Overview:

- Introduction/Historical Remarks
- Physical Picture of 'The First Three Minutes'
- Theoretical Description
- Observations
- Comparison/Discussion
Introduction
The Standard Hot Big-Bang Model

Established by (key phenomena):

- Hubble ($z \approx 10$)
- CMB ($z \approx 10^3$)
- BBN ($z \approx 10^{10}$)
Big-Bang Nucleosynthesis

- Production of light elements (D, $^3$He, $^4$He, $^7$Li, ...)
- Starting point: $T \approx 0.3$MeV (D stable against photo-dissociation)
  End point: $T \approx 0.1$MeV (Coulomb-barrier suppression)
- Radiation dominated expansion
  → abundances depend only on baryon-to-photon ratio
  → one parameter theory
**Historical Remarks**

1946: Gamov suggested big-bang origin of light elements.

1948: Alpher indicated radiation dominance.

1949: Turkevich and Fermi showed, that BBN stops at \( A = 7 \).

1953: Alpher, Herman and Follin set up BBN picture as it is today.

1965-1967: Peebles; Wagoner, Fowler and Hoyle simulated detailed reaction network.

1973: Measurement of Deuterium restricted \( \Omega_B < 0.1 \).

From 1998: Keck and WMAP opened new precision area.
Physical Picture of 'The First Three Minutes'
Nonrelativistic nuclear species with mass number $A$ and charge $Z$ in/with

- kinetic equilibrium: $n_A = g_A \left( \frac{m_A T}{2\pi} \right)^{\frac{3}{2}} \exp \left( \frac{\mu_A - m_A}{T} \right)$,
- chemical equilibrium: $\mu_A = Z \mu_p + (A - Z) \mu_n$,
- binding energy: $B_A = Z m_p + (A - Z) m_n - m_A$.

$$\Rightarrow \quad n_A = g_A A^{\frac{3}{2}} 2^{-A} \left( \frac{2\pi}{m_N T} \right)^{3(A-1)/2} n_p n_{n}^{A-Z} \exp \left( \frac{B_A}{T} \right)$$
Define mass fraction $X_A$ contributed by nuclear species $A(Z)$:

$$X_A := \frac{n_A A}{\sum_{A', n_{A', A'}} n_{A', A'}}$$

$$= \frac{g_A \zeta(3)(A-1) \pi^{1-A} 2^{3A-5} A^2}{(m_N)} \left( \frac{T}{m_N} \right)^\frac{3(A-1)}{2}$$

$$\times \eta^{A-1} X_p X_n^{A-Z} \exp \left( \frac{B_A}{T} \right),$$

where

$$\eta = \frac{n_B}{n_\gamma} \quad (\approx 6 \times 10^{-10})$$

is the baryon-to-photon ratio ($\propto \Omega_B h^2$).
Step 1: Initial Condition

- $t = 10^{-2} \text{s}, \ T = 10\text{MeV}$
- relativistic degrees of freedom: $e^\pm, \nu_i, \gamma$
- weak interactions keep up equilibrium due to high temperature
- light elements in NSE because of small $\eta$
The Three Steps of BBN 2

Step 2: Freeze-out’s and $e^\pm$-annihilation

- $\nu_i$ decouple at $T = 2.7\text{MeV}$.
- $e^\pm$-annihilation at $T = 1\text{MeV}$. ($\rightarrow T_\gamma = \left(\frac{11}{4}\right)\frac{1}{3} T_\nu$)
- Equilibrium between neutrons and protons freezes out at $T = 0.8\text{MeV}$:
  \[
  \frac{n}{p} = \frac{X_n}{X_p} = \exp\left(-\frac{m_n - m_p}{T} + \frac{\mu_e - \mu_\nu}{T}\right) \approx \exp\left(-\frac{m_n - m_p}{T}\right) \approx \frac{1}{6}.
  \]
  - Reaction rate $\Gamma \approx G_F^2 T^5$ for weak the interactions
    \[
    n \leftrightarrow p + e^- + \bar{\nu},
    \]
    \[
    \nu + n \leftrightarrow p + e^-,
    \]
    \[
    e^+ + n \leftrightarrow p + \bar{\nu}
    \]
    (determined by neutron lifetime $\tau_n$).
  - Expansion rate of the universe: $H \approx 1.66 g^*_\ast T^2 / m_{pl}$.
    \[
    \Rightarrow \frac{\Gamma}{H} \approx \left(\frac{T}{0.8\text{MeV}}\right)^3
    \]
The Three Steps of BBN 3

Step 3: Nucleosynthesis

- At $T = 0.3\text{MeV}$ and $t = 1\text{min}$, Deuterium becomes stable against photo-dissociation (Saha-condition, note: $B_2 = 2.2\text{MeV}$). Deuterium ‘bottleneck’ opens up and light elements abundances evolve towards their NSE value.

