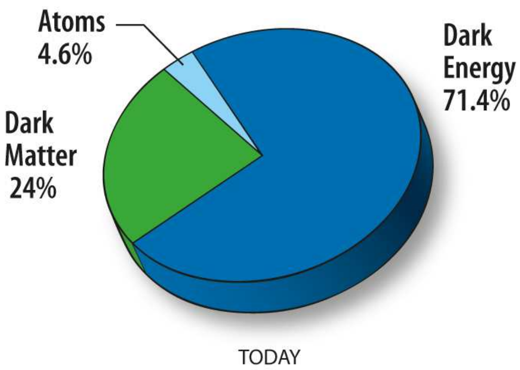
Axion: Mass Dark Matter Abundance Relation

- Mystery 1: Dark Matter
- Mystery 2: T-symmetry of QED and QCD
- The Axion: a solution to both mysteries?
- Early Universe cosmology of the axion
- How to predict the axion mass if it's the dark matter

Dark Matter: a Cosmic Mystery



Atoms: Standard Model.

Dark Energy: Cosmological Constant.

Dark Matter: MYSTERY! NOT SM!

We only know 3 things about dark matter:

- It's **Matter**: gravitationally clumps.
- It's **Dark**: negligible electric charge, interactions too feeble to be detected except by gravity
- It's **Cold**: negligible pressure by redshift $z = 3000$

Another mystery: T-symmetry in QED and QCD

T symmetry: "when you run a movie backwards, the *microphysics* is correct."

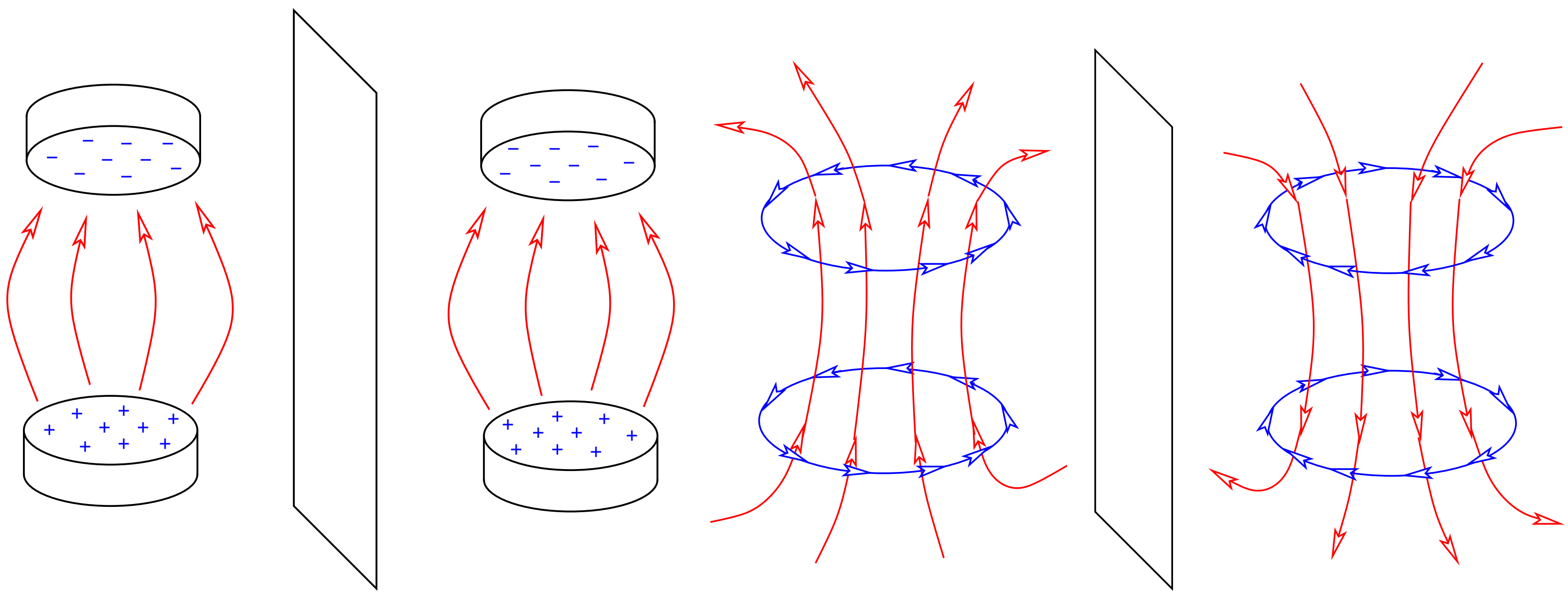
Statistical mechanics breaks T.
But microphysics very nearly obeys it!

Weak physics breaks T, but only through very small CKM effects. Observed in handful of experiments, all involving neutral meson oscillation.

No evidence for T viol in E&M or Strong interactions.

T in E&M

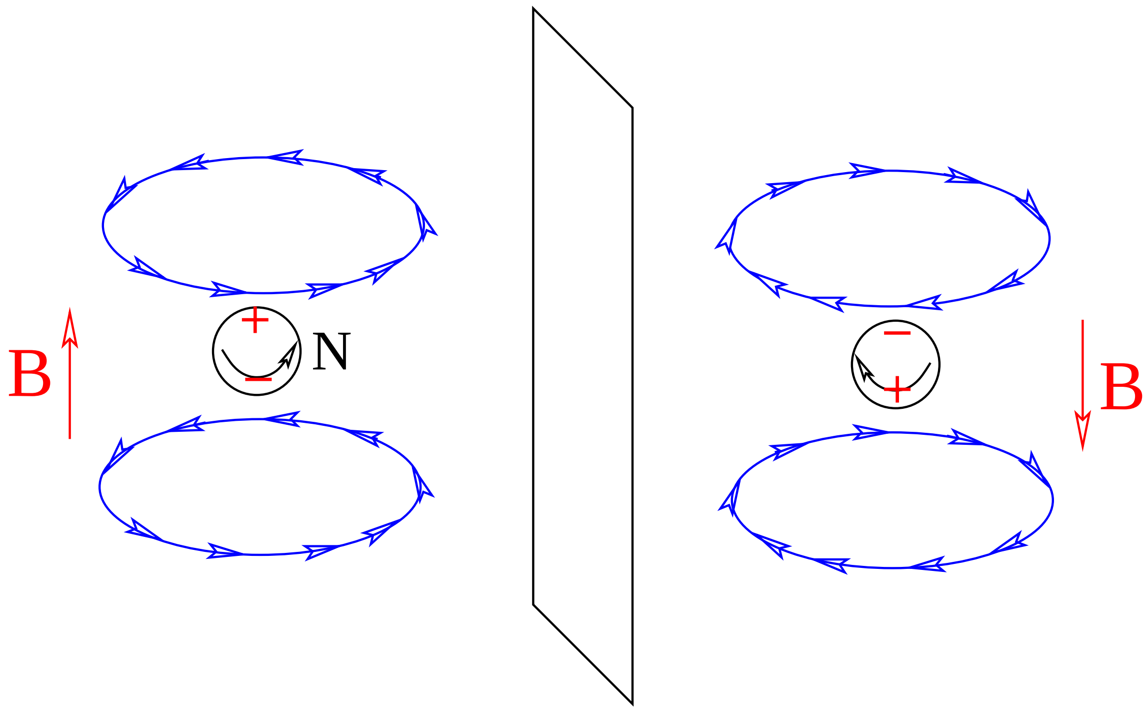
How do E, B fields change when you run movie backwards?



