

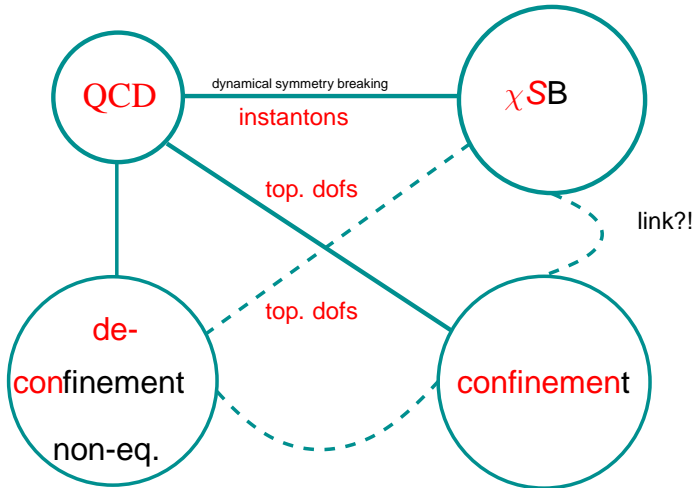
Confinement, chiral symmetry breaking and the QCD phase diagram

Jan Martin Pawłowski

Institute for Theoretical Physics
Heidelberg University

Quarks and Hadrons in strong QCD, St Goar, March 17th, 2008



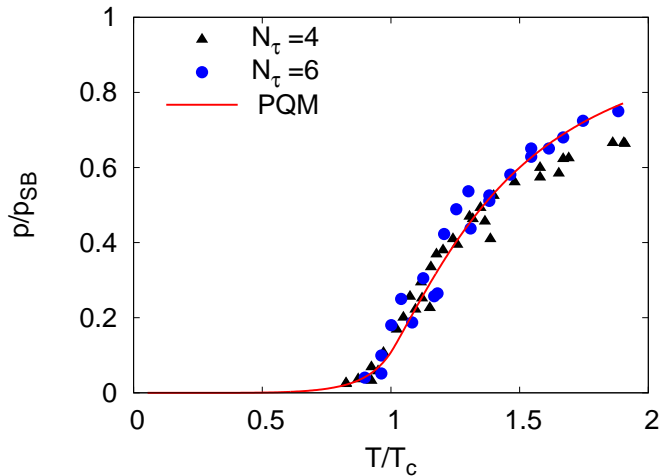


some questions

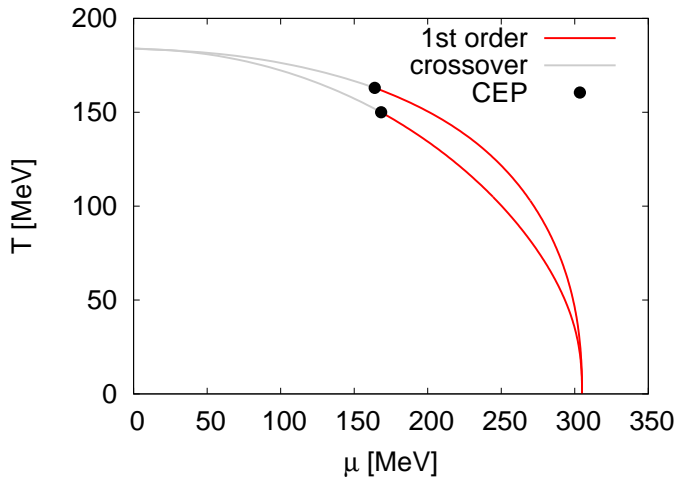
- chiral symmetry breaking
 - mechanism & critical temperature
 - bound state spectrum
- confinement-deconfinement
 - mechanism & critical temperature
 - spectrum, mass gap
- finite density
 - phase diagram & critical point
- dynamics

some answers: compute Green functions

- chiral symmetry breaking
 - $\langle q(x)\bar{q}(y) \rangle, \dots$
- confinement-deconfinement
 - $\langle A(x)A(y) \rangle, \langle C(x)\bar{C}(y) \rangle, \dots$
 - $\langle 1/N_{\text{ctr}} \mathcal{P} \exp i \int_0^\beta dt A_0 \rangle, \dots$
- dynamics with functional methods
- gauge fixing is mostly a benefit, not a liability
 - Landau gauge & Polyakov gauge

lattice data taken from Ali Khan et al. (CP-PACS), Phys. Rev. D **64** (2001)

see talk of B.-J. Schaefer



see talk of B.-J. Schaefer

1 Functional RG

- properties
- topology

2 Landau gauge QCD

- Signatures of confinement
- Infrared asymptotics & finite volume effects

3 QCD at finite temperature

- confinement-deconfinement phase transition

1 Functional RG

- properties
- topology

2 Landau gauge QCD

- Signatures of confinement
- Infrared asymptotics & finite volume effects

3 QCD at finite temperature

- confinement-deconfinement phase transition

$$k\partial_k\Gamma_k[\phi] = \frac{1}{2}\text{Tr}\frac{1}{\Gamma_k^{(2)}[\phi] + k^2} 2k^2$$

$$k\partial_k\Gamma_k[\phi] = \frac{1}{2} \text{Tr} \frac{1}{\Gamma_k^{(2)}[\phi] + R_k(p^2)} k\partial_k R_k(p^2)$$

$$k\partial_k\Gamma_k[\phi] = \frac{1}{2} \text{Tr} \frac{1}{\Gamma_k^{(2)}[\phi] + R_k(p^2)} k\partial_k R_k(p^2)$$

