Charmonium in the QGP

Debye screening a' la Matsui & Satz

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Overview

(1) Charmonium: an Introduction
(2) Rehersion: Debye Screening
(3) Charmonium in the QGP
(4) Color Screening – Models and Predictions
(5) Summary / Outlook
Introduction - Charmonium

- Charmonium is a bound state of a charm quark – antiquark pair: \((c\bar{c})\)

- Stable states as long as strong decay is forbidden, i.e. Mass of charmonium is below mass of possible light-heavy mesons \((M \geq 1.9\text{GeV})\)

<table>
<thead>
<tr>
<th>state</th>
<th>(\eta_c)</th>
<th>(J/\psi)</th>
<th>(\chi_{c0})</th>
<th>(\chi_{c1})</th>
<th>(\chi_{c2})</th>
<th>(\psi')</th>
</tr>
</thead>
<tbody>
<tr>
<td>mass [GeV]</td>
<td>2.98</td>
<td>3.10</td>
<td>3.42</td>
<td>3.51</td>
<td>3.56</td>
<td>3.69</td>
</tr>
<tr>
<td>(\Delta E) [GeV]</td>
<td>0.75</td>
<td>0.64</td>
<td>0.32</td>
<td>0.22</td>
<td>0.18</td>
<td>0.05</td>
</tr>
</tbody>
</table>

stable Charmonium states and the binding Energies [Satz2006]

- states above \(\psi'\) possible but not stable, will not be discussed
- of importance to us: Quarkonia-states a much more tightly bound than lighter hadrons because of heavy quark masses
Charmonium in the QGP

- Consider heavy quark $Q$ and antiquark $\bar{Q}$ in a colorsinglet state
  (for this talk mainly charm-quarks, but bottom quarks are qualatively equal)
  Assume quarks are heavy and static so that any energy changes in the system indicate change in the binding energy.

- What happens as we increase Temperature $T$?
Charmonium below $T_c$

- Start at $T=0$ and vacuum: Free energy has string form:
  \[ F(r) \sim \sigma r \]
  resonance experiments determine $\sigma \approx 0.16 \text{GeV}^2$

- $Q\bar{Q}$-state breaks if $F(r)$ high enough that two light quarks can be formed:
  \[ F_0 = 2(M_D - m_c) \approx 1.2 \text{GeV} \]
  \[ r_0 \approx 1.2 \text{GeV}/\sigma \approx 1.5 \text{fm} \]

In vacuum $Q\bar{Q}$ break if they are separated by more than 1.5 fm
Charmonium below $T_c$

- $T>0$: Light mesons are in our system:

Flip flop recoupling may occur, decreasing the large distance $Q\bar{Q}$ potential $F(\infty, T)$ with increasing $T$.

String breaking through recoupling [Satz2006]
Focus on – Deconfinement

- QCD predicts that the strong interaction weakens at small distances, resulting in asymptotic freedom.

- Interesting implication: At very high pressures or temperatures quarks come close enough to each other and form a deconfined phase of matter.

Simplified idea: At large distances overall color charge has to be neutral. We need

a) three quarks of different color i.e. \((rgb)\)

b) quarkonium state \((r\bar{r})\)

At very small distances one quark always sees enough quarks of suitable colors so that overall charge cancels out:
Charmonium in QGP

- At $T > T_c$, light quarks and gluons become deconfined which leads to color screening.

Quarkonium can be used as temperature probe, ranging from vacuum string breaking, then dissociation through recoupling to colorscreening.

- This talk: Screening theory, later talks: experimental implications
Charmonium in the QGP

Since 2000

impr. stagg., p4, $N_f=2$
Karsch et al, NPB 605 (01) 579

impr. Wilson, $N_f=2$
CP-PACS, PRD 63 (01) 034502

Wilson. impr., $N_f=2$
Nakamura et al, hep-lat/0409153

impr. stagg., asqtad, $N_f=2+1$
MILC, PRD 71 (05) 034504

impr. stagg., HYP, $N_f=2+1$
Petreczky, J. Phys. G 30 (04) S1259

std. stagg., $N_f=2+1$
Fodor.Katz, JHEP 0404 (04) 050

Critical Temperatures obtained from different lattice studies[Petreczky]
Debye-Screening: Rehersion

- In order to understand Debye-Screening in QGP, first look at screening in classical Plasma

- Plasma: charged particles with added test charge:

\[ \frac{d^2 \phi}{dx^2} = -\frac{\rho}{\varepsilon_0} \]  
relation of electrostatic potential \( \phi \) and charge density \( \rho \) (1-D)

- thermal equilibrium, particle density distribution follows Boltzmann distr.:

\[ n(x) = n_0 e^{-U(x)/kT} \]  

\( U(x) \): potential Energy  
\( k \) : Boltzmann constant
Debye Screening: Rehersion

- Now assume ions charge \( Z q_e \) Potential Energy becomes

\[
U(x) = Z q_e \phi(x) \quad (2.3)
\]