Q's unchanged, but J's flip. E same, but B flips.

Looking for T: Neutron EDM

Put neutron in $\vec{B}$ field – spin lines up with $\vec{B}$.



Is there an Electric Dipole Moment (EDM) aligned with spin?

If so: looks different when movie runs backwards, T viol!

T and the E&M Action

Action $S \Rightarrow$ all physics. Local field thy: $S = \int \mathcal{L}\, d^4x$.
$\mathcal{L}$ a singlet (gauge symm) and spacetime scalar (Lorentz):

$$\mathcal{L} = \frac{\vec{B}^2 - \vec{E}^2}{2e^2} + \frac{\Theta}{4\pi^2}\vec{E}\cdot\vec{B} + \text{(electrons...)}$$

T flip: $\vec{E} \to \vec{E}$ and $\vec{B} \to -\vec{B}$:
$(B^2 - E^2) \to (B^2 - E^2)$ **BUT** $E \cdot B \to -E \cdot B$.

$$\mathcal{L} \xrightarrow{T} \frac{\vec{B}^2 - \vec{E}^2}{2e^2} - \frac{\Theta}{4\pi^2}\vec{E}\cdot\vec{B} + \text{(electrons...)}$$

Nonvanishing Θ is a T violation!

E&M T violation is Illusory!

The $\Theta \vec{E} \cdot \vec{B}$ term has no *consequences*!

$$\vec{E} \cdot \vec{B} = \frac{1}{4} \epsilon_{\mu\nu\alpha\beta} F^{\mu\nu} F^{\alpha\beta} = \partial^\mu K_\mu \,, \quad K^\mu \equiv \frac{1}{2} \epsilon_{\mu\nu\alpha\beta} A^\nu F^{\alpha\beta}$$

I can integrate $\vec{E} \cdot \vec{B}$ to a boundary term.
Vanishes if $F^{\alpha\beta}$ vanishes on boundary. Alternately, EOM:

$$0 = \partial_\mu \left(\frac{1}{e^2} F^{\mu\nu} + \frac{\Theta}{8\pi^2} \epsilon_{\mu\nu\alpha\beta} \partial^\alpha A^\beta \right)$$

Second term zero by antisymmetry (**if** Θ constant)

QCD and its Lagrangian

QCD is like 8 copies of E&M, but with non-linearities:

$$\text{Field strength}: \quad F_a^{\mu\nu} = \partial^\mu A_a^\nu - \partial^\nu A_a^\mu + g f_{abc} A_b^\mu A_c^\nu \,,$$

g: coupling. $a = 1\ldots 8$. f_{abc} "structure constants"

$$S = \int dt \int d^3x \left(\frac{\vec{E}_a^2 - \vec{B}_a^2}{2g^2} + \frac{\Theta}{8\pi^2} \vec{E}_a \cdot \vec{B}_a \right)$$

where $\vec{E}_a \cdot \vec{B}_a$ still a total derivative:

$$\vec{E}_a \cdot \vec{B}_a = \partial^\mu K_\mu \,, \qquad 2K_\mu = \epsilon_{\mu\nu\alpha\beta} \left(A_a^\nu F_a^{\alpha\beta} + \frac{g f_{abc}}{3} A_a^\nu A_b^\alpha A_c^\beta \right)$$

Last term *need not* vanish on boundary even if $\vec{E}_a = 0 = \vec{B}_a$ there!
It's always $8\pi^2 N_I$ with N_I integer. So Θ mod 2π has *physical consequences*
G. 't Hooft, PRL 37, 8(1976); R. Jackiw and C. Rebbi, PRL 37, 172 (1976);

Gallan Dashen and Gross, Phys Lett 63B, 334 (1976)

Neutron EDM and Θ

Theory: Neutron electric dipole moment should exist,

$$d_n = -3.8 \times 10^{-16}\, e\,\mathrm{cm} \times \Theta$$

so long as Θ is not zero! Guo *et al*, arXiv:1502.02295, assumes Θ, modulo 2π, is small

Experiment: Consistent with zero! Baker *et al* (Grenoble), arXiv:hep-ex/0602020

$$|d_n| < 2.9 \times 10^{-26}\, e\,\mathrm{cm}$$

Either $|\Theta| < 10^{-10}$ by (coincidence? accident?) or there is something deep going on here.

Why an accident seems unlikely

- T is not a fundamental symmetry. We have observed its violation (Kaon, B-meson, possibly D-meson physics)
- More T violation almost surely out there – otherwise, tough to explain why Universe is filled with matter!
- Renormalization: T viol. one place finds its way into other places, including Θ. Θ does *not get smaller* as you go from high to low scales R.G. marginal – feels T viol from all scales.

Axion: an explanation for $\Theta = 0$

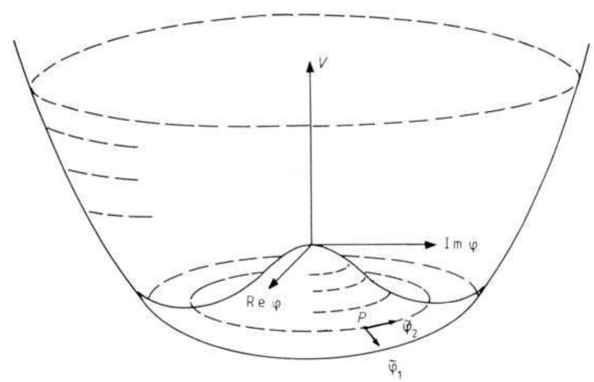
Hypothesize an extra *complex scalar* field $\varphi = \phi e^{i\theta_A}$

Assume a *symmetry*: $\varphi \to e^{i\theta}\varphi$ $[\theta_A = \theta_A + \theta]$. Lagrangian:

$$\mathcal{L} = g^{\mu\nu}\partial_\mu\varphi^*\partial_\nu\varphi + \lambda\left(\varphi^*\varphi - f_a^2/2\right)^2 \text{ (+interactions)}$$

Spontaneous symmetry breaking with VEV f_a.

