Meson spectroscopy from the Bethe-Salpeter equation

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QHQCD, St. Goar, 17th March 2008

Work with

A. Höll, C. D. Roberts, S. V. Wright (ANL, PHY)

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- Austrian Research Foundation
- Argonne National Laboratory
- University of Graz

Outline

Motivation
- QCD and Hadrons
- DSEs

Equations and Solutions
- Quark DSE
- Bethe-Salpeter Equation
- BSE Solution Strategies

Results
- Exact Results
- Example Calculations
- Example Results

Conclusion
- Summary
- Outlook

QCD and Hadrons

- Dyson Schwinger Equations:
  a modern method in relativistic QFT

  P. Maris and C. D. Roberts, Int. J. Mod. Phys. E **12** (2003) 297
  R. Alkofer and L. von Smekal, Phys. Rept. **353** (2001) 281
  C. D. Roberts and S. M. Schmidt, Prog. Part. Nucl. Phys. 45 (2000) S1
  A. Holl, C. D. Roberts, S. V. Wright, nucl-th/0601071
  C. S. Fischer, J. Phys. G **32** (2006) R253

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- Study hadrons as composites of
  quarks and gluons . . .
- . . . including:
  - Chiral symmetry and $D\chi SB$
  - correct perturbative limit (via $\alpha_p(Q^2)$)
  - quark and gluon confinement
  - Poincaré covariance
- Propagators and Bethe-Salpeter amplitudes
  $\longrightarrow$ can be used to calculate observables

DSEs

- Euclidean Green functions (also calculated on the lattice) satisfy the [Dyson, Schwinger] equations
- Each function satisfies integral equation involving other functions ⇒
- Infinite set of coupled integral equations
- Truncation scheme necessary ⇒
- Generating tool for perturbation theory

FWF

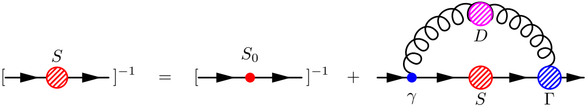
DSEs

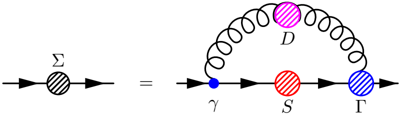
- Euclidean Green functions (also calculated on the lattice) satisfy the [Dyson, Schwinger] equations
- Each function satisfies integral equation involving other functions $\Rightarrow$
- Infinite set of coupled integral equations
- Truncation scheme necessary $\Rightarrow$

- Nonperturbative truncation scheme
- Respect symmetries
- Prove exact (model independent) results
- Devise (sophisticated) models to illustrate them
- Perform reliable calculations of hadron properties

Gap Equation

$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$





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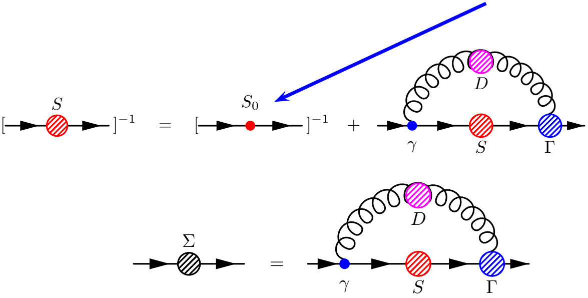
current quark mass m_ζ



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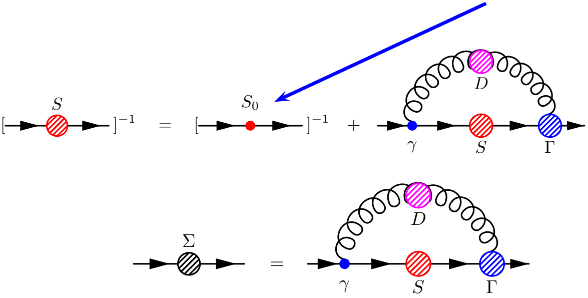

