

# Hyperon-nucleon interactions from chiral EFT

Johann Haidenbauer

IAS & JCHP, Forschungszentrum Jülich, Germany

Hadrons from Quarks and Gluons, Hirschegg,  
January 12-18, 2014

- 1 Introduction
- 2  $YN$  in chiral effective field theory
- 3  $YN$  Results
- 4 Three- and four-body systems

## The $YN$ ( $YY$ ) interaction

- study the role of strangeness in low and medium energy nuclear physics
- test  $SU(3)_{\text{flavor}}$  symmetry
- H dibaryon
  - Jaffe (1977)  $\rightarrow$  deeply bound 6-quark state with  $I = 0$ ,  $J = 0$ ,  $S = -2$
  - many experimental searches but no convincing signal
  - Lattice QCD (2010)  $\rightarrow$  evidence for a bound H dibaryon
- prerequisite for studies of ( $\Lambda$ ,  $\Sigma$ ) hypernuclei
- quest for  $\Lambda\Lambda$  hypernuclei and  $\Xi$  hypernuclei
  - $\rightarrow$  J-PARC, FAIR
- implications for astrophysics
  - $\rightarrow$  hyperon stars
  - stability/size of neutron stars

# $\Upsilon N$ in chiral effective field theory

## Advantages:

- Power counting  
systematic improvement by going to higher order
- Possibility to derive two- and three baryon forces and external current operators in a consistent way

Obstacle:  $\Upsilon N$  data base is rather poor

- about 35 data points, all from the 1960s
- 10 data points from the KEK-PS E251 collaboration (1999-2005)  
(cf.  $> 4000$  NN data for  $E_{lab} < 350$  MeV!)
- constraints from hypernuclei
- no polarization data  $\Rightarrow$  no phase shift analysis

$\rightarrow$  impose  $SU(3)_f$  constraints

We\* follow the scheme of S. Weinberg (1990)  
in complete analogy to the study of NN in  $\chi$ EFT by E. Epelbaum et al.

\*J.H., N. Kaiser, S. Petschauer, U.-G. Meißner, A. Nogga, W. Weise

$$V_{\text{eff}} \equiv V_{\text{eff}}(\mathbf{Q}, g, \mu) = \sum_{\nu} (\mathbf{Q}/\Lambda)^{\nu} \mathcal{V}_{\nu}(\mathbf{Q}/\mu, g)$$

- $\mathbf{Q}$  ... soft scale (baryon three-momentum, Goldstone boson four-momentum, Goldstone boson mass)
- $\Lambda$  ... hard scale
- $g$  ... pertinent low-energy constants
- $\mu$  ... regularization scale
- $\mathcal{V}_{\nu}$  ... function of order one
- $\nu \geq 0$  ... chiral power

Leading order (LO):  $\nu = 0$

- a) non-derivative four-baryon contact terms
- b) one-meson (Goldstone boson) exchange diagrams

Next-to-leading order (NLO):  $\nu = 2$

- a) four-baryon contact terms with two derivatives
- b) two-meson (Goldstone boson) exchange diagrams

e.g., LO contact terms for  $BB$ :

$$\begin{aligned}\mathcal{L} &= C_i (\bar{N}\Gamma_i N) (\bar{N}\Gamma_i N) \Rightarrow \mathcal{L}^1 = \tilde{C}_i^1 \langle \bar{B}_a \bar{B}_b (\Gamma_i B)_b (\Gamma_i B)_a \rangle, \\ \mathcal{L}^2 &= \tilde{C}_i^2 \langle \bar{B}_a (\Gamma_i B)_a \bar{B}_b (\Gamma_i B)_b \rangle, \\ \mathcal{L}^3 &= \tilde{C}_i^3 \langle \bar{B}_a (\Gamma_i B)_a \rangle \langle \bar{B}_b (\Gamma_i B)_b \rangle\end{aligned}$$

$$B = \begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & \frac{-\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & n \\ \underline{\Xi^-} & \underline{\Xi^0} & \underline{-\frac{2\Lambda}{\sqrt{6}}} \end{pmatrix}$$

$a, b \dots$  Dirac indices of the particles

$$\Gamma_1 = 1, \Gamma_2 = \gamma^\mu, \Gamma_3 = \sigma^{\mu\nu}, \Gamma_4 = \gamma^\mu \gamma_5, \Gamma_5 = \gamma_5$$

