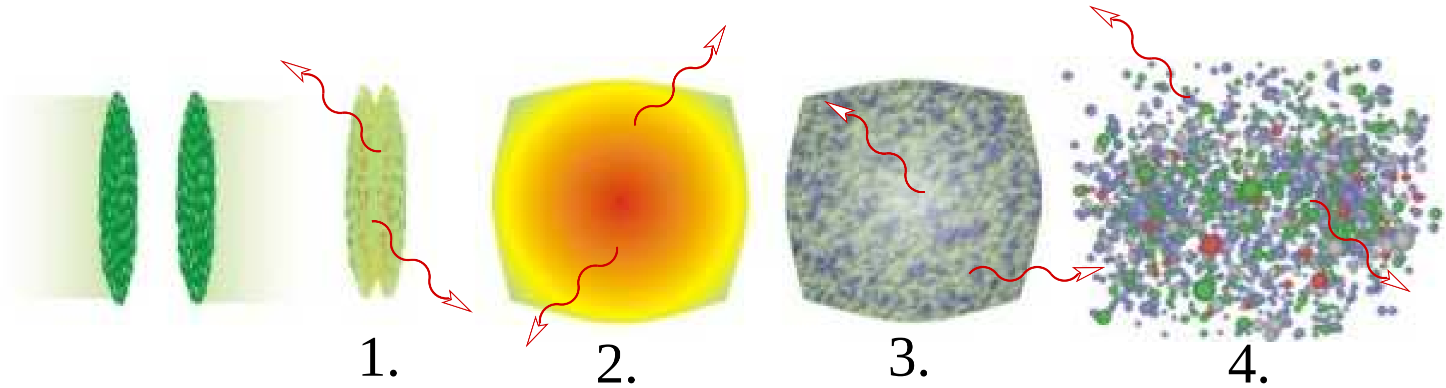
QGP Photon production: Beyond Leading Order

with Jacopo Ghiglieri, Juhee Hong, Aleksi Kurkela, Egang Lu, Derek Teaney

- Introduction and motivation
- Approaches and their Problems
- Perturbation theory: Leading, Next-to-Leading Order
- The Miracles of Lightlike momenta, Analyticity
- Results and prospects

Stages of a Heavy Ion Collision



1. Ions collide, making q, g, photons "primary"
2. q, g rescatter as QGP, make photons "thermal"?
3. Hadrons form, scatter, make photons "Hadronic"
4. Hadrons escape, some decay to photons "decay"

Photon re-interaction rare ($\alpha_{\mathrm{EM}} \ll 1$): direct info.

Thermal photons *may* act as a thermometer for QGP.

We should try to understand production rate!

How Photons Get Made

Since $\alpha_{\rm EM} \ll 1$, work to lowest order in $\alpha_{\rm EM}$:

- assume photon production *Poissonian* Find single-photon production
- neglect back-reaction on system cooling insignificant...

Single-photon production at $\mathcal{O}(\alpha_{\rm EM})$

$$2k^0 \frac{d\mathrm{Prob}}{\bar{d}^3 k} = \sum_X \mathrm{Tr}\, \rho\, U^\dagger(t) |X, \gamma(k)\rangle\langle X, \gamma(k)| U(t)$$

$U(t)$ time evolution operator, ρ density matrix.

Expand $U(t)$ in EM interaction picture:

$$U(t) = 1 - i\int^t dt' \int d^3x\, eA^\mu(x,t')J_\mu(x,t') + \mathcal{O}(e^2)$$

A^μ produces the photon. Get

$$\frac{d\mathrm{Prob}}{đ^3k} = \frac{e^2}{2k^0}\int d^4Y d^4Z e^{-iK\cdot(Y-Z)} \sum_X \mathrm{Tr}\,\rho\, J^\mu(Y)|X\rangle\langle X|J_\mu(Z)$$

Assume slow-varying, near-equilibrium: $\int d^4Z \to Vt$:

Get rate per 4-volume:

$$\frac{d\Gamma}{đ^3k} = \frac{e^2}{2k^0}G^<(K)\,, \quad G^<(K) \equiv \int d^4Y e^{-iK\cdot Y}\left\langle J^\mu(Y)J_\mu(0)\right\rangle_\rho$$

Success of Hydro – but not true at early times ....

Calculational Approaches

No first-principles, *nonperturbative* way to compute $\langle J^\mu J_\mu \rangle(K)$ directly. Instead we have

1. Lattice techniques (uncontrolled analytic continuation)

2. Weak-coupling techniques (uncontrolled extrapolation from $\alpha_{\rm s} < 0.1$)

3. Strong-coupling $\mathcal{N}{=}4$ SYM (Uncontrolled relation to QCD)

I will mention 1. and 3. but concentrate on 2.

Lattice

Lattice: find correlators at unequal *Euclidean* time τ.

How can I use that? Trade $G^<(K)$ for spectral function

$$\sigma(K) \equiv \int d^4Y e^{-iK\cdot Y} \left\langle \left[J^\mu(X) \, , \, J_\mu(0) \right] \right\rangle = \frac{1}{n_b(k^0)} G^<(K)$$

Related to Euclidean-frequency correlator via Kramers-Kronig

$$G_{\rm E}(\omega_{\rm E}, k) = \int \frac{dk^0}{2\pi} \, \frac{\sigma(k^0, k)}{k^0 - i\omega_{\rm E}}$$

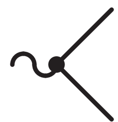
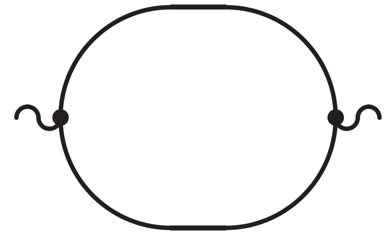
Transform to commonly measured $G_{\rm E}(\tau, k)$:

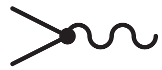
$$G_{\rm E}(\tau, k) = \int \frac{dk^0}{2\pi} \, \frac{\sigma(k^0)}{k^0} \; \times \; \frac{k^0 \cosh(k^0(\tau - 1/2T))}{\sinh(k^0/2T)}$$

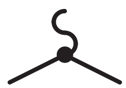
Problem: I want $\sigma(k^0)$, I know $G_E(\tau)$ with errors.

What do I expect for σ?

