THE UNIVERSITY OF ADELAIDE AUSTRALIA

SPECIAL RESEARCH CENTRE FOR THE SUBATOMIC STRUCTURE OF MATTER

Infrared Exponents and the Strong Coupling Limit in Lattice Landau Gauge

St Goar, March 2008

Wilhelm & Else Heraeus Seminar: Quarks and Gluons in Strong QCD

Faculty of Sciences
School of Chemistry & Physics

Lorenz von Smekal

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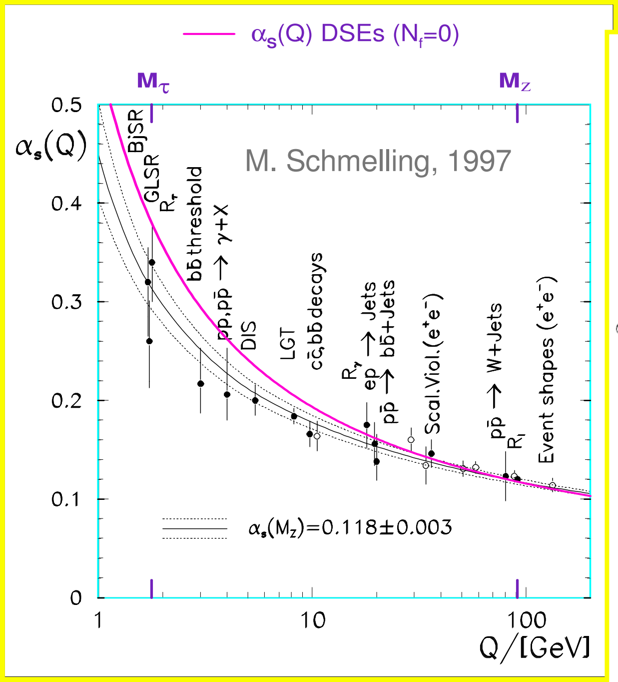
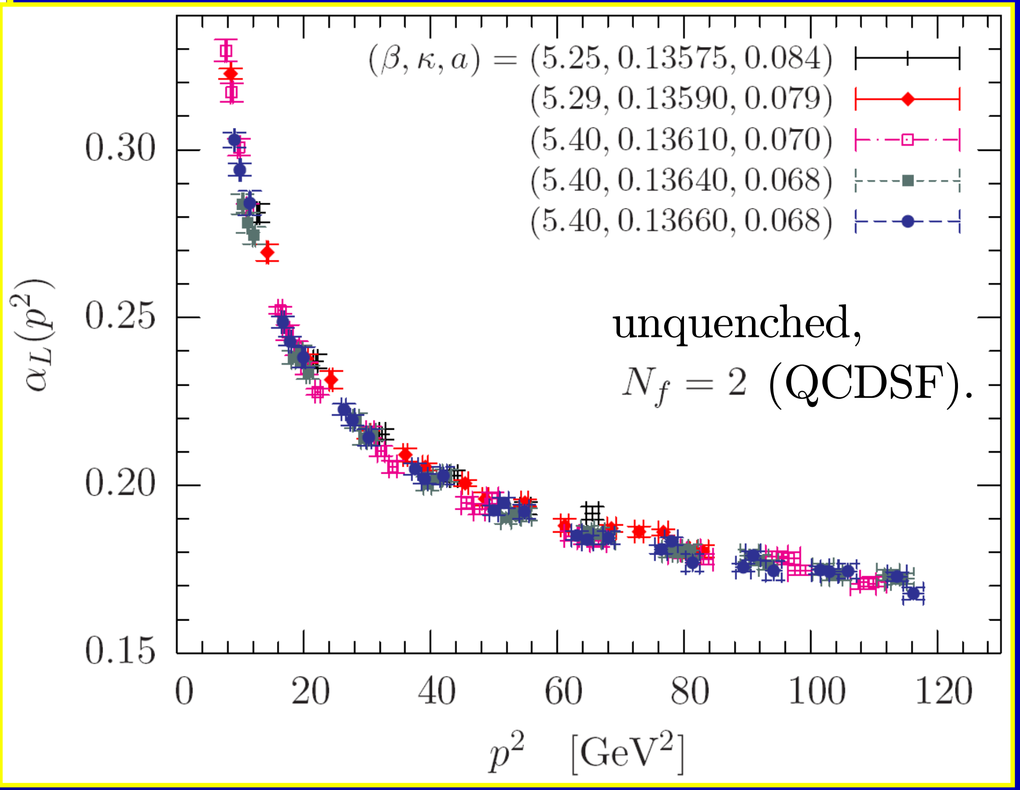
Running Coupling

- Determine α_S from lattice Landau-gauge ghost/gluon propagators:

$$\alpha_S^{\mathrm{minMOM}}(q^2) = \frac{g^2(a)}{4\pi} Z_L(q^2, a^2) G_L^2(q^2, a^2) + O(a^2)$$

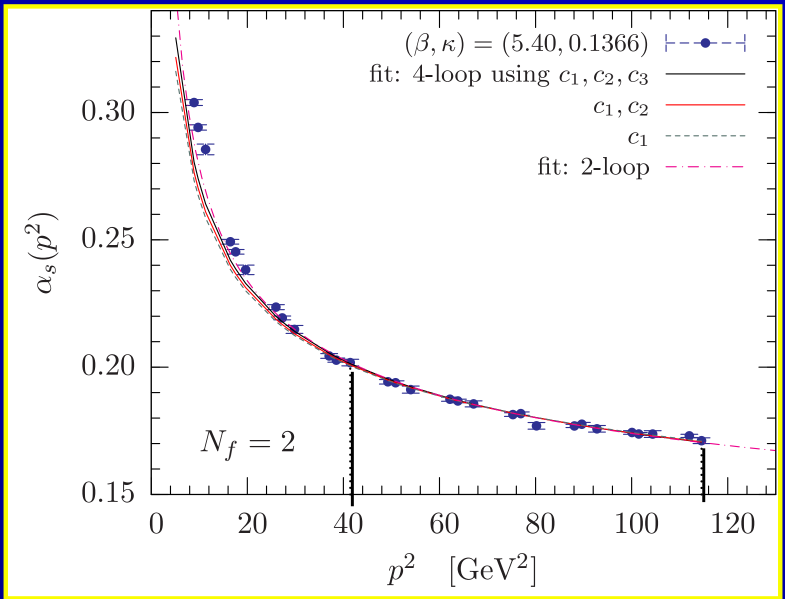


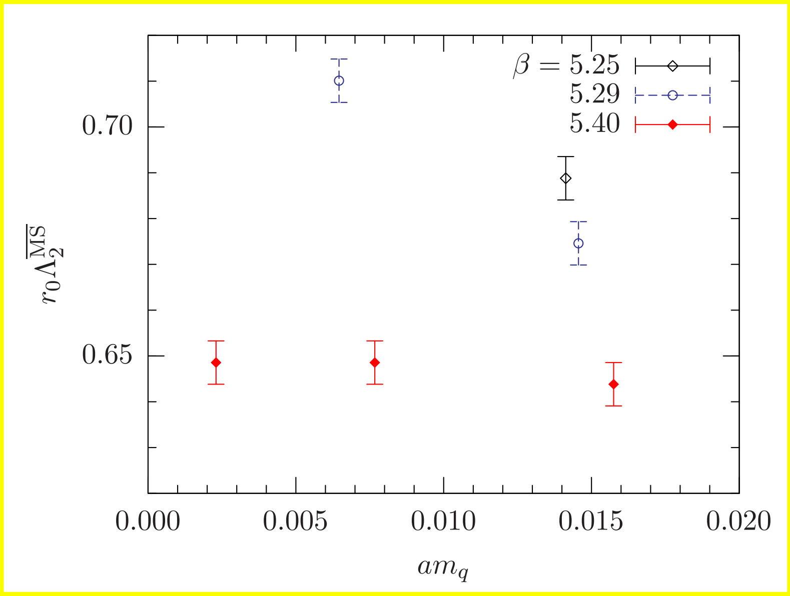


A. Sternbeck, K. Maltman, L. von Smekal, A. G. Williams, E. M. Ilgenfritz and M. Müller-Preussker, PoS (LATTICE 2007) 256, `arXiv:0710.2965 [hep-lat]`.

Running Coupling

- Minimom scheme $\alpha_S^{\mathrm{minMOM}}$ known to 4 loops (Maltman, Sternbeck, LvS, in prep):





- Compare observables in PT:

Baikov, Chetyrkin & Kühn, `arXiv:0801.182[hep-ph]`.

$$4\pi^2 D_0(Q^2) = 1 + \frac{\alpha_s^{\overline{\mathrm{MS}}}}{\pi} + 1.64\left(\frac{\alpha_s^{\overline{\mathrm{MS}}}}{\pi}\right)^2 + 6.37\left(\frac{\alpha_s^{\overline{\mathrm{MS}}}}{\pi}\right)^3 + 49.1\left(\frac{\alpha_s^{\overline{\mathrm{MS}}}}{\pi}\right)^4 + \ldots$$

$$= 1 + \frac{\alpha_s^{\mathrm{mM}}}{\pi} - 1.05\left(\frac{\alpha_s^{\mathrm{mM}}}{\pi}\right)^2 - 4.82\left(\frac{\alpha_s^{\mathrm{mM}}}{\pi}\right)^3 + 3.13\left(\frac{\alpha_s^{\mathrm{mM}}}{\pi}\right)^4 + \ldots\,.$$

Contents

- Landau Gauge QCD Propagators (DSEs, lattice LG,...).
- Strong Coupling Limit in Lattice Landau Gauge.
- Modified Lattice Landau Gauge.
- Summary and Conclusions.

