What (little) we Know about The SU(3) Sphaleron Rate

- Reminder: physics of Sphaleron Rate
- Correlation function defining Sphaleron Rate
- Difficulty with Analytic Continuation
- Evaluation at weak coupling
- Gaps in our knowledge, and Which Gaps we can Fill

Recall, a fermion's axial current

$$J^\mu_{\rm A} \equiv \bar\psi \gamma^\mu \gamma_5 \psi$$

is anomalous in SU($N_{\rm c}$) gauge theory:

$$\begin{aligned} \partial_\mu J^\mu_{\rm A} &= 2m\bar\psi\gamma_5\psi - \frac{g^2}{16\pi^2} F^{\mu\nu}\tilde F_{\mu\nu} \,, \\ \tilde F_{\mu\nu} &\equiv \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}F^{\alpha\beta} \end{aligned}$$

Violation due to mass AND due to anomaly!

Change in net chiral quark number associated with

$$N_{\rm cs} \equiv \int d^4x \; \frac{g^2}{32\pi^2} F^{\mu\nu}\tilde F_{\mu\nu} = \int d^4x \; \frac{g^2}{8\pi^2} \mathbf{E}\cdot\mathbf{B}$$

Random motion

Amplitude to go from an initial config $|\psi_1(t_1)\rangle$ to final config $\langle\psi_2(t_2)|$:

$$A_{12} \equiv \langle\psi_2(t_2)| \; |\psi_1(t_1)\rangle$$

Amplitude times change in $N_{\rm cs}$:

$$AN_{12} \equiv \int_{t_1}^{t_f} \int d^3x \, \langle\psi_2| \; \frac{g^2}{32\pi^2} F_{\mu\nu}\tilde{F}^{\mu\nu}(x,t) \; |\psi_1\rangle$$

Squared change, times probability of process:

$$|AN_{12}|^2 \equiv \int d^4x d^4y \, \langle\psi_1| \; \frac{g^2}{32\pi^2} F_{\mu\nu}\tilde{F}^{\mu\nu}(x) \; |\psi_2\rangle \, \langle\psi_2| \; \frac{g^2}{32\pi^2} F_{\alpha\beta}\tilde{F}^{\alpha\beta}(y) \; |\psi_1\rangle$$

Sum over possible final states: Mean squared $N_{\rm cs}$ change

$$\langle(\Delta N_{\rm cs})^2\rangle = \int d^4x d^4y \, \langle\psi_1| \; \frac{g^2}{32\pi^2} F_{\mu\nu}\tilde{F}^{\mu\nu}(x) \frac{g^2}{32\pi^2} F_{\alpha\beta}\tilde{F}^{\alpha\beta}(y) \; |\psi_1\rangle \; .$$

Diffusion and Relaxation

Mean-square change per 4-volume:

$$\Gamma_{\rm Sphal} \equiv \frac{\langle (\Delta N_{\rm cs})^2 \rangle}{Vt} = \int d^4x \left\langle \frac{g^2}{32\pi^2} F_{\mu\nu}\tilde{F}^{\mu\nu}(x) \frac{g^2}{32\pi^2} F_{\alpha\beta}\tilde{F}^{\alpha\beta}(0) \right\rangle$$

Diffusion rate of topological number.

Fluctuation–Dissipation: Relaxation rate for $Q_5 \equiv \langle J^0_5 \rangle$:

(Giudice-Shaposhnikov hep-ph/9311367, Moore hep-ph/9705248)

$$\frac{dQ_5}{dt} = -(Q_5 - Q_{5,\rm equil}) \frac{(2N_{\rm f})^2}{\chi_Q} \frac{\Gamma_{\rm Sphal}}{2T} \simeq -Q_5 \frac{6N_{\rm f}}{N_{\rm c}} \frac{\Gamma_{\rm Sphal}}{T^3}$$

(Using free-theory Q_5 susceptibility $\chi_Q = \frac{1}{3} N_{\rm f} N_{\rm c} T^2$)

nonperturbative relationship (so far)

Note hidden assumption:

Sphaleron transition generates Q_5. Associated free energy (chem potential) pushes $N_{\rm cs}$ back.