- Eq. (2.2) yields:

Density of ions

\[
n_i(x) = n_0 e^{-\frac{Z q_e \phi(x)}{k T_i}} \quad (2.4a)
\]

Density of electrons

\[
n_e(x) = n_0 e^{\frac{q_e \phi(x)}{k T_e}} \quad (2.4b)
\]

- Total charge density is:

\[
\rho = Z q_e n_i - q_e n_e \\
\Longleftrightarrow \rho = q_e n_0 \left( e^{-\frac{Z q_e \phi}{k T_e}} - e^{\frac{q_e \phi}{k T_i}} \right) \quad (2.5)
\]
Debye-Screening: Rehersion

- Eq: (2.5) in (2.1) \( \iff \phi \) must satisfy

\[
\frac{d^2 \phi}{dx^2} = -\frac{q_e n_0}{\epsilon_0} \left( e^{-Zq_e \phi / kT_i} - e^{+q_e \phi / kT_e} \right)
\]

(2.6)

- In plasma \( T \gg \phi \) \( \iff \) \( e^{\pm q_e \phi / kT} = 1 \pm \frac{q_e \phi}{kT} \)

\[
\Rightarrow \frac{d^2 \phi}{dx^2} = \frac{q_e^2 n_0}{\epsilon_0 k} \left( \frac{Z}{T_i} + \frac{1}{T_e} \right) \phi
\]

\[
\Leftarrow \frac{q_e^2 n_0}{\epsilon_0 k} \frac{Z T_e + T_i}{T_e T_i} \phi
\]

(2.7)
Debye Screening: Rehersion

- Solution of (2.7) is of type $\phi = Ae^{-x/\lambda_D} + Be^{+x/\lambda_D}$
  with $B=0$, else potential would grow to infinity for large $x$

\[ \lambda_D = \sqrt{\frac{\epsilon_0 k T_i T_e}{n_0 q_e^2 (T_i + ZT_e)}} \]

- for $T_i \ll T_e$

\[ \lambda_D = \sqrt{\frac{\epsilon_0 k T_e}{n_0 q_e^2}} \]

- The Coulomb-potential of a test charge drops to $1/e$ at $\lambda_D$. Thus it is effectively screened from charges outside the Debye-sphere

[Feynmann, Hofmann]
**Color Screening in the QGP**

- Similar to Debye-Screening discussed earlier.

  color charge instead of electrical charge

  \[ \leftrightarrow \]

  Charge-carriers are now quarks and gluons

- Won't work if charge-carriers can't move freely

  \[ \leftrightarrow \]

  Need for deconfined medium: QGP
Color Screening in the QGP

- Due to screening Charmonium will dissociate, if Debye sphere around one charm quark is smaller than the binding radius of the charmonium. ([Matsui] $0.5 \leq r_{J/\psi}^{\text{max}} \leq 1.3\,\text{fm}$ for non-relativistic interaction)

- Screening radius is inversely proportional to the density and thus temperature.

What does this imply?
Color Screening in QGP

- Expect that different Charmonium states will dissociate at different temperatures, depending on how tightly bound.

\[ [\text{Satz2006}] \]

- To compare with experiments quantitative calculations required.
Color Screening in QGP - Models

- Three approaches possible:

1) Model heavy quark potential $V(r, T)$ as function of $T$ and solve Schrödinger-equation

2) Use internal energy as potential $V(r, T) = U(r, T)$ and then solve Schrödinger-equation

$$U(r, T) = -T^2 \left( \frac{\delta [F(r, T)/T]}{\delta T} \right) = F(r, T) - T \left( \frac{\delta F(r, T)}{\delta T} \right) \quad (3.1)$$

3) Directly calculate quarkonium spectrum from finite temperature lattice QCD still to complicated

[Satz2006]
Potential Model Approach-Schwinger Form

- Simplest confining potential for $Q \bar{Q}$ is Cornell-potential

\[ V(r) = \sigma r - \frac{\alpha}{r} \]

\[ \sigma : \text{string tension} \]
\[ \alpha : \text{gauge coupling constant} \]

Added Debye-screening yields

\[ V(r, T) \propto \sigma r \left\{ \frac{1 - e^{-\mu r}}{\mu r} \right\} - \frac{\alpha}{r} e^{-\mu r} = \frac{\sigma}{\mu} \left\{ 1 - e^{\mu r} \right\} - \frac{\alpha}{r} e^{-\mu r} \quad (3.2) \]

$\mu(T)$: inverse Debye-radius, determined from Free Energy Fits
Potential Model Approach-Schwinger Form

- Input into Schrödinger-equation

\[
\left\{ 2m_e - \frac{1}{m_e} \nabla^2 + V(r, T) \right\} \phi_i = \Delta E_i \phi_i(r) \quad (3.3)
\]

\[\psi' \text{ and } \chi_c \text{ dissociate at } T \approx T_c \text{, } J/\psi \text{ holds until about } 1.2T_c\]

- Shortcomings: Schwinger Form is for 1-D, reality is 3-D
  Screening Mass is assumed in high energy form, but behaves different at \( T_c \)
Internal Energy Model

- [Dixit1990] suggests screening can be evaluated for a given free Energy

\[ F \propto r^q \]

in Cornell-potential \( q=1 \) for string term, \( q=-1 \) for gauge term, in \( d \) dimensions.

