

# Confinement and chiral symmetry breaking in Landau gauge QCD

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C.F., F. Llanes-Estrada, R. Alkofer, K. Schwenzer, in preparation.

C. Kellermann and C.F., arXiv:0801.2697

C.F. and J. M. Pawłowski, Phys. Rev. D **75** (2007) 025012.

C.F., J.Phys.G32:R253-R291 (2006).

# Outline

- **Introduction**
- Infrared properties of  $SU(N)$  Yang-Mills theory
- Infrared slavery in quenched QCD

# Propagators of QCD: Covariant Gauge

★ Faddeev-Popov method: quark, gluon, ghost

$$\mathcal{Z}_{QCD} = \int \mathcal{D}[A, c, \Psi] \exp \left\{ \int \bar{\Psi} (i\not{D} - m) \Psi - \frac{1}{4} (F_{\mu\nu}^a)^2 + \frac{(\partial^\mu A_\mu^a)^2}{2\xi} + \bar{c}^a (-\partial^\mu D_\mu) c^a \right\}$$

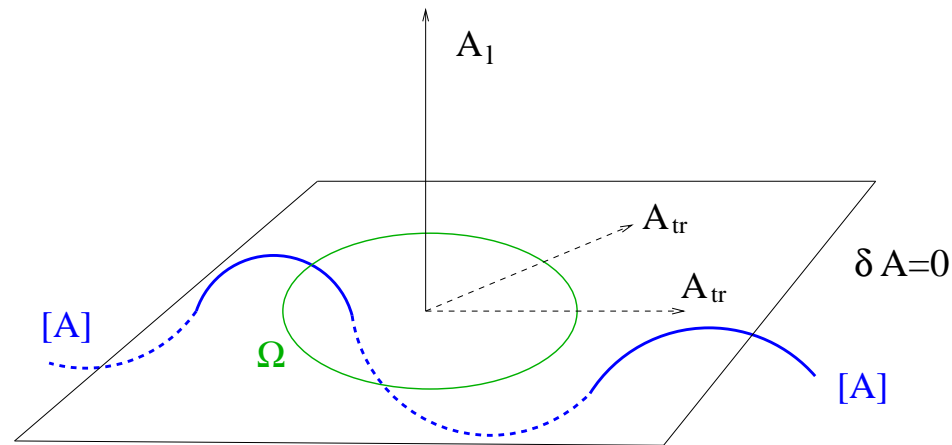
★ Landau gauge propagators in momentum space,

$$D_{\mu\nu}^{\text{Gluon}}(p) = \frac{\mathbf{Z}(p^2)}{p^2} \left( \delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right)$$

$$D^{\text{Ghost}}(p) = -\frac{\mathbf{G}(p^2)}{p^2}$$

$$S^{\text{Quark}}(p) = \frac{Z_f(p^2)}{-i\not{p} + M(p^2)}$$

# Gauge fixing and Horizon condition



- Problem with gauge fixing: Gribov-copies
- (Partial) Solution: Integrate only over gauge configurations in Gribov region  $\Omega$ :

$$\Omega = \{A : \partial A = 0 \wedge -\partial D \geq 0\}$$

- $G(p^2) \xrightarrow{p^2 \rightarrow 0} \infty$ ,  $\frac{Z(p^2)}{p^2} \xrightarrow{p^2 \rightarrow 0} 0$       Horizon conditions

D. Zwanziger, Phys. Rev. D69 (2004) 016002.

# Confinement a la Kugo-Ojima

- ▷ BRST-charge  $Q_B$  defines physical subspace:

$$\mathcal{V}_{phys} = \{ |phys\rangle : Q_B |phys\rangle = 0 \}$$

- ▷ No cluster decomposition in  $\mathcal{V}_{phys}$
- ▷ **Conserved color charge**  $Q^a \Rightarrow \langle phys | Q^a | phys' \rangle = 0$

T. Kugo and I. Ojima, Prog. Theor. Phys. **66**,1 (1979)

**Landau gauge:** Well defined  $Q^a \Leftrightarrow \left( G(p^2) \xrightarrow{p^2 \rightarrow 0} \infty \right)$

T. Kugo, arXiv:hep-th/9511033.

# 1PI Green's functions

Dressing functions of propagators of QCD,

$$G(p^2), Z(p^2), M(p^2), Z_f(p^2)$$

- are connected to Kugo-Ojima/Gribov-Zwanziger:

$$G(p^2) \xrightarrow{p^2 \rightarrow 0} \infty, \quad \frac{Z(p^2)}{p^2} \xrightarrow{p^2 \rightarrow 0} 0$$

- determine running coupling
- indicate dynamical chiral symmetry breaking
- are ingredients for hadron phenomenology

# Outline

- Introduction
- Infrared properties of  $SU(N)$  Yang-Mills theory
- Infrared Slavery in quenched QCD

# Dyson-Schwinger equations (DSEs)

$$\begin{aligned}
 & \overset{-1}{\text{diagram}} = \overset{-1}{\text{diagram}} - \frac{1}{2} \text{diagram} \\
 & - \frac{1}{2} \text{diagram} - \frac{1}{6} \text{diagram} \\
 & - \frac{1}{2} \text{diagram} + \text{diagram} \\
 & \overset{-1}{\text{diagram}} = \overset{-1}{\text{diagram}} - \text{diagram}
 \end{aligned}$$

The diagrams represent various terms in the Dyson-Schwinger equations for the quark and ghost propagators in Landau gauge QCD. The shaded circles represent ghost loops, and the white circles represent quark loops. The lines are either solid (quarks) or dashed (ghosts).



# General Infrared Structure

Scaling analysis: One external scale  $p^2 \ll \Lambda_{QCD}$

- Dressing function with  $n$  external ghost legs and  $m$  external gluon legs:

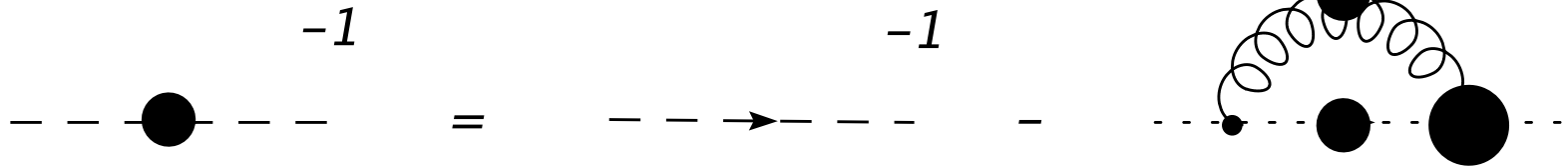
$$\Gamma^{n,m}(p^2) \sim (p^2)^{(n/2-m)\kappa}$$