- Weak coupling expansion reproduces every diagram in perturbation theory, but:
- Perturbation theory: $m_\zeta = 0 \Rightarrow M(p^2) \equiv 0$

Quark Mass Function

Solution of gap equation: $S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$

P. Maris, C. D. Roberts, Phys. Rev. **C56**, 3369 (1997)

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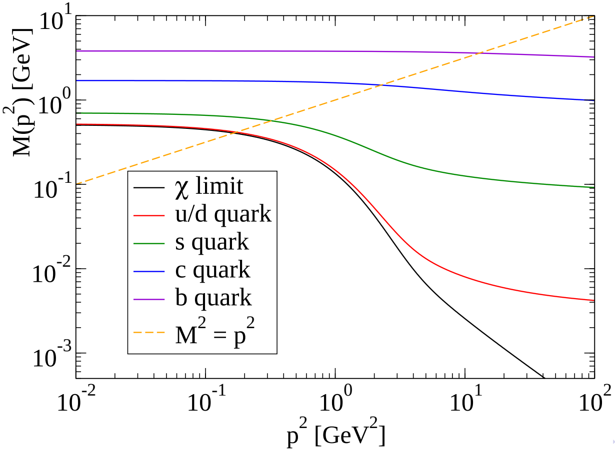

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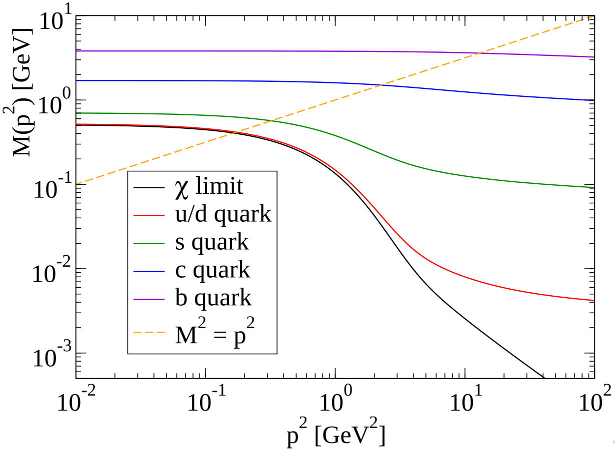
$M^2(p^2) = p^2 \;\Rightarrow$ Euclidean constituent quark mass M_E



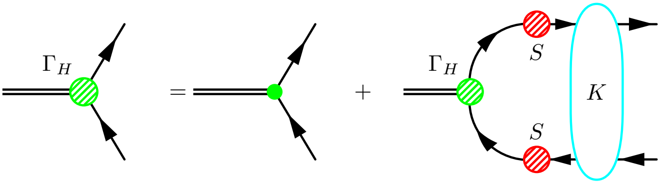
q	M_E/m_ζ
χ	∞
u/d	100
s	7
c	1.7
b	1.2

$\rightarrow D\chi SB$

Inhomogeneous BSE

- BSE for $q\bar{q}$ or qq bound states ($\chi = S\,\Gamma_H\,S$)

$$\Gamma_H(p;P) = \text{d. t.} + \int d^4q\,\chi(q;P)\,K(q,p;P)\,.$$

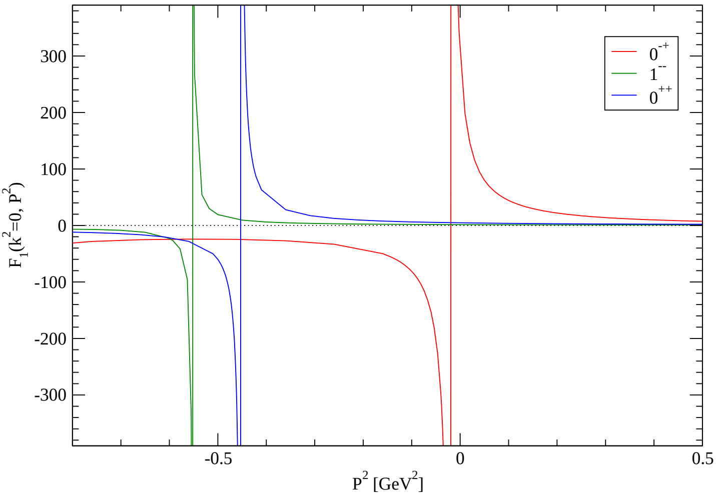


- Gap eq. output → BSE input
- Bound state at $P^2 = -m_H^2$:

$$\Gamma_H(q;P) = \frac{r_H\,\Gamma_h(q;P)}{P^2 + m_H^2} + \text{regular terms}$$