We will want $f_a \sim 10^{11}$GeV.



Peccei Quinn PRL 38, 1440 (1977);

J. E. Kim, PRL 43, 103 (1979); Shifman Vainshtein and Zakharov, NPB 166, 493 (1980)

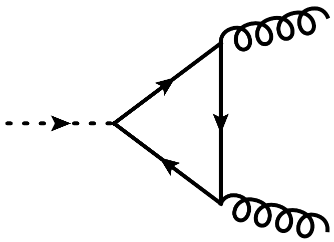
Axion: Indirect QCD coupling

φ is singlet under QCD.

But add some heavy physics which couples it indirectly

(such as heavy quarks which get mass due to $|\varphi|$)

$$\mathcal{L} = \ldots + \frac{\mathrm{Arg}\,\varphi}{8\pi^2} \vec{E}_a \cdot \vec{B}_a$$



Phase of $\varphi = f_a e^{i\theta_A}$ turns into part of $\bar{\Theta}$:

$$\bar{\Theta}_{\mathrm{tot}} = \Theta + \theta_A$$

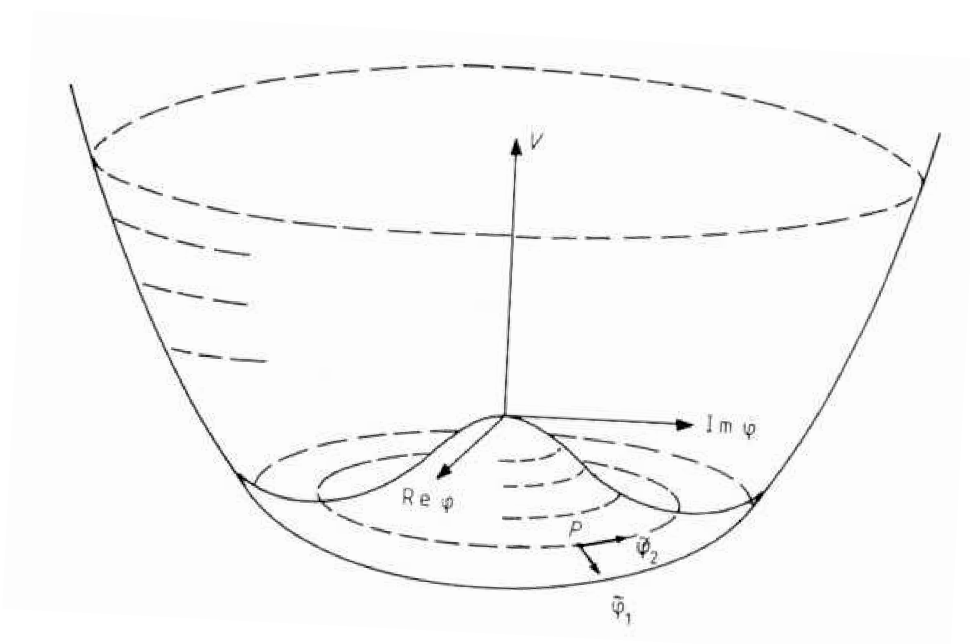
QCD cares about this total, not Θ alone.

QCD Chooses θ_A

Free energy $F = -T \ln Z$, and

$$Z = \int \mathcal{D}A_\mu e^{-S_{QCD} - S_{\text{axion}}} \;\times\; e^{-i(\Theta + \theta_A)\vec{E}_a \cdot \vec{B}_a / 8\pi^2}$$

Z reduced, F raised, by phase cancellations unless $\Theta + \theta_A = 0$:



Axion potential tilted

by phase cancelations in Z

Min. where $\Theta + \theta_A = 0$

$V_{\text{tilt}} = \chi(1 - \cos\theta_A)$.

Field finds T-symm point!

Axion and Instantons

Angular fluctuation θ_A is the axion.

Mass $m_a^2 = \chi / f_a^2$ with *topological susceptibility*

$$\chi(T) = \int_0^{1/T} dt_{\mathrm{Eucl.}} \int d^3x \left\langle \frac{E_a \cdot B_a(x)}{8\pi^2} \, \frac{E_a \cdot B_a(0)}{8\pi^2} \right\rangle \simeq \frac{\langle N_I^2 \rangle}{\beta V}$$

Strongly temperature dependent.

Determination of T dependence for $T \sim 1$GeV very important, but we'll leave that for another day.

Axion in cosmology

Assume first that φ starts homogeneous [inflation]

Classical axion field!

Starts oscillating around

$t = \pi m_a^{-1}$. Damped:

- Hubble drag
- effect of dm_a/dt

Acts Like Dark Matter!

Modern axion energy density larger if field starts oscillating later: $\varepsilon_{\rm axion} \propto f_a^{0.84} \propto m_a^{-0.84}$.

Initial state of φ field?

Maybe it's the same everywhere in space.

But most likely, it's randomly different in different places!

- Inflation stretches quantum fluctuations to classical ones: $\Delta\varphi \sim H_{\mathrm{infl.}}$. If $N_{\mathrm{efolds}} H^2 > f_a^2$, scambles field.

  If not: need $H < 10^{-5} f_a$ to avoid excess "isocurvature" fluctuations in axion field

- Gets scrambled *after* inflation if Universe was ever really hot $T > f_a \sim 10^{11}$ GeV.

