

Quark Model

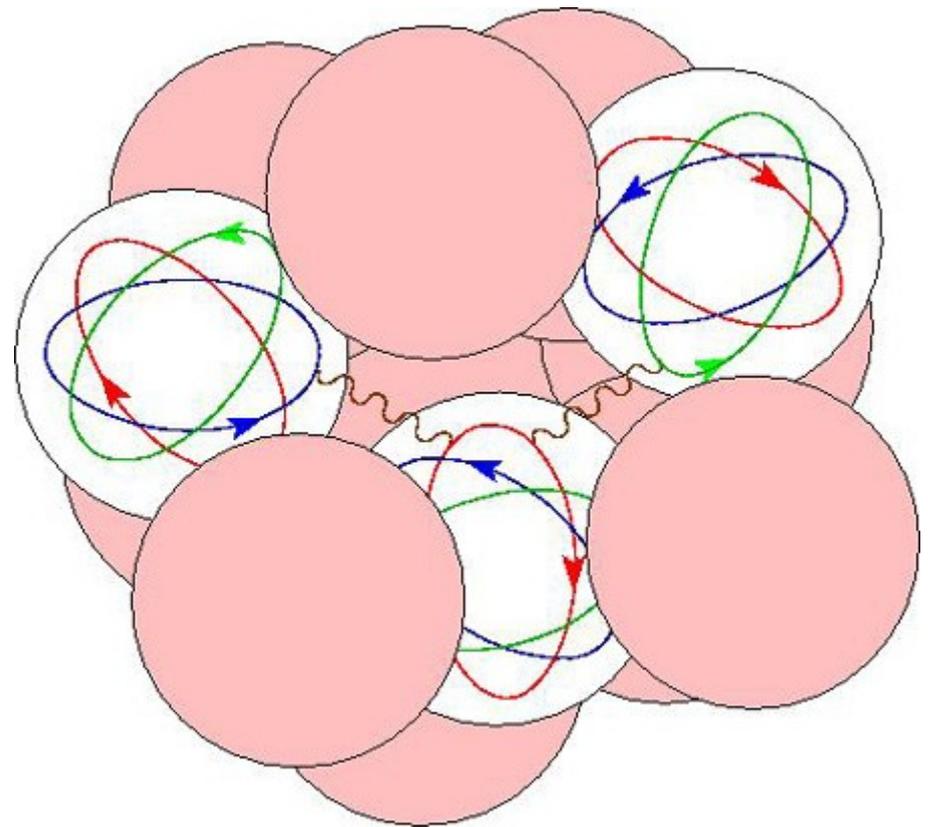
History and current status



TECHNISCHE
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Heavy-Ion Seminar 2013



Outline

Introduction

- ▶ Motivation and historical development

Group theory and the Quark Model

- ▶ Basics of group theory
- ▶ $SU(2)$ and $SU(3)$
- ▶ Existence of Quarks

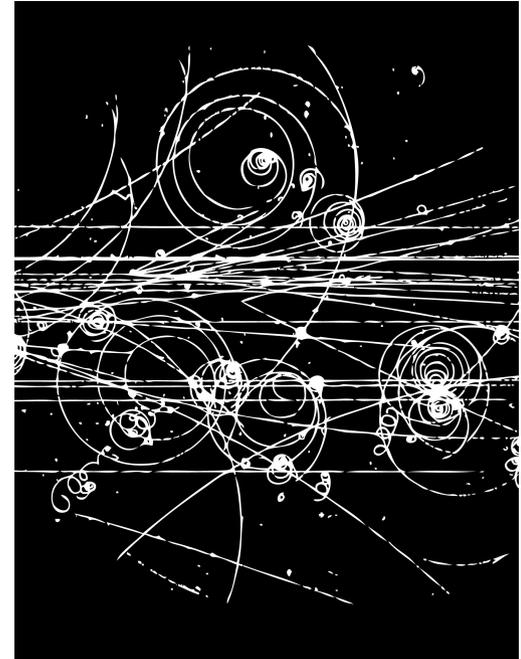
From Quark Model towards QCD

Heavy Quarks in the Quark Model

Actual status

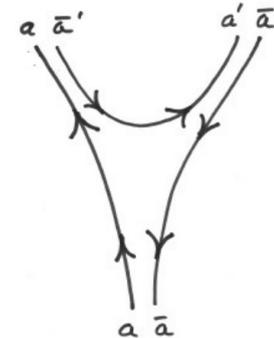
Historical background

- ▶ Particle zoo (bubble chambers, ...)
- ▶ 1949: Fermi + Yang: Pion not elementary particle
- ▶ 1950's: Isospin $SU(2)$ symmetry of strong int.
- ▶ 1953 Exp.: additional quantum number: strangeness
- ▶ 1956: Sakata proposes that pion consists of three particles (n,p, Λ)
- ▶ 1961: Gell-Mann: Eightfold way: $SU(3)$ symmetry

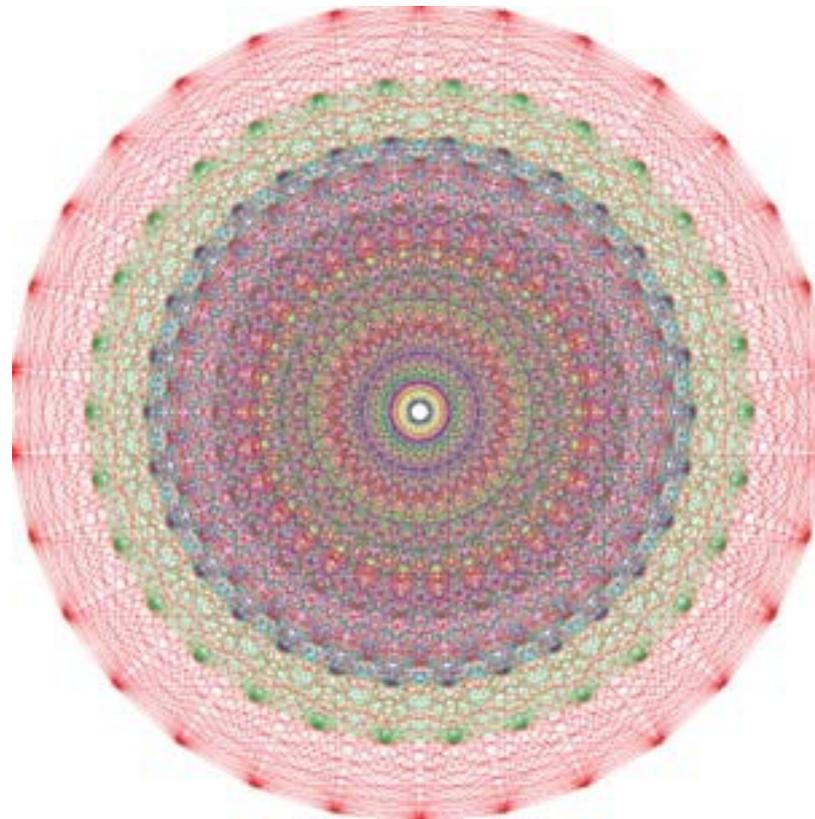


Historical background

- ▶ 1962: Prediction of Ω^- particle (measured 1964)
- ▶ 1964: Gell-Mann, Zweig: Quark Model (u,d,s)
- ▶ 1964: Greenberg: Color as quantum number
- ▶ 1964: Zweig Rule
- ▶ 1967-73: Measurements confirm substructure of nucleons
- ▶ 1974: Discovery of charm Quark (Ψ/J)
- ▶ 1977: Discovery of bottom Quark
- ▶ 1978: Discovery of the Gluon
- ▶ 1995: Discovery of the Top Quark