R. Alkofer, C. F., F. Llanes-Estrada, Phys. Lett. B 611 (2005)

- solves whole tower of DSEs
- solves STIs

# Infrared Structure of YM-theory I

- Example: Ghost-propagator



- Selfconsistency:

$$\Gamma^{n,m}(p^2) \sim (p^2)^{(n/2-m)\kappa}$$

$$G(p^2) \sim (p^2)^{-\kappa}, \quad Z(p^2) \sim (p^2)^{2\kappa}, \quad \Gamma^{2,1}(p^2) \sim (p^2)^0$$

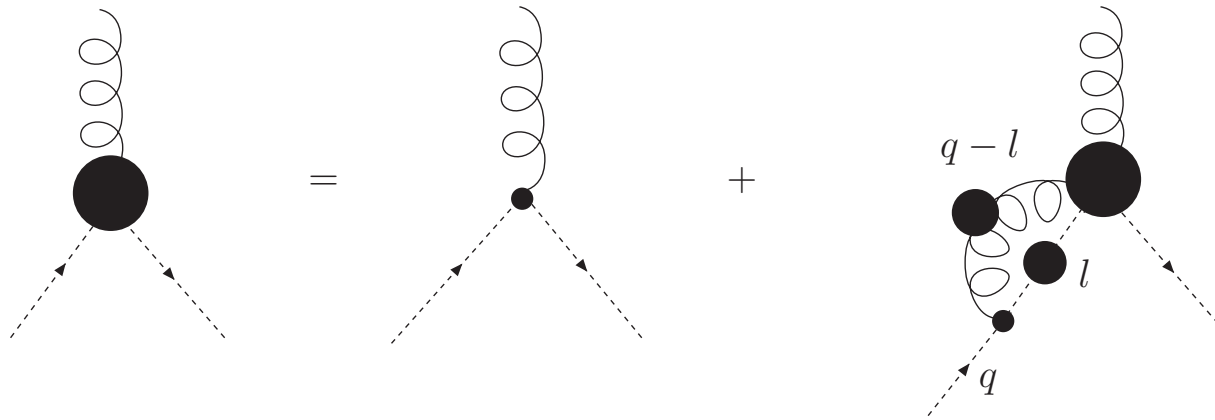
L. v. Smekal, A. Hauck, R. Alkofer, Phys. Rev. Lett. **79** (1997) 3591

C. Lerche, L. v. Smekal, Phys. Rev. D **65** (2002) 125006.

D. Zwanziger Phys. Rev. D **65** (2002) 094039.

# Infrared Structure of YM-theory II

- Example: Ghost-Gluon-Vertex



- Selfconsistency:

$$G(p^2) \sim (p^2)^{-\kappa}, \quad Z(p^2) \sim (p^2)^{2\kappa}, \quad \Gamma^{2,2}(p^2) \sim (p^2)^{-\kappa}$$
$$\Gamma^{2,1}(p^2) \sim (p^2)^0$$

J. C. Taylor, Nucl. Phys. B 33 (1971) 436.

A. Cucchieri, A. Maas and T. Mendes, PRD 74 (2006) 014503, arXiv:0803.1798

# Infrared Structure of YM-theory III

● Four-gluon vertex:

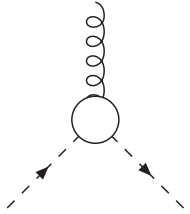
The diagram shows the infrared structure of the four-gluon vertex. On the left is the tree-level four-gluon vertex, represented by a central grey circle with four wavy lines (gluons) extending from it. This is equal to the sum of several terms:

- A tree-level diagram with a central black dot and four wavy lines.
- A term with a coefficient  $\frac{1}{2}$  multiplied by a diagram with a central grey circle, a loop of wavy lines, and four wavy lines.
- A term with a minus sign followed by a diagram with a central grey circle, a dashed loop of wavy lines, and four wavy lines.
- A term with a coefficient  $\frac{1}{2}$  multiplied by a diagram with a central grey circle, a loop of wavy lines, and four wavy lines.
- A term with a coefficient  $\frac{1}{6}$  multiplied by a diagram with a central grey circle, a loop of wavy lines, and four wavy lines.

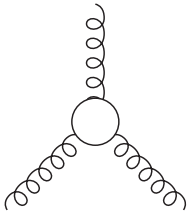
The equation is:

$$\text{Tree-level vertex} = \text{Tree-level vertex} + \frac{1}{2} \text{Loop diagram} - \text{Dashed loop diagram} + \frac{1}{2} \text{Loop diagram} + \frac{1}{6} \text{Loop diagram},$$

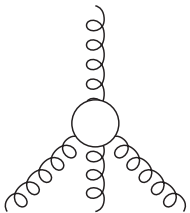
# Running Coupling: IR-Universality



$$\alpha^{gh-gl}(p^2) = \alpha_\mu G^2(p^2) Z(p^2) \sim \mathbf{const}/N_c$$



$$\alpha^{3g}(p^2) = \alpha_\mu [\Gamma^{3g}(p^2)]^2 Z^3(p^2) \sim \mathbf{const}/N_c$$



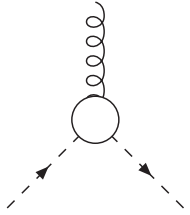
$$\alpha^{4g}(p^2) = \alpha_\mu \Gamma^{4g}(p^2) Z^2(p^2) \sim \mathbf{const}/N_c$$

with

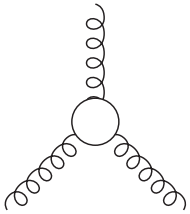
$$\Gamma^{3g}(p^2) \sim (p^2)^{-3\kappa}, \quad \Gamma^{4g}(p^2) \sim (p^2)^{-4\kappa}$$

$$G(p^2) \sim (p^2)^{-\kappa}, \quad Z(p^2) \sim (p^2)^{2\kappa}$$

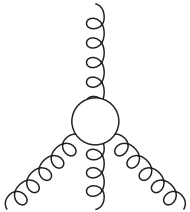
# Running Coupling: IR-fixed points



$$\alpha^{gh-gl}(p^2) = \alpha_\mu G^2(p^2) Z(p^2) \sim \mathbf{8.92}/N_c$$



$$\alpha^{3g}(p^2) = \alpha_\mu [\Gamma^{3g}(p^2)]^2 Z^3(p^2) \sim \mathbf{const}/N_c$$

