High-tension Global Strings for Axions

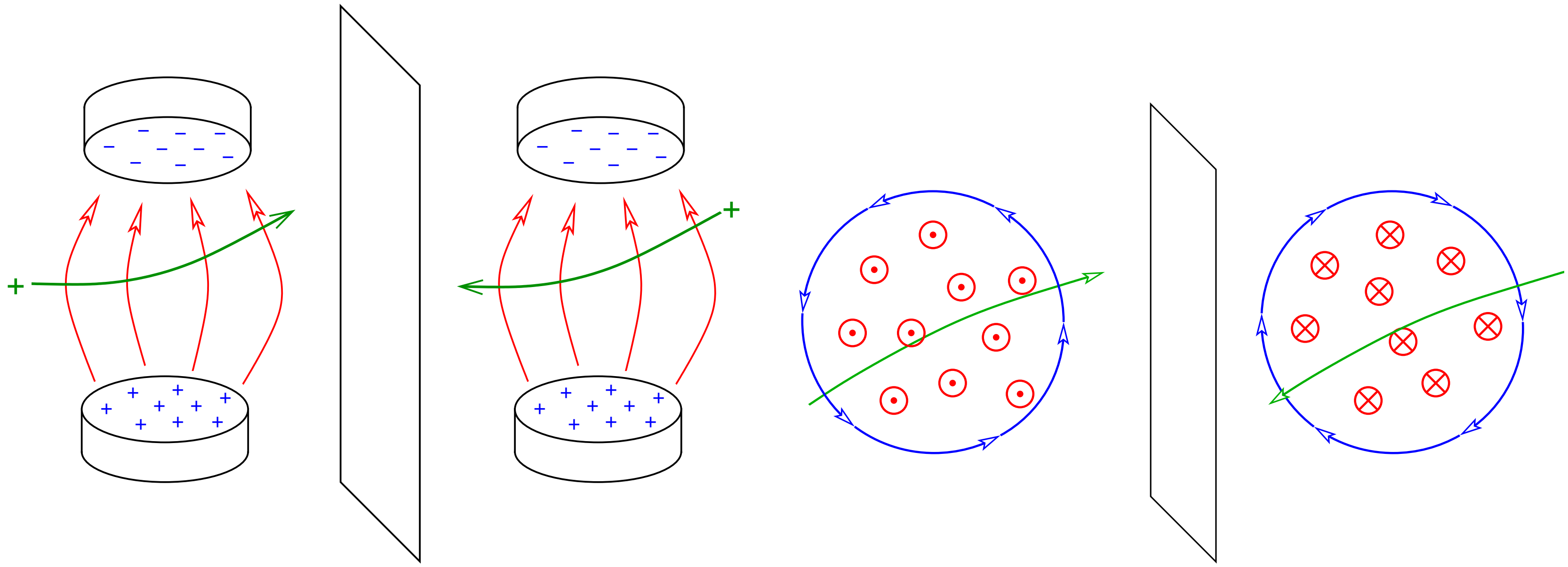
Guy Moore, TU Darmstadt

With Vincent Klaer, Leesa Fleury

- Axions: summary and review
- String and string-wall networks in axion fields
- The global-string, multi-scale $\rightarrow$ high-tension issue
- Effective theory description
- My proposed solution
- Results so far

$\mathbf{T}$ in E&M

The physics of EM is $\mathbf{T}$ symmetric:



Q's unchanged, but J's flip. E same, but B flips.

QCD and its Lagrangian

QCD Lagrangian can have two distinct "field-strength" terms:

$$S = \int dt \int d^3x \left(\frac{\vec{E}_a^2 - \vec{B}_a^2}{2g^2} + \frac{\Theta}{8\pi^2} \vec{E}_a \cdot \vec{B}_a \right)$$

The Θ term is $\mathbf{T}$ odd!
$\vec{E}_a \cdot \vec{B}_a$ is a total derivative:

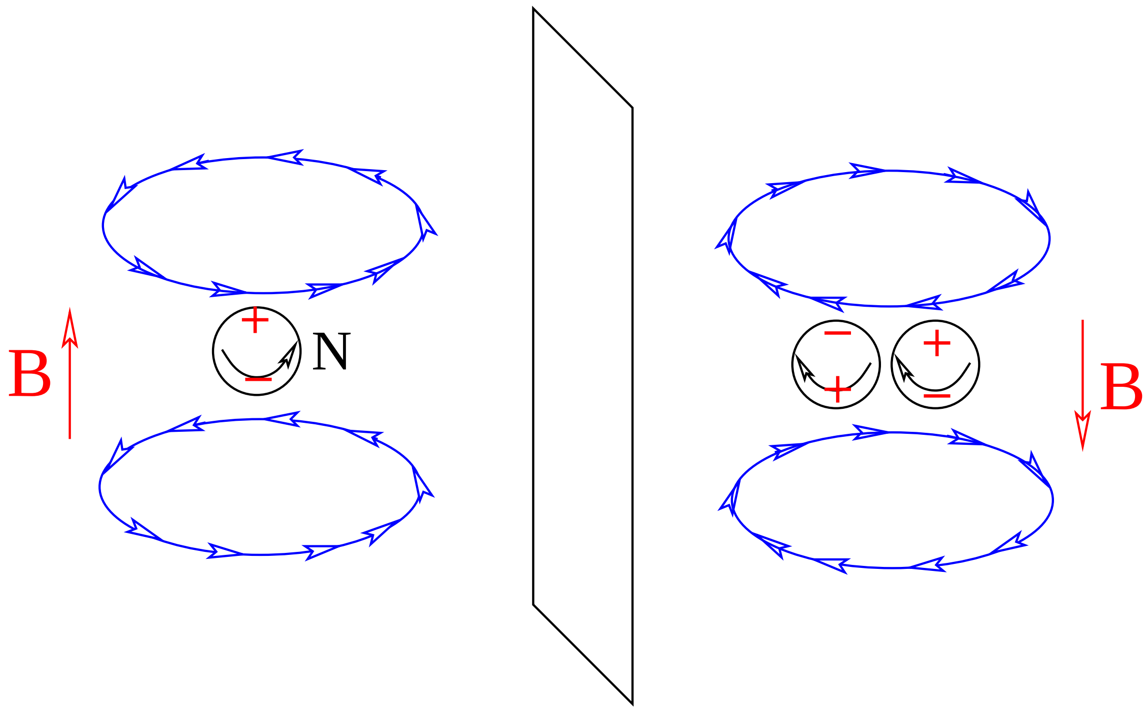
$$\vec{E}_a \cdot \vec{B}_a = \partial^\mu K_\mu \,, \qquad 2K_\mu = \epsilon_{\mu\nu\alpha\beta} \left(A_a^\nu F_a^{\alpha\beta} + \frac{g f_{abc}}{3} A_a^\nu A_b^\alpha A_c^\beta \right)$$

