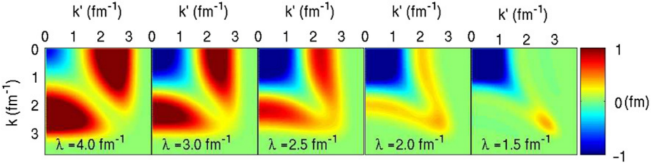
Similarity Renormalization Groups (SRG) for nuclear forces

Nuclear structure and nuclear astrophysics

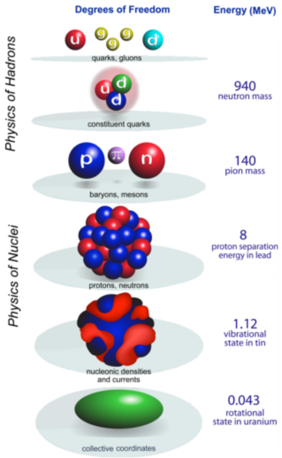
Philipp Dijkstal

12.5.2016



Content

Resolution scale in nuclear physics

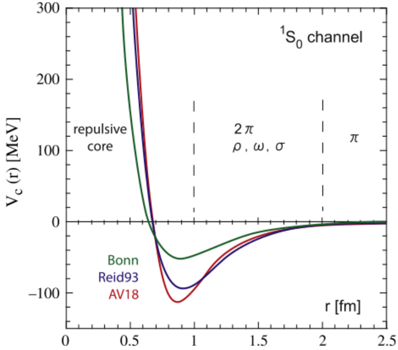


Bogner et. al., PPNP 65 (2010),94-147

Separation of scales

- Systematic restriction on relevant degrees of freedom
  $\hookrightarrow$ p, n
- Typical momenta within large nuclei: 200 MeV $\approx$ 1 fm^{-1}

Phenomenological NN potentials (purely local)



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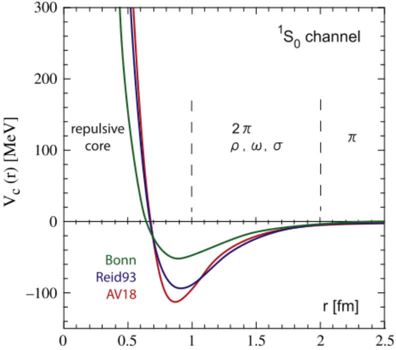

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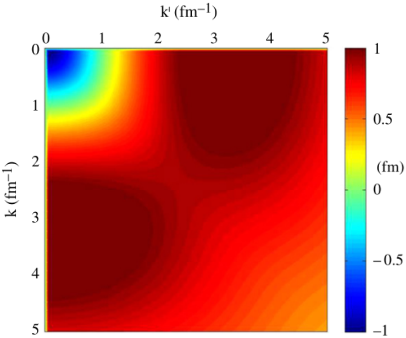

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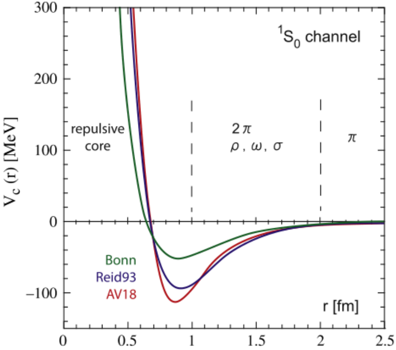

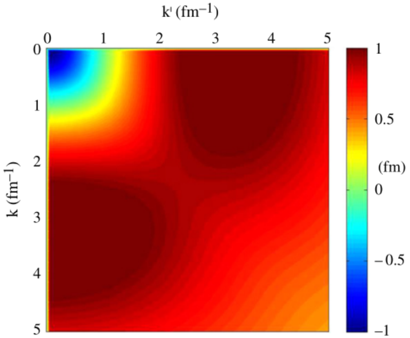

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Coupling in Lippmann-Schwinger equation for T matrix:

$$T_l(k,k',E) = V_l(k,k') + \frac{2}{\pi}\int q^2 dq \frac{V_l(k,q)\,T_l(q,k',E)}{E - q^2/2\mu + i\epsilon}$$