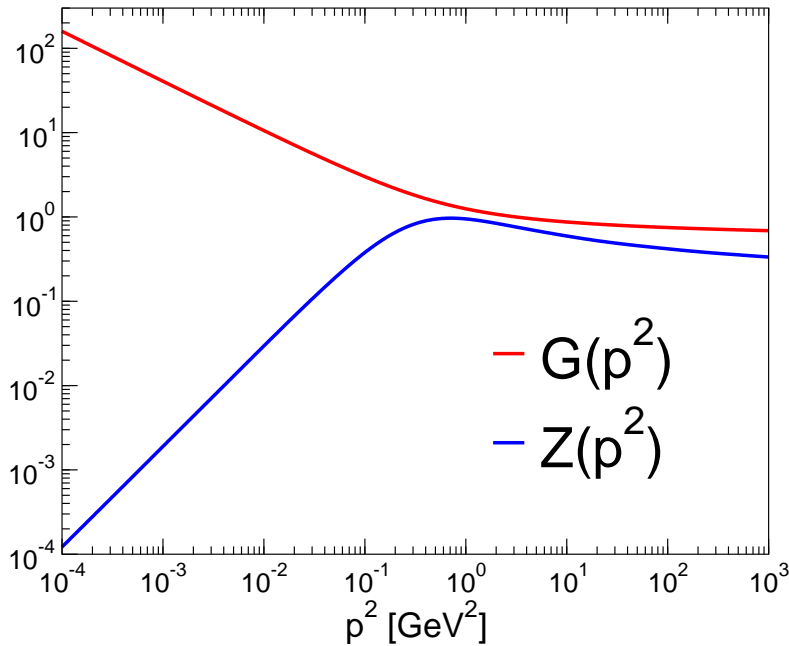


$$\alpha^{4g}(p^2) = \alpha_\mu \Gamma^{4g}(p^2) Z^2(p^2) \sim \mathbf{0.0086}/N_c$$

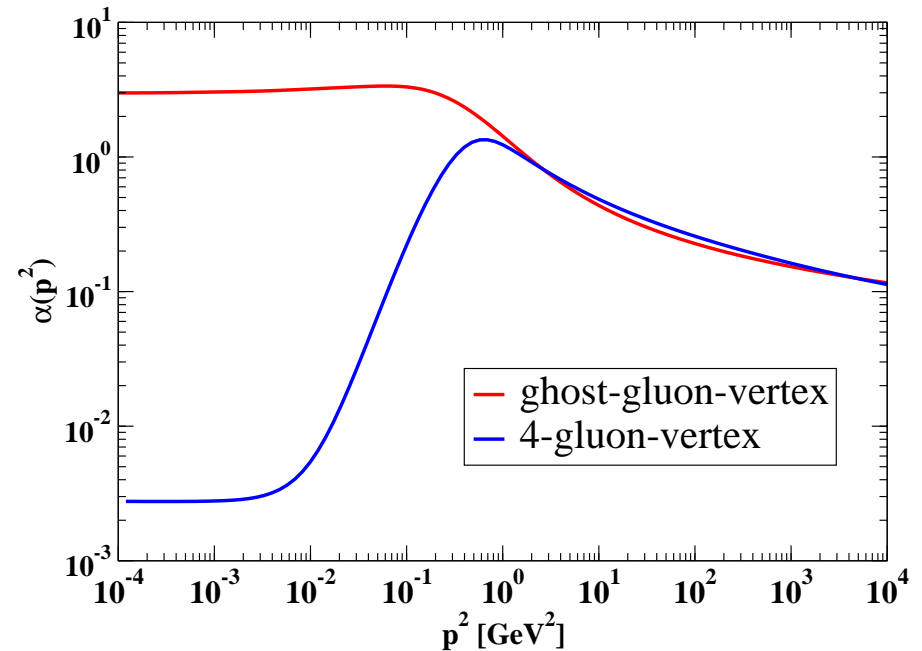
Lerche and Smekal, PRD **65** (2002) 125006

Kellermann and CF, arXiv:0801.2697

# Ghost, Glue and Coupling



CF and Alkofer, PLB 536 (2002) 177.



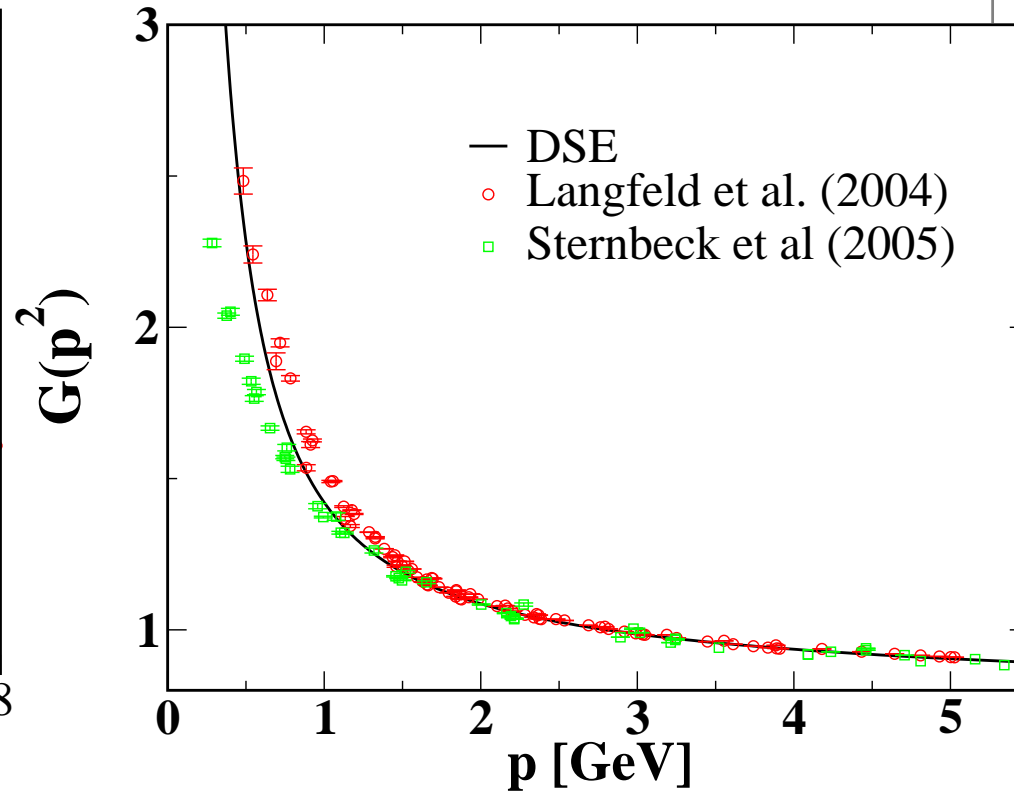
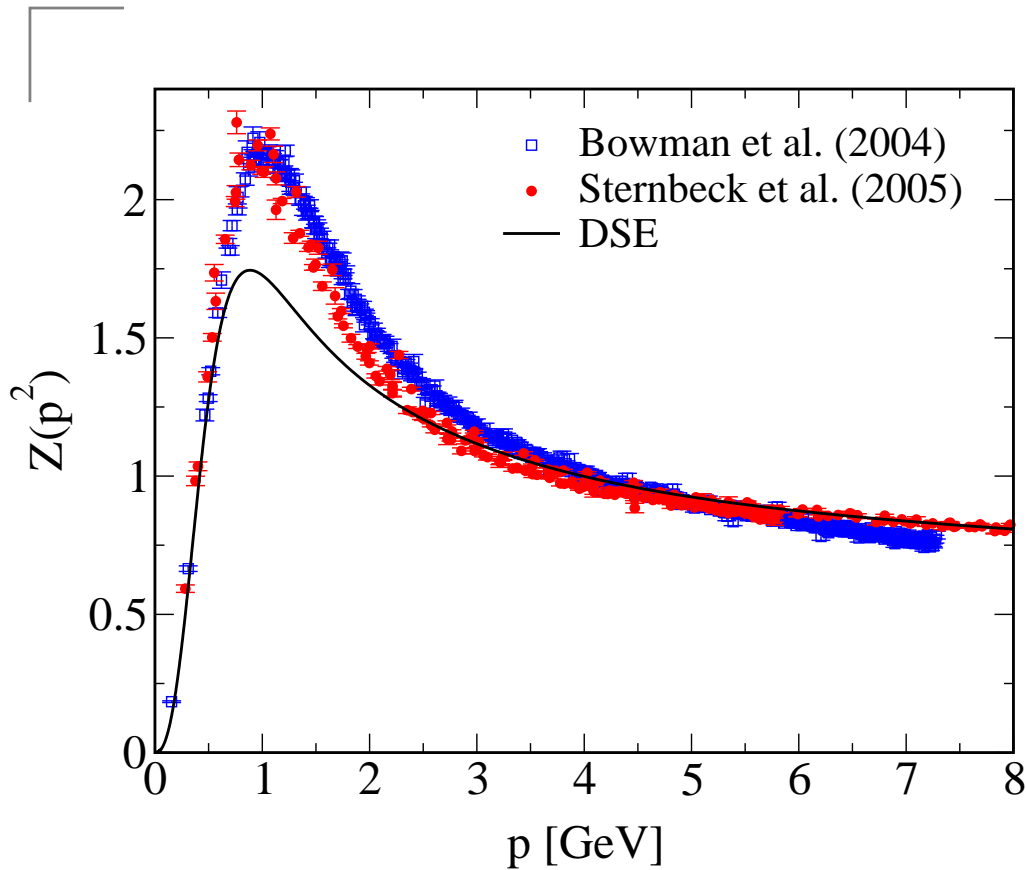
Kellermann and CF, arXiv:0801.2697

