2 Single-particle states

Let us now assume that the field ϕ creates a single-particle state of mass $m_{\rm phy}$ from the vacuum state as well as creating other multiparticle states. Then, separating out the single-particle state with the energy-momentum relation

$$p^0 = E(\mathbf{p}) = \sqrt{\mathbf{p}^2 + m_{\rm phy}^2} \qquad (6.1.42)$$

from the sum (6.1.15) that defines the spectral weight,

$$\begin{aligned}\rho(-p^2) = \int \frac{(d^3\mathbf{p}')}{(2\pi)^3}\frac{1}{2E(\mathbf{p}')}(2\pi)^3\delta^{(4)}(p-p')|\langle 0|\phi(0)|p'\rangle|^2 \\ + \sum_{\text{multiparticles}} (2\pi)^3\delta^{(4)}(p-p_n)|\langle 0|\phi|n\rangle|^2 . \qquad (6.1.43)\end{aligned}$$

Here the same Lorentz invariant normalization for the single-particle state $|p'\rangle$ is used that has been previously employed in the free-field theory. Hence

$$|\langle 0|\phi(0)|p'\rangle|^2 = Z > 0 \qquad (6.1.44)$$

defines a positive constant‡ Z. This is a *finite* wave-function renormalization constant which must appear since, although the field ϕ is the renormalized field, it is defined by a minimal subtraction scheme, and it does not couple the vacuum state to the single-particle state with unit strength. With $p^0 > 0$,

$$\begin{aligned}\int (d^3\mathbf{p}')\frac{1}{2E(\mathbf{p}')}\delta^{(4)}(p-p') &= \frac{1}{2p^0}\delta(p^0 - E(\mathbf{p})) \\ &= \delta(p^2 + m_{\rm phy}^2) , \qquad (6.1.45)\end{aligned}$$

and so

$$\rho(-p^2) = Z\delta(-p^2 - m_{\rm phy}^2) + \bar\rho(-p^2) , \qquad (6.1.46)$$

where now $\bar\rho(-p^2)$ stands for the contribution of the multiparticle states with

$$\bar\rho(s) = 0 , \qquad s < M_{\rm th}^2 . \qquad (6.1.47)$$

The threshold mass $M_{\rm th}$ must be larger than the particle mass $m_{\rm phy}$, $M_{\rm th}^2 > m_{\rm phy}^2$, for otherwise the single particle created by ϕ would decay into the particles associated with $M_{\rm th}$; the particle would not be stable

‡ By Lorentz invariance, Z must be a function only of p'^2. But $-p'^2 = m_{\rm phy}^2$, and so Z must be a constant.

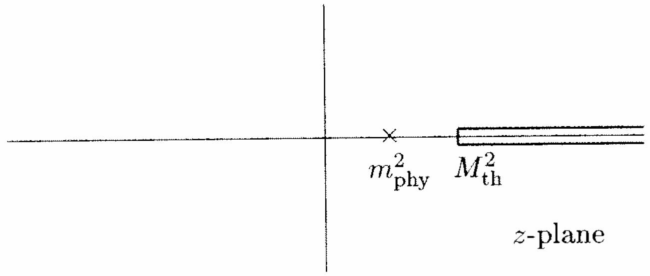

Fig. 6.3. Analytic structure of the Green's function with a single-particle pole.

and so it would not enter into the spectrum of physical states. Thus

$$G_+(p) = \frac{Z}{p^2 + m_{\rm phy}^2} + \int_{M_{\rm th}^2}^{\infty} ds \frac{\bar\rho(s)}{p^2 + s} . \qquad (6.1.48)$$

The physical mass of the particle is the position of pole of the two-point Green's function. Moreover, the positive residue of this pole is a finite wave function renormalization in addition to the minimal but infinite renormalization that relates the renormalized field ϕ to the bare field ϕ_0. The analytic structure is now described by the picture in Fig. 6.3.

6.2 Reduction formula

The Green's functions of a quantum field theory yield all the (n-particle) scattering amplitudes of the theory. To see how this goes, we first examine the construction of single-particle states $|p\rangle$ in terms of the field operator $\phi(x)$ acting on the vacuum state $|0\rangle$.

