_e^3) = Qn_{\mathrm{ion}}$

  $\Rightarrow$ Coulomb energy $E_C^{(\mathrm{WS})} = E_C^{(\mathrm{sph})} + \Delta E_C^{(\mathrm{WS})}$

Coulomb Interaction in Matter

- explicit potential A_γ only in systems with spatially inhomogeneous charge distribution, homogeneous approaches for EoS $\Rightarrow$ effective treatment of Coulomb effects
- crystal:
  lattice-periodic Coulomb potential $\longrightarrow$ potential in Wigner-Seitz approximation:
  single nucleus and electron background in spherical cell with
  size such that total charge vanishes $\Rightarrow$ screening of Coulomb potential
- analytical solution for homogeneously charged sphere (ion, radius R, charge Qe)
  and constant electron density $n_e = 3/(4\pi R_e^3) = Q n_{\text{ion}}$

  $\Rightarrow$ Coulomb energy $E_C^{(\text{WS})} = E_C^{(\text{sph})} + \Delta E_C^{(\text{WS})}$ with

  $E_C^{(\text{sph})} = \frac{3}{5}\frac{Q^2 e^2}{R}$ part of energy of nucleus

  $\Delta E_C^{(\text{WS})} = -\frac{9}{10}\frac{Q^2 e^2}{R_e}\left(1 - \frac{R^2}{3R_e^2}\right)$ energy shift with finite-size correction

  $\Rightarrow$ approximation for lattice Coulomb energy,
  often applied in EoS models in liquid phase (?)

One-Component Plasma (OCP) I

- N ions (point particles, charge $Qe > 0$) in homogeneous background of electrons (density $n_e = 3/(4\pi a_e^3)$) at temperature T

Stefan Typel

One-Component Plasma (OCP) I

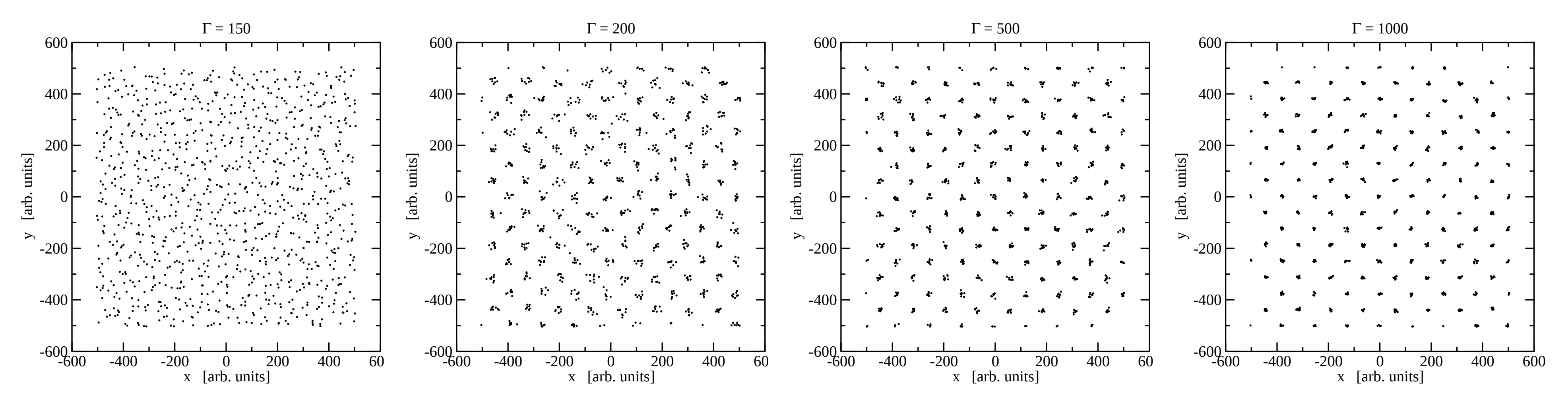
- N ions (point particles, charge $Qe > 0$) in homogeneous background of electrons (density $n_e = 3/(4\pi a_e^3)$) at temperature T
- classical model with screened Coulomb interaction between ions (calculation: Ewald method)
- internal energy of ions: $U_{\text{ion}} = U_{\text{kin}} + U_{\text{pot}}$ with $U_{\text{kin}} = \frac{3}{2}NT$

One-Component Plasma (OCP) I

- N ions (point particles, charge $Qe > 0$) in homogeneous background of electrons (density $n_e = 3/(4\pi a_e^3)$) at temperature T
- classical model with screened Coulomb interaction between ions (calculation: Ewald method)
- internal energy of ions: $U_{\text{ion}} = U_{\text{kin}} + U_{\text{pot}}$ with $U_{\text{kin}} = \frac{3}{2}NT$
- Monte Carlo simulation, only one relevant parameter $\Gamma = \dfrac{Q^2 e^2}{a_e T}$ for $U_{\text{pot}}/(NT)$

One-Component Plasma (OCP) I

- N ions (point particles, charge $Qe > 0$) in homogeneous background of electrons (density $n_e = 3/(4\pi a_e^3)$) at temperature T
- classical model with screened Coulomb interaction between ions (calculation: Ewald method)
- internal energy of ions: $U_{\text{ion}} = U_{\text{kin}} + U_{\text{pot}}$ with $U_{\text{kin}} = \frac{3}{2}NT$
- Monte Carlo simulation, only one relevant parameter $\Gamma = \dfrac{Q^2 e^2}{a_e T}$ for $U_{\text{pot}}/(NT)$
- example: 1024 ions in $8 \times 8 \times 8$ bcc lattice



One-Component Plasma (OCP) II

- limits:

  $\Gamma \to 0$: liquid phase $U_{\text{pot}}^{(L)}/(NT) \to -\frac{\sqrt{3}}{2}\Gamma^{3/2}$

  (Debye-Hückel)

  $\Gamma \to \infty$: solid phase $U_{\text{pot}}^{(S)}/(NT) \to \frac{3}{2} + C_M \Gamma$

  ($C_M^{(\text{bcc})} = -0.895929255682$ Madelung constant)

One-Component Plasma (OCP) II

- limits:

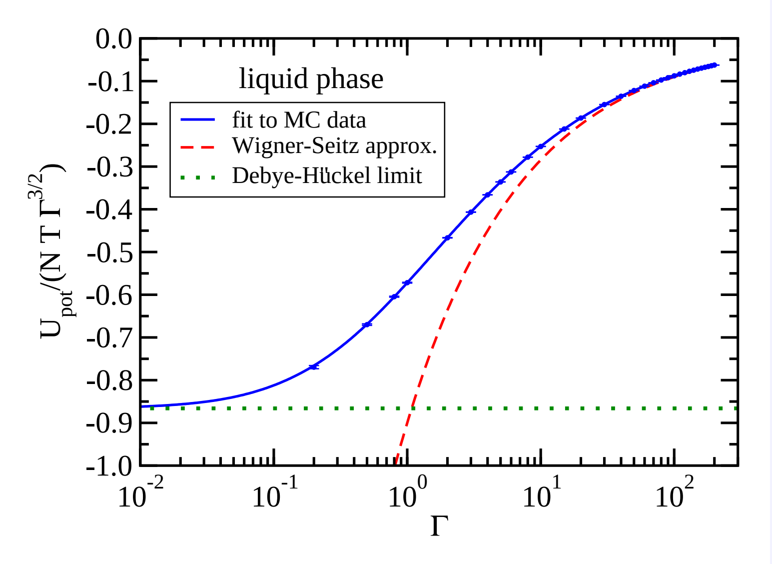
  $\Gamma \to 0$: liquid phase $U_{\text{pot}}^{(L)}/(NT) \to -\frac{\sqrt{3}}{2}\Gamma^{3/2}$

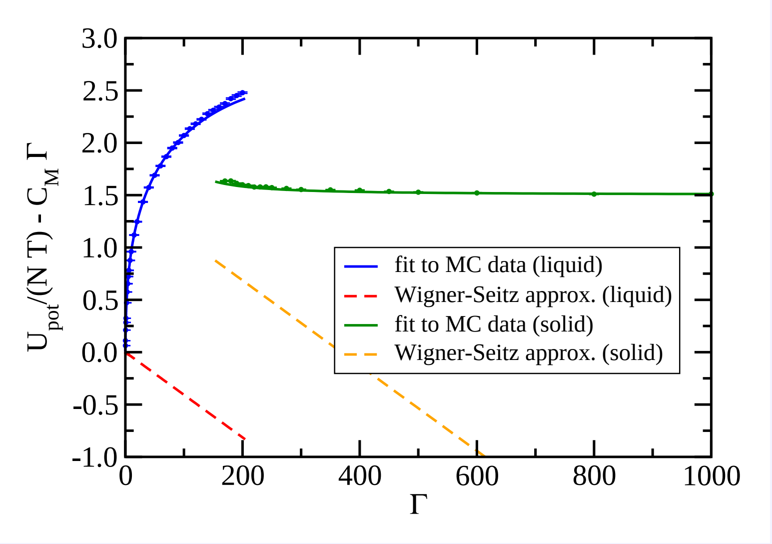
  (Debye-Hückel)