(Q_5 number diffuses, but on some timescale diffusion amplitude = thermal fluctuation. Longer times – excursions about zero, not fluctuations.)

I assumed time scale involved in $F\tilde{F}$ autocorrelation short compared to relaxation time.

If not the case: no well defined diffusion rate or relaxation time scale. May occur at strong coupling.

Euclidean methods???

I can calculate Euclidean version on the lattice. Not the same:

$$\int_0^\beta d\tau \int d^3x \langle F\tilde{F}(x,\tau)F\tilde{F}(0,0)\rangle \neq \int dt_{\rm mink} \int d^3x \langle F\tilde{F}(x,t)F\tilde{F}(0,0)\rangle$$

The Sphaleron Strikes Back, Peter Arnold and Larry McLerran, Phys.Rev.**D37**:1020,1988. 316 references

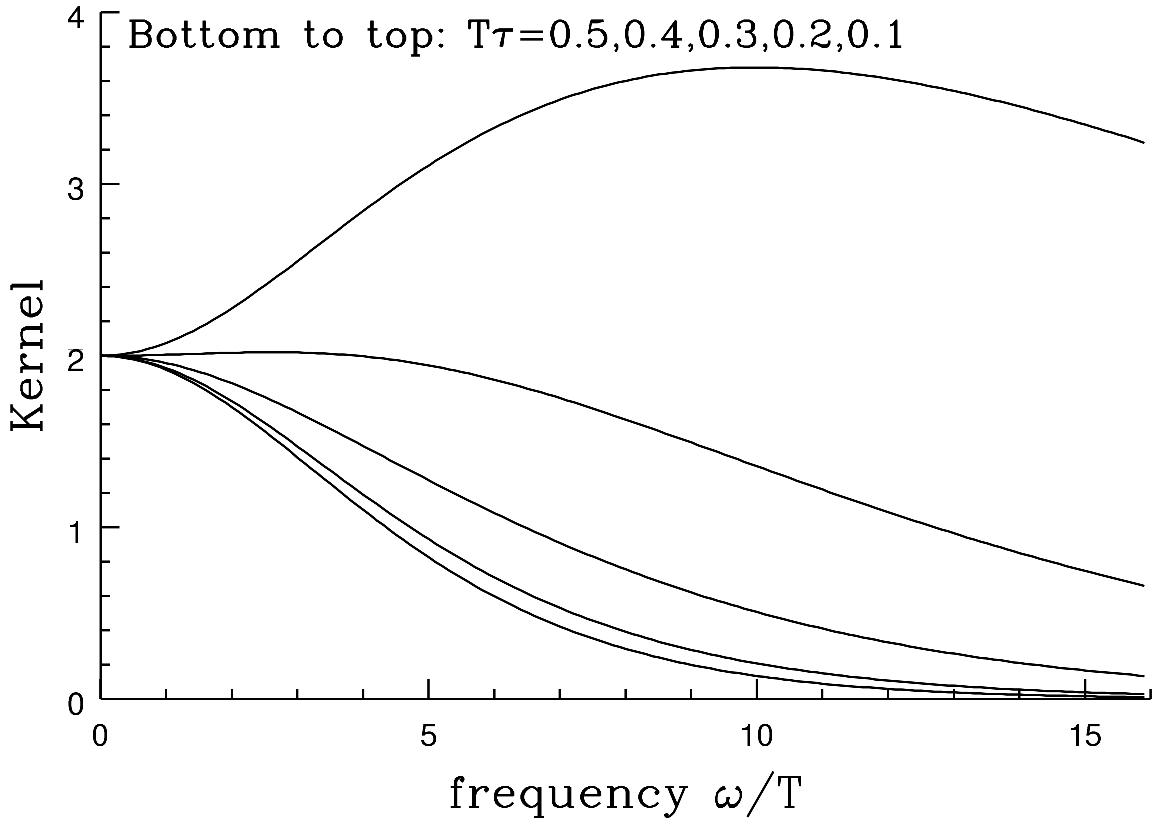
Consider rigid rotor or pendulum, coupled to thermal bath of SHO's or anharmonic oscillators. High temperature: $\left\langle (\int_0^\beta d\tau (d\theta/dt))^2 \right\rangle$ exponentially small but really rotor should spin like crazy.