Inhomogeneous BSE :: Solution

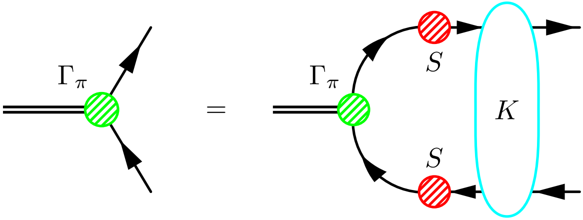
0^{-+}, 0^{++}, and 1^{--} meson amplitudes



Homogeneous BSE

- BSE for $q\bar{q}$ or qq bound states ($\chi = S\,\Gamma_h\,S$)

$$\Gamma_{h\ tu}(p;P) \qquad = \int d^4q\ [\chi(q;P)]_{sr}\ K^{tu}_{rs}(q,p;P)\,.$$



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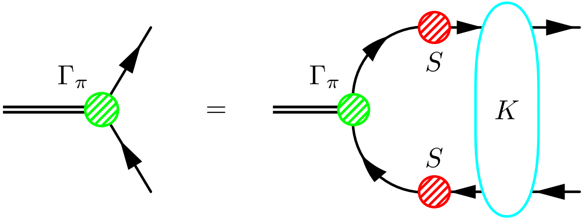
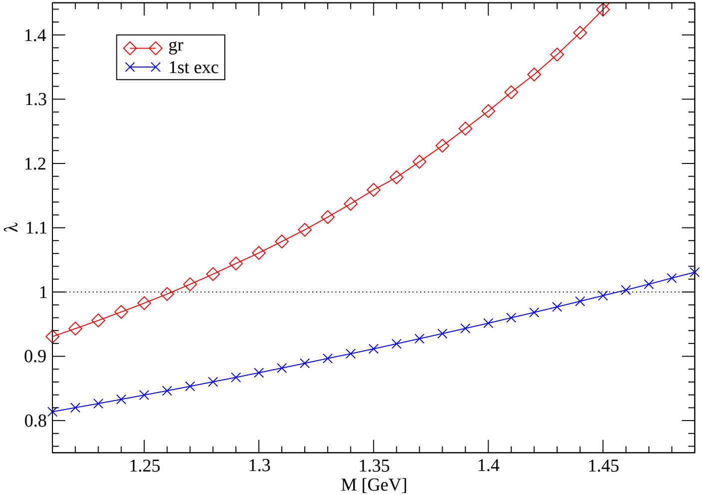


- homogeneous $\longrightarrow$ eigenvalue equation

Homogeneous BSE :: Solution Strategy

Solution strategy for homogeneous BSE



AV WTI

- Axial-vector Ward-Takahashi identity

$$P_\mu \Gamma^j_{5\mu}(k;P) = S^{-1}(k_+) i\gamma_5 \frac{\tau^j}{2} + i\gamma_5 \frac{\tau^j}{2} S^{-1}(k_-) - 2i\, m(\zeta)\, \Gamma^j_5(k;P),$$

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- Consequence: Gap and BSE kernels related

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- Consequence (residues): $f_{\pi_n} m^2_{\pi_n} = 2\, m(\zeta)\, \rho_{\pi_n}(\zeta)\,;$

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- Consequence (residues): $f_{\pi_n} m^2_{\pi_n} = 2\, m(\zeta)\, \rho_{\pi_n}(\zeta)$;
- valid for every pseudoscalar meson
- valid for every current quark mass
- $\Rightarrow$ GMOR, PCAC