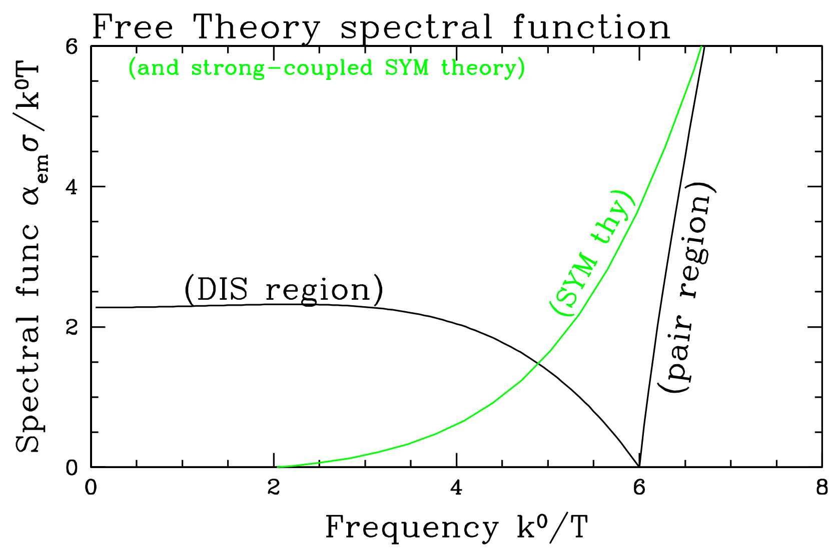
First consider lowest order perturbation theory:

$$J^\mu = \sum_{q=uds} e_q \bar{q} \gamma^\mu q \, :$$  so LO: $\langle JJ \rangle =$ 

Timelike K: pair annihilation  kinematically fine

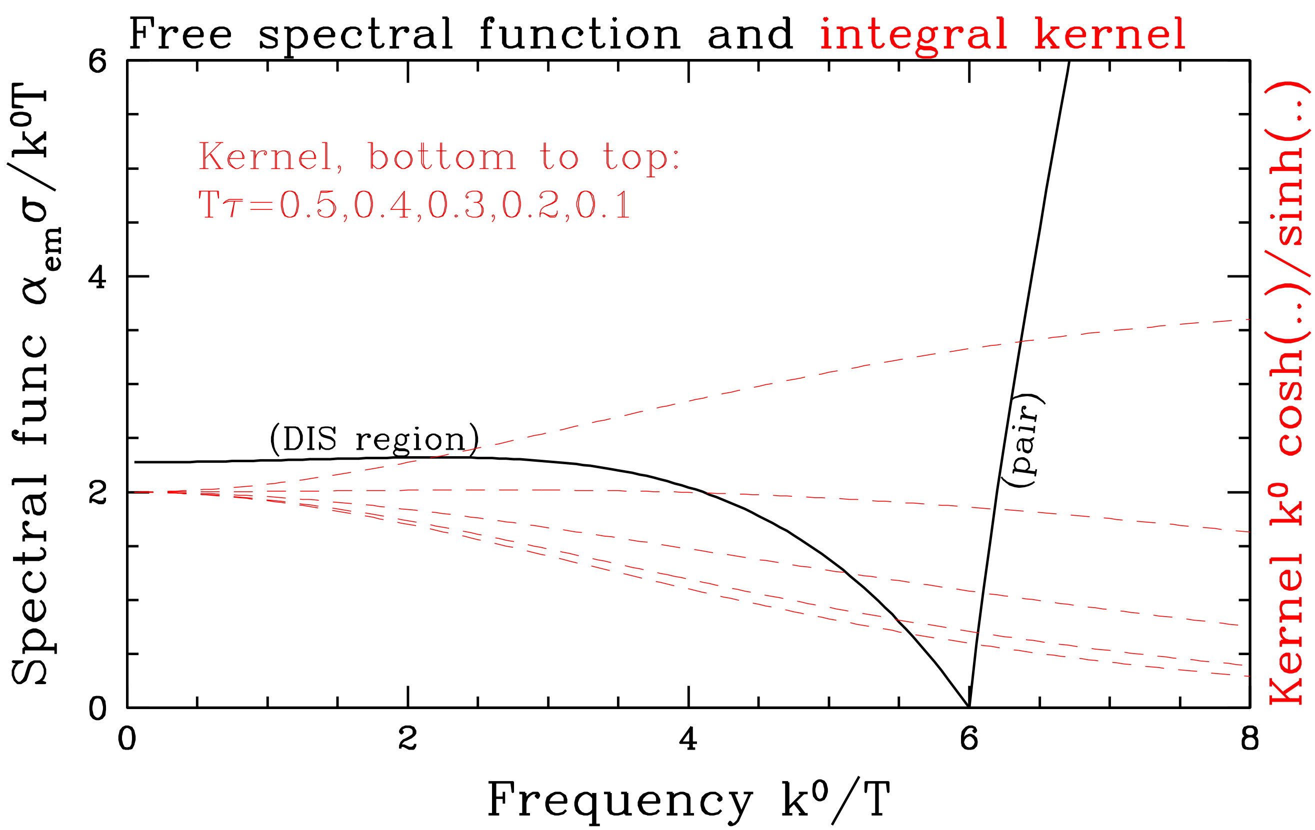
Spacelike K: DIS  also kinematically OK

Lightlike K: Neither allowed. Notch feature!



Lattice II

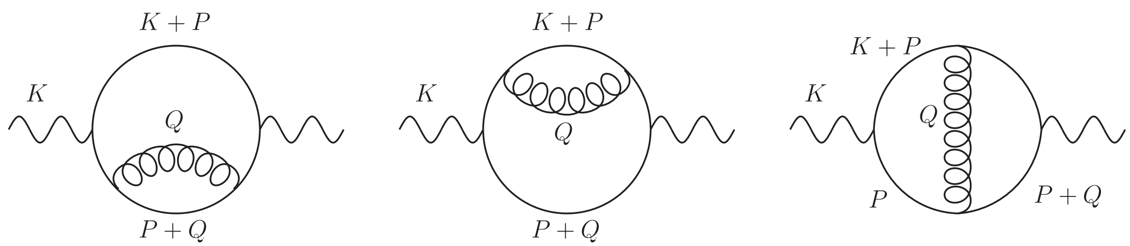
Can I capture sharp features with integral moments?



Not hopeful. Answer depends on input assumptions!

