QCD vacuum and hadrons

Yu.A.Simonov, ITEP, Moscow

Talk at the 415 th Wilhelm & Else Heraeus seminar:"Quarks and Hadrons in strong QCD", St. Goar, Germany, 17-20 March 2008

Contents

1. Introduction: Mysteries of QCD

1. QCD is a selfconstitent quantum theory defined by the only scale: Λ_{QCD}, or string tension σ, or ρ meson mass...
   How these different scales are connected? How dimensional transmutation works?
   Actually in QCD different dynamical scales occur: glueball mass $M_{gl} \gtrsim 2$ GeV, while ρ meson mass 0.75 GeV, $m_\pi \equiv 0.14$ GeV etc.
2. Potential models work nicely even for high excited meson states, while QCD sum rules fail (e.g. for ground state glueballs).
3. What is dynamics of confinement, and
4. Why chiral symmetry breaking accompanies confinement, and they disappear together at temperature phase transition.

It will be shown, that the basic role in answering these question lies in the fundamental quantity – Vacuum Field Correlator and its correlation length λ.

2. Field Correlators and QCD vacuum

As will be shown below, Green's function of any white system is proportional to the path integral of the Wilson loop.

For $q\bar{q}, G_{q\bar{q}} \sim \int (Dz)\langle trW(C)\rangle$... Therefore Wilson loop defines the dynamics (pert. and nonpert.) of light and heavy quarks.

Building blocks: Wegner-Wilson loops

$$W(C) = \mathsf{P}\exp ig\oint_C A^a_\mu(z)t^a dz_\mu \tag{1}$$

Parallel transporter

$$\Phi(x;y) = \mathsf{P}\exp ig\int_x^y A^a_\mu(z)t^a dz_\mu \tag{2}$$

Field strength

$$F_{\mu\nu}(x) = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu]$$

$$D^{(n)}_{\mu_1\nu_1\ldots\mu_n\nu_n}(x_1, ..., x_n) =$$

$$= \left(\frac{g}{\sqrt{N_c}}\right)^n \langle \mathsf{Tr}\, F_{\mu_1\nu_1}(x_1)\Phi(x_1, x_2)F_{\mu_2\nu_2}(x_2)...F_{\mu_n\nu_n}(x_n)\Phi(x_n, x_1)\rangle \tag{3}$$

Nonabelian Stokes Theorem and Cluster Expansion

$$\langle \mathsf{Tr}\, W(C)\rangle = \left\langle \mathsf{Tr}\, \mathcal{P} \exp ig \int\limits_S \Phi F_{\mu\nu}(z)\Phi d\sigma_{\mu\nu}(z) \right\rangle =$$

$$= \exp \sum_{n=2}^{\infty} (i)^n \Delta^{(n)}[S] = \exp(-V(R)T) \tag{4}$$

The basic element of Nonperturbative QCD – the correlator $D^{(2)}_{\mu\nu\rho\sigma}$.

$$\Delta^{(2)}[S] = \frac{1}{2}\int\limits_S d\sigma_{\mu\nu}(z_1)\int\limits_S d\sigma_{\rho\sigma}(z_2) D^{(2)}_{\mu\nu\rho\sigma}(z_1, z_2) \tag{5}$$

$$\Delta^{(2)}[S] = \sigma S$$

Gauge-invariant Field Correlators

$$D^{(2)}_{\mu\nu\rho\sigma}(z) = \frac{g^2}{N_c}\langle \mathrm{Tr}\, F_{\mu\nu}(x)\Phi F_{\rho\sigma}(y)\Phi\rangle \tag{6}$$

Two basic scalars: D and D_1 (Dosch+ Yu.S., ('88)).

$$D^{(2)}_{\mu\nu\rho\sigma}(z) = (\delta_{\mu\rho}\delta_{\nu\sigma} - \delta_{\mu\sigma}\delta_{\nu\rho})D(z) +$$
$$+\frac{1}{2}\left(\frac{\partial}{\partial z_\mu}(z_\rho\delta_{\nu\sigma} - z_\sigma\delta_{\nu\rho}) - \frac{\partial}{\partial z_\nu}(z_\rho\delta_{\mu\sigma} - z_\sigma\delta_{\mu\rho})\right) D_1(z) \qquad (7)$$

$D(x)$ is purely nonperturbative (pert. cancel-Shevchenko+Yu.S.'98).

Important: Dominance of Gaussian correlator $D^{(2)}(z) \to$ the QCD vacuum is almost $(>95\%)$ Gaussian (Bali '99, Shevchenko and Yu.S.'00).Check: Casimir scaling – $\Delta^{(2)} \sim C_2$, hence all $Q\bar{Q}$ potentials in different representations (j) are proportional to $C_2(j)$. Odd n correlations vanish on flat surfaces).

$$\Delta^{(2)}[S] \gg \sum_{n=3}^{\infty} \Delta^{(n)}[S] \qquad (8)$$

If (connected) average $D^{(n)}(x_1 - x_2, ...) \sim \exp(-\frac{|x_i - x_j|}{\lambda})$ for large $|x_i - x_j|$, then

$$\frac{\Delta^{(n+2)}[S]}{\Delta^{(n)}[S]} \approx \lambda^4 \langle F^2 \rangle \approx \sigma \lambda^2$$

It will be shown, that $\lambda \sim 0.1$fm, and expansion parameter is $\sigma\lambda^2 \sim 0.05$. Therefore all $\Delta^{(n)}$ with $n > 2$ contribute few percent.

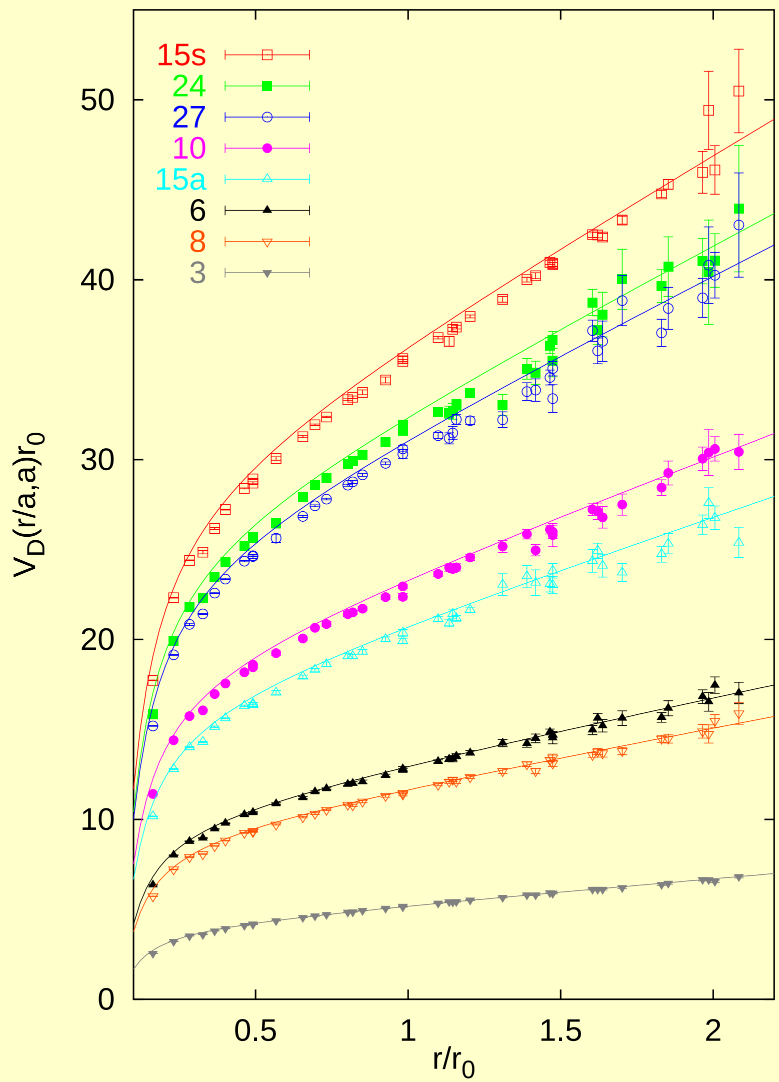

Figure 1:

From lattice and analytic data

$$D(x) \sim \exp(-|x|/\lambda),$$

Important feature of QCD vacuum! Vacuum correlator length λ
Campostrini, Di Giacomo, Olejnik ('86).
Di Giacomo et al. $\lambda \approx 0.2 \div 0.3$ fm
Bali, Brambilla, Vairo $\lambda \lesssim 0.2$ fm
Dosch et al. $\lambda \lesssim 0.2$ fm
Yu.S. $\lambda \approx 0.15$ fm.

Recently $D(x), D_1(x)$ were computed on lattice (Koma and Koma) in evaluating spin-dependent potentials. Results are compatible with $\lambda \lesssim 0.1$ fm.

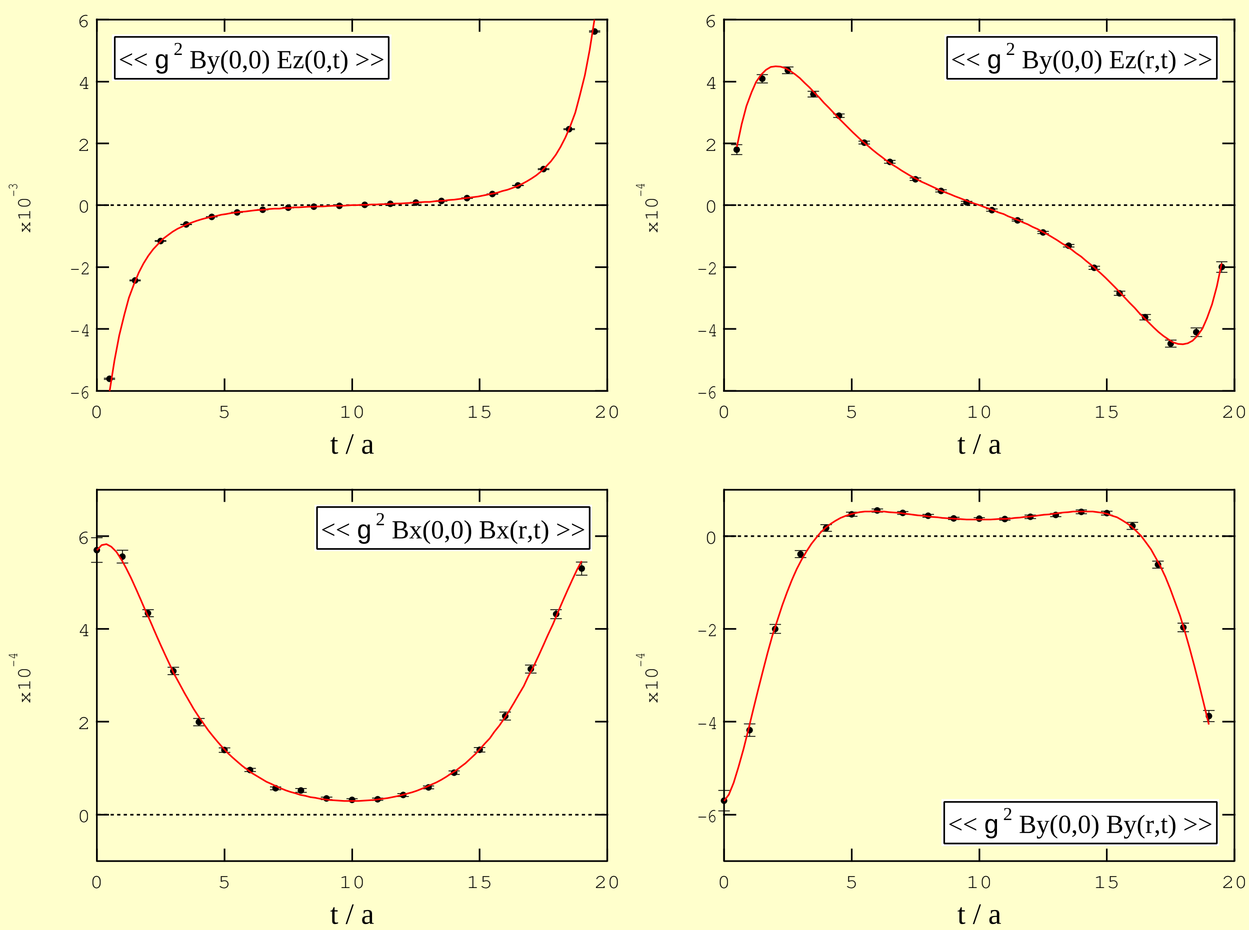

Figure 2: Field strength correlators at $\beta = 6.0$ on the 20^4 lattice for $r/a = 5$ as a function of t/a. The solid lines are the fit curves corresponding to Eqs. (**??**)-(**??**).