- Assuming screening can be calculated separately for each term yields total free energy of:

\[ F(r, T) = F_s(r, T) + F_c(r, T) = \sigma r f_s(r, T) - \frac{\alpha}{r} f_c(r, T) \]  \hspace{1cm} (3.4)

with boundary conditions

\[ f_s(r, T) = f_c(r, T) = 1 \quad \text{for} \quad T \to 0 \]

\[ f_s(r, T) = f_c(r, T) = 1 \quad \text{for} \quad r \to 0 \]  \hspace{1cm} (3.5)
Internal Energy Model

- Through Debye-Hückel theory this results in

\[
F_c (r, T) = -\frac{\alpha}{r} [ e^{-\mu r} + \mu r ]
\]

(3.6)

**gauge/coulomb -term**

**string term**

\[
F_s (r, T) = \frac{\sigma}{\mu} \left[ \frac{\Gamma (1/4)}{2^{3/2} \Gamma (3/4)} - \frac{\sqrt{\mu r}}{2^{3/2} \Gamma (3/4)} K_{1/4} \left[ (\mu r)^2 \right] \right]
\]

(3.7)

- large distance limit due to color screening
- gaussian cut-off

\[
K_{1/4} (x^2) \propto \exp (-x^2)
\]
Internal Energy Model

- Can now use (3.6) and (3.7) as expression for free energy.
Internal Energy Model

- Can now use (3.6) and (3.7) as expression for free energy.

- Recall (3.1):  
  \[ U(r, T) = -T^2 \left( \frac{\delta[F(r, T)/T]}{\delta T} \right) = F(r, T) - T \left( \frac{\delta F(r, T)}{\delta T} \right) \]
  
  this gives us binding potential which we put into the Schrödinger equation

- We want to know temperature \( T_i \) at which state i dissociates
  rewrite potential as

  \[ V(r, T) = V(\infty, T) + \tilde{V}(r, T) \]
Internal Energy Model

- \( V(\infty, T) = c_1 \frac{\sigma}{\mu} - \alpha \mu + T \frac{d \mu}{dT} \left[ c_1 \frac{\sigma}{\mu^2} + \alpha \right] \)

with \( c_1 = \Gamma(1/4)/2^{3/2} \Gamma(3/4) \) describes energy of cloud of quarks and gluons in Debye-sphere around heavy quark relative to cloud without heavy quark

- \( \tilde{V}(r, T) \) vanishes for \( r \to \infty \)
Internal Energy Model – Results

- Schrödinger Equation rewritten in terms of binding Energy of state i

\[
\left\{ \frac{1}{m_e} \nabla^2 + V(r, T) \right\} \phi_i = \Delta E_i(T) \phi_i(r)
\]

with \( \Delta E_i(T) = V(\infty, T) - M_i - 2m_c \)

- State i ceases to exist if \( \Delta E_i(T) = 0 \) giving us a critical temperature for each state, depending only on \( \mu(T) \)
Internal Energy Model - Results

Binding energy at different values of $T$ for different Charmonium states [Satz]

<table>
<thead>
<tr>
<th>state</th>
<th>$J/\psi(1S)$</th>
<th>$\chi_c(1P)$</th>
<th>$\psi'(2S)$</th>
<th>$\Upsilon(1S)$</th>
<th>$\chi_b(1P)$</th>
<th>$\Upsilon(2S)$</th>
<th>$\chi_b(2P)$</th>
<th>$\Upsilon(3S)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_d/T_c$</td>
<td>2.10</td>
<td>1.16</td>
<td>1.12</td>
<td>$&gt; 4.0$</td>
<td>1.76</td>
<td>1.60</td>
<td>1.19</td>
<td>1.17</td>
</tr>
</tbody>
</table>

Dissociation temperatures for Charmonium and Bottonium states [Satz]
First two approaches are model dependent, direct calculation using only QCD favourable.

To date solving full QCD calculations not possible due to lack of computer power. Quenched QCD does not include dynamical quark loops => uncertainties are introduced.

\[ J/\psi \text{ and } \chi_c \text{ spectral functions at different temperatures [Satz2006]} \]
Summary/ Outlook

- Charmonium suppression can be a useful probe for QGPs

- Screening models so far still include uncertainties, but near future will allow full lattice QCD calculations, which are truly model independent

- Models need to be experimentally verified. See future talks:
  - A. Marin - The experimental status of J/ψ measurements in nucleus-nucleus collisions (E)
  - Jun Nian - J/ψ suppression and enhancement - the statistical hadronization model (T)
  - M. Freudenberger - Quarkonium physics in nuclear collisions at the LHC (T)
References