If $\phi(x)$ is a free field, the single-particle state has the construction (Chapter 3, Section 2)

$$\langle p'| = \int (d^3\mathbf{x}) e^{-ip'x} i(\overrightarrow{\partial}_0 - \overleftarrow{\partial}_0)\langle 0|\phi(x) , \qquad (6.2.1)$$

with these states having the normalization

$$\langle p'|p\rangle = 2p^0(2\pi)^3\delta(\mathbf{p}' - \mathbf{p}) . \qquad (6.2.2)$$

For this free field, the wave function is given by

$$\langle p'|\phi(x)|0\rangle = e^{-ip'x} . \qquad (6.2.3)$$

For an interacting field theory which has a single-particle state with the energy-momentum relation

$$p'^{\mu} = \left(\sqrt{\mathbf{p}'^2 + m_{\rm phy}^2} , \mathbf{p}'\right) , \qquad (6.2.4)$$

we are motivated to try a construction akin to that given in Eq. (6.2.1) for a free particle. This construction is tested by examining the resulting "wave function" which is now given by an operation on the interacting two-point Green's function. Using the Lehmann representation,

$$\begin{aligned}
&\int (d^3\mathbf{x}) e^{-ip'x} i(\overrightarrow{\partial}_0 - \overleftarrow{\partial}_0)\langle 0|\phi(x)\phi(x')|0\rangle \\
&= \int (d^3\mathbf{x}) e^{-i\mathbf{p}'\cdot\mathbf{x}+ip'^0 t} i(\overrightarrow{\partial}_0 - \overleftarrow{\partial}_0) \int ds \left[Z\delta(s-m^2_{\text{phy}}) + \bar{\rho}(s)\right] \\
&\quad \int \frac{(d^3\mathbf{p})}{(2\pi)^3}\frac{1}{2E_s(\mathbf{p})} e^{i\mathbf{p}\cdot(\mathbf{x}-\mathbf{x}')-iE_s(p)(t-t')} \\
&= Ze^{-ip'x'} + \int_{M^2_{\text{th}}}^{\infty} ds\bar{\rho}(s)\frac{E_s(\mathbf{p}')+p'^0}{2E_s(\mathbf{p}')} e^{-iE_s(\mathbf{p}')(t-t')} e^{-i\mathbf{p}'\cdot\mathbf{x}'+ip'^0 t} .
\end{aligned} \tag{6.2.5}$$

The first term on the right-hand side of the last equality is just the free-particle wave function modified by the appearance of the finite wave-function renormalization constant Z. The second term represents the contribution of the continuum of states that is also produced when an interacting field acts on the vacuum state.

To understand the nature of the continuum contribution, it is worthwhile to first review the Riemann-Lebesgue Lemma. This lemma roughly states that if $f(\omega)$ is a "smooth" function which vanishes as $\omega \to \pm\infty$, then its Fourier transform vanishes in the infinite time limit,

$$t \to \infty : \qquad \int_{-\infty}^{\infty} d\omega f(\omega) e^{-i\omega t} \to 0 . \tag{6.2.6}$$

The point is that as $t \to \infty$, the exponential oscillates ever more rapidly and adjacent regions of the integration cancel. The proof follows by partial integration

$$\begin{aligned}
\int_{-\infty}^{\infty} d\omega f(\omega) e^{-i\omega t} &= \frac{i}{t}\int_{-\infty}^{\infty} d\omega f(\omega)\frac{\partial}{\partial\omega} e^{-i\omega t} \\
&= -\frac{i}{t}\int_{-\infty}^{\infty} d\omega \frac{df(\omega)}{d\omega} e^{-i\omega t} \\
&= \quad \cdots\cdots \\
&= \left(\frac{-i}{t}\right)^N \int_{-\infty}^{\infty} d\omega \frac{d^N f(\omega)}{d\omega^N} e^{-i\omega t} .
\end{aligned} \tag{6.2.7}$$

This process can be repeated until a discontinuity or singularity is obtained in the N-th derivative, in which case one has proved that the Fourier transform vanishes at least as rapidly as $1/t^N$. If all the derivatives of $f(\omega)$ exist, then the Fourier transform vanishes faster than any

power of $1/t$ as $t \to \infty$. In view of this lemma, we expect that

$$\lim_{|t-t'|\to\infty} \int_{M^2_{\text{th}}}^{\infty} ds\, \bar{\rho}(s)\, \frac{E_s + p'^0}{2E_s} e^{-iE_s(t-t')} = 0 , \tag{6.2.8}$$

so that the continuum contribution vanishes in the infinite time limit. However the integrand is not smooth at the various n-particle thresholds — at the points $s = M_n^2$ where the states containing n particles start to make their contribution. Hence, the continuum contribution does not vanish faster than any power of $1/|t-t'|$.