Static potentials from rectangular WW loop $(R \times T)$,

$$\langle trW(C)\langle = \exp(-TV(R))$$

$$V(R) = V_D(R) + V_1(R)$$

$$V_D(R) = 2\int_0^R (R-\rho)d\rho \int_0^\infty d\nu D(\sqrt{\rho^2+\nu^2}), \tag{9}$$

$$V_1(R) = \int_0^R \rho d\rho \int_0^\infty d\nu D_1(\sqrt{\rho^2+\nu^2}). \tag{10}$$

D ensures confinement

$$V_D(R) = \sigma R + \mathcal{O}(R^0) \quad ; \quad \sigma = \frac{1}{2}\int d^2 z D(z), R \to \infty \tag{11}$$

$$V_D(R) = cR^2 + O(R^4), \quad R \lesssim \lambda \tag{12}$$

D_1 contains all (but not confinement), $V_1(R) = V_1^{(pert)} + V_1^{(nonpert)}$

$$V_1^{(nonpert)}(R \to \infty) = const \sim 0.5 GeV \tag{13}$$

V_1 supports bound states $Q\bar{Q}$ in quark-gluon plasma (Yu.S.'91, '05)

$$V_1^{(pert)} = -\frac{4(\alpha_s + O(\alpha_s^2))}{3R}. \tag{14}$$

3. Visualizing the QCD strings

The gauge-invariant probe of fields in $W(C)$ with a small plaquette.

The field $\mathcal{F}_{\mu\nu}\ (x)$ is gauge invariant.

$$\mathcal{F}_{\mu\nu}(x) = \langle \mathsf{Tr}\, W(C)\rangle^{-1}\langle \mathsf{Tr}\, ig\Phi F_{\mu\nu}(x)\Phi W(C)\rangle. \tag{15}$$

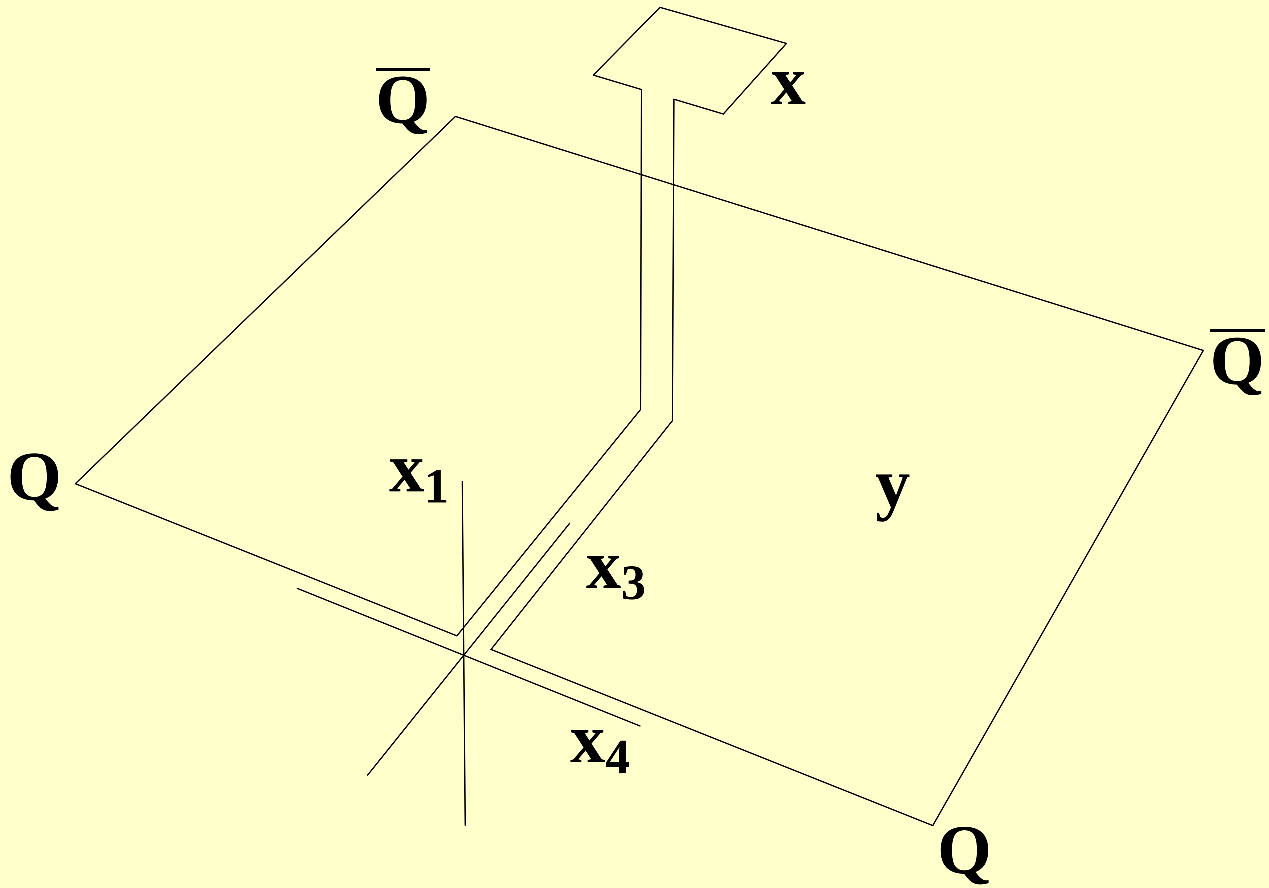

Figure 3: A connected probe for static quark and antiquark

$$\mathcal{F}_{\mu\nu}(x)\,\delta\sigma_{\mu\nu}(x) = \langle \mathsf{Tr}\, W(C)\rangle^{-1}(\langle \mathsf{Tr}\, W(C,C_P)\rangle -$$
$$-\langle \mathsf{Tr}\, W(C)\rangle) \equiv \tilde{M}(C,C_P) \tag{16}$$

Electric field in the probe $\mathcal{F}$; $\boldsymbol{\mathcal{E}}_k \equiv \mathcal{F}_{k4}$. Differentiating $\mathcal{F}$ one obtains equations of motion, i.e. Maxwell equations

$$\frac{\partial}{\partial x_\rho}\mathcal{F}_{\rho\mu}(x) = j_\mu(x) \tag{17}$$

Magnetic current

Maxwell equations with magnetic currents

$$\frac{1}{2}\,\epsilon_{\mu\rho\alpha\beta}\frac{\partial}{\partial x_\rho}\mathcal{F}_{\alpha\beta}(x) = k_\mu(x)\ ; \qquad \frac{\partial}{\partial x_\rho}\mathcal{F}_{\rho\mu}(x) = j_\mu(x), \tag{18}$$

Now we use Method of Field correlators, and $\mathcal{F}_{\mu\nu}$ is expressed through $D^{(2)}$

$$\mathcal{F}_{\mu\nu}(x) = \int_S d\sigma_{\alpha\beta}(y)\, D^{(2)}_{\alpha\beta\mu\nu}(x-y), D^{(2)} = \delta \cdot \delta \;\; D + (...)D_1 \tag{19}$$

We know $D(z)$, $D_1(z)$ both from lattice and from analytic calculations

$$D(z) = \frac{\sigma}{\pi\lambda^2} \exp\left(-\frac{|z|}{\lambda}\right). \tag{20}$$

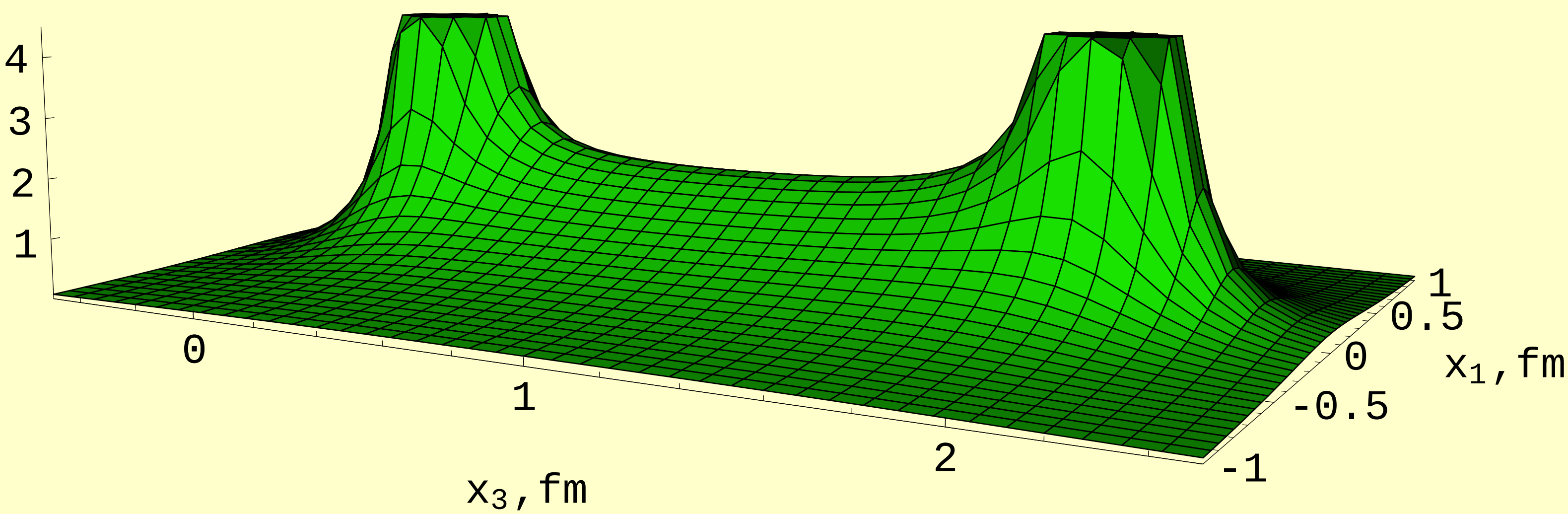

Figure 4: A distribution of the field $|\boldsymbol{\mathcal{E}}(x_1, 0, x_3)|$ at quark-antiquark separation 2 fm. Cutted peaks of color-Coulomb field and string between quark and antiquark are clearly distinguished. The standard values of parameters $\sigma = 0.18$ GeV^2, $\lambda = 0.2$ fm are used.

$$\mathcal{E}(\rho) = 2\sigma\left(1 + \frac{\rho}{\lambda}\right)\exp\left(-\frac{\rho}{\lambda}\right), \tag{21}$$

Radius of the string R_{str} is $R_{str} \sim \lambda$.