Last term *need not* vanish on boundary even if $\vec{E}_a = 0 = \vec{B}_a$ there!
It's always $8\pi^2 N_I$ with N_I integer. So Θ mod 2π has *physical consequences*
G. 't Hooft, PRL 37, 8(1976); R. Jackiw and C. Rebbi, PRL 37, 172 (1976);

Callan Dashen and Gross, Phys Lett 63B, 334 (1976)

Looking for $\mathbf{T}$: Neutron EDM

Put neutron in $\vec{B}$ field – spin lines up with $\vec{B}$.



Is there an Electric Dipole Moment (EDM) aligned with spin?

If so: looks different when movie runs backwards, $\mathbf{T}$ viol!

Neutron EDM and Θ

Theory: Neutron electric dipole moment should exist,

$$d_n = -3.8 \times 10^{-16}\, e\,\mathrm{cm} \times \Theta$$

so long as Θ is not zero! Guo *et al*, arXiv:1502.02295, assumes Θ, modulo 2π, is small

Experiment: Consistent with zero! Baker *et al* (Grenoble), arXiv:hep-ex/0602020

$$|d_n| < 2.9 \times 10^{-26}\, e\,\mathrm{cm}$$

Either $|\Theta| < 10^{-10}$ by (coincidence? accident?) or there is something deep going on here.

Θ from UV physics

Consider heavy Dirac quark $[Q^\alpha \; q_{\dot{\alpha}}]$ Two Weyl spinors
Q^α is 3, q^α is $\bar{3}$. Lagrangian:

$$\mathcal{L}(Q, q) = \frac{1}{2}\bar{Q}\not{D}Q + \frac{1}{2}\bar{q}\not{D}q + m q_\alpha Q^\alpha + m^* q^{\dot{\alpha}} Q_{\dot{\alpha}}$$

Mass m is in general complex.
Rotate $m = |m|e^{i\theta} \to |m|$ by rotating Q but not q.
Such a chiral rotation generates shift, $\Theta \to \Theta + \theta$.

Phase in mass of heavy quark becomes part of Θ_{QCD}.

Axion

Give Q^α, q^α different (global) U(1) charges (so $m = 0$)
Introduce complex φ with U(1) charge: can now write

$$\mathcal{L}_{\varphi q Q} = y\varphi q_\alpha Q^\alpha + y\varphi^* q^{\dot\alpha} Q_{\dot\alpha}$$

Symmetry-breaking potential for φ:

$$\mathcal{L}_\varphi = \mathcal{L}_{\varphi q Q} + \partial_\mu \varphi^* \partial^\mu \varphi + \frac{m^2}{8 f_a^2}\left(2\varphi^*\varphi - f^2\right)^2$$

Phase $\varphi = e^{i\theta_A} f_a$ becomes part of Θ: $\Theta_{\text{eff}} = \Theta + \theta_A$ or

$$\mathcal{L}_\varphi = \partial_\mu \varphi^* \partial^\mu \varphi + V(\varphi^*\varphi) + \theta_A \frac{F^a_{\mu\nu}\tilde{F}^{\mu\nu a}}{32\pi^2} \text{[dim-5]}$$

How the axion works

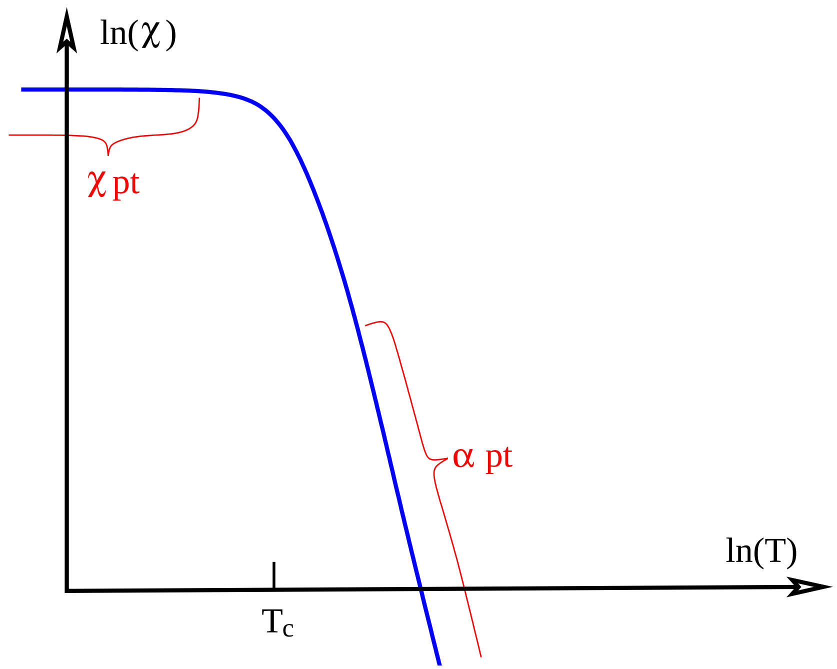
φ, therefore θ_A, can evolve. What value is (free) energetically preferred? $W = \Omega V_{\mathrm{eff}}(\varphi) = -T\ln(Z_{\mathrm{Eucl}})$, so