Gluon Propagator

<u>Lattice:</u>

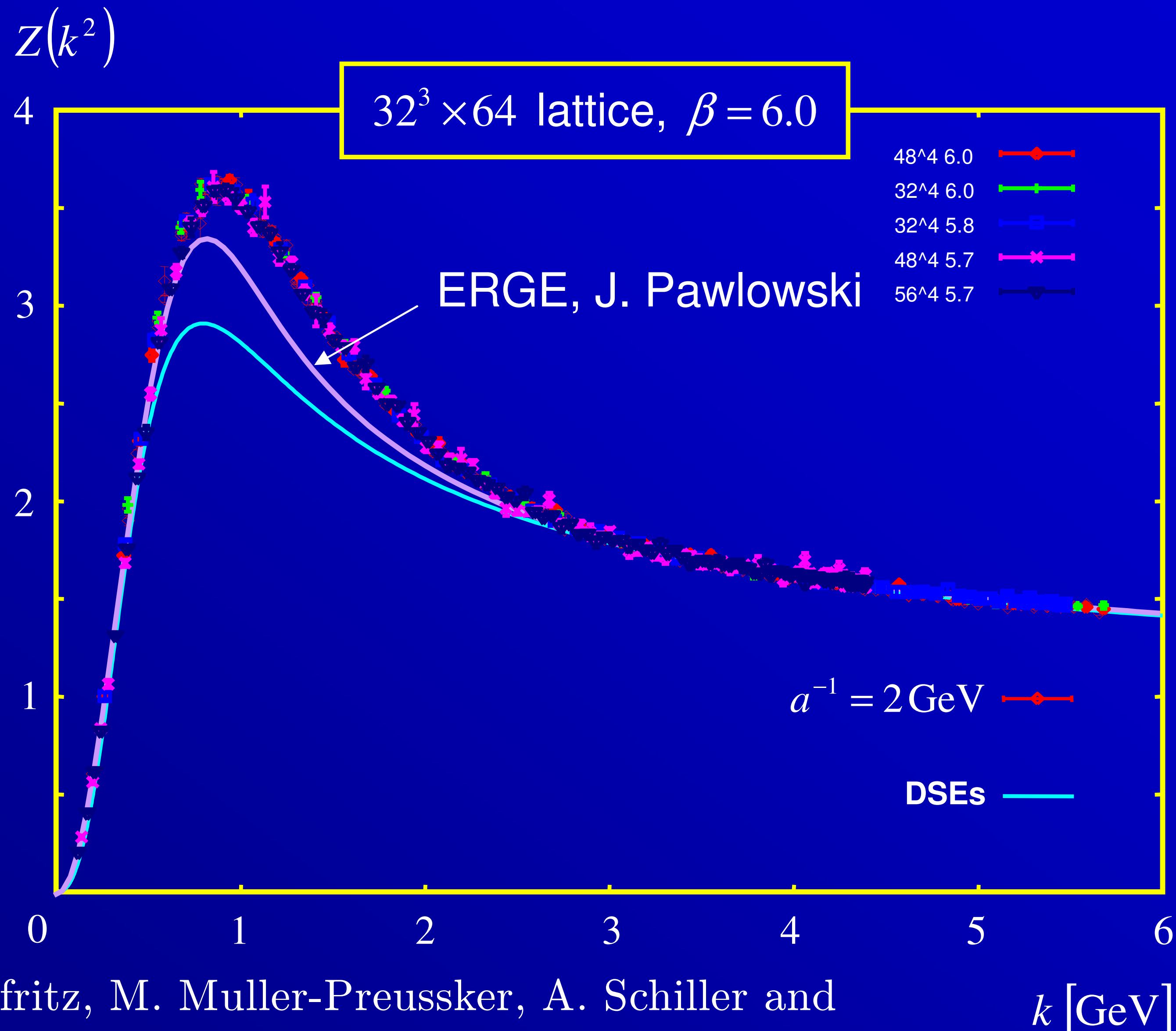
Leinweber *et al.*, 1998

<u>DSEs:</u>

$$\frac{Z(k^2)}{k^2} \overset{k^2 \to 0}{\propto} \frac{1}{k^2}\left(\frac{k^2}{\sigma}\right)^{2\kappa} \to 0, \quad 0.5 < \kappa < 1$$

L.v.S. *et al.*, 1997

Lerche & L.v.S., 2002
Fischer & Alkofer, 2002



A. Sternbeck, E. M. Ilgenfritz, M. Muller-Preussker, A. Schiller and I. L. Bogolubsky, PoS **LAT2006**, 076 (2006) [arXiv:hep-lat/0610053].

Running Coupling

$$\alpha(k^2) = \frac{g^2}{4\pi} Z(k^2) G^2(k^2)$$

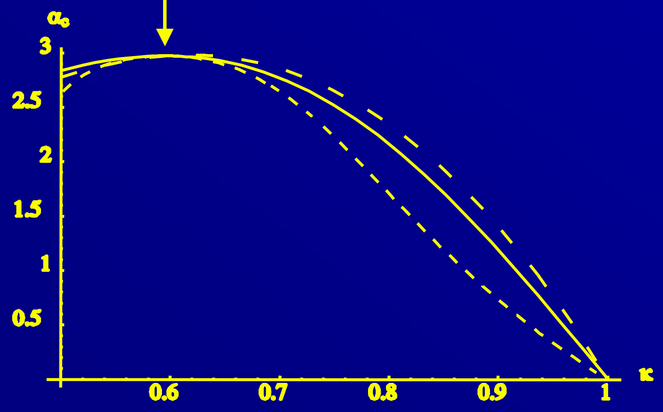
$$\to \alpha_c \,, \quad k^2 \to 0$$

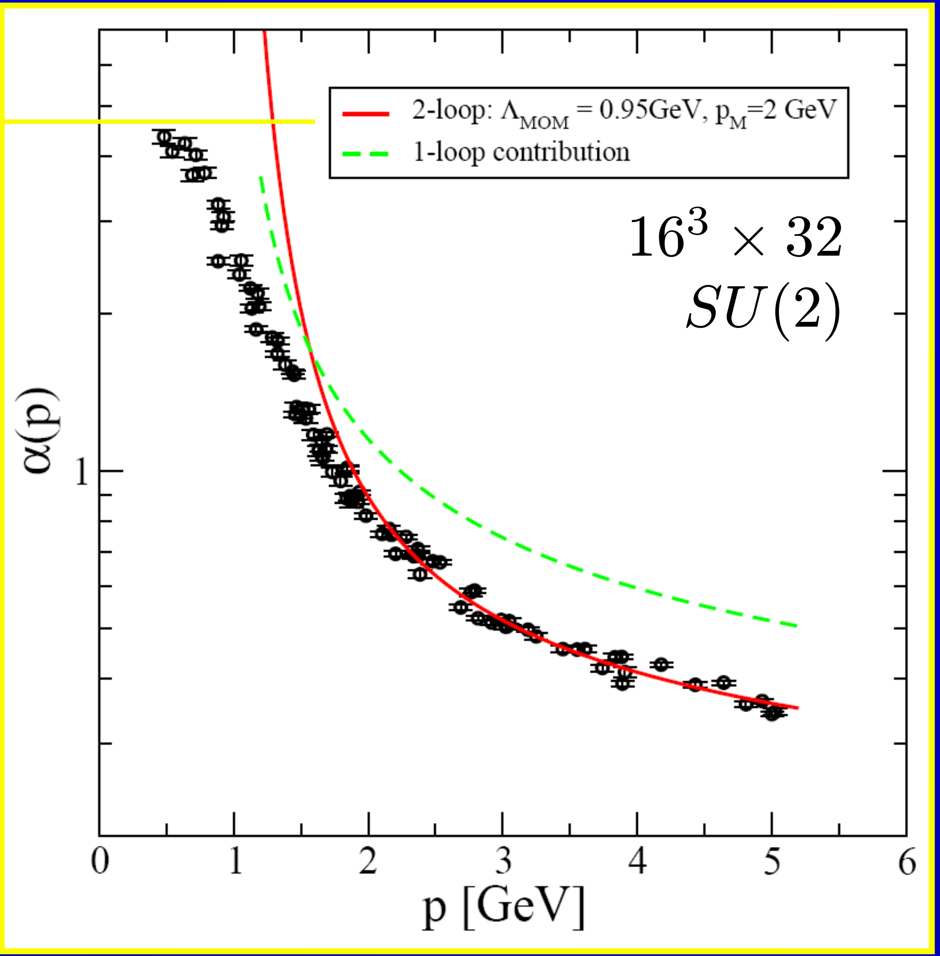
L.v.S. et al., 1997

$$\alpha_c = \frac{4\pi}{N_c I(\kappa)} = 4.46\ldots \,, \quad N_c = 2$$

Lerche & L.v.S., 2002

$$\alpha_c = 2.97\ldots \,, \quad N_c = 3$$





J. C. R. Bloch, A. Cucchieri, K. Langfeld and T. Mendes, Nucl. Phys. B **687** (2004) 76.

Infrared Dominance of Ghosts

Ghost Propagator

Lattice:

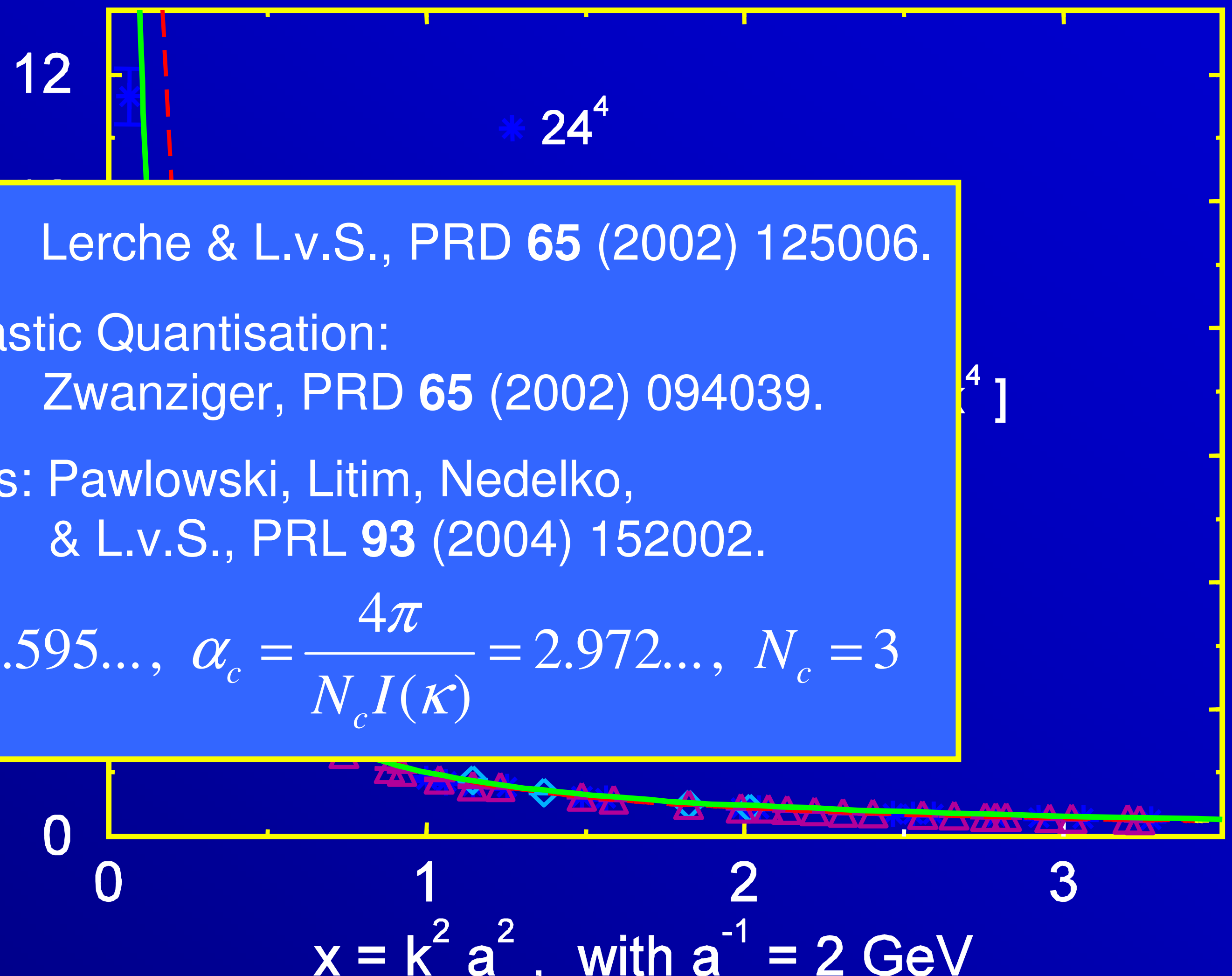
Suman & Schilling, 1996

DSEs:

$$-\frac{G(k^2)}{k^2} \overset{k^2\to 0}{\propto} -\frac{1}{k^2 \ldots}$$

L.v.S. et al., 1997

$$0.5 < \kappa < \ldots$$



- DSEs: Lerche & L.v.S., PRD **65** (2002) 125006.
- Stochastic Quantisation: Zwanziger, PRD **65** (2002) 094039.
- ERGEs: Pawlowski, Litim, Nedelko, & L.v.S., PRL **93** (2004) 152002.

$$\kappa = 0.595\ldots, \quad \alpha_c = \frac{4\pi}{N_c I(\kappa)} = 2.972\ldots, \quad N_c = 3$$