From Maxwell equations

$$\mathbf{k} = \mathrm{rot}\,\boldsymbol{\mathcal{E}}, \tag{22}$$

$$k_\varphi(\rho) = -\frac{2\sigma\rho}{\lambda^2}\exp\left(-\frac{\rho}{\lambda}\right). \tag{23}$$

Equivalent of (dual) Londons equation

$$\mathrm{rot}\ \mathbf{k} = \lambda^{-2}\boldsymbol{\mathcal{E}} \tag{24}$$

Hence dual Meissner effect: circular magnetic current squeeze (color)electric fluxes - strings are created-confinement.

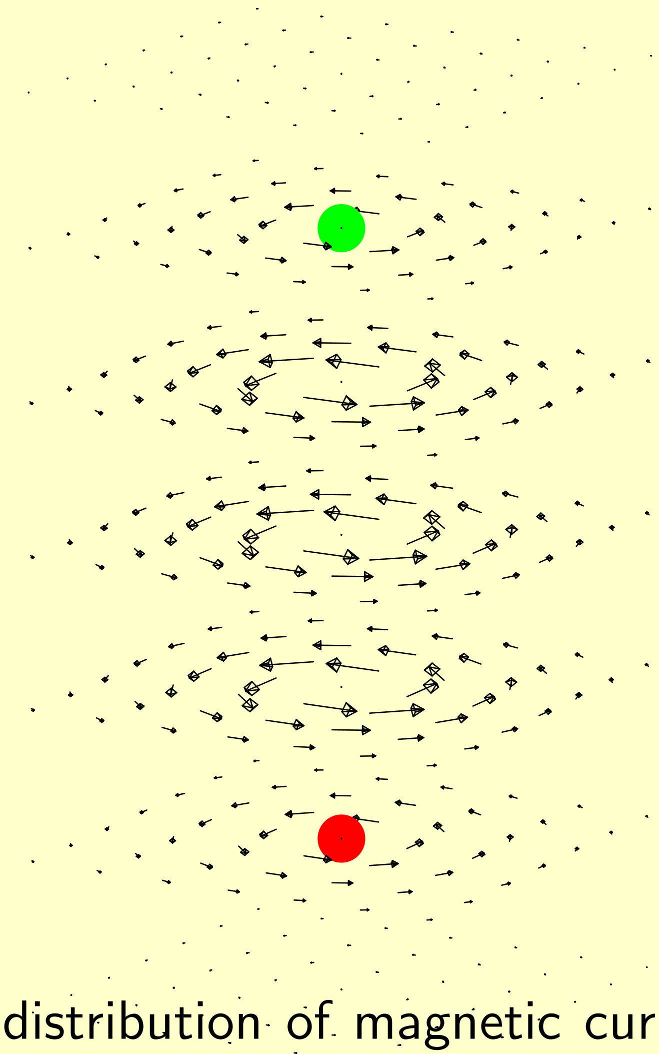

Figure 5: A vector distribution of magnetic currents at quark-antiquark separation 2 fm. Positions of quark and antiquark are shown by points.

Origin of Confinement

It is interesting what is the origin of the circular magnetic currents, i.e. the origin of confinement?

From the definition of $\mathbf{k}$, $k_\mu = \varepsilon_{\mu\alpha\beta\gamma}\partial_\alpha \mathcal{F}_{\beta\gamma} = \int(FFF) + \int(D\tilde{F})$. In nonabelian theory Bianchi identity holds, $D\tilde{F} = 0$.Hence triple condensate is at the base of confinement.
$k \sim \langle FFF \rangle = f^{abc} e_{ikl} \langle E_i^a E_k^b H_l^c \rangle$. Note, that triple condensate does not exist in Abelian theory.

In Abelian theory, $U(1)$, in confinement phase $D\tilde{F} \neq 0$ due to magnetic monopoles (lattice artefacts).

In a similar way one defines the distribution of colorelectric fields, $|\mathcal{E}|^2$ in a baryon

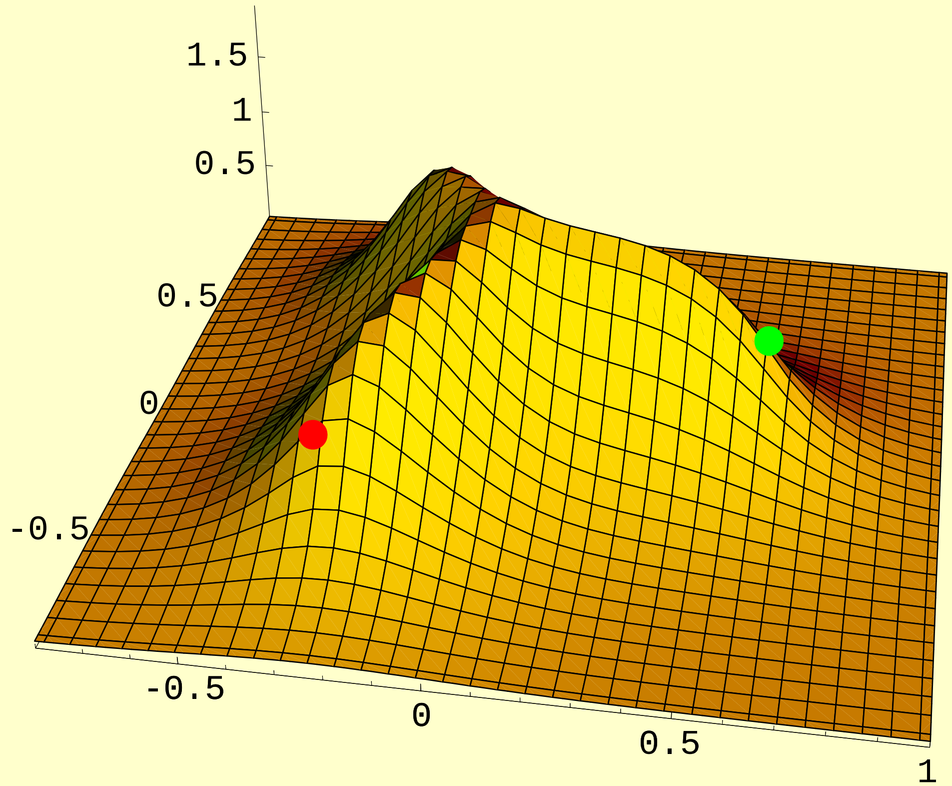

Figure 6: A distribution of the field $\boldsymbol{\mathcal{E}}^{(B)}$ in GeV/fm with the only correlator D contribution considered in the quark plane for equilateral triangle with the side 1 fm. Coordinates are given in fm, positions of quarks are marked by points.

The Y-type string is clearly seen.

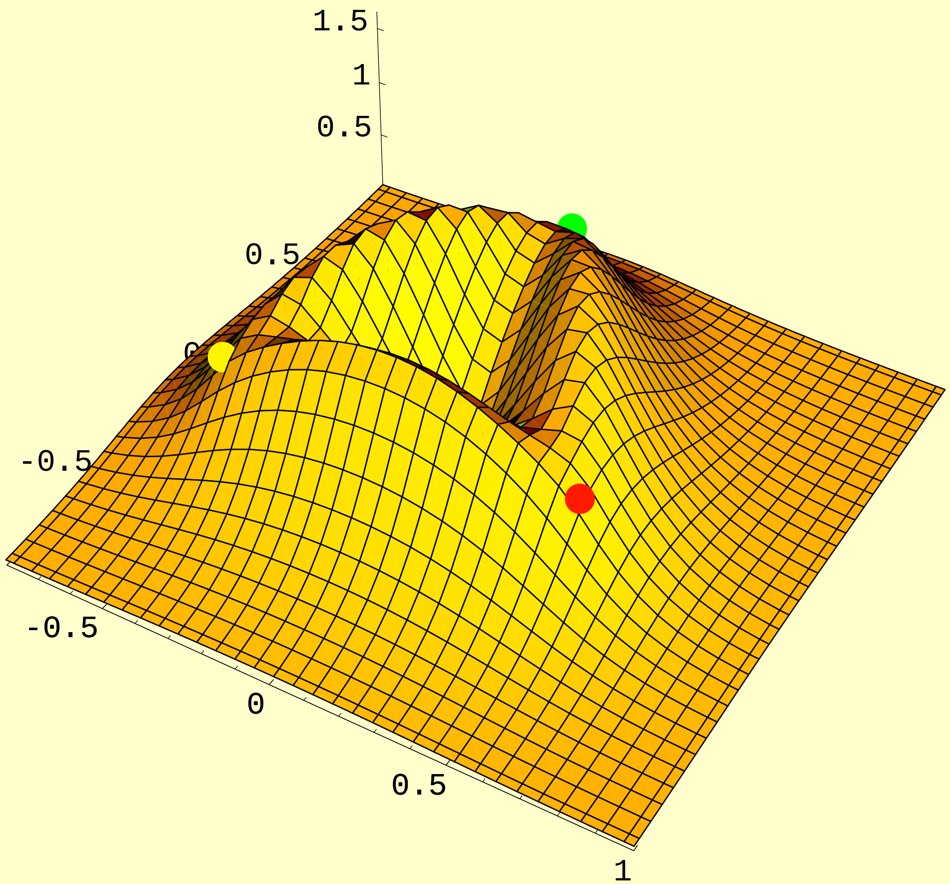

Figure 7: A distribution of the field $|\boldsymbol{\mathcal{E}}_{\Delta}^{(G)}(\mathbf{x})|$ in GeV/fm of the triangular glueball in the plane of valence gluons with separations 1 fm. Coordinates are given in fm, positions of valence gluons are marked by points.

4. Minimal strings and physical spectrum: mesons and glueballs

Quark Green's function (Euclidean)

$$S_q(x,y) = (m+\hat{D})^{-1} = (m-\hat{D})\int_0^\infty ds (Dz)_{xy} e^{-K}\Phi_F(x,y)$$

where all dependence on field A_μ is in

$$\Phi_F(x,y) = (P\exp ig\int_y^x A_\mu dz_\mu)(P\exp g\int_0^s d\tau \sigma_{\mu\nu}F_{\mu\nu}) \equiv \Phi\Sigma$$

Φ charge factor, Σ spin factor.

Green's function for $q\bar{q}$ (mesons) or gg (glueballs)

$G_M, G_{Gl} = \int\int$ integral measure $\langle W_\sigma \rangle$

Thus all dynamics is defined by the Wilson loop (with spin factor insertions).

Wilson loop with spin factors

$\langle trW_\sigma(C)\rangle = \exp(-\sigma Area)$ (spin factors)

$$Area = \int_0^T dt \int_0^1 d\beta \sqrt{\dot{w}^2 w'^2 - (\dot{w}w')^2};$$

Note: no DOF on the area after vacuum averaging. Minimal area $\rightarrow$ minimal strings without DOF except at the ends.