But they do have something to do with each other.
Minkowski quantity is $\lim_{\omega \to 0} T \frac{d}{d\omega} \sigma_{F\tilde{F},F\tilde{F}}(\omega, k=0)$ (spectral func)

$$\int d^3x \left\langle \langle F\tilde{F}(x,\tau)F\tilde{F}(0,0) \right\rangle_{\rm eucl} = \int_0^\infty \frac{d\omega}{\pi} \frac{\sigma_{F\tilde{F},F\tilde{F}}(\omega)}{\omega} \frac{\omega \cosh \omega(\tau - \beta/2)}{\sinh \omega\beta/2}$$

Can I invert this relation?

Inversion is Hard!



Each τ gives *same* info about $\omega \sim 0$ region.

Difference between peak, smooth feature at $\omega \lesssim T$ invisible!

Can I even do the lattice measurement?

Technically difficult: Reflection positivity:

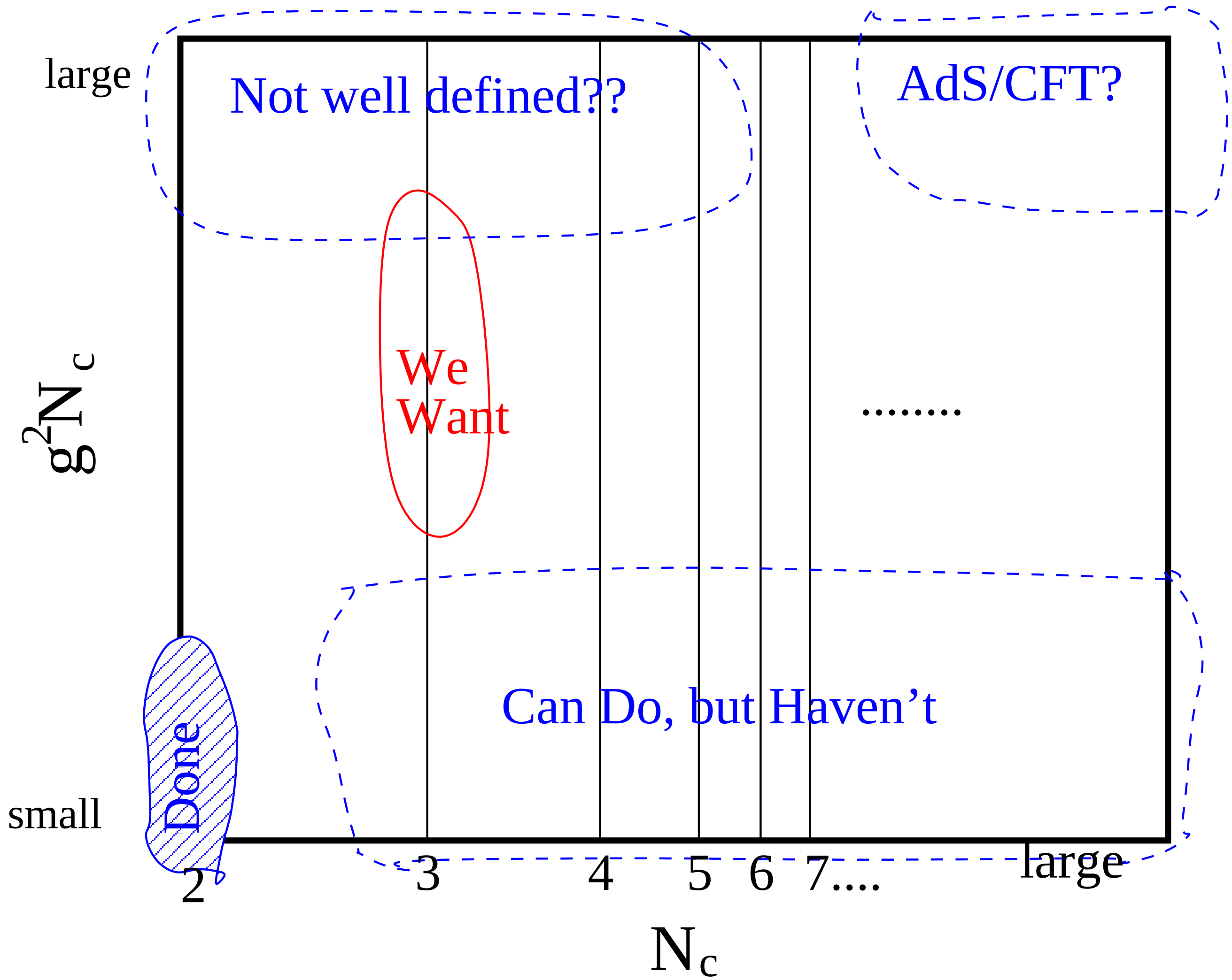
$$\int dx \left\langle \frac{g^2}{32\pi^2} F_{\mu\nu} \tilde{F}^{\mu\nu}(x,t) \frac{g^2}{32\pi^2} F_{\alpha\beta} \tilde{F}^{\alpha\beta}(0,0) \right\rangle \; < \; 0 \quad \forall\, t \neq 0$$