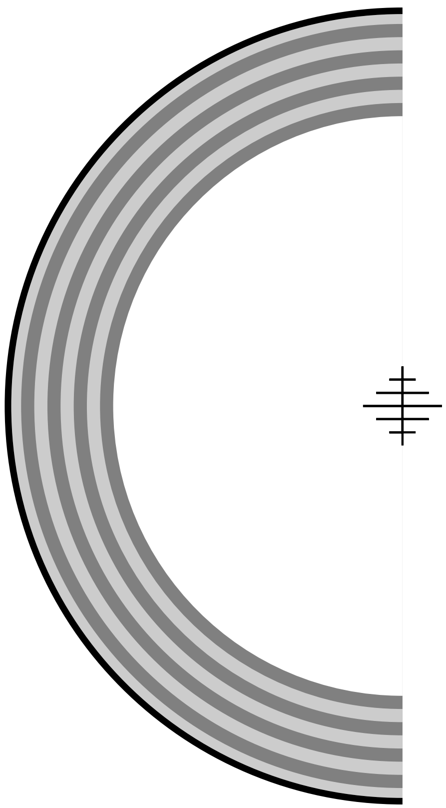
Predictive: *should be able* to predict DM abundance from m_a

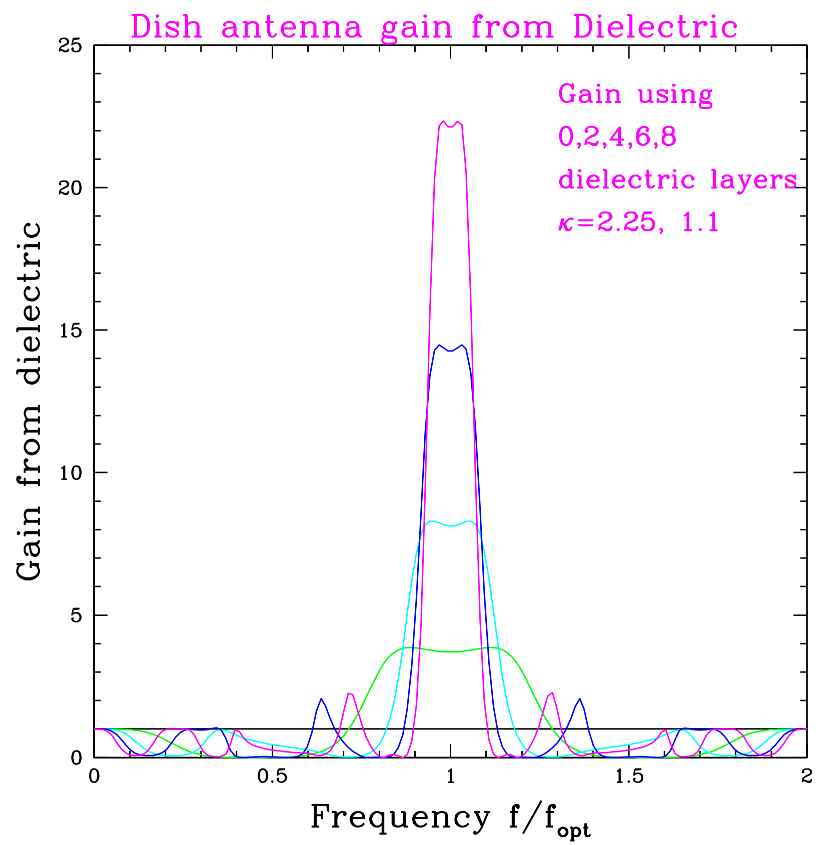
L. Visinelli and P. Gondolo, PRL 103, 011802 (2014)

Axion mass from DM abundance

Assume Axion=DM. Should predict m_a. Nice, but also helpful

Add dielectric layers on dish antenna: improve gain in f-window:





My Goal

Predict relation between **Dark Matter Density** and **Axion Mass** assuming random starting angle.

Challenges:

1. $\chi(T)$: needs Lattice Gauge Thy. Open, subdominant problem
2. Axion field dynamics: classical but with large scale hierarchy $f_a/H \sim 10^{30} \gg 1$

I will explain 2., and show progress (no solution yet)

Solving space-inhomogeneous case

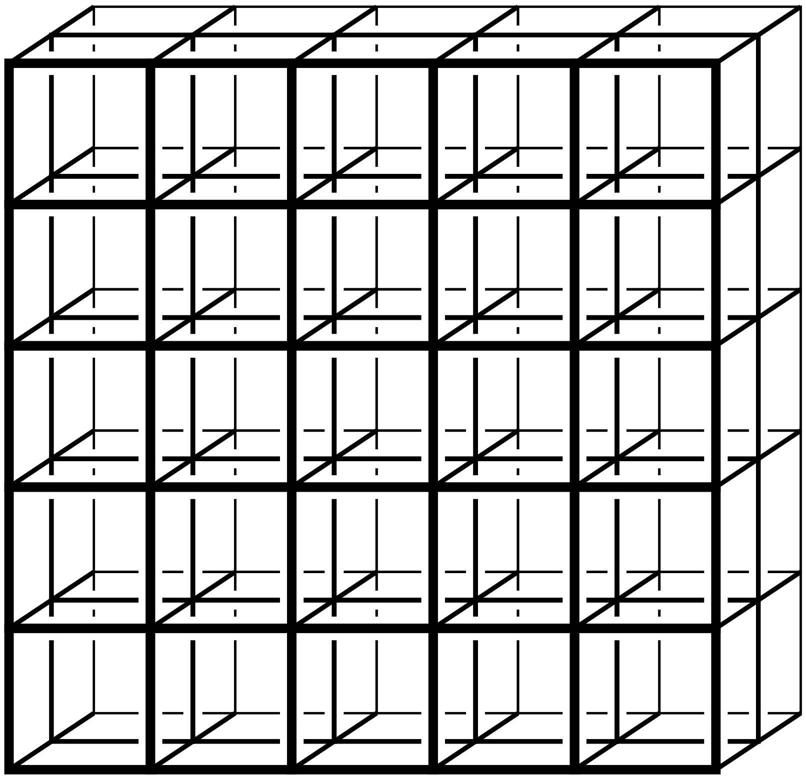
Fully nonlinear dynamics.

Nonperturbative approach.

Put it on a lattice,

Solve class. spacetime evol.

Easy enough to do!



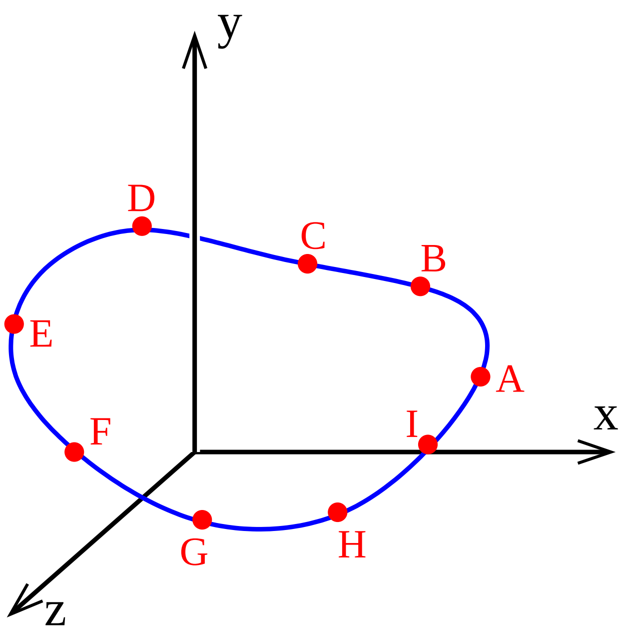
What do you see?