  $\Gamma \to \infty$: solid phase $U_{\text{pot}}^{(S)}/(NT) \to \frac{3}{2} + C_M\Gamma$

  ($C_M^{(\text{bcc})} = -0.895929255682$ Madelung constant)

- parametrization of Monte Carlo results





(parametrization: H.E. DeWitt and W. Slattery, Contrib. Plasma Phys. 39 (1999) 97)

One-Component Plasma (OCP) II

- limits:

  $\Gamma \to 0$: liquid phase $U_{\text{pot}}^{(L)}/(NT) \to -\frac{\sqrt{3}}{2}\Gamma^{3/2}$

  (Debye-Hückel)

  $\Gamma \to \infty$: solid phase $U_{\text{pot}}^{(S)}/(NT) \to \frac{3}{2} + C_M \Gamma$

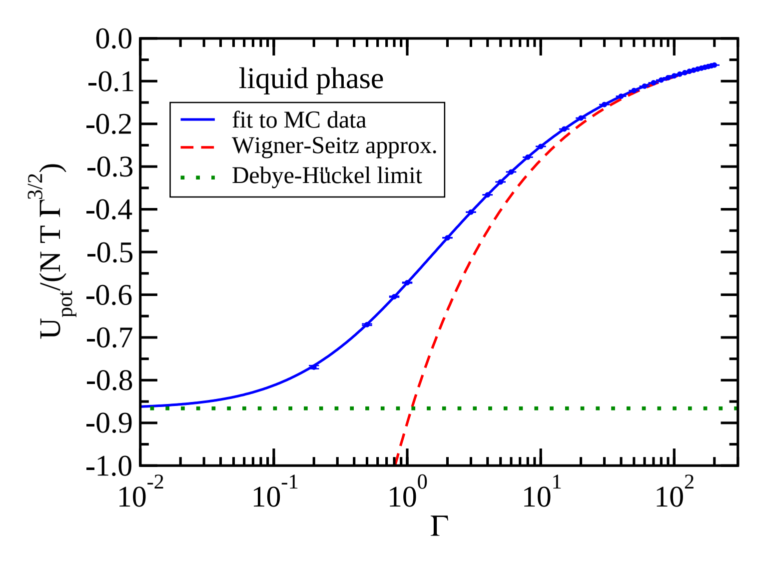
  ($C_M^{(\text{bcc})} = -0.895929255682$ Madelung constant)

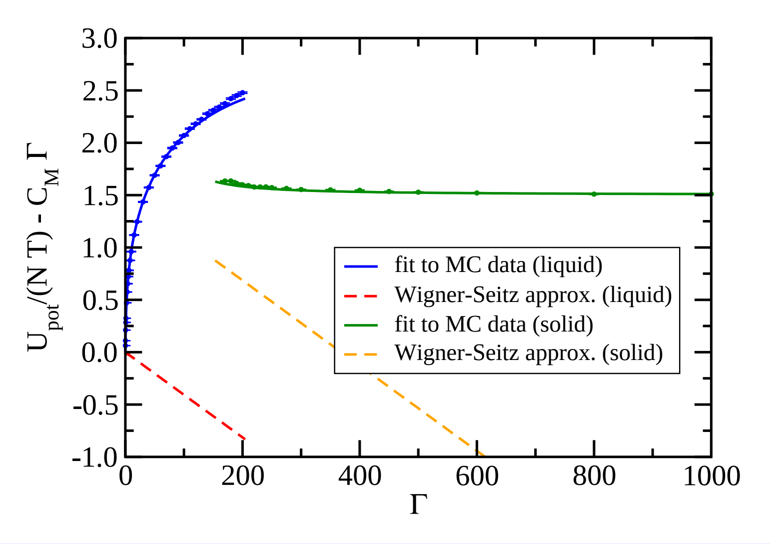
- parametrization of Monte Carlo results

- free energies: $F_{\text{pot}}^{(L)}$, $F_{\text{pot}}^{(S)}$ from integration

  $$\frac{F_{\text{pot}}^{(\text{L})}(\Gamma)}{NT} = \int_0^\Gamma \frac{d\Gamma'}{\Gamma'} \frac{U_{\text{pot}}(\Gamma')}{NT} \qquad \frac{F_{\text{pot}}^{(\text{S})}(\Gamma)}{NT} = \dots$$

  $\Rightarrow F^{(\text{L})}$, $F^{(\text{S})}$ (integration constants !)





(parametrization: H.E. DeWitt and W. Slattery, Contrib. Plasma Phys. 39 (1999) 97)

One-Component Plasma (OCP) II

- limits:

  $\Gamma \to 0$: liquid phase $U_{\rm pot}^{(L)}/(NT) \to -\frac{\sqrt{3}}{2}\Gamma^{3/2}$

  (Debye-Hückel)

  $\Gamma \to \infty$: solid phase $U_{\rm pot}^{(S)}/(NT) \to \frac{3}{2} + C_M \Gamma$

  ($C_M^{\rm (bcc)} = -0.895929255682$ Madelung constant)

- parametrization of Monte Carlo results

- free energies: $F_{\rm pot}^{(L)}$, $F_{\rm pot}^{(S)}$ from integration

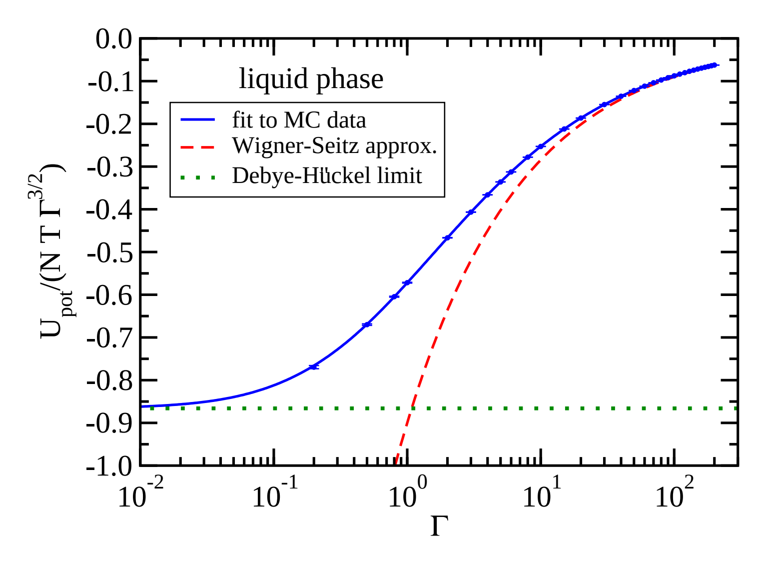
  $$\frac{F_{\rm pot}^{\rm (L)}(\Gamma)}{NT} = \int_0^\Gamma \frac{d\Gamma'}{\Gamma'} \frac{U_{\rm pot}(\Gamma')}{NT} \qquad \frac{F_{\rm pot}^{\rm (S)}(\Gamma)}{NT} = \dots$$

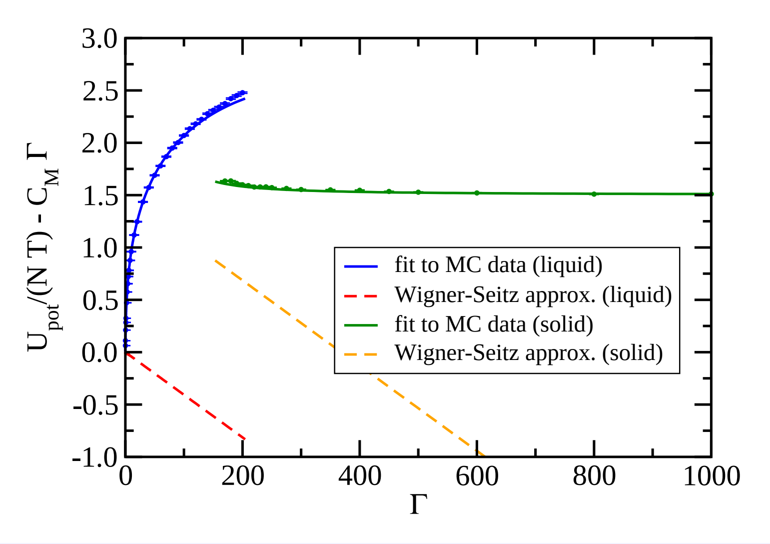
  $\Rightarrow F^{\rm (L)}$, $F^{\rm (S)}$ (integration constants !)