P. Maris, C. D. Roberts, Phys. Rev. **C56**, 3369 (1997)

A. Höll, A. K., and C. D. Roberts, Phys. Rev. C **70**, 042203 (2004)

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Mass Formula

- Investigate the chiral limit of $f_{\pi_n} m_{\pi_n}^2 = 2\, m(\zeta)\, \rho_{\pi_n}(\zeta)\,;$

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- Ground state pion:
  - $m(\zeta) \longrightarrow 0$
  - $f_{\pi_{gr}}$ finite
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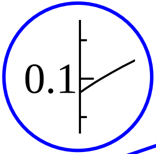

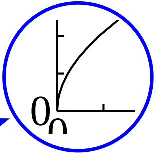

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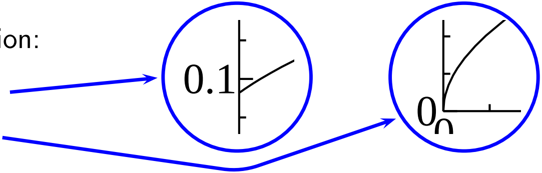
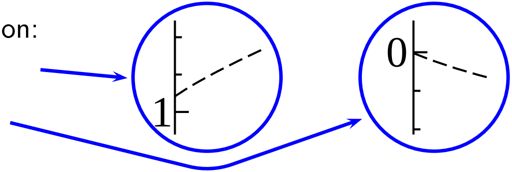
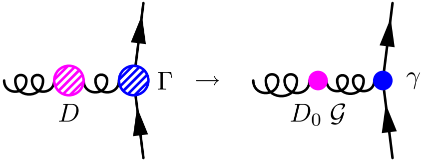


- Excited state pion:
  - $m(\zeta) \longrightarrow 0$
  - $m_{\pi_{exc1}}$ finite
  - $f_{\pi_{exc1}} \longrightarrow 0$



Rainbow-Ladder (RL) Truncation

- Rainbow approximation for gap equation
- Ladder approximation for BSE



- Bare quark-gluon vertex γ_ν
- Bare gluon propagator $D_{\mu\nu}^{\mathrm{free}}(p-q)$
- Effective coupling $\mathcal{G}$
- Input needed for $\mathcal{G}$: modeling

Effective Coupling

- What do we know?
- Effective running coupling $\mathcal{G}(Q^2)$
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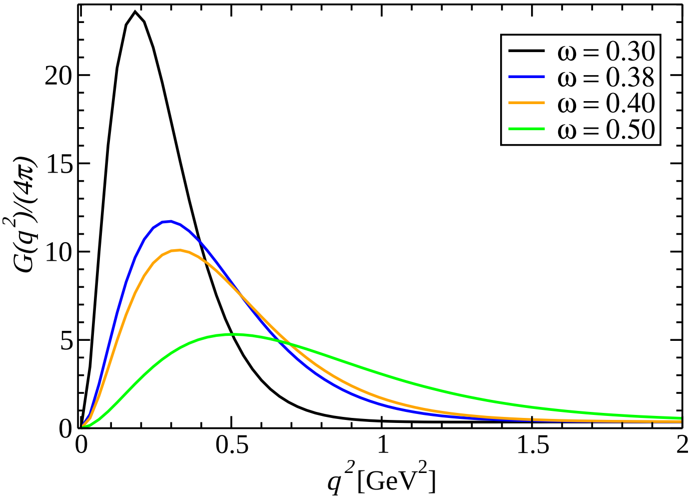
- Precise form at low Q^2 $\rightarrow$ model
- IR: two-parameters via Gaussian: strength D and width ω
- perturbative α in the UV region

Model Details

- Effective coupling $\mathcal{G}(Q^2)$: $\omega\, D$ =const.