$C_i, \tilde{C}_i \dots$  low-energy constants

# Contact terms for $BB$

spin-momentum structure of the contact term potential:

$BB$  contact terms without derivatives (LO):

$$V_{BB \rightarrow BB}^{(0)} = C_{S, BB \rightarrow BB} + C_{T, BB \rightarrow BB} \vec{\sigma}_1 \cdot \vec{\sigma}_2$$

$BB$  contact terms with two derivatives (NLO):

$$\begin{aligned} V_{BB \rightarrow BB}^{(2)} &= C_1 \vec{q}^2 + C_2 \vec{k}^2 + (C_3 \vec{q}^2 + C_4 \vec{k}^2) (\vec{\sigma}_1 \cdot \vec{\sigma}_2) \\ &+ \frac{i}{2} C_5 (\vec{\sigma}_1 + \vec{\sigma}_2) \cdot (\vec{q} \times \vec{k}) + C_6 (\vec{q} \cdot \vec{\sigma}_1) (\vec{q} \cdot \vec{\sigma}_2) \\ &+ C_7 (\vec{k} \cdot \vec{\sigma}_1) (\vec{k} \cdot \vec{\sigma}_2) + \frac{i}{2} C_8 (\vec{\sigma}_1 - \vec{\sigma}_2) \cdot (\vec{q} \times \vec{k}) \end{aligned}$$

note:  $C_i \rightarrow C_{i, BB \rightarrow BB}$

$\vec{q} = \vec{p}' - \vec{p}$ ;  $\vec{k} = (\vec{p}' + \vec{p})/2$

# $SU(3)$ symmetry

10 independent spin-isospin channels in  $NN$  and  $YN$  (for  $L=0$ )  
( $NN$  ( $l=0$ ),  $NN$  ( $l=1$ ),  $\Lambda N$ ,  $\Sigma N$  ( $l=1/2$ ),  $\Sigma N$  ( $l=3/2$ ),  $\Lambda N \leftrightarrow \Sigma N$ )

$\Rightarrow$  in principle (at LO), 10 low-energy constants

$SU(3)$  symmetry  $\Rightarrow$  only 5 independent low-energy constants

$SU(3)$  structure for scattering of two octet baryons:  
direct product:

$$8 \otimes 8 = 1 \oplus 8_a \oplus 8_s \oplus 10^* \oplus 10 \oplus 27$$

$C_{S,i}$ ,  $C_{T,i}$ ,  $C_{1,i}$ , etc., can be expressed by the constants  
corresponding to the  $SU(3)_f$  irreducible representations:  
 $C^1$ ,  $C^{8_a}$ ,  $C^{8_s}$ ,  $C^{10^*}$ ,  $C^{10}$ ,  $C^{27}$



# $SU(3)$ structure of contact terms for $BB$

	Channel	$l$	$V_{1S0}, V_{3P0,3P1,3P2}$	$V_{3S1}, V_{3S1-3D1}, V_{1P1}$	$V_{1P1-3P1}$
$S = 0$	$NN \rightarrow NN$	0	–	$C^{10^*}$	–
	$NN \rightarrow NN$	1	$C^{27}$	–	–
$S = -1$	$\Lambda N \rightarrow \Lambda N$	$\frac{1}{2}$	$\frac{1}{10} (9C^{27} + C^{8_s})$	$\frac{1}{2} (C^{8_a} + C^{10^*})$	$\frac{-1}{\sqrt{20}} C^{8_s 8_a}$
	$\Lambda N \rightarrow \Sigma N$	$\frac{1}{2}$	$\frac{3}{10} (-C^{27} + C^{8_s})$	$\frac{1}{2} (-C^{8_a} + C^{10^*})$	$\frac{3}{\sqrt{20}} C^{8_s 8_a}$
	$\Sigma N \rightarrow \Sigma N$	$\frac{1}{2}$	$\frac{1}{10} (C^{27} + 9C^{8_s})$	$\frac{1}{2} (C^{8_a} + C^{10^*})$	$\frac{-1}{\sqrt{20}} C^{8_s 8_a}$
	$\Sigma N \rightarrow \Sigma N$	$\frac{3}{2}$	$C^{27}$	$C^{10}$	$\frac{3}{\sqrt{20}} C^{8_s 8_a}$
	$\Sigma N \rightarrow \Sigma N$				–

Number of contact terms:

$NN$ : 2 (LO) 7 (NLO)

$YN$ : +3 (LO) +11 (NLO)

$YY$ : +1 (LO) + 4 (NLO)

$\Rightarrow C^1$  contributes only to  $l = 0, S = -2$  channels!!