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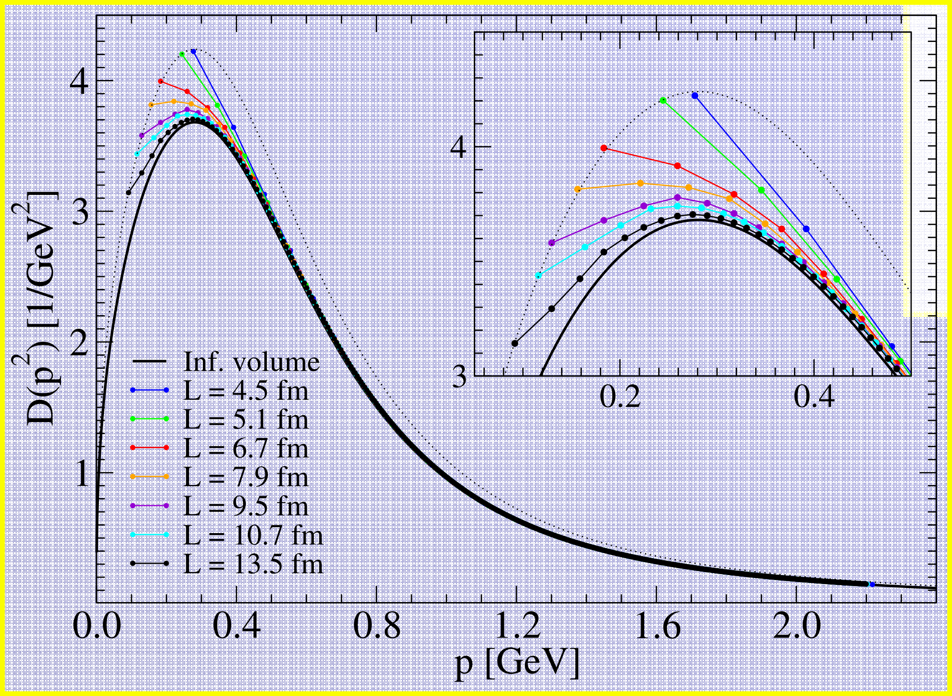
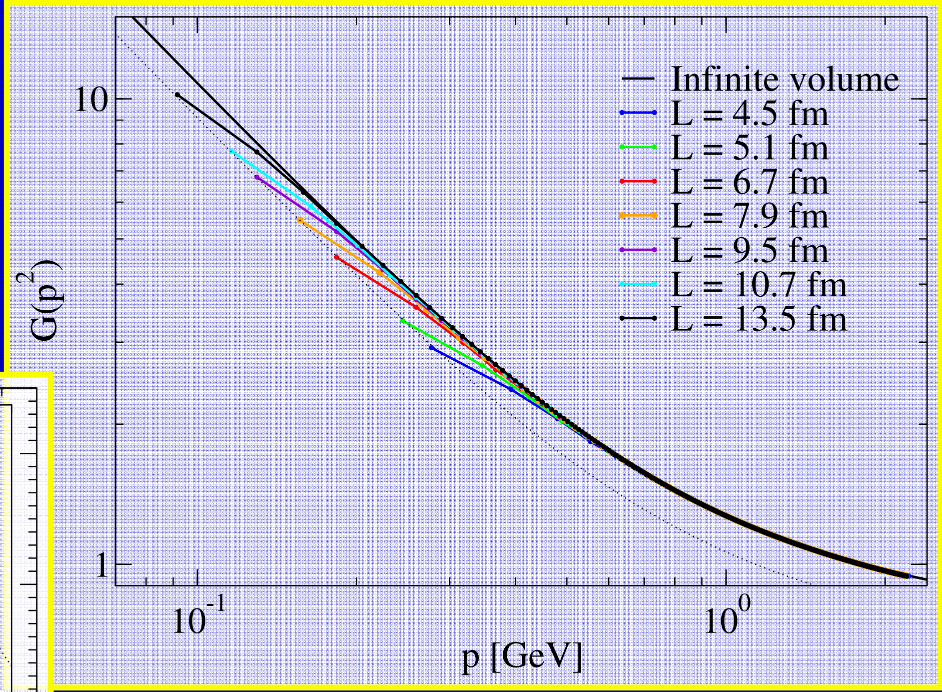
Finite Volume DSE Solutions

gluon



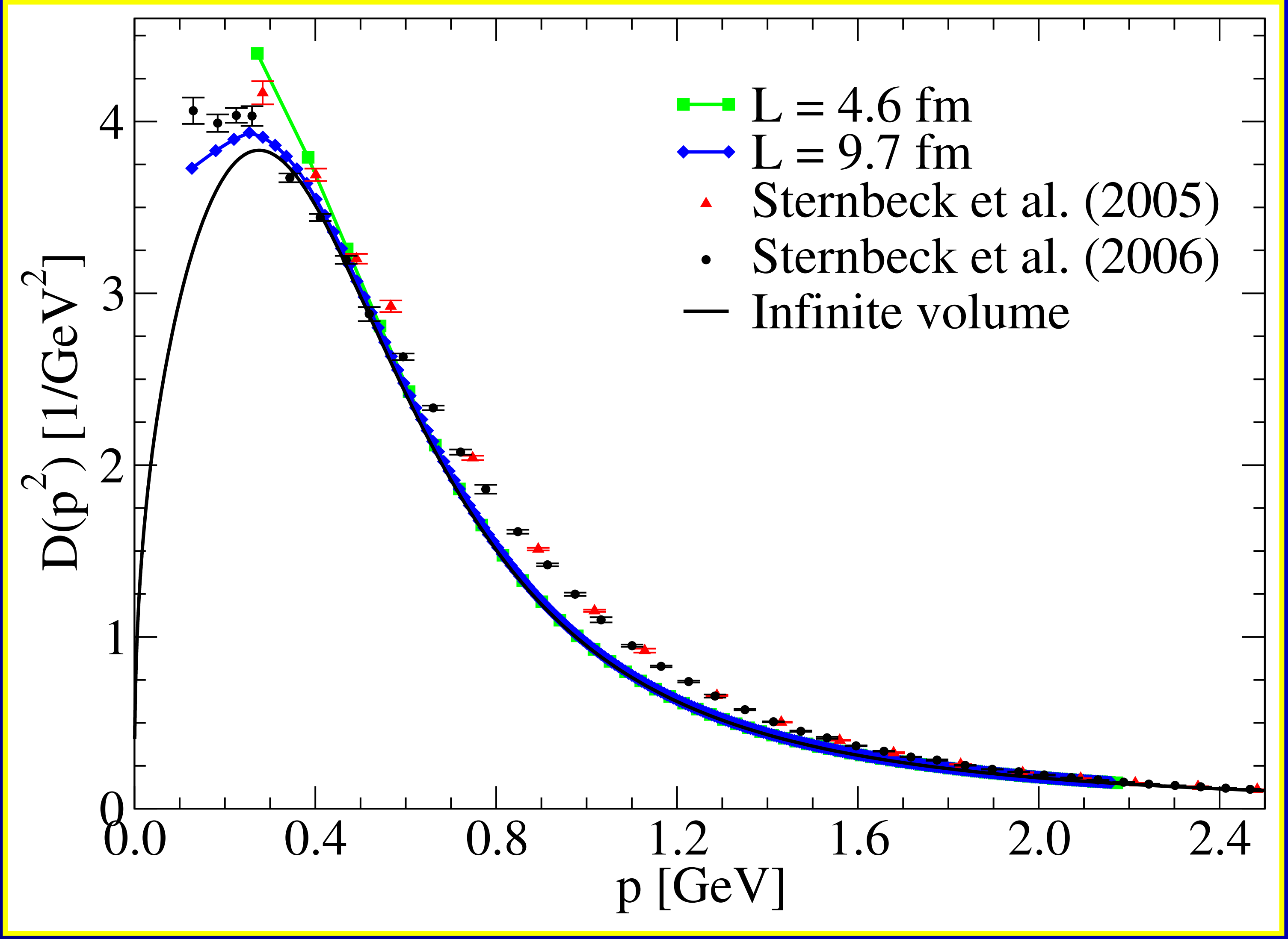


ghost

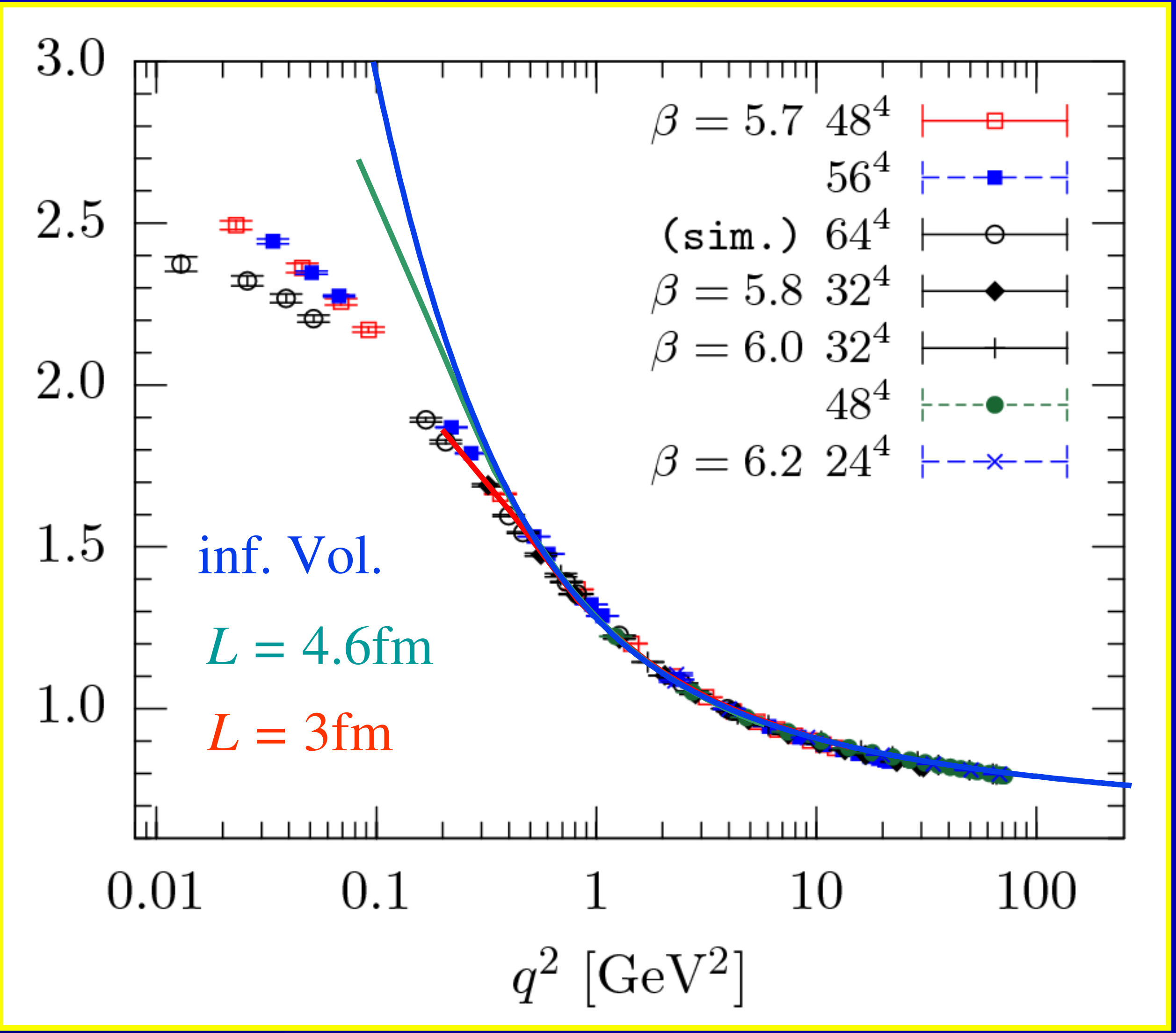
need: $\pi/L \ll p \ll \Lambda_{\rm QCD}$

Fischer, Maas, Pawlowski & L.v.S., Ann. Phys. **322** (2007) 2916.

