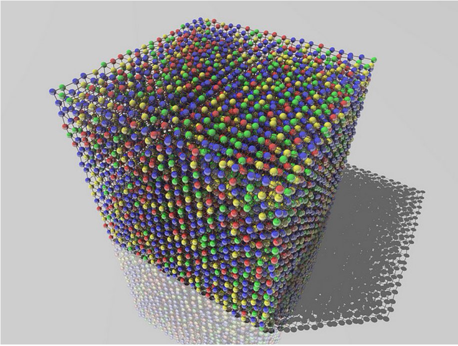
Lattice QCD

David Scheffler

Seminar: Relativistische Schwerionenphysik



by “fdecomite” on flickr

Outline

How to formulate lattice QCD

QCD thermodynamics

Methods for $\mu > 0$

Summary

TECHNISCHE UNIVERSITÄT DARMSTADT

What is lattice QCD?

- non-perturbative method
- solve QCD path integral
- numerical simulation on discretized 4d spacetime
- regularization by finite lattice spacing
- introduced by K. Wilson (1974)

Continuum QCD

Langrangian

$$\mathcal{L}_{QCD} = -\frac{1}{4} F_a^{\mu\nu} F^a_{\mu\nu} + \bar{\Psi} \left(i\gamma^\mu D_\mu - m \right) \Psi$$

with

$$F_a^{\mu\nu} = \partial^\mu A_a^\nu - \partial^\nu A_a^\mu + g f_a^{bc} A_b^\mu A_c^\nu$$

$$D^\mu = \partial^\mu - i\frac{g}{2} A_a^\mu \lambda^a$$

Continuum QCD

TECHNISCHE
UNIVERSITÄT
DARMSTADT

Langrangian

$$\mathcal{L}_{QCD} = -\frac{1}{4} F_a^{\mu\nu} F_{\mu\nu}^a + \bar{\Psi} \left(i\gamma^\mu D_\mu - m \right) \Psi$$

with

$$F_a^{\mu\nu} = \partial^\mu A_a^\nu - \partial^\nu A_a^\mu + g f_a^{bc} A_b^\mu A_c^\nu$$

$$D^\mu = \partial^\mu - i\frac{g}{2} A_a^\mu \lambda^a$$

Generating functional and expectation value

$$Z_{QCD} = \int DA\, D\bar{\Psi}\, D\Psi\; e^{\,i \int d^4x \mathcal{L}_{QCD}[A,\bar{\Psi},\Psi]}$$

$$\langle O \rangle = \frac{1}{Z_{QCD}} \int DA\, D\bar{\Psi}\, D\Psi\; O\; e^{\,i \int d^4x \mathcal{L}_{QCD}[A,\bar{\Psi},\Psi]}$$

Lattice formulation

TECHNISCHE UNIVERSITÄT DARMSTADT

- Wick rotation from Minkowski to Euclidean spacetime:

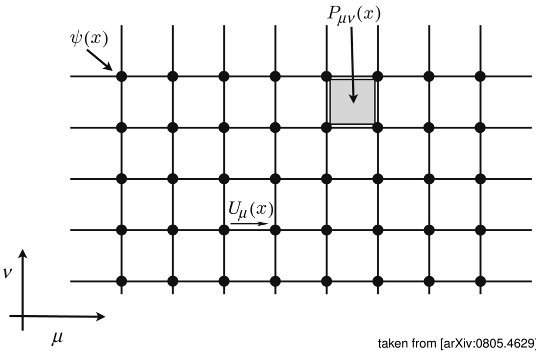
$$\mathcal{L}_E = \frac{1}{4} F_a^{\mu\nu} F_{\mu\nu}^a + \bar{\Psi} \left(\gamma^\mu D_\mu^E + m \right) \Psi$$

$$Z_E = \int DA \, D\bar{\Psi} \, D\Psi \, e^{-\int \mathrm{d}^4 x \mathcal{L}_E[A,\bar{\Psi},\Psi]}$$

- equivalent to statistical theory $\Rightarrow$ apply Monte Carlo algorithms
- 4d lattice: $N_s^3 \times N_\tau$ with lattice spacing a
- discrete imaginary time: $x_0 = -ix_4$
- temperature: $T = \frac{1}{N_\tau a}$

Degrees of freedom

TECHNISCHE UNIVERSITÄT DARMSTADT



taken from [arXiv:0805.4629]