# Pseudoscalar-meson exchange

$SU(3)_f$ -invariant pseudoscalar-meson-baryon interaction Lagrangian:

$$\mathcal{L} = - \left\langle \frac{g_A(1-\alpha)}{\sqrt{2}F_\pi} \bar{B} \gamma^\mu \gamma_5 \{ \partial_\mu P, B \} + \frac{g_A \alpha}{\sqrt{2}F_\pi} \bar{B} \gamma^\mu \gamma_5 [ \partial_\mu P, B ] \right\rangle$$

$$f = g_A/(2F_\pi); \quad g_A \simeq 1.26, \quad F_\pi \approx 93 \text{ MeV}$$

$$\alpha = F/(F + D) \text{ with } g_A = F + D$$

$$P = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2\eta}{\sqrt{6}} \end{pmatrix}$$

$$\begin{array}{lll} f_{NN\pi} = f & f_{NN\eta_8} = \frac{1}{\sqrt{3}}(4\alpha - 1)f & f_{\Lambda NK} = -\frac{1}{\sqrt{3}}(1 + 2\alpha)f \\ f_{\Xi\Xi\pi} = -(1 - 2\alpha)f & f_{\Xi\Xi\eta_8} = -\frac{1}{\sqrt{3}}(1 + 2\alpha)f & f_{\Xi\Lambda K} = \frac{1}{\sqrt{3}}(4\alpha - 1)f \\ f_{\Lambda\Sigma\pi} = \frac{2}{\sqrt{3}}(1 - \alpha)f & f_{\Sigma\Sigma\eta_8} = \frac{2}{\sqrt{3}}(1 - \alpha)f & f_{\Sigma NK} = (1 - 2\alpha)f \\ f_{\Sigma\Sigma\pi} = 2\alpha f & f_{\Lambda\Lambda\eta_8} = -\frac{2}{\sqrt{3}}(1 - \alpha)f & f_{\Xi\Sigma K} = -f \end{array}$$

# Pseudoscalar-meson (boson) exchange

One-pseudoscalar-meson exchange ( $V^{OBE}$ ) [LO]

$$V_{B_1 B_2 \rightarrow B'_1 B'_2}^{OBE} = -f_{B_1 B'_1 P} f_{B_2 B'_2 P} \frac{(\vec{\sigma}_1 \cdot \vec{q})(\vec{\sigma}_2 \cdot \vec{q})}{\vec{q}^2 + m_P^2}$$

$f_{B_1 B'_1 P}$  ... coupling constants

$m_P$  ... mass of the exchanged pseudoscalar meson

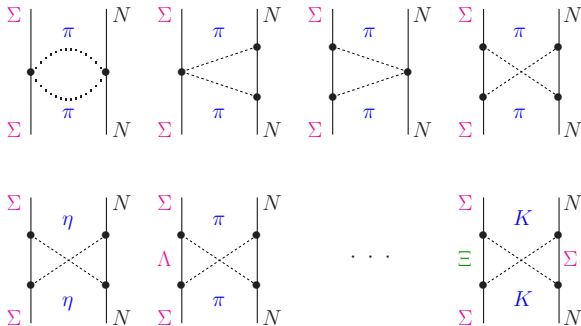
- dynamical breaking of  $SU(3)$  symmetry due to the mass splitting of the ps mesons  
( $m_\pi = 138.0$  MeV,  $m_K = 495.7$  MeV,  $m_\eta = 547.3$  MeV)  
taken into account already at LO!

Details:

(H. Polinder, J.H., U.-G. Meißner, NPA 779 (2006) 244; PLB 653 (2007) 29)

# Two-pseudoscalar-meson exchange diagrams

Two-pseudoscalar-meson exchange diagrams ( $V^{TBE}$ ) [NLO]



$\Rightarrow$  J.H., S. Petschauer, N. Kaiser, U.-G. Meißner, A. Nogga, W. Weise,  
NPA 915 (2013) 24