● IR:  $G(p^2) \sim (p^2)^{-\kappa}$  ,  $Z(p^2) \sim (p^2)^{2\kappa}$  ,  $\Gamma^{4g} \sim (p^2)^{-4\kappa}$

$$\kappa \approx 0.595$$

(dependent on truncation!)

# DSEs vs Lattice



- Physical  $p^2$ : Systematic improvement possible for **DSEs**
- Deep infrared: Subtle problems cp Continuum vs **Lattice**  
**Discussed in other talks**



# Uniqueness of IR-solution I

Ghost-DSE:

$$\text{---} \bullet \text{---}^{-1} = \text{---}^{-1} - \text{---} \circ \text{---}$$

The diagram shows the Dyson-Schwinger equation for the ghost propagator. On the left, a dashed line with a black dot and a superscript -1. This is equal to a bare dashed line with a superscript -1, minus a diagram where a dashed line with a black dot is connected to a ghost loop (a circle of wavy lines with two black dots) which is then connected to another dashed line with a black dot.

Ghost-FRG:

$$k \partial_k \text{---} \bullet \text{---}^{-1} = \text{---} \circ \text{---}^{-1} + \text{---} \circ \text{---}^{-1} - \frac{1}{2} \text{---} \circ \text{---}^{-1} + \text{---} \circ \text{---}^{-1}$$

The diagram shows the flow equation for the ghost propagator. On the left, a dashed line with a black dot and a superscript -1, multiplied by  $k \partial_k$ . This is equal to a sum of four diagrams: 1) a ghost loop with a cross on the top vertex, 2) a ghost loop with a cross on the bottom vertex, 3) a ghost loop with a cross on the left vertex, and 4) a ghost loop with a cross on the right vertex. The third diagram is multiplied by  $-\frac{1}{2}$ .

IR-Analysis of whole tower of equations  $\Rightarrow$

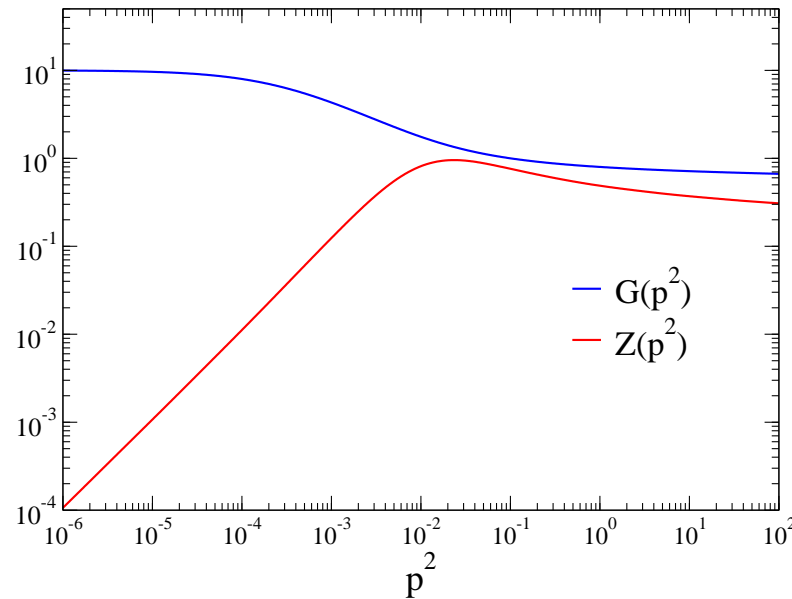
$$\Gamma^{n,m}(p^2) \sim (p^2)^{(n/2-m)\kappa}$$

is unique scaling solution.

C.F. and J. M. Pawłowski, Phys. Rev. D **75** (2007) 025012.

# Exception: decoupling scenario

Decoupling: Massive gluon propagator;  
all other Green's functions IR finite



● Renormalisation condition(s),  $G(\mu)$ ,  $Z(\mu)$  crucial!

CF, in prep.

Boucaud, Leroy, Yaouanc, Micheli, Pene, Rodriguez-Quintero, arXiv:0801.2721.

Aguilar, Binosi, Papavassiliou, arXiv:0802.1870.

