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Formelsammlung

1. Mechanik

$$m\ddot{\vec{r}} = \vec{F} = \frac{d\vec{p}}{dt} \qquad \vec{F}_{ij} = -\vec{F}_{ji} \qquad \vec{F} = -\vec{\nabla} V$$

$$V = -\int_{\vec{0}}^{\vec{r}} d\vec{r}\,' \cdot \vec{F}(\vec{r}\,') \qquad W = \int_C d\vec{r} \cdot \vec{F}(\vec{r}) \qquad \frac{\partial W}{\partial t} = \vec{F} \cdot \vec{v}$$

$$L = T - V \qquad \frac{d}{dt}\frac{\partial L}{\partial \dot{q}_j} - \frac{\partial L}{\partial q_j} = 0 \text{ für } j = 1, ..., s$$

$$p_i = \frac{\partial L}{\partial \dot{q}_i} \qquad H = T + V = \sum_{i=1}^{s} p_i \dot{q}_i - L \qquad \dot{p}_i = -\frac{\partial H}{\partial q_i} \qquad \dot{q}_i = \frac{\partial H}{\partial p_i}$$

$$\{F_1, F_2\} = \sum_{i=1}^{s} \left(\frac{\partial F_1}{\partial q_i}\frac{\partial F_2}{\partial p_i} - \frac{\partial F_1}{\partial p_i}\frac{\partial F_2}{\partial q_i} \right) \qquad \{q_j, H\} = \frac{\partial H}{\partial p_j} \qquad \{p_j, H\} = -\frac{\partial H}{\partial q_j}$$

$$L = \frac{1}{2}(\dot{\xi}^T \underline{M} \dot{\xi} - \xi^T \underline{K} \xi) \qquad det(\underline{K} - \omega^2 \, \underline{M}) = 0$$

$$\underline{M}_{ij} = \frac{\partial^2 T}{\partial \dot{q}_i \partial \dot{q}_j}\Big|_{\vec{q}=\vec{q}_0} \qquad \underline{K}_{ij} = \frac{\partial^2 V}{\partial q_i \partial q_j}\Big|_{\vec{q}=\vec{q}_0}$$

$$\sum_{j=1}^{s} (\underline{M}_{ij}\ddot{\xi}_j + \underline{K}_{ij}\xi_j) = 0 \qquad \ddot{\eta}_\alpha + \omega_\alpha^2 \eta_\alpha = 0$$

$$p_i = \frac{\partial F_1(q,Q)}{\partial q_i} \quad P_i = -\frac{\partial F_1(q,Q)}{\partial Q_i} \qquad p_i = \frac{\partial F_2(q,P)}{\partial q_i} \quad Q_i = \frac{\partial F_2(q,P)}{\partial P_i}$$

$$q_i = -\frac{\partial F_3(p,Q)}{\partial p_i} \quad P_i = -\frac{\partial F_3(p,Q)}{\partial Q_i} \qquad q_i = -\frac{\partial F_4(p,P)}{\partial p_i} \quad Q_i = \frac{\partial F_4(p,P)}{\partial P_i}$$

$$H(P, Q, t) = H(p, q, t) + \frac{\partial F_i}{\partial t} \quad \text{für } i = 1..4$$

$$H\left(q_1, ..., q_s, \frac{\partial F_2}{\partial q_1}, ..., \frac{\partial F_2}{\partial q_s}, t\right) + \frac{\partial F_2}{\partial t} = 0$$

2. Elektrodynamik

$$\vec{\nabla} \cdot \vec{D} = \rho \qquad \vec{\nabla} \times \vec{E} = -\frac{\partial}{\partial t}\vec{B}$$

$$\vec{\nabla} \cdot \vec{B} = 0 \qquad \vec{\nabla} \times \vec{H} = \vec{j} + \frac{\partial}{\partial t}\vec{D}$$

$$\vec{D} = \epsilon_0\, \epsilon_r\, \vec{E} \qquad \vec{B} = \mu_0 \mu_r\, \vec{H}$$

$$\int_{\partial V} d\vec{f} \cdot \vec{D} = Q \qquad \oint_{\partial F} d\vec{s} \cdot \vec{E} = -\frac{\partial}{\partial t}\int_F d\vec{f} \cdot \vec{B}$$

$$\int_{\partial V} d\vec{f} \cdot \vec{B} = 0 \qquad \oint_{\partial F} d\vec{s} \cdot \vec{H} = I + \frac{\partial}{\partial t}\int_F d\vec{f} \cdot \vec{D}$$

$$Q = \sigma F$$

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi}\int d^3r'\, \vec{j}(\vec{r}') \times \frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|^3}$$