- Due to neutron decay $\frac{n}{p} \rightarrow \frac{1}{t}$
  (compare to equilibrium value of $\frac{1}{74}$).

  $Y_p := X_4 = \frac{4(n/2)}{n + p} \approx 0.25$

  (independent of nuclear reaction rates)

- At $T = 0.1\text{MeV}$ and $t = 3\text{min}$ the nucleosynthesis stops (Coulomb-barrier suppression).
Theoretical Description
Nuclear Reaction Network in an expanding Box

- Idea: Integrate up starting conditions according to thermal rates.
- Small baryon-to-photon ratio:
  - Need only to consider 2-body nuclear reactions.
  - Expansion driven by radiation.
- Thermal reaction rates from averaging cross sections:

\[ \lambda = N_A \langle \sigma v \rangle = N_A \left( \frac{8}{\pi \mu (kT)^3} \right)^{\frac{1}{2}} \int_0^\infty \sigma(E)E \exp \left( -\frac{E}{kT} \right) dE. \]

- State of the art: Systematic error estimation.
- Result: BBN abundances computed from measurements in a laboratory.
$N_A \sigma v$ (also called R-factor) for $p(n, \gamma)d$: Four data points and R-matrix calculation with 1\sigma error.
The Reaction Network

- 11 reaction rates
- no stable $A = 5, 8$
- 2 branches for $^7\text{Li}$ production
Left: Theoretical Prediction with 1\( \sigma \) error bars as function of \( \eta \).
Right: Relative uncertainties in percent of the light element predictions.
• The light element abundances depend on $\tau_n$, $\eta$, $g_*$, $G_N$ and 11 nuclear reaction rates.

• Larger $\tau_n$ or $g_*$ means earlier $\frac{n}{p}$ freeze-out, larger $\eta$ higher baryon density and all therefore more $^4\text{He}$.

• The numerical evaluation at $\eta = 6.14 \times 10^{-10}$ gives:

$$Y_p = 0.24849 \left( \frac{10^{10} \eta}{6.14} \right)^{0.39} \left( \frac{\tau_n}{\tau_{n,0}} \right)^{0.72} \left( \frac{G_N}{G_{N,0}} \right)^{0.35},$$

$$10^5 \frac{\text{D}}{\text{H}} = 2.558 \left( \frac{10^{10} \eta}{6.14} \right)^{-1.62} \left( \frac{\tau_n}{\tau_{n,0}} \right)^{0.41} \left( \frac{G_N}{G_{N,0}} \right)^{0.95} R_4^{-0.55} R_5^{-0.45} R_3^{-0.32} R_2^{-0.20},$$

$$10^{10} \frac{^7\text{Li}}{\text{H}} = 4.364 \left( \frac{10^{10} \eta}{6.14} \right)^{2.12} \left( \frac{\tau_n}{\tau_{n,0}} \right)^{0.44} \left( \frac{G_N}{G_{N,0}} \right)^{-0.72} R_2^{1.34} R_3^{0.96} R_8^{-0.76} R_1^{1.71} R_4^{0.71} R_3^{0.59} R_6^{-0.27},$$

where $R_i$ are renormalizations of nuclear reactions.

• Deuterium, which can also be measured most accurately, gets the role of the 'baryometer'. Helium-4 is sensible to $\tau_n$ and $g_*$. 

Observations

Idea: Measure present day abundances and extrapolate back to $t = 3\text{ min}$.
Problem: Chemical evolution of elements after BBN not always clear.
Deuterium

- Binding energy $B_2 = 2.2\text{MeV}$ and thus very fragile.
- No astrophysical source known.

$\Rightarrow$ Measurements will always give an upper bound.
- Measured via absorption features of distant HII-regions against more distant quasars.
Lyman-α absorption line (1216 Å) of a \( z = 3.572 \) HII-region. The line of D is shifted by 0.33(1 + \( z \)) Å.

Problems:
- Inloper
- Redshifted H-lines mimic D (complex velocity structure).
- How much D is destroyed?
- Only few quasars usable, small sample size.
- \( \chi^2_{world} = 4.1. \)

World average of the 5 best measurements: \( \frac{D}{H} = \left(2.78^{+0.44}_{-0.38}\right) \times 10^{-5}. \)

2 multiple absorption line system: \( \frac{D}{H} = \left(2.49^{+0.20}_{-0.18}\right) \times 10^{-5}. \)
- $^4$He is produced in stars.
- Measured by optical recombination lines in low-metallic HII-regions.
- Extrapolated to zero metallicity.

- 2 different values for $Y_p$, depending on treating emission and underlying absorption:

  $$Y_p = 0.238 \pm 0.002 \pm 0.005,$$

  $$Y_p = 0.244 \pm 0.002 \pm 0.005$$
• Evolution unclear, since it is destroyed and produced in stars.