- melting point: $F^{\rm (L)}(\Gamma_m) = F^{\rm (S)}(\Gamma_m)$

  $\Rightarrow$ $\Gamma_m \approx 175$

  - very sensitive to Coulomb correlations
  - Wigner-Seitz approximation fails





(parametrization: H.E. DeWitt and W. Slattery, Contrib. Plasma Phys. 39 (1999) 97)

Gas/Liquid Phase I

constituents (i):

- baryons (n, p, Λ, Σ^+, Σ^0, Σ^-, Ξ^0, Ξ^-, . . .) $\Rightarrow$ fermions $(\sigma_i = +1)$
- mesons $(\pi^+/\pi^-$, π^0, K^+/K^-, $K^0/\bar{K}^0$, ω, ρ, . . .) $\Rightarrow$ bosons $(\sigma_i = -1)$
- light nuclei (^{2}H, ^{3}H, ^{3}He, ^{4}He) $\Rightarrow$ fermions/bosons
- heavy nuclei $(^{A_i}Z_i)$, NN scattering correlations $\Rightarrow$ classical particles $(\sigma_i = 0)$
- leptons $(e^-/e^+$, μ^-/μ^+, $\nu_e/\bar{\nu}_e$, $\nu_\mu/\bar{\nu}_\mu$, . . .) $\Rightarrow$ fermions
- photons (γ) $\Rightarrow$ bosons

Gas/Liquid Phase I

constituents (i):

- baryons (n, p, Λ, Σ^{+}, Σ^{0}, Σ^{-}, Ξ^{0}, Ξ^{-}, . . .) $\Rightarrow$ fermions ($\sigma_i = +1$)
- mesons (π^{+}/π^{-}, π^{0}, K^{+}/K^{-}, $K^{0}/\bar{K}^{0}$, ω, ρ, . . .) $\Rightarrow$ bosons ($\sigma_i = -1$)
- light nuclei (^{2}H, ^{3}H, ^{3}He, ^{4}He) $\Rightarrow$ fermions/bosons
- heavy nuclei ($^{A_i}Z_i$), NN scattering correlations $\Rightarrow$ classical particles ($\sigma_i = 0$)
- leptons (e^{-}/e^{+}, μ^{-}/μ^{+}, $\nu_e/\bar{\nu}_e$, $\nu_\mu/\bar{\nu}_\mu$, . . .) $\Rightarrow$ fermions
- photons (γ) $\Rightarrow$ bosons

◦ consider particles ($\eta_i = +1$) and antiparticles ($\eta_i = -1$)

◦ degeneracy factors g_i

◦ distinguish individual constituents ($g_i =$ const.$, i \in \mathcal{I}$)
and effective constituents ($g_i(T, n_j), i \in \mathcal{E}$)

Gas/Liquid Phase I

constituents (i):

- baryons (n, p, Λ, Σ^+, Σ^0, Σ^-, Ξ^0, Ξ^-, ...) $\Rightarrow$ fermions $(\sigma_i = +1)$
- mesons $(\pi^+/\pi^-$, π^0, K^+/K^-, $K^0/\bar{K}^0$, ω, ρ, ...) $\Rightarrow$ bosons $(\sigma_i = -1)$
- light nuclei (^{2}H, ^{3}H, ^{3}He, ^{4}He) $\Rightarrow$ fermions/bosons
- heavy nuclei $(^{A_i}Z_i)$, NN scattering correlations $\Rightarrow$ classical particles $(\sigma_i = 0)$
- leptons $(e^-/e^+$, μ^-/μ^+, $\nu_e/\bar{\nu}_e$, $\nu_\mu/\bar{\nu}_\mu$, ...) $\Rightarrow$ fermions
- photons (γ) $\Rightarrow$ bosons

- ○ consider particles $(\eta_i = +1)$ and antiparticles $(\eta_i = -1)$
- ○ degeneracy factors g_i
- ○ distinguish individual constituents $(g_i = \text{const.}, i \in \mathcal{I})$ and effective constituents $(g_i(T, n_j), i \in \mathcal{E})$
- ○ quasi-particles with relativistic energy

$$e_i^{(\eta_i)}(k) = \sqrt{k^2 + (m_i - S_i)^2} + \eta_i V_i$$

S_i scalar potential, V_i vector potential
m_i rest mass in vacuum, k momentum

Gas/Liquid Phase II

interaction

- Lorentz scalar mesons $m \in \mathcal{S} = \{\sigma, \delta, \sigma_*, \ldots\}$
- Lorentz vector mesons $m \in \mathcal{V} = \{\omega, \rho, \phi, \ldots\}$

Gas/Liquid Phase II

interaction

- Lorentz scalar mesons $m \in \mathcal{S} = \{\sigma, \delta, \sigma_*, \ldots\}$
- Lorentz vector mesons $m \in \mathcal{V} = \{\omega, \rho, \phi, \ldots\}$

◦ represented by (classical) fields A_m with mass m_m

◦ coupling to constituents: $\Gamma_{im} = g_{im}\Gamma_m$
with scaling factors g_{im} and density dependent $\Gamma_m = \Gamma_m(\varrho)$, $\varrho = \sum_i B_i n_i$

Gas/Liquid Phase II

interaction

- Lorentz scalar mesons $m \in \mathcal{S} = \{\sigma, \delta, \sigma_*, \ldots\}$
- Lorentz vector mesons $m \in \mathcal{V} = \{\omega, \rho, \phi, \ldots\}$

- ◦ represented by (classical) fields A_m with mass m_m
- ◦ coupling to constituents: $\Gamma_{im} = g_{im}\Gamma_m$
  with scaling factors g_{im} and density dependent $\Gamma_m = \Gamma_m(\varrho)$, $\varrho = \sum_i B_i n_i$
- ◦ scalar potential $S_i = \sum_{m \in \mathcal{S}} \Gamma_{im} n_m^{(\mathrm{source})} - \Delta m_i$
  with medium-dependent mass shift $\Delta m_i(T, n_j)$
- ◦ vector potential $V_i = \sum_{m \in \mathcal{V}} \Gamma_{im} n_m^{(\mathrm{source})} + V_i^{(\mathrm{em})} + V_i^{(r)}$

Gas/Liquid Phase II

interaction

- Lorentz scalar mesons $m \in \mathcal{S} = \{\sigma, \delta, \sigma_*, \dots\}$
- Lorentz vector mesons $m \in \mathcal{V} = \{\omega, \rho, \phi, \dots\}$

- represented by (classical) fields A_m with mass m_m
- coupling to constituents: $\Gamma_{im} = g_{im}\Gamma_m$
  with scaling factors g_{im} and density dependent $\Gamma_m = \Gamma_m(\varrho)$, $\varrho = \sum_i B_i n_i$
- scalar potential $S_i = \sum_{m \in \mathcal{S}} \Gamma_{im} n_m^{(\mathrm{source})} - \Delta m_i$

  with medium-dependent mass shift $\Delta m_i(T, n_j)$
- vector potential $V_i = \sum_{m \in \mathcal{V}} \Gamma_{im} n_m^{(\mathrm{source})} + V_i^{(\mathrm{em})} + V_i^{(r)}$

  with electromagnetic contribution $V_i^{(\mathrm{em})} = T f_L(\Gamma_i)$ from fit of OCP data

  assuming linear mixing rule ($\Gamma_i = Q_i^{5/3}\Gamma_Q$, $\Gamma_Q = e^2/(a_Q T)$, $a_Q = [3/(4\pi n_Q)]^{1/3}$)

  and rearrangement contribution $V_i^{(r)} = B_i V^{(r)} + U_i^{(\mathrm{mass})} + U_i^{(\mathrm{em})} + U_i^{(\mathrm{deg})}$

  $V^{(r)} = \sum_{m \in \mathcal{V}} \Gamma'_m A_m n_m^{(\mathrm{source})} - \sum_{m \in \mathcal{S}} \Gamma'_m A_m n_m^{(\mathrm{source})}$, $\Gamma'_m = d\Gamma_m/d\varrho$

Gas/Liquid Phase III

effective density functional

- grand canonical potential density $\omega^{(L)} = \omega^{(L)}_{\mathrm{qp}} + \omega^{(L)}_{\mathrm{strong}} + \omega^{(L)}_{\mathrm{em}}$

Gas/Liquid Phase III

effective density functional

- grand canonical potential density $\omega^{(L)} = \omega_{\mathrm{qp}}^{(L)} + \omega_{\mathrm{strong}}^{(L)} + \omega_{\mathrm{em}}^{(L)}$
- contribution of quasi-particles

$$\omega_{\mathrm{qp}}^{(L)} = \sum_{i \in \mathcal{I}} g_i \left(\omega_i^{(r)} + \omega_i^{(p)} \delta_{\sigma_i,+1} + \omega_i^{(c)} \delta_{\sigma_i,-1} \right) + \sum_{i \in \mathcal{E}} \left(g_i \omega_i^{(r)} - U_i^{(\mathrm{deg})} n_i \right)$$