$$\begin{aligned} V_{\mathrm{eff}}(\theta_A) &= -\frac{T}{\Omega}\ln\int\mathcal{D}(A_\mu\bar{\psi}\psi)\,\mathrm{Det}\,(\not{D}+m)e^{-\int\frac{F^2}{4g^2}}\times e^{i(\Theta+\theta_A)\int\frac{F\tilde{F}}{32\pi^2}} \\ &\simeq \chi(T)(1-\cos[\Theta+\theta_A])\,, \\ \chi(T) &= \left\langle \int d^4x\frac{F\tilde{F}(x)}{32\pi^2}\,\frac{F\tilde{F}(0)}{32\pi^2}\right\rangle_\beta \end{aligned}$$

Nontrivial $\Theta+\theta_A$ ($\mathbf{T}$-violation) $\to$ phase cancellation V_{eff} minimized when $\Theta_{\mathrm{eff}}=0\to\mathbf{T}$ valid.

Peccei Quinn PRL 38, 1440 (1977);
J. E. Kim, PRL 43, 103 (1979); Shifman Vainshtein and Zakharov, NPB 166, 493 (1980)

$\chi(T)$: what we expect



Low T: χ-pt works.

$\chi \simeq \frac{m_u m_d}{(m_u+m_d)^2} m_\pi^2 f_\pi^2$

Hi T: standard

pert-thy works(??)

Low T: $\chi(T \ll T_c) = (76 \pm 1\,\mathrm{MeV})^4$ Cortona *et al*, arXiv:1511.02867

High T: $\chi(T \gg T_c) \propto T^{-8}$ Gross Pisarski Yaffe Rev.Mod.Phys.53,43(1981)

but with much larger errors.

Summary of the field theory stuff:

- Global U(1) symmetry, spontaneously broken.
  Goldstone mode θ_A is an angle $\theta_A \in [0, 2\pi]$

- Anomalous interaction with QCD: θ_A *is* QCD Θ-angle

$$-\mathcal{L} = \frac{1}{2f_a^2}\partial_\mu\theta_A\partial^\mu\theta_A + (\theta_A/32\pi^2)F^a_{\mu\nu}\tilde{F}^{\mu\nu}_a$$

- QCD physics explicitly breaks U(1), gives potential
  $V(\theta_A) = \chi(T)(1 - \cos\theta_A)$

- Potential is strongly T-dependent, vanishes at high-T

Initial state of φ field?

most likely: **randomly different in different places!**

- Inflation stretches quantum fluctuations to classical ones: $\Delta\varphi \sim H_{\mathrm{infl.}}$. If $N_{\mathrm{efolds}} H^2 > f_a^2$, scambles field.

  If not: need $H < 10^{-5} f_a$ to avoid excess "isocurvature" fluctuations in axion field

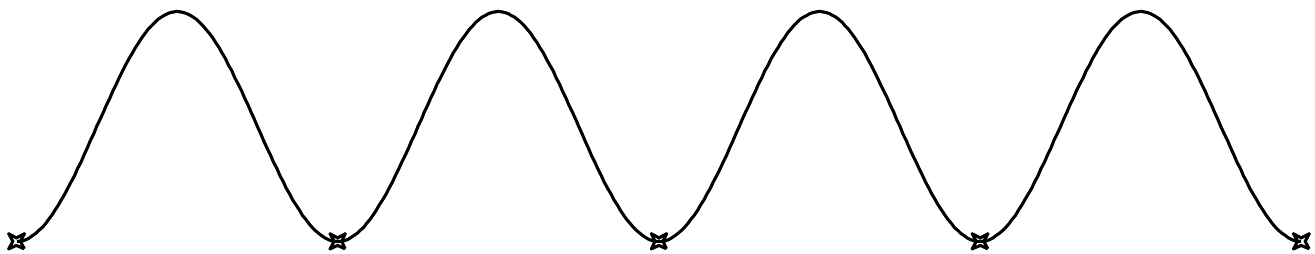
- Gets scrambled *after* inflation if Universe was ever really hot $T > f_a \sim 10^{11}$ GeV.

Random starting conditions for field breaking a global U(1) symmetry: expect a network of cosmic strings!

Network dynamics with explicit breaking

Spontaneously-broken U(1) $\implies$ Global cosmic strings

Small explicit breaking due to θ_A potential $\chi(T)(1 - \cos\theta_A)$



gives rise to domain walls as θ_A goes from 0 to 2π

One domain wall attached to each string

Domain wall tension increases as t rises, T falls

Walls pull together strings, leaving only fluctuations

Just what you see (local 2D region)

Just what you see (global network).

Why should I care?

Network collapses around $T = 500$ MeV into oscillations

Oscillations mildly nonrelativistic at formation

Quickly become matter – cold dark matter

Depending on

- Details of network breakup dynamics, axion production
- Parameters of model: f_a

could account for the Dark Matter of the Universe.

What if I rescale f_a?