- on sites: fermion fields $\psi(x)$
- on links: gauge fields $U_\mu(x) = \exp[iagA_\mu(x)]$
- build gauge independent combinations, e. g. plaquette
  $P_{\mu\nu}(x) = U_\mu(x)U_\nu(x + a\hat{\mu})U^\dagger_\mu(x + a\hat{\nu})U^\dagger_\nu(x)$

Gauge action

- interpretation of plaquette

$$P_{\mu\nu}(x) = U_\mu(x)\, U_\nu(x + a\hat{\mu})\, U_\mu^\dagger(x + a\hat{\nu})\, U_\nu^\dagger(x)$$

Gauge action

TECHNISCHE UNIVERSITÄT DARMSTADT

- interpretation of plaquette

$$P_{\mu\nu}(x) = e^{iagA_\mu(x)}\ e^{iagA_\nu(x+a\hat{\mu})}\ e^{-iagA_\mu(x+a\hat{\nu})}\ e^{-iagA_\nu(x)}$$

Gauge action

- interpretation of plaquette

$$P_{\mu\nu}(x) = e^{iagA_\mu(x)}\ e^{iagA_\nu(x+a\hat{\mu})}\ e^{-iagA_\mu(x+a\hat{\nu})}\ e^{-iagA_\nu(x)}$$
$$= \exp\left(ia^2 F_{\mu\nu}(x) + \mathcal{O}(a^3)\right)$$

(expand in small *a* and use Baker-Campbell-Hausdorff-formula)

Gauge action

TECHNISCHE UNIVERSITÄT DARMSTADT

- interpretation of plaquette

$$P_{\mu\nu}(x) = e^{iagA_\mu(x)}\, e^{iagA_\nu(x+a\hat{\mu})}\, e^{-iagA_\mu(x+a\hat{\nu})}\, e^{-iagA_\nu(x)}$$
$$= \exp\left(ia^2 F_{\mu\nu}(x) + \mathcal{O}(a^3)\right)$$

(expand in small *a* and use Baker-Campbell-Hausdorff-formula)

- standard (Wilson) plaquette action:

$$S_g[U] = \sum_x \sum_{\mu\leq\nu} \beta \left(1 - \frac{1}{3}\mathrm{Re}\,\mathrm{Tr}\, P_{\mu\nu}(x)\right) \qquad \beta = \frac{2N_c}{g^2}$$

Gauge action

TECHNISCHE UNIVERSITÄT DARMSTADT

- interpretation of plaquette

$$P_{\mu\nu}(x) = e^{iagA_\mu(x)}\, e^{iagA_\nu(x+a\hat{\mu})}\, e^{-iagA_\mu(x+a\hat{\nu})}\, e^{-iagA_\nu(x)}$$
$$= \exp\left(ia^2 F_{\mu\nu}(x) + \mathcal{O}(a^3)\right)$$

(expand in small *a* and use Baker-Campbell-Hausdorff-formula)

- standard (Wilson) plaquette action:

$$S_g[U] = \sum_x \sum_{\mu\leq\nu} \beta \left(1 - \frac{1}{3}\mathrm{Re}\,\mathrm{Tr}\, P_{\mu\nu}(x)\right) \qquad \beta = \frac{2N_c}{g^2}$$
$$= \frac{a^4}{2g^2} \sum_x \sum_{\mu,\nu} \left(\mathrm{Tr}\, F^2_{\mu\nu}(x) + \mathcal{O}(a^2)\right)$$

Gauge action

- interpretation of plaquette

$$P_{\mu\nu}(x) = e^{iagA_\mu(x)}\ e^{iagA_\nu(x+a\hat{\mu})}\ e^{-iagA_\mu(x+a\hat{\nu})}\ e^{-iagA_\nu(x)}$$
$$= \exp\left(ia^2 F_{\mu\nu}(x) + \mathcal{O}(a^3)\right)$$

(expand in small a and use Baker-Campbell-Hausdorff-formula)

- standard (Wilson) plaquette action:

$$S_g[U] = \sum_x \sum_{\mu\leq\nu} \beta \left(1 - \frac{1}{3}\mathrm{Re}\,\mathrm{Tr}\, P_{\mu\nu}(x)\right) \qquad \beta = \frac{2N_c}{g^2}$$
$$= \frac{a^4}{2g^2} \sum_x \sum_{\mu,\nu} \left(\mathrm{Tr}\, F^2_{\mu\nu}(x) + \mathcal{O}(a^2)\right)$$

- improved lattice actions use further terms to reduce discretization errors

Including fermions

- fermion fields $\bar{\psi}(x)$, $\psi(x)$ need to be anti-commuting
- Grassmann numbers: $\eta_1\eta_2 = -\eta_2\eta_1$, $\eta^2 = 0$