Group Theory and the Quark Model



Group Theory: Basic definitions

Definition: Group

Set with assignment, satisfying

$$(I) f, g \in G \Rightarrow fg \in G$$

$$(II) f, g, h \in G \Rightarrow (fg)h = f(gh)$$

(III) neutral element

(IV) inverse element

Definition: Representation

Mapping D that projects elements of G on linear operators $GL(V)$, with following properties:

$$(I) D(e) = 1$$

$$(II) D(g_1)D(g_2) = D(g_1g_2)$$

Definition: Invariant subspace

$D : G \rightarrow GL(V)$. A subspace $U \subset V$ is called invariant, if
 $D(g)U \subset U, \quad \forall g \in G$

Group Theory: Basic definitions

Definition: Reducible representation

A representation is reducible, if it has an invariant subspace.

It is then equiv. to:

$$\begin{pmatrix} D_1(g) & 0 & \dots \\ 0 & D_2(g) & \\ \dots & & D_3(g) \end{pmatrix}$$

$$D = D_1 \oplus D_2 \oplus D_3 \oplus \dots$$

SU(2)

- ▶ Well known group (Quantum mechanics,...)
- ▶ Special Unitary group: fundamental representation in 2 dimensions:

$$M = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \quad \det M \stackrel{!}{=} 1 \quad MM^\dagger = M^\dagger M = 1$$

- ▶ Those matrices have following properties:

- Can be written as: $M = e^{-\frac{i}{2}\vec{\sigma}\vec{\alpha}}$

- **Pauli matrices:**

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

- Commutation relation: $[\sigma_i, \sigma_j] = i\epsilon_{ijk}\sigma_k$

- Define **raising and lowering** operators: $\sigma_\pm = \sigma_x \pm i\sigma_y$

$$\sigma_+ \begin{pmatrix} 0 \\ 1 \end{pmatrix} \propto \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \sigma_- \begin{pmatrix} 1 \\ 0 \end{pmatrix} \propto \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Group Theory: Lie Groups

Definition: Lie Group

g depends on continuous parameter α .

Natural representation:

$$D(\alpha) = e^{i\alpha_a X_a}$$

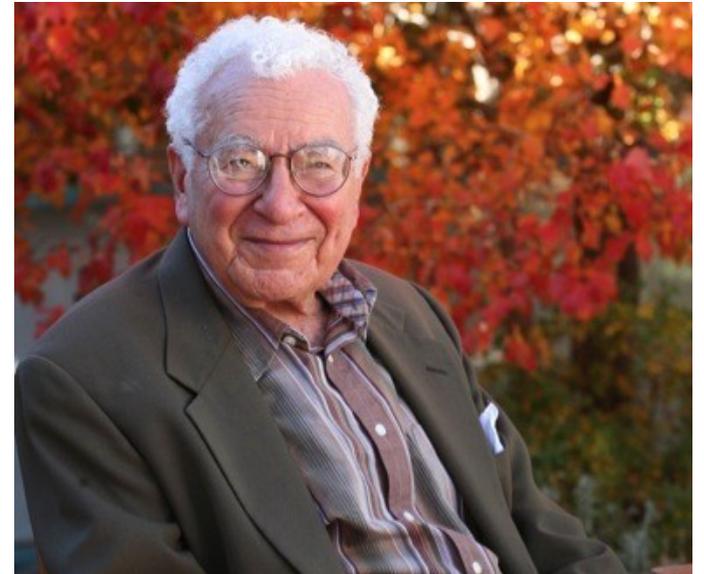
with the generators: $X_a = -i \frac{\partial}{\partial \alpha_a} D(\alpha) |_{\alpha=0}$

Lie Algebra: $[X_a, X_b] = i f_{abc} X_c$ Adjoint representation

Group Theory: SU(3)

$$\begin{aligned}\lambda_1 &= \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} & \lambda_2 &= \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} & \lambda_3 &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \\ \lambda_4 &= \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} & \lambda_5 &= \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} & \lambda_6 &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \\ \lambda_7 &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} & \lambda_8 &= \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}\end{aligned}$$

Gell-Mann matrices, analogous to Pauli matrices



Group Theory: SU(3)

Define **raising and lowering** operators for the eigenvalues of the diagonal matrices:

$$e_{\pm}^1 = \frac{1}{2}(\lambda_1 \pm i\lambda_2)$$

$$e_{\pm}^2 = \frac{1}{2}(\lambda_6 \pm i\lambda_7)$$

$$e_{\pm}^3 = \frac{1}{2}(\lambda_4 \pm i\lambda_5)$$

Eigenvalues of λ_3	Eigenvalues of λ_8
1	$1/\sqrt{3}$
-1	$1/\sqrt{3}$
0	$-2/\sqrt{3}$

$\left. \begin{matrix} e_{\pm}^1 \\ e_{\pm}^2 \end{matrix} \right\} e_{\pm}^3$

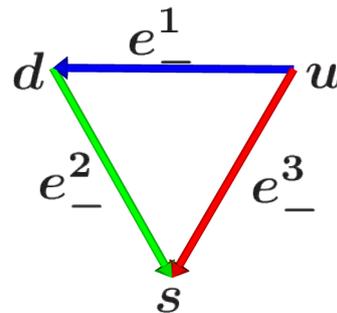
Vector ($EV_{\lambda_3}, EV_{\lambda_8}$) is called **weight**

Group Theory: SU(3) representations

Examples for **irreducible representations**:

1) **Fundamental representation**: (3-dimensional)

states	weight
$u=(1,0,0)$	$(1, 1/\sqrt{3})$
$d=(0,1,0)$	$(-1, 1/\sqrt{3})$
$s=(0,0,1)$	$(0, -1/\sqrt{3})$



To construct **weight diagram**, apply highest weight procedure:

I) Determine highest weight: $e_+^i |\psi\rangle = 0$

II) Apply lowering operators on it

This representation is called the **3**

Group Theory: SU(3) representations

2) Adjoint representation: (8 dimensional)