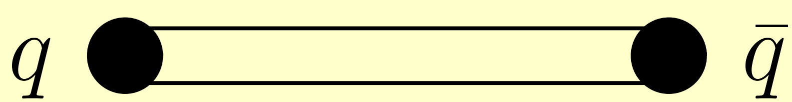

Figure 8:

Hamiltonian of minimal strings with quarks (gluons) at the ends

Last step: from path integral to Hamiltonian

$$G_{q\bar{q}}(x,y) = \langle x|\exp(-HT)|y\rangle \tag{25}$$

For equal current masses $m_q = m_{\bar{q}} = m, \quad \mu_1 = \mu_2 = \mu$

$$H_0 = \frac{m^2+\mathbf{p}^2}{\mu} + \mu + \frac{\hat{L}^2/r^2}{\mu + 2\int_0^1 d\beta(\beta-\frac{1}{2})^2\nu(\beta)} +$$

$$+\frac{\sigma^2 r^2}{2}\int_0^1 \frac{d\beta}{\nu(\beta)} + \int_0^1 \frac{\nu(\beta)}{2} d\beta. \tag{26}$$

$$\frac{\partial H_0}{\partial \mu_i}|_{\mu_i=\mu_i^{(0)}} = 0, \quad \frac{\partial H_0}{\partial \nu}|_{\nu=\nu^{(0)}} = 0. \tag{27}$$

$\mu_i^{(0)}$ play role of constituent mass of particle i, $\mu_i^{(0)} = \langle\sqrt{m_i^2 + \mathbf{p}^2}\rangle$

$$H_0(L=0) = \sum_{i=1}^{2} \sqrt{m_i^2 + \mathbf{p}^2} + \sigma r. \qquad (28)$$

For large $L, L \to \infty$ one obtains a free bosonic string.

$$H_0^2 \approx 2\pi\sigma\sqrt{L(L+1)}, \quad \nu^{(0)}(\beta) = \sqrt{\frac{8\sigma L}{\pi}} \frac{1}{\sqrt{1-4(\beta-\frac{1}{2})^2}}. \qquad (29)$$

Constituent masses $\mu_i^{(0)}$ are calculated through σ and m_i.

For quarks, $m = 0$ $\mu_q = c_n\sqrt{\sigma} = 0.34$ GeV(ground state).

For gluons $\mu_g = \sqrt{C_2}\mu_q = \frac{3}{2}\mu_q = 0.5$ GeV. (Note: This mass is not connected with IR freezing of α_s.)

Total Hamiltonian

$$H = H_0 + H_{self} + H_{spin} + H_{Coul} + H_{rad} + H_{mix}. \qquad (30)$$

For H_0 only, $m = 0$

$$M_0^2 \approx 8\sigma L + 4\pi\sigma\left(n + \frac{3}{4}\right), \quad n = 0, 1, 2, ...$$

The input is minimal:

1. Quark current masses m_1, m_2 (pole masses if H_{pert} is used).
2. String tension σ.
3. Background strong coupling $\alpha_B(r)$.

In momentum space in one loop appr.

$$\alpha_B^{(1)}(Q) = \frac{4\pi}{\beta_0}\frac{1}{\ln\frac{(M_0^2+Q^2)}{\Lambda_{QCD}^2}}$$

To be derived later.

Resulting spectra of light mesons are shown.

Orbital excitations (Regge trajectories) vs experiment (Badalian, Bakker).

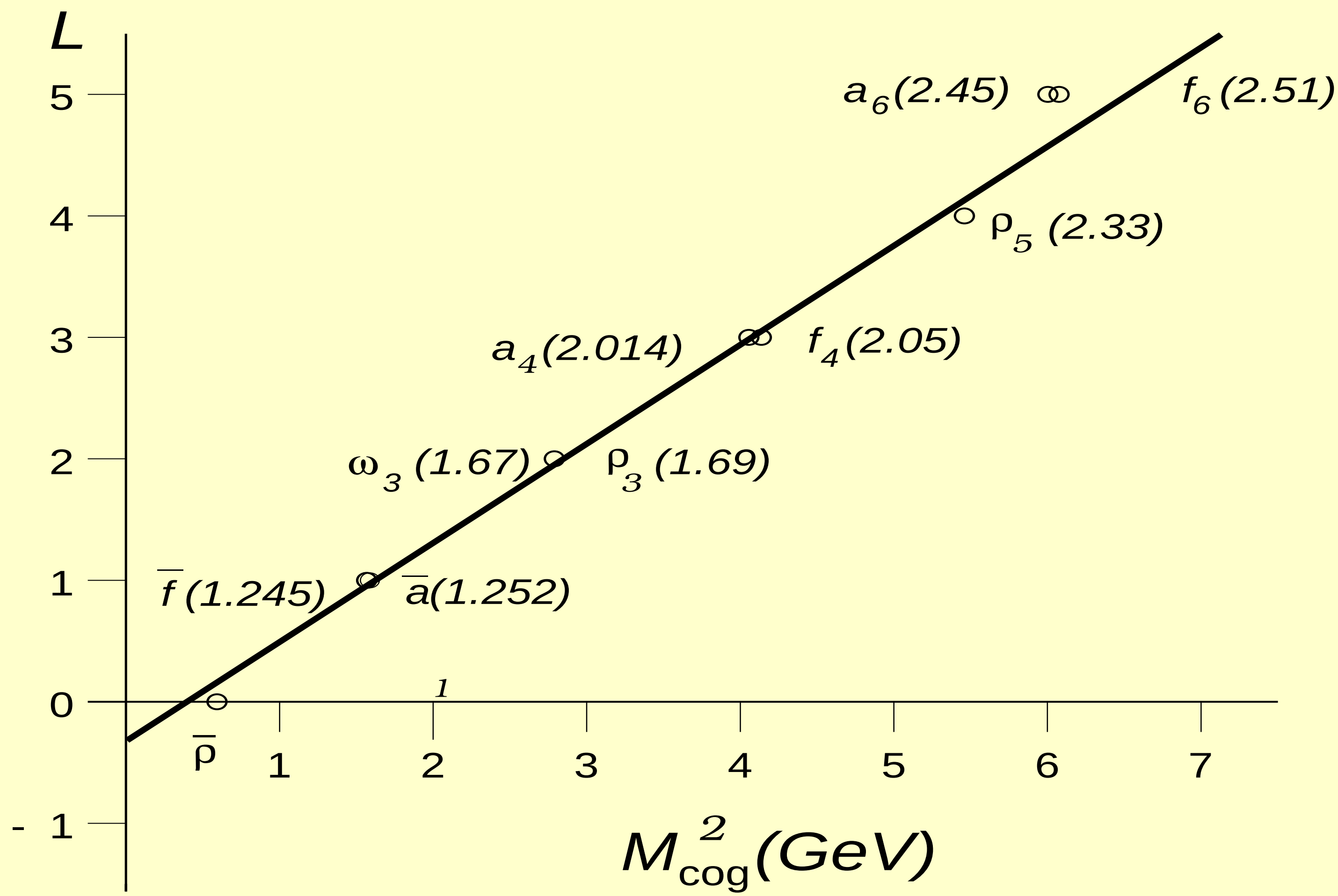

Figure 9: The Regge L-trajectory for the light mesons.

Table 1

Comparison of calculated glueball masses (in GeV) with lattice data ($\sigma_f = 0.18$ GeV^2, $\alpha_s = 0.3$ ($\alpha_s = 0.2$ in parentheses))

J^{PC}	M_{theory} this work	M_{lat} [22]	[23]	[24]
0^{++}	(1.61) 1.41	1.53±0.10	1.53±0.04	1.52±0.13
2^{++}	(2.21) 2.30	2.13±0.12	2.20±0.07	2.12±0.15
0^{++*}	(2.72) 2.41	2.38±0.25	2.79±0.09	
2^{++*}	(3.13) 3.32	2.93±0.14	2.85±0.28	
0^{-+}	2.28	2.30±0.15	2.11±0.24	2.27±0.15
0^{-+*}	3.35	3.24±0.2		
2^{-+}	2.70	2.76±0.16	3.0±0.28	2.70±0.19
2^{-+*}	3.73	3.46±0.21		

Glueballs: Kaidalov+Yu.S.('00,'05).

5. String excitation: hybrids.

A Standard approach: in QCD string excitation is described by Nambu-Goto spectrum. For fixed-end string the spectrum is

$$E_n^{NG}(R) = \frac{\pi N}{R}, N = 1, 2, ...$$

B In BPTh hybrids (minimal string excitations) are described by valence gluons "sitting" on the string.
Spectrum (at large R)

$$E_{n_\perp}^B(R)(transverse) = \frac{\sqrt{12}}{R}(n_\perp + \Lambda + 1) = \frac{\sqrt{12}}{R} N$$

$$E_{n_z}^B(R)(longitud.) = \frac{3}{2^{1/3}} \left(\frac{\sigma}{R}\right)^{1/3} (n_z + \frac{1}{2})^{2/3}$$

Small $R, R \to 0 (\sigma R^2 \ll 1)$

$$E_0^B(R) = M_{gluelump} + \frac{\sigma R^2}{2}\sqrt{\frac{\sigma}{3}} =$$

$$= 2\sqrt{3\sigma} + \frac{\sigma R^2}{2}\sqrt{\frac{\sigma}{3}} + O(R^4)$$

In what follows we outline the formalism for hybrids and compare $\mathbf{A}$ and $\mathbf{B}$ with lattice data, arguing that $\mathbf{B}$ is favored over $\mathbf{A}$.

Hybrid Green's function

$$G_{hybrid}(x,y) = \Gamma \langle tr(\Gamma S_q(x,y) G_g(x,y) S_{\bar{q}}(x,y) \bar{\Gamma}) \rangle_A$$

G_g is the gluon Green's function in BPTh. In the background Feynman gauge

$$A_\mu = B_\mu + a_\mu; \quad \langle a_\mu(x) a_\nu(y) \rangle_a \equiv G_g(x,y)$$

$$G_g(x,y) = -(\hat{D}^2 \delta_{\mu\nu} + 2ig\hat{F}_{\mu\nu})^{-1}_{x,y}$$

Using path-integral form of FFSR, one has

$$G_{hybrid}(x,y) = \int (D\gamma)_{q\bar{q}g} e^{-K-\bar{K}-K_g} W_{q\bar{q}g}$$

$W_{q\bar{q}g}$ – Wilson loop on paths of $q, \bar{q}, g$ with spin insertions, Fig. 9

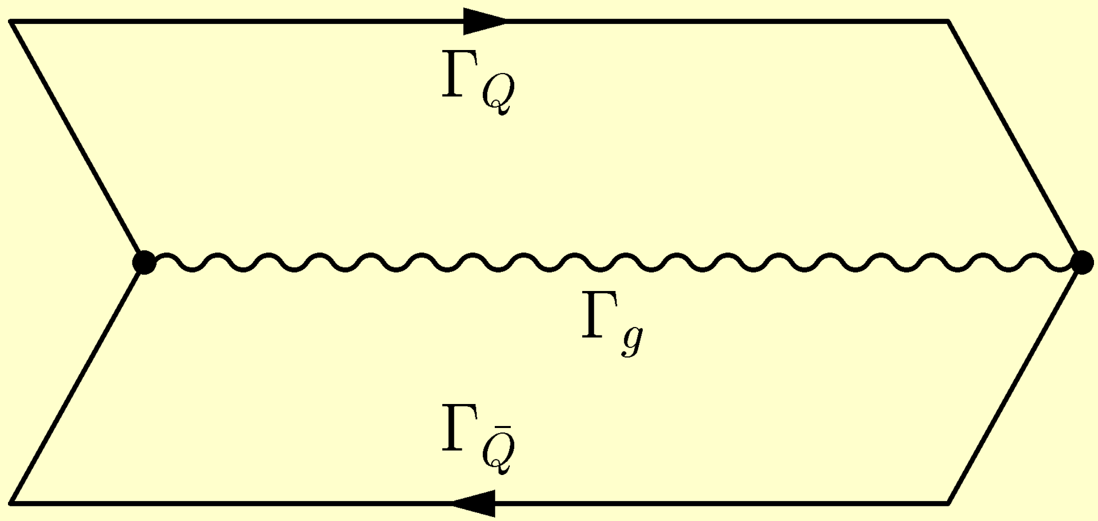

Figure 10: Hybrid Wilson loop.