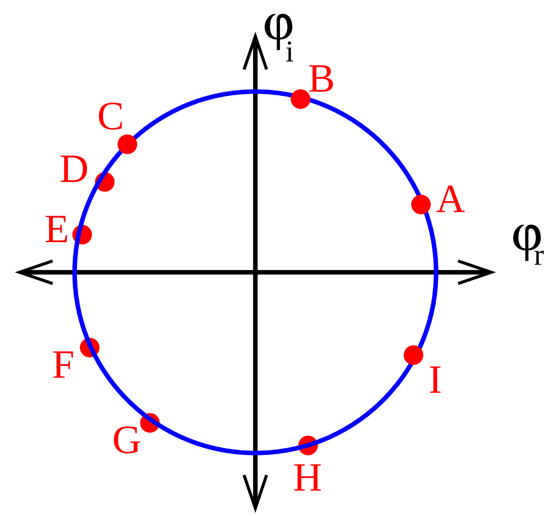
Energy density has string-like maxima, complex dynamics:

ε has stringlike structures: Annihilate when V tilts:

Axions and Topology I

Consider the value of φ around a circle in *space*:





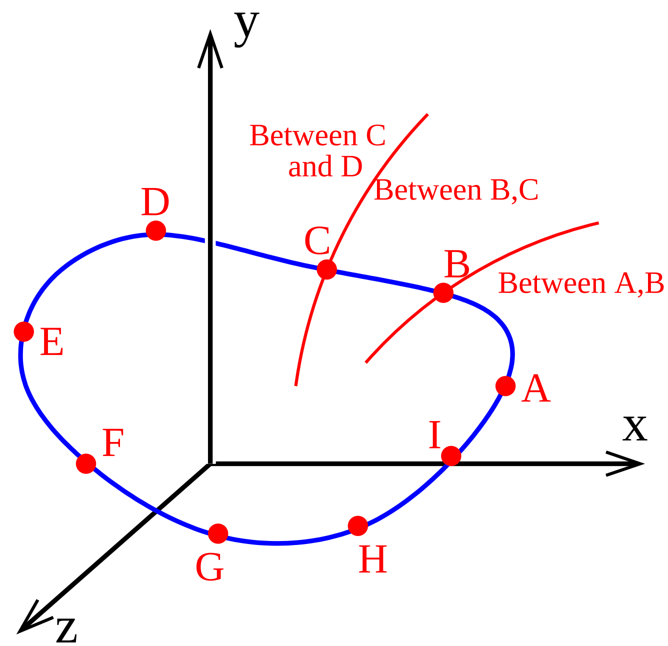
Each point maps to a point in field-space.

These may make a circuit around field-min. circle

Continuity $\rightarrow$ somewhere in middle, field "goes over top"

Axions and Topology II

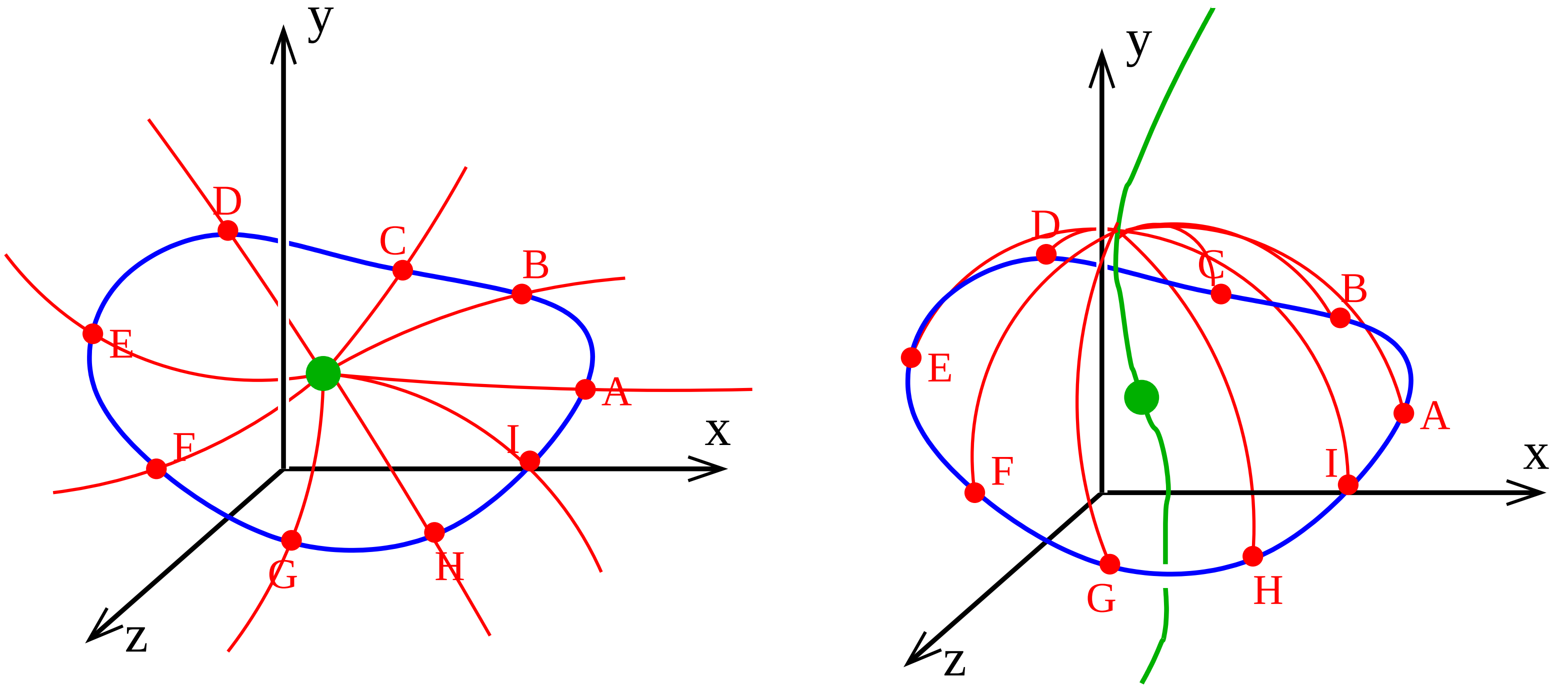
What are points *inside* our space-circle with the same field-values? Use continuity . . . .



These lines can only cross where field is discontinuous
Or where it "leaves the ring." Energy cost!
Minimize energy – have all lines meet at one point.