Gluon Propagator



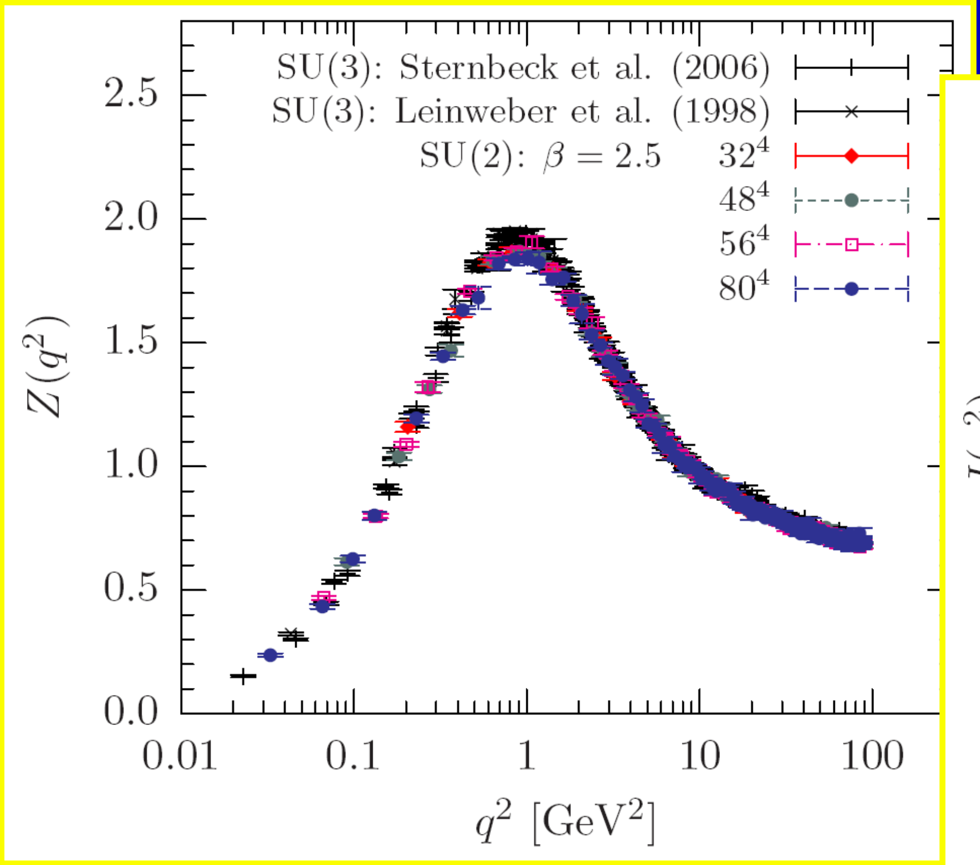
Ghost Propagator

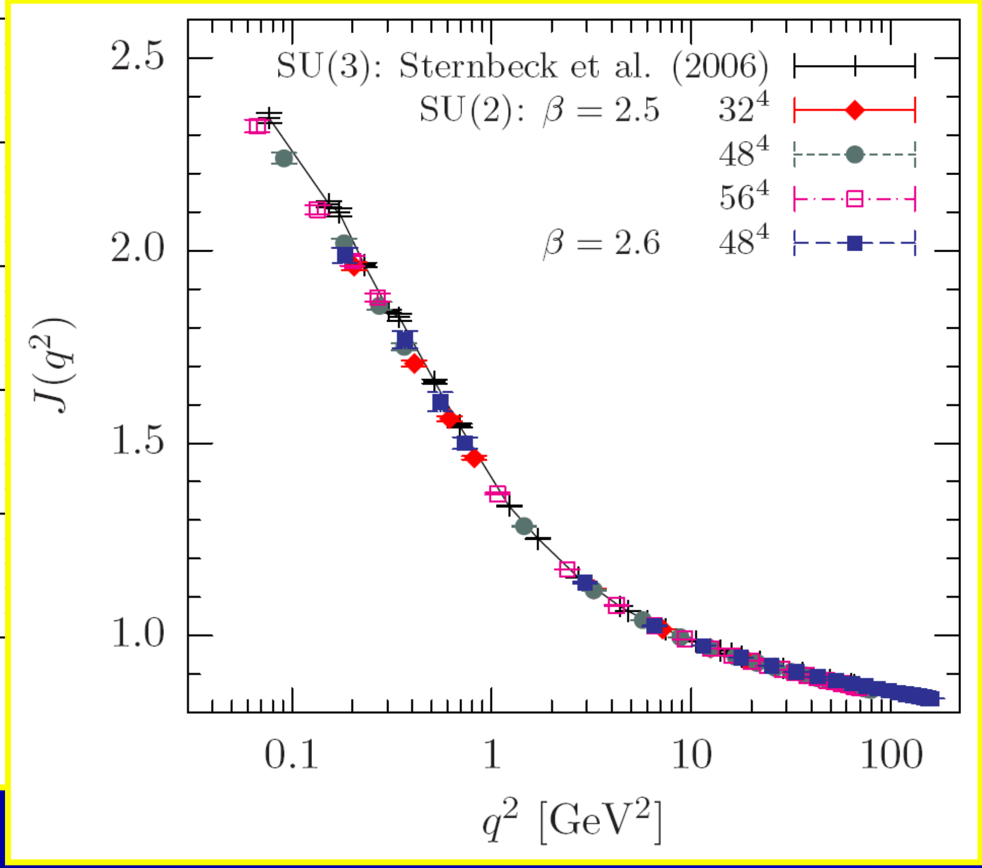


Sternbeck *et al.*, Adelaide–Berlin–Moscow, unpublished (preliminary).

SU(3) vs *SU*(2)