Need to know local overlap value accurately!

No lattice definition of $F\tilde{F}$ is topological; topology only recovered in $\int d^4x$ limit. Spectral function approach requires you make sense of x, t dependence of correlation function – contaminated by nontopological operators.

My opinion: Euclidean techniques are not available.

The $N_c - g^2N_c$ Plane



Large Coupling

D. Son and A. Starinets hep-th/0205051 calculated $\Gamma_{\rm sphal}$ in $\mathcal{N}{=}4$ SYM theory in the $N_{\rm c} \to \infty$, $(g^2 N_{\rm c}) \gg 1$ limit and found

$$\Gamma_{\rm sphal}(\mathcal{N}{=}4{\rm SYM}) = \frac{(g^2 N_{\rm c})^2}{256\pi^3} T^4 \,.$$

Note, $\chi_Q \sim N_{\rm c} N_{\rm f} T^2$, $\Gamma_{\rm sphal}/(\chi_Q T^2) \ll 1$.

Guess: behavior at reasonable $N_{\rm c}$, $g^2 N_{\rm c}$ similar.

Plausible that at $N_{\rm c} \sim 3$, $g^2 N_{\rm c} \sim 1$, $\Gamma_{\rm sphal}/(\chi_Q T^2) \sim 1$.
(May not be well defined in that case ...)

Weak coupling?

Sphaleron rate is nonperturbative physics:
$g^2 \int (\text{something}) = 1$.

Only way to get nonperturbative physics at weak coupling is IR large-occupancy effects.

Large occupancy:

$$\frac{1}{2} + n_b(\omega) = \frac{T}{\hbar\omega} + \frac{1}{12}\frac{\hbar\omega}{T} + \ldots$$

Classical equipartition value T/ω. IR physics classical.

Evolution of classical fields determines $\Gamma_{\rm sphal}$.

Grigoriev Rubakov, NPB 299 p 67; Grigoriev Rubakov Shaposhnikov, PLB216 p 172

only it's not that simple

dynamics of soft nonperturbative fields *influenced* by everybody else, who are not soft.

Argument:

(Arnold Son Yaffe hep-ph/9609481, hep-ph/9810216; Bödeker hep-ph/9801430; GDM hep-ph/9810313

$$\mathbf{D} \times \mathbf{B} - \mathrm{D}_t \mathbf{E} = \mathbf{J} = \sigma \mathbf{E}$$

First order in time, color conductivity $\sigma \sim T$ dominated by $p \sim T$ "particle" fluctuations.

Volume to get nonpert. physics: $(1/g^2 T)^3$.
Time for fields to evolve: $1/g^4 T$ due to large σ.

$$\Gamma_{\rm sphal} \sim g^{10} T^4 \ln(1/g^2)$$

Three scale problem

Three distinct inverse length scales play a role:

- T scale: where quantum physics kicks in
- gT scale: where screening effects get big
- g^2T scale: nonperturbative physics of interest.

Classical approx: $T \gg g^2T$.
Large medium effects: $gT \gg g^2T$.