Similarity Renormalization Groups

Unitary transformation

$$
\begin{aligned}
U^\dagger U &= \mathbb{1} \\
E &= \langle \Psi | H | \Psi \rangle \\
&= (\langle \Psi | U^\dagger) U H U^\dagger (U | \Psi \rangle) \\
&= \langle \widetilde{\Psi} | \widetilde{H} | \widetilde{\Psi} \rangle
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Desired properties of $\widetilde{H}$:

- Dependent on resolution scale with parameter s or $\lambda = {}^{1}/{s^{1/4}}$
- Steady evolution
  $U = U_s$, $\widetilde{H} = H_s$
  $U_0 = \mathbb{1}$

Parameter flow equation

Differential equation to evolve V:

$$H_s = U_s H U_s^\dagger$$

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$$\begin{aligned}
\frac{\mathrm{d}H_s}{\mathrm{d}s} = \frac{\mathrm{d}H_s}{\mathrm{d}s} &= \frac{\mathrm{d}U_s}{\mathrm{d}s} H U_s^\dagger + U_s H \frac{\mathrm{d}U_s^\dagger}{\mathrm{d}s} \\
&= \frac{\mathrm{d}U_s}{\mathrm{d}s} \underbrace{U_s^\dagger U_s}_{\mathbb{1}} H U_s^\dagger + \underbrace{U_s H U_s^\dagger}_{H_s} U_s \frac{\mathrm{d}U_s^\dagger}{\mathrm{d}s} \\
&= \eta_s H_s + H_s \eta_s^\dagger = [\eta_s, H_s] \\
\eta_s &= \frac{\mathrm{d}U_s}{\mathrm{d}s} U_s^\dagger = -\eta_s^\dagger
\end{aligned}$$

Parameter flow equation

Generator G_s for η_s:

$$\begin{aligned}\eta_s &= [G_s, H_s] \\ \frac{\mathrm{d}H_s}{\mathrm{d}s} &= [[G_s, H_s], H_s] \\ &= G_s H_s H_s - 2 H_s G_s H_s + H_s H_s G_s\end{aligned}$$

Common choice: $G_s = T$ with $T|k\rangle = \epsilon_k|k\rangle$

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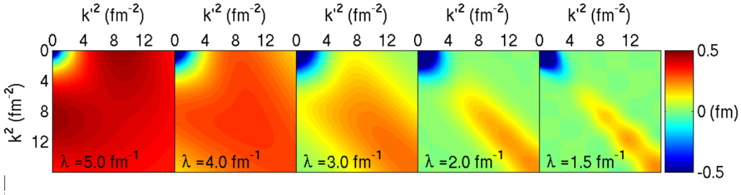
Common choice: $G_s = T$ with $T|k\rangle = \epsilon_k|k\rangle$

$$\frac{\mathrm{d}V_s}{\mathrm{d}s} = \frac{\mathrm{d}H_s}{\mathrm{d}s} = 2(TV_sT - V_sTV_s) + V_s^2T + TV_s^2 - V_sT^2 - T^2V_s$$

$$\frac{\mathrm{d}V_s}{\mathrm{d}s}(k,k') = \langle k'|V_s|k\rangle$$
$$= -(\epsilon_k'^2 - \epsilon_k)^2 V(k,k')$$
$$+ \frac{2}{\pi}\int_0^\infty q^2\mathrm{d}q(\epsilon_k + \epsilon_{k'} - 2\epsilon_q)V_s(k,q)V_s(q,k')$$

SRG Examples - Argonne ν_{18}



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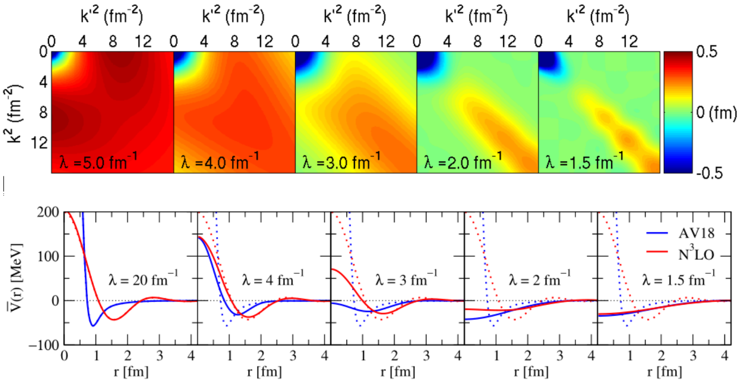
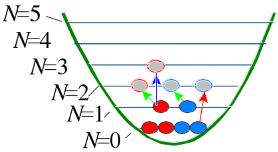