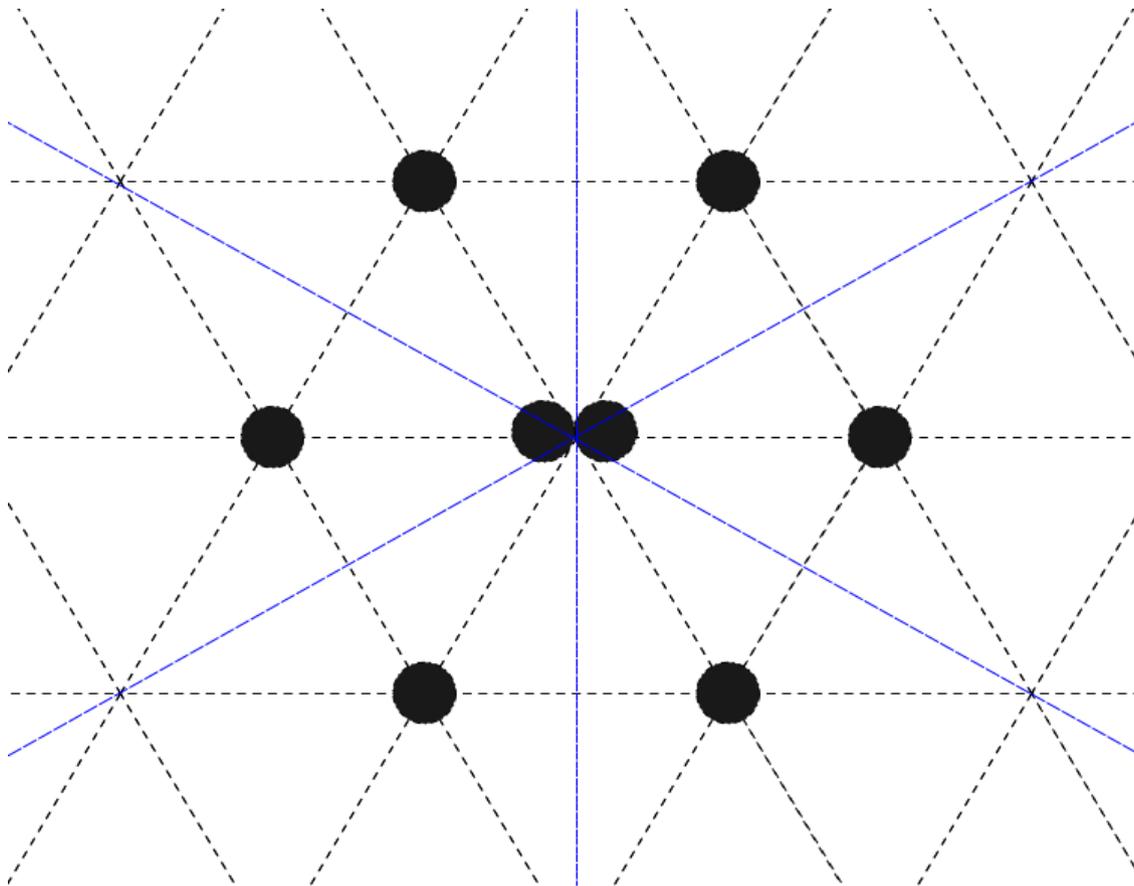
$$ad_x(y) = [x, y]$$

v	$[\lambda_3, v]$	$[\lambda_8, v]$	weight
λ_3	0	0	(0, 0)
λ_8	0	0	(0, 0)
e_+^1	e_+^1	0	(1, 0)
e_-^1	$-e_-^1$	0	(-1, 0)
e_+^2	$-1/2e_+^2$	$\sqrt{3}/2e_+^2$	$(-1/2, \sqrt{3}/2)$
e_-^2	$1/2e_-^2$	$-\sqrt{3}/2e_-^2$	$(1/2, -\sqrt{3}/2)$
e_+^3	$1/2e_+^3$	$\sqrt{3}/2e_+^3$	$(1/2, \sqrt{3}/2)$
e_-^3	$-1/2e_-^3$	$-\sqrt{3}/2e_-^3$	$(-1/2, -\sqrt{3}/2)$

← Highest weight

Group Theory: $SU(3)$ representations

Weight diagram for adjoint representation:

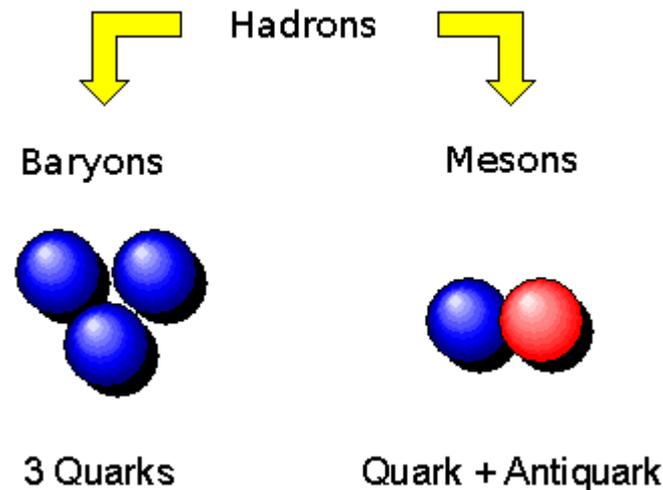


This representation is called the **8**

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Group Theory and the Quark Model

- ▶ 1961 Gell-Mann identifies Flavor- $SU(3)$ as symmetry group of strong interaction
- ▶ 1964 Gell-Mann and Zweig develop Quark Model
- ▶ Basis vectors of fundamental representations are Quark states
- ▶ Baryon: made of 3 Quarks
- ▶ Meson: made of 2 Quarks

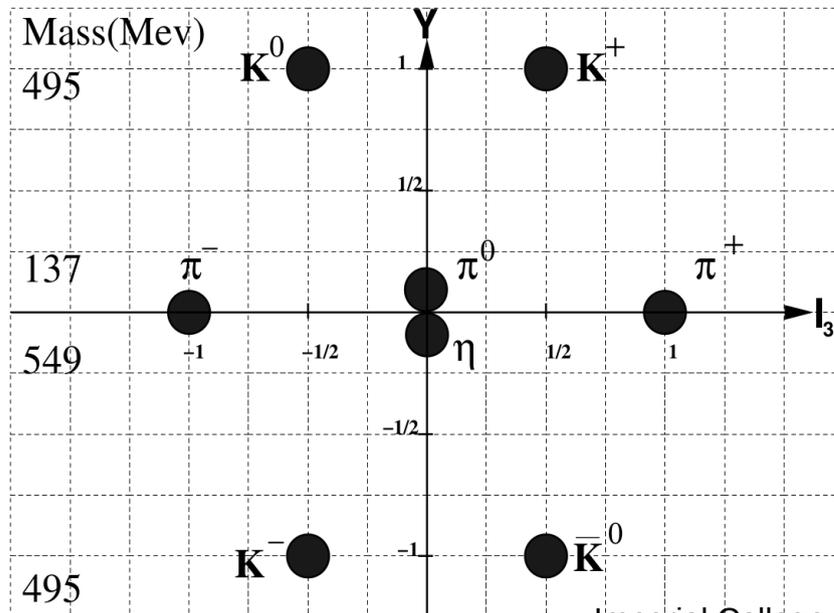


Group Theory and the Quark Model

► Classification of Mesons by plotting **Hypercharge Y** against **Isospin I_z** :