ghost

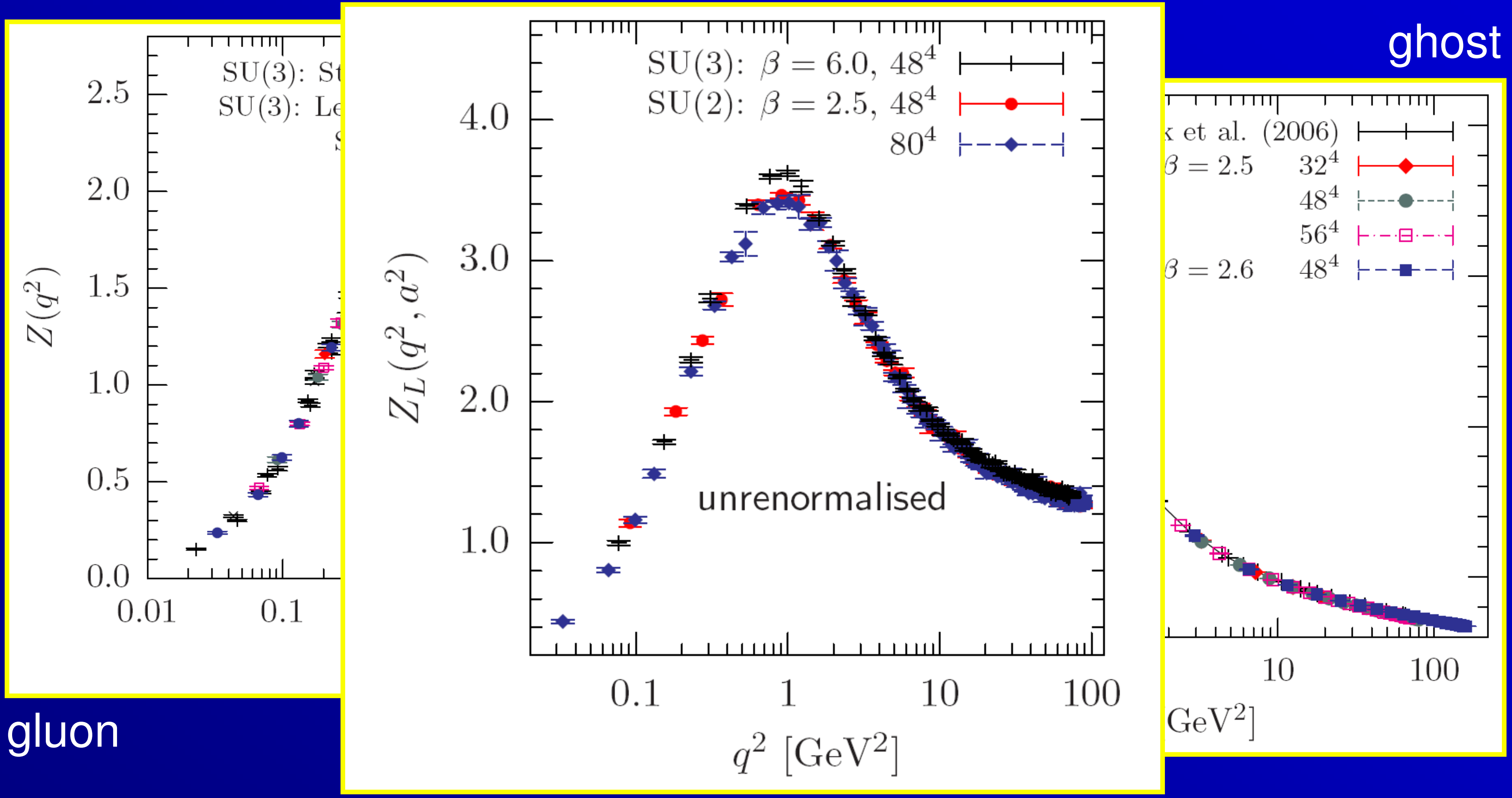




gluon

A. Sternbeck, L. von Smekal, D. B. Leinweber and A. G. Williams,
PoS (LATTICE 2007) 340, `arXiv:0710.1982 [hep-lat]`.

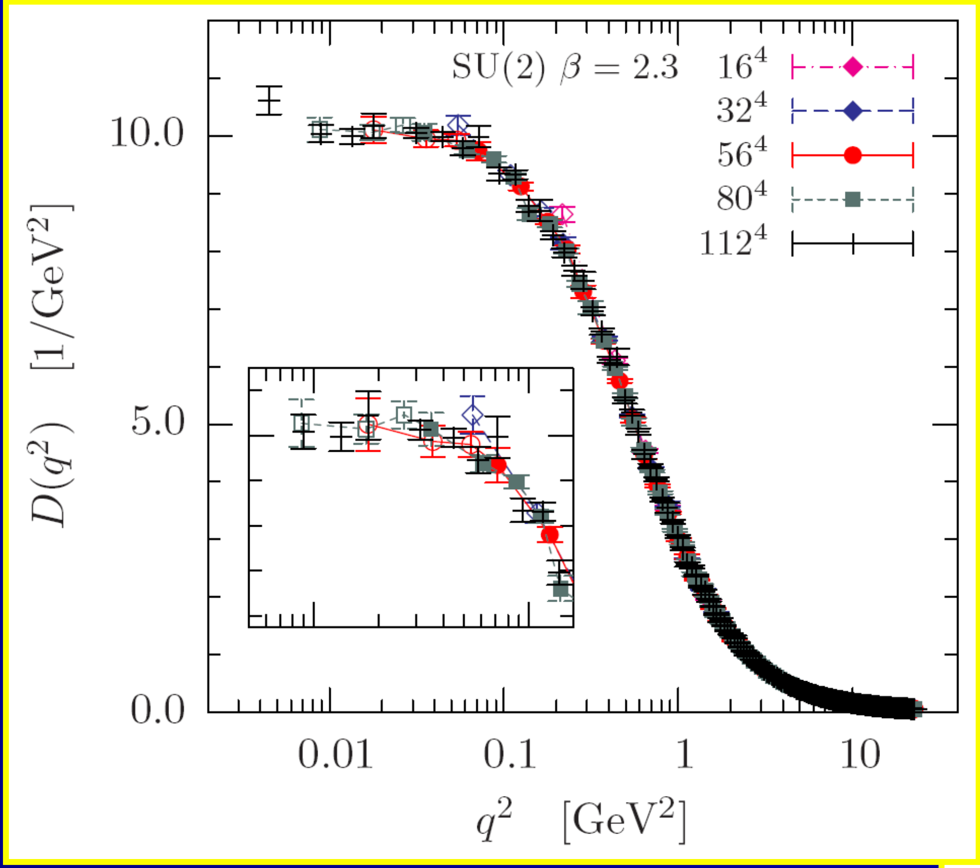
$SU(3)$ vs $SU(2)$



A. Sternbeck, L. von Smekal, D. B. Leinweber and A. G. Williams, PoS (LATTICE 2007) 340, `arXiv:0710.1982 [hep-lat]`.

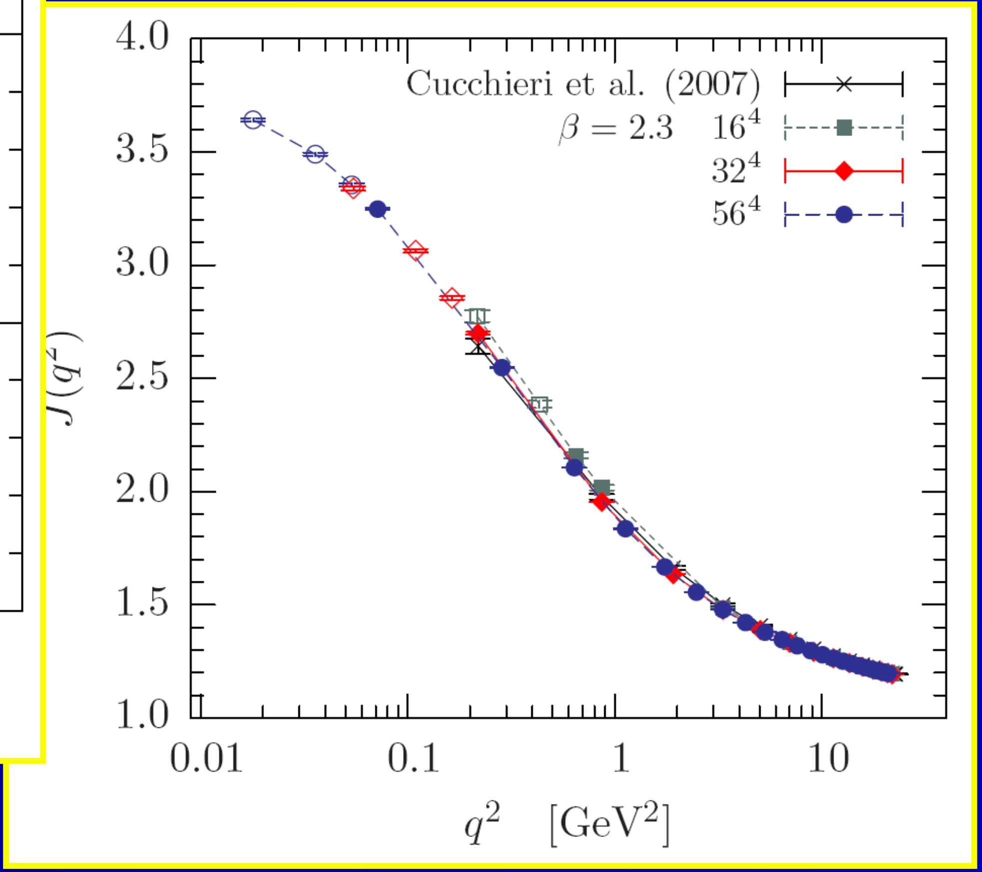
$SU(2)$ Propagators
volume dependence

fixed β, unrenormalised.