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot \vec{j} = 0$$

$$\vec{p} = \int d^3r\; \vec{r}\rho(\vec{r}) \qquad \vec{m} = \frac{1}{2}\int d^3r\; \vec{r} \times \vec{j}(\vec{r})$$

$$Q_{ij} = \int d^3r\, \rho(\vec{r})\,(3x_i x_j - r^2\delta_{ij}) \qquad q_{\ell m} = \int d^3r\; r^\ell\, Y_\ell^m(\theta,\varphi)\,\rho(\vec{r})$$

$$Y_\ell^m(\theta,\varphi) = \sqrt{\frac{2\ell+1}{4\pi}\frac{(\ell-m)!}{(\ell+m)!}}\, P_\ell^m(\cos\theta)\, e^{im\varphi} \qquad P_\ell^m(x) = (-1)^m\,(1-x^2)^{\frac{m}{2}}\,\frac{\partial^m}{\partial x^m}\,P_\ell(x)$$

$$P_0(x) = 1 \qquad P_1(x) = x \qquad P_2(x) = \frac{1}{2}(3x^2-1)$$

$$\vec{n}\left(\vec{D}_2 - \vec{D}_1\right) = \sigma \qquad \vec{n}\left(\vec{B}_2 - \vec{B}_1\right) = 0$$

$$(\vec{t} \times \vec{n})\left(\vec{E}_2 - \vec{E}_1\right) = 0 \qquad (\vec{t} \times \vec{n})\left(\vec{H}_2 - \vec{H}_1\right) = \vec{t} \cdot \vec{j}_F$$

$$\frac{\partial}{\partial t}\Big(W_{mech} + W_{Feld}\Big) = \int_V \left(\vec{j} \cdot \vec{E} + \frac{\partial w}{\partial t}\right) = -\int_V d^3r\; \vec{\nabla} \cdot \vec{S} = -\int_{\partial V} d\vec{f} \cdot \vec{S}$$

$$\frac{\partial}{\partial t}\Big(\vec{p}_{mech} + \vec{p}_{Feld}\Big)_i = \int d^3r\; \left(\left[\rho\vec{E} + \vec{j} \times \vec{B}\right] + \dot{\vec{D}} \times \vec{B}\right)_i = \int_{\partial V} df\, \sum_{k=1}^{3} n_k\, T_{ik}$$

$$w = \frac{1}{2}(\vec{E} \cdot \vec{D} + \vec{B} \cdot \vec{H}) \qquad \vec{S} = \vec{E} \times \vec{H}$$

$$T_{ik} = \left(\epsilon_0\epsilon_r E_i E_k + \frac{1}{\mu_0\mu_r}B_i B_k - \frac{1}{2}(\epsilon_0\epsilon_r \vec{E}^2 + \frac{1}{\mu_0\mu_r}\vec{B}^2)\,\delta_{ik}\right)$$

$$\vec{A}(t) = \vec{A}_0\, e^{i\omega t}\ , \quad \vec{B}(t) = \vec{B}_0\, e^{i\omega t} \qquad \Rightarrow \qquad \begin{aligned} \overline{Re\,\vec{A}(t)\ Re\,\vec{B}(t)} &= \frac{1}{2}\, Re(\vec{A}_0\, \vec{B}_0^*) \\ \overline{Re\,\vec{A}(t) \times Re\,\vec{B}(t)} &= \frac{1}{2}\, Re(\vec{A}_0\, \times\, \vec{B}_0^*) \end{aligned}$$

$$u = \frac{1}{\sqrt{\epsilon_r \epsilon_0 \mu_r \mu_0}} = \frac{c}{\sqrt{\epsilon_r \mu_r}} = \frac{c}{n}$$

$$\frac{\sin\vartheta_1}{\sin\vartheta_2} = \frac{k_2}{k_1} = \frac{n_2}{n_1}$$

$$\left(\frac{E_{02}}{E_{01}}\right)_\perp = \frac{2n_1\cos\vartheta_1}{n_1\cos\vartheta_1 + n_2\cos\vartheta_2}\,, \qquad \left(\frac{E_{01r}}{E_{01}}\right)_\perp = \frac{n_1\cos\vartheta_1 - n_2\cos\vartheta_2}{n_1\cos\vartheta_1 + n_2\cos\vartheta_2}$$

$$\left(\frac{E_{02}}{E_{01}}\right)_{||} = \frac{2n_1\cos\vartheta_1}{n_2\cos\vartheta_1 + n_1\cos\vartheta_2}\,, \qquad \left(\frac{E_{01r}}{E_{01}}\right)_{||} = \frac{n_2\cos\vartheta_1 - n_1\cos\vartheta_2}{n_2\cos\vartheta_1 + n_1\cos\vartheta_2}$$

$$G_0^{ret}(\vec{r} - \vec{r}', t - t') = \frac{1}{4\pi|\vec{r} - \vec{r}'|}\delta(t_{ret} - t') \quad \text{mit} \quad t_{ret} = t - \frac{|\vec{r} - \vec{r}'|}{u}$$

$$\begin{aligned} \varphi(\vec{r}, t) &= \frac{1}{4\pi\epsilon_0\epsilon_r} \int d^3r'\, \frac{\rho(\vec{r}', t_{ret})}{|\vec{r} - \vec{r}'|} \\ \vec{A}(\vec{r}, t) &= \frac{\mu_0\mu_r}{4\pi} \int d^3r'\, \frac{\vec{j}(\vec{r}', t_{ret})}{|\vec{r} - \vec{r}'|} \end{aligned}$$