Perturbative Treatment

Diagrams:



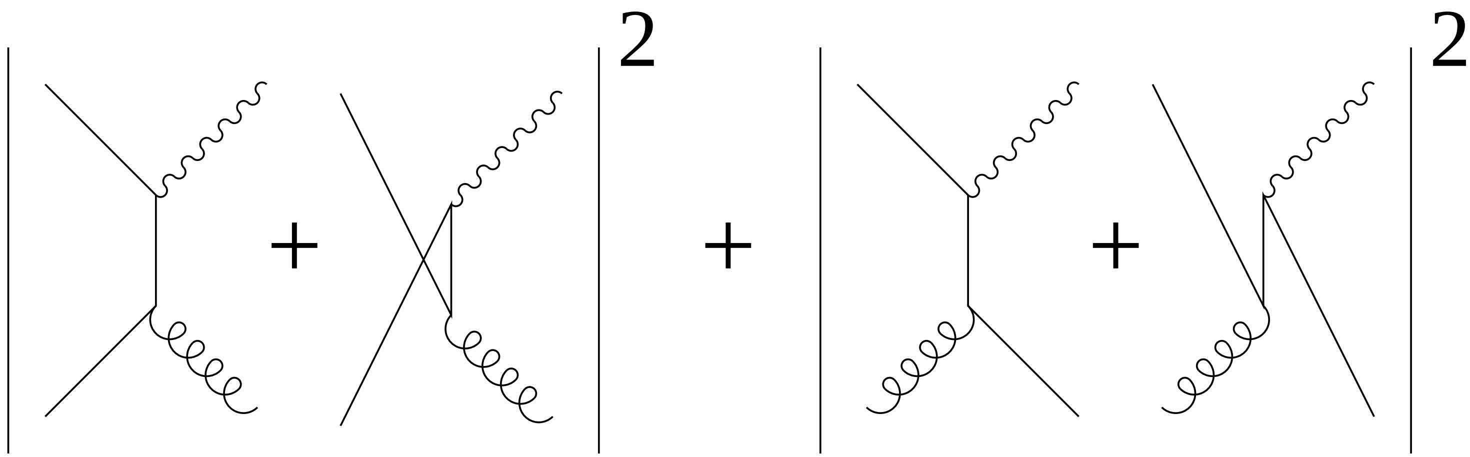
Kinematic regions (at leading order):

- K^μ, P^μ collinear, $Q \ll K$: collisionally broaden DIS and pair . . . . Aurenche Gelis Kobes Petitgirard Zaraket 1996-2000; AMY 2001
- P^μ off-shell and at large angle WRT K^μ: Usual $2 \leftrightarrow 2$

  Baier Nakagawa Niegawa Redlich 1992, Kapusta Lichard Seibert 1991

Leading order: $2 \leftrightarrow 2$

Ordinary photo-gluon scattering

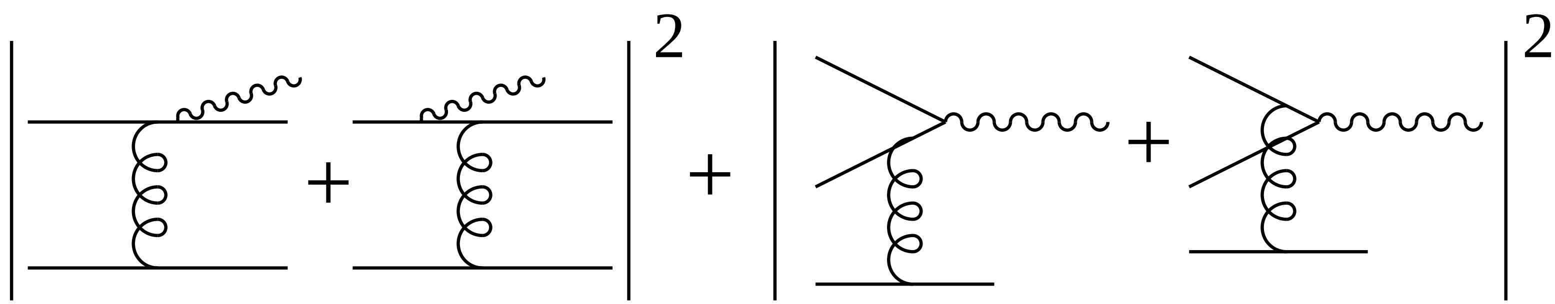


Log divergent as exchange quark P gets small

in $P \sim gT$ region, need (HTL) medium resummations

Previously done by brute-force integration with HTL self-energy...

Leading order: Collinear



Soft $Q \sim gT$ exchange with another hard πT particle

Stimulates emission (Bremsstrahlung or pair annihilation)

Small phase space but large near-on-shell enhancements

Must resum multiple scatterings (another talk)

Resummation possible by making collinear approximations

Exchanged propagator must (already) be (HTL) resummed

Complexities of Pert. Thy

- Thermal bath breaks Lorentz Symmetry.
  $m_{\rm gluon} = 0$ due to gauge+Lorentz.
  $m^2_{\rm gluon} \sim g^2 T^2$ $(g^2/\pi^2$ loop times $(\pi T)^2$ scale)
  But static magnetic fields still massless. Klimov,Weldon'82

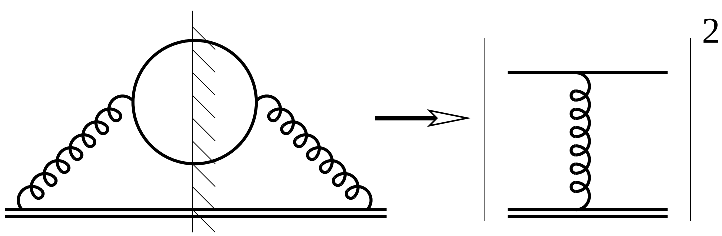
- Loop expansion modified: $\frac{1}{2} g^2 \longrightarrow \left(n_b(\omega) + \frac{1}{2} \right) g^2$
  Loop expansion breaks down for $\omega, k \lesssim g^2 T$. Linde'80

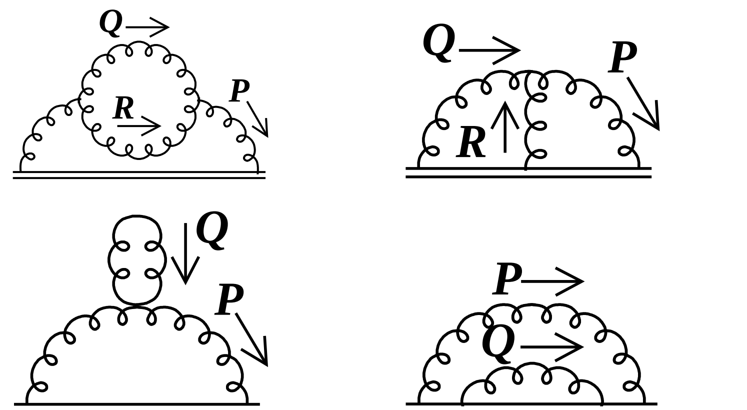
gT scale: need to resum masses – and vertices related by gauge invariance BraatenPisarski '91 – at this scale.
$g^2 T$ scale: pert. theory breaks down.