Including fermions

TECHNISCHE UNIVERSITÄT DARMSTADT

- fermion fields $\bar{\psi}(x)$, $\psi(x)$ need to be anti-commuting
- Grassmann numbers: $\eta_1\eta_2 = -\eta_2\eta_1$, $\eta^2 = 0$
- fermion action S_f with Dirac operator D_{xy}

$$S_f = \sum_{x,y} \bar{\psi}(x) D_{xy} \psi(y)$$

Including fermions

TECHNISCHE
UNIVERSITÄT
DARMSTADT

- fermion fields $\bar{\psi}(x)$, $\psi(x)$ need to be anti-commuting
- Grassmann numbers: $\eta_1\eta_2 = -\eta_2\eta_1$, $\eta^2 = 0$
- fermion action S_f with Dirac operator D_{xy}

$$S_f = \sum_{x,y} \bar{\psi}(x) D_{xy} \psi(y)$$

- Gaussian integration $\int D\bar{\eta} D\eta e^{-\bar{\eta}\mathcal{M}\eta} = \det \mathcal{M}$ leads to “Fermion determinant”

$$Z = \int DU\, D\bar{\psi}\, D\psi\, e^{-S_g[U] - S_f[U,\bar{\psi},\psi]} = \int DU \det \mathcal{M}\, e^{-S_g[U]}$$

(color and flavor indices are suppressed)

Fermion actions (I)

TECHNISCHE
UNIVERSITÄT
DARMSTADT

Naive discretization of the Dirac operator

- replace partial derivative:

$$\partial_\mu \psi(x) \to \frac{1}{2a}\big(\psi(x + a\hat{\mu}) - \psi(x - a\hat{\mu})\big)$$

Fermion actions (I)

TECHNISCHE UNIVERSITÄT DARMSTADT

Naive discretization of the Dirac operator

- replace partial derivative:

$$\partial_\mu \psi(x) \to \frac{1}{2a}\big(\psi(x + a\hat{\mu}) - \psi(x - a\hat{\mu})\big)$$

- include gauge links to make the derivative covariant:

$$D_{xy} = \sum_\mu \gamma_\mu \frac{1}{2a}\big(U_\mu(x)\delta_{x+a\hat{\mu},y} - U_{-\mu}(x)\delta_{x-a\hat{\mu},y}\big) + m\delta_{xy}$$

Fermion actions (II)

TECHNISCHE UNIVERSITÄT DARMSTADT

Fermion doubling

- unphysical poles in propagator
- 16 degenerate fermions (“doublers”)

Fermion actions (II)

Fermion doubling

- unphysical poles in propagator
- 16 degenerate fermions ("doublers")
- Nielsen-Ninomiya "No go" theorem: Impossible to have a
  - a) chirally invariant,
  - b) doubler-free,
  - c) local,
  - d) translation invariant
  - e) real bilinear

  fermion action on the lattice.

Fermion actions (III)

Wilson fermions

- additional Wilson term
- doublers gain mass $\sim 1/a$
- break chiral symmetry

Fermion actions (III)

TECHNISCHE UNIVERSITÄT DARMSTADT

Wilson fermions

- additional Wilson term
- doublers gain mass $\sim 1/a$
- break chiral symmetry

Staggered fermions

- diagonalization in Dirac space, use only one of four identical terms
- reduces doublers from 16 to 4 remaining "tastes"
- computationally cheaper
- further reduce doubling by rooting procedure
  e. g. for $N_f = 2 + 1$: $\int D\bar{\psi}\, D\psi\, e^{-S_f[U,\bar{\psi},\psi]} = \det \mathcal{M}_{ud}^{1/2} \det \mathcal{M}_s^{1/4}$

Sketch of simulation

TECHNISCHE
UNIVERSITÄT
DARMSTADT

generation of gauge configurations

- generate ensemble $\{U_i\}$ (e.g. via HMC)
- according to probability distribution $P(U) = \det \mathcal{M}[U]\, e^{-S_g[U]}$ (importance sampling)

Sketch of simulation

TECHNISCHE UNIVERSITÄT DARMSTADT

generation of gauge configurations

- generate ensemble $\{U_i\}$ (e.g. via HMC)
- according to probability distribution $P(U) = \det \mathcal{M}[U]\, e^{-S_g[U]}$ (importance sampling)

measurement

ensemble average:

$$\langle O \rangle = \frac{1}{N} \sum_{i=1}^{N} O(U_i)$$

Temperature

$$T = \frac{1}{N_\tau a(\beta)}$$

Change temperature

a) by varying β at fixed N_τ

b) by varying N_τ at fixed a:
"fixed scale approach"

