Classical Yang-Mills Theory Cascade

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arXiv:1207.1663 (Monday)

- Why look at classical Yang-Mills theory?
- Cascade towards UV, scaling of momentum and occupancy
- Approach to a *scaling solution*
- Infrared effects: screening and magnetic screening
- How would condensates behave?

What I really want to study: Quantum YM theory at $\alpha_{\rm s} = 0.3$ with intense-field inhomogeneous expanding initial conditions

What I want to study: Classical YM theory $+$ quantum fluctuations with intense-field inhomo. expanding init. condit.

What I would like to study: Classical YM theory with intense-field expanding initial conditions

What I will study for now: Classical YM, intense-field but non-expanding.

What does classical YM do?

Most theories seek equilibrium.

Classical field thy. in continuum has no equilibrium.

Unlimited UV phase space. Equipartition: energy should move into UV *forever*

Start with $f \sim \frac{1}{g^2 N_c}$ for $p \lesssim Q$, f small for $p \gg Q$.

Typical momentum scale $p_{\max}$ grows, typical occupancy $\tilde{f}$ shrinks, with time

Let's *rigorously define* my scales Q and $p_{\rm max}$:

$$\varepsilon = 2(N_{\rm c}^2-1)\int \frac{k^2 dk}{2\pi^2} k\, f(k) \quad \text{and} \quad \varepsilon \sim \frac{Q^4}{g^2 N_{\rm c}}\,, \quad f \sim \frac{1}{g^2 N_{\rm c}}$$

so we define

$$\varepsilon = \frac{2(N_{\rm c}^2-1)}{2\pi^2 N_{\rm c} g^2} Q^4 \quad \text{or} \quad Q^4 \equiv \frac{2\pi^2 N_{\rm c} g^2 \varepsilon}{2(N_{\rm c}^2-1)}$$

so that, to the extent f is well defined,

$$Q^4 = \int k^3 (g^2 N_{\rm c}\, f(k)) dk$$

Also define "typical momentum scale now":

$$p_{\rm max}^2 \equiv \frac{\langle (\nabla \times \mathbf{B})^2 \rangle}{\frac{1}{2}\langle \mathbf{E}^2 + \mathbf{B}^2 \rangle} \qquad p_{\rm max}^2 \simeq \frac{\int k^6\, f(k)\, dk}{\int k^4\, f(k)\, dk}$$

Dynamics: Expect collision rate Γ order $\Gamma t \sim 1$.

Estimate $\Gamma \sim g^4 f^2 p_{\rm max}$. Two expressions:

$$g^4 f^2 p_{\rm max} t \sim 1\,, \qquad p_{\rm max}^4 g^2 f \sim Q^4 \text{ time independent}$$

Solving,

$$p_{\rm max} \sim Q(Qt)^{\frac{1}{7}}\,, \qquad f \sim \frac{1}{g^2 N_{\rm c}} (Qt)^{\frac{-4}{7}}$$

see Kurkela and GM arXiv:1107:5050, Blaizot *et al.* arXiv:1107:5296

What about particle number? $\Gamma_{\rm number\ chg} \sim g^4 f^2 p_{\rm max}$.
Number change could keep up – or there might be condensates??

Questions we want to ask

Do we observe expected $p_{\max} \simeq Q(Qt)^{\frac{1}{7}}$ scaling?

Does $f(p,t)$ approach *scaling solution*?

$$f(p,t) = (Qt)^{\frac{-4}{7}} \tilde{f}(p(Qt)^{\frac{-1}{7}}) \quad \text{Time-independent}$$