To estimate the long-time limit of the continuum contribution, we note that the spectral weight $\bar{\rho}(s)$ has infinitely many thresholds corresponding to 2-particle, 3-particle, ... intermediate states. With s near the n-particle threshold, $s = M_n^2$, $\bar{\rho}(s)$ is proportional to the phase space for the production of n-particles (with additional factors of $\sqrt{s - M_n^2}$ appearing if the particles have spin). A straightforward calculation (see Problem 2) shows that near the n-particle threshold*

$$\bar{\rho}(s) \propto \left(\sqrt{s - M_n^2}\right)^{3n-5} . \tag{6.2.9}$$

The leading term in the long-time limit which arises from the threshold behavior of an n-particle intermediate state is obtained by the approximation

$$E_s(\mathbf{p}') = \sqrt{\mathbf{p}'^2 + s} \simeq E_{M_n}(\mathbf{p}') + \frac{s - M_n^2}{2E_{M_n}(\mathbf{p}')} , \tag{6.2.10}$$

with

$$E_{M_n}(\mathbf{p}') = \sqrt{\mathbf{p}'^2 + M_n^2} , \tag{6.2.11}$$

which is valid in the threshold region where s is near M_n. Thus

$$\begin{aligned}
&\int_{M_n^2}^{\infty} ds\, \bar{\rho}(s) \frac{E_s(\mathbf{p}') + p'^0}{2E_s(\mathbf{p}')} e^{-iE_s(\mathbf{p}')(t-t')} \\
&\sim \int_{M_n^2}^{\infty} ds (s - M_n^2)^{\frac{3n-5}{2}} \exp\left\{-\frac{i}{2}\frac{s-M_n^2}{E_{M_n}(\mathbf{p}')}(t-t')\right\} \exp\left\{-iE_{M_n}(\mathbf{p}')(t-t')\right\} \\
&\sim \left(\frac{1}{M|t-t'|}\right)^{\frac{3}{2}(n-1)} \exp\left\{-iE_{M_n}(\mathbf{p}')(t-t')\right\} ,
\end{aligned} \tag{6.2.12}$$

where M is some characteristic elementary particle mass scale.

Taking, say, $M \sim 1$ GeV corresponds to $M^{-1} \sim 10^{-23}$ sec. Therefore for $(t-t') \gg M^{-1} \sim 10^{-23}$ sec the continuum contribution is negligible.

* The simple square root behavior for a two particle threshold is evident from the calculation of the dispersion relation for the "bubble" graph found in Problem 2 of Chapter 4.

For example, with $t-t' \sim 10^{-13}$ sec, the contribution of an $n=2$-particle threshold is on the order of 10^{-15}. Although the continuum contribution vanishes only as an inverse power for large time difference, the scale is set by the very short elementary particle time M^{-1}. Hence

$$\lim_{|t-t'|\to\infty} \int (d^3\mathbf{x}) e^{-ip'x} i(\overrightarrow{\partial}_0 - \overleftarrow{\partial}_0)\langle 0|\phi(x)\phi(x')|0\rangle = Z e^{-ip'x'}, \quad (6.2.13)$$

where, in physical terms, the "infinite limit" means time differences $|t-t'|$ on the order of 10^{-13} sec or so.