Axions, Topology III

Point where lines meet – field leaves min-energy circle



Think about any other surface bounded by the circle:

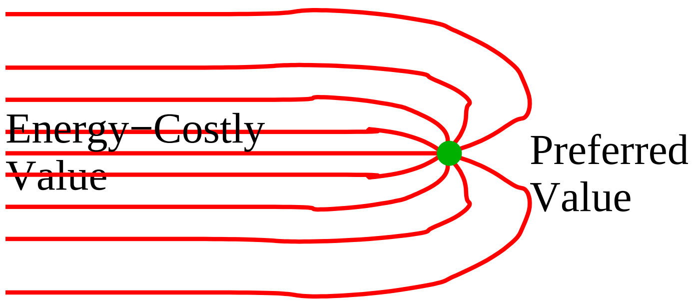
Consider locus of all such points.

φ leaves vacuum circle along a "string" (line)

Strings and Walls

Random starting conditions $\Rightarrow$ network of these "strings"
Network evolves, strings straighten out, find each other.

Potential tilts: $\theta_A \to 0$ where it can. Near string: explores all values. String has "domain wall" attached.



Pulls strings towards each other, speeds breakup of network.
Complex string – domain wall network dynamics.

This dynamics produces Axions!

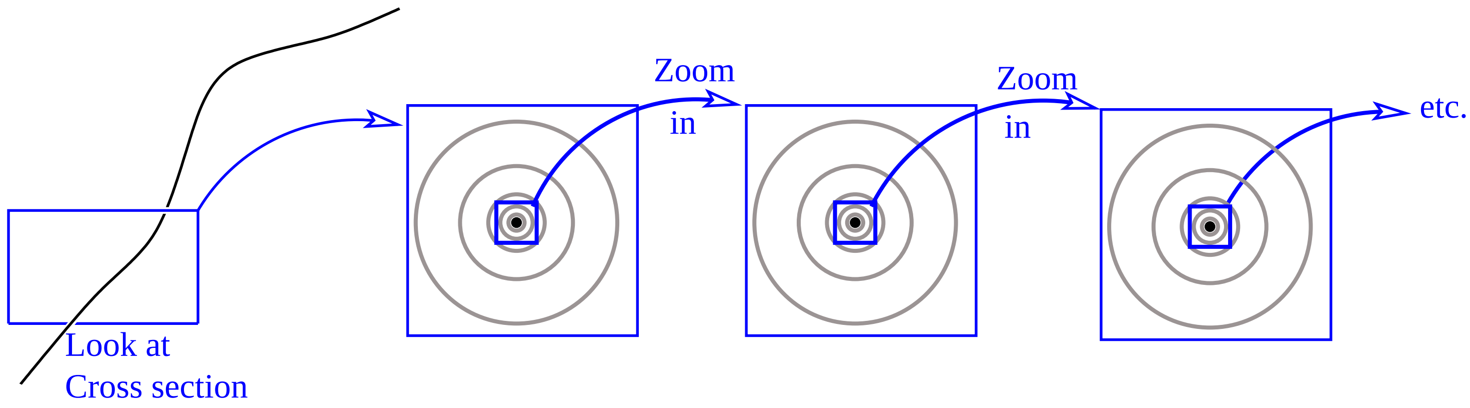
Big complication

These strings are like onions (or scallions)



Layers! Innermost "core" has radius $r \sim f_a^{-1} \sim 10^{-27}$m. Outer size $\sim H^{-1} \sim 10^2$m. Energy stored logarithmically with radius $\varepsilon \propto \int \frac{r dr}{r^2}$. Name $\ln(f_a/H) \equiv \kappa \sim 70$.

Layers of String Energy



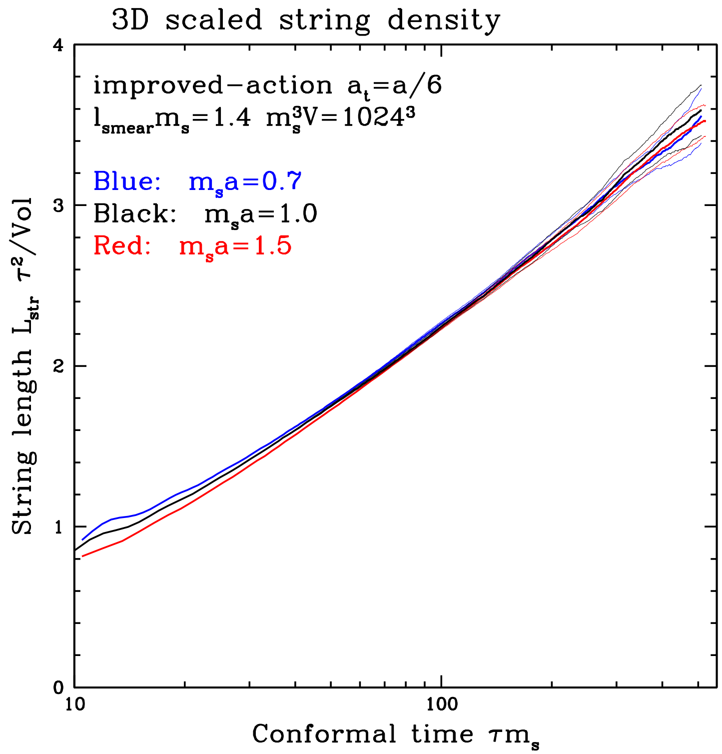
Series of "sheaths" around string:
equal energy in each $\times 2$ scale, 10^{30} scale range!

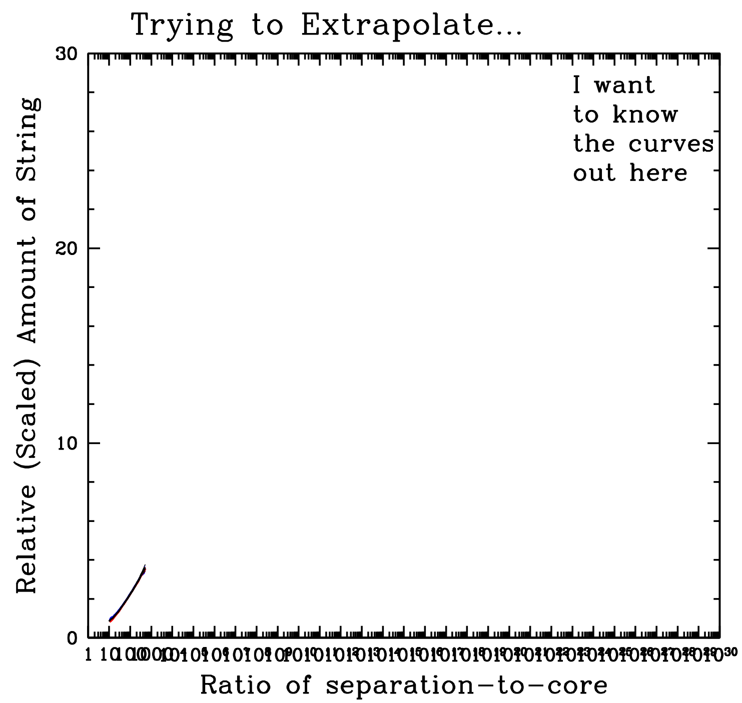
More "sheaths" $\rightarrow$ outermost layer less important:

- inter-string interactions less important
- Radiation from strings less efficient

Both "thin out" string network. Big scale separation $\rightarrow$ dense network

Does it matter?