Gas/Liquid Phase III

effective density functional

- grand canonical potential density $$\omega^{(L)} = \omega^{(L)}_{\text{qp}} + \omega^{(L)}_{\text{strong}} + \omega^{(L)}_{\text{em}}$$
- contribution of quasi-particles
$$\omega^{(L)}_{\text{qp}} = \sum_{i \in \mathcal{I}} g_i \left(\omega_i^{(r)} + \omega_i^{(p)} \delta_{\sigma_i,+1} + \omega_i^{(c)} \delta_{\sigma_i,-1} \right) + \sum_{i \in \mathcal{E}} \left(g_i \omega_i^{(r)} - U_i^{(\text{deg})} n_i \right)$$
    - regular contribution $$\omega_i^{(r)} = -\frac{T}{\sigma_i} \int \frac{d^3k}{(2\pi)^3} \sum_{\eta_i} \ln[1 + \sigma_i \exp(-E_i^{(\eta_i)}/T)]$$
      with $E_i^{(\eta_i)} = e_i^{(\eta_i)} - \mu_i$
    - pairing contribution $\omega_i^{(p)} = \ldots$
    - condensate contribution $\omega_i^{(c)} = \ldots$

Gas/Liquid Phase III

effective density functional

- grand canonical potential density $\omega^{(L)} = \omega^{(L)}_{\text{qp}} + \omega^{(L)}_{\text{strong}} + \omega^{(L)}_{\text{em}}$
- contribution of quasi-particles

$$\omega^{(L)}_{\text{qp}} = \sum_{i\in\mathcal{I}} g_i \left(\omega_i^{(r)} + \omega_i^{(p)}\delta_{\sigma_i,+1} + \omega_i^{(c)}\delta_{\sigma_i,-1}\right) + \sum_{i\in\mathcal{E}} \left(g_i\omega_i^{(r)} - U_i^{(\text{deg})} n_i\right)$$

  - regular contribution $\omega_i^{(r)} = -\frac{T}{\sigma_i}\int \frac{d^3k}{(2\pi)^3}\sum_{\eta_i} \ln[1+\sigma_i \exp(-E_i^{(\eta_i)}/T)]$

    with $E_i^{(\eta_i)} = e_i^{(\eta_i)} - \mu_i$
  - pairing contribution $\omega_i^{(p)} = \ldots$
  - condensate contribution $\omega_i^{(c)} = \ldots$
- contribution from strong interaction

$$\omega^{(L)}_{\text{strong}} = \sum_{m\in\mathcal{S}} m_m^2 A_m^2 - \sum_{m\in\mathcal{V}} m_m^2 A_m^2 - V^{(r)}\varrho - \sum_{i\in\mathcal{I}\cup\mathcal{E}} U_i^{(\text{mass})} n_i$$

Gas/Liquid Phase III

effective density functional

- grand canonical potential density $\omega^{(L)} = \omega^{(L)}_{\text{qp}} + \omega^{(L)}_{\text{strong}} + \omega^{(L)}_{\text{em}}$
- contribution of quasi-particles

$$\omega^{(L)}_{\text{qp}} = \sum_{i\in\mathcal{I}} g_i \left(\omega_i^{(r)} + \omega_i^{(p)}\delta_{\sigma_i,+1} + \omega_i^{(c)}\delta_{\sigma_i,-1}\right) + \sum_{i\in\mathcal{E}} \left(g_i\omega_i^{(r)} - U_i^{(\text{deg})} n_i\right)$$

  - regular contribution $\omega_i^{(r)} = -\frac{T}{\sigma_i}\int \frac{d^3k}{(2\pi)^3}\sum_{\eta_i} \ln[1+\sigma_i \exp(-E_i^{(\eta_i)}/T)]$

    with $E_i^{(\eta_i)} = e_i^{(\eta_i)} - \mu_i$
  - pairing contribution $\omega_i^{(p)} = \ldots$
  - condensate contribution $\omega_i^{(c)} = \ldots$
- contribution from strong interaction

$$\omega^{(L)}_{\text{strong}} = \sum_{m\in\mathcal{S}} m_m^2 A_m^2 - \sum_{m\in\mathcal{V}} m_m^2 A_m^2 - V^{(r)}\varrho - \sum_{i\in\mathcal{I}\cup\mathcal{E}} U_i^{(\text{mass})} n_i$$

- contribution from electromagnetic interaction

$$\omega^{(L)}_{\text{em}} = -\sum_{i\in\mathcal{I}\cup\mathcal{E}} U_i^{(\text{em})} n_i$$

Gas/Liquid Phase IV

fermions $\Rightarrow$ pairing correlations

- pairing potential $v_i(k, k')$

Gas/Liquid Phase IV

fermions $\Rightarrow$ pairing correlations

- pairing potential $v_i(k,k')$
- pairing contribution to $\omega_{\text{qp}}^{(L)}$

$$\omega_i^{(p)} = \int \frac{d^3k}{(2\pi)^3} \sum_{\eta_i} \{\tfrac{1}{2}[e_i^{(\eta_i)}(k) - \mu_i - E_i^{(\eta_i)}(k)] + \Delta_i^{(\eta_i)}(k)\nu_i^{(\eta_i)}(k)\}$$

$$+\tfrac{1}{2} \int \frac{d^3k}{(2\pi)^3} \int \frac{d^3k'}{(2\pi)^3} \sum_{\eta_i} \nu_i^{(\eta_i)}(k) v_i^{(\eta_i)}(k,k') \nu_i^{(\eta_i)}(k')$$

$E_i^{(\eta_i)} = \pm\sqrt{[e_i^{(\eta_i)} - \mu_i]^2 + [\Delta_i^{(\eta_i)}]^2}$, $\Delta_i^{(\eta_i)}(k)$ pairing gap

$\nu_i^{(\eta_i)}(k) = \frac{\Delta_i^{(\eta_i)}(k)}{2E_i^{(\eta_i)}(k)}[1 - 2f_{+1}(E_i^{(\eta_i)})]$ anomalous distribution function,

$f_{+1}(E) = [\exp(E) + 1]^{-1}$ Fermi-Dirac distribution function

Gas/Liquid Phase IV

fermions $\Rightarrow$ pairing correlations

- pairing potential $v_i(k,k')$
- pairing contribution to $\omega_{\mathrm{qp}}^{(L)}$

$$\omega_i^{(p)} = \int \frac{d^3k}{(2\pi)^3} \sum_{\eta_i} \{\tfrac{1}{2}[e_i^{(\eta_i)}(k) - \mu_i - E_i^{(\eta_i)}(k)] + \Delta_i^{(\eta_i)}(k)\nu_i^{(\eta_i)}(k)\}$$

$$+\tfrac{1}{2} \int \frac{d^3k}{(2\pi)^3} \int \frac{d^3k'}{(2\pi)^3} \sum_{\eta_i} \nu_i^{(\eta_i)}(k) v_i^{(\eta_i)}(k,k') \nu_i^{(\eta_i)}(k')$$

$E_i^{(\eta_i)} = \pm\sqrt{[e_i^{(\eta_i)} - \mu_i]^2 + [\Delta_i^{(\eta_i)}]^2}$, $\Delta_i^{(\eta_i)}(k)$ pairing gap

$\nu_i^{(\eta_i)}(k) = \frac{\Delta_i^{(\eta_i)}(k)}{2E_i^{(\eta_i)}(k)}[1 - 2f_{+1}(E_i^{(\eta_i)})]$ anomalous distribution function,

$f_{+1}(E) = [\exp(E) + 1]^{-1}$ Fermi-Dirac distribution function

- $\partial\omega^{(L)}/\partial\Delta_i^{(\eta_i)}(k) = 0 \Rightarrow$ gap equation

$$\Delta_i^{(\eta_i)}(k) + \int \frac{d^3k'}{(2\pi)^3} v_i^{(\eta_i)}(k,k')\nu_i^{(\eta_i)}(k') = 0$$