Furnstahl, Nucl. Phys. B. (Proc. Suppl.) 228 (2012) 139-175

- $^3S_1$ channel
- Non-local evolved potentials

Matrix dimensions in nuclear physics calculations

No Core Shell Model calculations



- H in HO basis with N_{max} shells
- Numerical diagonalization
- Max matrix dim. $\approx 10^9$

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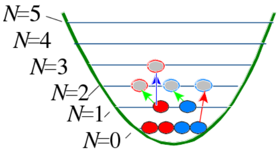

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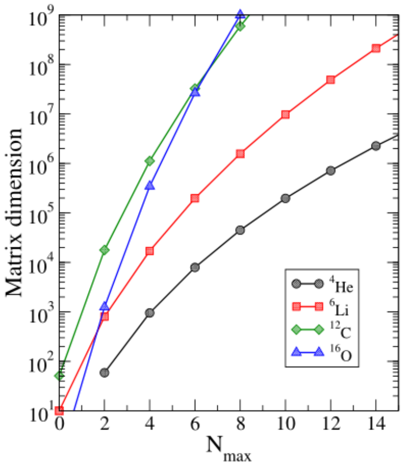

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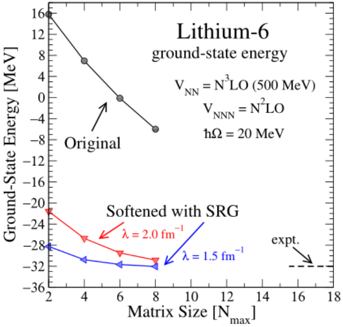
SRG Examples - ^{6}Li



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- NCFC calculations
- Based of EFT potential
- Convergence possible with SRG

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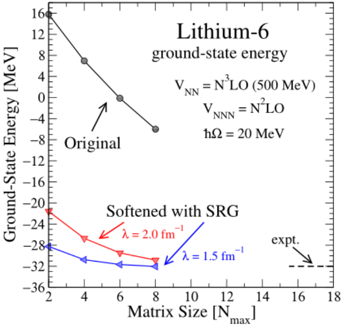

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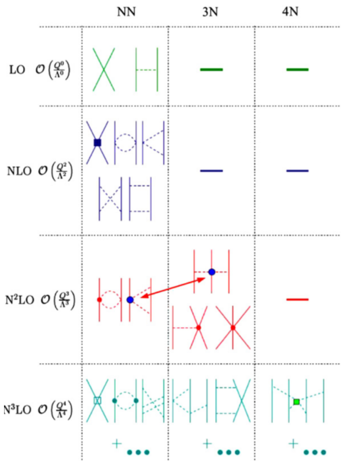
↪ 3-body forces?

Hierarchy of forces in chiral Effective Field Theory



Bogner et al., Progress in Particle and Nucl. Phys. 65 (2010) 94-147

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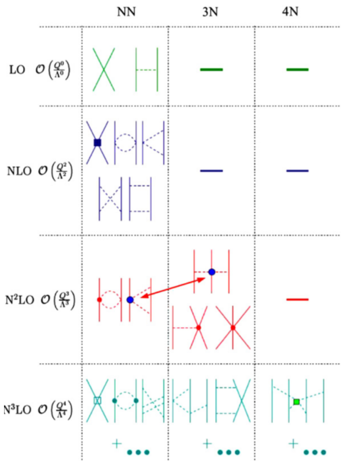


Bogner et al., Progress in Particle and Nucl. Phys. 65 (2010) 94-147

- Chiral EFT: Pions are included
- Expansion parameter : $\left(\frac{Q}{\Lambda}\right)$

$\hookrightarrow$ Multi-body forces arise naturally in EFT
$\hookrightarrow$ Consequence of neglecting higher order interaction processes

N-body forces from SRG

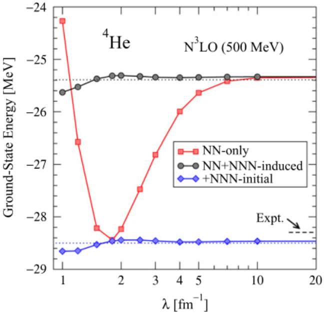
Why do 3-body forces and higher increase?