Complexities of NLO: Heavy Quarks

Caron-Huot and GM, 0708.4232; 0801.2173

Heavy quark near-rest, buffeted by soft $\mathbf{E}$ fields





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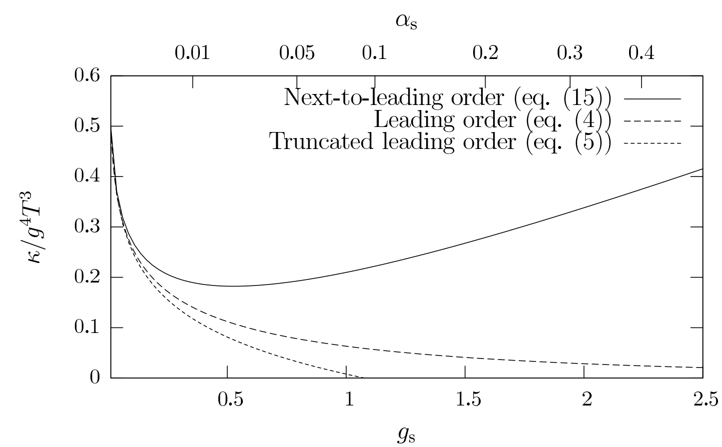
LO: single-scatter NLO:scatter off soft stuff .

Calc. difficult, convergence poor.



Why is convergence poor?

Scattering dominated by small Q exchange Coulomb logs

Small Q's require resummation Forward scattering off hard modes

$$G^{\mu\nu}(Q) = \frac{P_{\rm T}^{\mu\nu}}{q^2 - q_0^2 + m_{\rm D}^2 \Pi_{\rm T}(q^0/q)} + \frac{P_{\rm L}^{\mu\nu}}{q^2 + m_{\rm D}^2 \Pi_{\rm L}(q^0/q)}$$

$$m_{\rm D}^2 = \int \frac{d^3p}{p}\left(N_{\rm c} n_b(p) + \frac{N_{\rm f}}{2} n_f(p) \right)$$

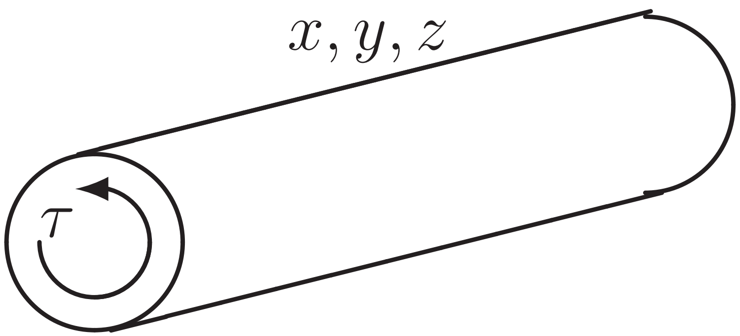
Problem: $m_{\rm D}^2$ "hard" loops dominated by gluon n_b term

Half this integral arises from $p \leq 1.1T$.

$m_{\rm D} = 1.1T$ for $\alpha_{\rm s} = .064$. Larger α: IR is gluey mess

But higher momenta might still be pert'ive!

Story for static properties



Time is compact; large distances act like 3D theory

3D theory describes massive A_0, massless F_{ij}

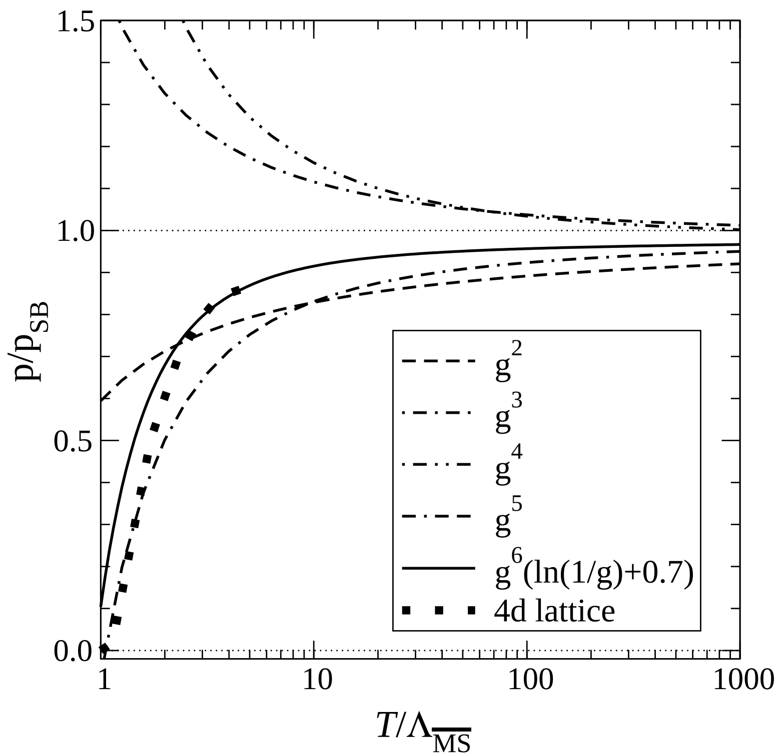
Coupling $g^2_{3\mathrm{D}} = g^2_{4\mathrm{D}} T$ dimensionful