Gas/Liquid Phase V

bosons $\Rightarrow$ condensation

- condensate contribution to $\omega_{\rm qp}^{(L)}$

$$\omega_i^{(c)} = \frac{1}{2}[\zeta_i^{(\eta_i)}]^2[(m_i - S_i)^2 - (\mu_i - V_i)^2]$$

with parameter $\zeta_i^{(\eta_i)}$

Gas/Liquid Phase V

bosons $\Rightarrow$ condensation

- condensate contribution to $\omega_{\mathrm{qp}}^{(L)}$

  $$\omega_i^{(c)} = \frac{1}{2}[\zeta_i^{(\eta_i)}]^2[(m_i - S_i)^2 - (\mu_i - V_i)^2]$$

  with parameter $\zeta_i^{(\eta_i)}$

- general condition on chemical potential μ_i

  $$|\mu_i - V_i| \leq m_i - S_i$$

Gas/Liquid Phase V

bosons $\Rightarrow$ condensation

- condensate contribution to $\omega_{\text{qp}}^{(L)}$

  $$\omega_i^{(c)} = \frac{1}{2}[\zeta_i^{(\eta_i)}]^2[(m_i - S_i)^2 - (\mu_i - V_i)^2]$$

  with parameter $\zeta_i^{(\eta_i)}$

- general condition on chemical potential μ_i

  $$|\mu_i - V_i| \leq m_i - S_i$$

- $\partial\omega^{(L)}/\partial\zeta_i^{(\eta_i)} = 0 \;\; \Rightarrow$ condition for condensation

  solutions:
  - $\zeta_i^{(\eta_i)} = 0$: no condensation
  - $\zeta_i^{(\eta_i)} \neq 0$, $\mu_i = V_i + m_i - S_i$: condensation of particles
  - $\zeta_i^{(\eta_i)} \neq 0$, $\mu_i = V_i - m_i + S_i$: condensation of antiparticles

  value of $\zeta_i^{(\eta_i)}$ determined by density of condensate state

Gas/Liquid Phase VI

densities $\Rightarrow$ usual form for quasiparticles

- net particle density

$$n_i = g_i \sum_{\eta_i} \{ \int \frac{d^3k}{(2\pi)^3} \eta_i f_i^{(\eta_i)}(k) + [\zeta_i^{(\eta_i)}]^2 (\mu_i - V_i) \delta_{\sigma_i,-1} \}$$

$$f_i^{(\eta_i)} = \frac{1}{2} \{ 1 - \frac{e_i^{(\eta_i)} - \mu_i}{E_i^{(\eta_i)}} [1 - 2 f_{\sigma_i}(E_i^{(\eta_i)})] \}, \; f_\sigma(E) = [\exp(E) + \sigma]^{-1}$$

Gas/Liquid Phase VI

densities $\Rightarrow$ usual form for quasiparticles

- net particle density

$$n_i = g_i \sum_{\eta_i} \{ \int \frac{d^3k}{(2\pi)^3} \eta_i f_i^{(\eta_i)}(k) + [\zeta_i^{(\eta_i)}]^2 (\mu_i - V_i) \delta_{\sigma_i,-1} \}$$

$$f_i^{(\eta_i)} = \frac{1}{2}\{1 - \frac{e_i^{(\eta_i)} - \mu_i}{E_i^{(\eta_i)}} [1 - 2 f_{\sigma_i}(E_i^{(\eta_i)})]\}, \; f_\sigma(E) = [\exp(E) + \sigma]^{-1}$$

- net scalar density

$$n_i^{(s)} = g_i \sum_{\eta_i} \{ \int \frac{d^3k}{(2\pi)^3} \frac{m_i - S_i}{\sqrt{k^2 + (m_i - S_i)^2}} f_i^{(\eta_i)}(k) + [\zeta_i^{(\eta_i)}]^2 (m_i - S_i) \delta_{\sigma_i,-1} \}$$

Gas/Liquid Phase VI

densities $\Rightarrow$ usual form for quasiparticles

- net particle density

$$n_i = g_i \sum_{\eta_i} \{ \int \frac{d^3k}{(2\pi)^3} \eta_i f_i^{(\eta_i)}(k) + [\zeta_i^{(\eta_i)}]^2 (\mu_i - V_i) \delta_{\sigma_i,-1} \}$$

$$f_i^{(\eta_i)} = \frac{1}{2} \{ 1 - \frac{e_i^{(\eta_i)} - \mu_i}{E_i^{(\eta_i)}} [1 - 2 f_{\sigma_i}(E_i^{(\eta_i)})] \}, \; f_\sigma(E) = [\exp(E) + \sigma]^{-1}$$

- net scalar density

$$n_i^{(s)} = g_i \sum_{\eta_i} \{ \int \frac{d^3k}{(2\pi)^3} \frac{m_i - S_i}{\sqrt{k^2 + (m_i - S_i)^2}} f_i^{(\eta_i)}(k) + [\zeta_i^{(\eta_i)}]^2 (m_i - S_i) \delta_{\sigma_i,-1} \}$$

- source densities
  - Lorentz scalar mesons, $m \in \mathcal{S}$

$$n_m^{(\mathrm{source})} = \sum_{i \in \mathcal{I} \cup \mathcal{E}} g_{im} n_i^{(s)}$$

  - Lorentz vector mesons, $m \in \mathcal{V}$

$$n_m^{(\mathrm{source})} = \sum_{i \in \mathcal{I} \cup \mathcal{E}} g_{im} n_i$$

Gas/Liquid Phase VII

thermodynamic consistency

- natural variables of $\omega^{(L)}$: T, μ_i, A_m, $\Delta_i^{(\eta_i)}(k)$, $\zeta_i^{(\eta_i)}$

  but $\omega^{(L)}$ depends explicitly on densities n_i, $n_i^{(s)}$ (already defined!)

Gas/Liquid Phase VII

thermodynamic consistency

- natural variables of $\omega^{(L)}$: $T,\ \mu_i,\ A_m,\ \Delta_i^{(\eta_i)}(k),\ \zeta_i^{(\eta_i)}$

  but $\omega^{(L)}$ depends explicitly on densities n_i, $n_i^{(s)}$ (already defined!)

- consistency criterion $n_j \stackrel{!}{=} -\frac{\partial}{\partial \mu_j}\omega^{(L)}(T, \mu_i, A_m, \Delta_i^{(\eta_i)}(k), \zeta_i^{(\eta_i)})\Big|_{T,\mu_{i\neq j},A_m,\Delta_i^{(\eta_i)}(k),\zeta_i^{(\eta_i)}}$

Gas/Liquid Phase VII

thermodynamic consistency

- natural variables of $\omega^{(L)}$: $T,\ \mu_i,\ A_m,\ \Delta_i^{(\eta_i)}(k),\ \zeta_i^{(\eta_i)}$

  but $\omega^{(L)}$ depends explicitly on densities n_i, $n_i^{(s)}$ (already defined!)

- consistency criterion $n_j \overset{!}{=} -\frac{\partial}{\partial \mu_j}\omega^{(L)}(T, \mu_i, A_m, \Delta_i^{(\eta_i)}(k), \zeta_i^{(\eta_i)})\Big|_{T,\mu_{i\neq j},A_m,\Delta_i^{(\eta_i)}(k),\zeta_i^{(\eta_i)}}$

  $\Rightarrow$ definition of rearrangement potentials

  - $U_i^{(\text{mass})} = \sum_{j\in\mathcal{I}\cup\mathcal{E}} \frac{\partial \Delta m_j}{\partial n_i} n_j^{(s)}$
  - $U_i^{(\text{em})} = \sum_{j\in\mathcal{I}\cup\mathcal{E}} \frac{\partial V_j^{(em)}}{\partial n_i} n_j$
  - $U_i^{(\text{deg})} = \sum_{j\in\mathcal{E}} \frac{\partial g_j}{\partial n_i} \omega_j^{(r)}$

Gas/Liquid Phase VII

thermodynamic consistency

- natural variables of $\omega^{(L)}$: $T,\ \mu_i,\ A_m,\ \Delta_i^{(\eta_i)}(k),\ \zeta_i^{(\eta_i)}$

  but $\omega^{(L)}$ depends explicitly on densities n_i, $n_i^{(s)}$ (already defined!)