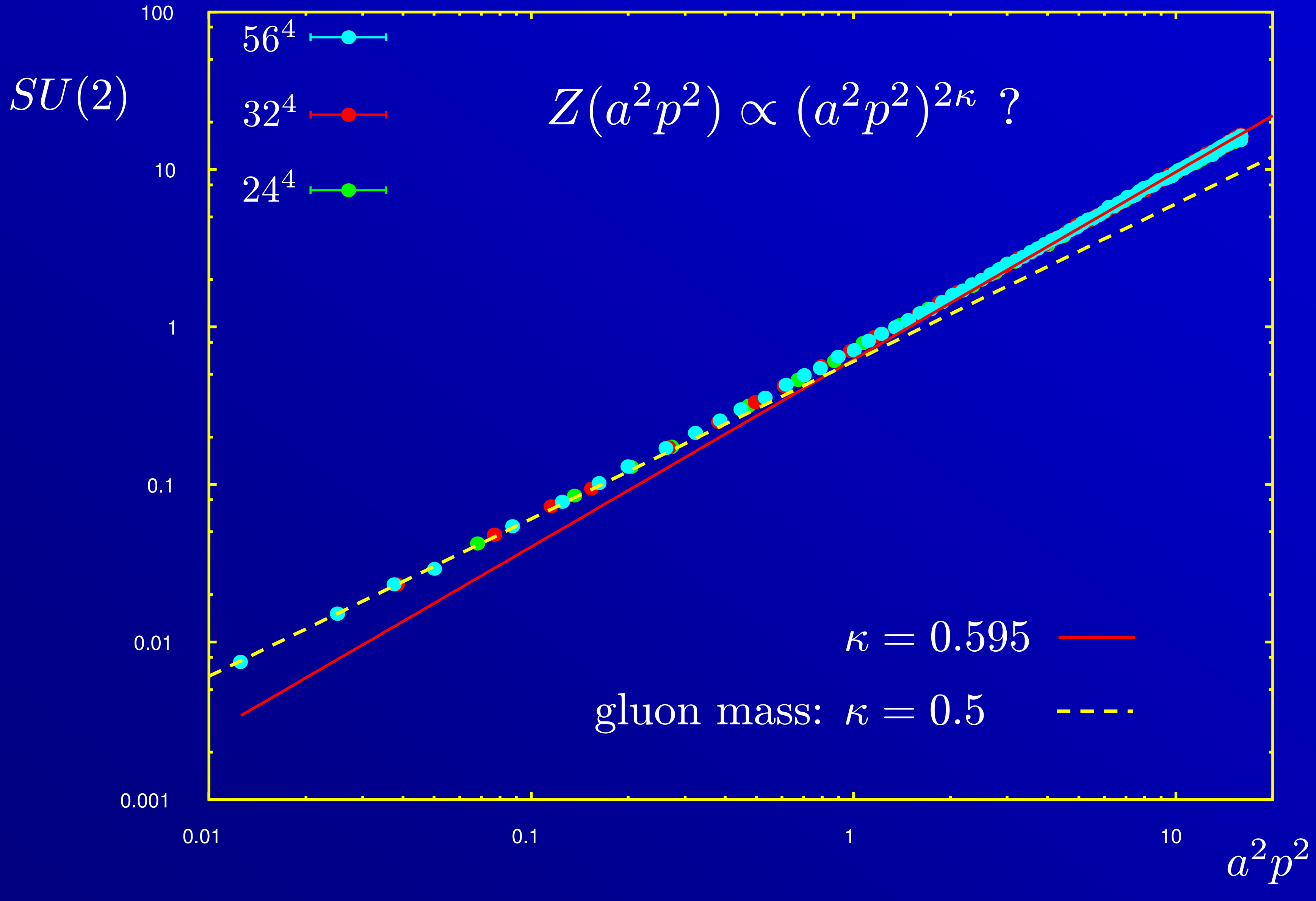
gluon

ghost

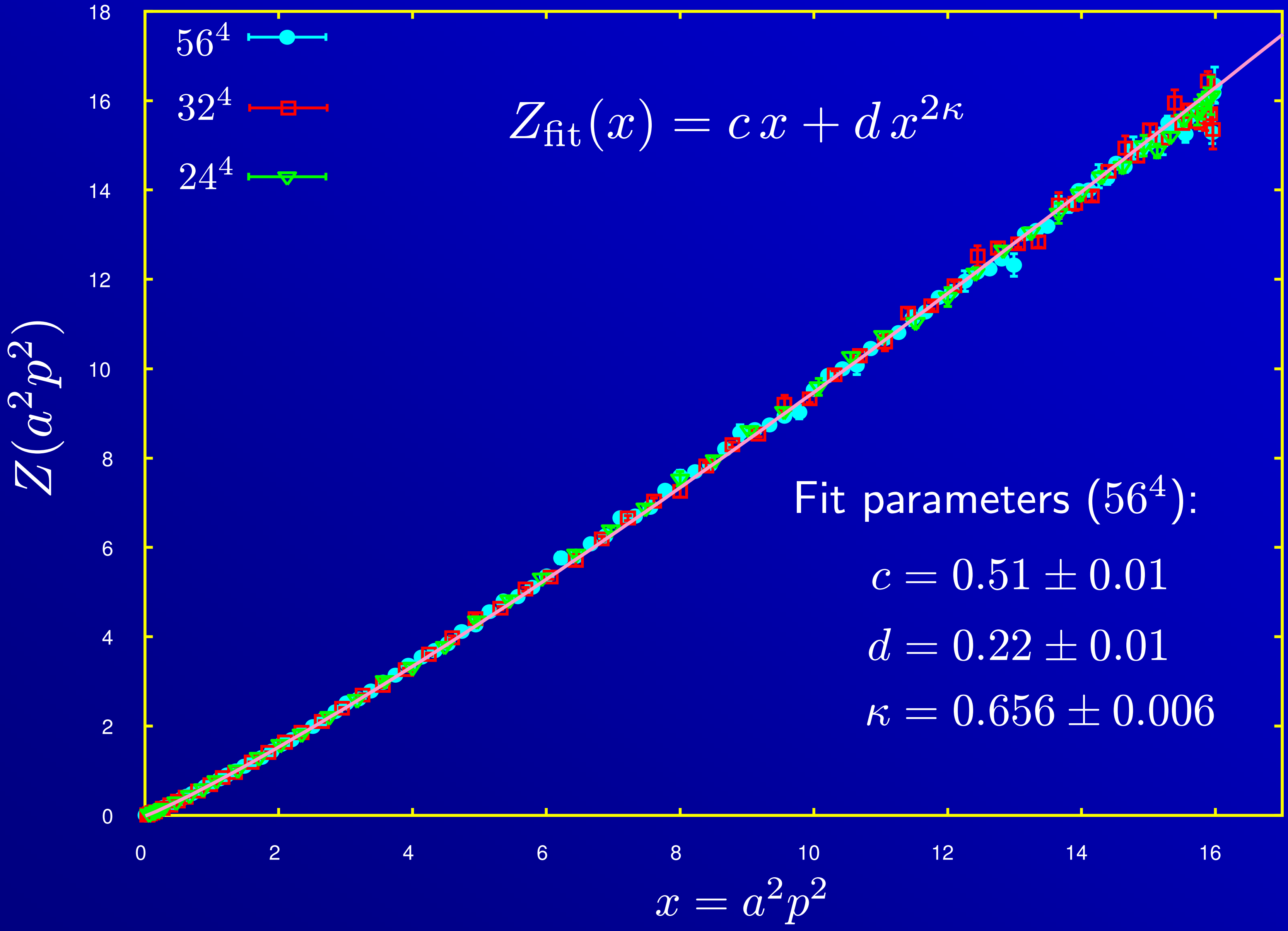


A. Sternbeck, L. von Smekal, D. B. Leinweber and A. G. Williams, PoS (LATTICE 2007) 340, arXiv:0710.1982 [hep-lat].

$\beta = 0$ — Gluon Propagator

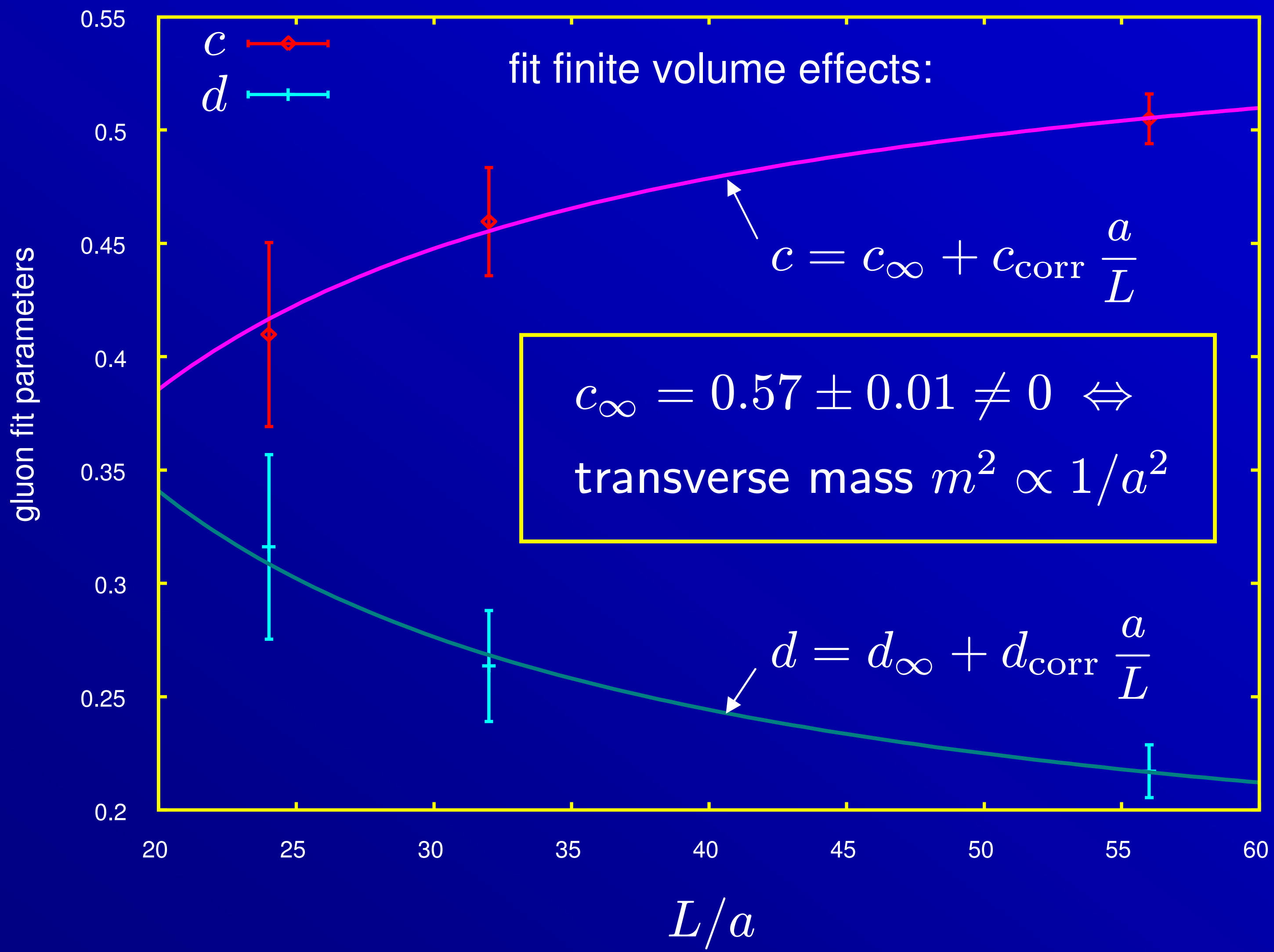


$\beta = 0$ — Gluon Propagator

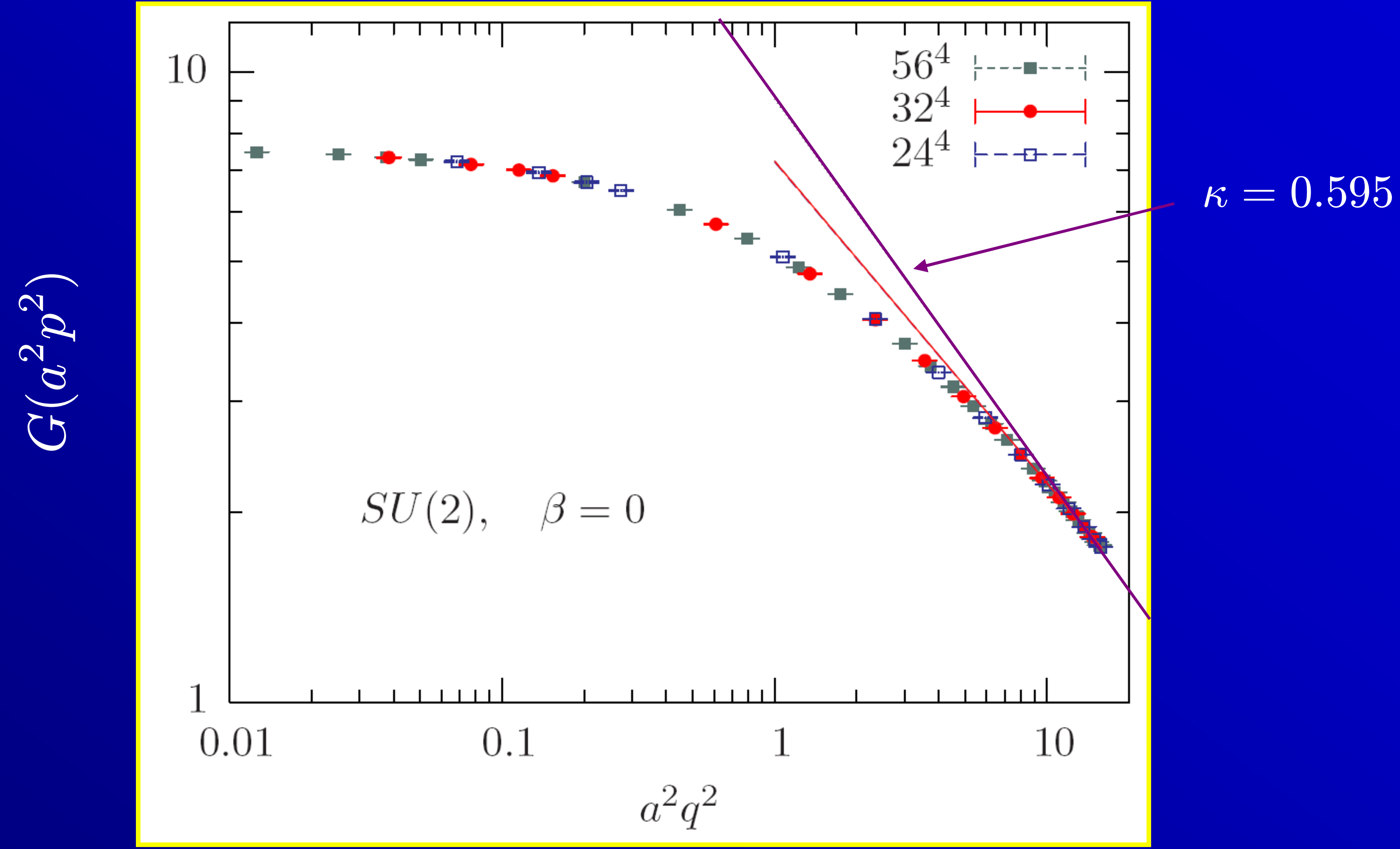


$\beta = 0$ — Gluon Propagator

$$Z_{\mathrm{fit}}(x) = c\,x + d\,x^{2\kappa}, \quad x = a^2p^2$$

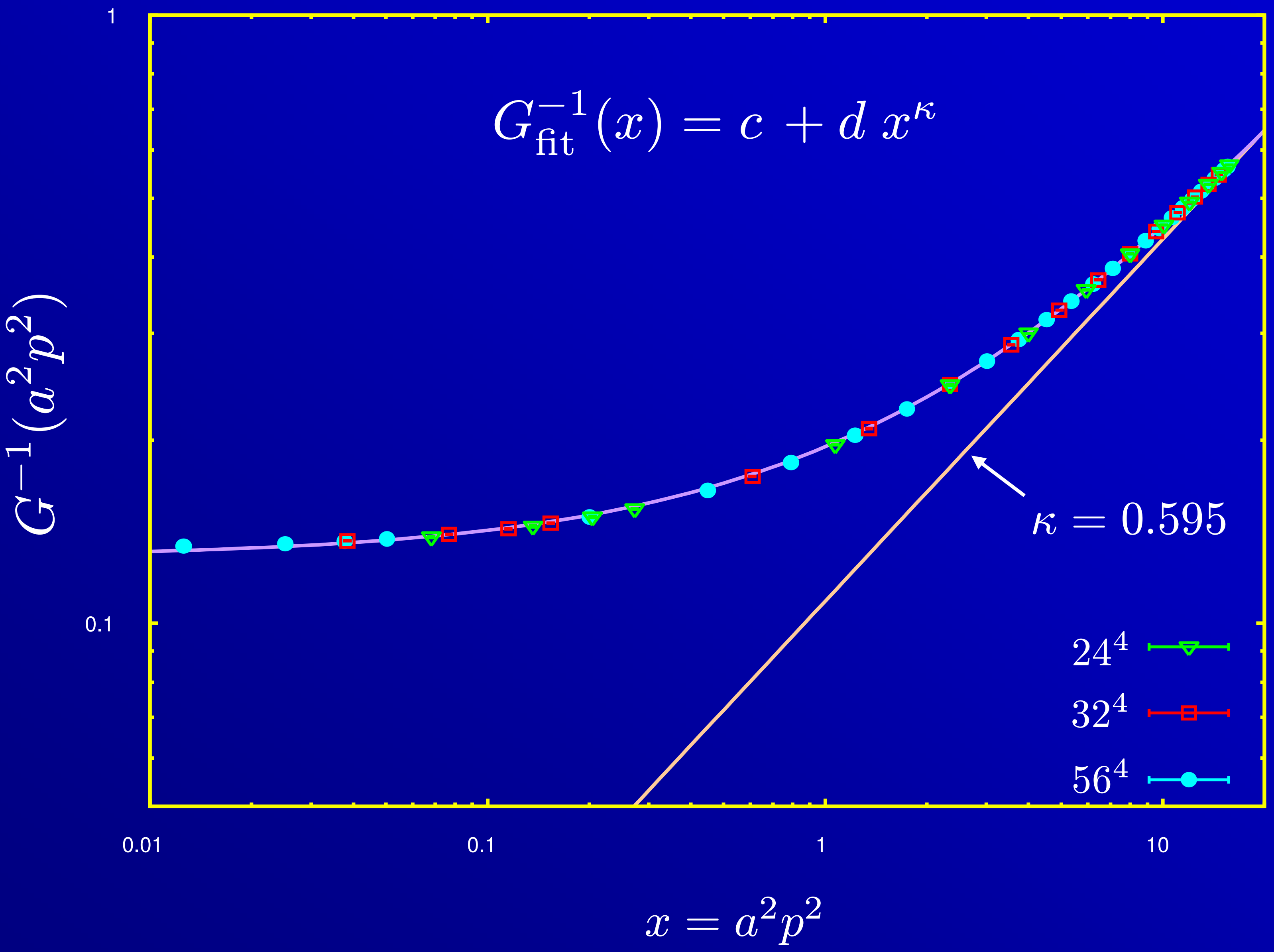


β = 0 — Ghost Propagator

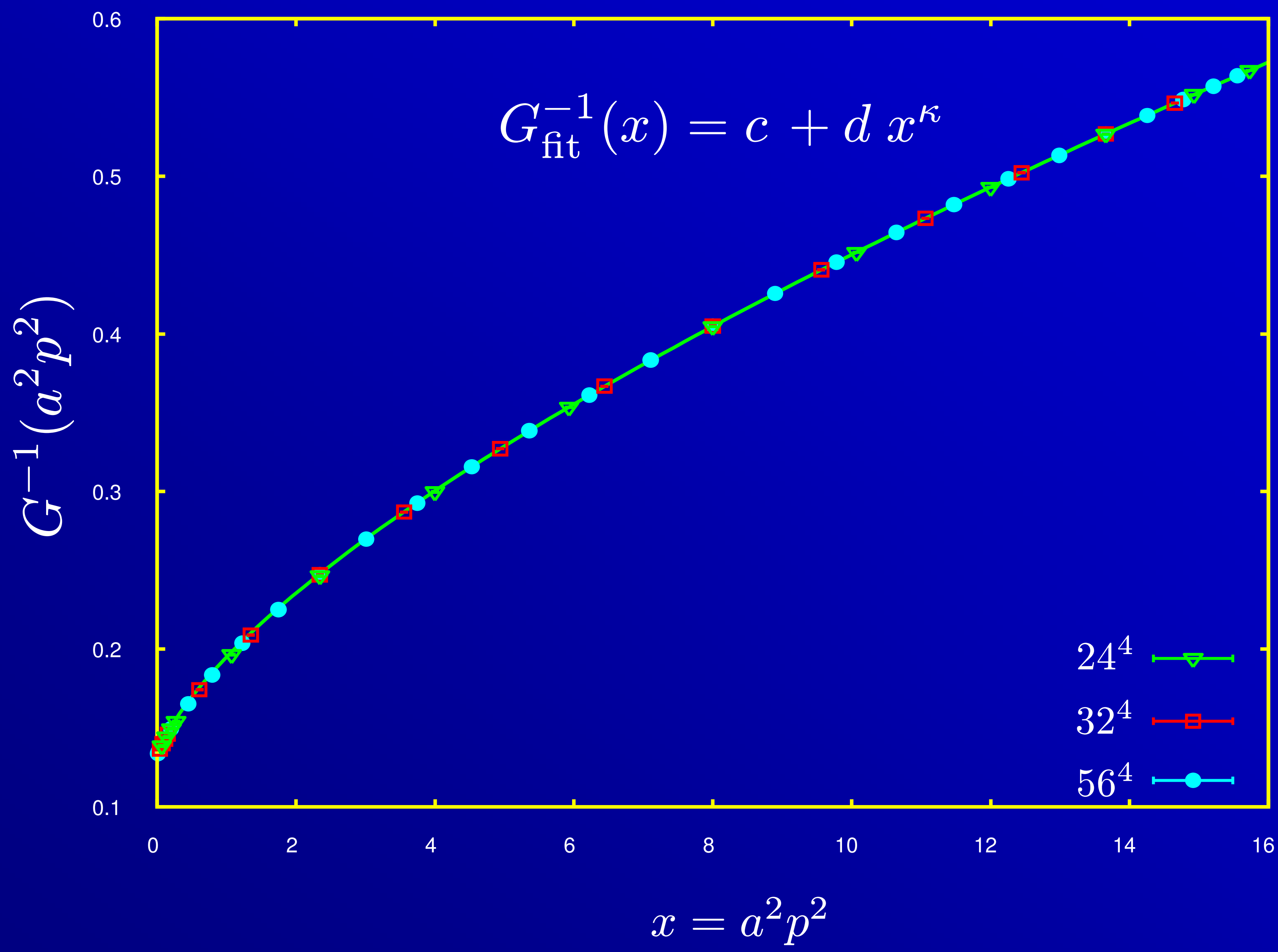


Not a finite volume effect!

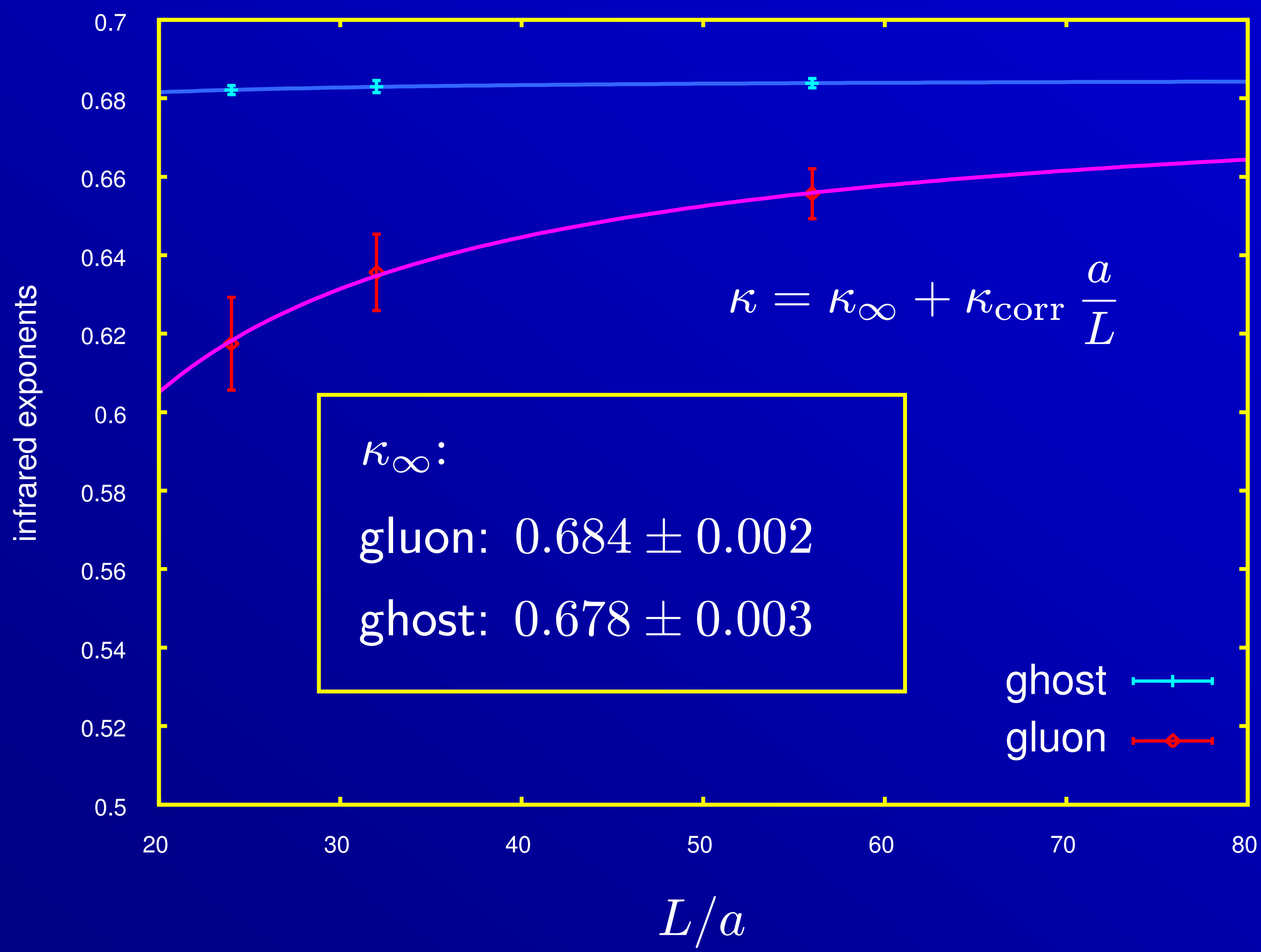
$\beta = 0$ — Ghost Propagator



$\beta = 0$ — Ghost Propagator



$\beta = 0$ — Infrared Exponents



$\beta = 0$ — Infrared Exponents

- alternative fit models:

$$Z_{\text{fit}}(x) = c\,x + d\,x^{2\kappa}, \qquad Z_{\text{fit}}(x) = c\,x\,(1 + d\,x)^{2\kappa-1},$$

$$G_{\text{fit}}^{-1}(x) = c\, + d\;x^{\kappa}, \qquad G_{\text{fit}}^{-1}(x) = c\,(1 + d\;x)^{\kappa},$$

$$x = a^2 p^2 \qquad x = a^2 p^2$$

κ_∞:	κ_∞:
gluon: 0.684 ± 0.002	gluon: 0.571 ± 0.001
ghost: 0.678 ± 0.003	ghost: 0.569 ± 0.006

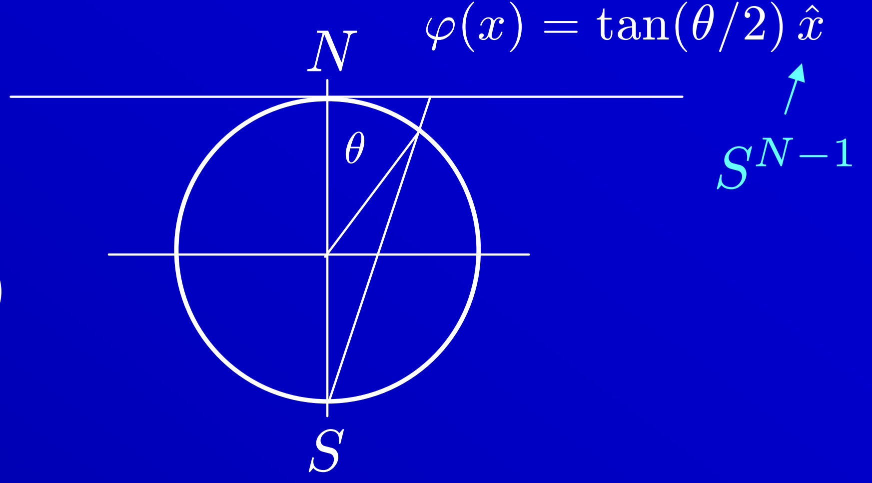
- in either case : $\kappa_{\text{gluon}} = \kappa_{\text{ghost}}$

Stereographic Projection

- Consider S^N

  with φ :

  $$(x_1, \ldots, x_{N+1}) \mapsto \frac{1}{1 + x_{N+1}} (x_1, \ldots, x_N)$$



- Example $SU(2)$:

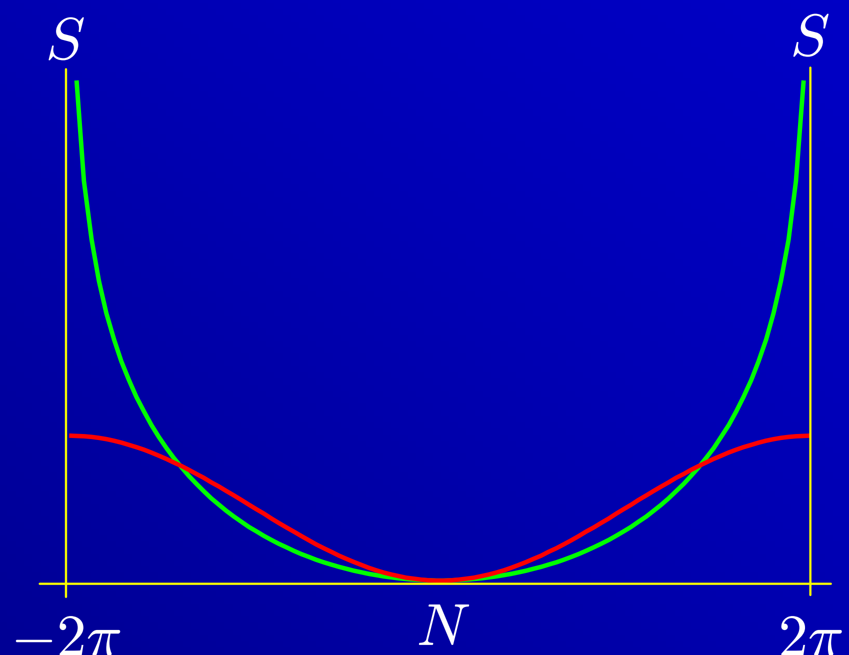
  replace

  $$\frac{1}{2}\,\mathrm{tr}\, U^g \;\to\; \ln\left(1 + \frac{1}{2}\,\mathrm{tr}\, U^g\right)$$

  in sum over links of gauge-fixing potential,

  suppresses South pole!