Oh yes! Network density rises with $\kappa =$log(scale).

Achievable scales nowhere near physically interesting regime!

A solution...

Need to include (simple) short-dist. string-core physics:

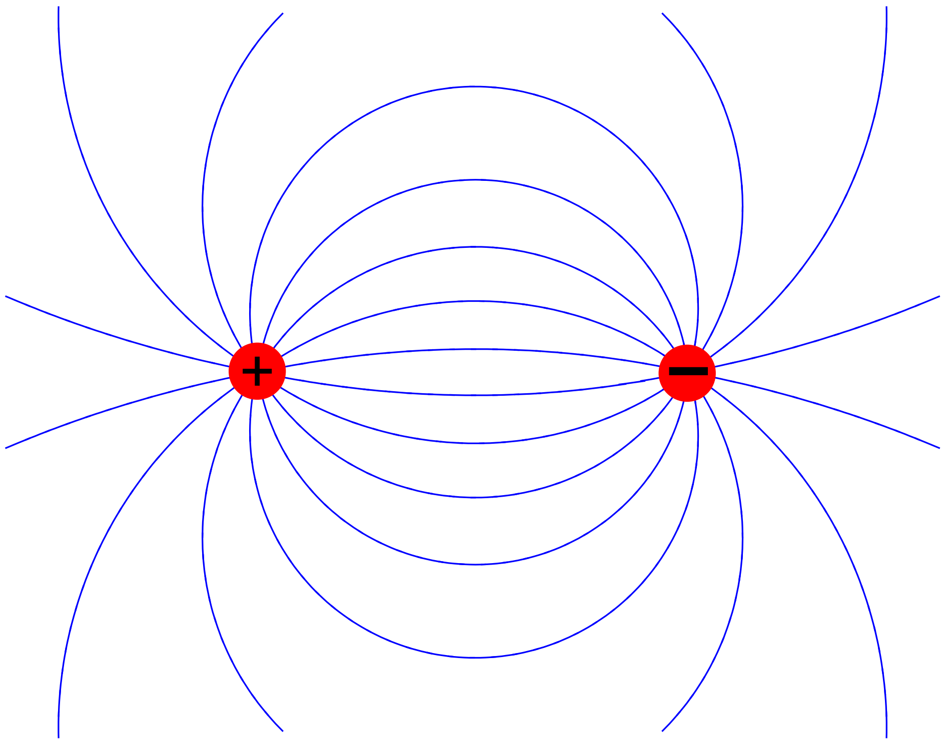
- "Cut out" the string cores (the short scale)
- Solve their physics explicitly – add explicit objects representing their effect to the lattice φ simulation
- "Sew together" the explicit cores and the φ fields to have the correct mutual interactions

Leaves only long-scale physics. No large scale ratio!

Sounds tricky. But 2+1D problem now solved!

Example: 2+1 dimensions

String is pointlike vortex. θ_A varies by $\pm 2\pi$ around.



Consider $+, -$ vortices:

Lines of constant θ_A

Look just like dipole

due to $+, -$ charges.

Actually identical!!!

constant-θ_A lines: $\vec{E}$-lines. $\partial_t \theta_A$: B (scalar in 2+1D)

2+1D: Electromagnetic Duality

Outside string core: $\varphi = f_a e^{i\theta_A}$, $\partial_\mu \partial^\mu \theta_A = 0$
Strict analogy to electromagnetism:

$$
\begin{aligned}
F^{\mu\nu} &\equiv \epsilon^{\mu\nu\alpha} \partial_\alpha \theta_A \\
\partial_\mu F^{\mu\nu} &= \epsilon^{\mu\nu\alpha} \partial_\mu \partial_\alpha \theta_A = 0 \\
\epsilon^{\alpha\mu\nu} \partial_\alpha F_{\mu\nu} &= -2 \partial_\alpha \partial^\alpha \theta_A = 0 \\
\oint \vec{E} \cdot d\hat{n} &= \oint \frac{d\theta_A}{dl} dl = 2\pi N_{\mathrm{wind}}
\end{aligned}
$$

replace strings with point-charges of charge $\pm 2\pi$
Short-dist: self-energy $\rightarrow$ mass, Lorentz force law

2+1D Axion Evolution

Discretize 2+1D E&M with smeared-charge Particle-in-Cell methods

Transform back to θ_A variables to include $V(\theta_A)$ tilt

String pairs bind off, inspiral, and annihilate . . . .

Strings: black squares

Walls: red lines

Tension turns on about halfway through

Strings annihilate inefficiently!

Persistent string “atoms”

It’s hard to radiate a massive field.