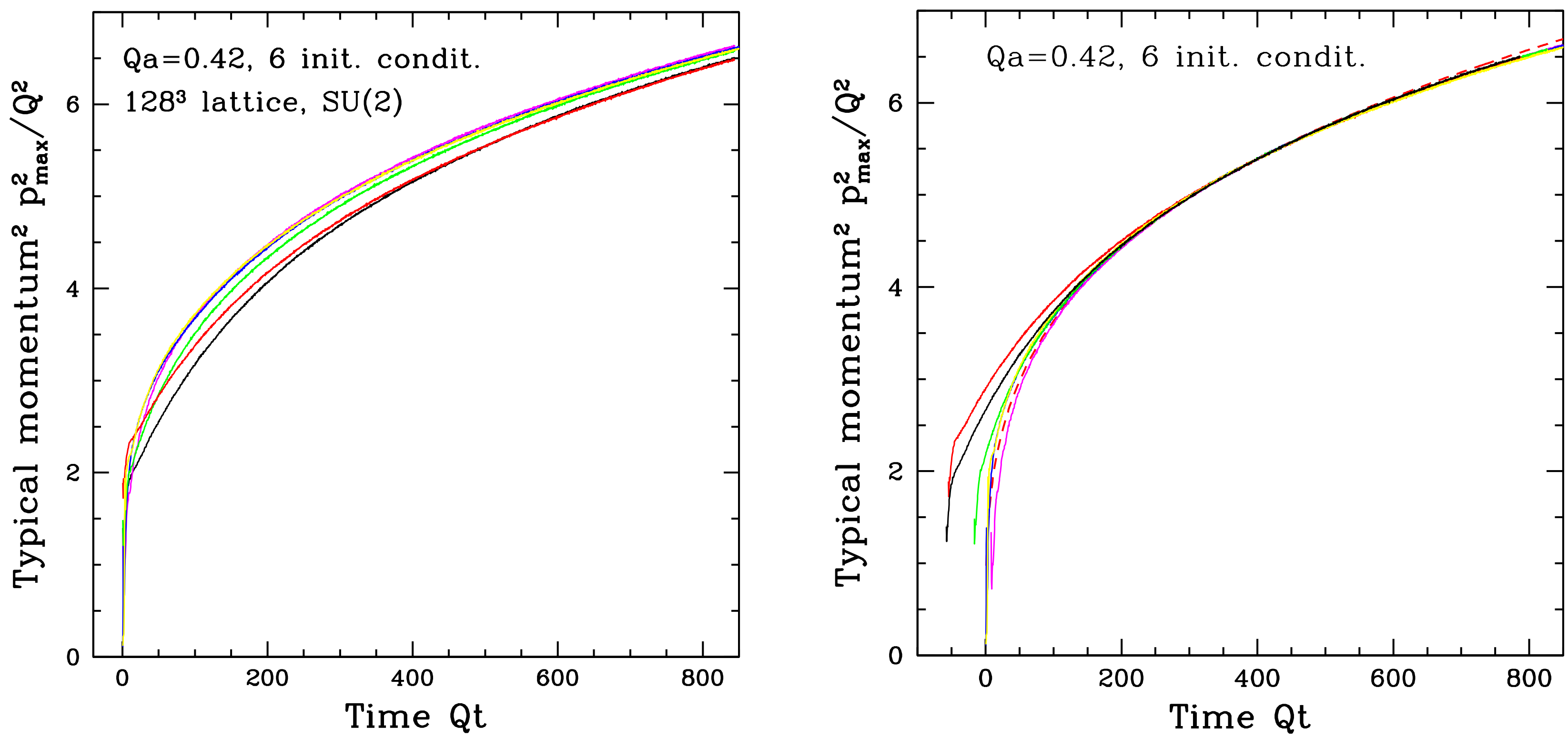
Behavior in infrared: $f \propto p^{-1}$, $f \propto p^{-\alpha}$ (4/3 or 3/2 or...)

Berges Schlichting Sexty

or is there a condensate? (larger IR occupancies than just power IR scaling)

If so, is it electric (plasmons) or magnetic?

Lattice study. gauge invar. measurables: $p^2_{\rm max}/Q^2$:



6 very different initial conditions converge, obey

$p_{\rm max} \sim Q(Qt)^{\frac{1}{7}}$

Occupancies? Fix to Coulomb gauge. Perturbatively,

$$\int d^3x\, e^{i\mathbf{p}\cdot\mathbf{x}} \langle A_a^i(x) A_b^j(0)\rangle = \frac{\delta_{ab}\mathcal{P}_{\rm T}^{ij}(\mathbf{p})}{|\mathbf{p}|} f(p)\,,$$

$$\int d^3x\, e^{i\mathbf{p}\cdot\mathbf{x}} \langle E_a^i(x) E_b^j(0)\rangle = \Big(\delta_{ab}\mathcal{P}_{\rm T}^{ij}(\mathbf{p})\,|\mathbf{p}|\Big) f(p)$$

(with $\mathcal{P}_{\rm T}^{ij} = \delta^{ij} - \hat{p}^i\hat{p}^j$) Then we could simply define:

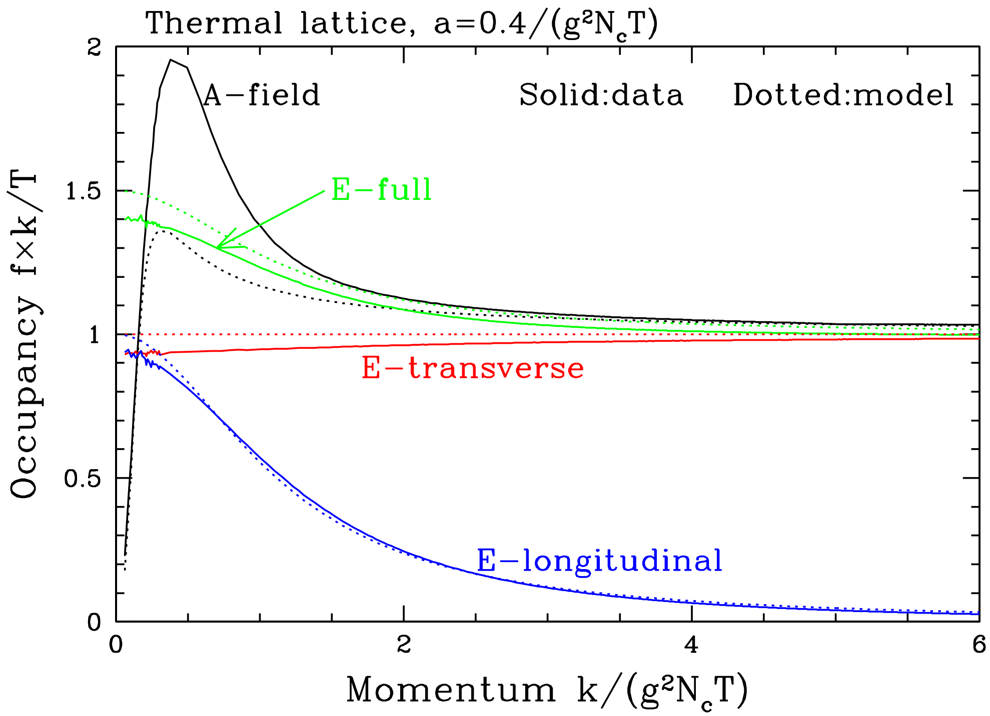
$$f_A(\mathbf{p}) = \frac{\delta_{ij}\delta_{ab}}{2(N_{\rm c}^2-1)}|\mathbf{p}| \int d^3x\, e^{i\mathbf{p}\cdot\mathbf{x}} \langle A_a^i(x) A_b^j(0)\rangle_{\rm coul}\,,$$