- consistency criterion $n_j \stackrel{!}{=} -\frac{\partial}{\partial \mu_j}\omega^{(L)}(T, \mu_i, A_m, \Delta_i^{(\eta_i)}(k), \zeta_i^{(\eta_i)})\Big|_{T,\mu_{i\neq j},A_m,\Delta_i^{(\eta_i)}(k),\zeta_i^{(\eta_i)}}$

  $\Rightarrow$ definition of rearrangement potentials

  - $U_i^{(\mathrm{mass})} = \sum_{j\in\mathcal{I}\cup\mathcal{E}} \frac{\partial \Delta m_j}{\partial n_i} n_j^{(s)}$
  - $U_i^{(\mathrm{em})} = \sum_{j\in\mathcal{I}\cup\mathcal{E}} \frac{\partial V_j^{(em)}}{\partial n_i} n_j$
  - $U_i^{(\mathrm{deg})} = \sum_{j\in\mathcal{E}} \frac{\partial g_j}{\partial n_i} \omega_j^{(r)}$

- non-standard contributions to entropy density $s = -\frac{\partial \omega^{(L)}}{\partial T}\Big|_{\mu_i,A_m,\Delta_i^{(\eta_i)}(k),\zeta_i^{(\eta_i)}}$

Solid Phase I

combination of models

- homogeneously distributed constituent particles
  - leptons, photons, neutrons, certain nuclei(?), . . .
    contribution to grand canonical potential as in gas/liquid phase

Solid Phase I

combination of models

- homogeneously distributed constituent particles
  - leptons, photons, neutrons, certain nuclei(?), . . .
    contribution to grand canonical potential as in gas/liquid phase
- nuclei on lattice sites, excitation of lattice vibrations/phonons
  - Einstein/Debye-like model, three branches $(\lambda = 0, 1, 2)$

    (extension of model by G. Chabrier, Ap. J. 414 (1993) 695)

Solid Phase I

combination of models

- homogeneously distributed constituent particles
  - leptons, photons, neutrons, certain nuclei(?), . . .
    contribution to grand canonical potential as in gas/liquid phase
- nuclei on lattice sites, excitation of lattice vibrations/phonons
  - Einstein/Debye-like model, three branches $(\lambda = 0, 1, 2)$

    (extension of model by G. Chabrier, Ap. J. 414 (1993) 695)

    – one longitudinal mode: $\omega_i(0, \vec{q}) = \alpha_0 \omega_i^{(p)}$

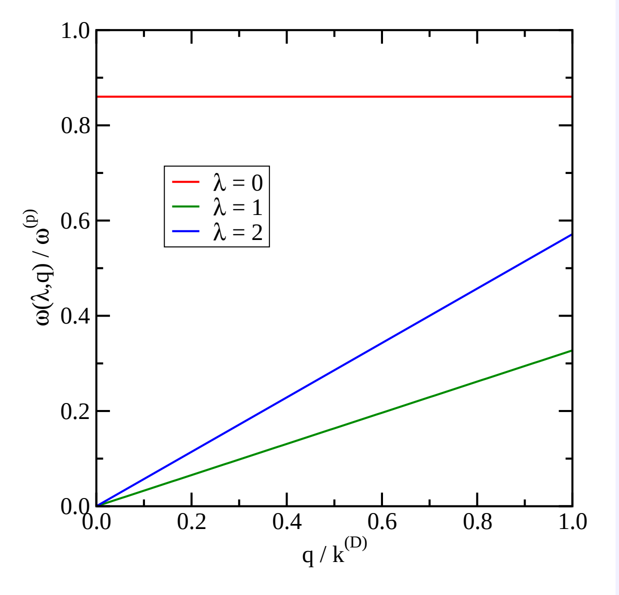
    – two transversal modes: $\omega_i(1, \vec{q}) = \alpha_1 \omega_i^{(p)} q/k_i^{(D)}$

    $\omega_i(2, \vec{q}) = \alpha_2 \omega_i^{(p)} q/k_i^{(D)}$

    plasma frequency $\omega_i^{(p)} = \sqrt{4\pi Q_i e^2 n_Q / m_i}$

    Debye wave number $k_i^{(D)} = (6\pi^2 n_i)^{1/3}$

    parameters α_0, α_1, α_2



Solid Phase II

- parameters α_0, α_1, α_2
  fitted to reproduce known frequency moments
  $\mu_n = \frac{1}{3}\sum_{\lambda,\vec{q}}[\omega_i(\lambda,\vec{q})/\omega_i^{(p)}]^n$ for $n = 1, 2$
  and consistency relation in classical limit $(3\bar{\mu} = \ln(\alpha_0\alpha_1\alpha_2) - 2/3)$

Solid Phase II

- parameters α_0, α_1, α_2
  fitted to reproduce known frequency moments

  $$\mu_n = \frac{1}{3}\sum_{\lambda,\vec{q}}[\omega_i(\lambda,\vec{q})/\omega_i^{(p)}]^n \quad \text{for } n = 1, 2$$

  and consistency relation in classical limit $(3\bar{\mu} = \ln(\alpha_0\alpha_1\alpha_2) - 2/3)$

  bcc lattice

	exact calculation*	model	significance
μ_{-2}	12.972	12.850	mean square displacement (classical)
μ_{-1}	2.79855	2.79031	mean square displacement (quantal)
μ_1	0.5113875	exact	zero-point oscillation energy
μ_2	$1/3$	exact	Kohn rule
μ_3	0.25031	0.24905	
$\bar{\mu}$	-0.831298	exact	classical limit of free energy

* D.A. Baiko, A.Y. Potekhin, D.G. Yakovlev, Phys. Rev. E 64 (2001) 057402

Solid Phase III

effective density functional

- canonical description ⇒ free energy density

$$f^{(S)} = \sum_{i\in\mathcal{S}} n_i[m_i + F_i^{(\mathrm{ph})} + F_i^{(\mathrm{em})} + F_i^{(\mathrm{mix})}]$$

Solid Phase III

effective density functional

- canonical description $\Rightarrow$ free energy density

$$f^{(S)} = \sum_{i \in \mathcal{S}} n_i [m_i + F_i^{(\mathrm{ph})} + F_i^{(\mathrm{em})} + F_i^{(\mathrm{mix})}]$$

  - contribution of phonons

$$F_i^{(ph)} = T\{\tfrac{3}{2}\mu_1 \eta_i + \sum_{\lambda=0}^{2} \ln[1-\exp(-\alpha_\lambda \eta_i)] - \tfrac{1}{3}\sum_{\lambda=1}^{2} D_3(\alpha_\lambda \eta_i)\}$$

with Debye function $D_3(x)$

essential parameters $\eta_i = \omega_i^{(p)}/T$

$\eta_i \to 0$: classical limit
$\eta_i \to \infty$: quantal effects

Solid Phase III

effective density functional

- canonical description ⇒ free energy density

$$f^{(S)} = \sum_{i \in \mathcal{S}} n_i [m_i + F_i^{(\mathrm{ph})} + F_i^{(\mathrm{em})} + F_i^{(\mathrm{mix})}]$$

  - ◦ contribution of phonons

$$F_i^{(ph)} = T\{\tfrac{3}{2}\mu_1 \eta_i + \sum_{\lambda=0}^{2} \ln[1 - \exp(-\alpha_\lambda \eta_i)] - \tfrac{1}{3}\sum_{\lambda=1}^{2} D_3(\alpha_\lambda \eta_i)\}$$

with Debye function $D_3(x)$

essential parameters $\eta_i = \omega_i^{(p)}/T$

$\eta_i \longrightarrow 0$: classical limit

$\eta_i \longrightarrow \infty$: quantal effects

  - ◦ contribution of electromagnetic interaction

$$F_i^{(\mathrm{em})} = T[C_M^{(\mathrm{bcc})}\Gamma_i + f_S(\Gamma_i)]$$ (from fit to OCP)

Solid Phase III

effective density functional

- canonical description $\Rightarrow$ free energy density

  $$f^{(S)} = \sum_{i \in \mathcal{S}} n_i [m_i + F_i^{(\mathrm{ph})} + F_i^{(\mathrm{em})} + F_i^{(\mathrm{mix})}]$$

  - contribution of phonons

    $$F_i^{(ph)} = T\{\frac{3}{2}\mu_1\eta_i + \sum_{\lambda=0}^{2} \ln[1-\exp(-\alpha_\lambda \eta_i)] - \frac{1}{3}\sum_{\lambda=1}^{2} D_3(\alpha_\lambda \eta_i)\}$$

    with Debye function $D_3(x)$

    essential parameters $\eta_i = \omega_i^{(p)}/T$

    $\eta_i \longrightarrow 0$: classical limit
    $\eta_i \longrightarrow \infty$: quantal effects

  - contribution of electromagnetic interaction

    $F_i^{(\mathrm{em})} = T[C_M^{(\mathrm{bcc})}\Gamma_i + f_S(\Gamma_i)]$ (from fit to OCP)