We can now see how to construct a single-particle state for an interacting field system, namely

$$\langle p| = \lim_{t\to\pm\infty} \int (d^3\mathbf{x}) e^{-ipx} i(\overrightarrow{\partial}_0 - \overleftarrow{\partial}_0)\langle 0|\frac{1}{\sqrt{Z}}\phi(x), \quad (6.2.14)$$

for this procedure gives the properly normalized wave function

$$\langle p|\phi(x)|0\rangle = \sqrt{Z}e^{-ipx}. \quad (6.2.15)$$

The Hermitian adjoint yields

$$|p\rangle = \lim_{t\to\pm\infty} \int (d^3\mathbf{x}) e^{+ipx}(-i)(\overrightarrow{\partial}_0 - \overleftarrow{\partial}_0)\frac{1}{\sqrt{Z}}\phi(x)|0\rangle. \quad (6.2.16)$$

Using the wave function (6.2.15), the scalar product of the two states constructed by Eqs. (6.2.14) and (6.2.16) — but with opposite time limits [$t\to+\infty$ for the bra, $t\to-\infty$ for the ket (or the other way around)] so as to suppress the continuum contribution — is just that which has been used for free particles:

$$\langle p|p'\rangle = (2\pi)^3 2p^0 \delta^{(3)}(\mathbf{p}-\mathbf{p}'). \quad (6.2.17)$$

It should be emphasized that the infinite time limits used to construct the single-particle states picks out only the pole term in the Green's function — the continuum contributions are "washed out" in accordance with the Riemann-Lebesgue Lemma.

We shall use this method of state construction not only to obtain single-particle states but also to construct multiparticle states as well. This does not follow strictly from what we have done. We will not give a rigorous proof, but the result is so plausible that it is easy to accept. Suppose that there is an initial ϕ-particle wave packet which later collides with the wave packet of another particle or system of particles which we denote by ζ. Initially the two wave packets are separated by a great spatial distance and there is no interaction between them. Therefore, the initial state $|\zeta-\rangle$ containing the other system is essentially the vacuum state as far as the well-separated ϕ-particle wave packet is concerned. Hence the initial state with the additional ϕ-particle wave packet is constructed by replacing the vacuum state in Eq. (6.2.16) by $|\zeta-\rangle$ with the momentum p which appears

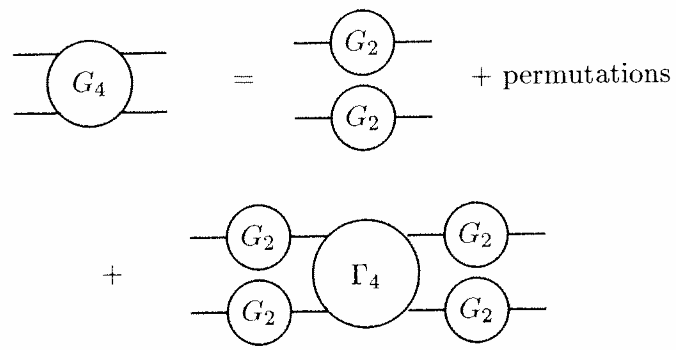

Fig. 6.4. Graphical description of the decomposition of the four-point Green's function into disconnected and single-particle irreducible parts.

in Eq. (6.2.16) integrated with the momentum-space wave function $f(p)$ which produces the well-separated ϕ-particle wave packet. In the limit in which the ϕ-particle wave packet becomes an incident plane-wave with momentum p, the initial state has the construction

$$|p,\zeta-\rangle = \lim_{t\to-\infty} \int (d^3\mathbf{x}) e^{ipx}(-i)(\overrightarrow{\partial}_0 - \overleftarrow{\partial}_0)\frac{1}{\sqrt{Z}}\phi(x)|\zeta-\rangle. \quad (6.2.18)$$

Similarly, the limit of the outgoing situation wave packet to a plane wave of momentum p' is given by

$$\langle p'\zeta'+| = \lim_{t\to+\infty} \int (d^3\mathbf{x}) e^{-ip'x} i(\overrightarrow{\partial}_0 - \overleftarrow{\partial}_0)\langle\zeta'+|\frac{1}{\sqrt{Z}}\phi(x). \quad (6.2.19)$$

Let us use this procedure to construct the transformation function $\langle p_1'p_2'+|p_1p_2-\rangle$ and thereby the two-particle scattering amplitude from the four-point Green's function

$$G(x_1,x_2,x_3,x_4) = \langle -T\phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4)\rangle. \quad (6.2.20)$$