Pseudoscalar meson octet:

$$B = 0 \quad J = 0$$

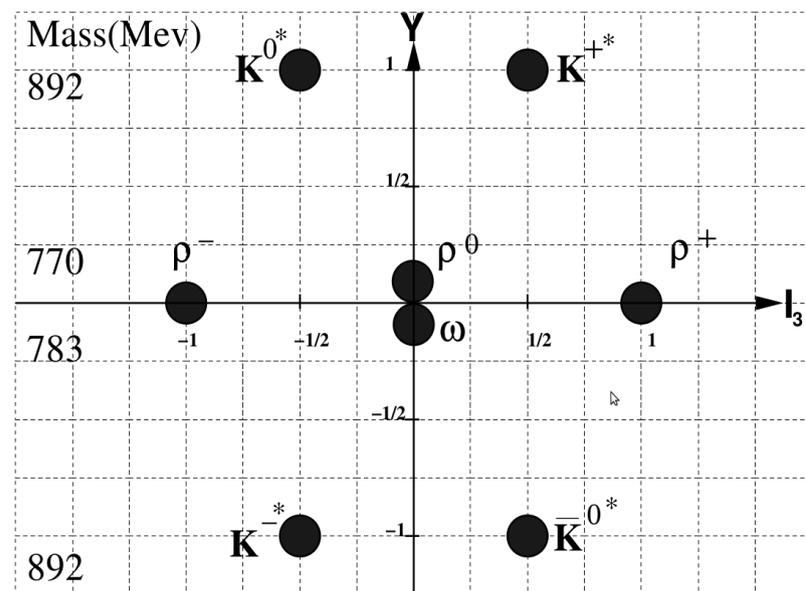


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+ J=0 meson **singlet** η'

Vector meson octet:

$$B = 0 \quad J = 1$$



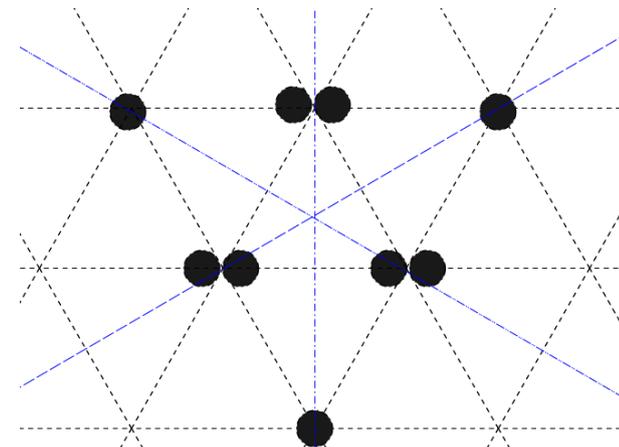
+ J=1 meson **singlet** ϕ

Group Theory and the Quark Model

▶ Mesons composed by 2 Quarks: $3 \otimes 3$

▶ Tensor product gives:

Quark content and weights for $3 \otimes 3$	
<i>Quark Content</i>	<i>Weight</i>
$u \otimes u$	$(1, \frac{1}{\sqrt{3}})$
$d \otimes d$	$(-1, \frac{1}{\sqrt{3}})$
$s \otimes s$	$(0, -\frac{2}{\sqrt{3}})$
$u \otimes d, d \otimes u$	$(0, \frac{1}{\sqrt{3}})$
$u \otimes s, s \otimes u$	$(\frac{1}{2}, -\frac{1}{2\sqrt{3}})$
$d \otimes s, s \otimes d$	$(-\frac{1}{2}, -\frac{1}{2\sqrt{3}})$



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Not an irreducible representation of SU(3)!

➡ Split into sums of irred. Rep.:

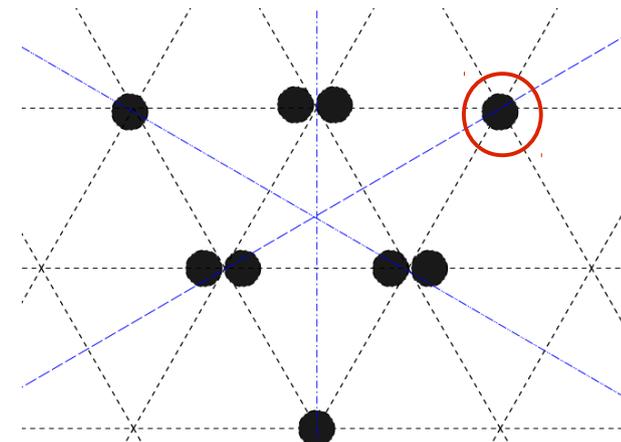
- 1) Find highest weight
- 2) Apply lowering operators
- 3) Erase those points of the diagram

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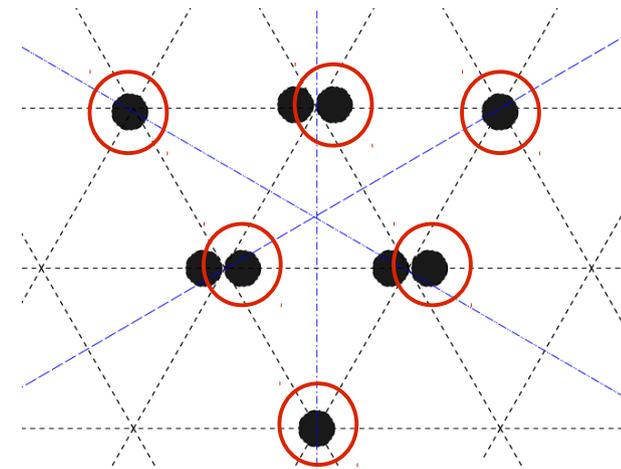
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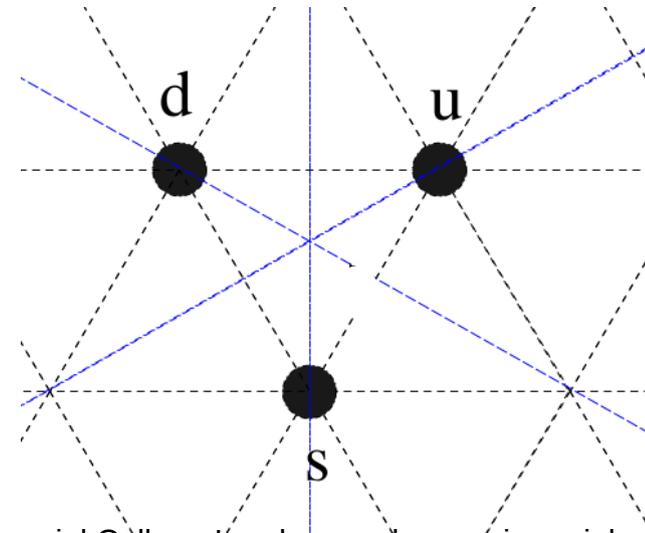
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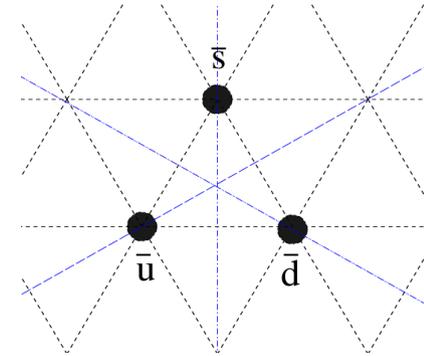
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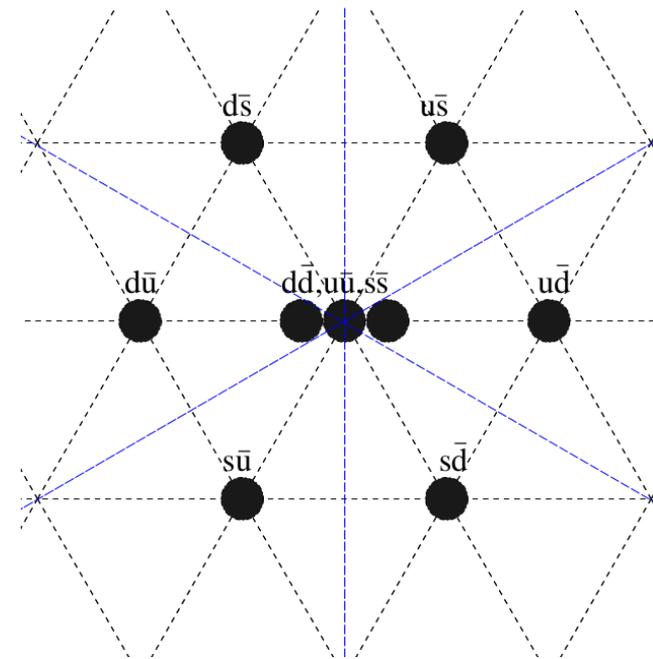
Group Theory and the Quark Model