$$f_E(\mathbf{p}) = \frac{\delta_{ij}\delta_{ab}}{2(N_{\rm c}^2-1)|\mathbf{p}|} \int d^3x\, e^{i\mathbf{p}\cdot\mathbf{x}} \langle E_a^i(x) E_b^j(0)\rangle_{\rm coul}\,.$$

Two estimates of occupancy: $\mathbf{A}$-field and $\mathbf{E}$-field.

Trust, but verify

Equilibrium behavior for these "occupancies" 256^3 SU(2)



f_A: peak (fake?) and fall $f \leq 6/(g^2 N_c)$ (magnetic screening?)

f_E: rise in IR (Longitudinal occupancy!)

We made an assumption

We assumed $\langle \mathbf{EE}(\mathbf{k}) \rangle$ remains transverse!

It doesn't: $\mathbf{D} \cdot \mathbf{E} = 0$, not $\boldsymbol{\nabla} \cdot \mathbf{E}$.
Fluctuations: effective random charge density.

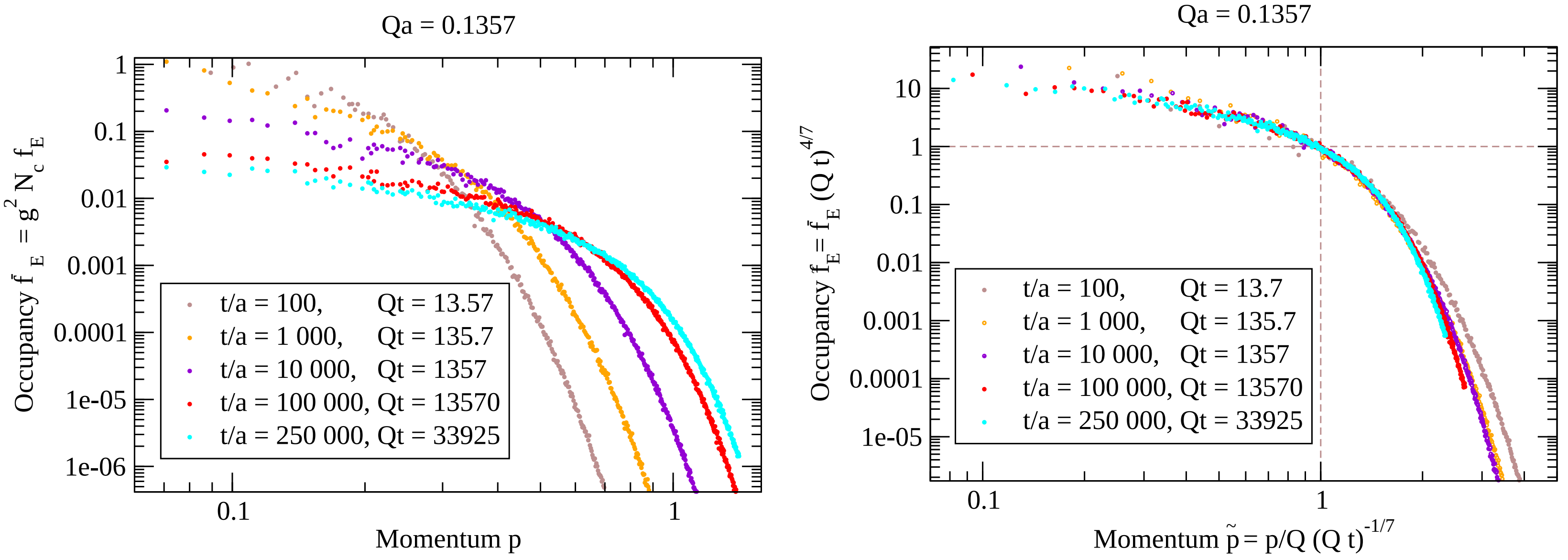
perturbatively but working a bit harder,

$$\int d^3x\, e^{i\mathbf{p}\cdot\mathbf{x}} \langle E_a^i(x) E_b^j(0) \rangle_{\rm eq} = \delta_{ab} T \left(\mathcal{P}_{\rm T}^{ij}(\mathbf{p}) + \frac{m_{\rm D}^2}{m_{\rm D}^2 + p^2} \hat{p}^i \hat{p}^j \right)$$