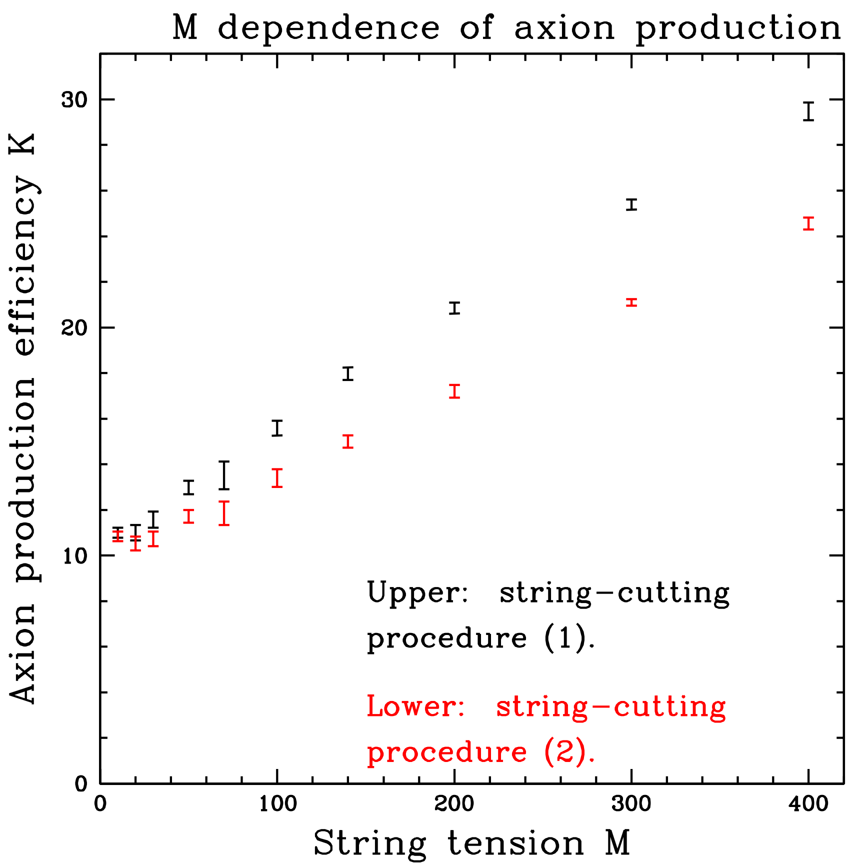
Harder if you are nonrelativistic (special to 2+1D)

Strings annihilate except for tightly bound $+, -$ “atoms”

Energy in axions $\gg$ energy in these “atoms,” but efficiency of atom $\rightarrow$ axion conversion not determined.

Either throw them out (lower bound) or turn all their energy into axions (upper bound)

Axion Production vs Scale Hierarchy



x-axis: $\pi\kappa$

y-axis: axion prod.

Linear dependence.

Axion number doubles

as go from fields-only

to physical hierarchy

Shown: dim'less efficiency. Need to scale in $\chi(T)$, f_a.

Results (2+1 Dim)

Axion prod. efficiency plus:

- $\chi(T)$ from Instanton Liquid model Shellard Wantz NPB829, 110 (2010),
- $\rho_{DM}/s = 0.39$eV from Planck Ade et al, Planck XIII
- Lattice EOS around $T \sim 1$GeV Borsanyi et al, PLB 730, 99 (2014)

gives DM density = observed density for
$f_a = 1.6 \times 10^{11}$GeV, $m_a = 36\mu$eV, $T_* = 1.72$GeV.

2+1D really $\neq$ 3+1D

Results show string tension matters! But 2+1D not 3+1D:

- 2+1D string $v^2 \simeq \kappa^{-1}$ nonrelativistic.
  3+1D strings always relativistic (string curvature)
- 2+1D string network density scales as $\kappa^{-3/2}$
  3+1D scales as κ^{-1}, then saturates $\rightarrow$ const.
- 2+1D: radiate mostly ultra-short wavelength axions
  3+1D expected to radiate mostly longer wavelengths
- 2+1D: when m_a turns on, radiation turns off
  3+1D: may happen (long-lived loops?)! But not clear!

3+1D Simulations?

Sure! $\partial_\mu \theta_A$ dual to Kalb-Ramond field

$$\begin{aligned} 6\partial_\mu \theta_A &= \epsilon_{\mu\nu\alpha\beta} H^{\nu\alpha\beta} \,, \\ \partial^\mu H_{\mu\alpha\beta}(x) &= j_{\alpha\beta}(x) = -2\pi \int d\sigma \delta^3(x - y(\sigma))(v_\alpha y'_\beta - y'_\alpha v_\beta) \end{aligned}$$

Force on string from θ_A field is

$$dF^\mu_{\mathrm{str}} = -2\pi F^{\mu\nu\alpha} v_\nu y'_\alpha = 2\pi f_a \epsilon^{\mu\nu\alpha\beta} v_\nu y'_\alpha \partial_\beta \theta_A$$

JUST need string update algorithm . . . it's coming

Conclusions

- Dark matter and T in QCD both mysteries
- Axion could explain both!
- Axion dynamics in early Universe: string defects
- Field-only simulations insufficient
- Simulations, explicit strings: 2+1D complete, 3+1D formulated
- I hope a tight prediction of the axion mass can be made!

Why is $\chi(T \sim 1\text{GeV})$ Hard?

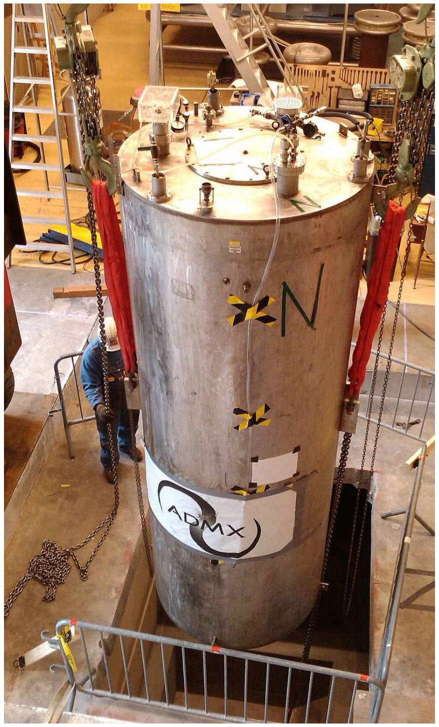
Lattice Monte-Carlo, find fraction of configs with instanton. But

- Instantons get rare fast, $\chi \sim T^{-7\ldots-8}$. Statistics??
- Slowing-down of algorithm to change N_I
- Instanton counting must have very low false-positive rate!

Chiral limit *not* a problem: $\chi \propto (m_u m_d m_s / T^3)$, if $m \ll T$.
Use $T \gg m \gg m_u$, multiply by $m_u m_d m_s / m^3$.

How do experimentalists look for Axions?

Most sensitive method: resonant cavity in magnetic field



Microwave cavity inside superconducting solenoid. $\vec{E}$ field of cavity mode aligned with $\vec{B}$ of solenoid. Cavity oscillation: oscillating $\vec{E} \cdot \vec{B}$. If cavity resonance matches $m_A/\hbar$: cavity resonance driven. Squid readout – tuneable cavity ...

What about Anthropic Principle?

Trendy Explanation for “coincidences” or “tunings”

Why is Cosmological Constant so small?
If it were 100 times bigger, matter would fly apart or collapse before life could evolve. Nature plays dice, universes with all values occur, but only universes with life get observed.

Why does **QCD** respect T symmetry?
If **QCD** *violated* T*, something would go wrong with nuclear physics, which would make life impossible. Nature plays dice, only universes where life is possible get observed.*
Except that life is fine in a world where $\Theta = 10^{-2}$!