• Abundances measured in atmosphere of old (population II) stars in halo of our galaxy:
  – Lithium plateau (‘Spite plateau’) for massive stars.
  – Large systematical errors.

Current measurements:

\[
\frac{^7\text{Li}}{\text{H}} = \left(1.23^{+0.68}_{-0.32}\right) \times 10^{-10}.
\]
Comparison/Discussion
Treating all observations equally:

\[ 1 \lesssim 10^{10} \eta \lesssim 7. \]

For Deuterium only:

\[ 10^{10} \eta = 6.28^{+0.34}_{-0.35}, \]

\[ 10^{10} \eta = 5.92^{+0.55}_{-0.58}, \]

for multiple absorption and world average respectively.
Comparison with CMB

WMAP determines baryon density and therefore baryon-to-photon ratio $\eta$ more precisely:

$$\Omega_B h^2 = 0.0224 \pm 0.0009,$$

$$\eta = (6.14 \pm 0.25) \times 10^{-10}.$$

<table>
<thead>
<tr>
<th>Elements</th>
<th>Observations</th>
<th>WMAP Predictions</th>
<th>$\chi^2_{eff}$</th>
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</thead>
<tbody>
<tr>
<td>D/H</td>
<td>$(2.49^{+0.20}_{-0.18}) \times 10^{-5}$</td>
<td>$(2.55^{+0.21}_{-0.20}) \times 10^{-5}$</td>
<td>0.045</td>
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<tr>
<td>D/H</td>
<td>$(2.78^{+0.44}_{-0.38}) \times 10^{-5}$</td>
<td>$(2.55^{+0.21}_{-0.20}) \times 10^{-5}$</td>
<td>0.281</td>
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<tr>
<td>$Y_p$</td>
<td>$0.238 \pm 0.007$</td>
<td>$0.2485 \pm 0.0005$</td>
<td>3.77</td>
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<tr>
<td>$Y_p$</td>
<td>$0.244 \pm 0.007$</td>
<td>$0.2485 \pm 0.0005$</td>
<td>0.692</td>
</tr>
<tr>
<td>$^7\text{Li}/H$</td>
<td>$(1.23^{+0.64}_{-0.46}) \times 10^{-10}$</td>
<td>$(4.26^{+0.91}_{-0.86}) \times 10^{-10}$</td>
<td>10.72</td>
</tr>
<tr>
<td>$^7\text{Li}/H$</td>
<td>$(2.19^{+0.46}_{-0.38}) \times 10^{-10}$</td>
<td>$(4.26^{+0.91}_{-0.86}) \times 10^{-10}$</td>
<td>4.50</td>
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</table>
Number of neutrinos: $Y_p$ insensitive to $\eta$, but sensitive on expansion rate, i.e.

$$g_* = 5.5 + \frac{7}{4} N_\nu.$$ 

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<th>Observations</th>
<th>$N_{\nu,eff}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>D/H</td>
<td>$(2.49^{+0.20}_{-0.18}) \times 10^{-5}$</td>
<td>$2.78^{+0.87}_{-0.76}$</td>
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<tr>
<td>D/H</td>
<td>$(2.78^{+0.44}_{-0.38}) \times 10^{-5}$</td>
<td>$3.65^{+1.46}_{-1.30}$</td>
</tr>
<tr>
<td>$Y_p$</td>
<td>$0.238 \pm 0.007$</td>
<td>$2.26^{+0.37}_{-0.36}$</td>
</tr>
<tr>
<td>$Y_p$</td>
<td>$0.244 \pm 0.007$</td>
<td>$2.67^{+0.40}_{-0.38}$</td>
</tr>
</tbody>
</table>

Here: $N_\nu = 2$ (yellow), 3 (blue), 4 (red)
Chemical potential of neutrinos $\mu_\nu$ in

$$\frac{n}{p} \approx \left( \frac{n}{p} \right)_{\mu_\nu=0} \exp \left( -\frac{\mu_\nu}{T} \right)$$

allows to dial in any desired freeze-out ratio.

Inhomogeneities in $\eta$, for example from a first order QCD phase transition:
- Diffusion of neutrons leads to isospin inhomogeneity.
- Only possible, if typical fluctuation distance compares to neutron diffusion length ($d = 200m - 500km$ during BBN).
- For same average $\eta$, usually overproduction of light elements.
Summary

- Standard BBN well established and in concordance with measured light element abundances and WMAP and spans nine orders of magnitude.
- Described by ’reaction network in an expanding box’, with the parameters $\eta, \tau_n, g_*$ and 11 reaction rates.
- Still difficulties in determining primordial abundances.
- Insight into chemical evolution of universe (especially HII-regions) and astrophysics of stars.
References