- ▶ Now we know that: $3 \otimes 3 = 6 \oplus 3$
- ▶ Obviously not describing mesons, which are categorized by octet states
- ▶ Solution: Quark + Antiquark: $3 \otimes \bar{3}$

$\bar{3}$ Representation:



Quark content and weights for $3 \otimes \bar{3}$	
Quark Content	Weight
$u \otimes \bar{s}$	$(\frac{1}{2}, \frac{\sqrt{3}}{2})$
$u \otimes \bar{d}$	$(1, 0)$
$d \otimes \bar{s}$	$(-\frac{1}{2}, \frac{\sqrt{3}}{2})$
$u \otimes \bar{u}, d \otimes \bar{d}, s \otimes \bar{s}$	$(0, 0)$
$d \otimes \bar{u}$	$(-1, 0)$
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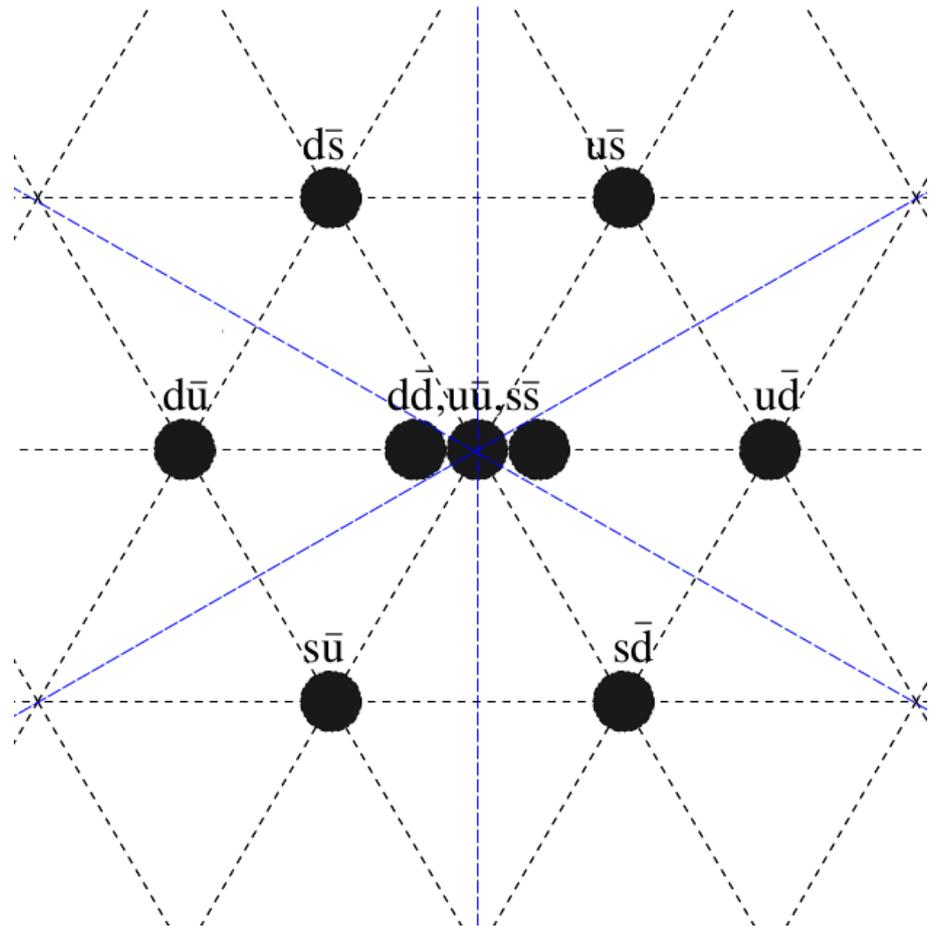
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Group Theory and the Quark Model

Apply highest weight procedure:

- 1) Find highest weight
- 2) Apply lowering operators
- 3) Erase those points of the diagram

Quark content and weights for $\mathbf{3} \otimes \bar{\mathbf{3}}$	
Quark Content	Weight
$u \otimes \bar{s}$	$(\frac{1}{2}, \frac{\sqrt{3}}{2})$
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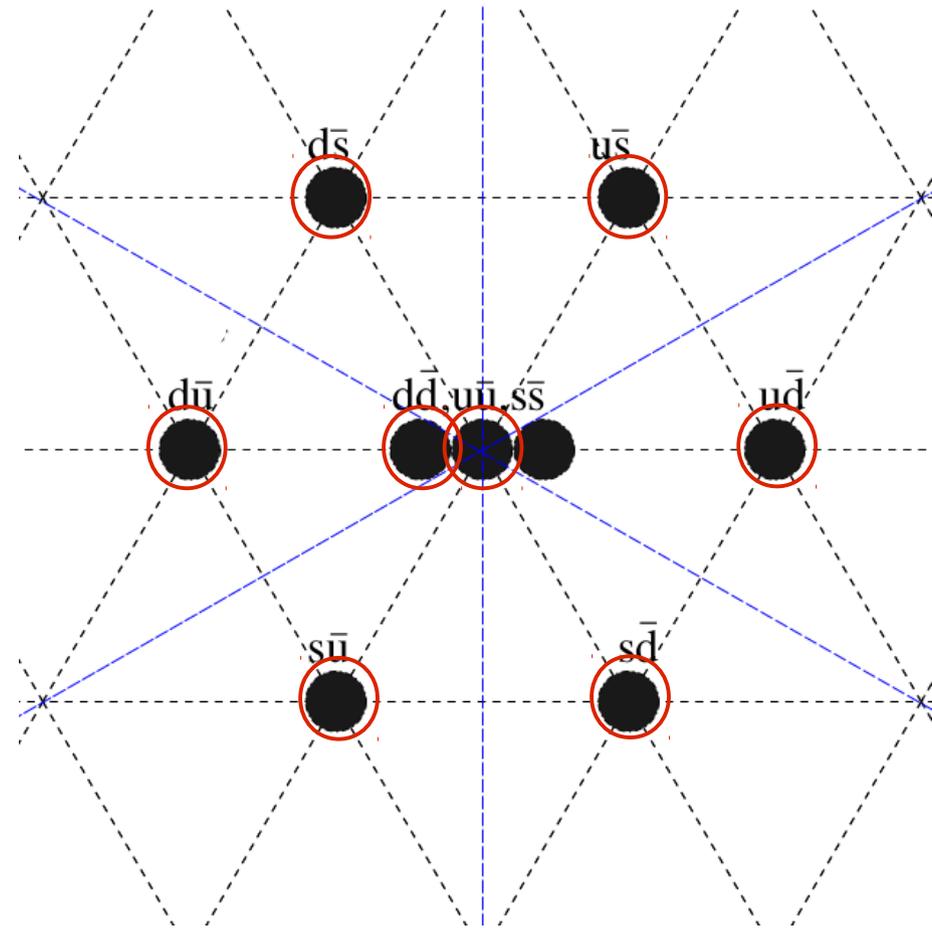
Group Theory and the Quark Model