Using minimal area law, one obtains the "excited string", approximated by straight-line pieces with a gluon between.

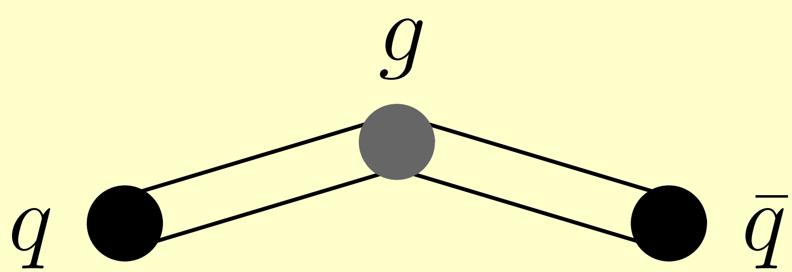

Figure 11:

From G_{hybrid} one derives the Hamiltonian.

Hybrid Hamiltonian $(q+\bar{q}+g)$

$$H_0^{(hyb)} = \frac{m_1^2}{2\mu_1} + \frac{m_2^2}{2\mu_2} + \frac{\mu_1+\mu_2+\mu_g}{2} + \frac{\mathbf{p}_\xi^2+\mathbf{p}_\eta^2}{2\mu} + $$
$$+\sigma\sum_{i=1}^{2}|\mathbf{r}_g-\mathbf{r}_i| + H_{str} + H_{SE} + H_{spin} + H_c. \quad (31)$$

Spectrum of light $q\bar{q}+g$ classified with hyperspherical $K=0,1,...$

$$K=0, (\pi+\mathbf{g})1^{+-},\ (\boldsymbol{\rho}+\mathbf{g})(2^{++},1^{++},0^{++}) \quad (32)$$

$$K = 1, (\pi + (\nabla_x \mathbf{g}))1^{--},\ (\boldsymbol{\rho} + (\nabla_x \mathbf{g}))(2^{-+}, 1^{-+}, 0^{-+}) \tag{33}$$

$$M_0(K = 0) \cong 1.42\text{GeV} \tag{34}$$

$$M_0(K = 1) \cong 1.9\text{GeV} \tag{35}$$

$$M_0(K = 2) \cong 2.45\text{GeV}. \tag{36}$$

Hybrids with two static quarks and one gluon (Yu.Kalashnikova et al(2001, 2002, 2003) , Yu.S. ('98, 05).

No fitting parameters: only σ and $\alpha_s \to$ static hybrid spectrum.

Asymptotic values of levels at large R

$$M^{(long)}_{hybrid} = \frac{3}{2^{1/3}} \left(\frac{\sigma}{R}\right)^{1/3} \left(n_z + \frac{1}{2}\right)^{2/3}, \quad M^{(trans)}_{hybrid} = \frac{\sqrt{12}}{R}(n_\perp + \Lambda + 1), \tag{37}$$

small R

$$M_{hybrid}(R) = 2\sqrt{3\sigma} + \frac{\sigma R^2}{2}\sqrt{\frac{\sigma}{3}} + O(R^4). \tag{38}$$

Notations: $\Lambda = \mathbf{J}_g \frac{\mathbf{R}}{R}, \quad \mathbf{R} = \mathbf{r}_q - \mathbf{r}_{\bar{q}}$

$$\Lambda = 0, 1, 2, 3, ...; \quad \eta_{CP} = +1, \quad g$$

$$\Sigma\ \Pi\ \Delta\ \Phi, ...; \qquad\qquad -1, \quad u$$

η_{CP} – insertion about midpoint of $\mathbf{R}$ times charge conjugation.

$\Sigma^{(\pm)}$ for even (odd) under reflection in the plane containing $\mathbf{R}$.

Yu.Kalashnikova and D.Kuzmenko calculated states in the Table.

Table 1: Quantum numbers of levels

(a)	$j = 1,\ l = 1,\ \Lambda = 0, 1$	$\Sigma_u^-,\ \Pi_u$
(b)	$j = 1,\ l = 2,\ \Lambda = 0, 1$	$\Sigma_g^+,\ \Pi_g$
(c)	$j = 2,\ l = 2,\ \Lambda = 0, 1, 2$	$\Sigma_g^-,\ \Pi_g,\ \Delta_g$
(d)	$j = 2,\ l = 3,\ \Lambda = 0, 1, 2$	$\Sigma_u^+,\ \Pi_u,\ \Delta_u$

The resulting E_i are in Fig in comparison with lattice data of Juge, Kuti, Morningstar(99)

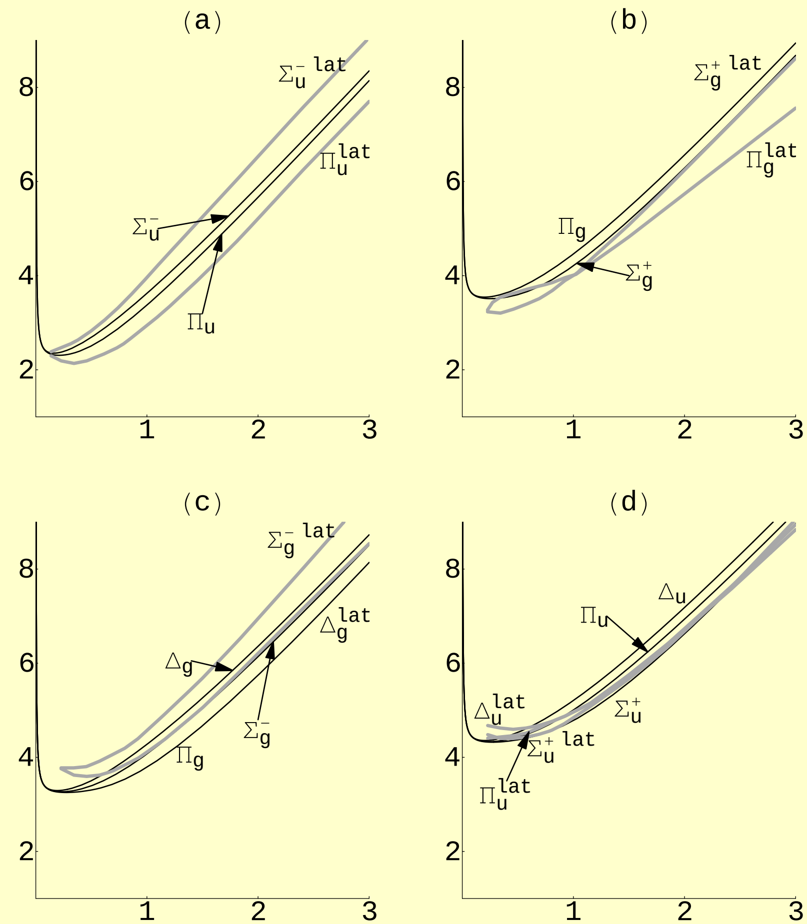

Figure 12: Hybrid potentials in full QCD string model (thin solid curves) compared to lattice results (thick light curves). $Q\bar{Q}$ distance R is in units $2r_0 \approx 1$ fm and potentials V are in units $1/r_0$.

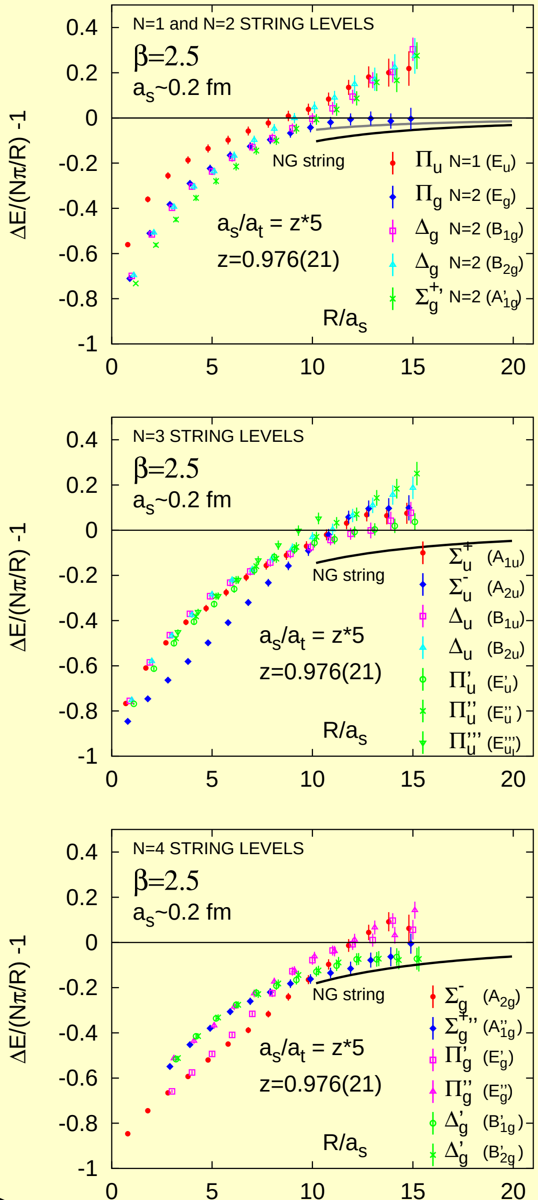

Figure 13: Energy gaps ΔE above Σ_g^+ are shown in string units for quantum numbers in continuum and lattice notation. The Nambu-Goto string

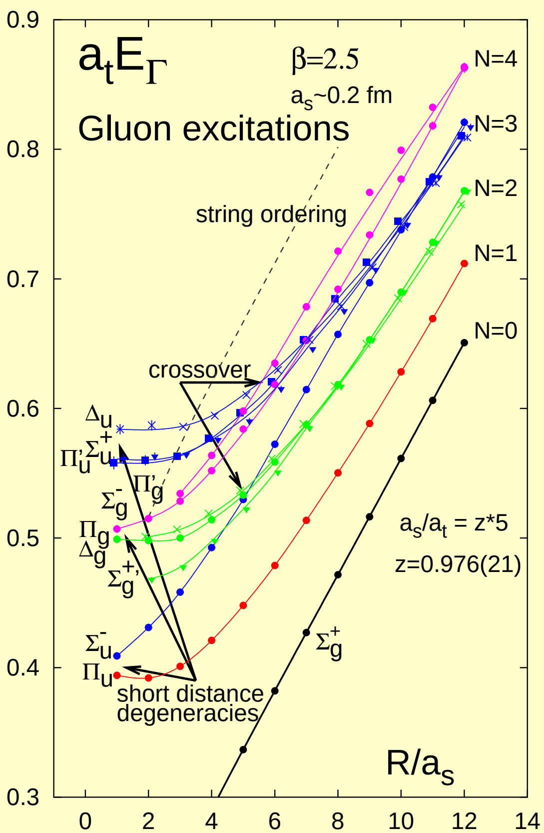

Figure 14: Short-distance degeneracies and crossover in the spectrum. The solid curves are only shown for visualization. The dashed line marks a lower bound for the onset of mixing effects with glueball states which requires careful interpretation.

