Confinement and chiral symmetry breaking in Landau gauge QCD

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C.F., F. Llanes-Estrada, R. Alkofer, K. Schwenzer, in preparation.

C. Kellermann and C.F., arXiv:0801.2697

C.F. and J. M. Pawlowski, Phys. Rev. D **75** (2007) 025012.

C.F., J.Phys.G32:R253-R291 (2006).

Outline

- **Introduction**
- Infrared properties of SU(N) Yang-Mills theory
- Infrared slavery in quenched QCD

Propagators of QCD: Covariant Gauge

★ Faddeev-Popov method: quark, gluon, ghost

$$\mathcal{Z}_{QCD} = \int \mathcal{D}[A, c, \Psi] \exp\left\{ \int \bar{\Psi} \left(i \not{D} - m \right) \Psi - \frac{1}{4} \left(F^a_{\mu\nu} \right)^2 + \frac{(\partial^\mu A^a_\mu)^2}{2\xi} + \bar{c}^a (-\partial^\mu D_\mu) c^a \right\}$$

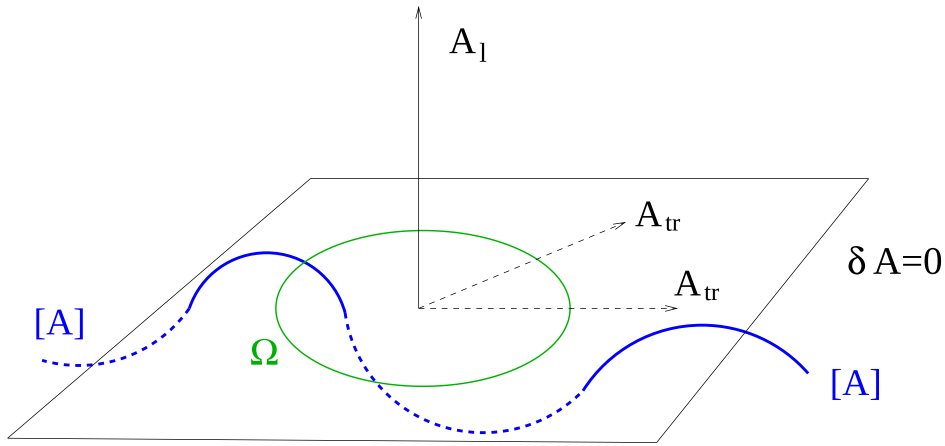
★ Landau gauge propagators in momentum space,

$$D^{\text{Gluon}}_{\mu\nu}(p) = \frac{\mathbf{Z}(p^2)}{p^2} \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right)$$

$$D^{\text{Ghost}}(p) = -\frac{\mathbf{G}(p^2)}{p^2}$$

$$S^{\text{Quark}}(p) = \frac{Z_f(p^2)}{-i\not{p} + M(p^2)}$$

Gauge fixing and Horizon condition



- Problem with gauge fixing: Gribov-copies
- (Partial) Solution: Integrate only over gauge configurations in Gribov region Ω:

$$\Omega = \{A: \ \partial A = 0 \ \wedge \ -\partial D \geq 0\}$$

- $G(p^2) \stackrel{p^2 \to 0}{\longrightarrow} \infty\,, \quad \frac{Z(p^2)}{p^2} \stackrel{p^2 \to 0}{\longrightarrow} 0$ Horizon conditions

D. Zwanziger, Phys. Rev. **D69** (2004) 016002.

Confinement a la Kugo-Ojima

- ▷ BRST-charge Q_B defines physical subspace:

$$\mathcal{V}_{phys} \;=\; \{|phys\rangle : Q_B|phys\rangle = 0\}$$

- ▷ No cluster decomposition in $\mathcal{V}_{phys}$
- ▷ Conserved color charge $Q^a \Rightarrow \langle \mathrm{phys}|Q^a|\mathrm{phys}'\rangle = 0$

T. Kugo and I. Ojima, Prog. Theor. Phys. **66**,1 (1979)

Landau gauge: Well defined $Q^a \Leftrightarrow \left(G(p^2) \stackrel{p^2 \to 0}{\longrightarrow} \infty \right)$

T. Kugo, arXiv:hep-th/9511033.

1PI Green's functions

Dressing functions of propagators of QCD,
$G(p^2)$, $Z(p^2)$, $M(p^2)$, $Z_f(p^2)$

- are connected to Kugo-Ojima/Gribov-Zwanziger:

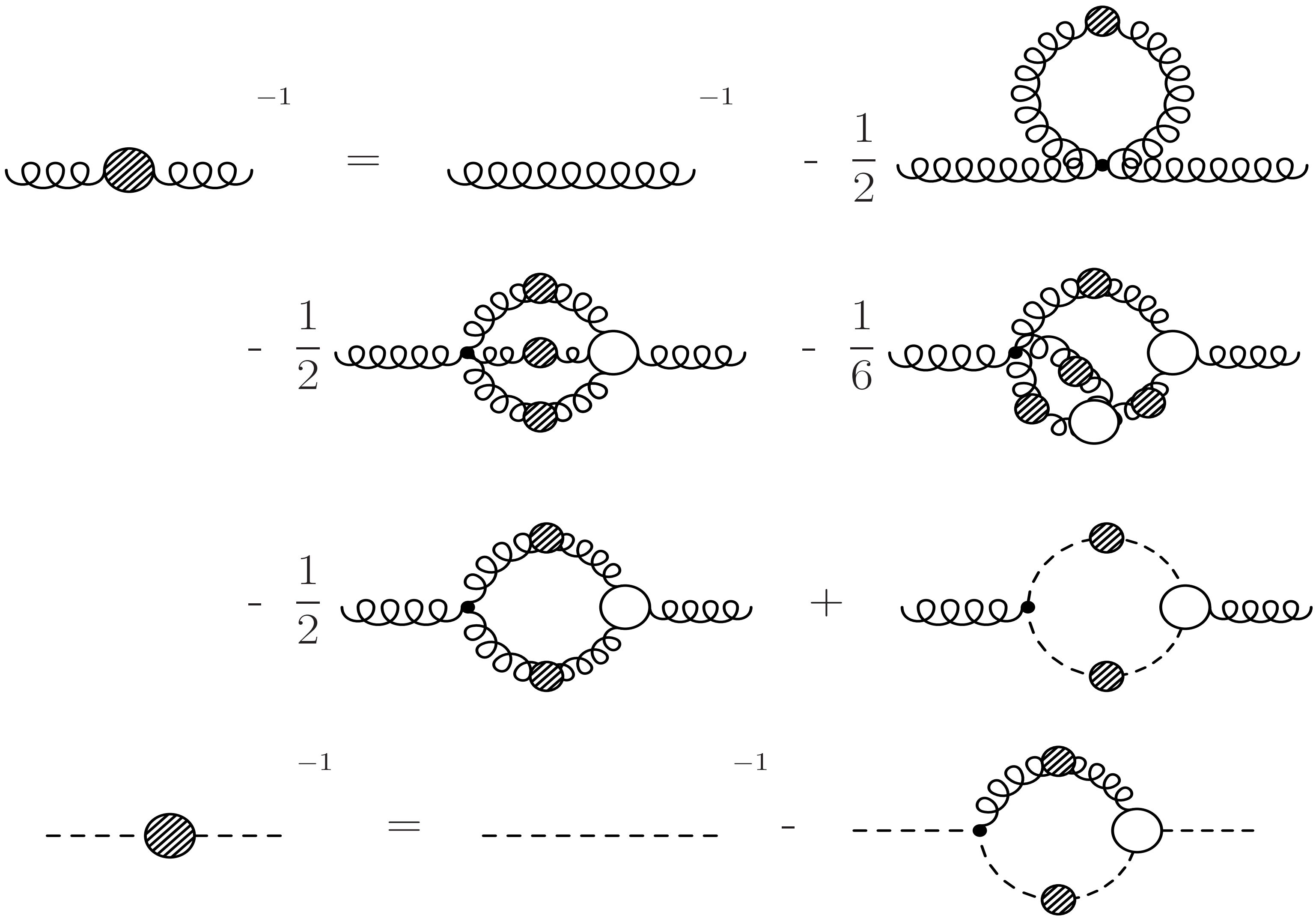
$$G(p^2) \stackrel{p^2\to 0}{\longrightarrow} \infty \,, \qquad\qquad \frac{Z(p^2)}{p^2} \stackrel{p^2\to 0}{\longrightarrow} 0$$