- in Yang-Mills theory

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \left(\text{Diagram 1} - \text{Diagram 2} \right)$$

$$k\partial_k\Gamma_k[\phi] = \frac{1}{2} \text{Tr} \frac{1}{\Gamma_k^{(2)}[\phi] + R_k(p^2)} k\partial_k R_k(p^2)$$

- self-similarity, reparameterisation & projections
- fermions straightforward though 'physically' complicated
 - no sign problem numerics as in scalar theories!
 - chiral fermions reminder: Ginsparg-Wilson fermions from RG argument!
 - bound states via (re-)bosonisation effective field theory techniques applicable!

$$k\partial_k\Gamma_k[\phi] = \frac{1}{2} \text{Tr} \frac{1}{\Gamma_k^{(2)}[\phi] + R_k(p^2)} k\partial_k R_k(p^2)$$

- self-similarity, reparameterisation & projections
- fermions straightforward
- flows in Landau gauge QCD

Ellwanger, Hirsch, Weber '96

Bergerhoff, Wetterich '97

Pawlowski, Litim, Nedelko, von Smekal '03

Kato '04

Gies, Fischer '04

Pawlowski '05

$$k\partial_k\Gamma_k[\phi] = \frac{1}{2} \text{Tr} \frac{1}{\Gamma_k^{(2)}[\phi] + R_k(p^2)} k\partial_k R_k(p^2)$$

- self-similarity, reparameterisation & projections
- fermions straightforward
- functional methods in Landau gauge QCD (IR)
 - Dyson-Schwinger equations
 - stochastic quantisation
 - flows in Landau gauge QCD
 - quark confinement from Landau gauge propagators

von Smekal, Hauck, Alkofer '97

Zwanziger '02

Pawlowski, Litim, Nedelko, von Smekal '03

Braun, Gies, Pawlowski '07

topology ?

- tunneling in QM
- instanton-induced terms in QCD
- $\mathcal{N} = 2$ susy Yang-Mills

Zappala, Phys. Lett. A **290** (2001) 35

Pawlowski, Phys. Rev. D **58** (1998) 045011

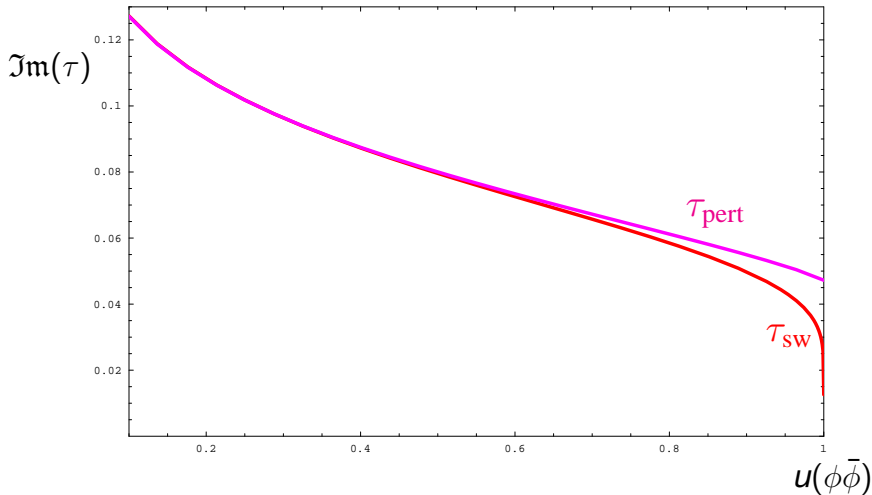
Dolan, Pawlowski, unpublished work

$$\tau = \frac{i}{g^2} + \frac{\theta}{8\pi^2} = \frac{i}{4\pi^2} \left(\ln \frac{\phi\bar{\phi}}{\Lambda_{\text{QCD}}^2} + 3 \right) + \text{top.}$$

- anomalies, solitons ...

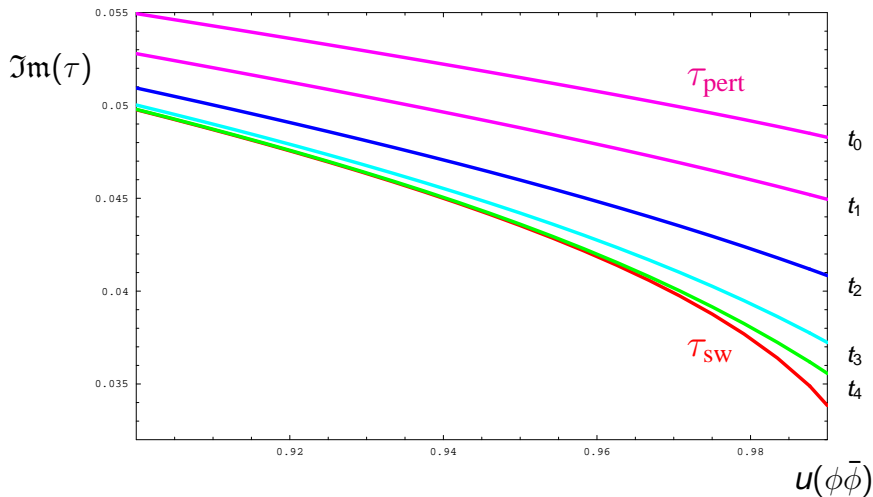
Coupling τ in $\mathcal{N} = 2$ susy Yang-Mills (Seiberg-Witten)

Dolan, Pawłowski, unpublished work



Coupling τ in $\mathcal{N} = 2$ susy Yang-Mills (Seiberg-Witten)