Asymptotics is in agreement with our spectrum, since transverse modes satisfy

$$E^B_{n_\perp}(R) = \frac{\sqrt{12}}{R}(n_\perp + \Lambda + 1) \approx \frac{\pi}{R}N$$

The NG string (solid line) corresponds to the Arvis result

$$E_n = \sigma R \sqrt{1 - \frac{\pi(D-2)}{12\sigma R^2} + \frac{2\pi N}{\sigma R^2}}$$

Important conclusion

Lattice spectrum clearly shows the splitting of levels, which is explained by the gluon degrees of freedom: spin splittings due to LS forces of gluon spin, string rotation corrections etc.

Also behaviour of spectrum for $R \lesssim 1.5$ fm is far from NG string.

This is closer to the picture of the gluon sitting on the string and difficult to explain by the NG string.

Another check of the nature of QCD strings:

Nambu-Goto vs QCD (minimal) strings

Nambu-Goto: Massless string with dynamical degree of freedom at each point of the string. Consequence: Lüscher term or Arvis potential

$$V_{arvis}(r) = \sqrt{\sigma^2 r^2 - \frac{(d-2)\pi}{12}\sigma} \cong$$

$$\cong \sigma r - \frac{\pi}{12r} \;\; (d=4).$$

No Casimir scaling, Lüscher term is present.

QCD (minimal) Strings – minimal length strings between gluons sitting on the string. Dynamical DOF – only on gluons. Consequence: Lüscher term is unprobable,

$$V(r) = \sigma r - C_2 \frac{\alpha_s(r)}{r}$$

Satisfies Casimir scaling.

Bali: 0.2$\leq r \leq 1.2$ fm.

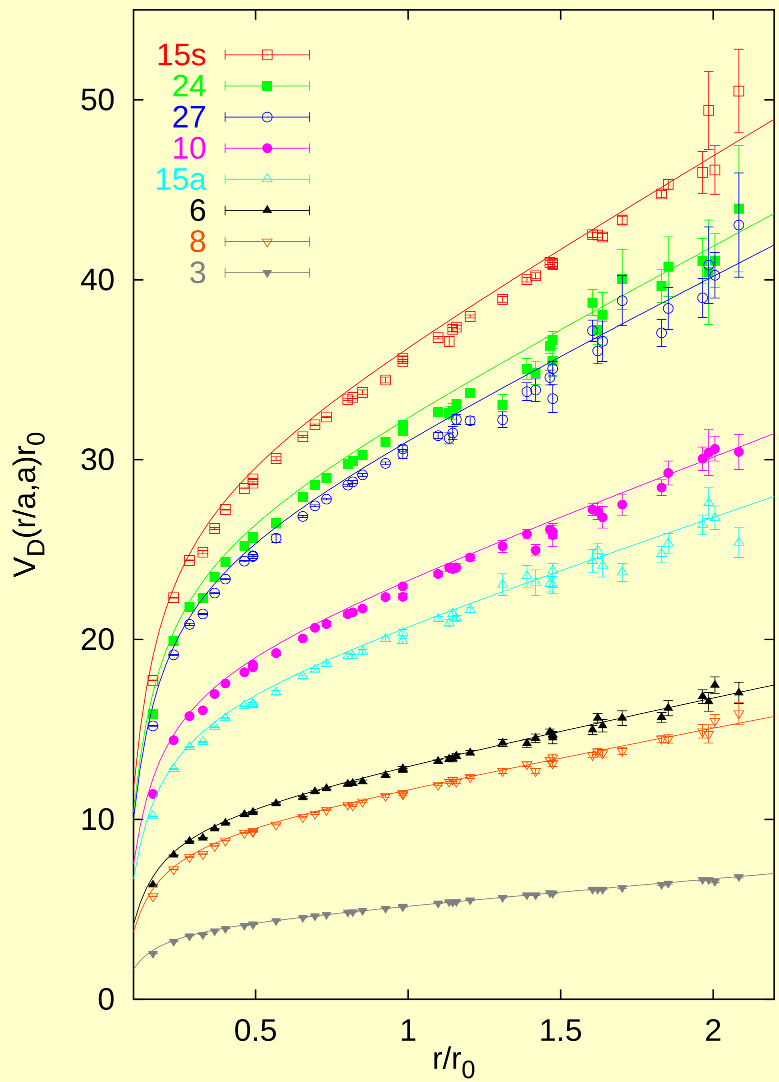

Figure 15:

Hari Dass + Pushan Majumdar

$$c(r) = \frac{r^3}{2}\frac{d^2V(r)}{dr} = -\frac{\pi}{12} \quad (Luescher)$$

for Lüscher term Experimentum crucis:

Measure $c(r)$ for adjoint sources with good accuracy. If $c_{adj}(r) = c_{fund}(r), r$ large – Lüscher term survives – NG strings.

If $c_{adj} = \frac{9}{4}c_{fund}$ – QCD (minimal) strings.

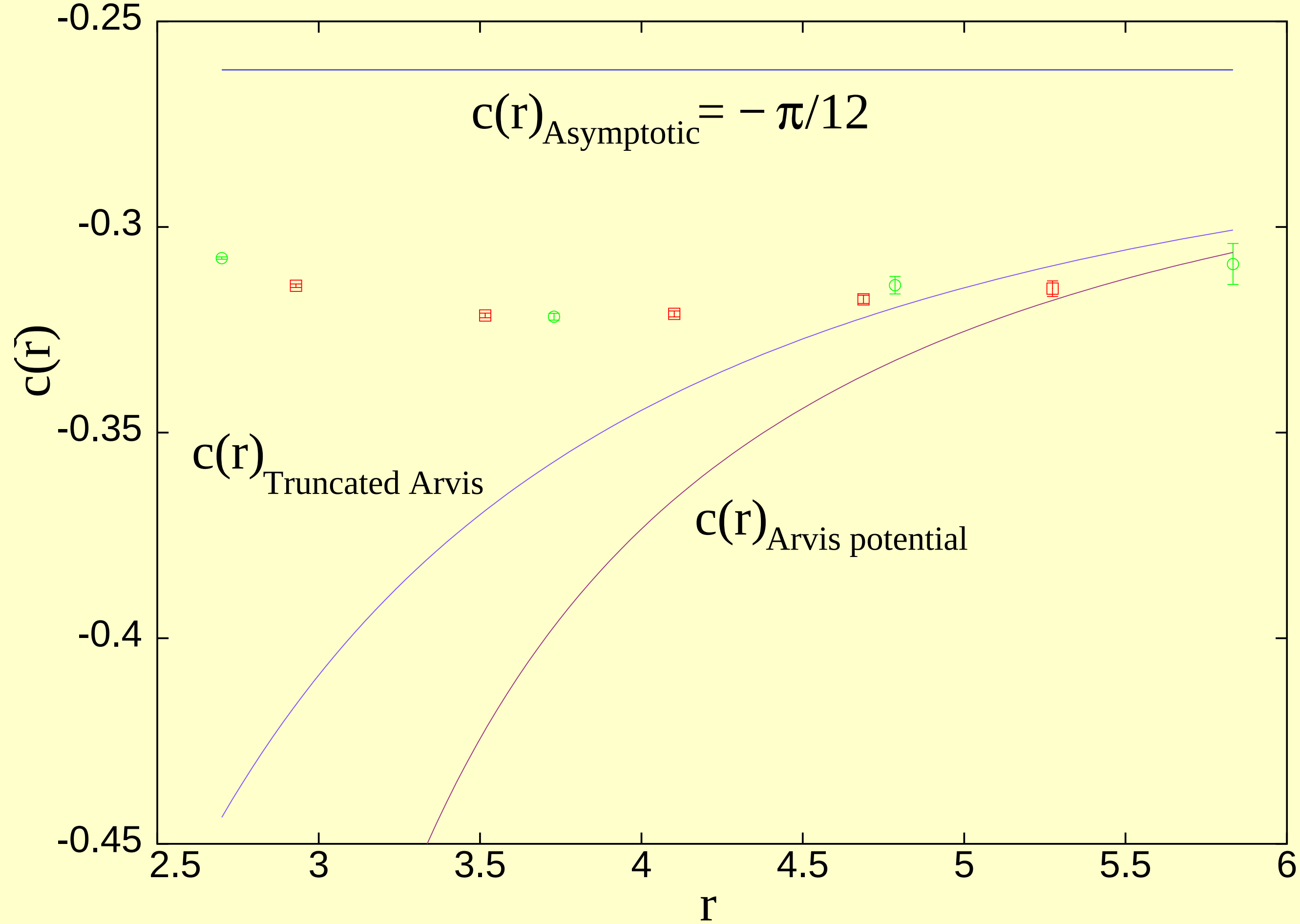

Figure 16: Scaled second derivative for d=4 SU(3) case.

6. Calculating confinement analytically

In this section we calculate D, D_1 analytically via gluelump Green's functions. Physical idea: Nonabelian mean field approach yields confining background field B_μ, with a^a_μ -quanta of gluonic field – propagating in vacuum with a fixed color index a, while $B_\mu \sim A^b_\mu$, $b \neq a$.

$$A_\mu = B_\mu + a_\mu$$

When averaging over B_μ one obtains confining string for a^a_μ.

As a result (Yu.S. '05, Antonov '05) to the lowest order in α_s

$$D_1(x) = -\frac{2g^2}{N_c^2} \frac{dG^{(1)}(x)}{dx^2}$$

$$D(x) = \frac{g^4(N - c^2 - 1)}{2} G^{(2)}(x)$$

and $G^{(1)}(x)$ is the one-gluon gluelump Green's function, $G^{(2)}$ - the same for two gluons.

$$G^{(1)}(x-y) = \frac{1}{4}\langle tr_a a_\mu(x)\hat{\Phi}(x,y)a_\mu(y)\rangle$$

$$G^{(2)}(x-y) = tr_a tr_{i,k\ldots}\langle a_i(x)a_k(x)\hat{\Phi}(x,y)a_i(y)a_k(y)\rangle.$$

Both $G^{(1)}$ and $G^{(2)}$ can be computed analytically [Yu.S'00, D.Antonov '05] and the corresponding masses are expressed via σ – string tension of background field.

$M^{(1)} = 1.5$ GeV for $\sigma = 0.18$ GeV^2

$M^{(2)} = 2.56$ Gev.

From this we have correlation lengths $\lambda^{(1)} = 0.13$fm, $\lambda^{(2)} \equiv \lambda = 0.08$fm.