# Decoupling scenario - Interpretation

- $\left( G(p^2) \xrightarrow{p^2 \rightarrow 0} \infty \right) \Leftrightarrow \text{Unbroken } Q^a$
- $G(p^2) = (p^2) \langle F P^{-1} \rangle$

	Confinement	Higgs phase
global gauge symm.	unbroken	broken
Eigenvalue density of FP at $\lambda = 0$	enhanced	not enhanced
$G(p^2 \rightarrow 0)$	$\infty$	finite

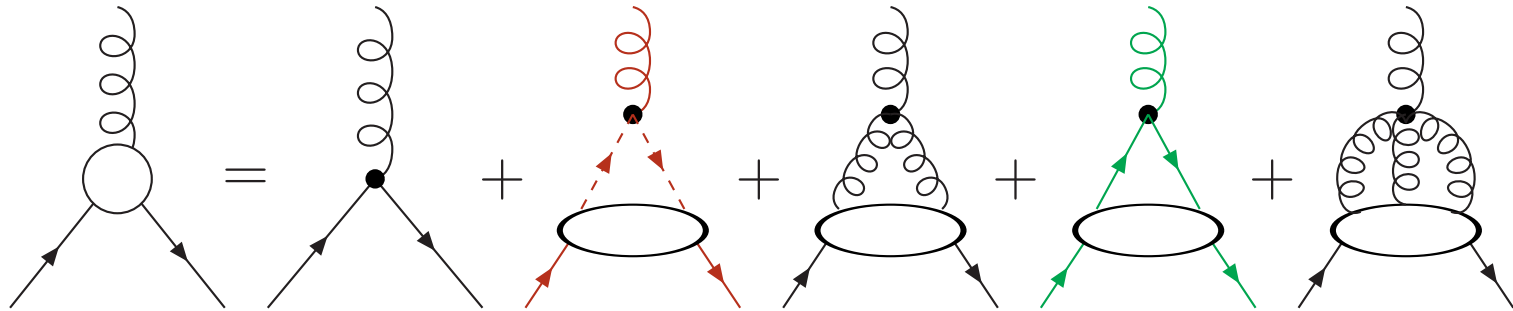
Greensite, Olejnik and Zwanziger, PRD **69** (2004) 074506, JHEP **0505** (2005) 070.

# Outline

- Introduction
- Infrared properties of SU(N) Yang-Mills theory
- **Infrared Slavery in quenched QCD**

# Infrared Structure of QCD I

Quark-gluon vertex:



- **Quark diagram:** Hadronic contributions ('unquenching')  
→ **Talk of Richard Williams**

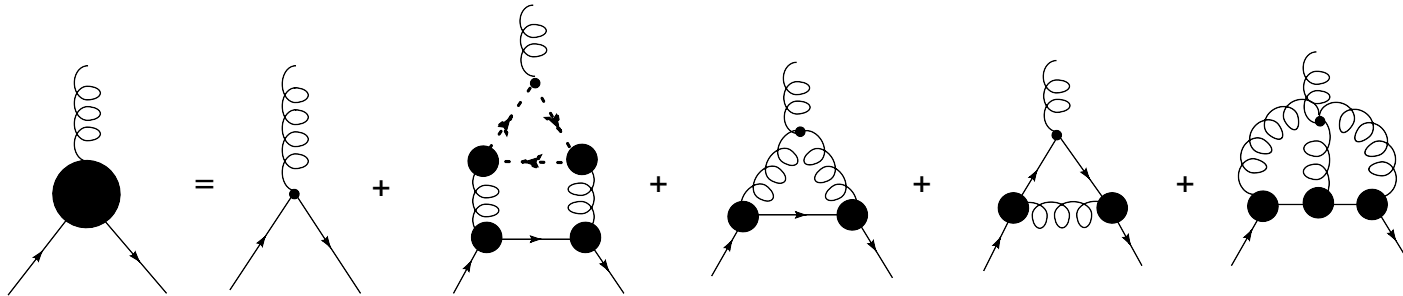
C.F, D. Nickel and J. Wambach, PRD 76 (2007) 094009

- **Ghost diagram:** Infrared leading

R. Alkofer, C.F., F. Llanes-Estrada, hep-ph/0607293.

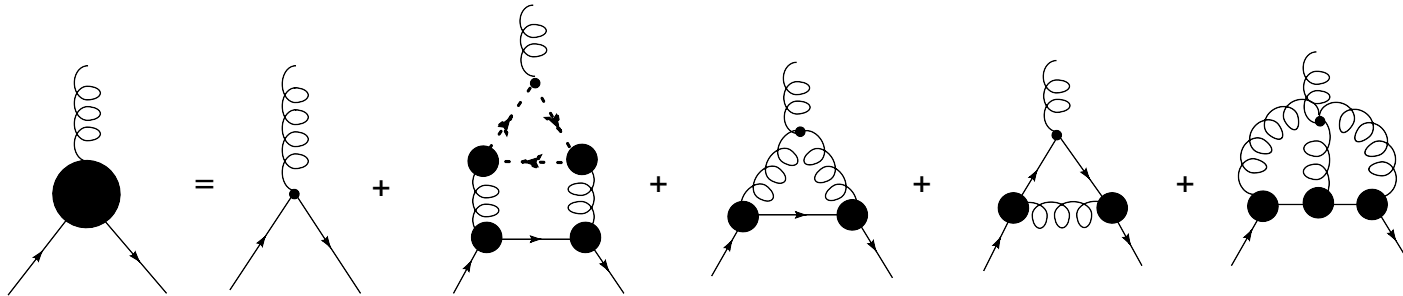
# Infrared Structure of QCD II

- Quark-gluon vertex: lowest order in skeleton expansion



# Infrared Structure of QCD II

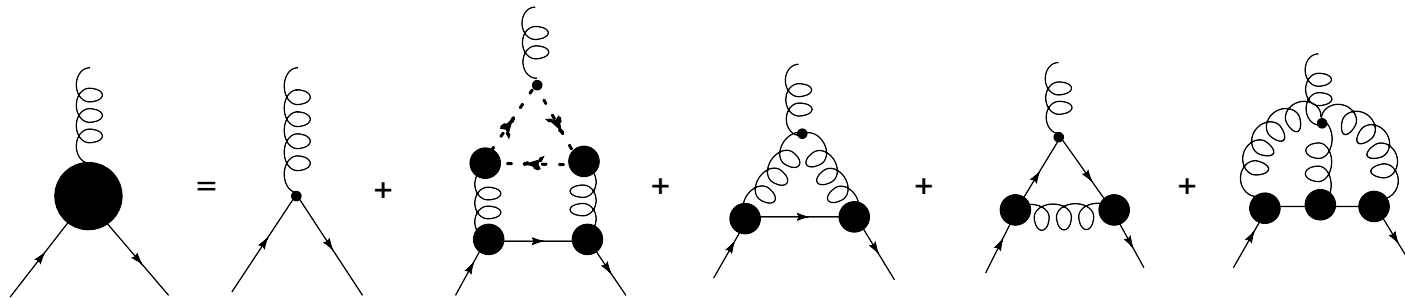
- Quark-gluon vertex: lowest order in skeleton expansion