Dolan, Pawłowski, unpublished work



1 Functional RG

- properties
- topology

2 Landau gauge QCD

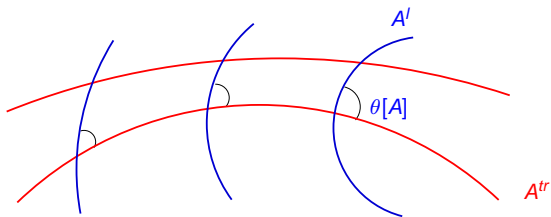
- Signatures of confinement
- Infrared asymptotics & finite volume effects

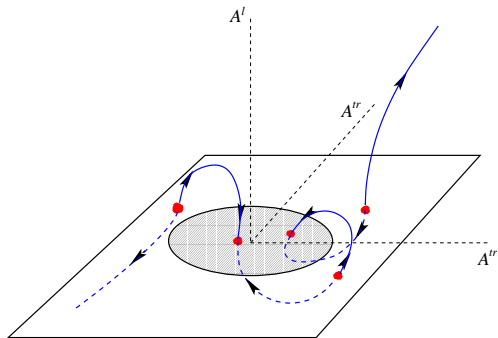
3 QCD at finite temperature

- confinement-deconfinement phase transition

$$S_{\text{cl}} = \frac{1}{2} \int \text{tr} F^2 = \frac{1}{2} \int A_\mu^a (p^2 \delta_{\mu\nu} - p_\mu p_\nu) A_\nu^a + \dots$$

gauge fixing ensures the existence of the gauge field propagator





Gribov problem

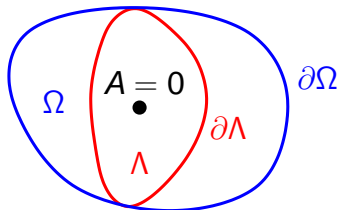
confinement scenario

$$\Omega = \{A \mid \partial_\mu A_\mu = 0, -\partial_\mu D_\mu \geq 0\}$$

- entropy

$$\int dA \det(-\partial D) e^{-S}$$

- entropy ($\int dA$)
 - $\partial\Omega(\cap\partial\Lambda)$ dominates IR
 - ghost IR-enhanced
 - gluonic mass-gap: **confined gluons**



non-renormalisation of ghost-gluon vertex

confinement scenario

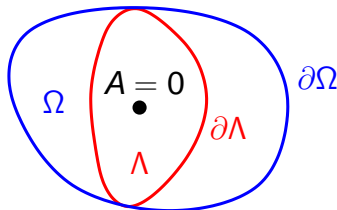
$$\Omega = \{A \mid \partial_\mu A_\mu = 0, -\partial_\mu D_\mu \geq 0\}$$

- entropy

$$\int dA \det(-\partial D) e^{-S}$$

- entropy ($\int dA$)

- $\partial\Omega(\cap\partial\Lambda)$ dominates IR
- ghost IR-enhanced
- gluonic mass-gap: **confined gluons**



non-renormalisation of ghost-gluon vertex

-
- Kugo-Ojima (in BRST-extended configuration space)

- gluonic mass-gap + no Higgs mechanism

functional RG

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \left(\text{Diagram 1} - \text{Diagram 2} \right)$$

- mode cut-off

$$R_k(p^2) \propto \Gamma_0^{(2)}(p^2) \delta(p^2 - k^2)$$

functional RG

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \left(\text{Diagram 1} - \text{Diagram 2} \right)$$

- mode cut-off

$$R_k(p^2) \propto \Gamma_0^{(2)}(p^2) \delta(p^2 - k^2)$$

- physics unchanged

functional RG

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \left(\text{Diagram 1} - \text{Diagram 2} \right)$$

- mode cut-off

$$R_k(p^2) \propto \Gamma_0^{(2)}(p^2) \delta(p^2 - k^2)$$

- physics unchanged
- loop integration

$$\frac{1}{\Gamma_k^{(2)} + R_k} (k \partial_k R_k) \frac{1}{\Gamma_k^{(2)} + R_k} \simeq \frac{1}{\Gamma_0^{(2)}} k^2 \delta'(p^2 - k^2)$$

functional RG

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \left(\text{Diagram 1} - \text{Diagram 2} \right)$$

The diagram shows the flow equation for the effective action Γ_k . The right-hand side is the difference of two diagrams, each representing a self-energy correction to the propagator. The first diagram is a solid circle with 16 small ovals along its perimeter, a solid black dot at the bottom, and a crossed circle at the top. The second diagram is a dashed circle with the same solid black dot at the bottom and crossed circle at the top, but without the ovals.

functional RG

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \left(\text{Diagram 1} - \text{Diagram 2} \right)$$

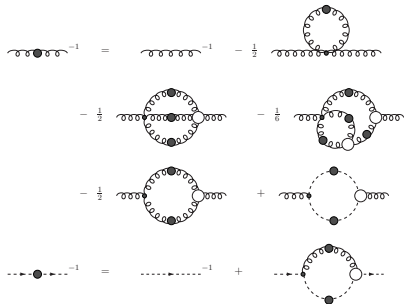
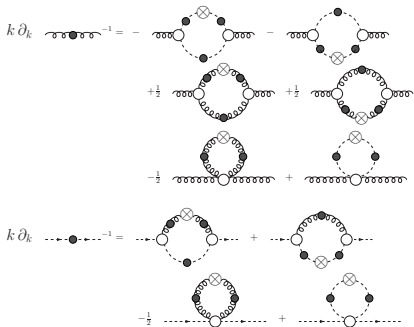
functional DSE