- determine running coupling
- indicate dynamical chiral symmetry breaking
- are ingredients for hadron phenomenology

Outline

- Introduction
- **Infrared properties of SU(N) Yang-Mills theory**
- Infrared Slavery in quenched QCD

Dyson-Schwinger equations (DSEs)



General Infrared Structure

Scaling analysis: One external scale $p^2 << \Lambda_{QCD}$

- Dressing function with n external ghost legs and m external gluon legs:

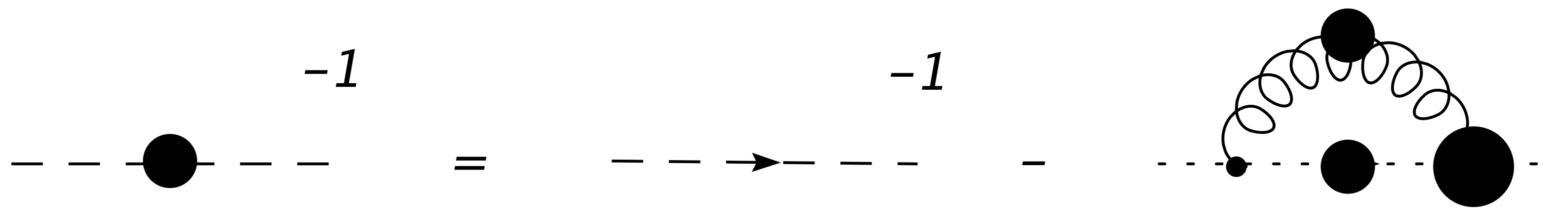
$$\Gamma^{n,m}(p^2) \sim (p^2)^{(n/2-m)\kappa}$$

R. Alkofer, C. F., F. Llanes-Estrada, Phys. Lett. B **611** (2005)

- solves whole tower of DSEs
- solves STIs

Infrared Structure of YM-theory I

- Example: Ghost-propagator



- Selfconsistency:

$$\Gamma^{n,m}(p^2) \sim (p^2)^{(n/2-m)\kappa}$$

$$G(p^2) \sim (p^2)^{-\kappa}\,, \quad Z(p^2) \sim (p^2)^{2\kappa}\,, \quad \Gamma^{2,1}(p^2) \sim (p^2)^0$$

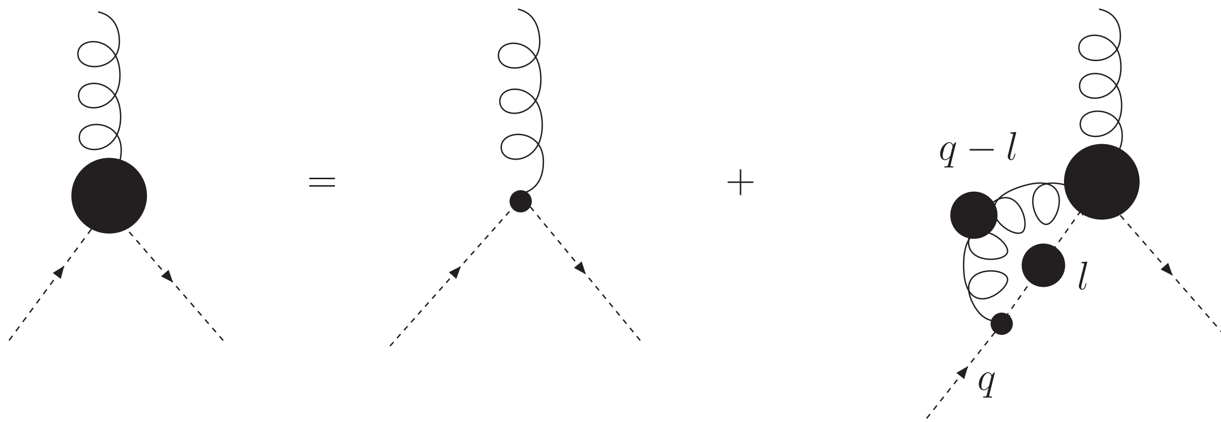
L. v. Smekal, A. Hauck, R. Alkofer, Phys. Rev. Lett. **79** (1997) 3591

C. Lerche, L. v. Smekal, Phys. Rev. D **65** (2002) 125006.

D. Zwanziger Phys. Rev. D **65** (2002) 094039.

Infrared Structure of YM-theory II

- Example: Ghost-Gluon-Vertex



- Selfconsistency:

$$G(p^2) \sim (p^2)^{-\kappa}\,, \quad Z(p^2) \sim (p^2)^{2\kappa}\,, \quad \Gamma^{2,2}(p^2) \sim (p^2)^{-\kappa}$$