- Consider mixing terms in the 2nd quantized flow equation
- Original V_{NN} only contains 2-body forces
- T is one-body operator

$$
\begin{aligned}
\frac{\mathrm{d}V_s}{\mathrm{d}s} &= [[T, V_{NN}], H_{NN}] \\
&= [[\sum a^\dagger a, \sum a^\dagger a^\dagger aa], \sum a^\dagger a^\dagger aa] \\
&= \underbrace{\sum a^\dagger a^\dagger aa}_{\text{2 body}} + \underbrace{\sum a^\dagger a^\dagger a^\dagger aaa}_{\text{3 body}} + \ldots
\end{aligned}
$$

$$
V_{s'} = V_s + \frac{\mathrm{d}V_s}{\mathrm{d}s} \cdot \ \Delta s
$$

Importance of N-body forces



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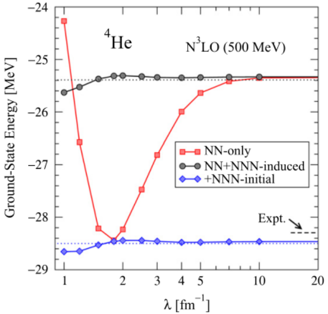


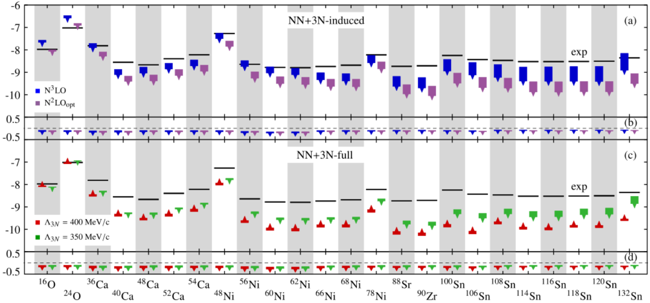
Furnstahl, Nuc. Phys. B (Proc. Suppl.) 228 (2012) 139-175

- Binding energy in ^{4}He
- Evolution from right to left
- NN-only fails

↪ 3-body forces and higher increase from SRG

↪ 4-body contribution significant

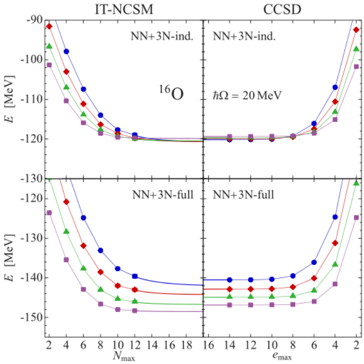
Heavy nuclei



Binder et al., Phys. Lett. B 736 (2014) 119-123

- Coupled Cluster
- Ground state energy per N
- (b) and (d): Triple correction

Comparison of CC and NCSM



Furnstahl and Hebeler, Rep. Prog. Phys. 76 (2013)

$s = 0.04\text{fm}^4$
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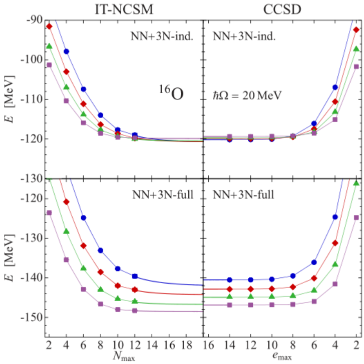

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$s = 0.04 \text{fm}^4$
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- CC completely different from NCSM
- Still great agreement

Conclusion

- Basics of low-energy nuclear physics
  - Separation of scales
  - Hierarchy of forces
- Phenomenological potentials
  - Repulsive core
  - coupling
- SRG
  - Flow equations
  - decoupling
  - Higher order forces
  - Applications
- Open Questions
  - 4-body forces
  - Alternative Generators