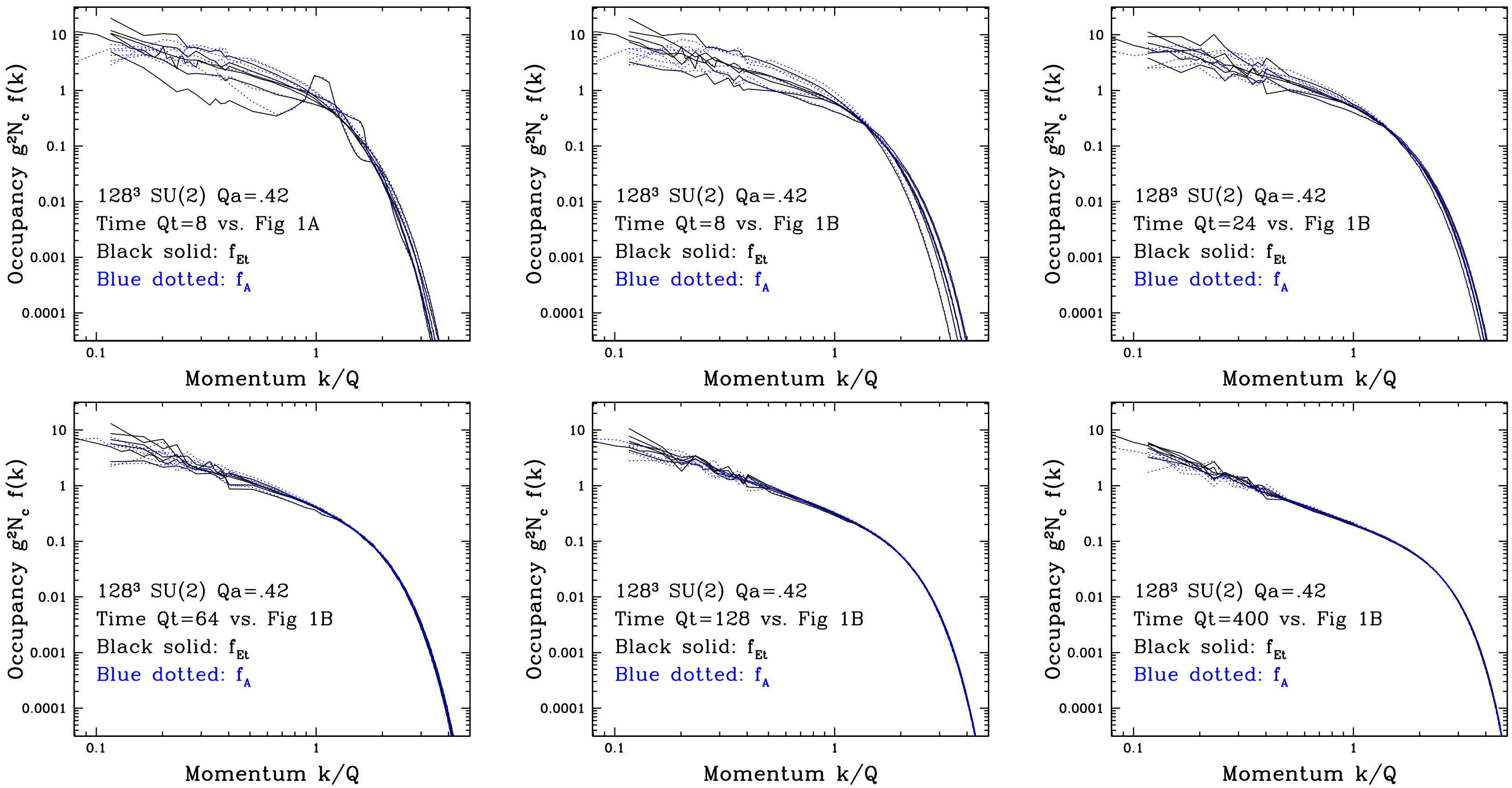
Below scale $m_{\rm D}$, significant *longitudinal* contrib.

Best solution: separate into $f_{E_t}(k)$, $f_{E_l}(k)$, believe f_{E_t}.
(Works in equilibrium, at least....)

Scaling works!