# Coupled channels Lippmann-Schwinger Equation

$$T_{\rho' \rho}^{\nu' \nu, J}(p', p) = V_{\rho' \rho}^{\nu' \nu, J}(p', p) + \sum_{\rho'', \nu''} \int_0^\infty \frac{dp'' p''^2}{(2\pi)^3} V_{\rho' \rho''}^{\nu' \nu'', J}(p', p'') \frac{2\mu_{\nu''}}{p^2 - p''^2 + i\eta} T_{\rho'' \rho}^{\nu'' \nu, J}(p'', p)$$

$$\rho', \rho = \Lambda N, \Sigma N$$

LS equation is solved for **particle channels** (in **momentum space**)

**Coulomb** interaction is included via the **Vincent-Phatak method**

The potential in the **LS** equation is cut off with the **regulator function**:

$$V_{\rho' \rho}^{\nu' \nu, J}(p', p) \rightarrow f^\Lambda(p') V_{\rho' \rho}^{\nu' \nu, J}(p', p) f^\Lambda(p); \quad f^\Lambda(p) = e^{-(p/\Lambda)^4}$$

consider values  $\Lambda = 450 - 700$  MeV [500 - 650 MeV]

## Pseudoscalar-meson exchange

- All one- and two-pseudoscalar-meson exchange diagrams are included
- $SU(3)$  symmetry is broken by using the physical masses of the pion, kaon, and eta
- $SU(3)$  breaking in the coupling constants is ignored  
 $F_\pi = F_K = F_\eta = F_0 = 93 \text{ MeV}$ ;  $g_A = 1.26$
- assume that  $\eta \equiv \eta_8$  (i.e.  $\theta_P = 0^\circ$  and  $f_{BB\eta_1} \equiv 0$ )
- assume that  $\alpha = F/(F + D) = 2/5$   
(semi-leptonic decays  $\Rightarrow \alpha \approx 0.364$ )
- Correction to  $V^{OBE}$  due to baryon mass differences are ignored
- (A fit with two-pion-meson exchange diagrams is possible!)
- (A fit with physical values for  $F_\pi, F_K, F_\eta$  is possible!)

# Contact terms

- $SU(3)$  symmetry is assumed
- (at NLO  $SU(3)$  breaking corrections to the LO contact terms arise!)
- 10 contact terms in  $S$ -waves  
no  $SU(3)$  constraints from the  $NN$  sector are imposed!
- 12 contact terms in  $P$ -waves and in  ${}^3S_1$ - ${}^3D_1$   
 $SU(3)$  constraints from the  $NN$  sector are imposed!
- 1 contact term in  ${}^1P_1$ - ${}^3P_1$  (singlet-triplet mixing) is set to zero
  
- contact terms in  $S$ -waves: can be fairly well fixed from data
- some correlations between NLO and LO LECs

# Contact terms in $P$ -waves

- contact terms in  $P$ -waves are much less constrained
- use  $SU(3)$  and fix some (5) LECs from  $NN$
- the others (7) are fixed from “bulk” properties:
  - (1)  $\sigma_{\Lambda p} \approx 10$  mb at  $p_{lab} \approx 700 - 900$  MeV/c
  - (2)  $d\sigma/d\Omega_{\Sigma^- p \rightarrow \Lambda n}$  at  $p_{lab} \approx 135 - 160$  MeV/c

## Other (future) options:

- consider matter properties:

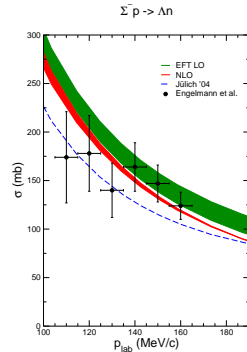
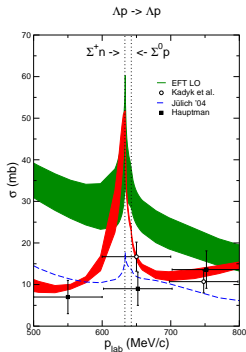
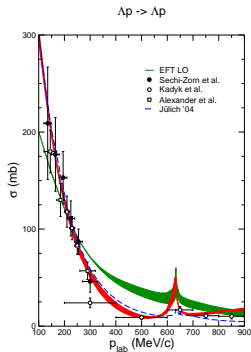
use spin-orbit splitting of the  $\Lambda$  single particle levels in nuclei

Consider the Scheerbaum factor  $S_{\Lambda}$  calculated in nuclear matter to relate the strength of the  $\Lambda$ -nucleus spin-orbit potential to the two body  $\Lambda N$  interaction