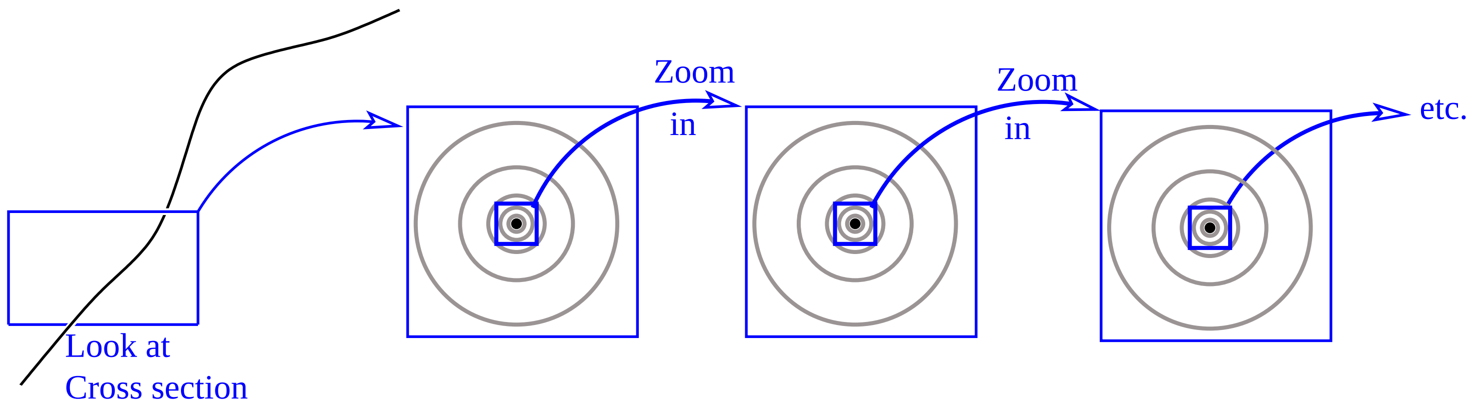
- String tension increases as f_a^2, but
- Inter-string forces and string radiation increases as f_a^2
- Domain wall tension rises as $f_a\sqrt{\chi(T)}$ linear in f_a

time for network collapse changes.
Scaling that *out*: conformally, same dynamics.

Total axion production $\rho_{\rm ax} \propto f_a^{7/8}$.
Solve dynamics (once): known $\rho_{\rm DM}$ *predicts* f_a, m_a!

m_h to m_a ratio does **not** scale out

$$E_{\rm str} = \int dz \int d\phi \int r\,dr \left(\nabla\phi^*\nabla\phi \simeq f_a^2/2r^2\right) \simeq \pi \ell f_a^2 \int_{m_h^{-1}}^{\sim H^{-1}} \frac{r\;dr}{r^2}$$



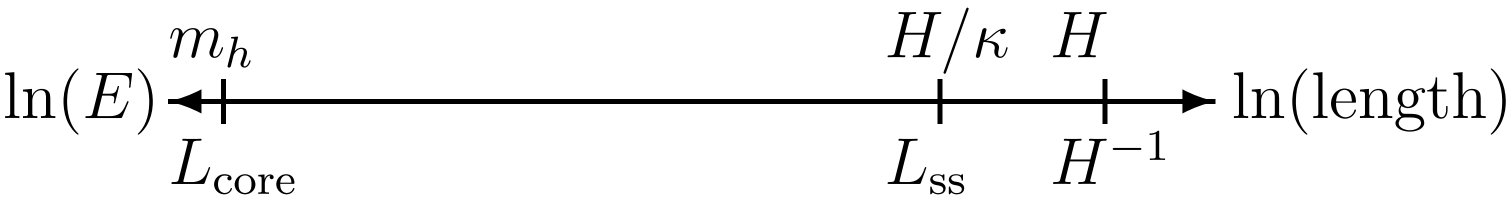
Series of "sheaths" around string:

equal energy in each $\times 2$ scale.

Assume $m_h \sim f_a$ and $H \sim T^2/m_{pl}$: $m_h/H \sim 10^{30}$.

Log-large string tension $T_{\rm str} = \pi f_a^2 \ln(10^{30}) \equiv \pi f_a^2 \kappa$

A three-scale problem!



Strings can radiate! But radiation suppressed, relative to tension, by a factor of $\kappa = \ln(m_h/H) \sim 70$. 3 scales:

- UV: Higgs-mass or core size scale (m_h or L_{core})
- IR: Inter-string spacing, axion mass (H or H^{-1})
- In between H^{-1}/κ: scale of string structure (L_{ss})

Getting string tension correct MATTERS!

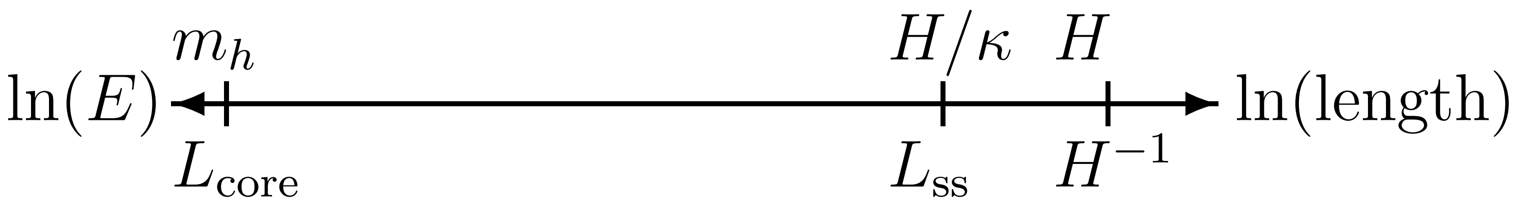
String dynamics are controlled by:

- String tension and inertia: $\propto \kappa\pi f_a^2$ FACTOR of κ
- String radiation, inter-string interactions, domain walls: $\propto \pi f_a^2$ NO factor of κ

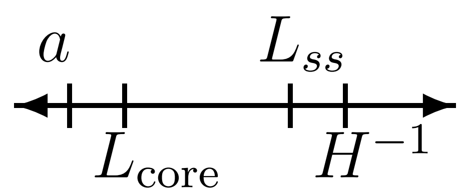
Large κ: denser network. Larger m_a/H before walls collapse network. Physically different – no rescaling cures this!