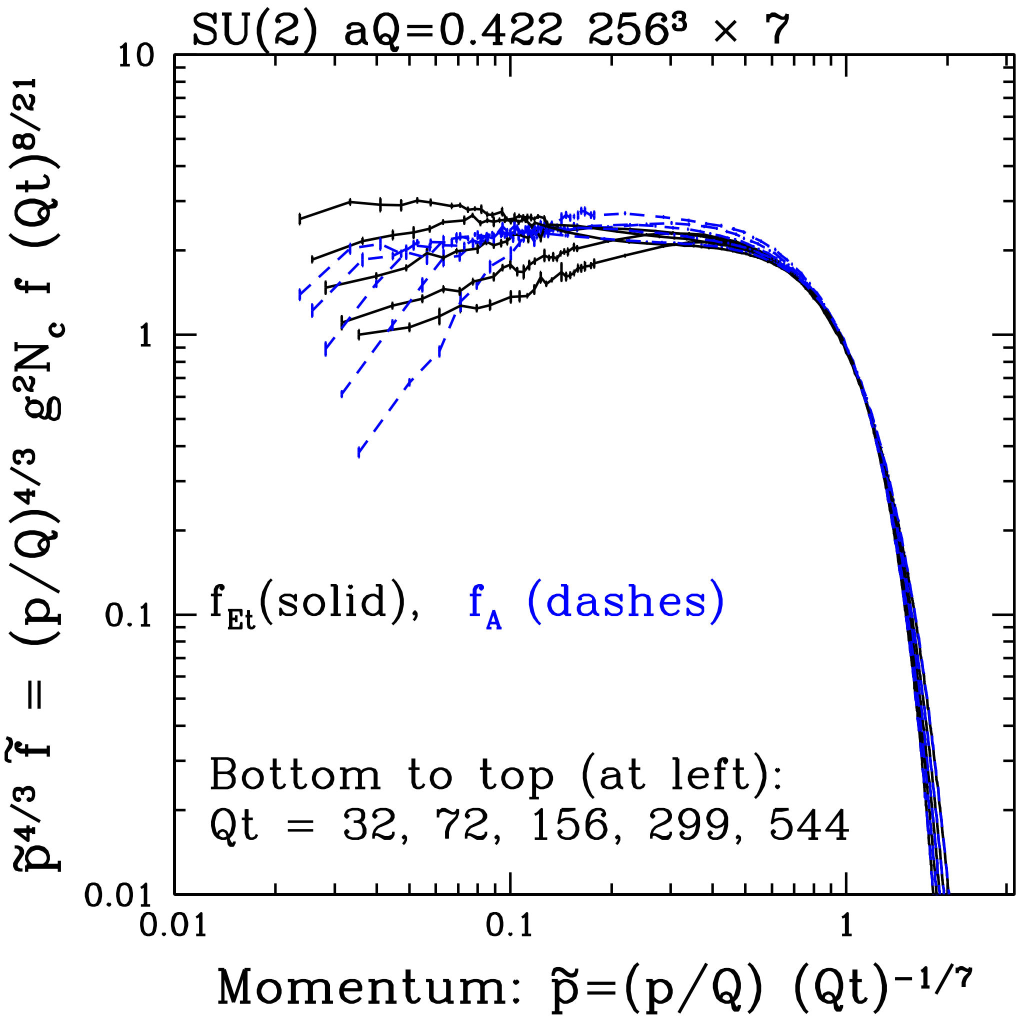


Rather rapid convergence to scaling solution:



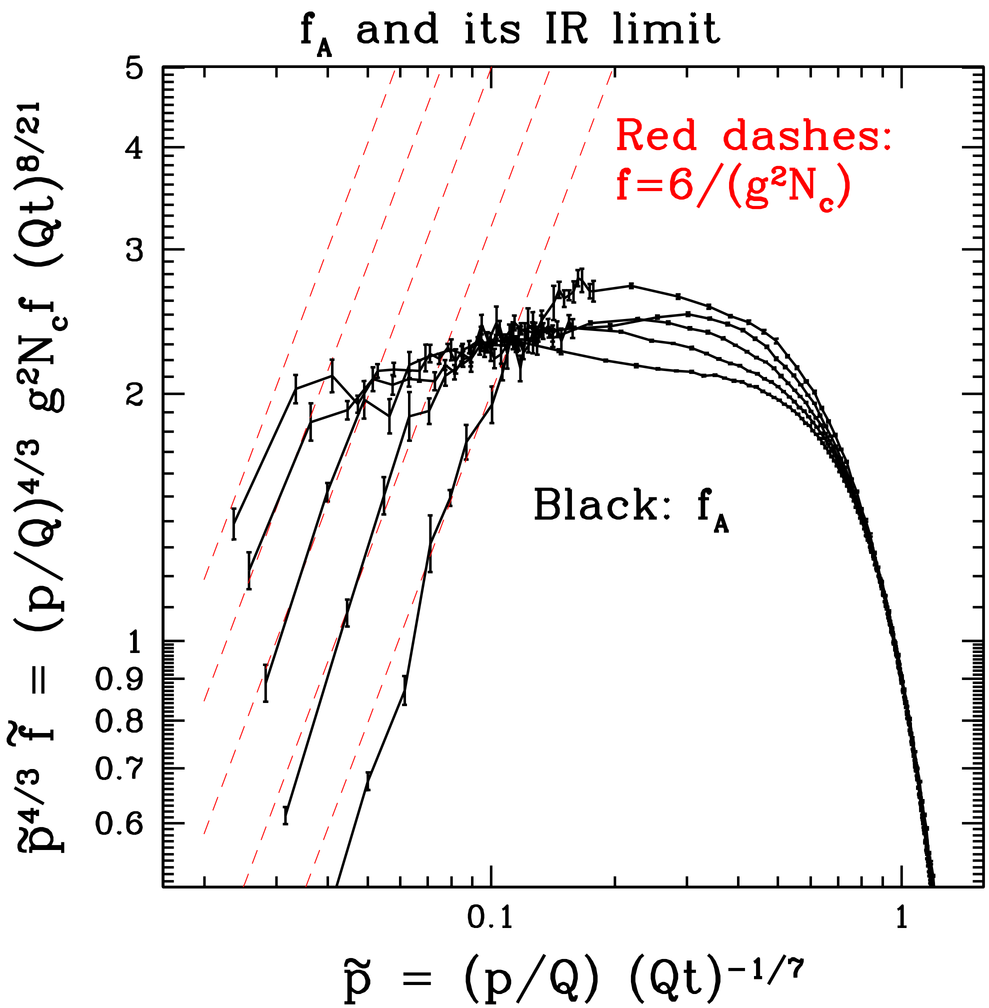
6 distinct initial conditions, but soon they all look same.