$$\frac{\delta \Gamma_k[\phi]}{\delta A} = \frac{\delta S[\phi]}{\delta A} + \text{Diagram 1} + \text{Diagram 2} + \text{Diagram 3}, \quad \frac{\delta \Gamma_k[\phi]}{\delta C} = \frac{\delta S[\phi]}{\delta C} + \text{Diagram 4}$$

$$\begin{aligned}
 k \partial_k \text{ (wavy line with dot) }^{-1} = & - \text{ (loop with dashed lines, top cross) } - \text{ (loop with dashed lines, bottom cross) } \\
 & + \frac{1}{2} \text{ (loop with solid lines, top cross) } + \frac{1}{2} \text{ (loop with solid lines, bottom cross) } \\
 & - \frac{1}{2} \text{ (loop with solid lines, top cross) } + \text{ (loop with dashed lines, top cross) }
 \end{aligned}$$

$$\begin{aligned}
 k \partial_k \text{ (dashed line with dot) }^{-1} = & \text{ (loop with dashed lines, top cross) } + \text{ (loop with solid lines, top cross) } \\
 & - \frac{1}{2} \text{ (loop with solid lines, top cross) } + \text{ (loop with dashed lines, top cross) }
 \end{aligned}$$

$$\begin{aligned}
 \text{Diagram 1}^{-1} &= \text{Diagram 2}^{-1} - \frac{1}{2} \text{Diagram 3} \\
 &- \frac{1}{2} \text{Diagram 4} - \frac{1}{6} \text{Diagram 5} \\
 &- \frac{1}{2} \text{Diagram 6} + \text{Diagram 7} \\
 \text{Diagram 8}^{-1} &= \text{Diagram 9}^{-1} + \text{Diagram 10}
 \end{aligned}$$



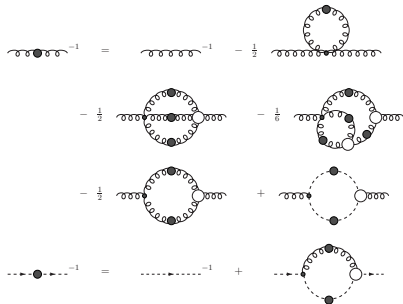
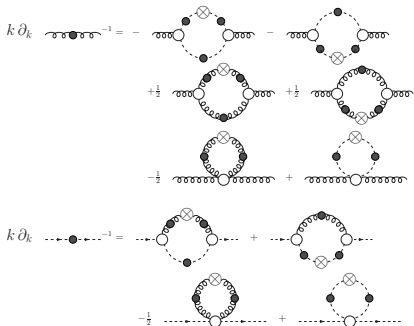
Unique infrared asymptotics in Landau gauge QCD

- conformal scaling

$$\Gamma^{(2n,m)}(\lambda p_1, \dots, \lambda p_{2n+m}) = \lambda^{\kappa_{2n,m}} \Gamma^{(2n,m)}(p_1, \dots, p_{2n+m})$$

- decoupling: $\kappa_{n,m} = 0$ & massive gluon

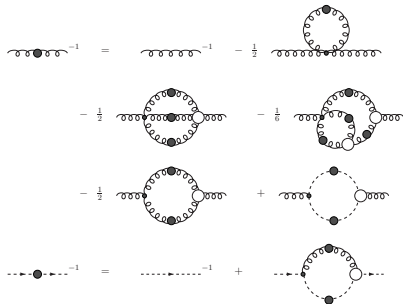
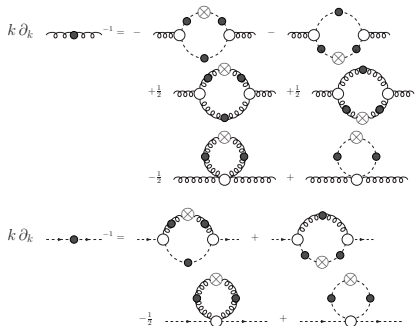
no confinement!?



Unique infrared asymptotics in Landau gauge QCD

$$\Gamma(2n, m) \sim p^{2(n-m)} \kappa_C \quad \text{with} \quad \kappa_C \geq 0$$

$\Gamma(2n, m)$: vertex with n ghost and anti-ghost lines, m gluons



Unique infrared asymptotics in Landau gauge QCD

$$\Gamma(2n, m, \text{quarks}) \sim p^{2(n-m)\kappa_C + \text{quarks}}$$

QCD: work in progress; QED₃: Nedelko, Pawłowski, in preparation

UV-IR flow

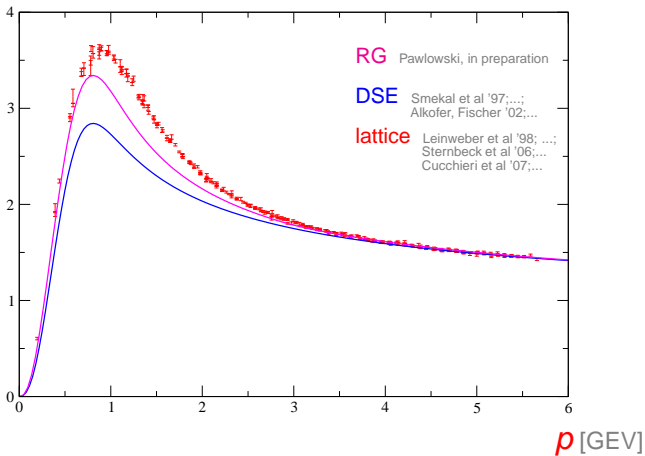
- full momentum dependence of propagators
- vertices momentum-dependent RG-dressing
- optimisation
- functional relations between diagrams