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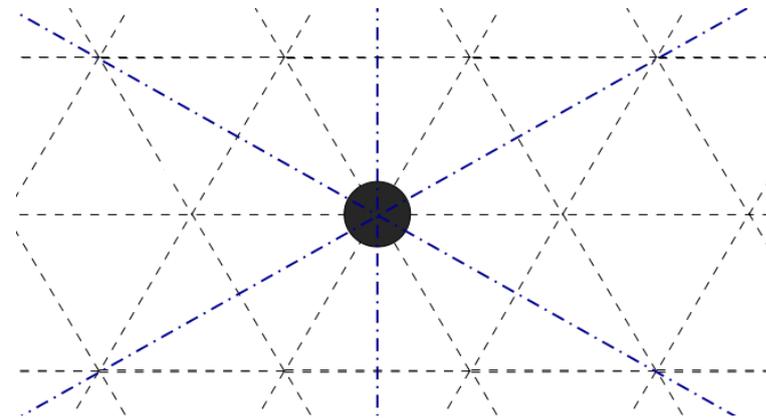
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Group Theory and the Quark Model

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So, that shows that:

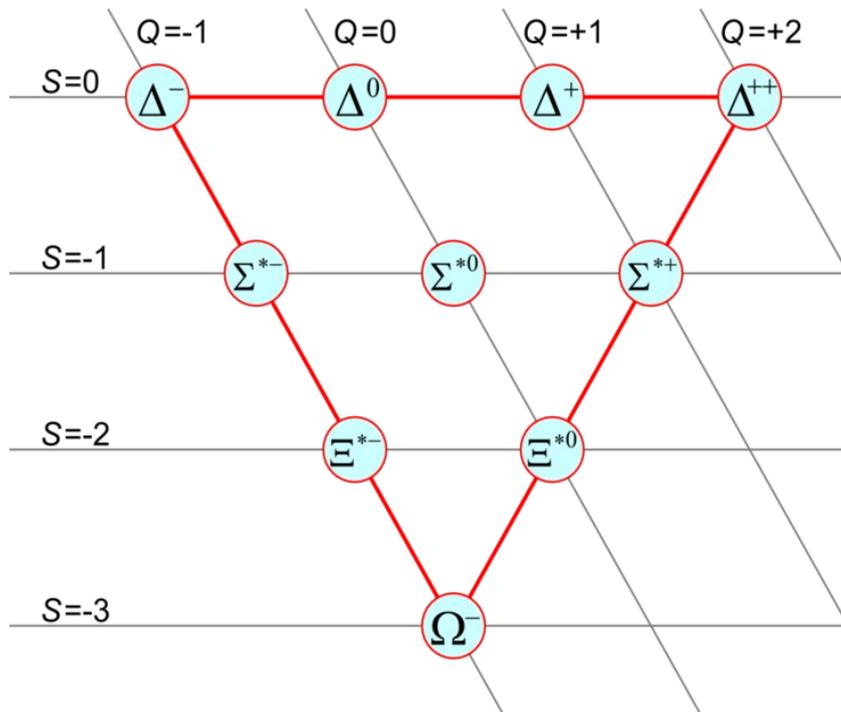
$$\mathbf{3} \otimes \bar{\mathbf{3}} = \mathbf{8} \oplus \mathbf{1}$$

Mesons are made of a **Quark** and **Antiquark**

Group Theory and the Quark Model

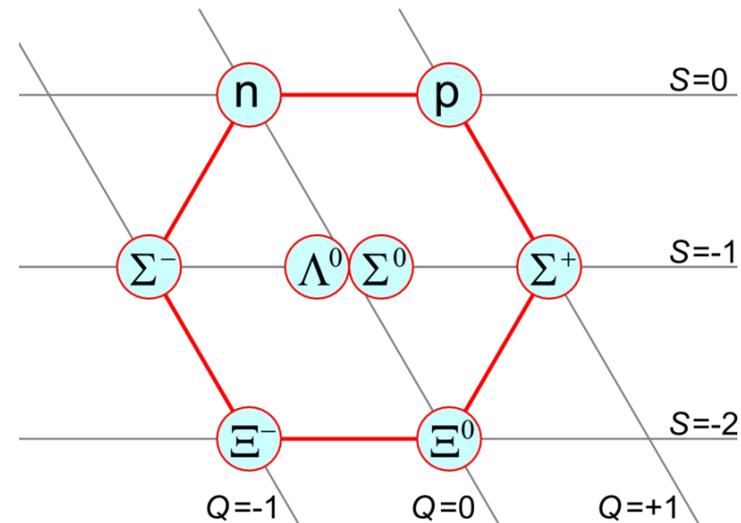
- ▶ Same procedure can be done for **Baryons**: $3 \otimes 3 \otimes 3 = 10 \oplus 8 \oplus 8 \oplus 1$
- ▶ The weight diagrams then look like:

Baryon decuplet: B=1, J=3/2



Particle data group pdg.lbl.gov/

Baryon octet: B=1 J=1/2



+ **Baryon singlet: B=1, J=1/2** Λ^{0*}

Gell-Mann-Okubo Mass formula

► Calculate masses of nucleons

► Assumptions:

- Binding energy independent of Flavor
- Quark mass difference is responsible for mass difference in SU(3) representations
- Exact SU(2): $m_u = m_d$