Temperature

TECHNISCHE UNIVERSITÄT DARMSTADT

$$T = \frac{1}{N_\tau a(\beta)}$$

Change temperature

a) by varying β at fixed N_τ

b) by varying N_τ at fixed a:
"fixed scale approach"

Line of constant physics

- vary (bare) lattice parameters
- but keep physical masses constant
- $\Rightarrow m = m(\beta)$

Setting the scale

- performed at zero temperature ($N_\tau \gg N_s$)
- measure quantity (e.g. meson mass) in lattice units
- equate to physical units
- but: only possible at physical point

Setting the scale

TECHNISCHE UNIVERSITÄT DARMSTADT

“Sommer scale” r_0

- Wilson loop

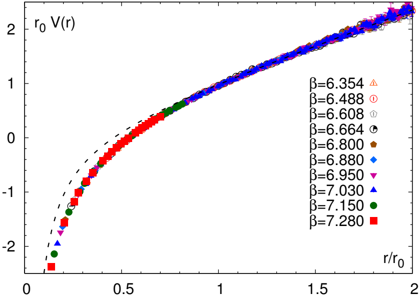
$$W(C) = \mathrm{Tr} \prod_{U_\mu \in C} U_\mu(x)$$

- static potential

$$V(r) = \lim_{t \to \infty} \frac{1}{t} \ln \langle W(C) \rangle$$

- fit to expected form

$$V(r) = C + B/r + \sigma r$$



from HotQCD [arXiv:1111.1710]

- r_0 defined via characteristic property:

$$\left(r^2 \frac{dV(r)}{dr} \right)_{r=r_0} = 1.65$$

Challenges

TECHNISCHE UNIVERSITÄT DARMSTADT

finite lattice spacing

- leads to discretization errors
- reduced by optimized actions
- extrapolate to $a \to 0$
  (continuum limit)
- finer lattices necessary

Challenges

TECHNISCHE UNIVERSITÄT DARMSTADT

finite lattice spacing

- leads to discretization errors
- reduced by optimized actions
- extrapolate to $a \to 0$ (continuum limit)
- finer lattices necessary

finite lattice volume

- leads to finite volume effects
- compare results from different N_s values
- larger lattices necessary

Challenges

TECHNISCHE UNIVERSITÄT DARMSTADT

finite lattice spacing

- leads to discretization errors
- reduced by optimized actions
- extrapolate to $a \to 0$ (continuum limit)
- finer lattices necessary

finite lattice volume

- leads to finite volume effects
- compare results from different N_s values
- larger lattices necessary

computationally intensive

- most complicated: inversion of fermion matrix
- worse for smaller quark masses
- high performance computing (CPU/GPU clusters) necessary

Challenges

TECHNISCHE UNIVERSITÄT DARMSTADT

finite lattice spacing

- leads to discretization errors
- reduced by optimized actions
- extrapolate to $a \to 0$ (continuum limit)
- finer lattices necessary

finite lattice volume

- leads to finite volume effects
- compare results from different N_s values
- larger lattices necessary

computationally intensive

- most complicated: inversion of fermion matrix
- worse for smaller quark masses
- high performance computing (CPU/GPU clusters) necessary

quark masses

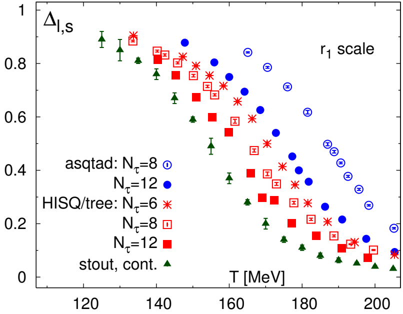
- extrapolate $m \to m_{phys}$

Observables

Chiral condensate

$$\langle \bar{\psi}\psi \rangle \sim \mathrm{Tr} \langle D^{-1} \rangle$$

- order parameter for chiral symmetry breaking
- chiral symmetry explicitly broken by finite quark mass



from HotQCD [arXiv:1111.1710]