UV-IR flow

$$k \partial_k \text{ (wavy line with dot) }^{-1} = - \text{ (loop with dashed line and cross) } - \text{ (loop with dashed line and dot) } + \frac{1}{2} \text{ (loop with wavy line and cross) } + \frac{1}{2} \text{ (loop with wavy line and dot) } - \frac{1}{2} \text{ (loop with wavy line and cross) } + \text{ (loop with dashed line and dot) }$$

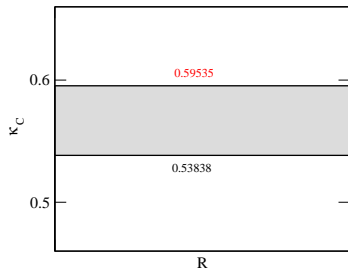
$$k \partial_k \text{ (dashed line with dot) }^{-1} = \text{ (loop with dashed line and cross) } + \text{ (loop with dashed line and dot) } - \frac{1}{2} \text{ (loop with wavy line and cross) } + \text{ (loop with dashed line and dot) }$$

$$\frac{p^2}{\Gamma_A^{(2)}(p)}$$



$$p^2 \langle A(p)A(-p) \rangle = \frac{p^2}{\Gamma_A^{(2)}(p)} \xrightarrow{p \rightarrow 0} (p^2)^{-2\kappa_C}$$

$$\stackrel{\text{DSE}}{=} \frac{D(p^2)}{p^2}$$



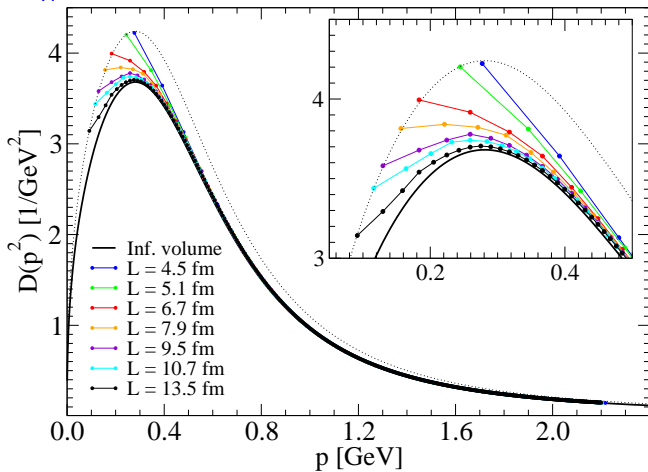
Pawlowski, Litim, Nedelko, von Smekal, Phys. Rev. Lett. **93** (2004) 152002

- optimisation: $\kappa_C = 0.59535\dots$, $\alpha_S = 2.9717\dots$

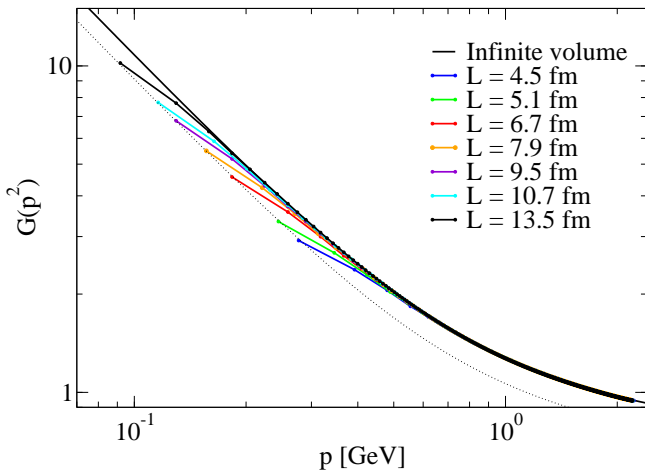
equals DS/StochQuant-result: Lerche, von Smekal, Phys. Rev. D **65** (2002) '02
D. Zwanziger, Phys. Rev. D **65** (2002)

RG-confirmation: C. S. Fischer and H. Gies, JHEP **0410** (2004)

$$D(p^2) = \frac{1}{\Gamma_A^{(2)}(p)}$$



$$G(p^2) = \frac{p^2}{\Gamma_C^{(2)}(p)}$$



Functional methods—lattice puzzle

- lower dimensions
 - quantitative agreement in $d = 2$ Maas '07
 - qualitative agreement in $d = 3$ talk of A. Maas
- large volumes on the lattice
 - in $d = 4$ up to 128^4 at $\beta = 2.2$ Cucchieri et al '07
- gauge fixings
 - improved gauge fixing procedures Bogolubsky et al '07, von Smekal et al '07, talk of A. Maas
 - stochastic quantisation with D. Spielmann, I.O. Stamatescu
- $SU(2)$ versus $SU(3)$ Cucchieri et al '07, Sternbeck et al '07
- $\beta = 0$: evidence for gauge fixing/finite size problems talk of L. von Smekal

1 Functional RG

- properties
- topology

2 Landau gauge QCD

- Signatures of confinement
- Infrared asymptotics & finite volume effects

3 QCD at finite temperature

- confinement-deconfinement phase transition

Order parameter

- Polyakov loop $\Phi(\vec{x}) = \langle L[A_0] \rangle$

$$L[A_0](\vec{x}) = \frac{1}{N_c} \text{tr} \mathcal{P} e^{i \int_0^\beta dt A_0}$$

with $\langle \Phi \rangle \simeq e^{-F_q}$

- confinement: $F_q = \infty$
- deconfinement: F_q finite

Order parameter

- Polyakov loop $\Phi(\vec{x}) = \langle L[A_0] \rangle$

$$L[A_0](\vec{x}) = \frac{1}{N_c} \text{tr} \mathcal{P} e^{i \int_0^\beta dt A_0}$$

with $\langle \Phi \rangle \simeq e^{-F_q}$

- confinement: $F_q = \infty$
 - deconfinement: F_q finite
- string tension

$$\langle L(\vec{x}) L^\dagger(\vec{y}) \rangle \simeq e^{-F_{q\bar{q}}(\vec{x} - \vec{y})}$$