"Scaling" solution evolves in IR



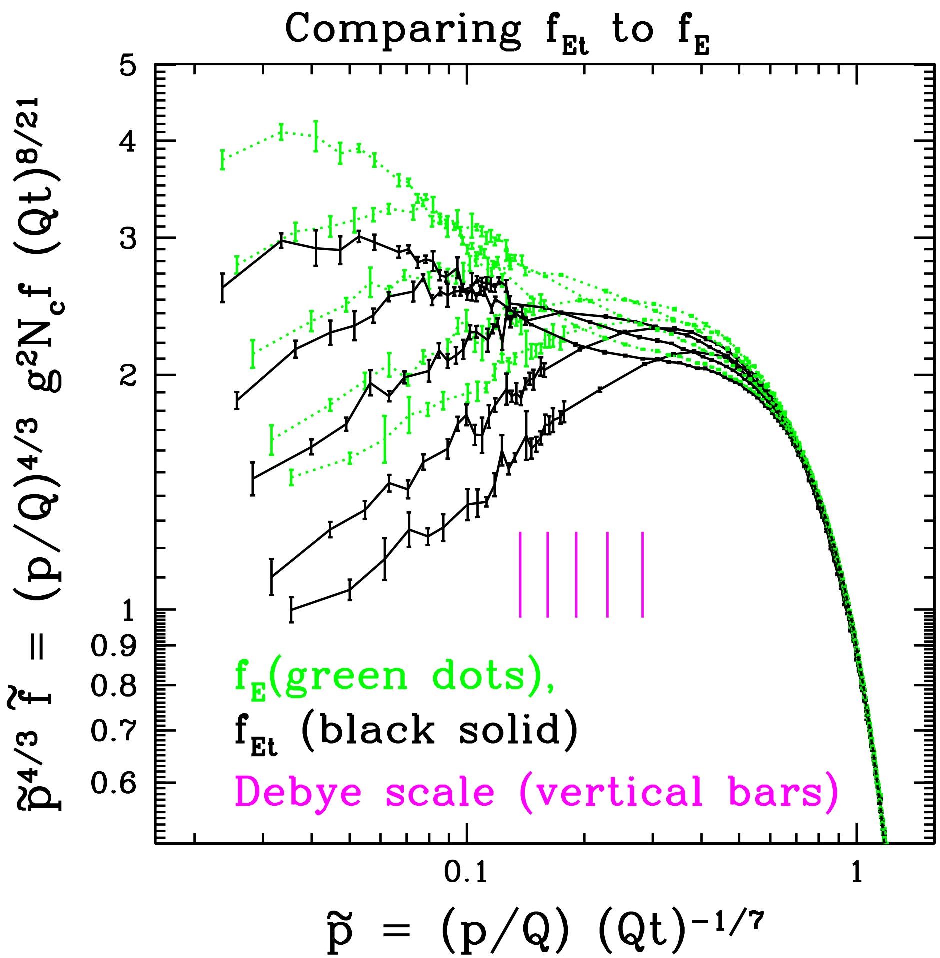
f_A, f_{E_t} tell same story except in IR

First look at f_A



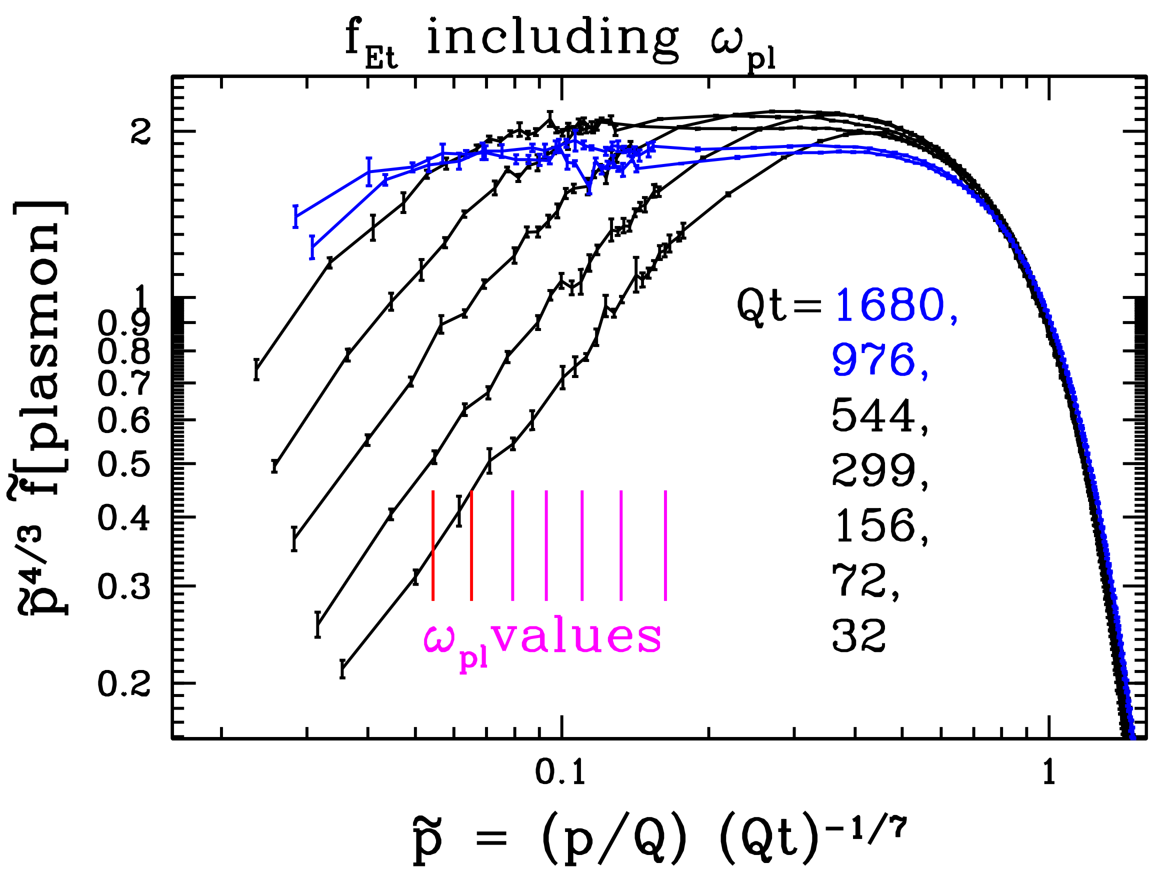
f_A rises, reaches $6/(g^2 N_{\rm c})$, saturates.