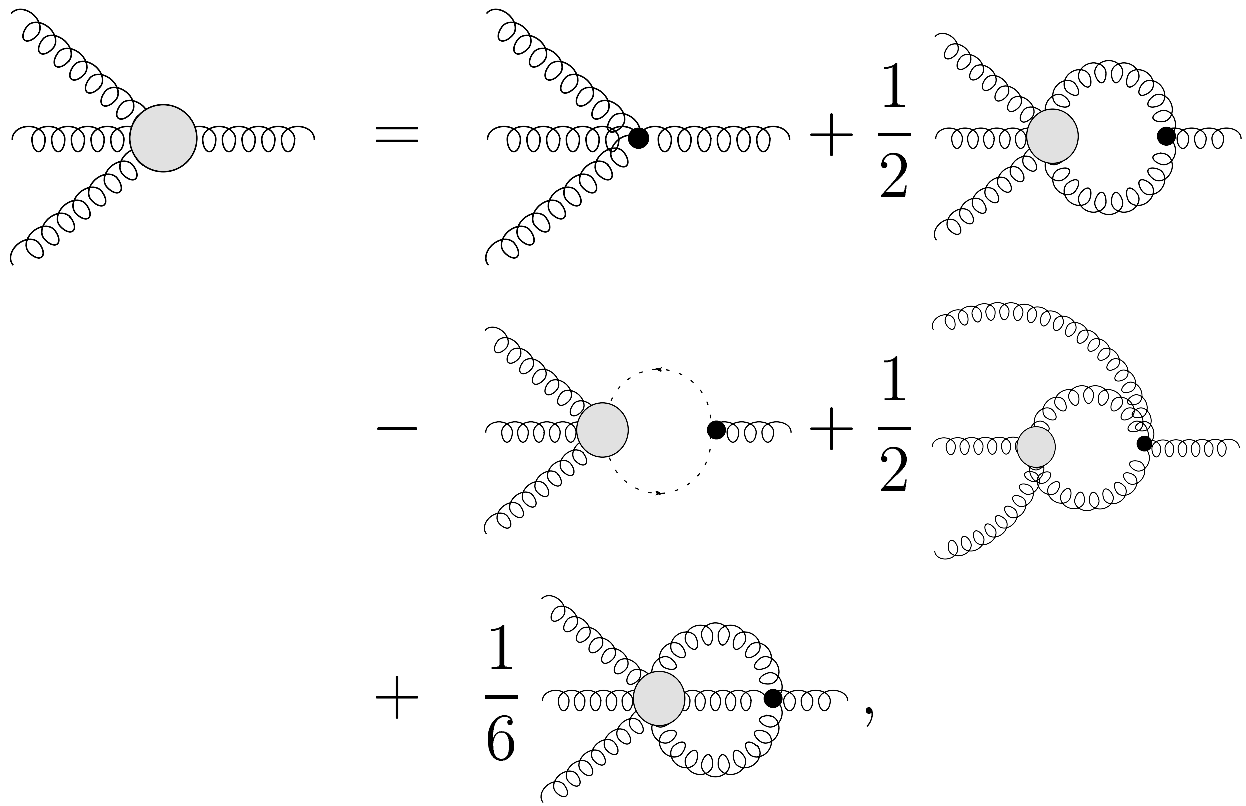
$$\Gamma^{2,1}(p^2) \sim (p^2)^{0}$$

J. C. Taylor, Nucl. Phys. B **33** (1971) 436.

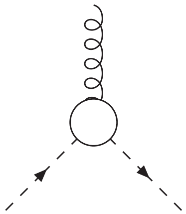
A. Cucchieri, A. Maas and T. Mendes, PRD **74** (2006) 014503, arXiv:0803.1798

Infrared Structure of YM-theory III

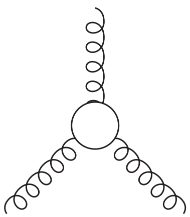
- Four-gluon vertex:



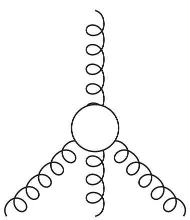
Running Coupling: IR-Universality



$$\alpha^{gh-gl}(p^2) \;=\; \alpha_\mu\, G^2(p^2)\, Z(p^2) \;\sim\; \mathbf{const}/N_c$$



$$\alpha^{3g}(p^2) \;=\; \alpha_\mu\, [\Gamma^{3g}(p^2)]^2\, Z^3(p^2) \;\sim\; \mathbf{const}/N_c$$



$$\alpha^{4g}(p^2) \;=\; \alpha_\mu\, \Gamma^{4g}(p^2)\, Z^2(p^2) \;\sim\; \mathbf{const}/N_c$$

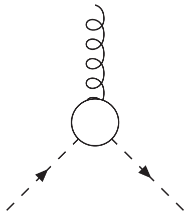
with

$$\Gamma^{3g}(p^2) \sim (p^2)^{-3\kappa}\,, \quad \Gamma^{4g}(p^2) \sim (p^2)^{-4\kappa}$$

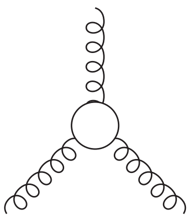
$$G(p^2) \sim (p^2)^{-\kappa}\,, \quad Z(p^2) \sim (p^2)^{2\kappa}$$

R. Alkofer, C. F., F. Llanes-Estrada, Phys. Lett. B **611** (2005)

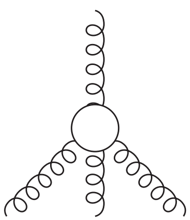
Running Coupling: IR-fixed points



$$\alpha^{gh-gl}(p^2) \;=\; \alpha_\mu\, G^2(p^2)\, Z(p^2) \;\sim\; \mathbf{8.92}/N_c$$



$$\alpha^{3g}(p^2) \;=\; \alpha_\mu\, [\Gamma^{3g}(p^2)]^2\, Z^3(p^2) \;\sim\; \mathbf{const}/N_c$$

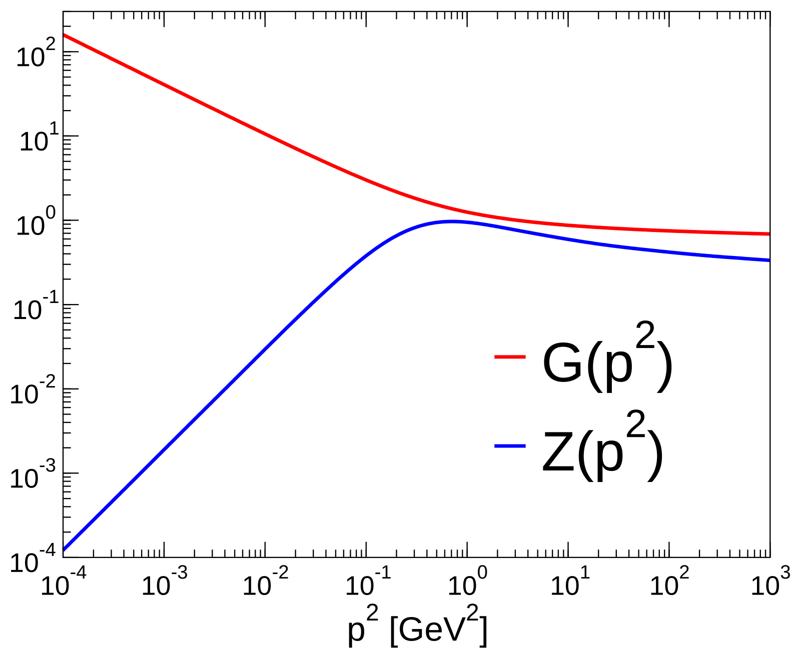


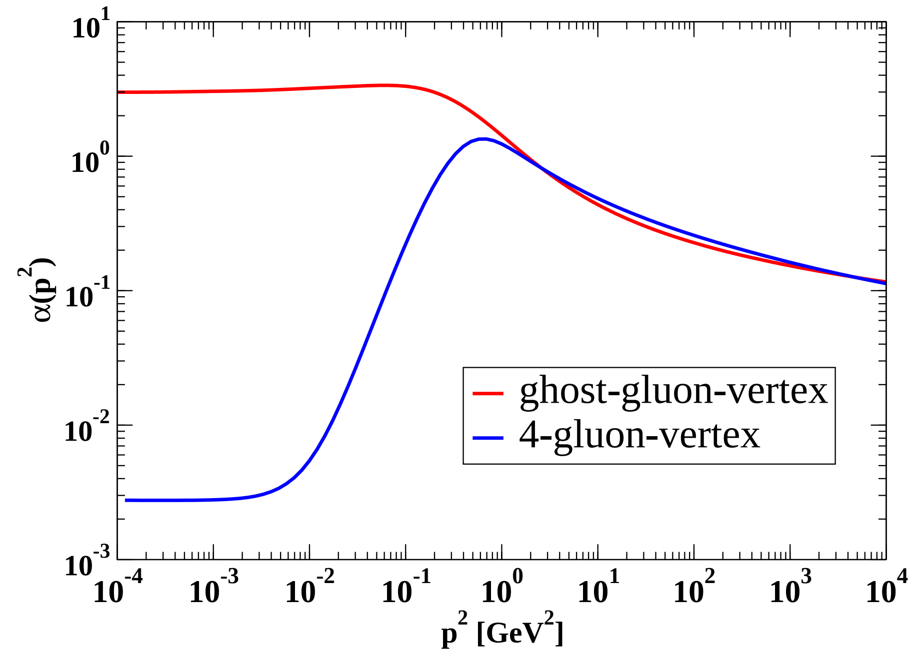
$$\alpha^{4g}(p^2) \;=\; \alpha_\mu\, \Gamma^{4g}(p^2)\, Z^2(p^2) \;\sim\; \mathbf{0.0086}/N_c$$

Lerche and Smekal, PRD **65** (2002) 125006

Kellermann and CF, arXiv:0801.2697

Ghost, Glue and Coupling





CF and Alkofer, PLB 536 (2002) 177.