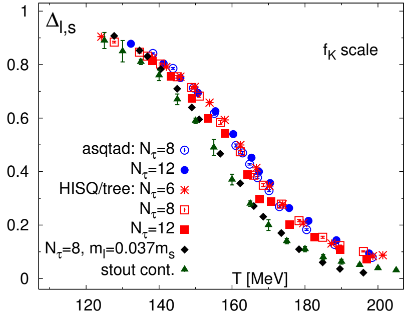
Observables

TECHNISCHE UNIVERSITÄT DARMSTADT

Chiral condensate

$$\langle \bar{\psi}\psi \rangle \sim \text{Tr} \langle D^{-1} \rangle$$

- order parameter for chiral symmetry breaking
- chiral symmetry explicitly broken by finite quark mass



from HotQCD [arXiv:1111.1710]

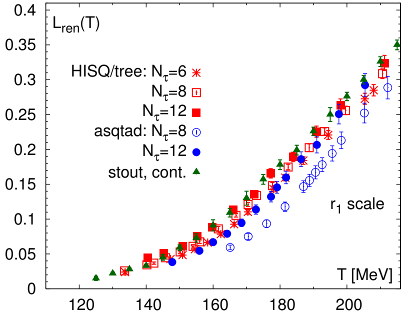
TECHNISCHE UNIVERSITÄT DARMSTADT

Polyakov loop

$$L = \left\langle \prod_{x_0}^{N_\tau} U_0(x) \right\rangle$$

- order parameter for confinement
- centre symmetry explicitly broken by presence of dynamical quarks
- relates to free energy of static quark

$$< L > \sim e^{-F_Q/T}$$



from HotQCD [arXiv:1111.1710]

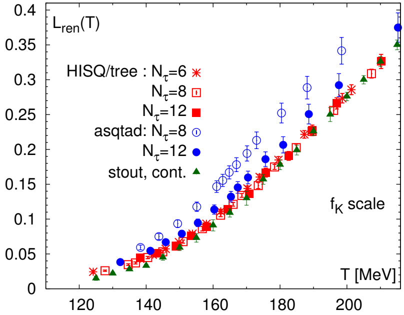
Observables

Polyakov loop

$$L = \left\langle \prod_{x_0}^{N_\tau} U_0(x) \right\rangle$$

- order parameter for confinement
- centre symmetry explicitly broken by presence of dynamical quarks
- relates to free energy of static quark

$$< L >\sim e^{-F_Q/T}$$



from HotQCD [arXiv:1111.1710]

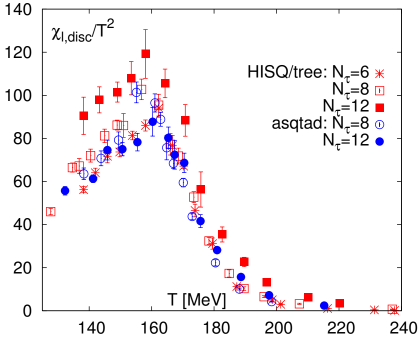
Observables

TECHNISCHE UNIVERSITÄT DARMSTADT

Chiral susceptibility

$$\chi_{l,disc} \sim \left\langle (\mathrm{Tr}\, D^{-1})^2 \right\rangle - \left\langle \mathrm{Tr}\, D^{-1} \right\rangle^2$$

- peak at transition temperature
- describes fluctuations of the order parameter



from HotQCD [arXiv:1111.1710]

Transition temperatures T_c at $\mu = 0$

collaboration	comments	T_c in MeV
Wuppertal-Budapest	infl. point of $\left\langle \bar{\psi}\psi \right\rangle_R$	155(3)(3)
Wuppertal-Budapest	infl. point of $\Delta_{l,s}$	157(3)(3)
Wuppertal-Budapest	peak of renorm. ch. susceptibility	147(2)(3)
HotQCD	HISQ/tree action, O(4) scaling	154(8)(1)
BMW	Polyakov loop	170(4)(3)

Transition temperatures T_c at $\mu = 0$

TECHNISCHE UNIVERSITÄT DARMSTADT

collaboration	comments	T_c in MeV
Wuppertal-Budapest	infl. point of $\langle\bar{\psi}\psi\rangle_R$	155(3)(3)
Wuppertal-Budapest	infl. point of $\Delta_{l,s}$	157(3)(3)
Wuppertal-Budapest	peak of renorm. ch. susceptibility	147(2)(3)
HotQCD	HISQ/tree action, O(4) scaling	154(8)(1)
BMW	Polyakov loop	170(4)(3)
RBC-Bielefeld	p4 action, 2006	192(7)(4)
MILC	asqtad action, 2004, peak of χ_{ch}, ch. limit	169(12)(4)

Explanation for previous differences

- large cut-off effects (taste splitting) in asqtad and p4 action
- extrapolation in $1/N_\tau^2$ not yet applicable for $N_\tau = 4, 6$