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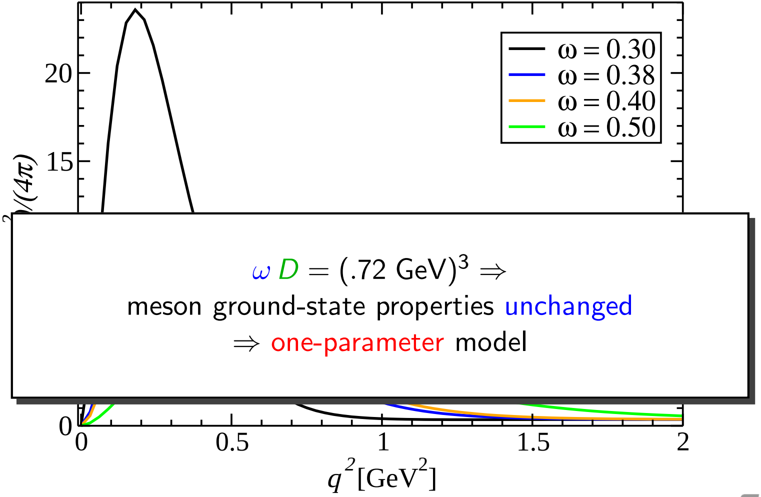

$\omega\ D = (.72\ \text{GeV})^3 \Rightarrow$
meson ground-state properties unchanged
$\Rightarrow$ one-parameter model

So Far :: Ground States

- P. Maris, P. C. Tandy: series of papers following

  P. Maris and P. C. Tandy, Phys. Rev. C **60**, 055214 (1999).

- Successful description of light pseudoscalar and vector mesons
  - Spectra
  - Leptonic decay constants
  - Electromagnetic properties (FF, radii)

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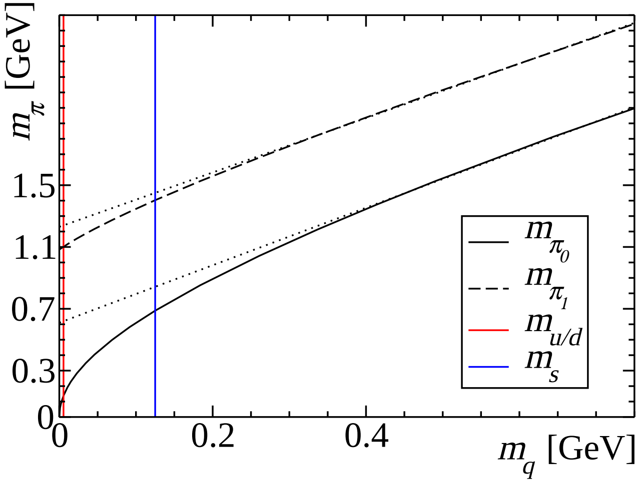
  - Spectra
  - Leptonic decay constants
  - Electromagnetic properties (FF, radii)
- Now:
  - Radial excitations
  - Scalar mesons
  - Axial vector mesons
- Study long range part of the strong interaction between light quarks