$$S(p) = i\not{p} \frac{Z_f}{p^2 + M^2} + \frac{Z_f M}{p^2 + M^2}$$

# Infrared Structure of QCD II

- Quark-gluon vertex: lowest order in skeleton expansion



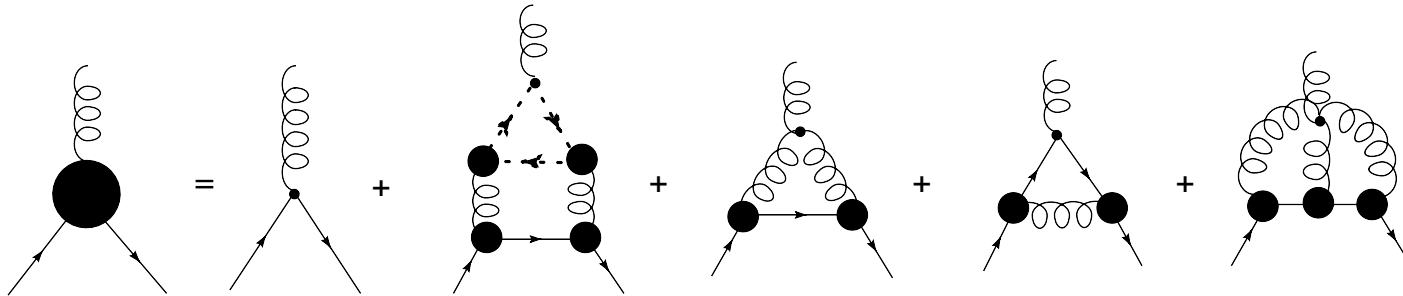
$$S(p) = i\hat{p} \frac{Z_f}{p^2 + M^2} + \frac{Z_f M}{p^2 + M^2}$$

$$\Gamma_\mu = ig \sum_{i=1}^4 \lambda_i G_\mu^i : \quad G_\mu^1 = \gamma_\mu, \quad G_\mu^2 = \hat{p}_\mu, \quad G_\mu^3 = \hat{p} \hat{p}_\mu, \quad G_\mu^4 = \hat{p} \gamma_\mu$$



# Infrared Structure of QCD II

- Quark-gluon vertex: lowest order in skeleton expansion

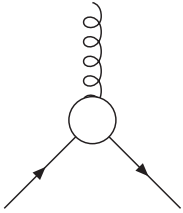


$$S(p) = i\hat{p} \frac{Z_f}{p^2 + M^2} + \frac{Z_f M}{p^2 + M^2}$$

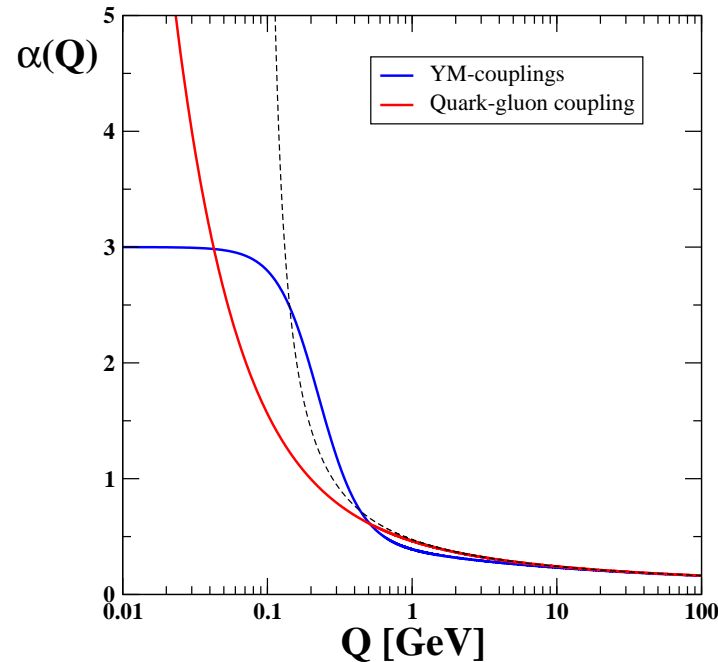
$$\Gamma_\mu = ig \sum_{i=1}^4 \lambda_i G_\mu^i : \quad G_\mu^1 = \gamma_\mu, \quad G_\mu^2 = \hat{p}_\mu, \quad G_\mu^3 = \hat{p} \hat{p}_\mu, \quad G_\mu^4 = \hat{p} \gamma_\mu$$

$$\lambda_{1,2,3,4} \sim (p^2)^{-1/2-\kappa} \quad \Leftrightarrow \quad \lambda_{1,3} \sim (p^2)^{-\kappa}$$

# Running Coupling: IR-slavery



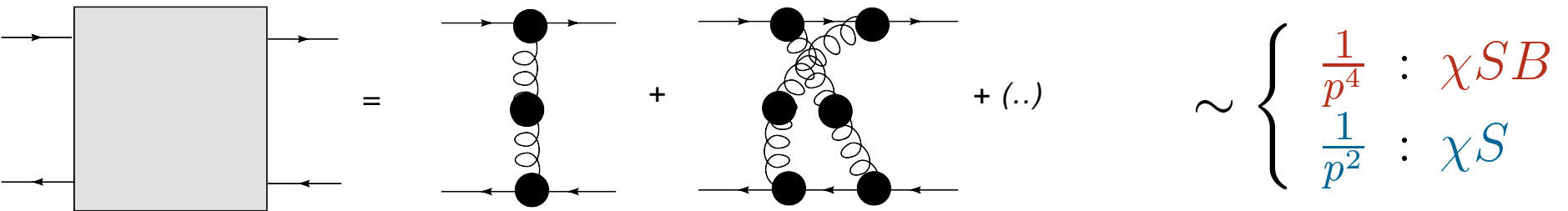
$$\alpha^{qg}(p^2) = \alpha_\mu [\Gamma^{qg}(p^2)]^2 [Z_f(p^2)]^2 Z(p^2) \sim \begin{cases} \frac{1}{p^2} : \chi^{SB} \\ const : \chi^S \end{cases}$$