Thermodynamics on the lattice

TECHNISCHE UNIVERSITÄT DARMSTADT

integral method

- cannot compute f or Z directly
- compute derivatives of free energy density f and integrate along line of constant physics

$$\frac{f}{T^4} = -\frac{N_\tau^3}{N_s^3} \ln Z$$

$$\left.\frac{f}{T^4}\right|_{(\beta_0, m_0)}^{(\beta, m)} = -\frac{N_\tau^3}{N_s^3} \int_{\beta_0, m_0}^{\beta, m} \left(\mathrm{d}\beta \, \frac{\partial \ln Z}{\partial \beta} + \mathrm{d}m \, \frac{\partial \ln Z}{\partial m} \right)$$

integral method

- cannot compute f or Z directly
- compute derivatives of free energy density f and integrate along line of constant physics

$$\frac{f}{T^4} = -\frac{N_\tau^3}{N_s^3} \ln Z$$

$$\left.\frac{f}{T^4}\right|_{(\beta_0,m_0)}^{(\beta,m)} = -\frac{N_\tau^3}{N_s^3} \int_{\beta_0,m_0}^{\beta,m} \left(\mathrm{d}\beta \left\langle \frac{\partial \ln Z}{\partial \beta} \right\rangle + \mathrm{d}m \left\langle \frac{\partial \ln Z}{\partial m} \right\rangle \right)$$

integral method

- cannot compute f or Z directly
- compute derivatives of free energy density f and integrate along line of constant physics

$$\frac{f}{T^4} = -\frac{N_\tau^3}{N_s^3} \ln Z$$

$$\left.\frac{f}{T^4}\right|_{(\beta_0,m_0)}^{(\beta,m)} = -\frac{N_\tau^3}{N_s^3} \int_{\beta_0,m_0}^{\beta,m} \left(\mathrm{d}\beta \left[\left\langle \frac{\partial \ln Z}{\partial \beta} \right\rangle - \left\langle \frac{\partial \ln Z}{\partial \beta} \right\rangle_{T=0} \right] + \mathrm{d}m \left[\left\langle \frac{\partial \ln Z}{\partial m} \right\rangle - \left\langle \frac{\partial \ln Z}{\partial m} \right\rangle_{T=0} \right] \right)$$

Thermodynamics on the lattice

integral method

- cannot compute f or Z directly
- compute derivatives of free energy density f and integrate along line of constant physics

- $\left\langle \frac{\partial \ln Z}{\partial \beta} \right\rangle \sim$ gauge action
- $\left\langle \frac{\partial \ln Z}{\partial m} \right\rangle \sim$ chiral condensate

$$\frac{f}{T^4} = -\frac{N_\tau^3}{N_s^3} \ln Z$$

$$\left.\frac{f}{T^4}\right|_{(\beta_0,m_0)}^{(\beta,m)} = -\frac{N_\tau^3}{N_s^3} \int_{\beta_0,m_0}^{\beta,m} \left(\mathrm{d}\beta \left[\left\langle \frac{\partial \ln Z}{\partial \beta} \right\rangle - \left\langle \frac{\partial \ln Z}{\partial \beta} \right\rangle_{T=0} \right] + \mathrm{d}m \left[\left\langle \frac{\partial \ln Z}{\partial m} \right\rangle - \left\langle \frac{\partial \ln Z}{\partial m} \right\rangle_{T=0} \right] \right)$$

Thermodynamics on the lattice

TECHNISCHE UNIVERSITÄT DARMSTADT

- trace anomaly/interaction measure

$$\frac{I(T)}{T^4} = -N_\tau^4 \left(a\frac{d\beta}{da}\langle -S_g\rangle^{sub} + \sum_f a\frac{dm_f}{da}\langle \bar{\psi}_f\psi_f\rangle^{sub} \right)$$