We really need to get this physics right!

"Standard" simulations get it wrong!



Above: what we want.
Below: what we get.



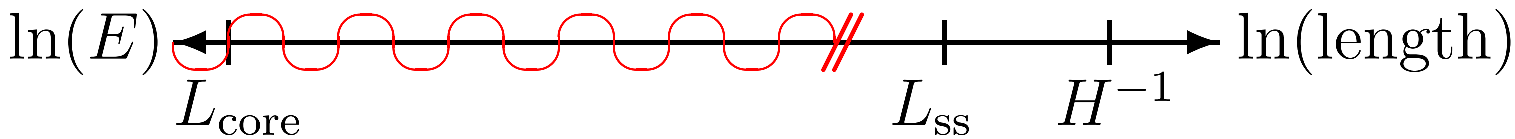
Numerics require: $a \leq m_h^{-1} \ll L_{\mathrm{ss}} < H^{-1} < Na$

Hierarchy is missing: $\kappa \equiv \ln(m_h/H)$ factor-10 too small.

Radiation factor-10 larger relative to string tension

String structure is missing, network behavior is changed.

An effective description



Scales from m_h to just-under $L_{\rm ss}$ are boring:

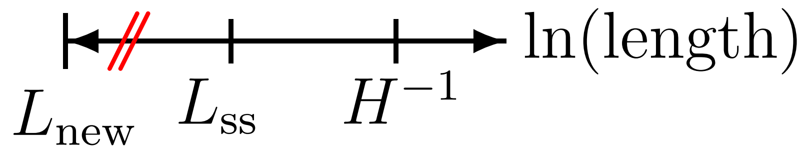
Just straight global string with log-scale tension.

Integrate them out *a la* **Abholkar and Quashnock**:

- Strings, tension $\mathbf{T} = \kappa' \pi f_a^2$, $\kappa' = \ln(m_h/m_{\rm reg})$
- Goldstone θ_A fields throughout space
- Kalb-Ramond coupling between string,Goldstone:

$$\mathcal{L} = \mathcal{L}_{NG} + f_a^2 \partial_\mu \theta_A \partial^\mu \theta_A / 2 + \mathcal{L}_{KR}$$

How does that help?



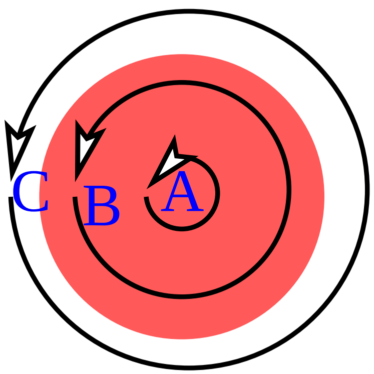
Put *some* other physics at scale $L_{\text{new}} \ll L_{\text{ss}}$, which generates *high-tension* strings + KR-coupling

Approaches right physics at L_{ss} up to power-suppressed corrections $(L_{\text{new}}/L_{\text{ss}})^P$ probably $P = 1$

Abelian Higgs Model: Tension-Only Strings

$$\mathcal{L}(\varphi, A_\mu) = \frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 + (D_\mu \varphi)^*(D^\mu \varphi) + \frac{\lambda}{8}\left(2\varphi^*\varphi - f_a^2\right)^2$$

with $D_\mu = \partial_\mu - ieA_\mu$ covariant derivative



$$\oint \partial_\phi \varphi \, d\phi = 2\pi f_a \quad \text{but}$$

$$\oint D_\phi \varphi \, d\phi = (2\pi - B_{\rm encl}) f_a$$

A: full $\nabla\varphi$ energy.

B: partial. C: cancels.

Outside string, B compensates $\nabla\varphi$.

Finite tension $T \simeq \pi f_a^2$. No long-range interactions.

Trick: global strings, local cores

Hybrid theory with A_μ and two scalars

$$\begin{aligned}\mathcal{L}(\varphi_1, \varphi_2, A_\mu) \;=\; & \frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 \\ & + \frac{\lambda}{8}\left[(2\varphi_1^*\varphi_1 - f^2)^2 + (2\varphi_2^*\varphi_2 - f^2)^2\right] \\ & + |(\partial_\mu - iq_1 e A_\mu)\varphi_1|^2 + |(\partial_\mu - iq_2 e A_\mu)\varphi_2|^2\end{aligned}$$