R. Alkofer, C. F., F. Llanes-Estrada, hep-ph/0607293.

# The quark-antiquark potential

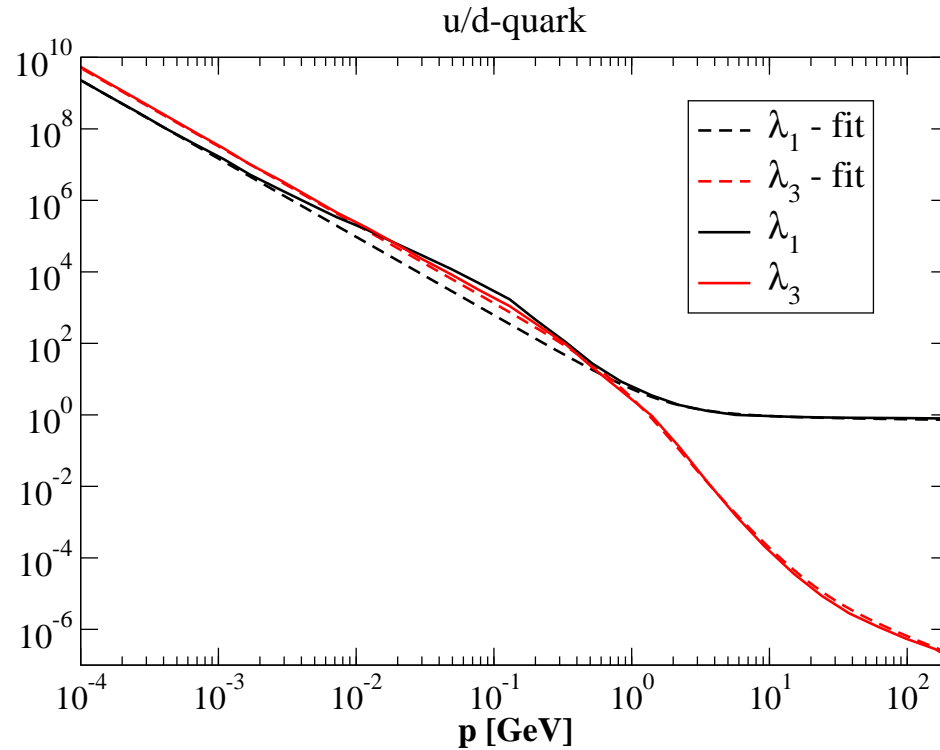
● quenched QCD



$$V(\mathbf{r}) = \frac{1}{(2\pi)^3} \int d^3p e^{i\mathbf{p}\mathbf{r}} \square \sim \begin{cases} |r| : D\chi SB \\ \frac{1}{|r|} : \chi S \end{cases}$$

Quark confinement  $\leftrightarrow \chi SB$

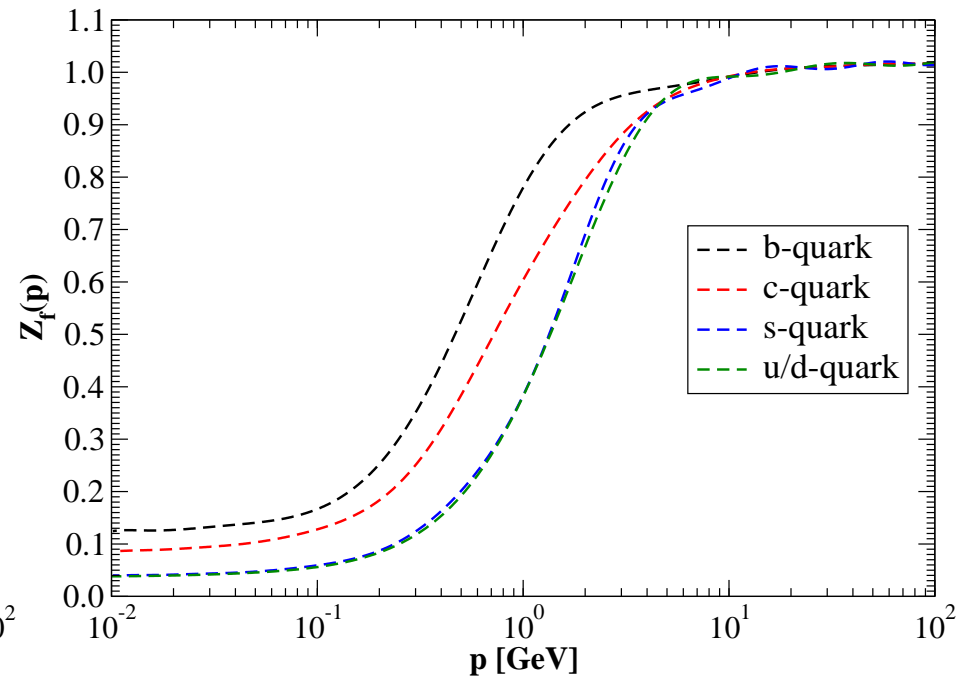
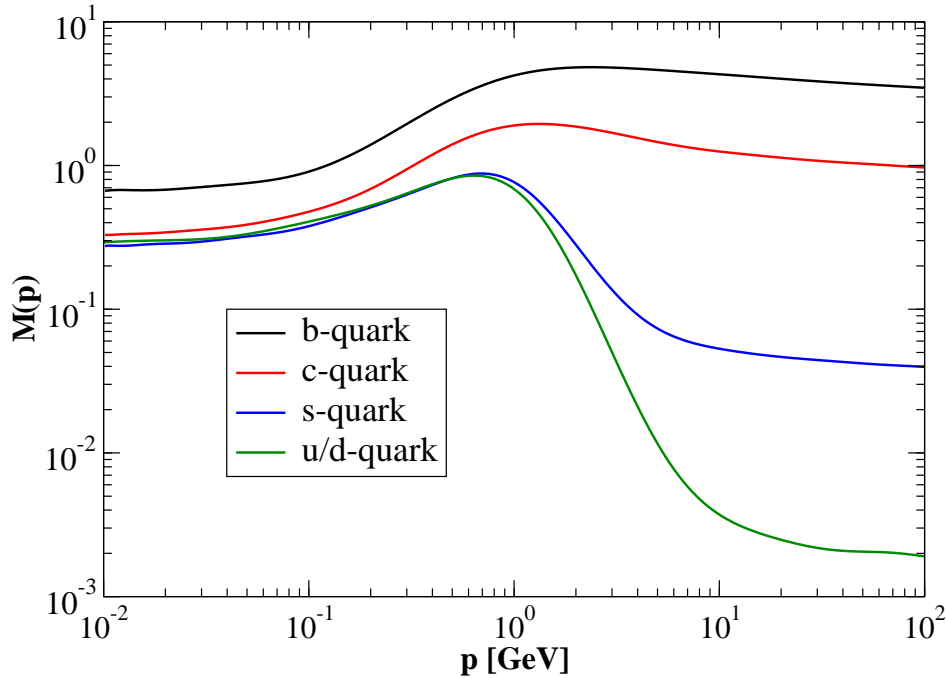
# Numerical results: Vertex