Classical simulations *require* $T \gg g^2T$.
Relative value of gT, g^2T controllable

Bödeker Effective Theory

Limit $gT \gg g^2T$. Langevin dynamics.
Theoretically clean – lattice sim. has well defined target

Lattice plus Particles

Choose gT/g^2T ratio by putting in right number of DOF.
Number of DOF is number you put in, plus $1/a$ of effective particles from lattice modes.

gT/g^2T as big as you want. Small only by making a big.

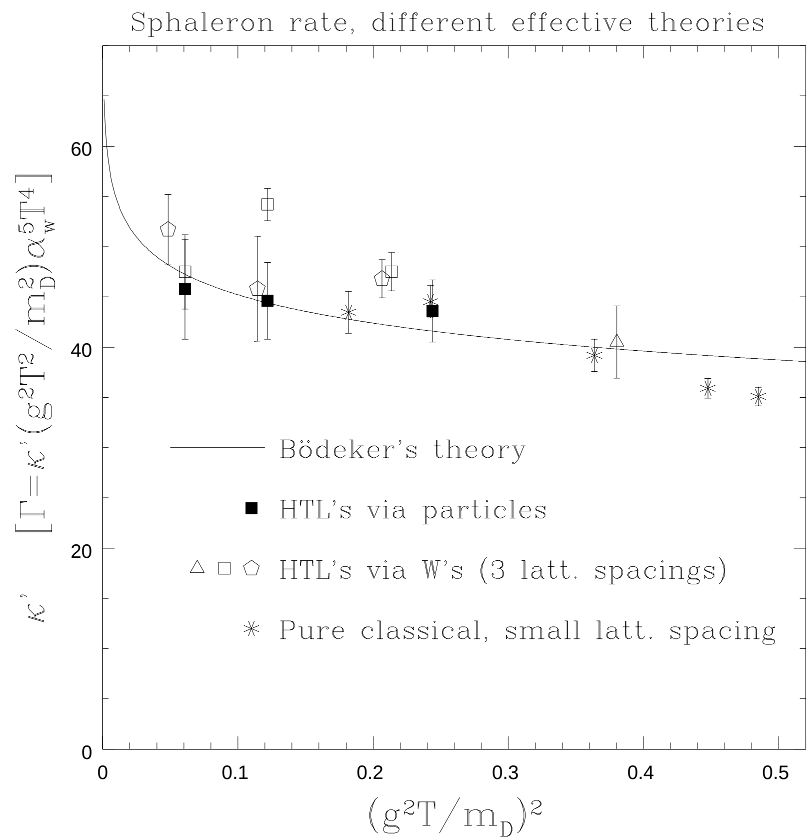
Case of SU(2)

Bödeker's Effective Theory:

$$\Gamma_{\rm sphal}[\mathrm{SU}(2)] = 10.0 \left(\frac{g^2}{4\pi}\right)^5 T^4 \left(3.041 + \ln \frac{2\pi\sqrt{\frac{4+N_{\rm f}}{6}} \ln\ x}{g}\right)$$

Comparison with particle simulations: good agreement.

Can do same for SU(3) but we haven't



Larger $N_{\rm c}$: speculations

I once speculated (hep-ph/0009161) that:

$$\Gamma \sim .24 N_{\rm c}^3 (N_{\rm c}^2 - 1) \left(\frac{N_{\rm c} g^2 T^2}{m_{\rm D}^2} \right) \alpha^5 T^4 \left[\ln \frac{m_{\rm D}}{\frac{g^2 N_{\rm c}}{4\pi} \ln x} + 3.041 \right] .$$

Argument was that $N_{\rm cs}$ fluctuates randomly inside regions of correlation length $2\pi / g^2 N_{\rm c} T$.

Problem: region too small to hold a sphaleron.
Get further and further from $\Delta N_{\rm cs} = 1$ as $N_{\rm c}$ grows.
Picture right? Or is there large $N_{\rm c}$ suppression?

Need to go back and do other $N_{\rm c}$ values!

Larger couplings?

α_s is not small like α_w is.
Need to treat not-so-small couplings.