3. Allgemeine mathematische Formeln

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} r\sin\theta\cos\varphi \\ r\sin\theta\sin\varphi \\ r\cos\theta \end{pmatrix} \qquad \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \rho\cos\varphi \\ \rho\sin\varphi \\ z \end{pmatrix} \qquad \dot{\vec{r}} = \dot{r}\vec{e}_r + r\dot{\theta}\vec{e}_\theta + r\sin\theta\dot{\varphi}\vec{e}_\varphi$$

$$f(r) = f(0) + \sum_n \frac{d^n f(r)}{dr^n}\Big|_{r=0} \frac{r^n}{n!}$$

$$\int_0^{2\pi} dx\, \sin^2 x = \int_0^{2\pi} dx\, \cos^2 x = \pi \qquad \int_0^{2\pi} dx\, \sin x\, \cos x = 0$$

$$\int dx\, \frac{1}{\sqrt{x^2+a^2}} = \ln(x + \sqrt{x^2+a^2}) + C \qquad \int dx\, \frac{1}{(x^2+a^2)^{3/2}} = \frac{x}{a^2\sqrt{x^2+a^2}} + C$$

$$\int dx\, \frac{x^3}{(x^2+a^2)^{3/2}} = \frac{x^2+2a^2}{\sqrt{x^2+a^2}} + C \qquad \int dx \frac{1}{x} = \ln|x| + C$$

$$\int_a^\infty dx\, \frac{b}{x\sqrt{x^2-b^2}} = \arcsin\left(\frac{b}{a}\right) \qquad \text{falls} \quad a > b > 0$$

$$\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a}\cdot\vec{c})\cdot\vec{b} - (\vec{a}\cdot\vec{b})\cdot\vec{c}$$

$$\int_F d\vec{f}\cdot(\vec{\nabla}\times\vec{A}) = \oint_{\partial F} d\vec{s}\cdot\vec{A} \qquad \int_V d^3r\, \vec{\nabla}\cdot\vec{A} = \int_{\partial V} d\vec{f}\cdot\vec{A}$$

$$\begin{aligned}
\vec{\nabla} f &= \frac{\partial f}{\partial\rho}\,\vec{e}_\rho + \frac{1}{\rho}\frac{\partial f}{\partial\varphi}\,\vec{e}_\varphi + \frac{\partial f}{\partial z}\,\vec{e}_z \\
\vec{\nabla}\cdot\vec{F} &= \frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho F_\rho) + \frac{1}{\rho}\frac{\partial}{\partial\varphi}F_\varphi + \frac{\partial}{\partial z}F_z \\
\vec{\nabla}\times\vec{F} &= \Big(\frac{1}{\rho}\frac{\partial}{\partial\varphi}\,F_z - \frac{\partial}{\partial z}\,F_\varphi\Big)\,\vec{e}_\rho + \Big(\frac{\partial}{\partial z}\,F_\rho - \frac{\partial}{\partial\rho}\,F_z\Big)\,\vec{e}_\varphi + \Big(\frac{1}{\rho}\frac{\partial}{\partial\rho}\,(\rho\,F_\varphi) - \frac{1}{\rho}\frac{\partial}{\partial\varphi}\,F_\rho\Big)\,\vec{e}_z \\
\triangle f &= \frac{\partial^2 f}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial f}{\partial\rho} + \frac{1}{\rho^2}\frac{\partial^2 f}{\partial\varphi^2} + \frac{\partial^2 f}{\partial z^2}
\end{aligned}$$

$$\begin{aligned}
\vec{\nabla} f &= \frac{\partial f}{\partial r}\,\vec{e}_r + \frac{1}{r}\frac{\partial f}{\partial\theta}\,\vec{e}_\theta + \frac{1}{r\sin\theta}\frac{\partial f}{\partial\varphi}\,\vec{e}_\varphi \\
\vec{\nabla}\cdot\vec{F} &= \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 F_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\,F_\theta) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\varphi}F_\varphi \\
\vec{\nabla}\times\vec{F} &= \frac{1}{r\sin\theta}\Big(\frac{\partial}{\partial\theta}\,(\sin\theta\,F_\varphi) - \frac{\partial}{\partial\varphi}\,F_\theta\Big)\,\vec{e}_r + \frac{1}{r}\Big(\frac{1}{\sin\theta}\frac{\partial}{\partial\varphi}\,F_r - \frac{\partial}{\partial r}\,(rF_\varphi)\Big)\,\vec{e}_\theta \\
&\quad + \frac{1}{r}\Big(\frac{\partial}{\partial r}\,(r\,F_\theta) - \frac{\partial}{\partial\theta}\,F_r\Big)\,\vec{e}_\varphi \\
\triangle f &= \frac{1}{r^2}\frac{\partial}{\partial r}(r^2\frac{\partial f}{\partial r}) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\,\frac{\partial f}{\partial\theta}) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 f}{\partial\varphi^2}
\end{aligned}$$