● Infrared singularity  $(p^2)^{-1/2-\kappa}$

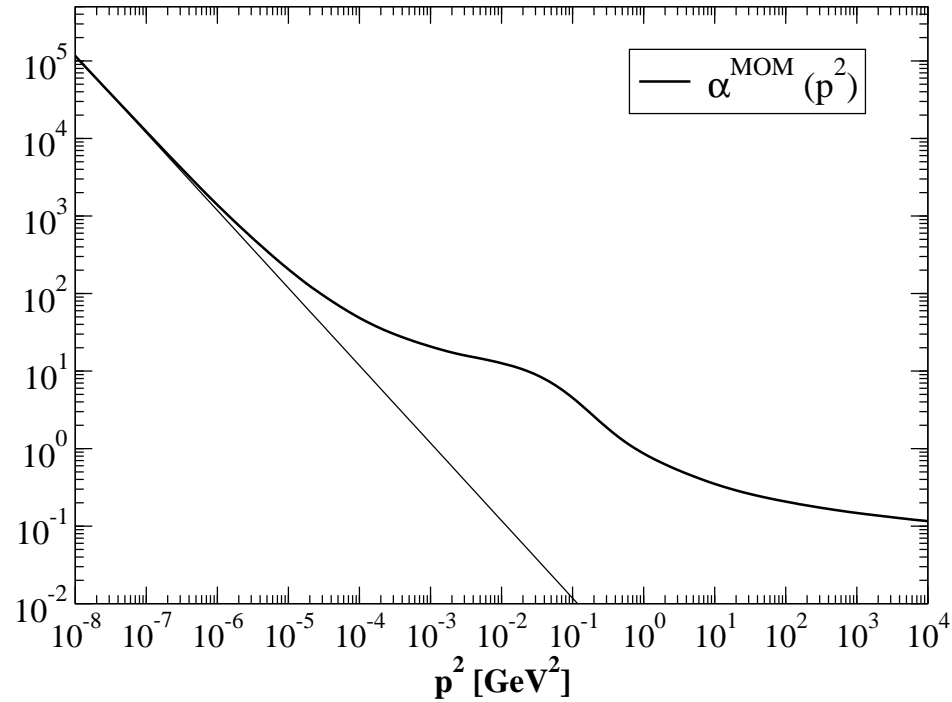
C.F., F. Llanes-Estrada, R. Alkofer, K. Schwenzer, in preparation

# Numerical results: Quark propagator



● Interesting mass dependence in IR!

# Numerical results: Coupling



- Infrared slavery:  $\alpha(p^2) \sim \frac{1}{p^2}$

# Summary

## Landau gauge Yang-Mills theory:

- 1PI-function with  $2n$  ghost and  $m$  gluon legs:

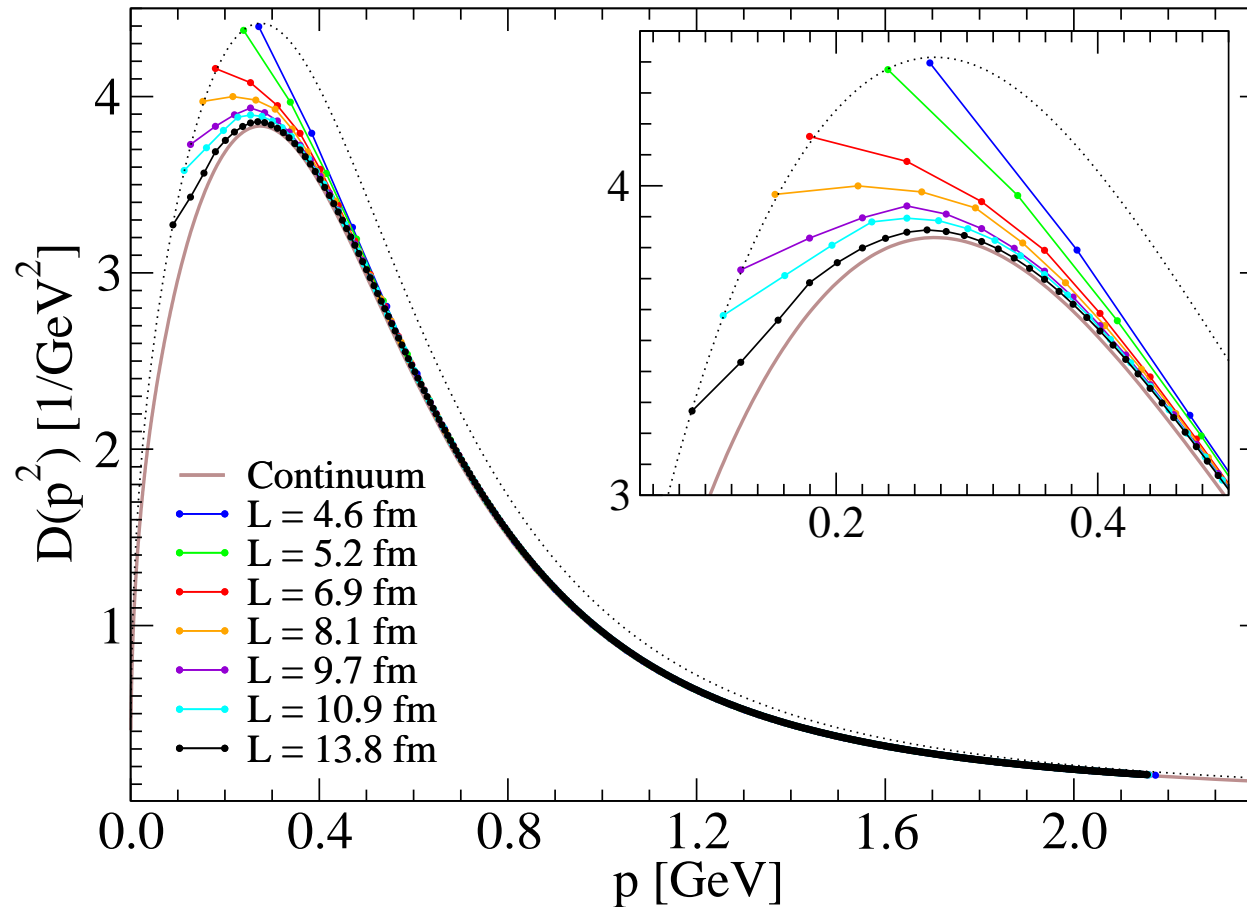
$$\Gamma^{n,m}(p^2) \sim (p^2)^{(n-m)\kappa}$$

- YM-couplings: **IR-fixed point**

## Landau gauge QCD (quenched):

- Quark-gluon-coupling: **Infrared slavery**
- Quark confinement  $\leftrightarrow \chi SB$

# DSEs on a torus: volume effects



- Contemporary lattices large enough to exclude large volume effects