Asymptotically for large $x \gg \lambda$,

$$D_1(x) = D_1^{pert.}(x) + D_1^{nonp.}(x)$$

$$D_1^{nonp.}(x) = \frac{C_2(f)\alpha_s 2M^{(1)}\sigma_{adj}}{|x|}$$

$$D_1^{pert}(x) = \frac{4C_2(f)\alpha_s}{\pi x^4} + O(\alpha_s^2)$$

$$D(x) = D^{pert.}(x) + D^{nonp.}(x)$$

$$D^{pert}(x) = \frac{\alpha_s^2(N_c^2-1)}{2\pi^2 x^4},$$

$$D^{nonp.}(x) = \frac{g^4(N_c^2-1)}{2} 0.108\sigma^2 e^{-M^{(2)}|x|}$$

Note: $D^{pert.}(x)$ chanels with pert. parts of higher correlators (V.Shevchenko+ Yu.S' 99)

Check of consistency

$$\sigma = \frac{1}{2}\int d^2x D(x) \approx \frac{1}{2}\int d^2x D^{nonp.}(x)$$

Taking $D^{nonp.}(x)$ one obtains relation

$$\sigma = \left(\frac{\alpha_s(\mu^{(2)})}{0.3}\right)^2 0.53\sigma + Const.\alpha_s^3\sigma + ...$$

$$\alpha(\mu^{(2)}) = 0.41 \cong \frac{4\pi}{\beta_0 ln\left(\frac{\mu^{(2)}}{\Lambda_{QCD}}\right)^2}$$

Here $\mu^{(2)}$ is typical scale of gluelump,

$$\mu^{(2)} \approx \sqrt{\langle \mathbf{p}^2 \rangle} \approx \frac{M^{(2)}}{3} = 0.84 GeV$$

hence

$$\Lambda_{QCD} \approx \mu^{(2)} e^{-\frac{2\pi}{0.41\beta_0}} \approx 0.21 GeV \quad (\Lambda_{\overline{MS}}(n_f = 0) = 0.24 GeV)$$

Here is an example of dimensional transmutation.

Expansion of $D(x), D_1(x)$ at small x

It was found (Yu.S '2005) that at small x the expansion is

$$D_1(x) = \frac{4C_2\alpha_s}{\pi}\left(\frac{1}{x^4} + \frac{\pi^2 G_2}{24N_c} + ...\right)$$

$$G_2 \equiv \frac{\alpha_s}{\pi} \langle F^a_{\mu\nu} F^a_{\mu\nu} \rangle$$

– standard gluonic condensate.

$$D(x) = \frac{g^4(N_c^2-1)}{2} \left(\frac{1}{(4\pi^2 x^2)^2} + const.G_2 + ... \right)$$

Hence 1) perturbative and nonperturbative separate at small distances 2) there is no term of the form $O\left(\frac{m^2}{x^2}\right)$.

Conclusion on section 6.

Thus one can fix the scale in the form:

$$\sigma = anything GeV^2 (or \Lambda_{QCD} = anything)$$

and calculate all spectra and Field Correlators $D(x), D_1(x)$ which yields all strong dynamics, except

1) chiral scale f_π, m_π

2) strong decay dynamics.

Note: confinement ($\sigma \neq 0$) is selfconsistent and selfsupporting: putting $\sigma = 0$ in gluelumps we get only perturbative terms in D_1, D and higher correlators, and obtain $\sigma = 0$ from them.

7. Why potential models succeed where QCD sum rules fail?

In what was discussed before we have

- taken interaction as minimal (straight-line) string plus OGE
- This means that string excitations are neglected, coupling to hybrids is neglected.
- Vacuum correlation time (length) was considered to be small as compared to the quark (or gluon) effective period of motion – T_q, so $T_q \gg \lambda$

  answers: size of hadron $R \sim T_q$ is much larger than λ.

  Examples: $R(\Upsilon(1S)) = 0.2$ fm$> \lambda = 0.08$ fm.

  Other mesons are larger.
- String excitation yields a gap ~ 1 GeV and can be neglected in first approximation.

  Coupling P_{HM} of mesons to hybrids in small

$P_{HM} = N_c g^2 \left(\frac{0.08GeV}{\Delta M}\right)^2$ if $\Delta M \gg 0.1$ GeV (Paris group' 85, Yu.S.'01)

- For QCD sum rules in general one needs condition $R \ll \lambda$. It is not clear, why this method is effective for some ground state mesons.

8. Chiral symmetry Breaking and Confinement. Effective Chiral Quark-pion Lagrangian

Bosonization of 4q Lagrangian obtained after averaging over NP gluonic fields,

$$S_{eff}(L) = -\frac{1}{2}\int d^4x d^4y \bar{\psi}_b \gamma_\mu \psi_a(x) \bar{\psi}_{a'}(y) \gamma_\nu \psi_{b'}(y)(\delta_{aa'}\delta_{bb'} - \frac{1}{N_c}\delta_{ab}\delta_{a'b'}) J_{\mu\nu} \tag{39}$$

leads to the effective quark-pion Lagrangian

$$\begin{gathered} S_{QM} = -\int d^4x d^4y [\bar{\psi}^f(x) i M_s(x,y) \hat{U}^{fg}(x,y) \psi^g(y) - \\ -2N_f (J_{\mu\nu}(x,y))^{-1} M_s^2(x,y)], \quad \hat{U} = \exp(i\gamma_5 t^a \phi_a(x,y)). \end{gathered} \tag{40}$$

in the partition function Z

$$Z = \int D\psi D\bar{\psi} e^{-(S_1+S_{QM})} DM_s D\phi_a d\rho(C_Q)\bar{W}(C_Q, L) \qquad (41)$$

Integrating out quark fields one obtains the Effective Chiral Lagrangian S_{ECL},

$$Z = \int DM_s D\phi_a d\rho(C_Q)\bar{W}(C_Q, L) e^{-S_{ECL}}, \qquad (42)$$

with

$$S_{ECL} = 2N_f \int d^4x d^4y J^{-1}_{\mu\mu} M_s^2(x,y) - W(\phi), \qquad (43)$$

and

$$W(\phi) = N_c tr \ln[i(\hat{\partial} + \hat{m} + M_s(x,y)\hat{U})] \qquad (44)$$

Integration over $DM_s D\phi_a$ is done using stationary point method, which yields stationary point solutions

$$\phi_a^{(0)} = 0, \quad M_s^{(0)}(x,y) = \frac{N_c}{4N_f} J_{\mu\mu}(x,y) Tr(S(x,y)) \qquad (45)$$

where $S(x,y) = S_\phi(x,y)|_{\phi=0}$, and

$$S_\phi(x,y) = \langle x|(i\hat{\partial} + i\hat{m} + iM_s^{(0)}\hat{U})^{-1}|y\rangle. \tag{46}$$

As a result in the quark propagator is possible emission of any number of NG mesons

$$M^{(0}_{s(x,y)} \approx \sigma|\mathbf{x} - \mathbf{x}_0(L)| \equiv M(\mathbf{x}). \tag{47}$$

To emit two pions (or two kaons) we need to expand $W(\phi)$

$$\begin{aligned} W(\phi) \cong N_c tr \ln[S^{-1} - M\gamma_5\hat{\phi} - \tfrac{i}{2}M\hat{\phi}^2] = \\ = W_0(\phi) + W_1(\phi) + W_2(\phi) + ..., \hat{\phi} \equiv t^a\phi^a = \tfrac{\varphi_a\lambda_a}{f_\pi}, \end{aligned} \tag{48}$$

$$W_2(\phi) = -\frac{N_c}{2} tr(iSM\hat{\phi}^2 + SM\hat{\phi}\gamma_5 S\gamma_5 M\hat{\phi}). \tag{49}$$

Inside heavy quarkonium $W_2(\phi) \to \langle W_2(\phi)\rangle_Q$

$$\langle W_2(\phi)\rangle_Q \equiv \int d\rho(C_q) W_2(\phi) W(C_Q, L) \tag{50}$$

it can be written as

$$W_2(\phi) = \frac{1}{2}\int \frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\phi_a(k_1)N(k_1,k_2)\phi_a(k_2) \tag{51}$$

where $N(k_1k_2)$ is

$$N(k_1,k_2) = \frac{N_c}{2}\left\{\int dx e^{i(k_1+k_2)x} tr(\Lambda M_s)_{xx} + \right.$$

$$\left. + \int d^4x d^4y e^{ik_1x+ik_2y} tr(\Lambda(x,y)M_s(y)\bar{\Lambda}(y,x)M_s(x))\right\} \tag{52}$$

and

$$\Lambda = (\hat{\partial} + m + M_s)^{-1}, \quad \bar{\Lambda} = (\hat{\partial} - m - M_s)^{-1}. \tag{53}$$

The diagrams for the first and second term in (21) are respectively

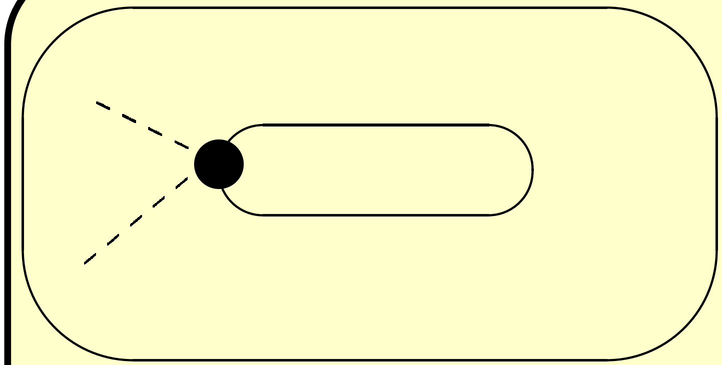

Fig 2 (a)

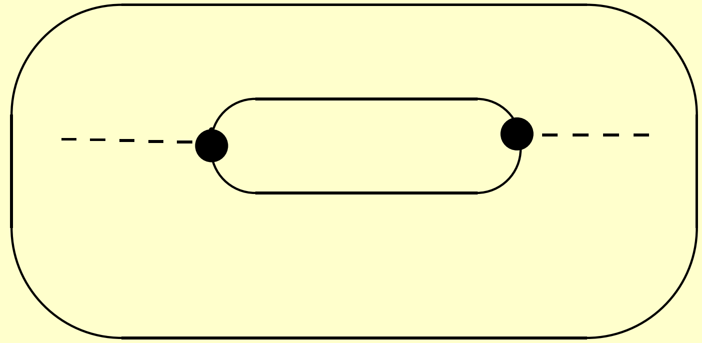

Fig 2 (b)

Now one can show that the expansion of $N(k_1, k_2)$ in powers of k_1, k_2 starts as

$$N(k_1, k_2) = \frac{m_\pi^2 f_\pi^2}{4N_c} + O(k_{1\mu}k_{2\mu}) + ...$$

where $m_\pi^2 f_\pi^2 = -\frac{m_q}{2N_c}\langle tr\bar{\psi}\psi\rangle$.

Hence in the chiral limit $(m_q \to 0)$ and for $k_{i\mu} \to 0, N \to 0$ (Adler zero). Note, that heavy quark loop acts as a spectator, and no definite channels are fixed in Fig. 2(a) and Fig 2(b).

$$\langle e^{\int \bar{\psi}\hat{A}\psi d^4x} \rangle \to e^{\mathcal{L}_4+\mathcal{L}_6+\dots} \tag{54}$$

$$\mathcal{L}_4 = \bar{\psi}\gamma_\mu\psi(x)\bar{\psi}\gamma_\nu\psi(y)J_{\mu\nu}(x,y) \to \bar{\psi}\bar{(}x)M(x,y)\psi(y) \tag{55}$$

$$\psi\bar{\psi} \to S(x,y) \ \ \text{(large } N_c, \text{ no NG)} \tag{56}$$

$$M(x,y) = \gamma_\mu iS(x,y)\gamma_\nu J_{\mu\nu}(x,y) \tag{57}$$

$$M^E(x,y) = \gamma_4 iS\gamma_4 J^E_{44} \tag{58}$$

$$M^H(x,y) = \gamma_i iS\gamma_k J^H_{ik} \tag{59}$$

$$J^E_{44}(x,4) = \int_0^x du \int_0^y dv D^E(u-v) \tag{60}$$

$$\sigma^E = \frac{1}{2}\int D^E(z)d^2z \tag{61}$$

C.S.B.: iS acquires scalar part.