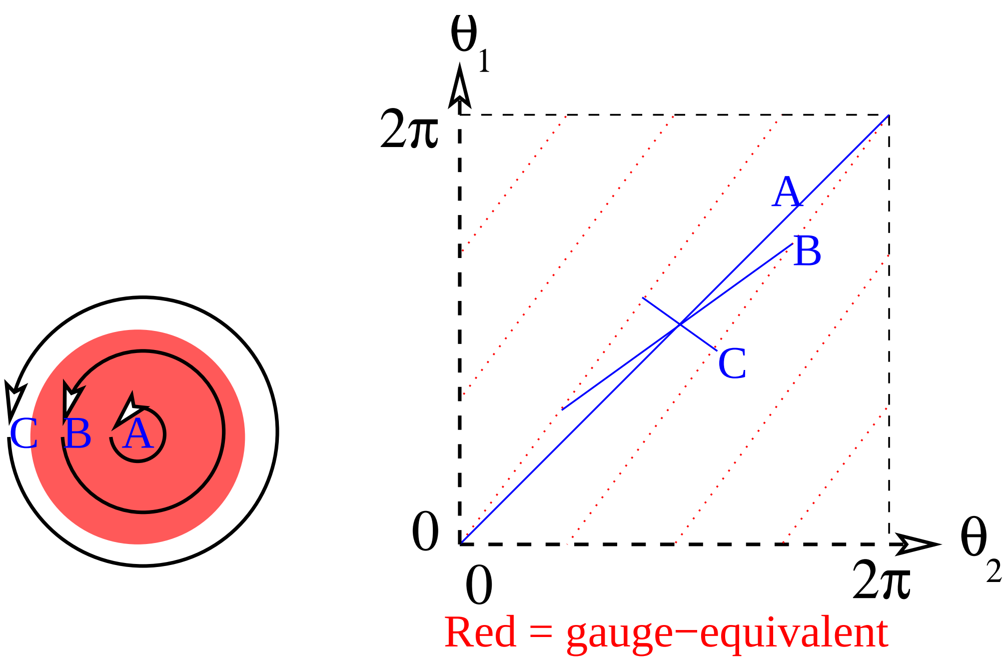
Pick $q_1 \neq q_2$, say, $q_1 = 4$, $q_2 = 3$.

Two rotation symmetries, $\varphi_1 \to e^{i\theta_1}\varphi_1$, $\varphi_2 \to e^{i\theta_2}\varphi_2$

$q_1\theta_1 + q_2\theta_2$ gauged, $q_2\theta_1 - q_1\theta_2$ global (Axion)

Two scalars, one gauge field

String where *each* scalar winds by 2π:



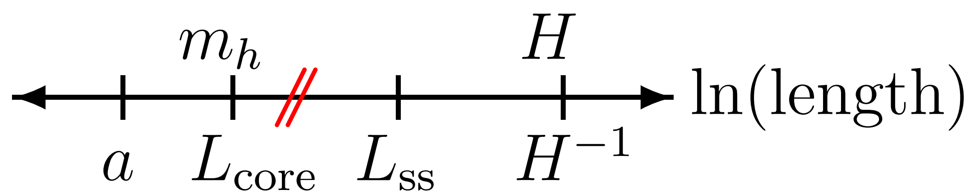
B-field *almost* compensates gradients outside string. $f_a^2 = f^2/(q_1^2+q_2^2)$.

$$T \simeq 2\pi f^2\,, \quad \frac{dF}{dl} = \frac{f^2}{(q_1^2+q_2^2)r}\,, \quad \kappa_{\rm eff} = 2(q_1^2+q_2^2)\,.$$

Two scalars, one gauge field

- Strings have Abelian-Higgs core $\rightarrow$ **Tension**
- Outside core: $q_1\theta_2 - q_2\theta_1 =$ **Axions**
- Ratio of tension to f_a tunable:
  can get string tension right!
- Bad news: extra (very heavy) DOF
  - $*$ Can change string interactions, cusps
  - $*$ Can propagate off strings