Lattice classical theory: large α_s means fewer UV DOF.
Bödeker theory breaks down first.
Lattice modes themselves: forced to coarse lattices.

Limit of lattice coarseness: topology, IR description break down. Worth exploring but large error bars.

Still larger couplings?

What if $\alpha_{\rm s} \sim 1$? (Realistic case!)

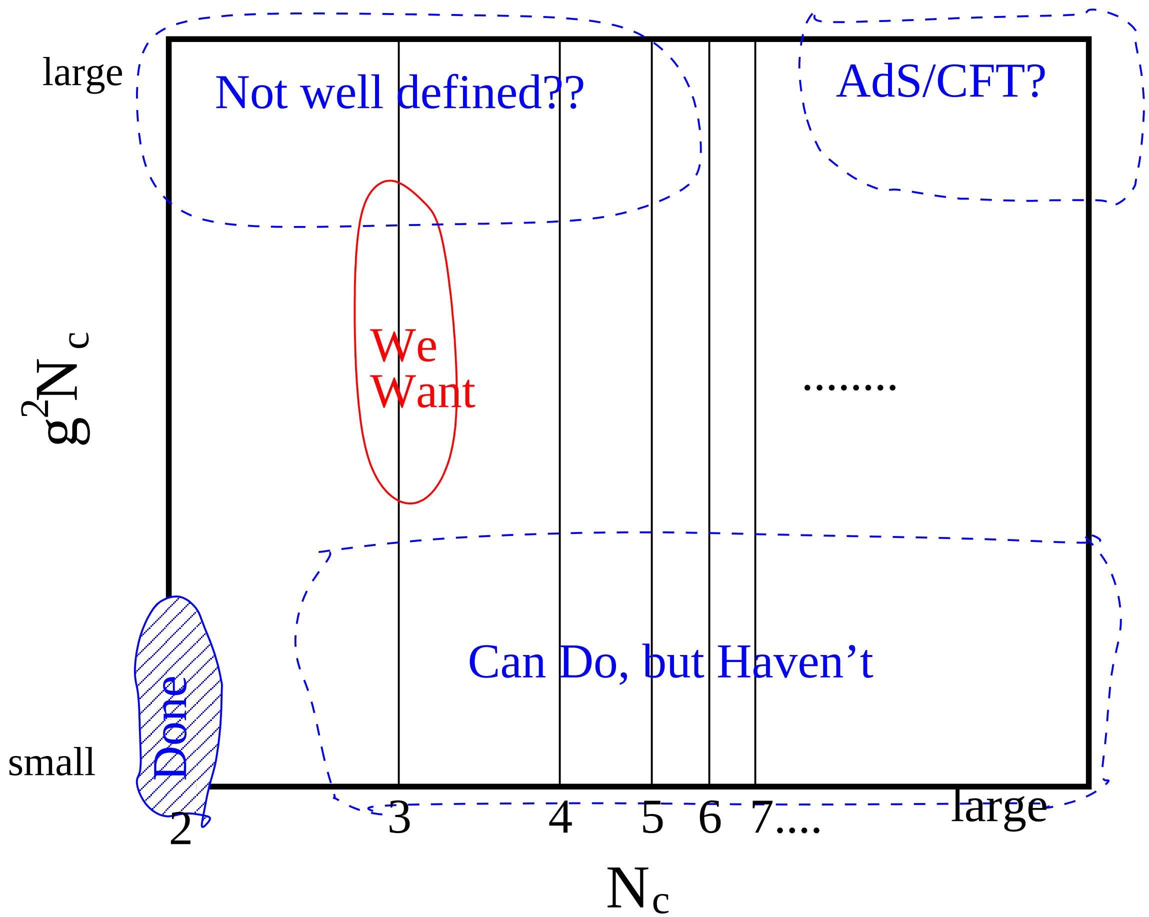
No classical description. Need another idea.

Effective theories? Models? Terra Incognita

Far from equilibrium system?

Good question, no answer. Sphaleron rate may not be well defined, doesn't characterize system well. Very weak coupling may be possible, stronger coupling also Terra Incognita

Conclusions



Should do weak coupling region. Need ideas in Red Circle!