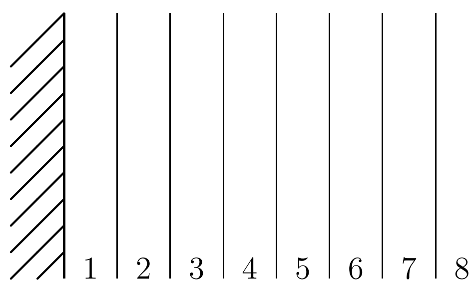
Lecture 2: from Pressure to Sound

The main goal of this lecture is to explain what a sound wave, in air, "is," by thinking about pressure and velocity in the air.

At times in this and future lectures I will make asides intended for some students with more background knowledge in a particular area. I will always put these in square brackets [], beginning with a note of who might find it useful, eg, [musicians: this interval is a perfect fifth]. In this lecture there will be a long aside for chemists and physicists.

I will work with the example of the little "siren" I demonstrate in class. A disk with holes in it spins in front of a disk with matching holes. The holes alternately line up and don't line up. A source (my mouth, for the one shown in class) presents compressed air on one side. We want to understand what happens when the air on the other side suddenly feels the higher pressure of the compressed air. To understand what is happening, mentally divide the air into a series of layers. Remember that air molecules only travel a tiny distance between scatterings, so molecules generally stay within their layer.

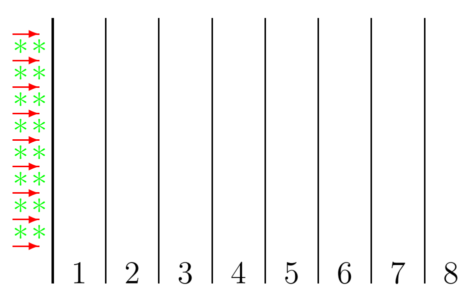
The starting picture looks like this:



Recall that

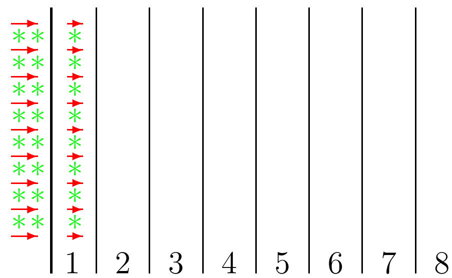
1. Denser air has higher pressure, that is, it pushes harder.
2. When the forces on an object do not balance, the object accelerates.
3. Once something is moving, it *keeps* moving until forces on it cause it to stop.

Now consider what happens when the wall is suddenly replaced by compressed, forwards moving air:

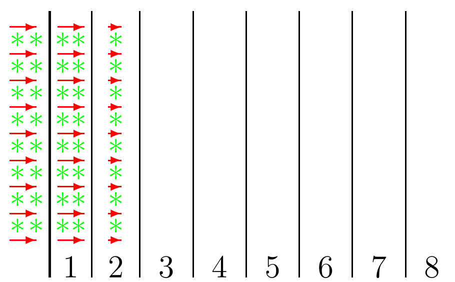


The green * are supposed to indicate that this air is compressed, that is, under higher pressure. The red arrows indicate what direction air is moving.

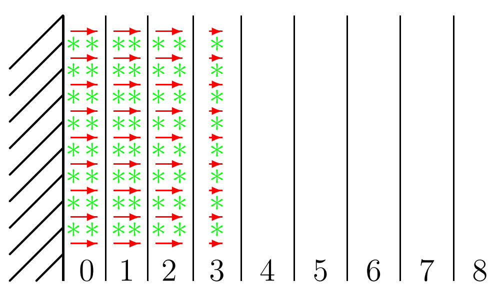
What happens to region 1? There is a larger pressure behind it than in front of it. Therefore the forces do not balance, and so it will start to move forward. Since the air behind it is moving forward, it will also get squeezed into a smaller space. Therefore, a moment later, the situation will look like this:



The first layer of air has become compressed, and has started to move forward. Since it is now compressed, it pushes harder on the second layer, than it did before. Since it is moving forward, it compresses the second layer. Therefore, a moment later the situation will look like,

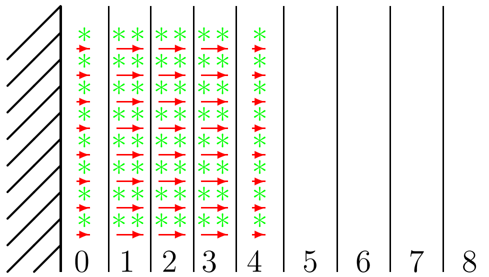


Now suppose a moment later that the barrier re-appears. Since the air moved forward, I will put it behind the new layer that got brought in. The situation looks like this:

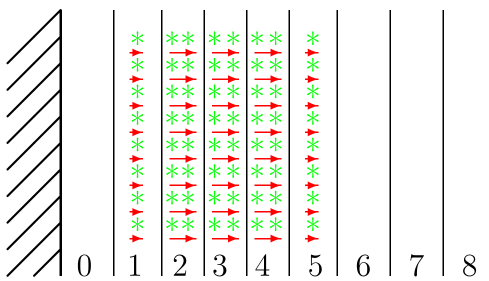


Notice that the air has only moved forward a bit–but the region where the air is compressed, is pretty big now.

Next what? The air in region 1 is feeling extra pressure in front but also behind. The pressures balance on it. But once something is moving forward, it keeps moving forward. Similarly, regions 0 and 2 keep moving forward, and regions 3 and 4 are being pushed forward and move faster. Since the wall is back in place, region 0 will now be getting stretched out, which means its pressure will fall. So we will get,



Now regions 4 and 5 will get compressed and pushed forward, but regions 0 and 1 feel more pressure on their fronts than their backs, and will keep moving forward (and hence, will de-compress) and will be slowed down. Therefore we will get,



From now on, the compressed region has nothing to do with the presence of a wall behind it. The front of the region sees a higher pressure behind than in front; so it speeds up. The back region sees a higher pressure behind than in front; so it slows down. Since the middle is moving, the front gets compressed and the back gets de-compressed. That is just the right thing to keep moving the pattern forward.