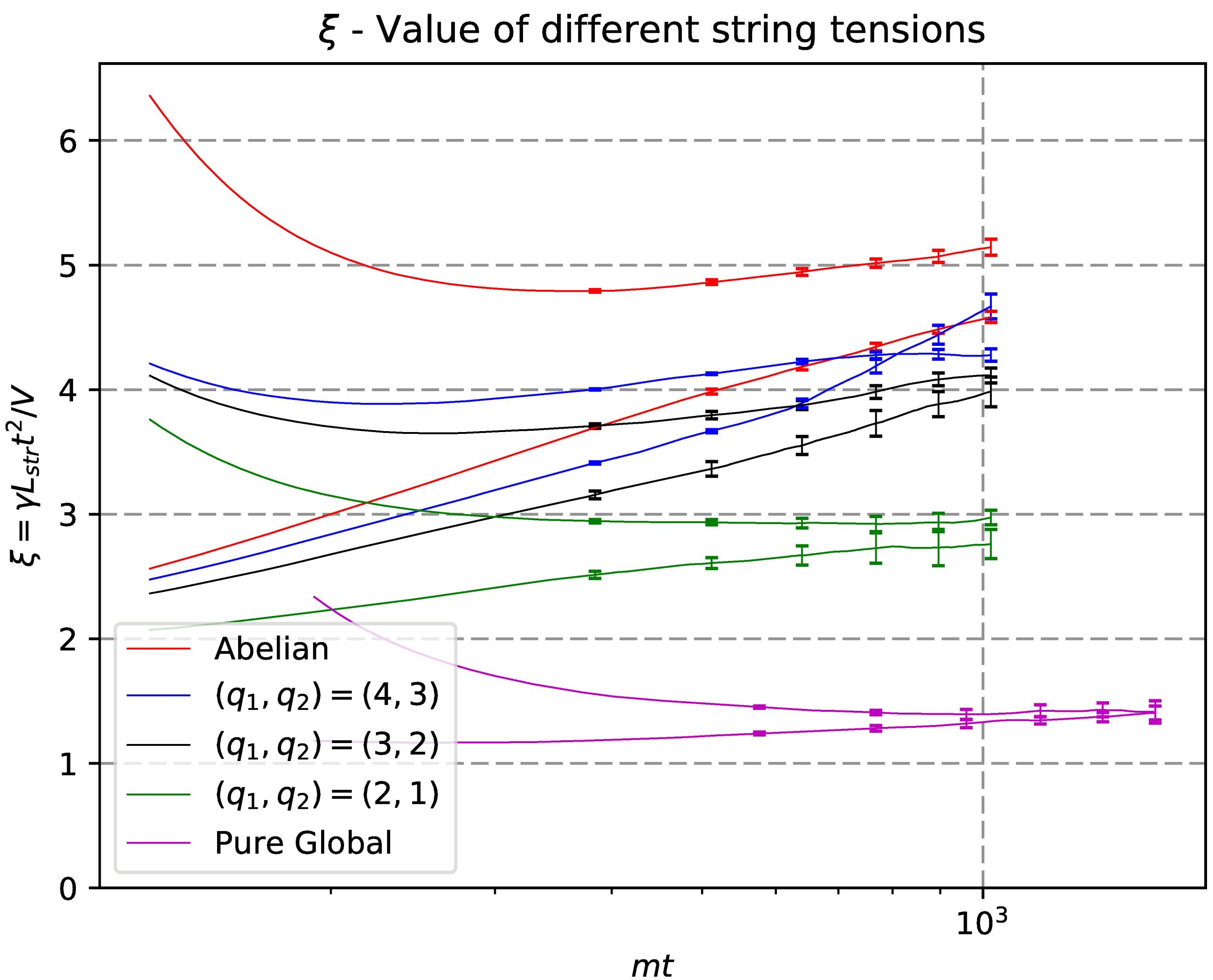
Scales in this model



Lattice spacing $a \ll m_h^{-1}$ micro-scale $\ll L_{\text{ss}} \ll H^{-1}$
and H^{-1} must fit in the box.
But this *should* work for 2048^3 boxes...

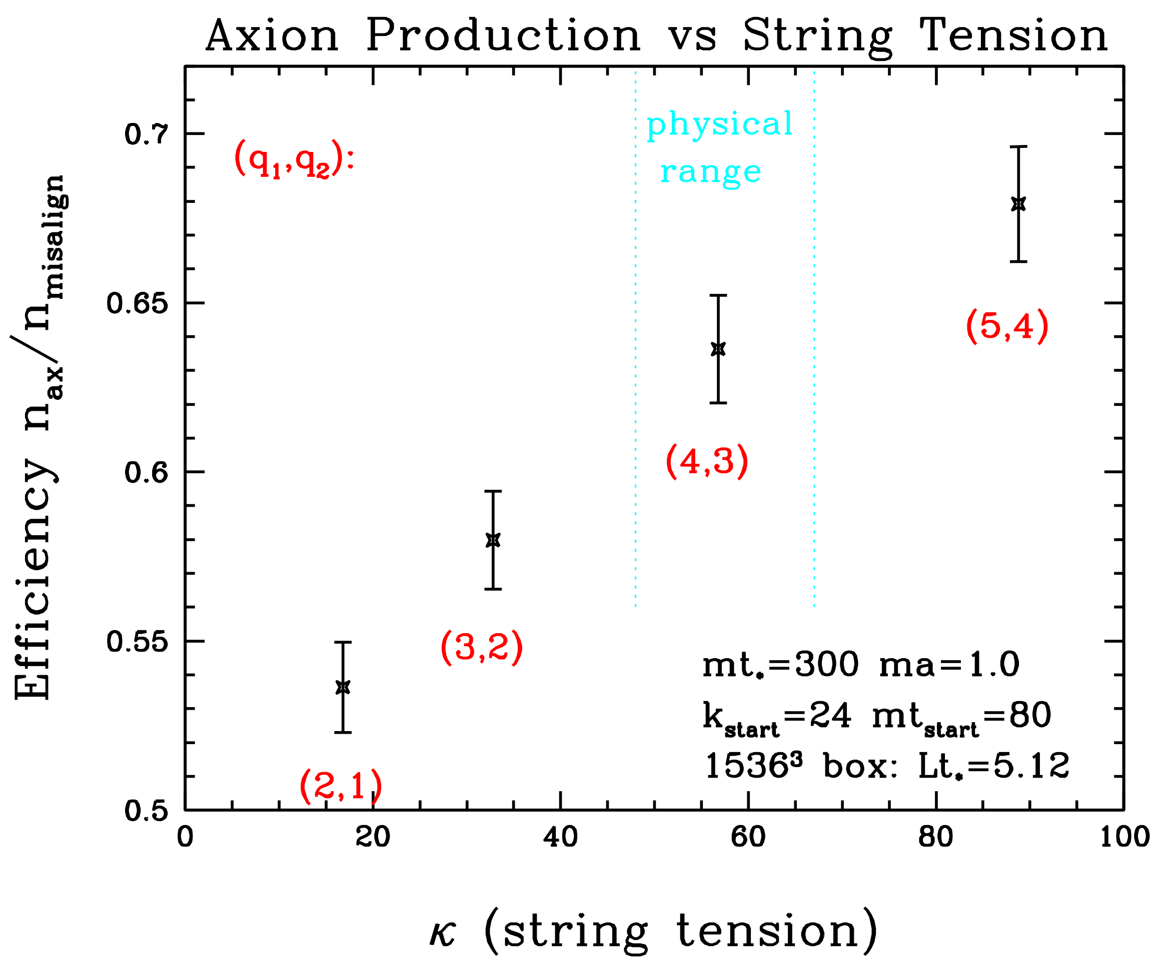
Need to systematically explore $m_h a \to 0$ and $m_h L_{\text{ss}} \to \infty$ limits.

String networks (no tilting)



Higher tension $=$ higher initial density, longer lasting, hardier loops

Results



Axions produced vary mildly with increasing string tension

Results

- $10\times$ string tension leads so $3\times$ network density but
- only 30% more axions than with axion-only simulation,
- **Fewer** (78%) axions than $\theta_{A\,\mathrm{init}}$-averaged misalignment
- Axionic string networks are *very bad* at making axions
- Results in less axion production.
  Must be compensated by ligher axion mass.