To do this, we first decompose the Green's function into disconnected and irreducible parts as shown by the graphical structure in Fig. 6.4. The first graphs represent the unscattered, "straight through" propagation of the particles, but with the "fully dressed" interacting propagators. The final graph represents the processes that give rise to the scattering, but again with the interacting propagator factors removed to define an amplitude Γ which is single-particle irreducible.† These graphs correspond to the

† It is clear that the sum of all the Feynman graphs fall into the categories illustrated in Fig. 6.4. It is not obvious, however, that they combine to form precisely the propagator factors G_2 that are shown. This is explicitly proven in Sections 4 and 5 below.

formula

$$G(x_1, x_2, x_3, x_4) = G(x_1 - x_2)G(x_3 - x_4) + G(x_1 - x_3)G(x_2 - x_4) + G(x_1 - x_4)G(x_2 - x_3)$$
$$+ \int (d\bar{x}_1) \cdots (d\bar{x}_4) G(x_1 - \bar{x}_1) \cdots G(x_4 - \bar{x}_4) \Gamma(\bar{x}_1, \bar{x}_2, \bar{x}_3, \bar{x}_4) , \quad (6.2.21)$$

or, in momentum space,

$$(2\pi)^4 \delta(\Sigma p) G(p_1 , p_2 , p_3 , p_4)$$
$$= (2\pi)^4 \delta(p_1 + p_2) G(p_1) \, (2\pi)^4 \delta(p_3 + p_4) G(p_3) + \text{perms}$$
$$+ (2\pi)^4 \delta(\Sigma p) G(p_1) G(p_2) G(p_3) G(p_4) \Gamma(p_1 , p_2 , p_3 , p_4) . \quad (6.2.22)$$

Using Eqs. (6.2.19) and (6.2.18) to construct the states, the transformation function is given by

$$\langle p_1' p_2' + | p_1 p_2 - \rangle = \int_{+\infty} (d^3\mathbf{x}_1) e^{-ip_1' x_1} (\overrightarrow{\partial}_{10} - \overleftarrow{\partial}_{10})$$
$$\cdots \int_{-\infty} (d^3\mathbf{x}_4) e^{ip_2 x_4} (\overrightarrow{\partial}_{40} - \overleftarrow{\partial}_{40})$$
$$\frac{1}{Z^2} \langle T\phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4) \rangle . \quad (6.2.23)$$

Note that the time-ordering in the matrix element orders the fields appropriately, with the field operators creating the outgoing state appearing on the left and the field operators creating the incoming state appearing on the right. Making use of the decomposition (6.2.21) we encounter terms of the form

$$\int_{+\infty} (d^3\mathbf{x}_1) e^{-ip_1' x_1} i (\overrightarrow{\partial}_{10} - \overleftarrow{\partial}_{10}) \frac{1}{\sqrt{Z}} G(x_1 - y)$$
$$= i \langle p_1' | \phi(y) | 0 \rangle = i\sqrt{Z} e^{-ip_1' y} , \quad (6.2.24)$$

and

$$\int_{-\infty} (d^3\mathbf{x}_3) e^{ip_1 x_3} (-i) (\overrightarrow{\partial}_{30} - \overleftarrow{\partial}_{30}) \frac{1}{\sqrt{Z}} G(x_3 - z)$$
$$= i \langle 0 | \phi(z) | p_1 \rangle = i\sqrt{Z} e^{ip_1 z} . \quad (6.2.25)$$

Using Eqs. (6.2.24) and (6.2.25) it is easy to check that the "straight through" propagation terms in the decomposition (6.2.21) connecting x_1 with x_2 and x_3 with x_4 do not contribute while the other "straight through" terms give single-particle transformation functions. Also using Eqs. (6.2.24) and (6.2.25) it is easy to see that the remaining scattering term involves the Fourier transform of the Γ amplitude. Thus one finds

that

$$\langle p_1' p_2' + | p_1 p_2 - \rangle = \langle p_1' | p_1 \rangle \langle p_2' | p_2 \rangle + \langle p_1' | p_2 \rangle \langle p_2' | p_1 \rangle$$
$$- i(2\pi)^4 \delta^{(4)}(p_1' + p_2' - p_1 - p_2) T , \quad (6.2.26)$$

in which

$$T = -iZ^2 \Gamma(p_1' , p_2' , -p_1 , -p_2) . \quad (6.2.27)$$

Note that as far as the scattering contribution described by T is concerned, the state construction just picks out the residue of the single-particle poles $(p^2 + m_{\text{phy}}^2)^{-1}$. A factor of $\sqrt{Z}$ is removed for each external particle. This leaves a finite wave function renormalization which must be taken into account, the factor Z^2 in relating the scattering amplitude to the irreducible vertex Γ. We have already learned in Chapter 3, Section 4 how to construct the scattering cross section from the structure (6.2.26).