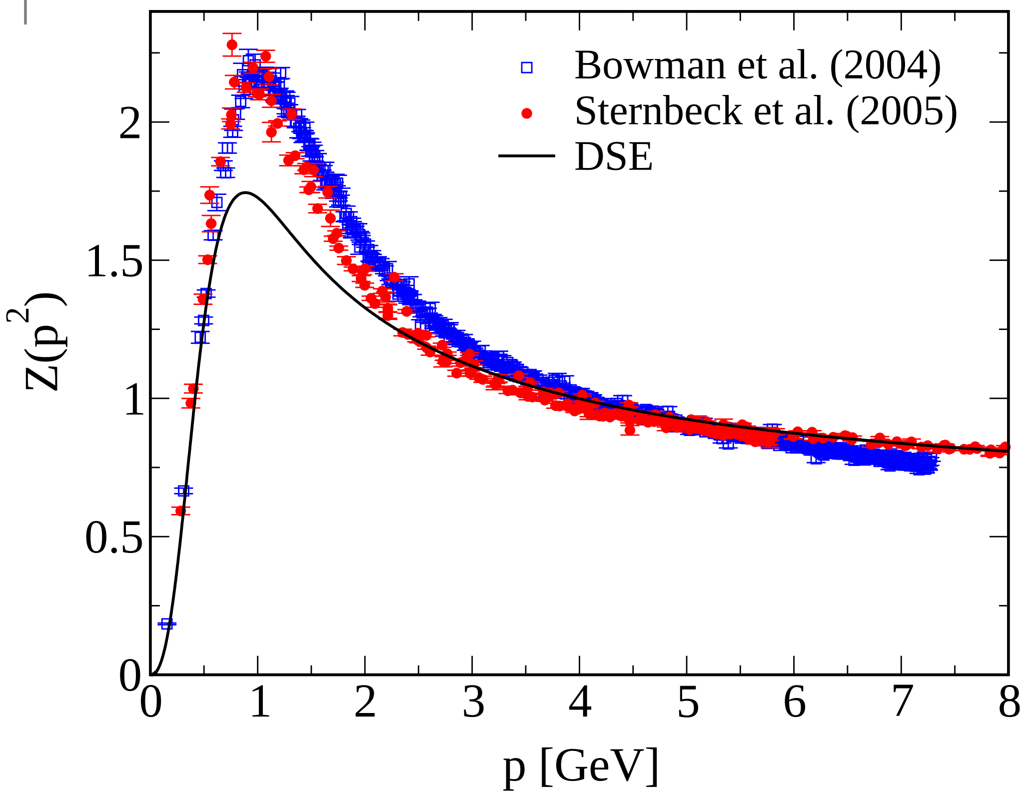
Kellermann and CF, arXiv:0801.2697

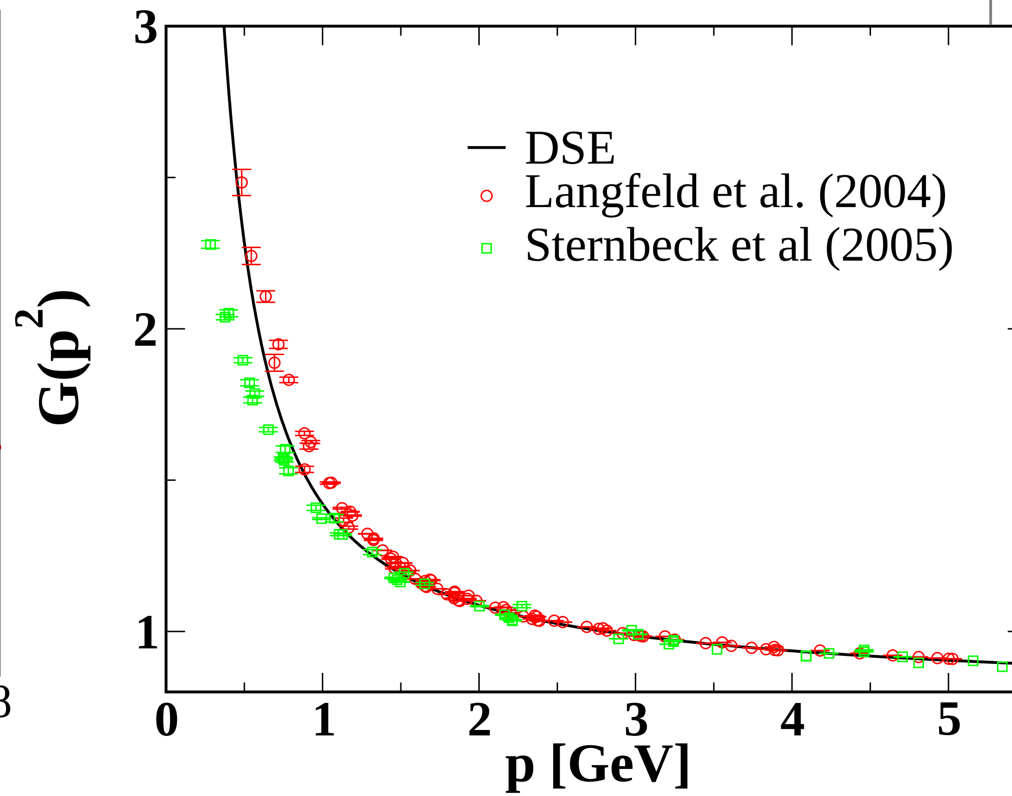
- IR: $G(p^2) \sim (p^2)^{-\kappa}$, $Z(p^2) \sim (p^2)^{2\kappa}$, $\Gamma^{4g} \sim (p^2)^{-4\kappa}$

$$\kappa \approx 0.595$$

(dependent on truncation!)