Put it all together

Axion production: $n_{\rm ax}(T=T_*) = (13 \pm 2) H(T_*) f_a^2$

Hubble law: $H^2 = \dfrac{8\pi\varepsilon}{3m_{\rm pl}^2}\,,$

Equation of state: $\varepsilon = \dfrac{\pi^2 T^4 g_*}{30}\,, \quad s = \dfrac{4\varepsilon}{3T}\,, \quad g_*(1{\rm GeV}) \simeq 73$

Susceptibility: $\chi(T) \simeq \left(\dfrac{1\,{\rm GeV}}{T}\right)^{7.6} (1.02(35) \times 10^{-11}{\rm GeV}^4)$

Dark matter: $\dfrac{\rho}{s} = 0.39\,{\rm eV}$

One finds $T_* = 1.54\,{\rm GeV}$ and $m_a = 26.2 \pm 3.4\,\mu{\rm eV}$

Conclusions

- Axions: well motivated physics needs global strings
- Global strings: 3 scales. "Standard" methods fail
- Effective description below L_{ss}
- Abelian 2-Higgs: Axion model with tuneable tension
- Higher string density, longer lived,
  but almost same axion production
- *Prediction:* **if** DM=Axions, **then** $m_a = 26.2 \pm 3.4\,\mu\mathrm{eV}$.

How to look for axions

Generally axion also couples to ordinary electromagnetism

$$\mathcal{L} = \ldots + \frac{\theta_A}{32\pi^2} F^{\mu\nu}_{\mathrm{QCD}} \tilde{F}^{\mu\nu}_{\mathrm{QCD}} + \frac{K\,\theta_A}{32\pi^2} F^{\mu\nu}_{\mathrm{EM}} \tilde{F}^{\mu\nu}_{\mathrm{EM}}$$

Since θ_A varies with time,

$$\begin{aligned} J^\nu &= \partial_\mu F^{\mu\nu} + \partial_\mu \left(\frac{K\,\theta_A}{8\pi^2} \epsilon^{\mu\nu\alpha\beta} \partial_\alpha A_\beta \right) \\ J^\nu &= \dot{E}_i + \nabla \times B_i + \frac{K\dot{\theta}_A}{8\pi^2} B_i \end{aligned}$$

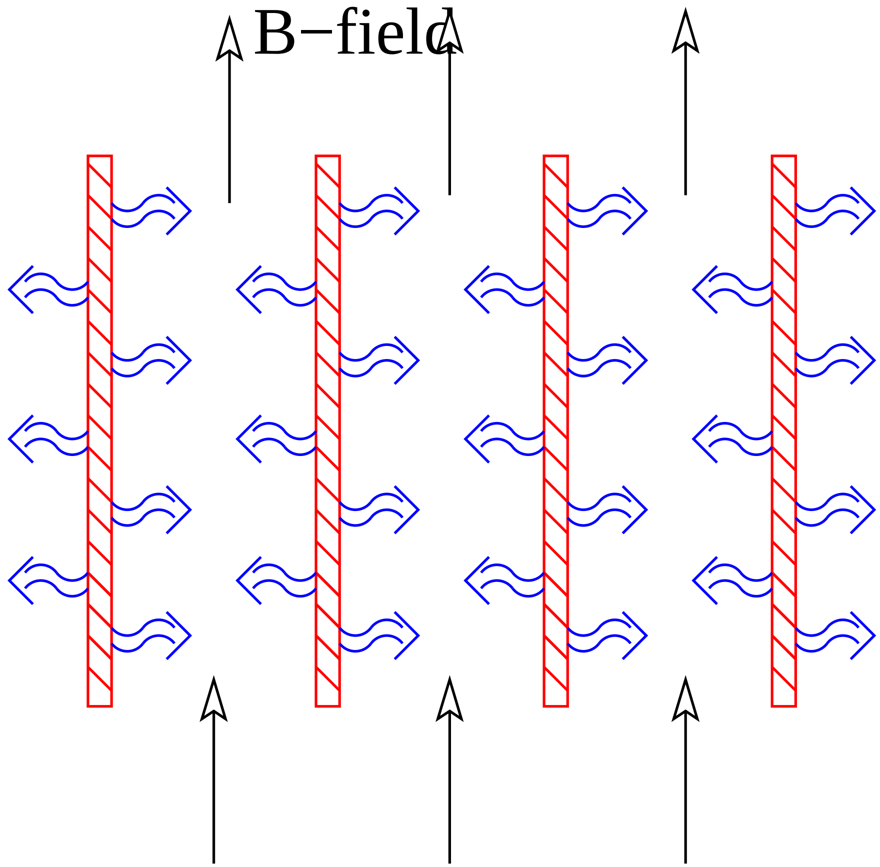
Axion turns B field into time-oscillating current!

MADMAX experiment Redondo et al 1611.05865

Spaced series of dielectrics, bathed in $\vec{B}$ field

Oscillating current along dielectric interface $\rightarrow$ microwave emission. Dielectric sheet spacing $\rightarrow$ constructive interference

$26\,\mu\mathrm{eV} \simeq 6\,\mathrm{Ghz} \simeq \lambda = 5\,\mathrm{cm}$



What about Anthropic Principle?

Trendy Explanation for “coincidences” or “tunings”

Why is Cosmological Constant so small?
If it were 100 times bigger, matter would fly apart or collapse before life could evolve. Nature plays dice, universes with all values occur, but only universes with life get observed.

Why does **QCD** respect $\mathbf{T}$ symmetry?
If **QCD** *violated* $\mathbf{T}$*, something would go wrong with nuclear physics, which would make life impossible. Nature plays dice, only universes where life is possible get observed.*
Except that life is fine in a world where $\Theta = 10^{-2}$!