We have gone through this "derivation" in some detail particularly to explain how the "straight-through-propagation" factors work out. Let us now give a simple but heuristic discussion which leads to the "reduction mnemonic". First we note that a particle of momentum p' is added to a final state by writing

$$\langle \zeta' p' + | \xi - \rangle = \int (d^3\mathbf{x}) e^{-ip'x} i (\overrightarrow{\partial}_0 - \overleftarrow{\partial}_0) \frac{1}{\sqrt{Z}} \langle \zeta' + | \phi(x) | \xi - \rangle , \quad (6.2.28)$$

where the limit $t = x^0 \rightarrow +\infty$ is understood. A partial integration presents this as

$$\langle \zeta' p' + | \xi - \rangle = \int_{-\infty}^{\infty} dt \, \partial_0 \int (d^3\mathbf{x}) e^{-ip'x} i (\overrightarrow{\partial}_0 - \overleftarrow{\partial}_0) \frac{1}{\sqrt{Z}} \langle \zeta' + | \phi(x) | \xi - \rangle$$
$$+ \int_{-\infty} (d^3\mathbf{x}) e^{-ip'x} i (\overrightarrow{\partial}_0 - \overleftarrow{\partial}_0) \frac{1}{\sqrt{Z}} \langle \zeta' + | \phi(x) | \xi - \rangle . \quad (6.2.29)$$

The second integration at $t = -\infty$ gives no "scattering" contribution — this term has a rapid phase variation that produces a vanishing result unless it corresponds to destroying a particle of momentum p' in the initial state, which gives a "straight-through-propagation" contribution. The overall time derivative on the right-hand side of the first line of the equation combines to give the form $i(\overrightarrow{\partial_0^2} - \overleftarrow{\partial_0^2})$. To this can be added the combination $\overrightarrow{\nabla^2} - \overleftarrow{\nabla^2}$ which gives no contribution as an integration by parts establishes. Hence

$$\langle \zeta' p' + | \xi - \rangle = \int (d^4x) e^{-ip'x} i (-\overrightarrow{\partial^2} + \overleftarrow{\partial^2}) \frac{1}{\sqrt{Z}} \langle \zeta' + | \phi(x) | \xi - \rangle$$
$$= \int (d^4 x) e^{-ip'x} i (-\overrightarrow{\partial^2} + m_{\text{phy}}^2) \frac{1}{\sqrt{Z}} \langle \zeta' + | \phi(x) | \xi - \rangle . \quad (6.2.30)$$

One now takes p' slightly off mass shell, $-p'^2 \neq m^2_{\rm phy}$. Then there are rapid phase variations at the space-time boundaries, and integrations by parts can be freely performed to give

$$\langle \zeta' p' + | \xi - \rangle = \frac{i}{\sqrt{Z}}(p'^2 + m^2_{\rm phy}) \int (d^4x) e^{-ip'x} \langle \zeta' + | \phi(x) | \xi - \rangle \, . \quad (6.2.31)$$

The physical limit $p'^2 + m^2_{\rm phy} \to 0$ picks out the residue of the $(p'^2 + m^2_{\rm phy})^{-1}$ pole of the Fourier transform of the amplitude in Eq. (6.2.31) and produces the physical state matrix element. Similarly, an additional particle in an initial state is constructed as

$$\langle \zeta' + | \xi p - \rangle = \lim_{p^2 + m^2_{\rm phy} \to 0} \frac{i}{\sqrt{Z}}(p^2 + m^2_{\rm phy}) \int (d^4x) e^{ipx} \langle \zeta' + | \phi(x) | \xi - \rangle \, . \quad (6.2.32)$$

As an example of the use of this reduction mnemonic, let us consider the previous example of the two-particle scattering amplitude. Applying the mnemonic to Eq. (6.2.22) we see again that that the scattering amplitude is identified as the residue in the $(p^2 + m^2_{\rm phy})^{-1}$ poles,

$$\begin{aligned} \langle p'_1 p'_2 + | p_1 p_2 - \rangle = & \text{ straight through propagation terms} \\ & -(2\pi)^4 \delta(p'_1 + p'_2 - p_1 - p_2) Z^2 \Gamma(p'_1 , p'_2 , -p_1 , -p_2) \, , \end{aligned} \quad (6.2.33)$$

in agreement with Eqs. (6.2.26) and (6.2.27).