- $\lim_{|\vec{x} - \vec{y}| \rightarrow \infty} F_{q\bar{q}}(\vec{x} - \vec{y}) \simeq \beta \sigma |\vec{x} - \vec{y}|$

- background field flow

$$k\partial_k\Gamma_k[\phi, A] = \frac{1}{2} \text{Tr} \frac{1}{\Gamma_k^{(2,0)}[\phi, A] + R_k(\Gamma_k^{(2,0)}[0, A])} k\partial_k R_k(\Gamma_k^{(2,0)}[0, A])$$

- fluctuation fields $\phi = (a, C, \bar{C})$
- background field A
- Landau-DeWitt gauge: $D_\mu(A)a_\mu = 0$

- background field flow for effective potential $V_{\text{eff}}[A_0] = \Gamma_k[0, A_0]$

$$k\partial_k V_{\text{eff}}[A_0] = \frac{1}{2} \text{Tr} \frac{1}{\Gamma_k^{(2,0)}[0, A_0] + R_k(\Gamma_k^{(2,0)}[0, A_0])} k\partial_k R_k(\Gamma_k^{(2,0)}[0, A_0])$$

- vanishing fluctuation fields $\phi = 0$

$$\Gamma_{k,A}^{(2,0)} = \frac{\delta^2 \Gamma_k}{\delta a^2} \neq \frac{\delta^2 \Gamma_k}{\delta A^2}$$

- background field flow for effective potential $V_{\text{eff}}[A_0] = \Gamma_k[0, A_0]$

$$k\partial_k V_{\text{eff}}[A_0] = \frac{1}{2} \text{Tr} \frac{1}{\Gamma_k^{(2,0)}[0, A_0] + R_k(\Gamma_k^{(2,0)}[0, A_0])} k\partial_k R_k(\Gamma_k^{(2,0)}[0, A_0])$$

- determination of propagator

$$\Gamma_k^{(2,0)}[0, A] = \Gamma_{k, \text{Landau}}^{(2)}(p^2 \rightarrow -D^2) + O(F)$$

- background field flow for effective potential $V_{\text{eff}}[A_0] = \Gamma_k[0, A_0]$

$$k\partial_k V_{\text{eff}}[A_0] = \frac{1}{2} \text{Tr} \frac{1}{\Gamma_k^{(2,0)}[0, A_0] + R_k(\Gamma_k^{(2,0)}[0, A_0])} k\partial_k R_k(\Gamma_k^{(2,0)}[0, A_0])$$

- determination of propagator

$$\Gamma_k^{(2,0)}[0, A] = \Gamma_{k, \text{Landau}}^{(2)}(p^2 \rightarrow -D^2) + O(F)$$

- Polyakov loop $\Phi(\vec{x}) = \langle L[A_0] \rangle$

$$L[A_0] = \frac{1}{N_c} \text{tr} \mathcal{P} e^{i \int_0^\beta dt A_0}$$

- background field flow for effective potential $V_{\text{eff}}[A_0] = \Gamma_k[0, A_0]$

$$k\partial_k V_{\text{eff}}[A_0] = \frac{1}{2} \text{Tr} \frac{1}{\Gamma_k^{(2,0)}[0, A_0] + R_k(\Gamma_k^{(2,0)}[0, A_0])} k\partial_k R_k(\Gamma_k^{(2,0)}[0, A_0])$$

- determination of propagator

$$\Gamma_k^{(2,0)}[0, A] = \Gamma_{k, \text{Landau}}^{(2)}(\mathbf{p}^2 \rightarrow -D^2) + O(F)$$

- Polyakov loop $\Phi(\vec{x}) = \langle L[A_0] \rangle$

$$L[\langle A_0 \rangle] \quad \text{from} \quad \left. \frac{\partial V_{\text{eff}}[A_0]}{\partial A_0} \right|_{A_0 = \langle A_0 \rangle} = 0$$

- background field flow for effective potential $V_{\text{eff}}[A_0] = \Gamma_k[0, A_0]$

$$k\partial_k V_{\text{eff}}[A_0] = \frac{1}{2} \text{Tr} \frac{1}{\Gamma_k^{(2,0)}[0, A_0] + R_k(\Gamma_k^{(2,0)}[0, A_0])} k\partial_k R_k(\Gamma_k^{(2,0)}[0, A_0])$$

- determination of propagator

$$\Gamma_k^{(2,0)}[0, A] = \Gamma_{k, \text{Landau}}^{(2)}(p^2 \rightarrow -D^2) + O(F)$$

- Polyakov loop $\Phi(\vec{x}) = \langle L[A_0] \rangle$

$$L[\langle A_0 \rangle] \geq \langle L[A_0] \rangle$$

- full effective action

$$\Gamma_0[0, A] = \frac{1}{2} \text{Tr} \ln \Gamma_0^{(2,0)}[0, A] + \mathcal{O}(\partial_t \Gamma_k^{(2,0)}) + \text{c.t.}$$

- full effective action

$$\Gamma_0[0, A] = \frac{1}{2} \text{Tr} \ln \Gamma_0^{(2,0)}[0, A] + \mathcal{O}(\partial_t \Gamma_k^{(2,0)}) + \text{c.t.}$$