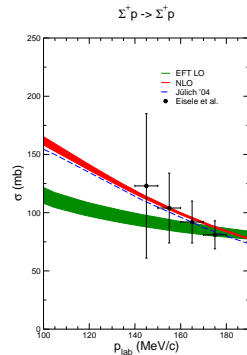
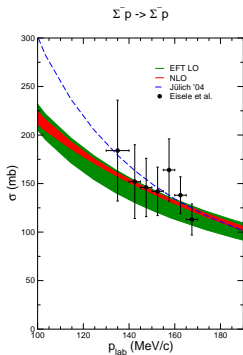
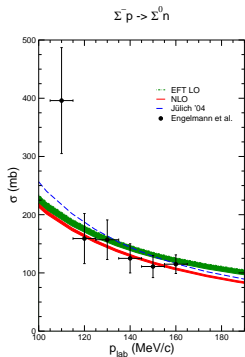
(R.R. Scheerbaum, NPA 257 (1976) 77)



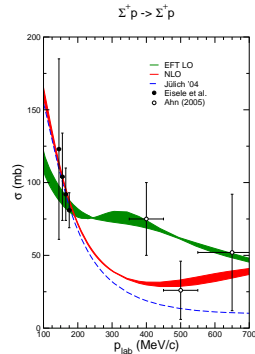
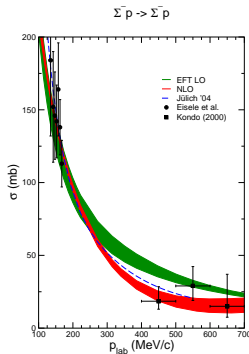
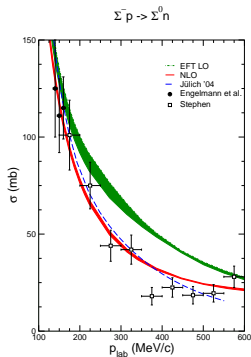
# $\Lambda N$ integrated cross sections



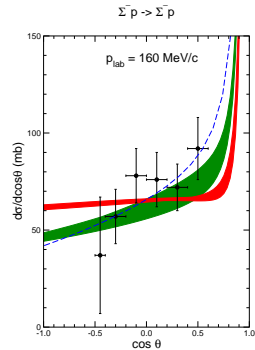
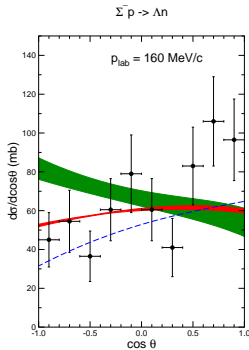
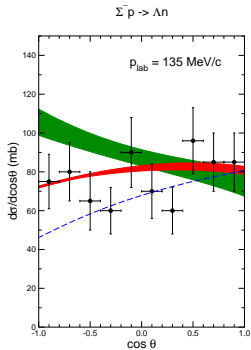
# $\Upsilon N$ integrated cross sections



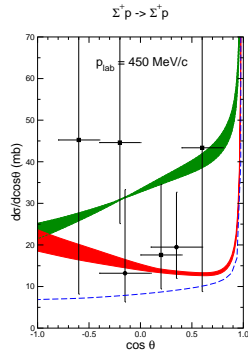
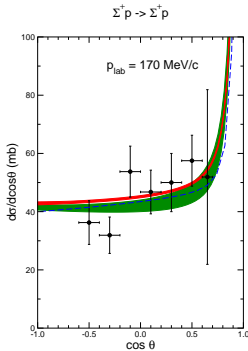
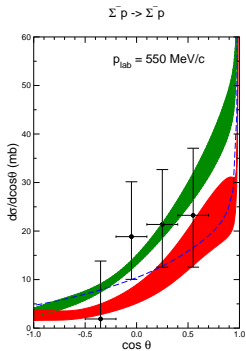
# $\Upsilon N$ integrated cross sections - higher energies