Masses

$m_{0^{-+}_{gr}}$ and $m_{0^{-+}_{exc1}}$ as functions of current quark mass



Leptonic Decay Constants

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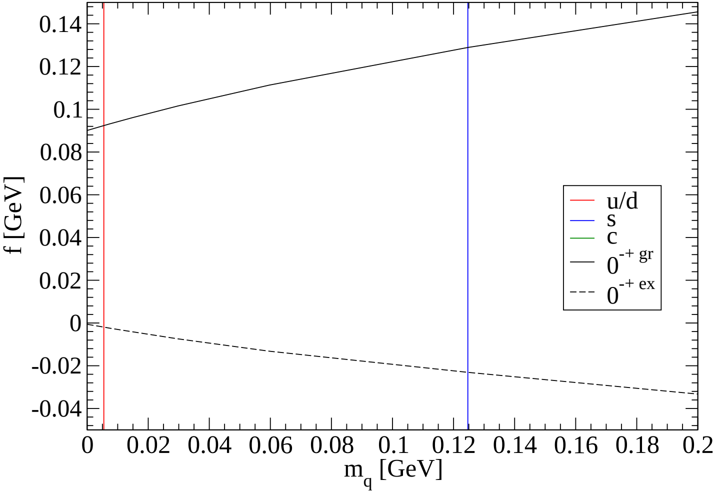
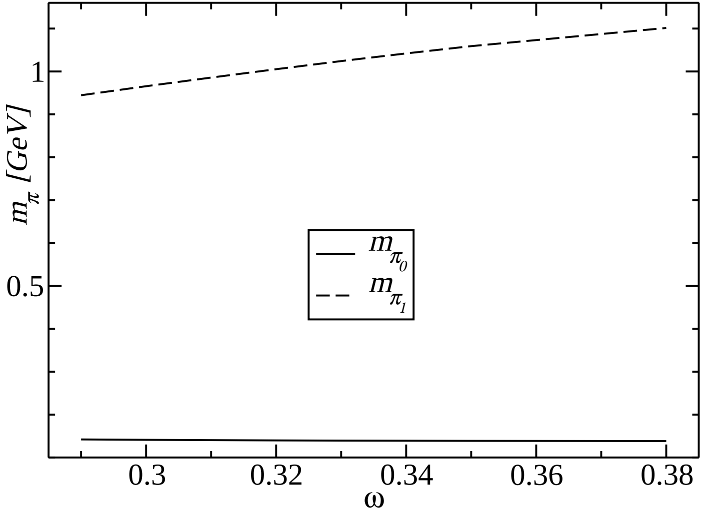


Model Parameter Dependence

$m_{\pi_{gr}}$ and $m_{\pi_{exc1}}$ as functions of ω



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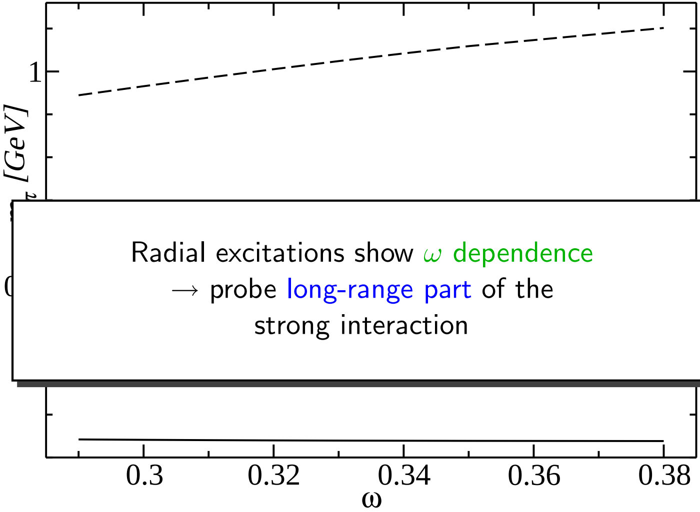