► Quark content of the **pseudoscalar mesons**:

$$\pi^+ \sim \bar{d}u, \quad \pi^0 \sim \frac{1}{\sqrt{2}}(\bar{u}u - \bar{d}d), \quad \pi^- \sim \bar{u}d$$

$$K^+ \sim \bar{s}u, \quad K^0 \sim \bar{s}d, \quad \bar{K}^0 \sim \bar{d}s, \quad K^- \sim \bar{u}s$$

$$\eta^0 \sim \frac{1}{\sqrt{6}}(\bar{u}u + \bar{d}d - 2\bar{s}s)$$

$$m_\pi^2 = B(m_0 + 2m_u)$$

$$m_K^2 = B(m_0 + m_u + m_s)$$

$$m_\eta^2 = B(m_0 + \frac{2}{3}(m_u + 2m_s))$$

$$B, m_0 = \text{const.}$$

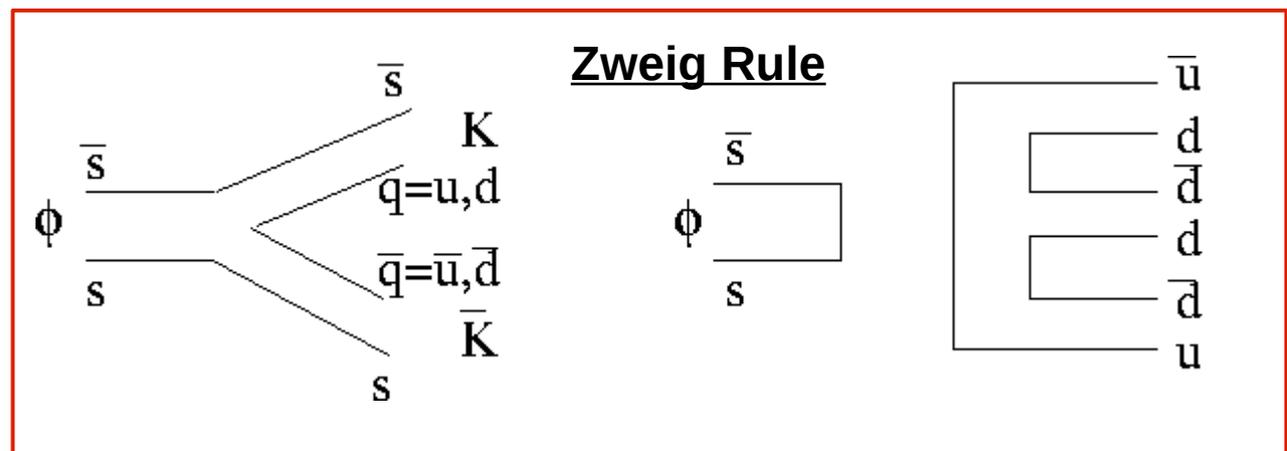
Mass formula:

$$4m_K^2 = m_\pi^2 + 3m_\eta^2$$

Gell-Mann-Okubo Mass formula

- ▶ For vector mesons: problems with experimental data
- ▶ Quark content the same as before $(\pi, K, \eta) \leftrightarrow (\rho, K^*, \omega)$
- ▶ Problem: ω mixes with singlet state ϕ
- ▶ Almost ideal mixing: $\omega = \frac{1}{\sqrt{2}}(\bar{u}u + \bar{d}d)$
 $\phi = \bar{s}s$

- ▶ Decay: $\omega \longrightarrow 3\pi$
 $\phi \longrightarrow \bar{K}K$



Are Quarks physical entities?

▶ Quark model describes and predicts particles correctly

▶ Properties of Quarks:

Point-like particles

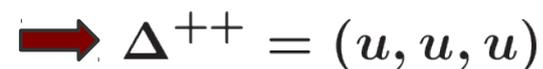
Spin $\frac{1}{2}$

Fractional charges: $u=2/3$, $d=-1/3$, $s=-1/3$

Strange Quark: $S=-1$

▶ Do they exist?

Pro Quark	Contra Quark
Why no mesons with $S=2$	Fractional charge
Anomalous magnetic moments of baryons	Can't measure 1 Quark
Mass split	Violate Pauli principle



Quark Model

- ▶ Solution to that problem:
additional quantum number: Color
- ▶ Need at least 3 Colors: $|r\rangle$, $|g\rangle$, $|b\rangle$
- ▶ Explains why mesons are only built of 1 Quark and 1 Antiquark
- ▶ Quantum numbers of Quarks:

	d	u	s
Q – electric charge	$-\frac{1}{3}$	$+\frac{2}{3}$	$-\frac{1}{3}$
I – isospin	$\frac{1}{2}$	$\frac{1}{2}$	0
I_z – isospin z -component	$-\frac{1}{2}$	$+\frac{1}{2}$	0
S – strangeness	0	0	-1

Quark Model

- ▶ Solution to that problem:
additional quantum number: Color
- ▶ Need at least 3 Colors: $|r\rangle, |g\rangle, |b\rangle$  QCD
- ▶ Explains why mesons are only built of 1 Quark and 1 Antiquark
- ▶ Quantum numbers of Quarks:

	d	u	s	c	b	t
Q – electric charge	$-\frac{1}{3}$	$+\frac{2}{3}$	$-\frac{1}{3}$	$+\frac{2}{3}$	$-\frac{1}{3}$	$+\frac{2}{3}$
I – isospin	$\frac{1}{2}$	$\frac{1}{2}$	0	0	0	0
I_z – isospin z -component	$-\frac{1}{2}$	$+\frac{1}{2}$	0	0	0	0
S – strangeness	0	0	-1	0	0	0
C – charm	0	0	0	+1	0	0
B – bottomness	0	0	0	0	-1	0
T – topness	0	0	0	0	0	+1

Excited states and exotic hadrons

- Categorization of (excited) mesons: J^{PC}

$$|l - s| \leq J \leq l + s$$

$$P = (-1)^{l+1} \quad C = (-1)^{l+s} \text{ ,for non-flavoured mesons}$$

- Allowed states and **forbidden states**:

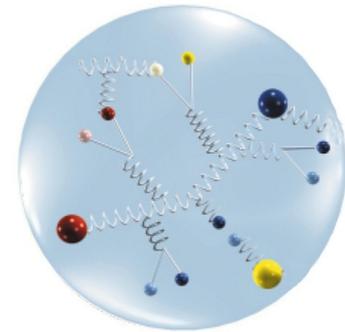
0--	0+-	0-+	0++
1--	1+-	1-+	1++
2--	2+-	2-+	2--
3--	3+-	3-+	3++

- Forbidden states called exotic states, **could exist!**

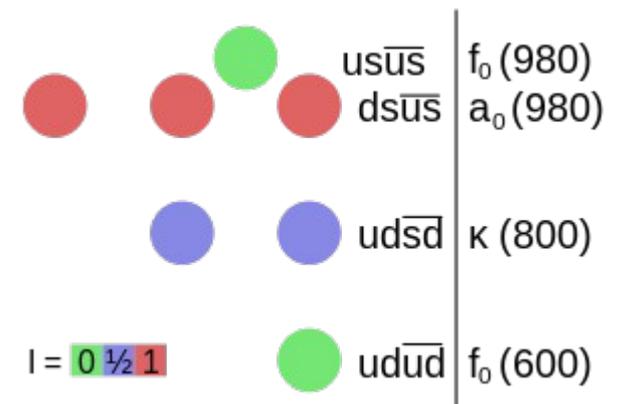
$$\pi_1(1400) \quad \pi_1(1600)$$

From the Quark Model to QCD

- ▶ Discovery of Color: **Color SU(3)** (Gauge degree of freedom)
- ▶ **Flavor SU(3) not fundamental**
- ▶ Color SU(3) implies gauge bosons: **Gluons**
- ▶ Quantum field theory of strong interactions: **QCD**
- ▶ Quarks not free in Hadrons
- ▶ Parton model, Gluons, Sea-Quarks, ...
- ▶ Exotic states could exist: Hybrids, Glueballs, etc.
(Lattice QCD)