DSEs vs Lattice

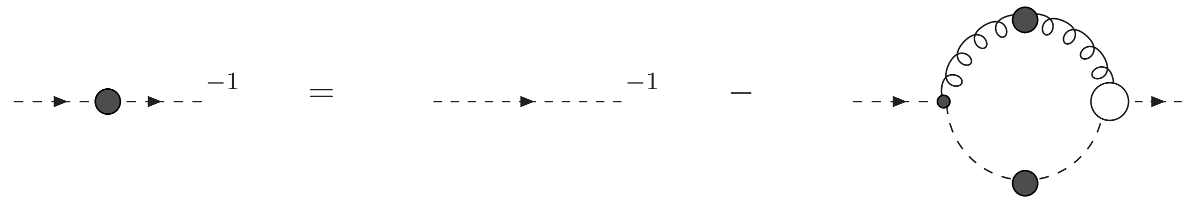




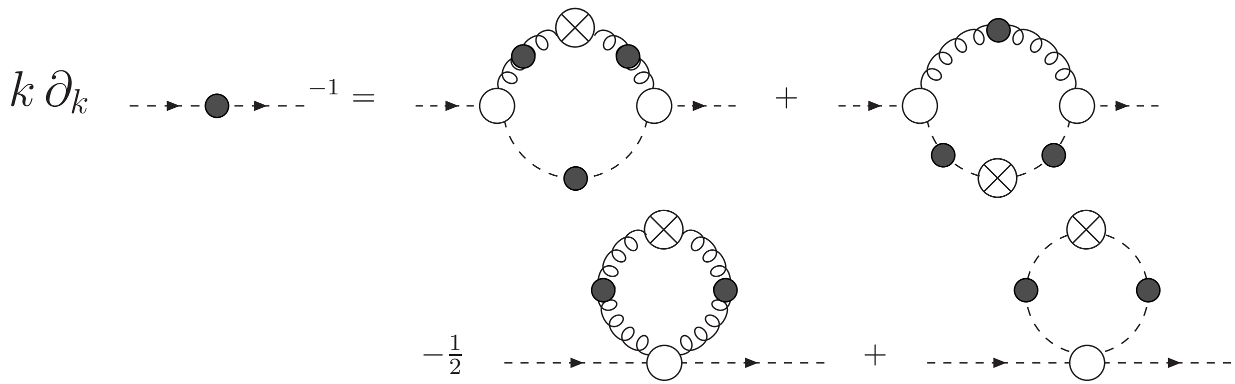
- Physical p^2: Systematic improvement possible for **DSEs**
- Deep infrared: Subtle problems cp Continuum vs **Lattice**
  Discussed in other talks

Uniqueness of IR-solution I

Ghost-DSE:



Ghost-FRG:



IR-Analysis of whole tower of equations $\Rightarrow$

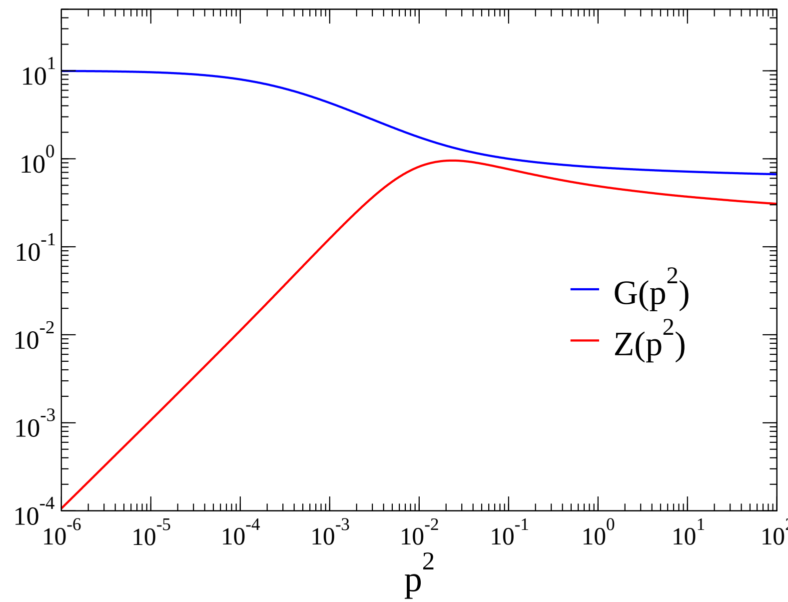
$$\Gamma^{n,m}(p^2) \sim (p^2)^{(n/2-m)\kappa}$$

is unique scaling solution.

C.F. and J. M. Pawlowski, Phys. Rev. D **75** (2007) 025012.

Exception: decoupling scenario

Decoupling: Massive gluon propagator;
all other Green's functions IR finite



- Renormalisation condition(s), $G(\mu), Z(\mu)$ crucial!

CF, in prep.

Boucaud, Leroy, Yaouanc, Micheli, Pene, Rodriguez-Quintero, arXiv:0801.2721.

Aguilar, Binosi,Papavassiliou, arXiv:0802.1870.

Decoupling scenario - Interpretation

- $\left(G(p^2) \xrightarrow{p^2 \to 0} \infty \right) \Leftrightarrow$ Unbroken Q^a
- $G(p^2) = (p^2)\langle FP^{-1} \rangle$

	Confinement	Higgs phase
global gauge symm.	unbroken	broken
Eigenvalue density of FP at $\lambda = 0$	enhanced	not enhanced
$G(p^2 \to 0)$	∞	finite

Greensite, Olejnik and Zwanziger, PRD **69** (2004) 074506, JHEP **0505** (2005) 070.

Outline

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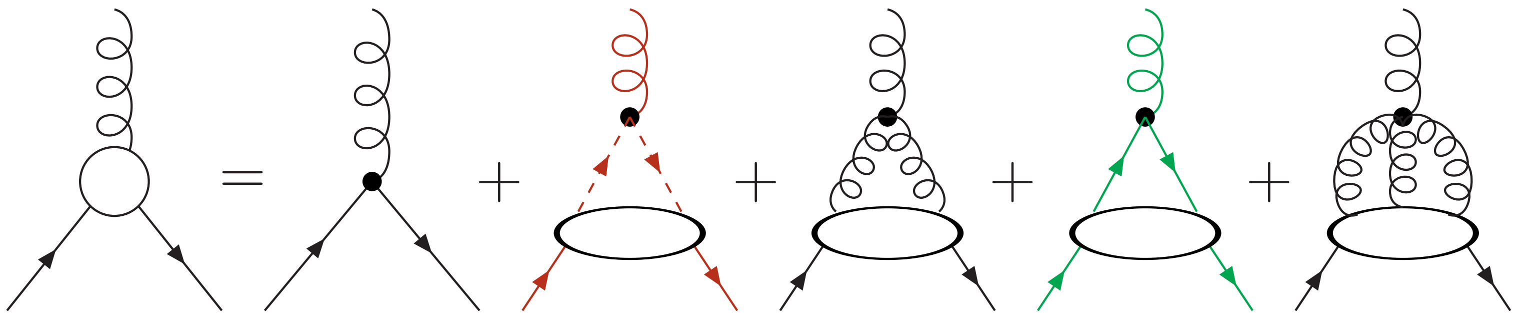
- **Infrared Slavery in quenched QCD**

Infrared Structure of QCD I

Quark-gluon vertex:



- Quark diagram: Hadronic contributions ('unquenching')
  $\rightarrow$ **Talk of Richard Williams**

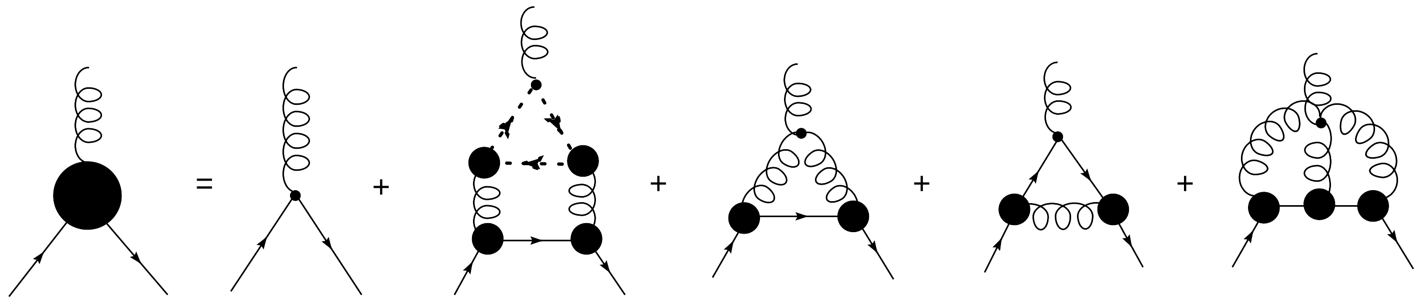
  C.F, D. Nickel and J. Wambach, PRD **76** (2007) 094009