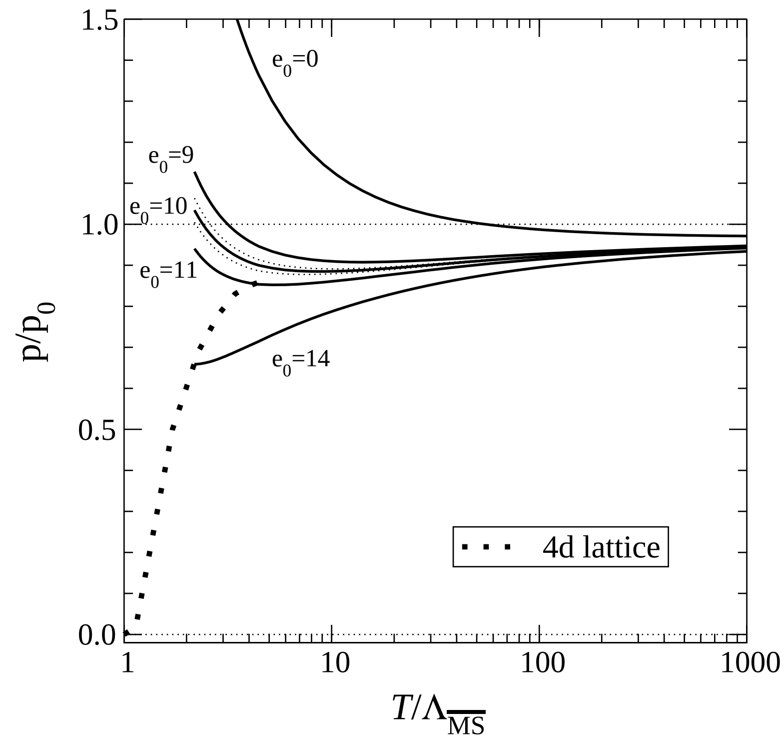
$g^2 T$ scale is where coupling becomes $\mathcal{O}(1)$

Reducing to 3D theory works for $T >$few T_c.

But must solve 3D theory nonperturbatively (or resum) unless T is *very* large.

Pressure





Order-by-Order Kajantie *et al* hep-ph/0211321 3D-resummed ibid hep-ph/0007109

Pert. theory of gT, g^2T scales works for $T > 100T_c$(?)

Pert.Thy. to get to 3D, solving 3D works to $3T_c$.

What about NLO photons?

Lines with gT momentum get corrected, and $Q \sim gT$ loops are only $\sim g$. This happens:

- $2 \leftrightarrow 2$: soft $P \sim gT$ exchanged quark Must resum at NLO!
- $2 \leftrightarrow 2$: soft $Q \sim gT$ emitted gluon
- Collinear: one fermion gets soft $P \sim gT$ or less collinear
- Collinear: NLO in scattering process ($d\Gamma/d^2q_\perp$ at NLO)

Need to treat each of these regions more carefully!

Medium Interactions at $v = 1$

Rate $d\Gamma/d^2q_\perp dt$ for $\mathbf{q}_\perp$ exchange with medium set by $\mathbf{b}_\perp \to \mathbf{q}_\perp$ transform of Tr Wilson loop, corners $(0,\mathbf{0},0), (0,\mathbf{b}_\perp,0), (l,\mathbf{b}_\perp,vl), (l,\mathbf{0},vl)$ Casalderry-Solana Teaney th/0701123

Heavy quark at rest: equal space unequal time.
Feels complicated medium dynamics.

If $v = \infty$: equal time Equilibrium Euclidean correlators.

What about $v = 1$? Too fast to feel medium dynamics.
Remarkable result: *Also* Euclidean correlators! Caron-Huot 0811.1603

We know NLO scattering in medium!

NLO Collinear

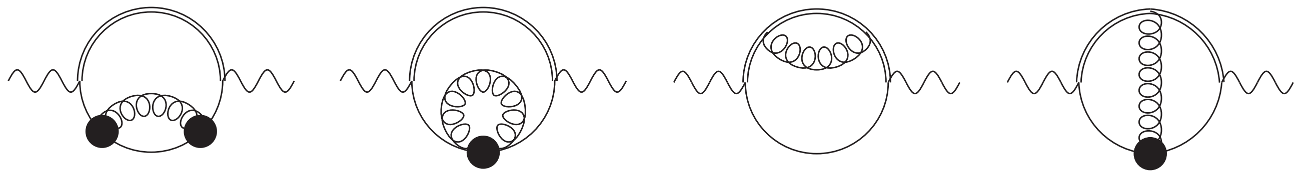
Redo the Leading-Order collinear calculation using new NLO scattering: Technology of AMY hep-ph/0109064, 0111107

$$\int dp_z \int d^2p_\perp \frac{d\Gamma_{\rm brem}}{d^3p} \qquad \frac{d\Gamma_{\rm brem}}{d^3p} = \text{see AMY hep-ph/0109064}$$

Collin approx breaks down, result is wrong for $p_z \sim gT$ Soft and for $p_\perp^2 \sim gT^2$ Semi-collinear each $\mathcal{O}(g)$ of total.

Integrate down to $p_z = 0$ and out past $p_\perp^2 \sim gT^2$.
Compute what above "naive" result *does* give there.
Then compute them "right" and *subtract* this extra.

Soft region

Diagrams: 

Single lines and bold vertices HTL resummed nasty

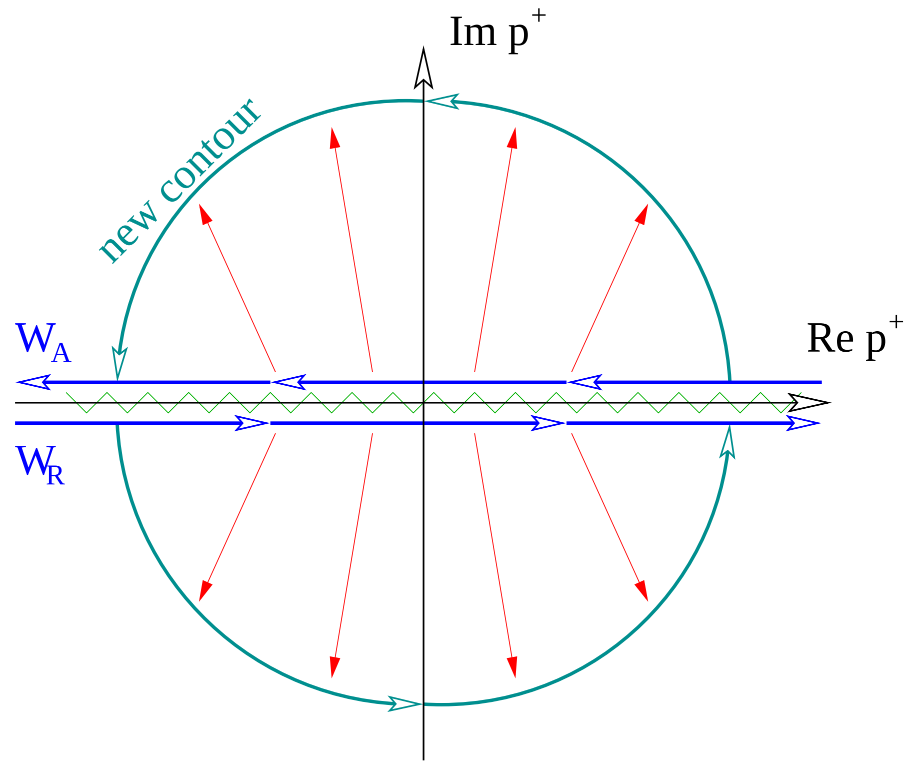
$$\int_{\sim gT} d^2 p_\perp dp^+ dp^- \int_{\sim gT} d^4 Q n_b(k^0)(G_{\rm R} - G_{\rm A})[\text{4 diagrams}]$$