- pressure

$$\frac{p}{T^4} = \int_{T_0}^{T} \mathrm{d}T \frac{I(T)}{T^5}$$

- energy density

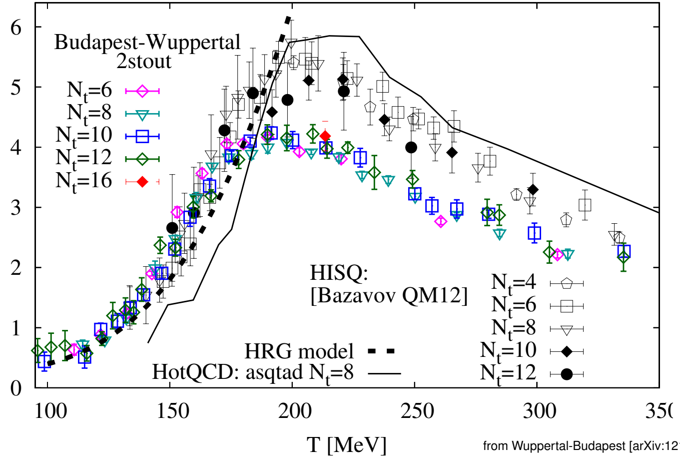
$$\epsilon = I + 3p$$

- entropy

$$s = \frac{\epsilon + p}{T}$$

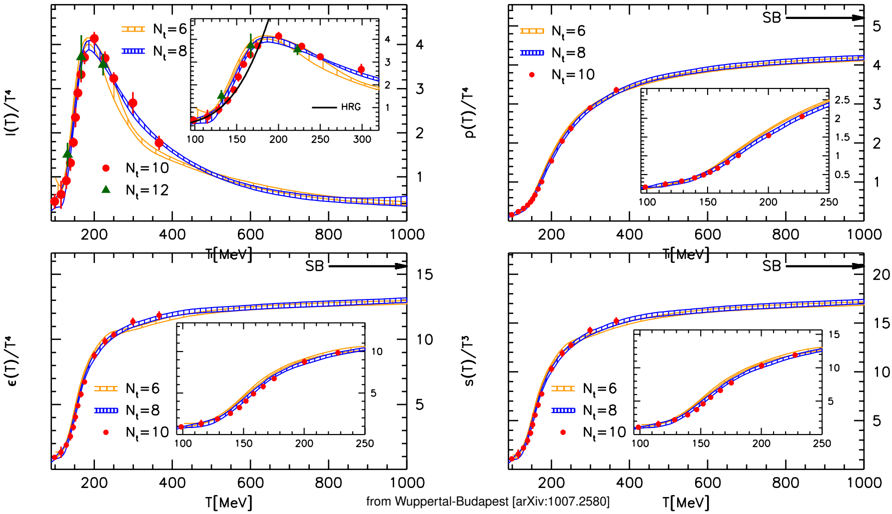
Trace anomaly

TECHNISCHE UNIVERSITÄT DARMSTADT



from Wuppertal-Budapest [arXiv:1210.6901]

Trace anomaly, pressure, energy density and entropy density



from Wuppertal-Budapest [arXiv:1007.2580]

Fermion sign problem

$$\det(\gamma_\mu D_\mu + m - \gamma_0 \mu) = \det{}^*(\gamma_\mu D_\mu + m + \gamma_0 \mu^*)$$

- fermion determinant is used as probability measure for importance sampling

Fermion sign problem

TECHNISCHE UNIVERSITÄT DARMSTADT

$$\det(\gamma_\mu D_\mu + m - \gamma_0\mu) = \det{}^*(\gamma_\mu D_\mu + m + \gamma_0\mu^*)$$

- fermion determinant is used as probability measure for importance sampling
- real if $\mathrm{Re}(\mu) = 0$
  a) vanishing chemical potential: $\mu = 0$
  b) finite imaginary chemical potential: $\mu = i\mu_i$
  c) finite isospin chemical potential: $\mu_d = -\mu_u$

Methods at finite densities

workarounds

- extrapolations
  a) Taylor expansion from $\mu = 0$
  b) analytic continuation from imaginary chemical potential ($\mu^2 < 0$)
- reweighting
- valid only for limited μ / T

Methods at finite densities

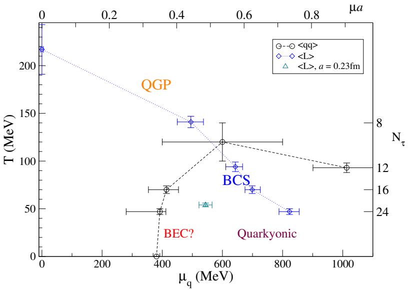
TECHNISCHE UNIVERSITÄT DARMSTADT

workarounds

- extrapolations
  a) Taylor expansion from $\mu = 0$
  b) analytic continuation from imaginary chemical potential ($\mu^2 < 0$)
- reweighting
- valid only for limited μ/T