Candidates for tetraquarks:

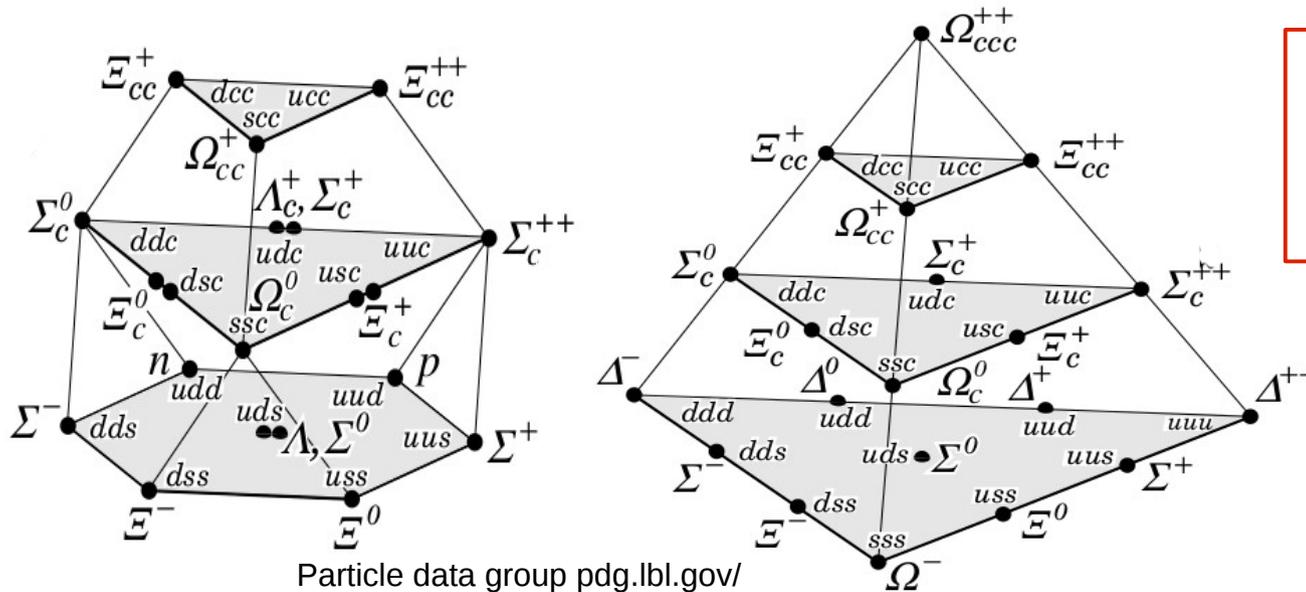


$I = 0 \quad \frac{1}{2} \quad 1$

Wikipedia

Heavy Quarks in the Quark Model

- ▶ Include charm quark in the Quark Model:
- ▶ Not a SU(4) symmetry, due to mass difference!



2 Light Quarks, heavy Quark:

$$3 \otimes 3 \otimes 1 = \bar{3} \oplus 6$$

J=1/2 baryons

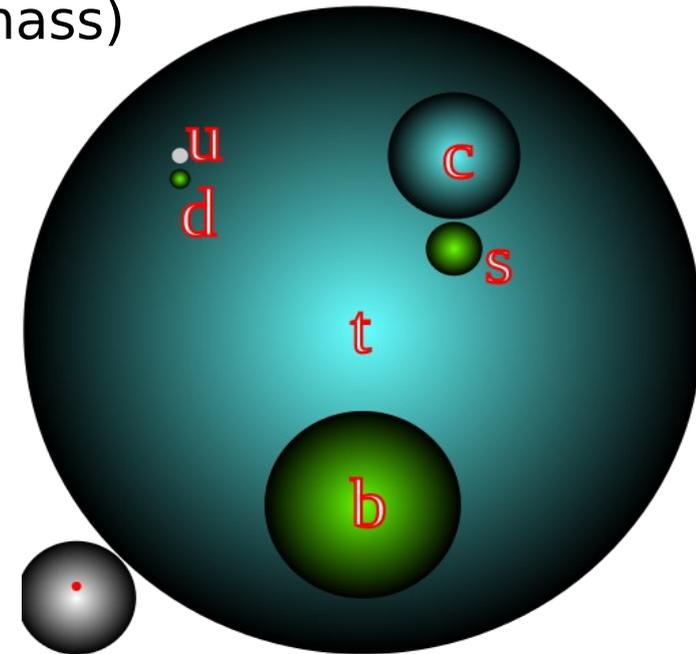
J=3/2 baryons

(analogous for bottom Quark)

Heavy Quarks in the Quark Model

- ▶ Six Quarks don't form SU(6) Flavor symmetry (mass)
- ▶ Top Quark doesn't form Hadrons (lifetime)

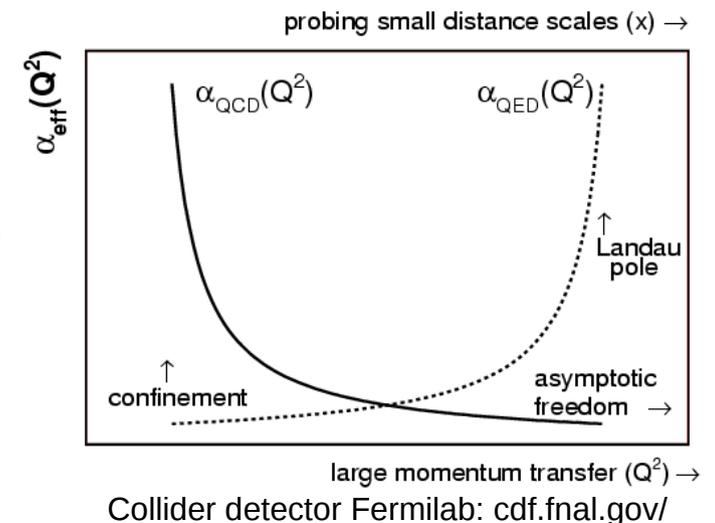
Quark	Mass in MeV
Up	1.7 - 3.1
Down	4.1 - 5.7
Strange	80 - 130
Charm	1120 - 1340
Bottom	4130 - 4370
Top	172 000 – 174 000



Wikipedia

Actual status

- ▶ Ground states of Hadrons well known
- ▶ Quark model for excited states is investigated
 - Introduce dynamics for Quarks
 - Relativistic / Nonrelativistic Potentials
- ▶ Heavy Quark expansion (effective field theory)
- ▶ QCD:
 - Confinement problem
 - Asymptotic freedom of QCD
 - Nonperturbative QCD: Lattice QCD, DSE, ...





Thanks for your attention!!!

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