Miracle 2: $K - P$ high-momentum line gives
$\delta((K-P)^2) \propto \delta(p^-)$. Need $\mathrm{R} - \mathrm{A}$ difference of $\int dp^+ \ldots$

Retarded function analytic in upper half ω plane.
And upper half [Any Timelike or Null Variable, eg p^+] plane.

Deform p^+ contour

into complex plane



Now $p^+ \gg p_\perp, Q$. (On mass-shell) Expand in $p^+ \gg p_\perp, Q$

$$G_{\rm R}[\text{4 diagrams}] = C_0 (p^+)^0 + C_1 (p^+)^{-1} + \ldots$$

C_0 is on-shell width, gives linear in p^+ divergence.

C_1 is on-shell dispersion correction, dp^+/p^+ gives const.

Soft region

Leading order: $(p^+)^0$ term absent, $(p^+)^{-1}$ term is

$$\frac{1}{p^+}(\text{Expected LeadLog coefficient})\frac{m_q^2}{m_q^2+p_\perp^2}$$

Gives known large-$p_\perp$ answer, explains old (numer.) results

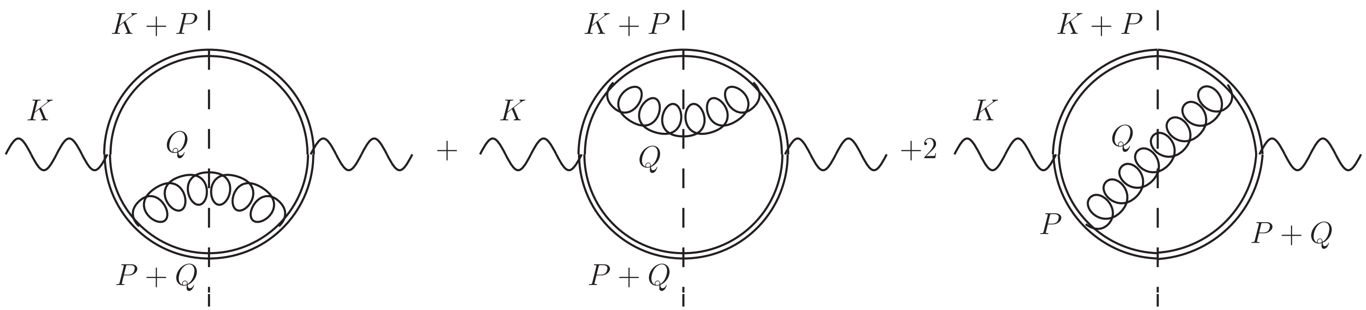
NLO: $(p^+)^{-1}$ answer is LO but with $m_q^2 \to m_q^2 + \delta m_q^2$

$$\frac{m_q^2+\delta m_q^2}{m_q^2+\delta m_q^2+p_\perp^2} = \frac{m_q^2}{m_q^2+p_\perp^2} + \frac{p_\perp^2\,\delta m_q^2}{(p_\perp^2+m_q^2)^2}$$

$(p^+)^0$ term, linear p^+ divergent, is exactly small-p^+ limit of collinear calculation (therefore subtracted)!

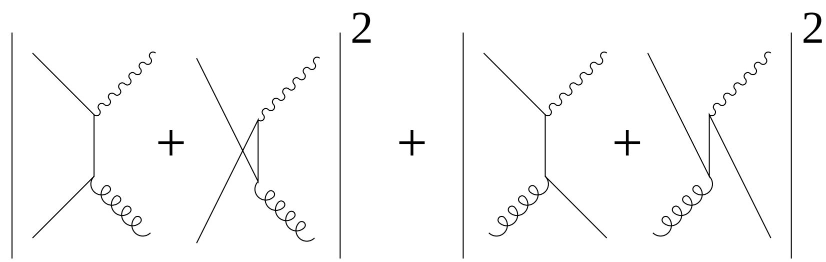
Semi-collinear region

Quarks at large momentum, gluon momentum small.



Gluon on shell:

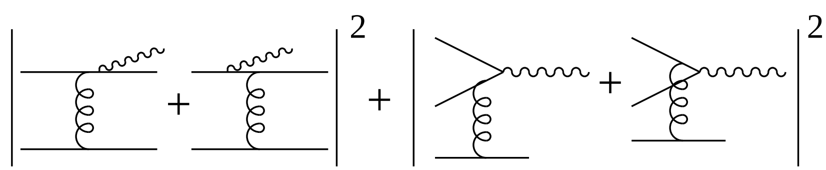
soft-glue correction to 2to2



Gluon in Landau cut:

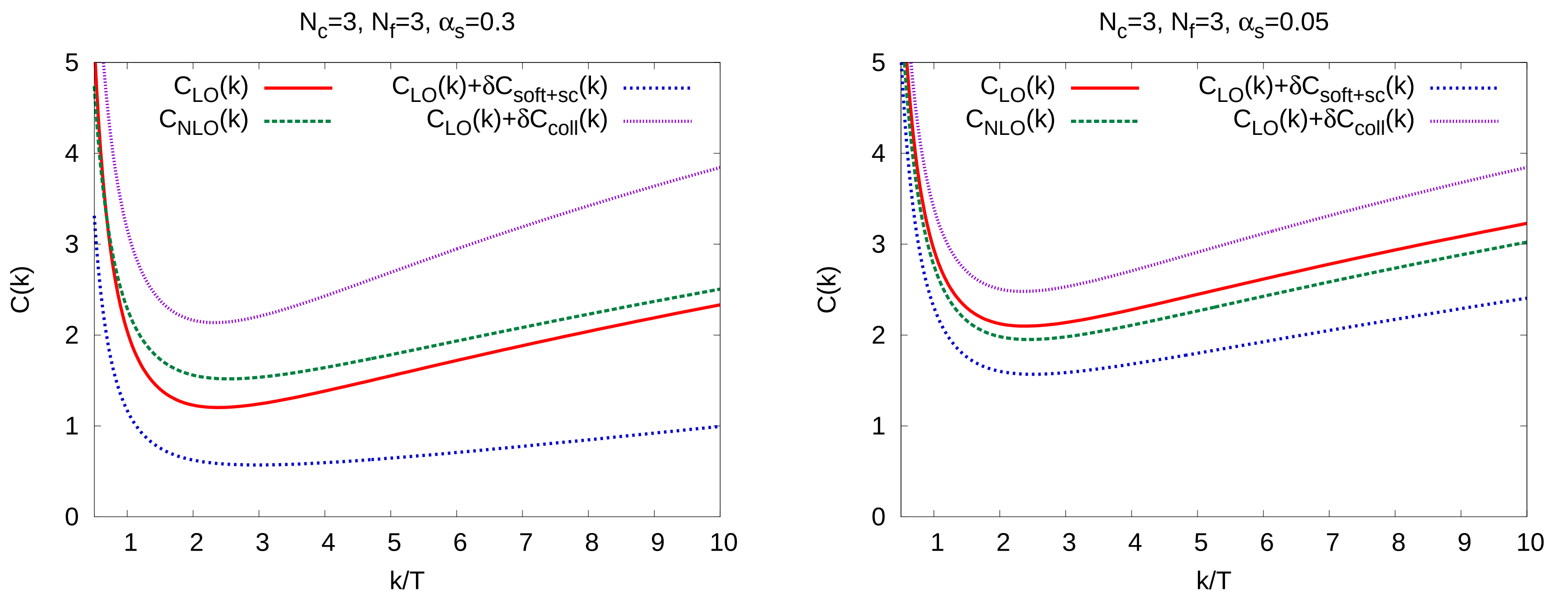
less-collinear correction

to pair+brem



Two subtractions. Contour trick works again.

Summing it up



more Soft scattering than LO. More collinear prod.

less $2 \leftrightarrow 2$: lost phase space from gluon "mass"

Each effect significant but not huge. Nearly cancel.

Why I am excited

Corrections not $that$ big, 100% not 500% Heavy quarks–bad luck?

Calculation easier than expected for structural reasons:

- Collinear: medium interaction $\Rightarrow$ Euclidean problem

  Definitely addressable on the lattice

- $2 \leftrightarrow 2$ involves asymptotic mass shift δm^2

  Smells like it can be done on the lattice

Key: high E photon $\Rightarrow$ high E, lightlike quark.

Lightlike v sees simple IR structure Existing correlation, not dynamics

Maybe we can do that on lattice, use Pert Thy to $3T_c$??

Why I am excited II

Other transport coefficients (shear...) dominated by $p \sim 5T$

Part. number $\int d^3p \to 3T$. Energy $pd^3p \to 4T$. Disequilibrium $p^2 d^3p \to 5T$

These move at $v \simeq 1$, should be pert've $T \sim \mathrm{few}\, T_c$

Main interactions with soft medium – nonpert've but if $v \simeq 1$, we find Euclidean methods may work there!

Hybrid approach may work for η etc as for pressure: "particles" perturbative, soft fields they feel are nonperturbative on the lattice. Devil in details

Aside: $\mathcal{N}$=4 SYM theory

QCD $+$ adjoint 4 Weyl quarks $+$ 6 real adjoint scalar quarks

plus very specific Yukawa, scalar interations, obeying SU(4) global R-symmetry

conformal for all values of α_s – can consider $\alpha_s \gg 1$

Can gauge $\mathrm{U}(1) \in \mathrm{SU}(4)_R$ as E&M, study γ production

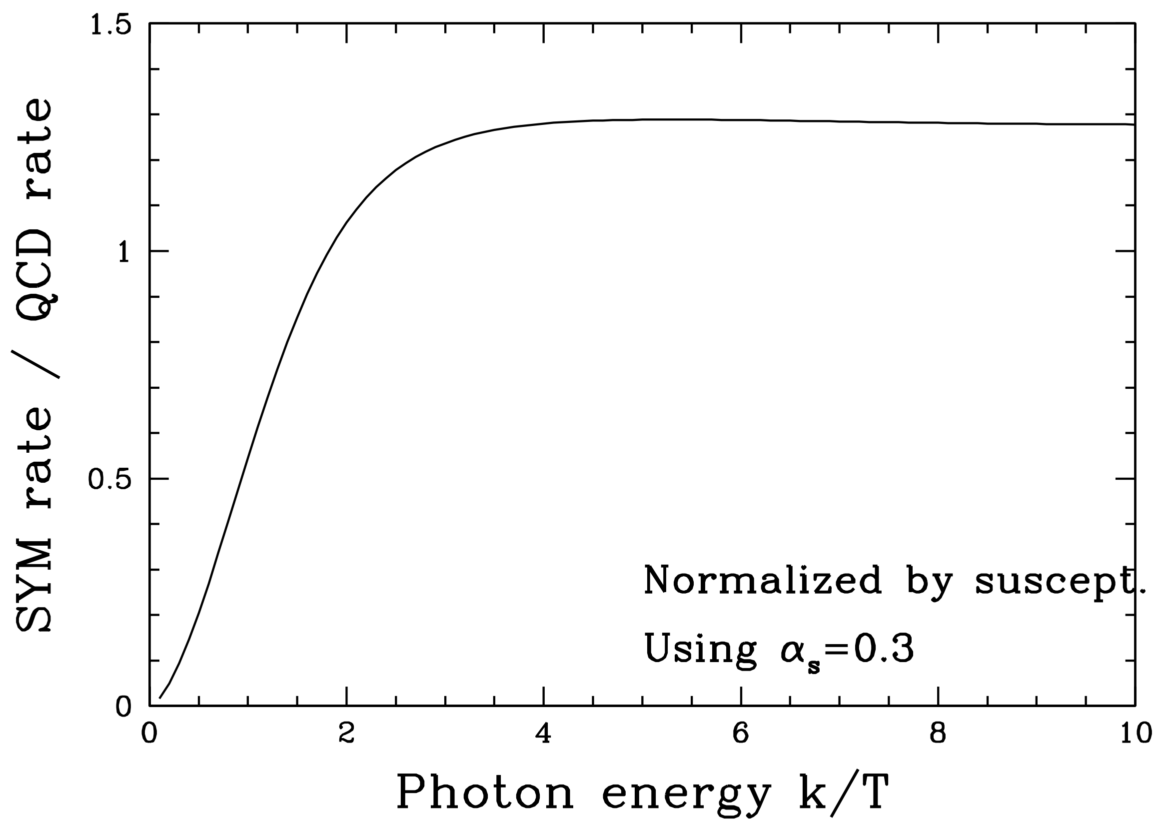
Caron-Huot Kovtun GM Starinets Yaffe th/0607237

How to relate to γ-prod rate in QCD?