This phenomenon is called a sound wave.

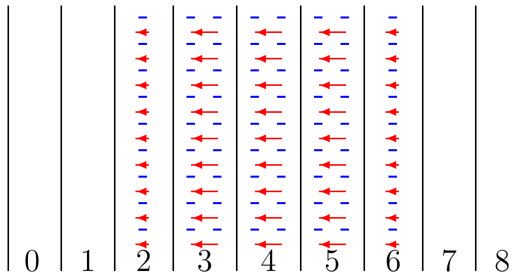
A *wave* is any phenomenon like this, where a disturbance–a phenomenon–moves forward, but the things which are causing the disturbance move very little. An ocean wave is the same thing. The wave is the phenomenon that there is a spot where the sea water is high and moving forward. If you swim in the sea, the wave carries you forward and the trough between waves carries you backwards. You, and the water, end up in the same place; but the wave certainly goes forward.

Another analogy is when you are waiting in a line. Someone at the front of the line gets to go to the counter. The person behind moves up, the person behind them moves up, so forth. The "moving up" is the wave. Each person moves a short distance, but the "moving up" phenomenon quickly goes the whole length of the line.

Let's clear up some easy to make mistakes about waves:

- The speed that the air moves, and the speed that the wave moves, need not be related and are usually very different, just as you may shuffle very slowly in the line analogy above, but the spot in the line where people are shuffling may move much faster.

- The direction the air moves, and the direction the wave moves, need not be the same. Think about what would happen here (blue dashes - represent pressure below normal):



  The regions in the middle are stretched out and moving backwards. The area in front, 6 and 7, feel *less* pressure behind and more in front, so they are accelerated backwards and start to move backwards. They are also stretched out, since the area behind them is moving away, and therefore become low pressure. At the other end, 2 and 3 are being pushed forward and squeezed, and so they regain normal pressure and stop moving. So a moment later, 7 will look like 6 looks now, 6 like 5 looks now, ... That means that the wave will move to the *right*, even though the motion of the air is to the *left*.

  What is true is that in a sound wave or part of a sound wave where pressure is higher, the wave and air move in the same direction. Where the pressure is low, the wave and air move in opposite directions.

How fast is a sound wave? That depends on two things:

- How hard does the air push or get pushed? The harder the air can push, that is, the higher the pressure, the faster the air can be made to move, and the faster the wave will propagate.

- How heavy is the air? That is, what is its density? The heavier the air, the more pushing it takes to get it to move, and he slower the wave will propagate.

From this argument, and just looking at the units, one can guess that the speed of sound is

$$v_{\rm sound} = C\sqrt{\frac{\rm Pressure}{\rm Density}} = C\sqrt{\frac{P}{\rho}}$$

(pressure and density are usually written with the letters P and ρ.) Here C is some constant which the argument I just gave is not quite enough to determine, and which turns out to be nearly but not quite 1. [Scientists: $C = \sqrt{c_P/c_V}$ with c_P and c_V being the fixed pressure and fixed temperature specific heats. These appear because the speed of sound is not $\sqrt{P/\rho}$, but actually $\sqrt{\partial P/\partial \rho}$ and the derivatives should be performed under adiabatic, not isothermal, conditions.]

Recall that the pressure of air is very large but the denisty is very small. That means $v_{\rm sound}$ will be large. It turns out that, at room temperature and for normal air,

$$v_{\rm sound} \simeq 344 \text{ m/s} .$$

If you prefer, this is 770 miles/hour or 1240 km/hour, and is also defined to be "Mach 1". The Mach number is just how many times faster than sound something is going; "Mach 3" means 3 times the speed of sound.

The speed of sound is about 1 km per 3 seconds or 1 mile per 5 seconds, hence the rule for lightning followed by thunder,

Five seconds, one mile.

Three seconds, one kilometer.

Recall that $P = nRT$. The density is $\rho = mn$, with m the mass of one molecule. That means that

$$v_{\rm sound} = C\sqrt{\frac{RT}{m}}$$

where R is another constant, as you recall. That means that

- Hot air has faster sound speed, cold air has a slower sound speed. (This causes problems in wind instruments, where pitch goes as sound speed.)
- High and low pressure air have the *same* sound speed, because the density of molecules n doesn't matter.
- Gas made of light atoms or molecules, like Helium, has a fast sound speed. Gas made of heavy atoms has a slow sound speed. Hence, people sound funny when their lungs are filled with Helium.

In a normal sound wave, the speed of the air and distance the air moves are tiny. For instance, at the threshold of hearing, the air speed is only 70 nm/s (nanometer/second), or .00000007 m/s. At the threshold of pain, the air moves a whopping 7 cm/s or .07 m/s. Similarly, the pressure changes only a tiny amount from the normal pressure in the environment (without the sound).

Sound can also occur in other materials besides air. For solids and liquids, since the atoms are pressed against each other, a compression leads to a much larger pressure than in a gas. Therefore the speed of sound is typically much larger in solid materials. For instance,

- v_{sound} in water: 1400 m/s
- v_{sound} in glass: 5000 m/s
- v_{sound} in steel: 5100 m/s

Because solids are so dense, when a sound wave in the air reaches the surface of a solid, only a tiny amount of the wave enters the solid and the rest is reflected. We will talk (much) more about this in later lectures.