- Ghost diagram: Infrared leading

  R. Alkofer, C.F., F. Llanes-Estrada, hep-ph/0607293.

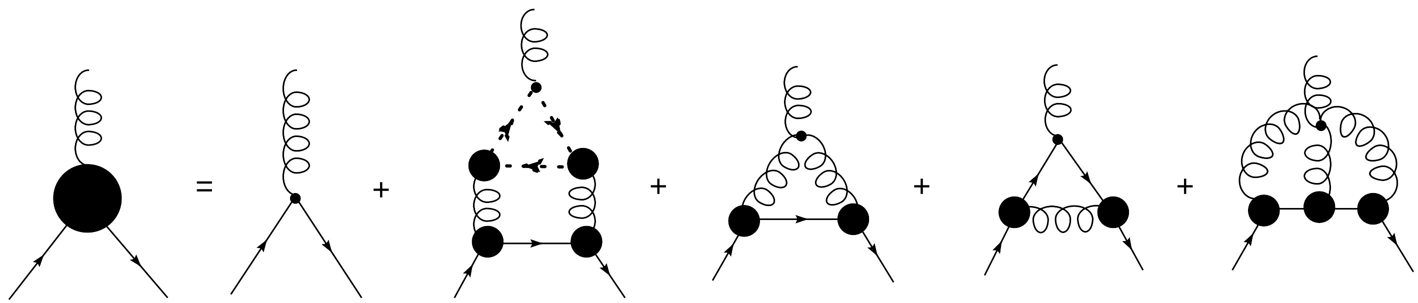
Infrared Structure of QCD II

- Quark-gluon vertex: **lowest order** in skeleton expansion



Infrared Structure of QCD II

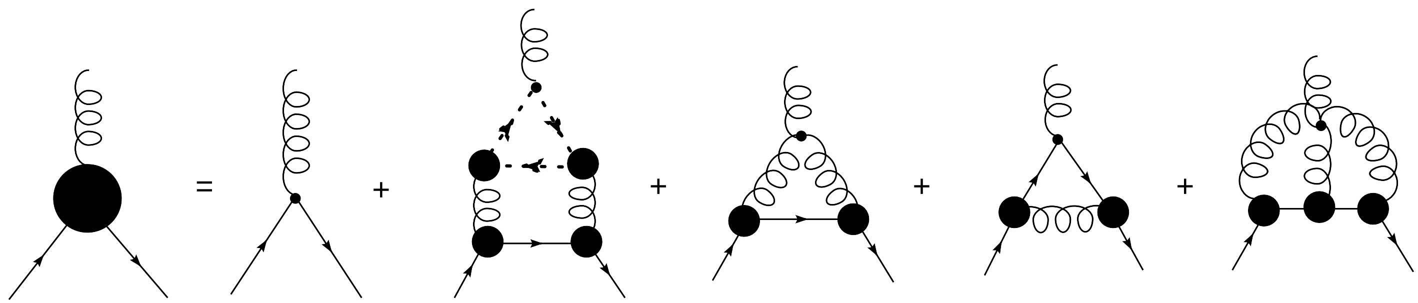
- Quark-gluon vertex: **lowest order** in skeleton expansion



$$S(p) = i\not{p}\,\frac{Z_f}{p^2 + M^2} + \frac{Z_f M}{p^2 + M^2}$$

Infrared Structure of QCD II

- Quark-gluon vertex: **lowest order** in skeleton expansion

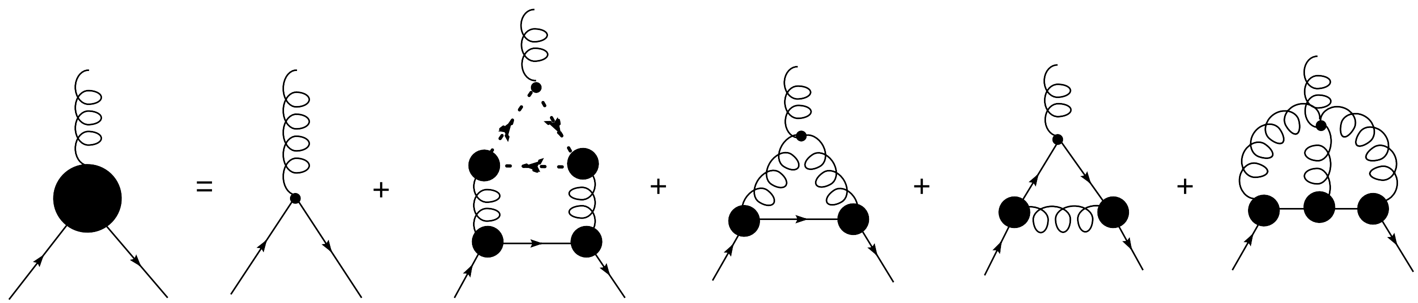


$$S(p) = i\not{p}\,\frac{Z_f}{p^2+M^2} + \frac{Z_f M}{p^2+M^2}$$

$$\Gamma_\mu = ig\sum_{i=1}^{4}\lambda_i G^i_\mu \ : \quad G^1_\mu = \gamma_\mu\,, \ \ G^2_\mu = \hat{p}_\mu\,, \ \ G^3_\mu = \hat{\not{p}}\,\hat{p}_\mu\,, \ \ G^4_\mu = \hat{\not{p}}\,\gamma_\mu$$

Infrared Structure of QCD II

- Quark-gluon vertex: **lowest order** in skeleton expansion

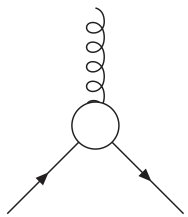


$$S(p) = i\not{p}\,\frac{Z_f}{p^2+M^2} + \frac{Z_f M}{p^2+M^2}$$

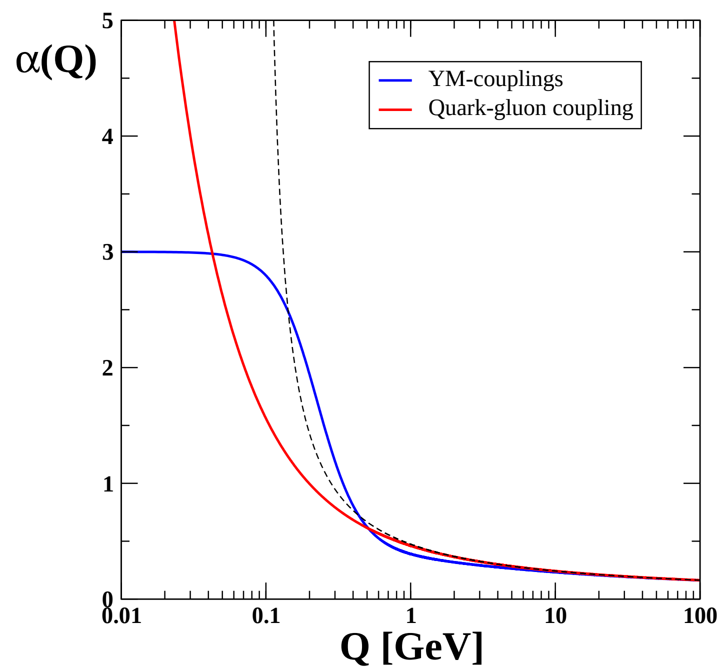
$$\Gamma_\mu = ig\sum_{i=1}^{4}\lambda_i G_\mu^i \;:\quad G_\mu^1=\gamma_\mu\,,\;\; G_\mu^2=\hat{p}_\mu\,,\;\; G_\mu^3=\hat{\not{p}}\,\hat{p}_\mu\,,\;\; G_\mu^4=\hat{\not{p}}\,\gamma_\mu$$

$$\lambda_{1,2,3,4}\sim (p^2)^{-1/2-\kappa} \qquad \leftrightarrow \qquad \lambda_{1,3}\sim (p^2)^{-\kappa}$$