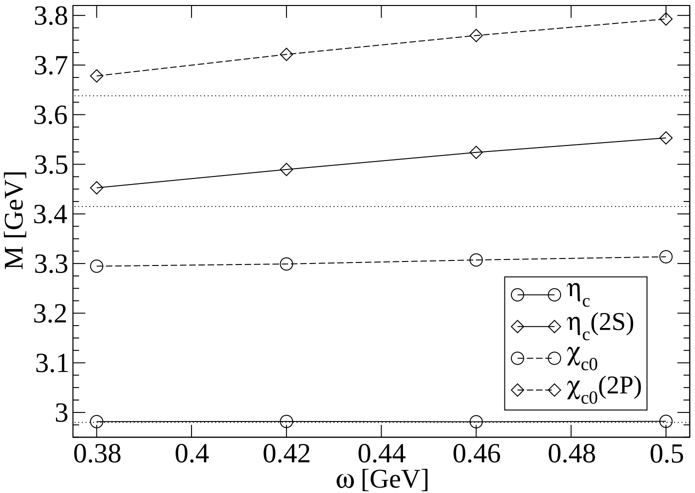
Radial excitations show ω dependence

$\rightarrow$ probe long-range part of the strong interaction

Ratios :: Charmonium

$m_{0^{-+}}$ and $m_{0^{++}}$ as functions of ω

A. K., C. D. Roberts, and S. V. Wright, Int.J.Mod.Phys.A22:424-431,2007



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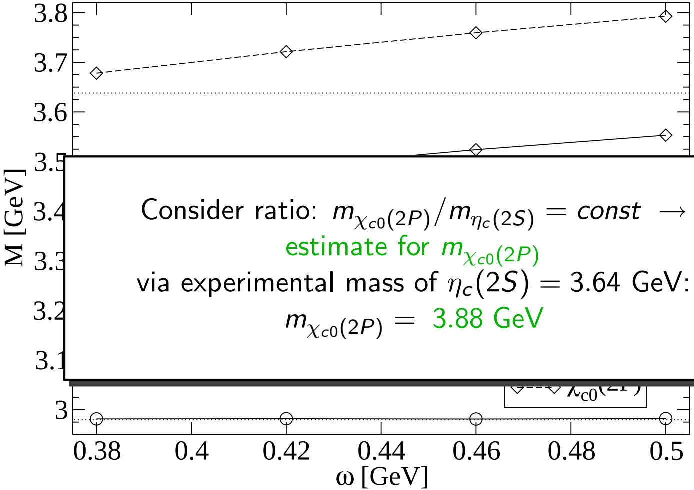

Consider ratio: $m_{\chi_{c0}(2P)}/m_{\eta_c(2S)} = const$ $\rightarrow$

estimate for $m_{\chi_{c0}(2P)}$

via experimental mass of $\eta_c(2S) = 3.64$ GeV:

$m_{\chi_{c0}(2P)} =$ 3.88 GeV

Meson Masses

Calculated masses for all mesons with $J = 0, 1$ for (equal) light and strange quark masses (MeV)

I. C. Cloet, A. K. , C. D. Roberts, arXiv:0710.5746 [nucl-th]

values for $\bar{c}c$ mesons added recently

J^{PC}	u/d	exp	s	exp	c	exp
0^{-+}	139	140	696	??	2906	2980
0^{--*}	860	??	1170	??	3161	??
0^{++}	673	??	1080	??	3248	3415
0^{+-*}	1040	??	1385	??	3413	??
1^{--}	745	770	1075	1020	3096	3097
1^{-+*}	1010	1376	1320	1600	3250	??
1^{++}	905	1260	1255	1426	3326	3511
1^{+-}	830	1235	1165	1386	3260	3526

* = exotic quantum numbers

Meson Masses :: $S = \pm 1$

Calculated masses for all strange mesons with $J = 0, 1$(MeV)

J^P	*gr*	*exp*	*exc*	*exp*
0^-	497	497	1032	$\sim$ 1460
0^+	894	672	1239	1414
1^-	935	892	1230	1414
1^+	1014	1272	1107	1403

Other Things and Elsewhere

- Finite temperature and density
- Diquark confinement (model-independent)
- Gluon propagator and quark-gluon vertex
- Comparison to lattice gauge QCD
- Baryon studies via quark-diquark Ansatz

WIP - Wish List

Work in progress

- Higher J (tensor mesons)
- Higher radial excitations
- Heavy quark sector
- Heavy-light mesons and radial excitations
- Hadronic decays, e. g. $\pi(1300) \rightarrow \varrho\,\pi$
- Nucleon properties (diquarks)

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Wish list

- Sophisticated meson model beyond RLT
- Good description of axial-vector mesons
- Study states with "exotic" quantum numbers
- ...

Conclusions

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- Dyson-Schwinger equations provide a nonperturbative continuum approach to QCD
- Bethe-Salpeter equation used to describe bound states in a manifestly covariant way
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Conclusions

- Step beyond Rainbow-Ladder truncation needed to go for axial vectors, scalars, exotics, radially excited states
- These provide means to study the long-range behavior of the strong interaction

The End

Thank you!