QCD-like theories

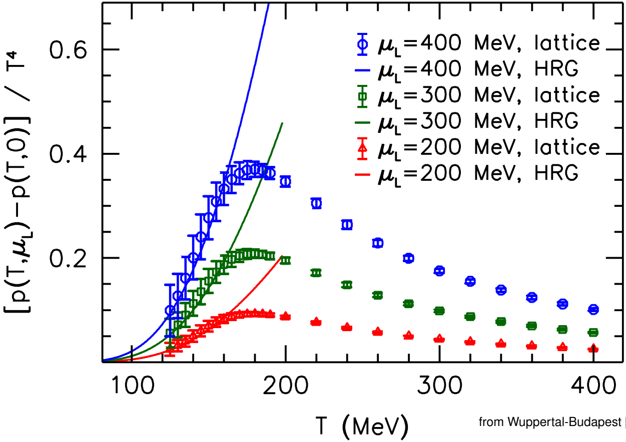
- free of sign-problem at $\mu > 0$
- e. g. two-color QCD



from S. Hands et al. [arXiv:1210.4496]

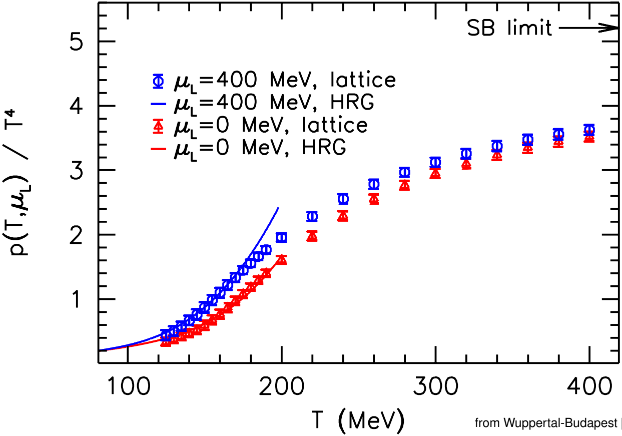
Pressure

TECHNISCHE UNIVERSITÄT DARMSTADT



from Wuppertal-Budapest [arXiv:1210.6710]

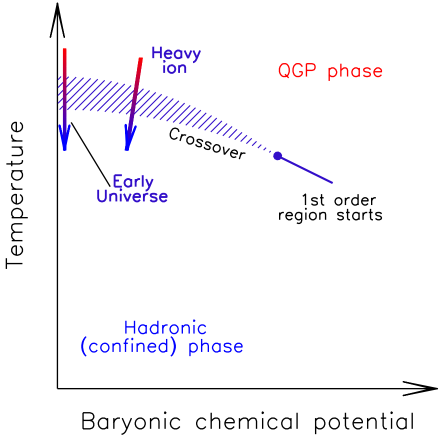
Pressure



from Wuppertal-Budapest [arXiv:1210.6710]

Phase diagram

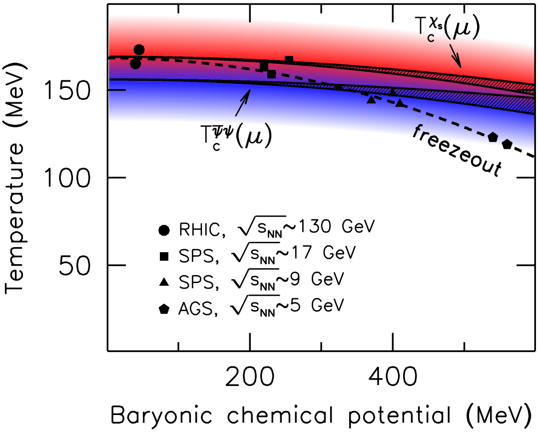
TECHNISCHE
UNIVERSITÄT
DARMSTADT



from Wuppertal-Budapest [arXiv:1102.1356]

Phase diagram

TECHNISCHE
UNIVERSITÄT
DARMSTADT



from Wuppertal-Budapest [arXiv:1102.1356]

Further active topics

- masses (spectroscopy)
- influence of external magnetic fields
- real-time quantities (e.g. transport coefficients)

Open challenges

- use of chiral fermions actions
- large densities

Thank you!

References

TECHNISCHE
UNIVERSITÄT
DARMSTADT

- Wuppertal-Budapest collaboration [arXiv: 1102.1356, 1210.6710, 1210.6901]
- HotQCD collaboration [arXiv:1111.1710, 1210.6312]
- C. Gattringer, C.B. Lang: Quantum Chromodynamics on the Lattice, Springer.
- O. Philipsen: The QCD equation of state from the lattice (Review) [arXiv:1207.5999]
- G. v. Hippel: Life on the Lattice, `latticeqcd.blogspot.de`