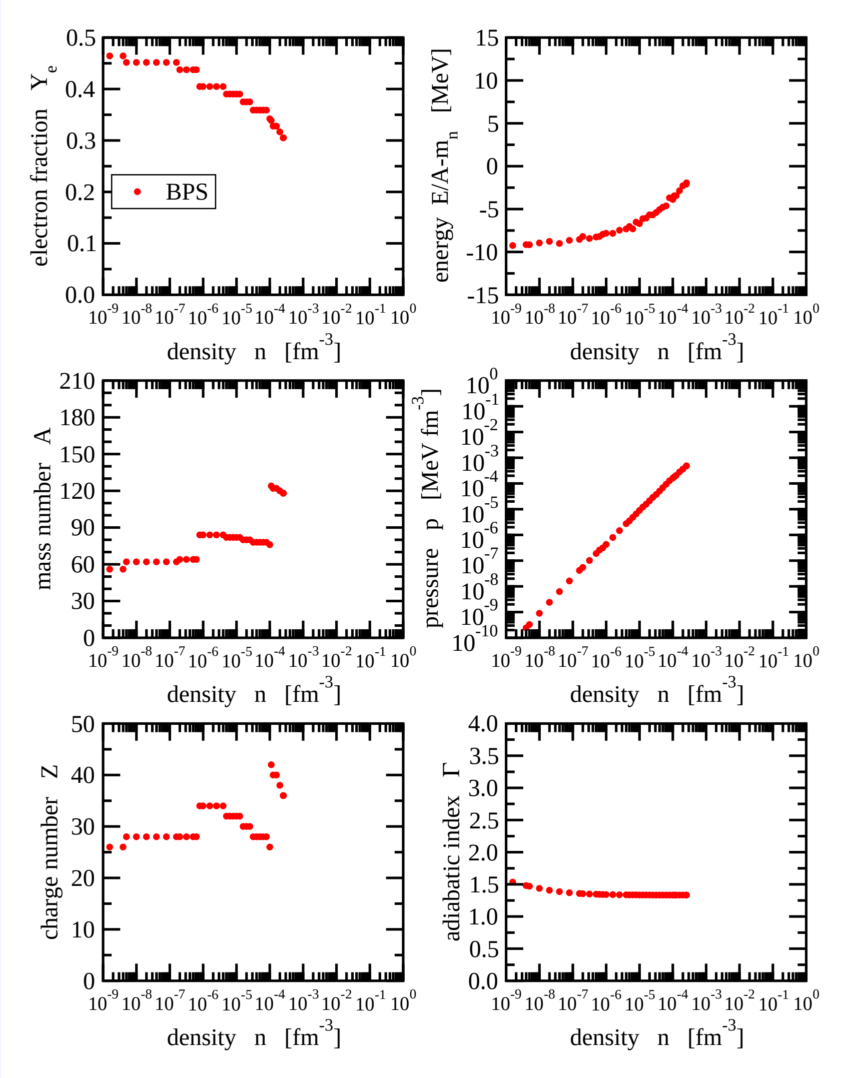
  - mixing contribution

    $F_i^{(\mathrm{mix})} = T \ln(\frac{Q_i n_i}{g_i n_Q})$ $n_Q = \sum_i Q_i n_i$

Solid Phase IV

- EoS of cold outer crust very well known (β equilibrium, $T = 0$ MeV)

β equilibrium, $T = 0$ MeV

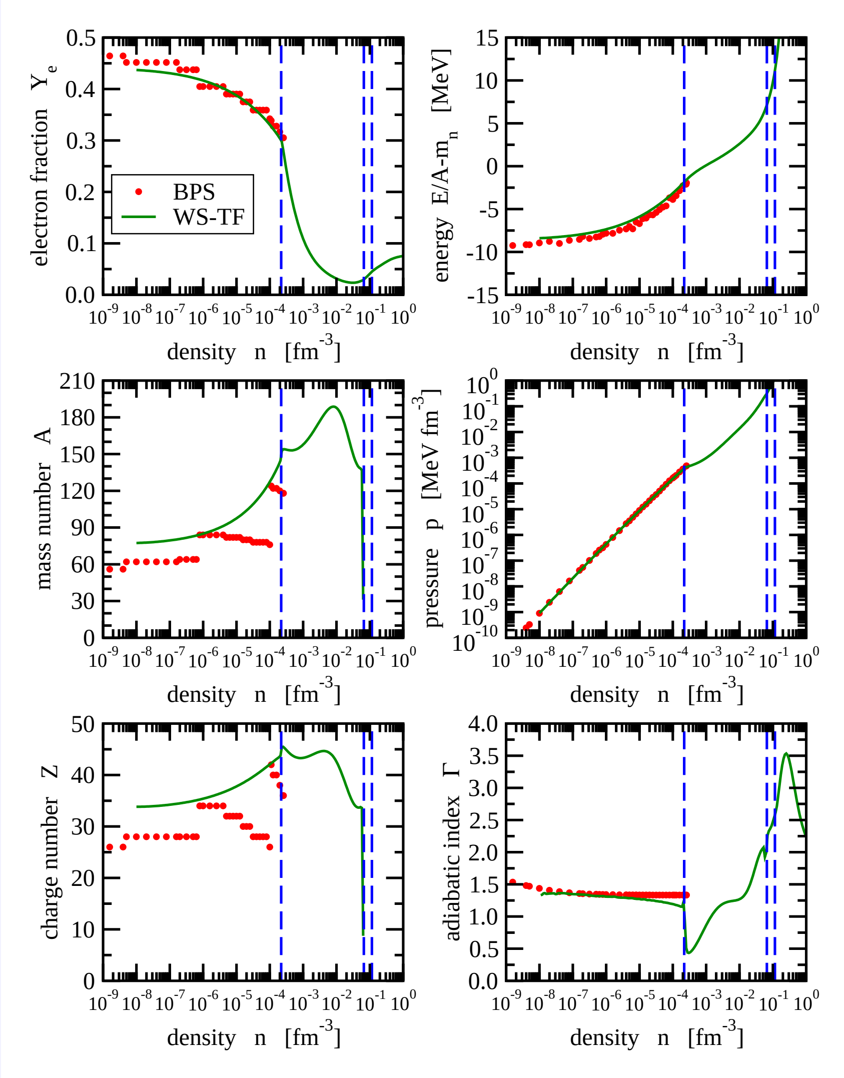


BPS: G. Baym, C. Pethick, P. Sutherland, Ap. J. 170 (1971) 299

Solid Phase IV

- EoS of cold outer crust very well known (β equilibrium, $T = 0$ MeV)
- calculation in Wigner-Seitz and Thomas-Fermi approximation (WS-TF) not sufficient

β equilibrium, $T = 0$ MeV

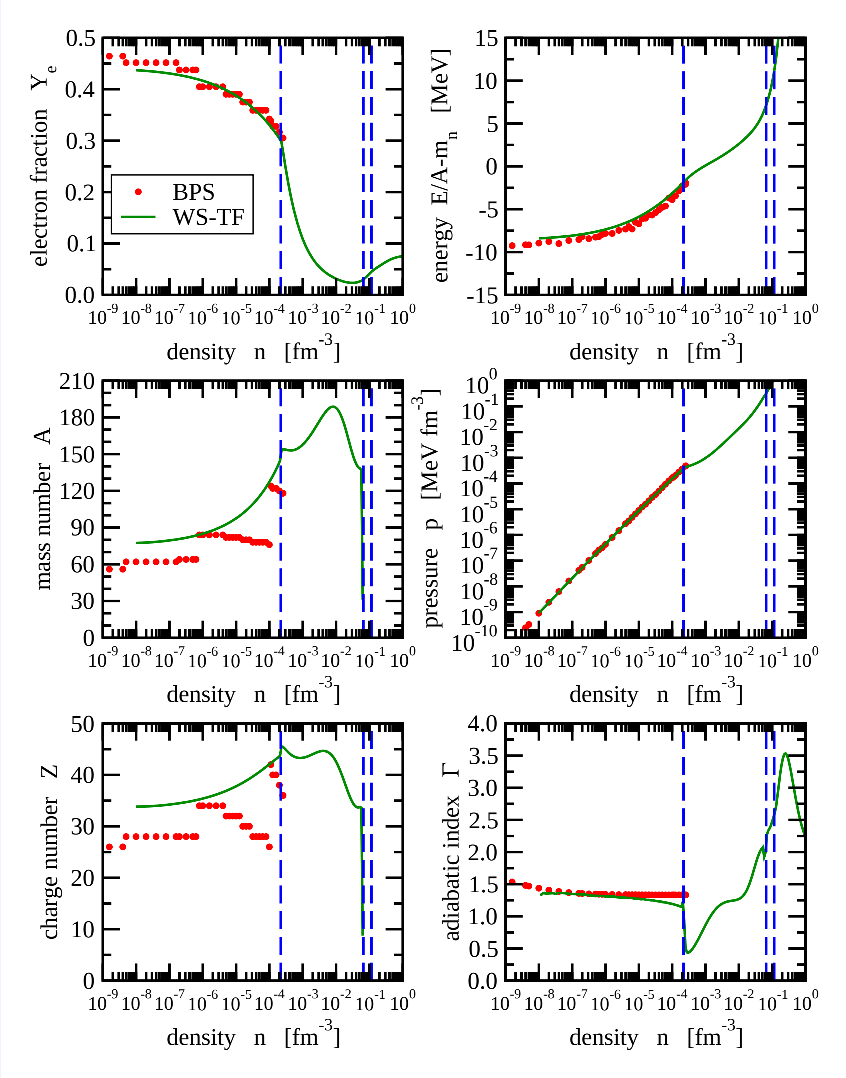


BPS: G. Baym, C. Pethick, P. Sutherland, Ap. J. 170 (1971) 299

Solid Phase IV

- EoS of cold outer crust very well known (β equilibrium, $T = 0$ MeV)
- calculation in Wigner-Seitz and Thomas-Fermi approximation (WS-TF) not sufficient
- effects of temperature
  - change of chemical composition
  - melting of crystal, solidification of gas/liquid