- full effective potential in the deep infrared, $\Gamma_{0,A/C}^{(2,0)} \sim (-D^2)^{1+\kappa_{A/C}}$

$$V^{\text{IR}}[\beta A_0] \simeq \left\{ \frac{d-1}{2} (1 + \kappa_A) + \frac{1}{2} - (1 + \kappa_C) \right\} \frac{1}{\Omega} \text{Tr} \ln (-D^2[A_0])$$

- full effective action

$$\Gamma_0[0, A] = \frac{1}{2} \text{Tr} \ln \Gamma_0^{(2,0)}[0, A] + O(\partial_t \Gamma_k^{(2,0)}) + \text{c.t.}$$

- full effective potential in the deep infrared

$$V^{\text{IR}}[\beta A_0] \simeq \left\{ 1 + \frac{(d-1)\kappa_A - 2\kappa_C}{d-2} \right\} V^{\text{UV}}[\beta A_0]$$

- full effective action

$$\Gamma_0[0, A] = \frac{1}{2} \text{Tr} \ln \Gamma_0^{(2,0)}[0, A] + \mathcal{O}(\partial_t \Gamma_k^{(2,0)}) + \text{c.t.}$$

- full effective potential in the deep infrared

$$V^{\text{IR}}[\beta A_0] \simeq \left\{ 1 + \frac{(d-1)\kappa_A - 2\kappa_C}{d-2} \right\} V^{\text{UV}}[\beta A_0]$$

- confinement criterion with sum rule $\kappa_A = -2\kappa_C - \frac{4-d}{2}$

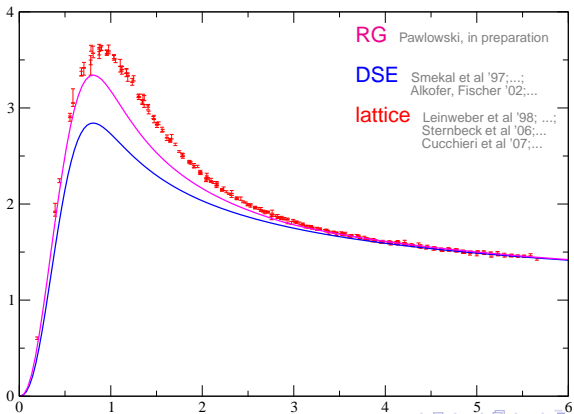
$$\kappa_C > \frac{d-3}{4}$$

no confinement with background field propagators $\delta^2 \Gamma_k / \delta A^2$

- determination of $L(\langle A_0 \rangle)$

$$\Gamma_0[0, A] = \frac{1}{2} \text{Tr} \ln \Gamma_0^{(2,0)}[0, A] + \mathcal{O}(\partial_t \Gamma_k^{(2,0)}) + \text{c.t.}$$

$$\frac{p^2}{\Gamma_A^{(2)}(p)}$$



p [GeV]

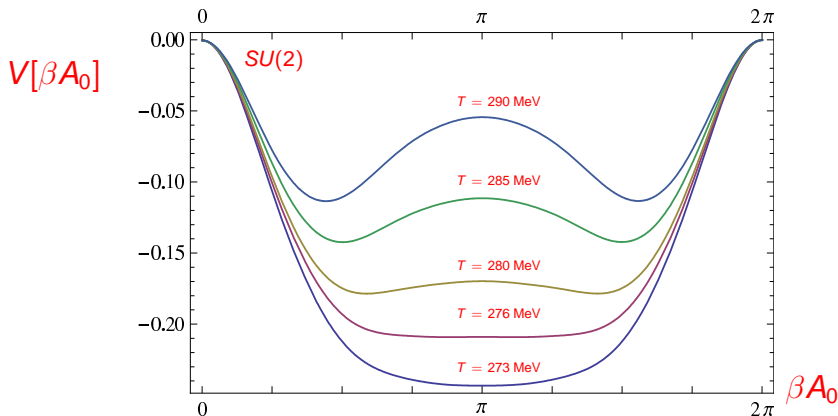
Polyakov loop potential, $SU(2)$

Braun, Gies, Pawłowski, arXiv:0708.2413 [hep-th]