Running Coupling: IR-slavery



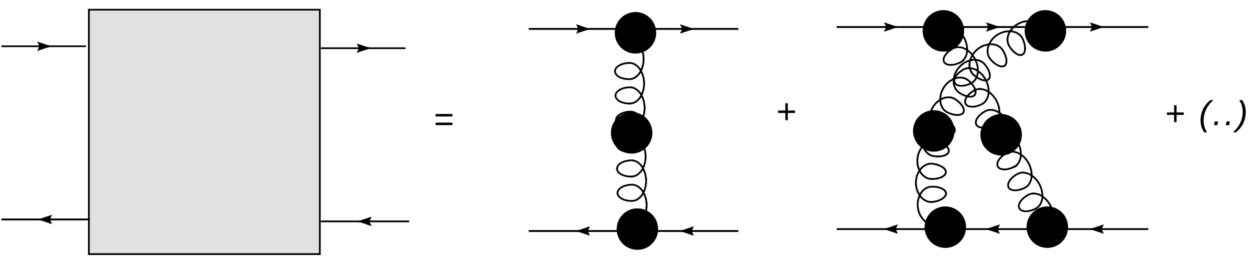
$$\alpha^{qg}(p^2) = \alpha_\mu \, [\Gamma^{qg}(p^2)]^2 \, [Z_f(p^2)]^2 \, Z(p^2) \sim \begin{cases} \frac{1}{p^2} : \chi SB \\ const : \chi S \end{cases}$$



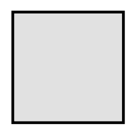
R. Alkofer, C. F., F. Llanes-Estrada, hep-ph/0607293.

The quark-antiquark potential

- quenched QCD

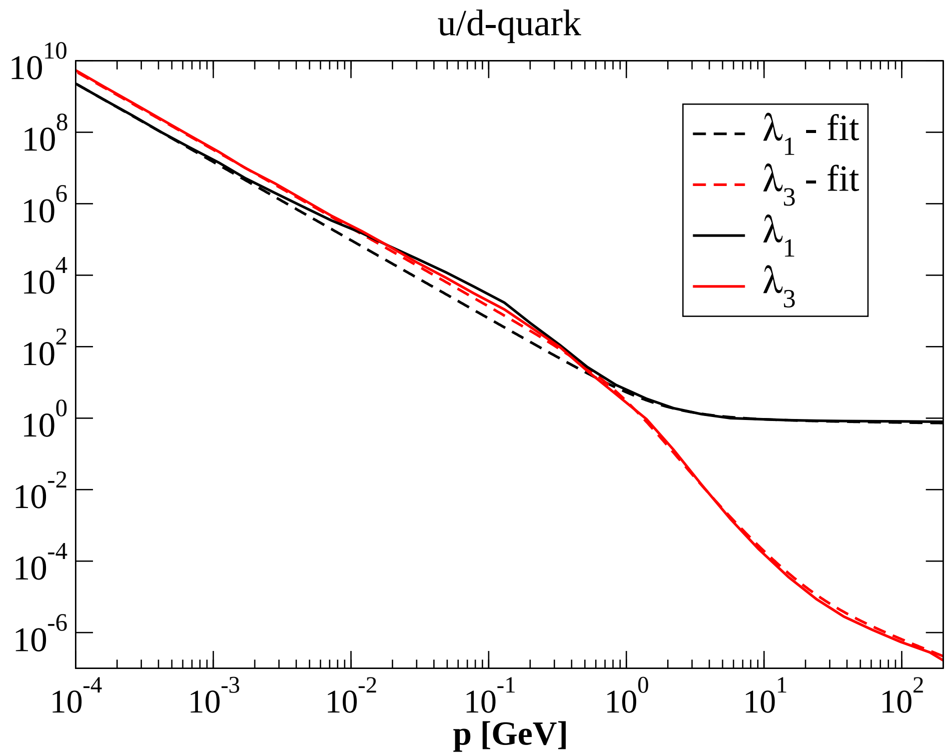


$$\sim \begin{cases} \frac{1}{p^4} & : \ \chi S B \\ \frac{1}{p^2} & : \ \chi S \end{cases}$$

$$V(\mathbf{r}) = \frac{1}{(2\pi)^3} \int d^3p \ e^{i\mathbf{p}\mathbf{r}}$$  $$\sim \begin{cases} |r| & : \ D\chi S B \\ \frac{1}{|r|} & : \ \chi S \end{cases}$$