Pert. Theory

$$iS^{(0)}(x) = \frac{\hat{x} - y}{(x-y)^4} + O(\gamma^{2k+1}) \tag{62}$$

Nonlinear eqs:

$$\begin{cases} M^E = \gamma_4 iS \gamma_4 J^E_{44} \\ iS = (\hat{\partial} + m + M^E)^{-1} \end{cases} \tag{63}$$

Solution: local approx.

$$M^E(x,y) \Rrightarrow M^E(x) = M^E(0) + \sigma_E|\mathbf{x}|; \tag{64}$$

with NG mesons $M^E(x,y) \Rightarrow (M^e(0) + \sigma_E|\mathbf{x}|)e^{i\varphi_\pi \gamma_5}$

$$\mathcal{L}_{eff} = \int \bar{\psi}(x)(\hat{\partial} + m + M^E(x))\psi(x)d^4x \tag{65}$$

$$M^E(0) = \ const\ \sigma_E \lambda; \quad const\ \approx 1 \tag{66}$$

$$D^E(x) = D^E(0)e^{-|x|/\lambda}; D^E(0) = \frac{\sigma_E}{\pi\lambda^2} \tag{67}$$

σ_E, λ define Chiral Dynamics $\sigma_E \approx 0.18\ GeV^2, \quad \lambda \cong 1\ GeV^{-1}$
scalar $M^E(x)$ means CSB (since it is scalar).

Indeed

$$f_n^2 = \frac{2N_c\langle Y_f\rangle|\bar{\varphi}_n(0)|^2}{\omega_1\omega_2 M_n} \tag{68}$$

$$\langle Y_{A_4}\rangle = (m_1 + M_1^E(0))(m_2 + M_2^E(0)) + \omega_1\omega_2 - \langle \mathbf{p}^2\rangle$$

for

$$m_1 = m_2 = 0, \quad \omega_i = \langle\sqrt{\mathbf{p}^2 + m_i^2}\rangle \to \sqrt{\mathbf{p}^2}; \tag{69}$$

$$\langle Y_{A_4}\rangle \to (M(0))^2$$

neglecting HF:

$$M_0 = 0.65\ GeV, \quad \varphi_0^2(0) = \frac{0.15\ GeV^3}{4\pi} \tag{70}$$

the same as for $\rho-$ meson.

$$\omega_1 = \omega_2 = \omega = 0.352\ GeV. \tag{71}$$

$$M^E(0) \cong 0.15\ GeV \quad f_\pi = 142\ MeV\ (vs\ 131\ MeV) \tag{72}$$

Interesting:

$$\omega \sim \sqrt{\sigma_E}; \quad \varphi_n^2(0) \sim \sigma_E^{3/2}, \quad M_n \sim \sqrt{\sigma^E} \tag{73}$$

$$f_\pi \sim M^E(0) \sim \sigma_E \lambda \sim 0.15\ GeV \tag{74}$$

f_π is order parameter for CSB

f_π disappears with σ_E!

$$-\langle \bar{q} q \rangle = N_c M^E(0) \sum_{n=0}^{\infty} \frac{|\varphi_n(0)|^2}{M_n} \tag{75}$$

$$|\langle \bar{q} q \rangle| \sim M^E(0) \sigma_E \sim \lambda \sigma_E^2 \sim (0.18)^2 \ \ GeV^3 \ \ \sim (0.32\ GeV)^3; \tag{76}$$

Conclusion: $\langle \bar{\mathbf{q}}\mathbf{q} \rangle$ and $\mathbf{f}_\pi$ disappear together with $\sigma_{\mathbf{E}} \sim \mathbf{D^E(x)} \sim \langle \mathbf{E^2} \rangle$

Fun with Gell-Mann-Oakes-Renner relation

$$f^2 m_\pi^2 = 2(m_u + m_d)|\bar{q}q|$$

$$f^2 m_k^2 = 2(m_u + m_s)|\bar{q}q|$$

at $\mu \cong 1\ GeV$

$$m_u = 4.2\ MeV; \quad m_d = 7.5\ MeV, \quad m_s \approx 170\ MeV$$

$$|\bar{q}q| \cong \lambda \sigma_E^2; \quad f \cong \lambda \sigma_E; \quad \lambda = 1\ GeV^{-1}$$

$$m_\pi^2 = \frac{2(m_u + m_d)}{\lambda}, \quad m_\pi \sim 0.15\ GeV;$$

$$m_k^2 = \frac{2(m_s + m_u)}{\lambda}, \quad m_k \sim 0.59\ GeV.$$

$$\frac{m_k^2}{m_\pi^2} = \frac{\bar{m} + m_s}{m_u + m_d} \cong 12;$$

σ_E cancels in GOR relation.

Chiral Lagrangians can be derived without confinement – e.g. in NJL or instanton model.

But: λ^{-1} gives the cutoff 1 GeV due to nonlocality in chiral perturbation theory.

Phase transition: Equation for $T_c(\mu)$ M.A.Trusov and Yu.S.

$$SVZ \quad \varepsilon_{vac} = 1/4\theta_{\mu\mu} = \frac{\beta(\alpha_s)}{16\alpha_s}\langle (F^a_{\mu\nu})^2\rangle \cong -\frac{(11-\frac{2}{3}n_f)}{32} G_2^{(n_f)} \tag{77}$$

$$(NSVZ) \quad G_2^{(n_f=2)} \approx \left(\frac{1}{3} \div \frac{1}{4}\right) G_2^{(n_f=0)} \tag{78}$$

$$\begin{gathered} G_2(0.02 \pm 0.005)\ GeV^4 \ \text{S.Narison} \\ G_2(0.01 \pm 0.002)\ GeV^4 \ \text{Andreev, Zakharov} \end{gathered} \tag{79}$$

$$P_1(T) = |\varepsilon_{vac}| + \frac{\pi^2}{30}T^4 + T\sum_k \frac{(2m_k T)^{3/2}}{8\pi^{3/2}} e^{-m_k/T} \equiv |\varepsilon_{vac}| + T^4\chi_1(T). \tag{80}$$

In the deconfined phase one can assume (later confirmed by lattice) (Yu.S. JETP Lett.'92), that

$$D^E(x) = 0 = \sigma_E; \quad D^H(x), D_1^H, D_1^E \neq 0. \tag{81}$$

$$P_2(T) = |\varepsilon_{vac}^{dec}| + T^4(p_{gl} + p_q) \tag{82}$$

Critical line $T_c(\mu)$

$$P_I = |\varepsilon_{vac}| + \chi_1(T) \to \frac{11}{32} G_2$$

$$P_{II} = \frac{11}{32} G_2^{dec} + (p_{gl} + p_q) T^2;$$

$$P_I(T_c) = P_{II}(T_c)$$

$$T_c(\mu) = \left(\frac{\frac{11}{32}\Delta G_2}{p_{gl} + p_q} \right)^{1/4},$$

within 10% $\Delta G_2 \approx \frac{1}{2} G_2$

$$P_{gl}(T_c) = \frac{16}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^4} L_{adj}^n \to \frac{16}{\pi^2} L_{adj}$$

$$p_q(T_c) = \frac{n_f}{\pi^2} \left[\Phi_\nu \left(\frac{\mu - \frac{V_!(T_C)}{2}}{T_c} \right) + \Phi_\nu \left(- \frac{\mu + \frac{V_!(T_C)}{2}}{T_c} \right) \right]$$

$$\Phi_\nu(a) = \int_0^\infty \frac{z^4 dz}{\sqrt{z^2 + \nu^2} \left(e^{\sqrt{z^2+\nu^2} - a} + 1 \right)}$$

$\nu = \frac{m_q}{T}$. Take $m_q = 0$.

Only 2 parameters (input)

ΔG_2 and $V_1(T_c)$.

$$1.\Delta G_2 = G_2(E,H) - G_2(0,H) \approx \frac{1}{2} G_2(E,H)(10\% acc)$$

$$G_2 = \frac{\pi^2}{36}(D^E(0) + D_1^E(0) + D^H(0) + D_1^H(0));$$

$$D_1^E(0) \approx 0.2 D^E(0) \quad (T=0)$$

2. $V_1(T_c) \cong 0.5$ GeV (lattice, analytic) within 10%.

Possible dependence on μ: weak for $\mu \ll$ dilaton mass ≈ 1.5 GeV
lattice data: $V_1 \Rightarrow F^1_{Q\bar{Q}}$

Two limiting cases for $T_c(\mu)$

1). $T_c(\mu \to 0)$: Expanding in $\frac{V_1(T_c)}{8T_c}$

$$T_c = T^{(0)} \left(1 + \frac{V_1(T_c)}{8T_c} + O\left(\frac{V_1(T_c)}{8T_c}\right)^2 \right)$$

with 3% accuracy

$$T_c(0) \approx \frac{1}{2} T^{(0)} \left(1 + \sqrt{1 + \frac{\kappa}{T^{(0)}}} \right), \quad T^{(0)} = \left(\frac{(11 - \frac{2}{3} n_f) \pi^2 \Delta G_2}{384 n_f} \right)^{1/4}. \tag{83}$$

$$\kappa \equiv \frac{1}{2} V_1(\infty, T_c) \cong \frac{1}{2} F^1_{Q\bar{Q}}(\infty, T_c) = 0.25 \;\; GeV.$$

2). End-point: $\mu_c(T \to 0)$

Using asymptotics

$$\Phi_0(a \to \infty) = \frac{a^4}{4} + \frac{\pi^2}{2} a^2 + \frac{7\pi^4}{60} + ...$$

one has

$$\mu_c(T \to 0) = \frac{V_1(T_c)}{2} + (48)^{1/4} T^{(0)} \left(1 - \frac{\pi^2}{2} \frac{T^2}{(\mu_c - \frac{V_1(T_c)}{2})} + \right)$$

For $V_1(T_c) = 0.5$ GeV and $n_f = 2$ one has.

$\frac{\Delta G_2}{0.01\ \mathrm{GeV}^4}$	0.191	0.341	0.57	1
$T_c(\ \mathrm{GeV})\quad n_f = 0$	0.246	0.273	0.298	0.328
$T_c(\ \mathrm{GeV})\quad n_f = 2$	0.168	0.19	0.21	0.236
$T_c(\ \mathrm{GeV})\quad n_f = 3$	0.154	0.172	0.191	0.214
$\mu_c(\ \mathrm{GeV})\quad n_f = 2$	0.576	0.626	0.68	0.742
$\mu_c(\ \mathrm{GeV})\quad n_f = 3$	0.539	0.581	0.629	0.686

9. Conclusions

1. Field correlator Method provides the explicit dynamical theory for Large-Distance QCD. The confinement is due to nonperturbative correlators of colorelectric fields, and for a flat (minimal) surface the lowest Gaussian correlator $D^E(x)$ plays the dominant role. Cluster expansion in n-th order correlators behaves as $\sim (\sigma\lambda^2)^n = (0.05)^n$.
2. Correlation length λ and correlators are calculated selfconsistently via gluelumps, $\lambda_D^E \approx 0.1$ fm. Thus one has a theory defined by the only parameter say σ (in addition to current quark masses)
3. CSB emerges together with scalar confinement and all CSB parameters are expressed via σ^E and λ, e.g., $f_\pi \approx \sigma^E \lambda = O(100 MeV)$.
4. Phase transition temperature T_c is expressed via gluonic condensate. CSB disappears at T_c together with confinement.