6.3 Unstable particles

Let us first recall the analytic structure of the two-point Green's function or propagator when the theory contains a particle whose physical mass we will now denote simply by m. It has a pole at $-p^2 = m^2$ and a branch cut starting at $-p^2 = M^2_{\rm th}$ as shown in Fig. 6.3. The Lehmann representation (6.1.40) implies that $G(p)$ has no singularities except for $-p^2$ real and positive. In particular a pole in $G(p)$ must lie on the real $-p^2$ axis. Suppose now that the parameters of the theory are changed. Then the pole and branch point may move towards one another. If further changes in the parameters are made the pole will move into the branch point. Then what? Since the pole cannot move onto the complex plane it must move into new Riemann sheets. As will be described shortly, the Green's function originally defined above the cut can be analytically continued into a second Riemann sheet below the cut. The final position of the pole in this second sheet is sketched in Fig. 6.5. Parameters giving poles on unphysical sheets produce an unstable particle if these poles are close to the real axis.

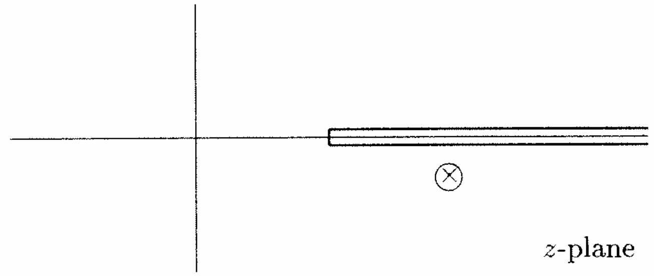

Fig. 6.5. When the parameters of the theory are varied, the pole at the mass of a stable particle can move through the branch point into other Riemann sheets. Shown here is the final position of the pole in the continuation into the second sheet which is described in the text.

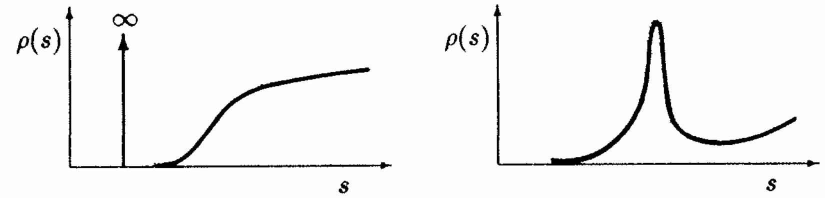

Fig. 6.6. Left: Spectral weight for a stable particle. Right: Spectral weight for an unstable particle.

The continuation of the mass parameter can also be cast in terms of the spectral weight of the Lehmann representation. Referring to Eq. (6.1.40) and using

$$\epsilon \to 0^+ \; : \qquad \mathrm{Im}\, \frac{1}{x \mp i\epsilon} = \pm \pi \, \delta(x) \, , \quad (6.3.1)$$

one sees that this weight is the imaginary part of the propagator,

$$\rho(-p^2) = \frac{1}{\pi} \mathrm{Im}\, G(p) \, . \quad (6.3.2)$$

Here $G(p)$ is the physical function obtained by the $\epsilon \to 0^+$ limit of Eq. (6.1.40). Note that the spectral weight also appears as the discontinuity across the cut in the analytic function $G(z)$ defined by Eq. (6.1.41),

$$G(x + i\epsilon) - G(x - i\epsilon) = 2\pi i \rho(x) \, . \quad (6.3.3)$$

In terms of the spectral weight, the stable and unstable particle cases are described by the sketches in Fig. 6.6. The presence of the unstable particle poles is reflected in a sharp peak in the spectral weight.

To describe the relevant "second sheet", we recall that the Green's func-