Now f_E versus f_{E_t}



f_E rises more than $4/3$ power, but ..

$$f_E = \frac{\mathcal{P}_{\rm T}^{ij}\delta_{ab}}{2(N_{\rm c}^2-1)\sqrt{p^2+\omega_{\rm pl}^2}} \int d^3x\, e^{i\mathbf{p}\cdot\mathbf{x}} \langle E_i^a(x) E_j^b(0)\rangle_{\rm coul}$$



So far, IR occupancy

$$f(p,t) \sim \frac{1}{g^2 N_\mathrm{c}} (Qt)^{\frac{-4}{7}} (p_\mathrm{max}/p)^{\frac{4}{3}}$$

For f_A, saturates at $f = 6/(g^2 N_\mathrm{c})$. f_E a bit lower.

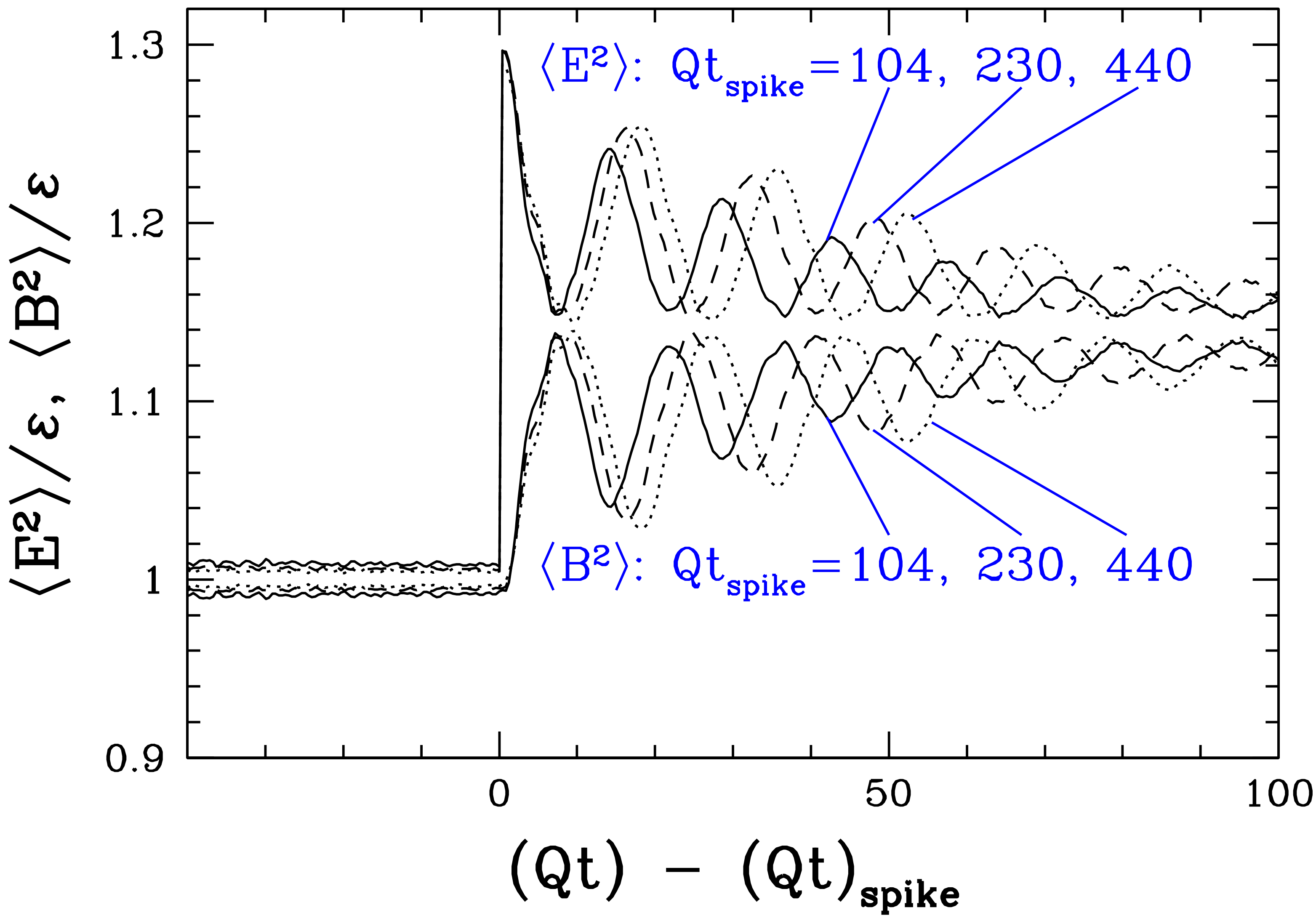
Part. number with $f \geq \frac{1}{g^2 N_\mathrm{c}}$ falls with time as $n_\mathrm{cond.}/n_\mathrm{tot} \sim (Qt)^{\frac{-5}{7}}$. Fairly small coefficient.