$$T_c \simeq 276 \pm 10 \text{ MeV}$$

$$T_c/\sqrt{\sigma} = 0.627 \pm 0.023$$

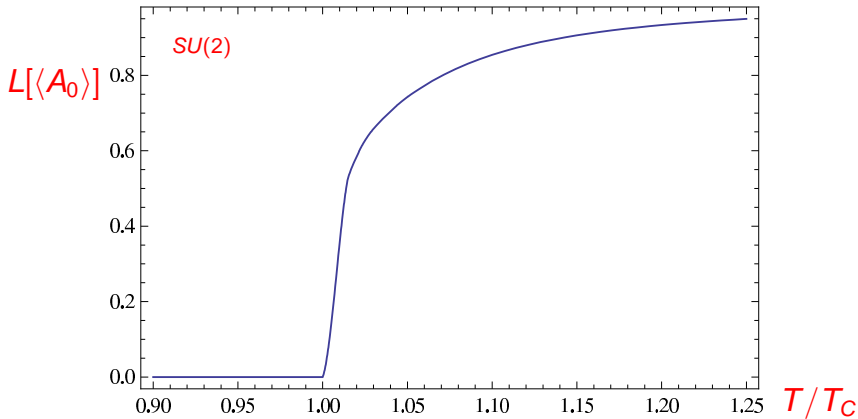
lattice: $T_c/\sqrt{\sigma} = .709$



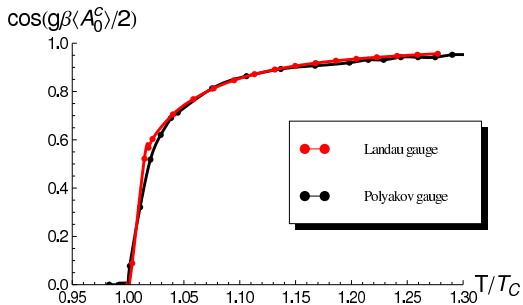
$$T_c \simeq 276 \pm 10 \text{ MeV}$$

$$T_c/\sqrt{\sigma} = 0.627 \pm 0.023$$

$$\text{lattice: } T_c/\sqrt{\sigma} = .709$$



flow in Polyakov gauge: $A_0 = A_0(\vec{x})\sigma_3$



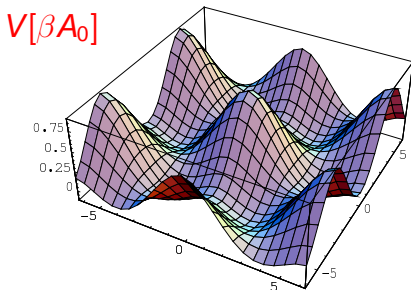
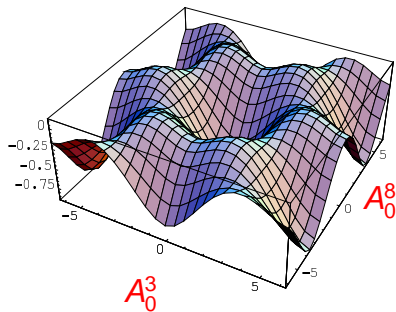
- —: Polyakov gauge
- —: Landau gauge propagators

Polyakov loop potential, $SU(3)$

Braun, Gies, Pawłowski, arXiv:0708.2413 [hep-th]

$$T_c \simeq 284 \pm 10 \text{ MeV}$$

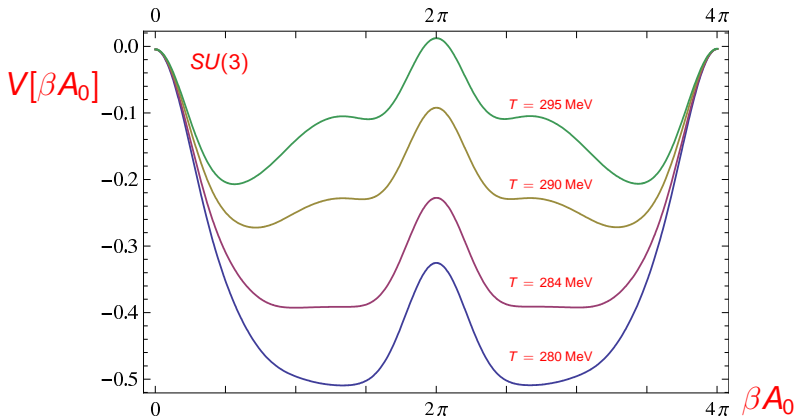
$$T_c/\sqrt{\sigma} = 0.646 \pm 0.023 \quad \text{lattice: } T_c/\sqrt{\sigma} = .646$$



$$T_c \simeq 284 \pm 10 \text{ MeV}$$

$$T_c/\sqrt{\sigma} = 0.646 \pm 0.023$$

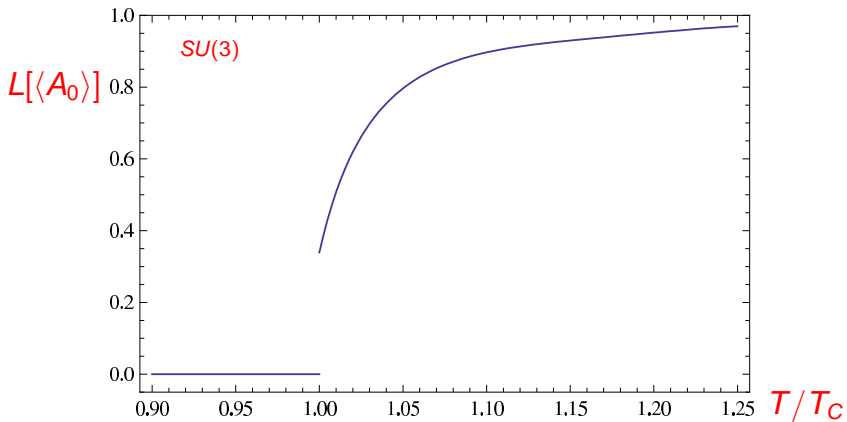
$$\text{lattice: } T_c/\sqrt{\sigma} = .646$$



$$T_c \simeq 284 \pm 10 \text{ MeV}$$

$$T_c/\sqrt{\sigma} = 0.646 \pm 0.023$$

$$\text{lattice: } T_c/\sqrt{\sigma} = .646$$



- results

- support for Kugo-Ojima/Gribov-Zwanziger scenario
- confinement-deconfinement phase transition from KO/GZ
- dynamical chiral symmetry breaking see talks of H. Gies, B.-J. Schaefer
- 'QCD phase diagram' from models see talk of B.-J. Schaefer

- challenges

- full QCD
- QCD at finite temperature & density
- flow of Wilson loops & Polyakov loops: area law

- results

- support for Kugo-Ojima/Gribov-Zwanziger scenario
- confinement-deconfinement phase transition from KO/GZ
- dynamical chiral symmetry breaking
- 'QCD phase diagram' from models

- challenges

- full QCD
- QCD at finite temperature & density
- flow of Wilson loops & Polyakov loops: area law