# $\Upsilon N$ differential cross sections



# $\Upsilon N$ differential cross sections



# $\Lambda N$ scattering lengths [fm]

	EFT LO	EFT NLO	Jülich '04	NSC97f	experiment*
$\Lambda$ [MeV]	550 ... 700	500 ... 650			
$a_s^{\Lambda p}$	-1.90 ... -1.91	-2.90 ... -2.91	-2.56	-2.51	$-1.8^{+2.3}_{-4.2}$
$a_t^{\Lambda p}$	-1.22 ... -1.23	-1.51 ... -1.61	-1.66	-1.75	$-1.6^{+1.1}_{-0.8}$
$a_s^{\Sigma^+ p}$	-2.24 ... -2.36	-3.46 ... -3.60	-4.71	-4.35	
$a_t^{\Sigma^+ p}$	0.60 ... 0.70	0.48 ... 0.49	0.29	-0.25	
$\chi^2$	$\approx 30$	15.7 ... 16.8	$\approx 25$	16.7	
$({}^3_{\Lambda}\text{H}) E_B^\dagger$	-2.34 ... -2.36	-2.30 ... -2.33	-2.27	-2.30	-2.354(50)

\* A. Gasparyan et al., PRC 69 (2004) 034006  $\Rightarrow$  extract from final-state interaction:

$pp \rightarrow K^+ \Lambda p$  (COSY, Jülich)

$\gamma d \rightarrow K^+ \Lambda n$  (SPRING-8)

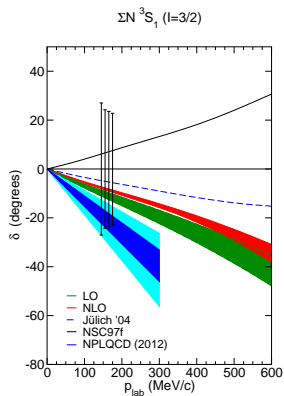
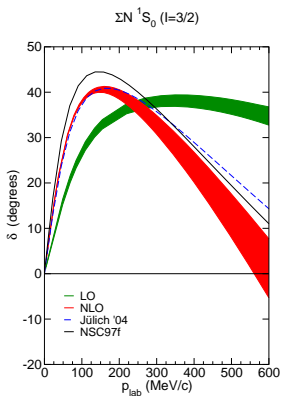
$^1S_0$ : test for  $SU(3)$  symmetry  $\rightarrow V_{NN} \equiv V_{\Sigma N}$

- LEC's that are fitted to the  $pp$   $^1S_0$  phase shift produce a bound state in  $\Sigma^+p$   
 $\rightarrow \sigma_{^1S_0} \approx 4 \times \sigma_{\Sigma^+p}$
- simultaneous fit is possible if we assume that there is  $SU(3)$  breaking in the LO contact term only.

$^3S_1$ - $^3D_1$ : decisive for  $\Sigma$  properties in nuclear matter

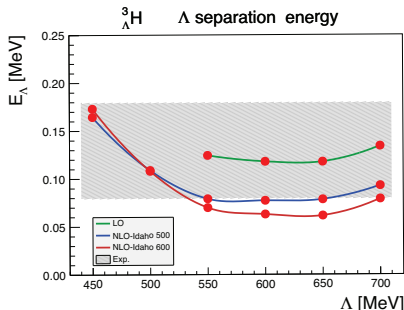
- A description of  $YN$  data is possible with an attractive as well as a repulsive  $^3S_1$ - $^3D_1$  interaction

# $\Sigma N$ ( $I=3/2$ ) phase shifts



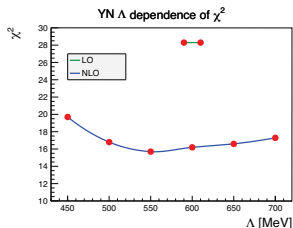


## Hypertriton separation energies



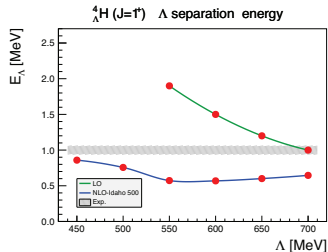
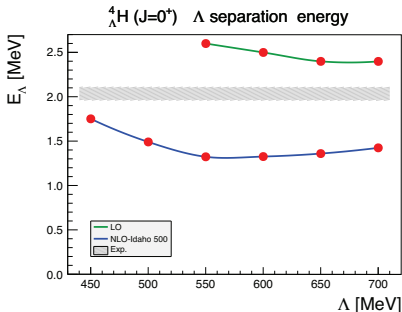
separation energies:

$$E_{\Lambda} = E(\text{core}) - E(\text{hypernucleus})$$



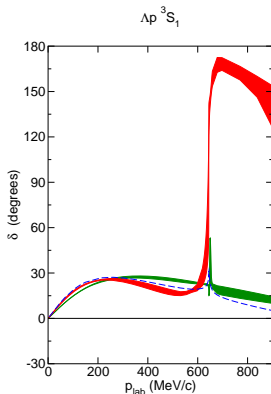
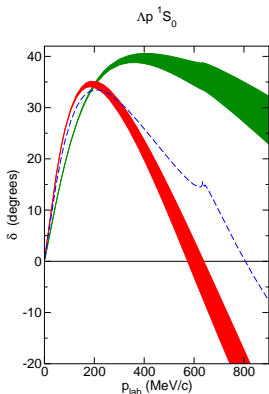
- singlet scattering length for one cutoff chosen so that hypertriton binding energy is OK
- cutoff variation
  - is **lower bound** for magnitude of higher order contributions
  - correlation with  $\chi^2$  of YN interaction ?
- long range 3BFs need to be explicitly estimated

## Separation energies for ${}^4_{\Lambda}\text{H}$



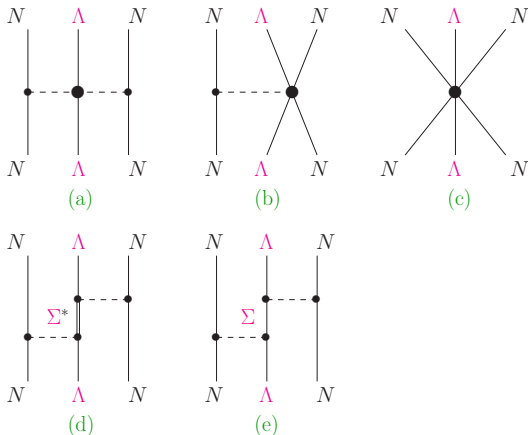
- LO/NLO results: LO uncertainty in  $0^+$  is underestimated by cutoff variation
- NLO results in line with model results, implies underbinding
- long range 3BFs need to be explicitly estimated
- **but:** for this version of NLO, results are **inconsistent** with experiment
  - note: this NLO does not allow for SU(3) breaking in contact part of YN
  - ad-hoc p-waves

# $\Lambda p$ S-wave phase shifts



⇒ less repulsion in  $^1S_0$  at short distances – and/or 3BFs ?

# Three-body forces



(a) - (c) appear at N<sup>2</sup>LO

(d) appears at NLO – in EFT that includes decuplet baryons

(e) is already included by solving coupled-channel Faddeev equations



## CSB at NLO & for model interactions



Contributions to the difference of  ${}^4_{\Lambda}\text{H} (0^+) - {}^4_{\Lambda}\text{He} (0^+)$  separation energies

$\Lambda$ [MeV]	450	500	550	600	650	700	Jülich 04	Nijm SC97	Nijm SC89	Expt.
$\Delta T$ [keV]	44	50	52	51	46	40	0	47	132	-
$\Delta V_{\text{NN}}$ [keV]	-3	-2	5	5	3	0	-31	-9	-9	-
$\Delta V_{\text{YN}}$ [keV]	-11	-11	-11	-10	-8	-7	2	37	228	-
tot [keV]	30	37	46	46	41	33	-29	75	351	350
$P_{\Sigma^-}$	1.0%	1.1%	1.2%	1.2%	1.1%	0.9%	0.3%	1.0%	2.7%	-
$P_{\Sigma^0}$	0.6%	0.6%	0.7%	0.7%	0.6%	0.5%	0.3%	0.5%	1.4%	-
$P_{\Sigma^+}$	0.1%	0.1%	0.2%	0.2%	0.2%	0.1%	0.3%	0.0%	0.1%	-

- **kinetic energy contribution is driven by  $\Sigma$  component**
- NN force contribution due to small deviation of Coulomb
- YN force contribution:
  - SC89 CSB is strong
  - NLO CSB is zero, only Coulomb acts ( $\Sigma$  component)

## $YN$ interaction based on chiral $EFT$

- approach is based on a modified **Weinberg power counting**, analogous to the  $NN$  case
- The potential (**contact terms**, **pseudoscalar-meson** exchanges) is derived imposing  $SU(3)_f$  constraints
- **Good description** of the empirical  $YN$  data was achieved already at **LO** (only **5 free parameters!**)
- Excellent results at **next-to-leading order (NLO)**
- $YN$  data are reproduced with a quality comparable to phenomenological models
- $SU(3)$  **symmetry** for the **LEC's** can be maintained in the  $YN$  system ( $\Lambda N$ ,  $\Sigma N$ ) but not between  $YN$  and  $NN$