Quark confinement $\leftrightarrow \chi S B$

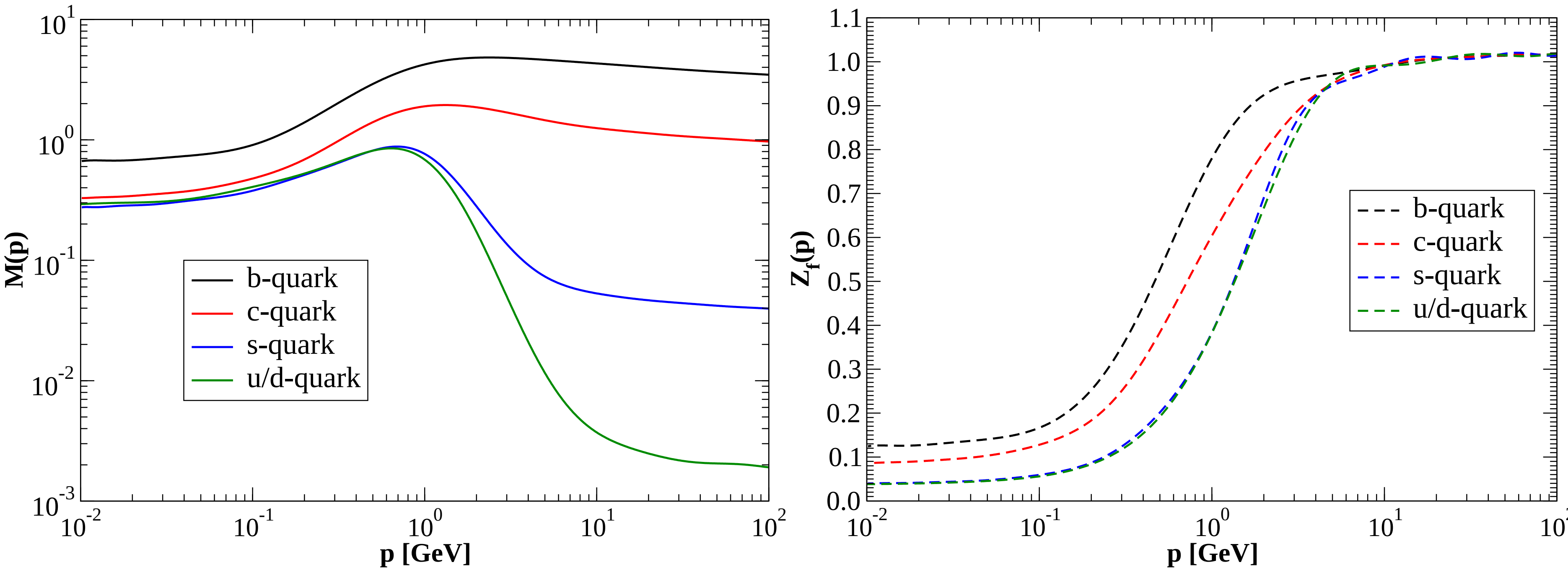
Numerical results: Vertex



- Infrared singularity $(p^2)^{-1/2-\kappa}$

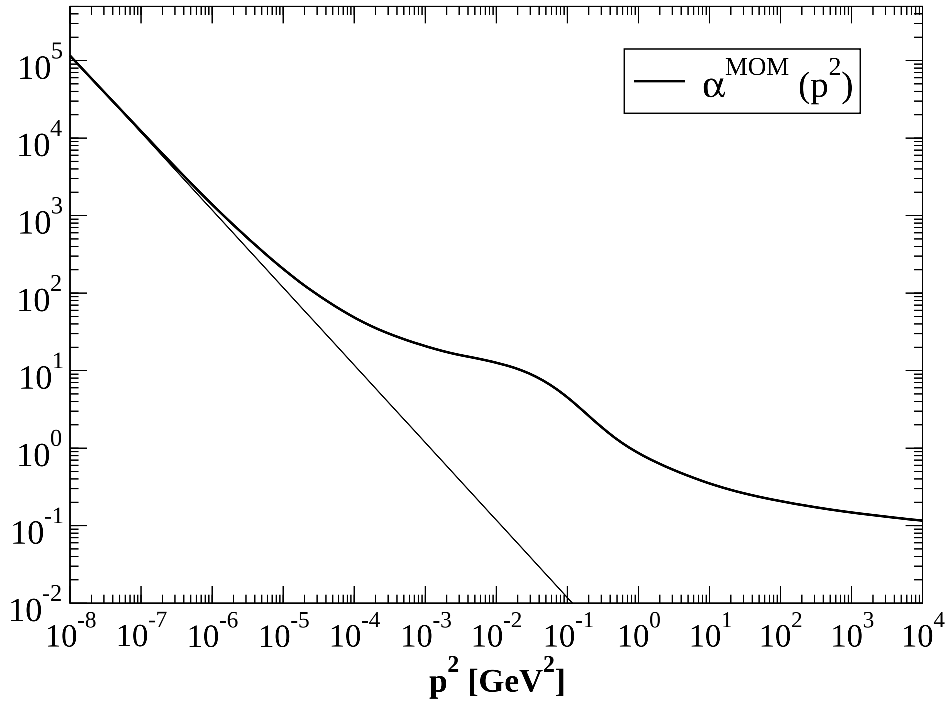
C.F., F. Llanes-Estrada, R. Alkofer, K. Schwenzer, in preparation

Numerical results: Quark propagator



- Interesting mass dependence in IR!

Numerical results: Coupling



- Infrared slavery: $\alpha(p^2) \sim \frac{1}{p^2}$

Summary

Landau gauge Yang-Mills theory:

- 1PI-function with $2n$ ghost and m gluon legs:

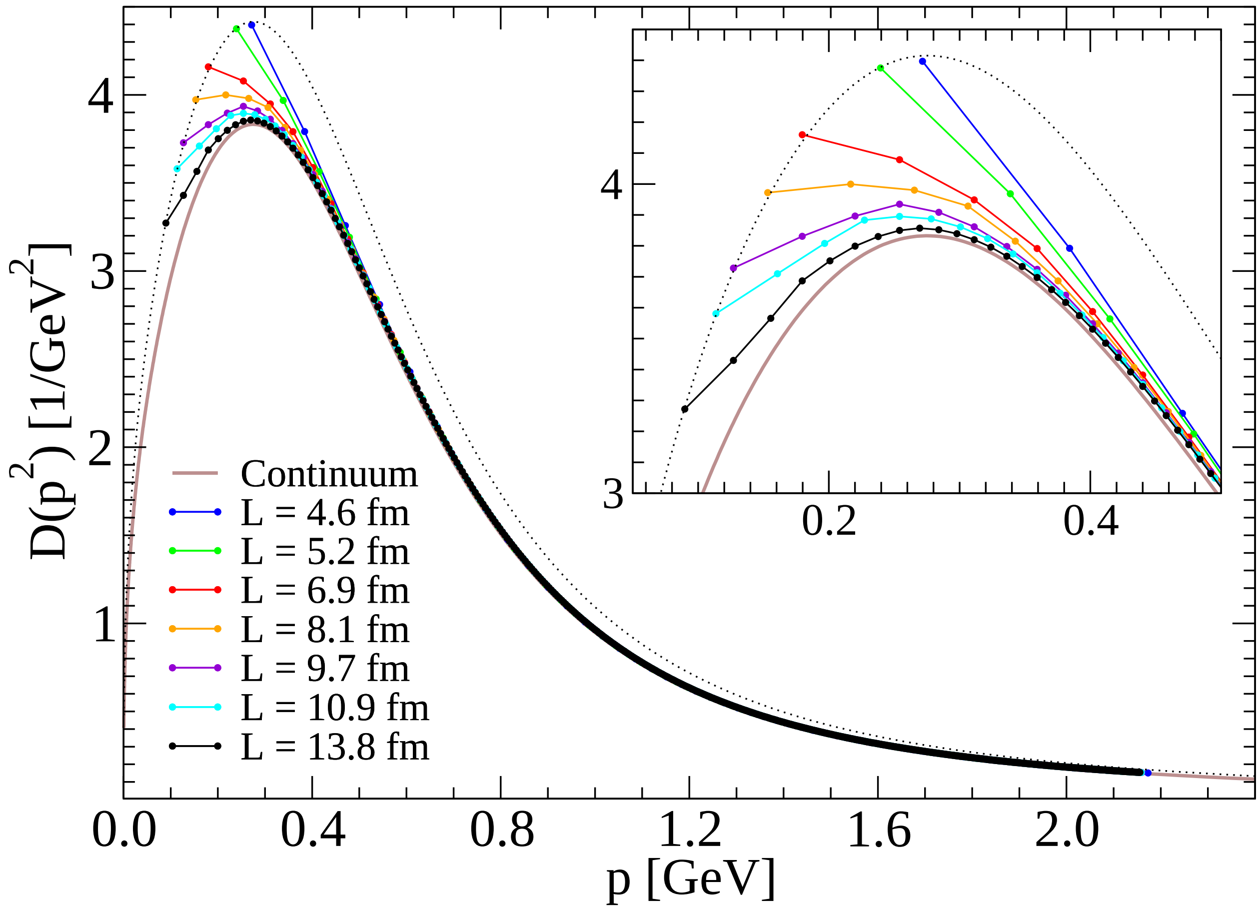
$$\Gamma^{n,m}(p^2) \sim (p^2)^{(n-m)\kappa}$$

- YM-couplings: IR-fixed point

Landau gauge QCD (quenched):

- Quark-gluon-coupling: Infrared slavery
- Quark confinement $\leftrightarrow \chi SB$

DSEs on a torus: volume effects



- Contemporary lattices large enough to exclude large volume effects