β equilibrium, $T = 0$ MeV



BPS: G. Baym, C. Pethick, P. Sutherland, Ap. J. 170 (1971) 299

Solid Phase IV

- EoS of cold outer crust very well known (β equilibrium, $T = 0$ MeV)

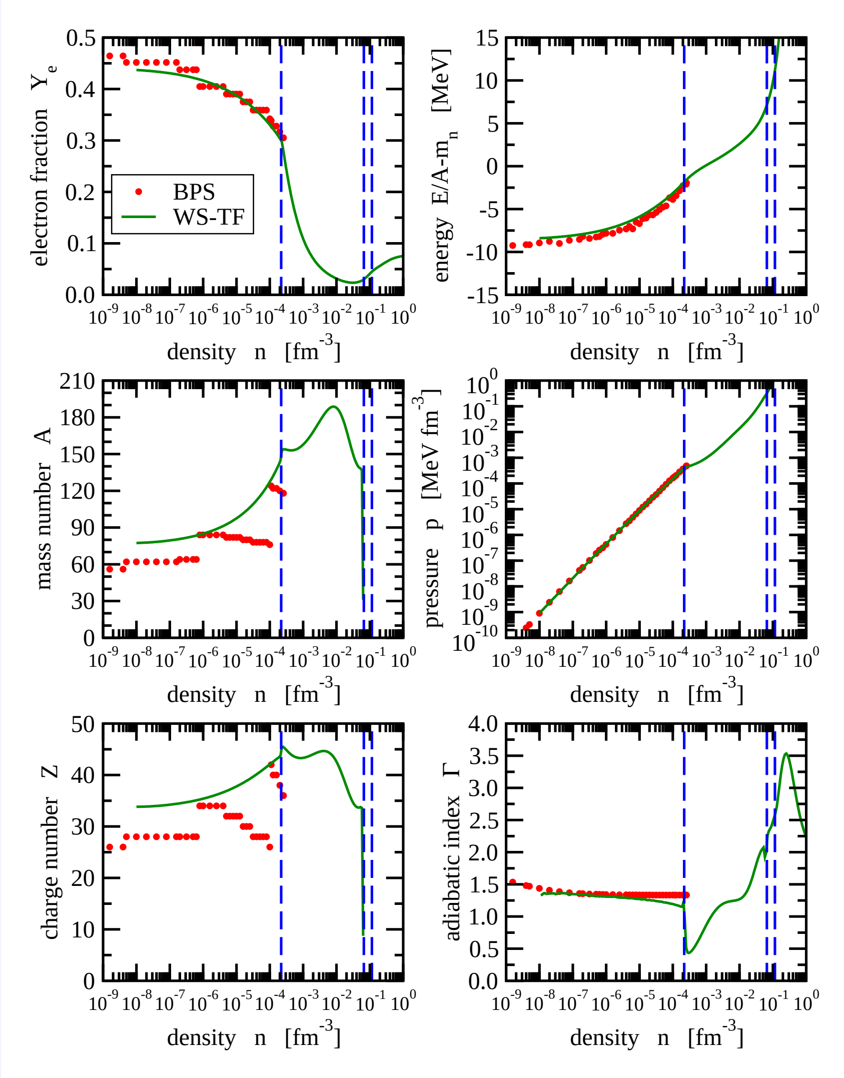
- calculation in Wigner-Seitz and Thomas-Fermi approximation (WS-TF) not sufficient

- effects of temperature
  - change of chemical composition
  - melting of crystal, solidification of gas/liquid

- general electron fraction
  - out of β equilibrium

  $\Rightarrow$ global EoS table

β equilibrium, $T = 0$ MeV



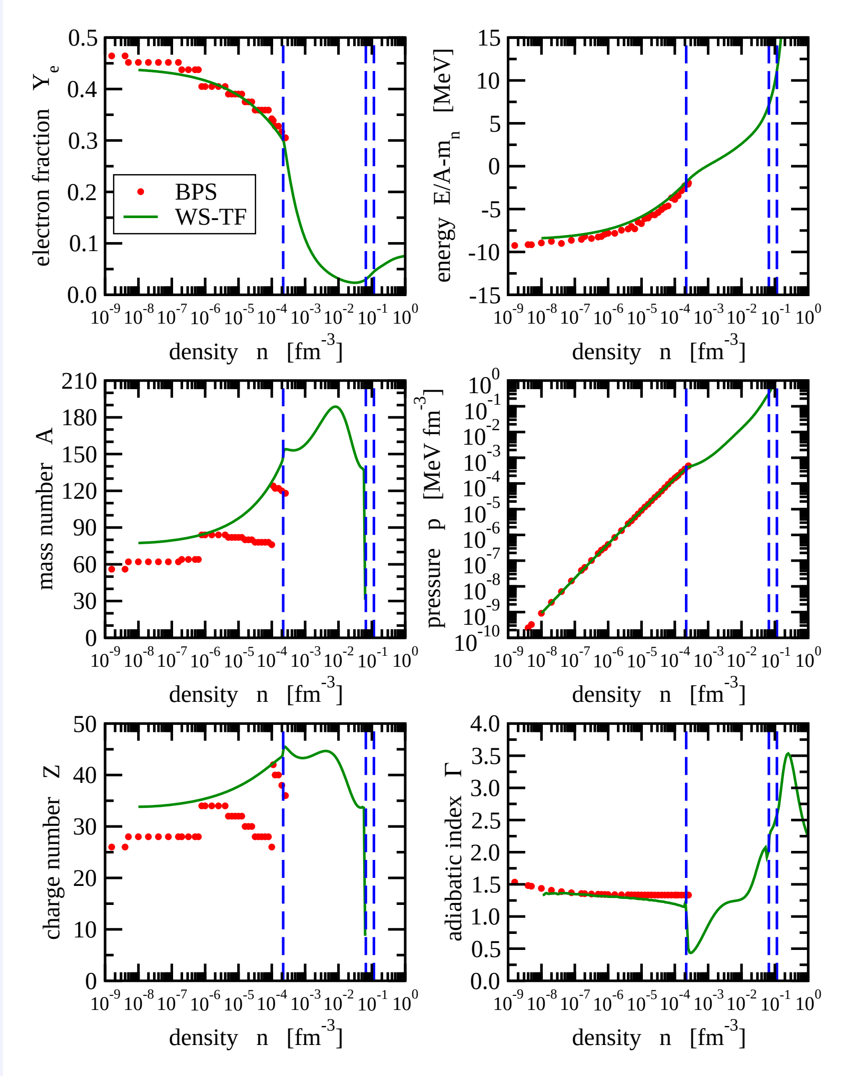
BPS: G. Baym, C. Pethick, P. Sutherland, Ap. J. 170 (1971) 299

Solid Phase IV

- EoS of cold outer crust very well known (β equilibrium, $T = 0$ MeV)
- calculation in Wigner-Seitz and Thomas-Fermi approximation (WS-TF) not sufficient
- effects of temperature
  - ◦ change of chemical composition
  - ◦ melting of crystal, solidification of gas/liquid
- general electron fraction
  - ◦ out of β equilibrium

  $\Rightarrow$ global EoS table
- details of phase transitions
- work in progress

β equilibrium, $T = 0$ MeV



BPS: G. Baym, C. Pethick, P. Sutherland, Ap. J. 170 (1971) 299

Summary

Summary

construction of **effective relativistic density functional** for dense matter

- extended set of constituents ⇒ nucleons, hyperons, mesons, nuclei, leptons, . . .
  ⇒ quasiparticles with medium dependent properties
- nuclear interaction ⇒ meson exchange with density dependent couplings
- electromagnetic interaction ⇒ effective potential from Monte Carlo simulations
- formation and dissolution of clusters
- rearrangement contributions for thermodynamic consistency
- phase transition liquid/gas ↔ solid
- well constrained parameters, correct limits
- work in progress

⇒ preparation of EoS tables for astrophysical applications

Thanks

- **to my collaborators**
  Gerd Röpke (Universität Rostock)
  Niels-Uwe Bastian (Universität Rostock)
  David Blaschke (Uniwersytet Wrocławski)
  Thomas Klähn (Uniwersytet Wrocławski)
  Hermann Wolter (Ludwig Maximilians-Universität München)
  Maria Voskresenskaya (GSI Darmstadt)

- **for support from**
  - Helmholtz Association (HGF)
    - Nuclear Astrophysics Virtual Institute (VH-VI-417)
    - Helmholtz International Center (HIC) for FAIR
  - Excellence Cluster 'Universe', Technische Universität München

- **to the organizers of the Hirschegg 2013 workshop**
  for the invitation and support

- **to you, the audience**
  for your attention and patience