I argue: normalize to charge susceptibility.

χ counts charges. $\chi^{-1} d\Gamma/d^3k$ is γ production per charge

Consider double ratio $\frac{(d\Gamma/d^3k)_{\rm SYM}/\chi_{\rm SYM}}{(d\Gamma/d^3k)_{\rm QCD}/\chi_{\rm QCD}}$



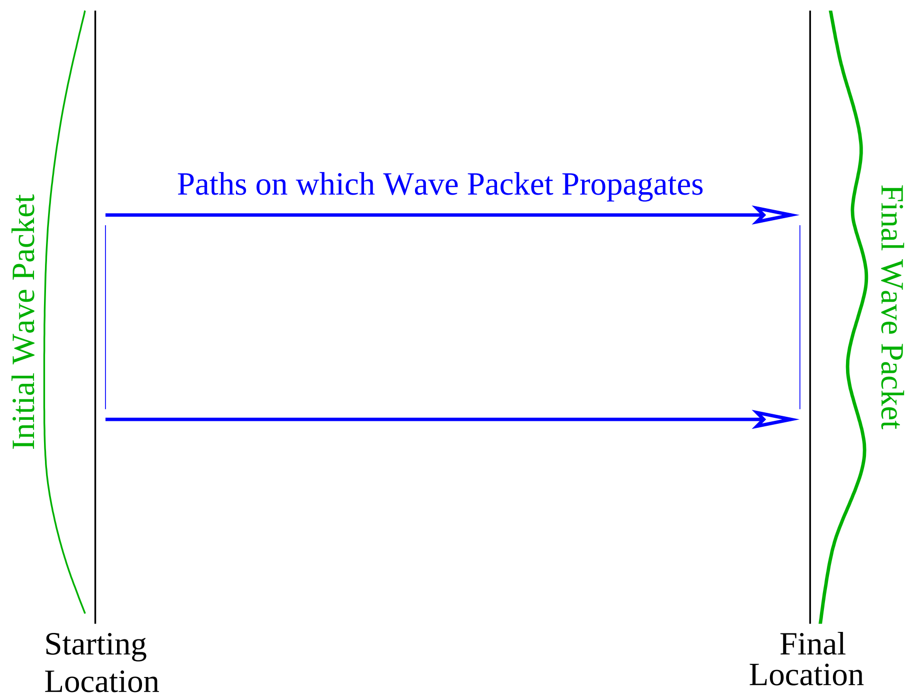
SYM not much larger than QCD!

Conclusions

- Photon rate is physically interesting
- Computation: $\langle JJ \rangle$ at lightlike momentum
- Phase space, perturbation theory subtle! Resummations
- NLO calculation: Analyticity, Euclidean methods key
- Maybe we can resum IR effects by lattice methods??!
- SYM case suggests strong-coupled rate not *much* larger than perturbative estimates

Backup: Wilson loop story

Consider high-energy particle, transverse wave packet:



Eikonal propagation. Phases from Wilson lines cause final wave packet to have phase variation

Fourier transform of final wave packet gives final transverse $\mathbf{p}_\perp$ distribution.

Hence $\ln \mathrm{Tr}\, W = L\, C(\mathbf{b}_\perp)$ where $C(\mathbf{b}_\perp)$ Fourier transforms to $C(\mathbf{q}_\perp)$

prob/length to gain transverse momentum $\mathbf{q}_\perp$

$v = 1$ versus equal time S. Caron-Huot

Consider correlator $G^<(x^0, \mathbf{x})$ with $x^z > |x^0|$. Fourier representation

$$G^<(x^0, \mathbf{x}) = \int d\omega \int dp_z d^2 p_\perp e^{i(x^z p^z + \mathbf{x}_\perp \cdot \mathbf{p}_\perp - \omega x^0)} G^<(\omega, p_z, p_\perp)$$

Use $G^<(\omega, \mathbf{p}) = n_b(\omega)(G_R(\omega, \mathbf{p}) - G_A(\omega, \mathbf{p})$ and define $\tilde{p}^z = p^z - (t/x^z)\omega$:

$$G^< = \int d\omega \int d\tilde{p}^z d^2 p_\perp e^{i(x^z \tilde{p}^z + \mathbf{x}_\perp \cdot \mathbf{p}_\perp)} n_b(\omega) \left(G_R(\omega, \tilde{p}^z + \omega \frac{x^0}{x^z}, \mathbf{p}_\perp) - G_A \right)$$

Perform ω integral: upper half-plane for G_R, lower for G_A, pick up poles from n_b:

$$G^<(x^0, \mathbf{x}) = T \sum_{\omega_n = 2\pi n T} \int dp^z d^2 p_\perp e^{i\mathbf{p}\cdot\mathbf{x}} G_E(\omega_n, p_z + i\omega_n (x^0/x^z), p_\perp)$$

Large separations: $n \neq 0$ exponentially small. $n = 0$ contrib. is x^0 independent!

Formal approach S. Caron-Huot 0811.1603

N-pt correlator with $t = \bar{v} x^z$ for all points: use density matrix $\rho = \exp(-\beta\bar{\gamma}(H + \bar{v}P^3))$
Time and x^z complex, periodicity $x'^\mu = x^\mu + i\bar{\gamma}(\beta, -\bar{v}\beta, 0_\perp)$
Matsubara: p^z is real but $\omega_n = 2\pi i n T/\bar{\gamma} - \bar{v}p^z$ complex

Now boost back to plasma rest frame: standard Matsubara formalism but with $\omega_E = 2\pi i n T$, $p^z_n = p^z + 2\pi i n T \bar{v}$.

Nonzero Matsubara propagators changed, zero Matsubara's are same.

Now integrate out $n \neq 0$: can deform p^z_n contour back to real axis without singularity, giving standard 3D theory (same as $\bar{v} = 0$)