Could there *be* a condensate? If so, how would it evolve?

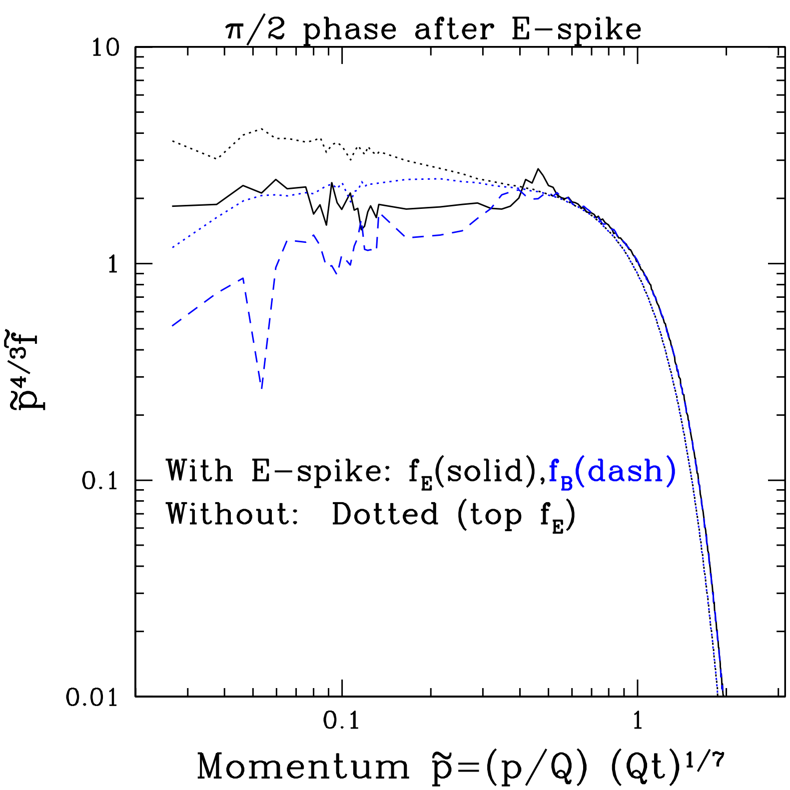
We can put one in by hand!

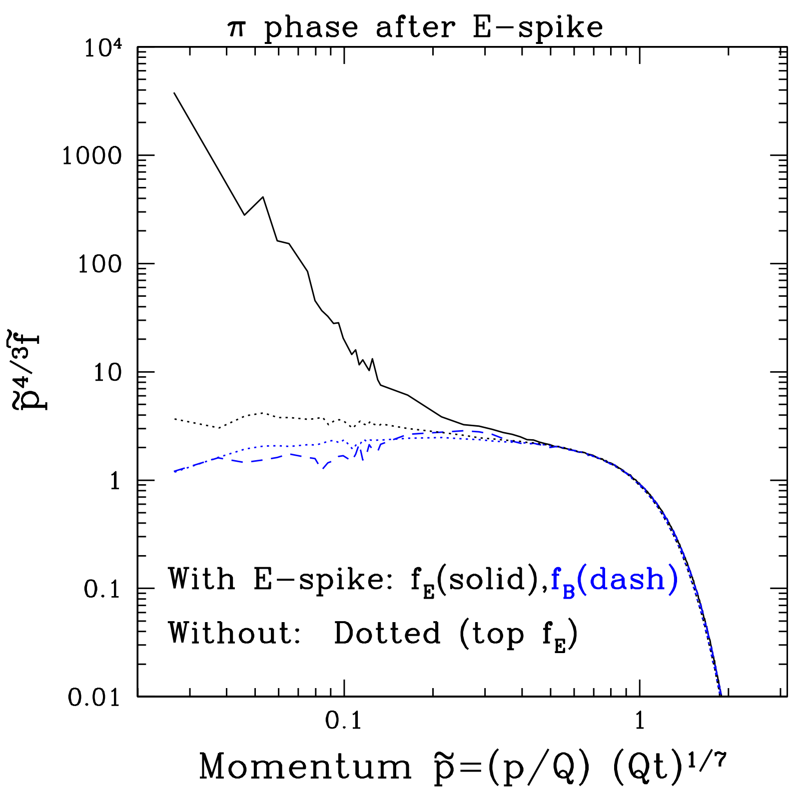
Evolve for a while, fix Coulomb gauge, insert uniform E field:

Energies with E-spike



Occupancies with E-spike





at $\mathbf{E}$-minimum

at $\mathbf{E}$-maximum

Conclusions

- Classical Yang-Mills dynamics: cascade to UV
- Scaling with time, $p_{\rm max} \sim Q(Qt)^{\frac{1}{7}}$ and $f \sim \frac{1}{g^2 N_{\rm c}}(Qt)^{\frac{-4}{7}}$
- $f(p,t)$ approaches scaling solution, with $f(p > p_{\rm max})$ exponential and $f(p < p_{\rm max}) \propto p^{\frac{-4}{3}}$
- IR corrections to scaling – mostly saturation of $\mathbf{A}$-fields at $f \sim 6/g^2 N_{\rm c}$ and screening effects on $\mathbf{E}$-fields
- No evidence of occupancies larger than above. Plasmon condensate possible, not realized and would decay fast