  $\rightsquigarrow$ modified Landau gauge



Modified Lattice Landau Gauge

- Compact $U(1)$:

links $U = e^{i\phi}$, with g.t. $\phi^\theta = \phi + d\theta$

standard Morse potential: $V[U^\theta] = \sum_{\text{links}} \cos\phi^\theta$

Landau gauge: $0 = F_i(\phi^\theta) = \dfrac{\partial}{\partial\theta_i} V = \sum_\mu \big(\sin\phi^\theta_{i,\,\mu} - \sin\phi^\theta_{i-\hat{\mu},\,\mu}\big)$

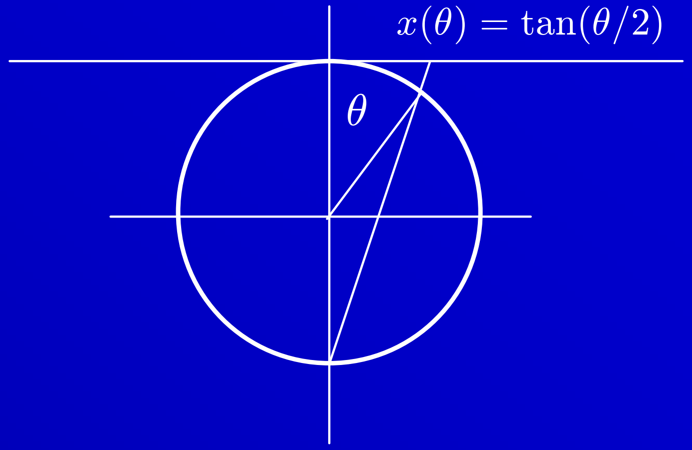
However,

$$Z_{gf} = \int \prod_{\text{sites}} d[\theta, \bar{c}, c, b]\ \exp\Big\{ -\sum_i s\Big(\bar{c}_i(F_i(\phi^\theta) - i\frac{\xi}{2} b_i)\Big)\Big\} = 0$$

$$\chi(S^{1\,\times\#\text{sites}}) = \chi(S^1)^{\#\text{sites}} = 0^{\#\text{sites}}$$

Modified Lattice Landau Gauge

- Use stereographic projection:



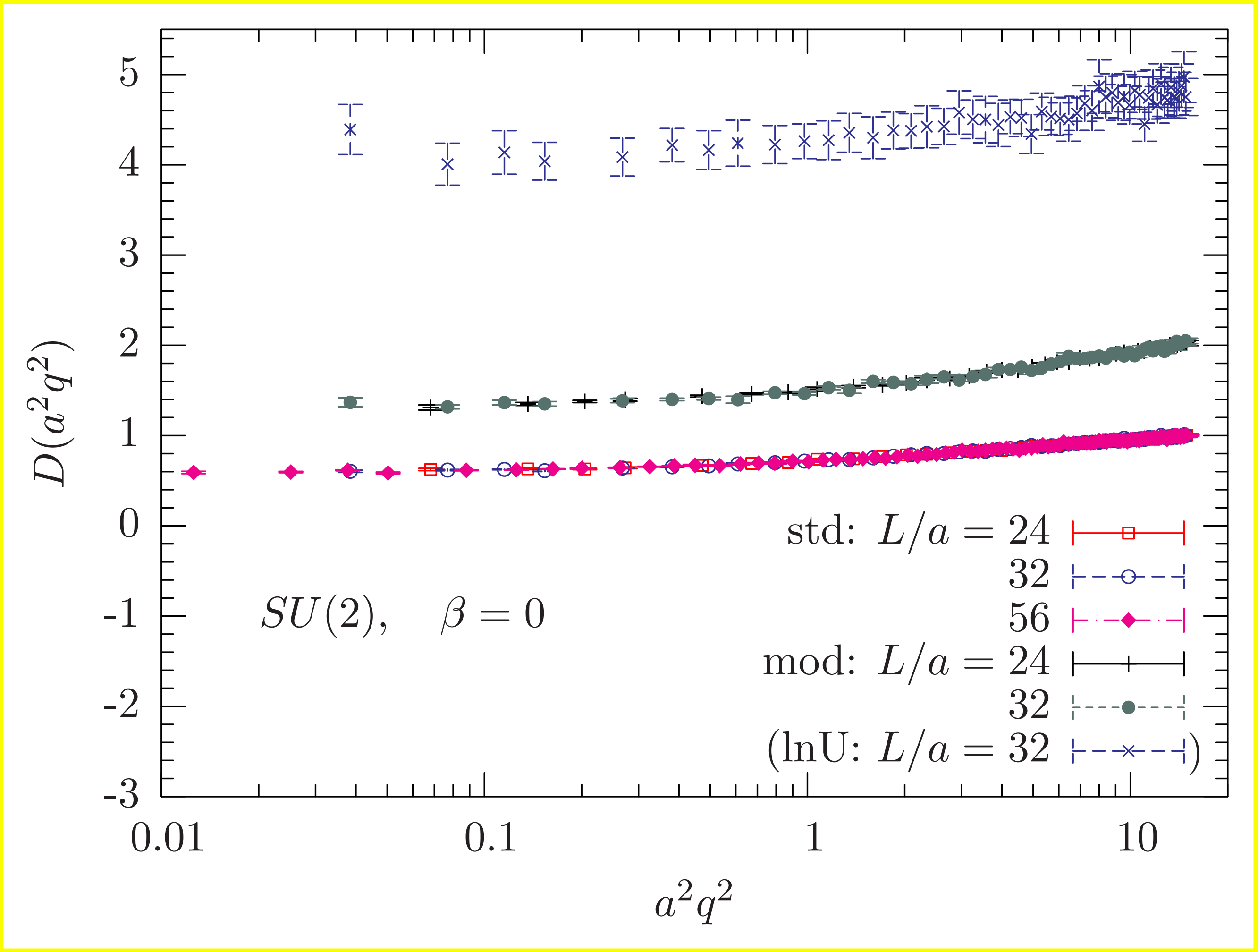
Morse potential: $V[U^\theta] = \sum_{\text{links}} \ln\left(1 + \cos\phi^\theta\right)$

and $$F_i(\phi^\theta) = \sum_\mu \left(\tan(\phi^\theta_{i,\,\mu}/2) - \tan(\phi^\theta_{i-\hat\mu,\,\mu}/2)\right)$$

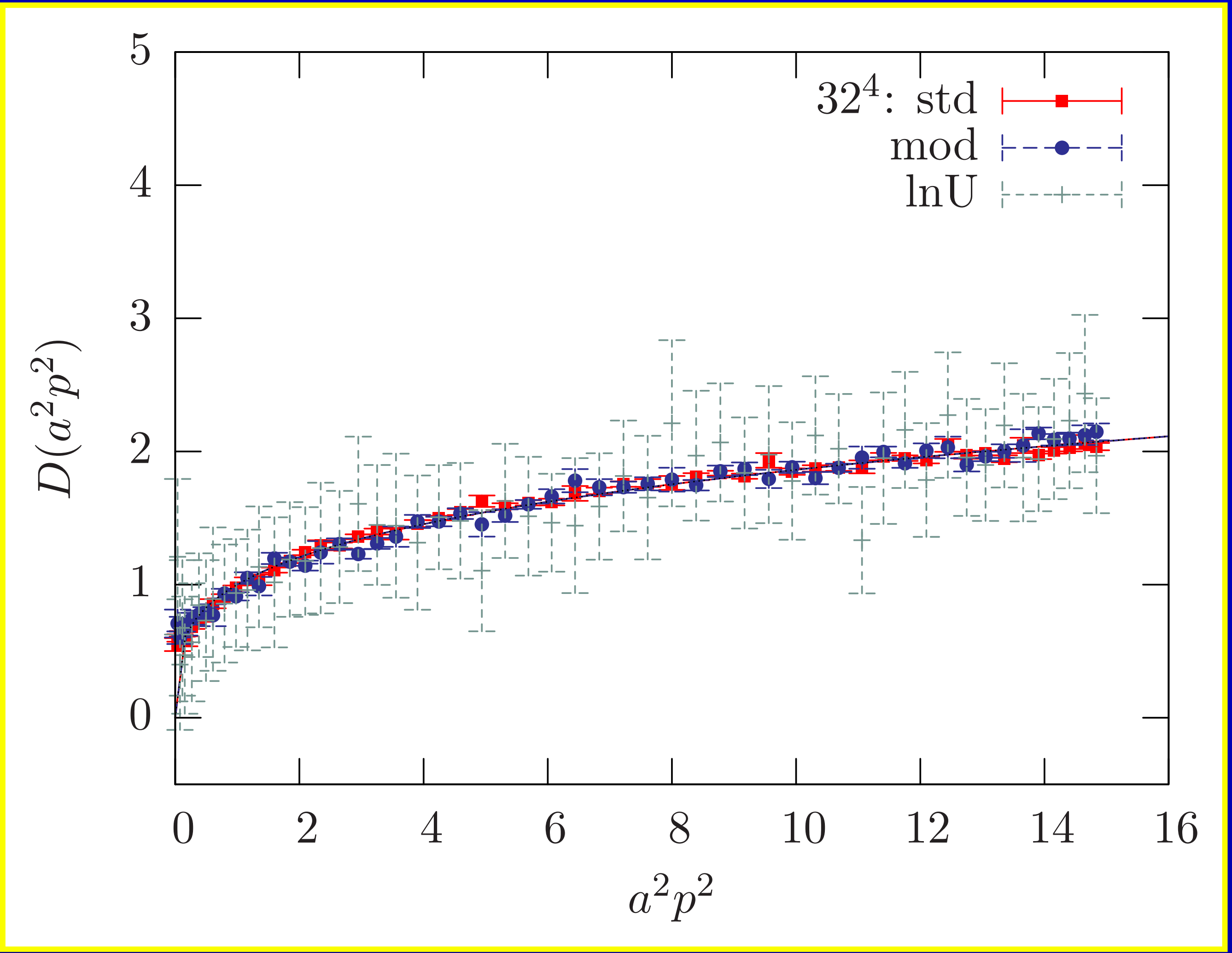
Explicitly worked out in 1-dim (eliminates all Gribov copies) and ≥ 2-dim (all but minima, 1^{st} Gribov region) compact $U(1)$.

L. von Smekal, D. Mehta, A. Sternbeck and A. G. Williams, PoS (LATTICE 2007) 382, `arXiv:0710.2410 [hep-lat]`.

SU(2) Gluon Propagator – *β* = 0



$SU(2)$ Gluon Propagator – $\beta = 0$



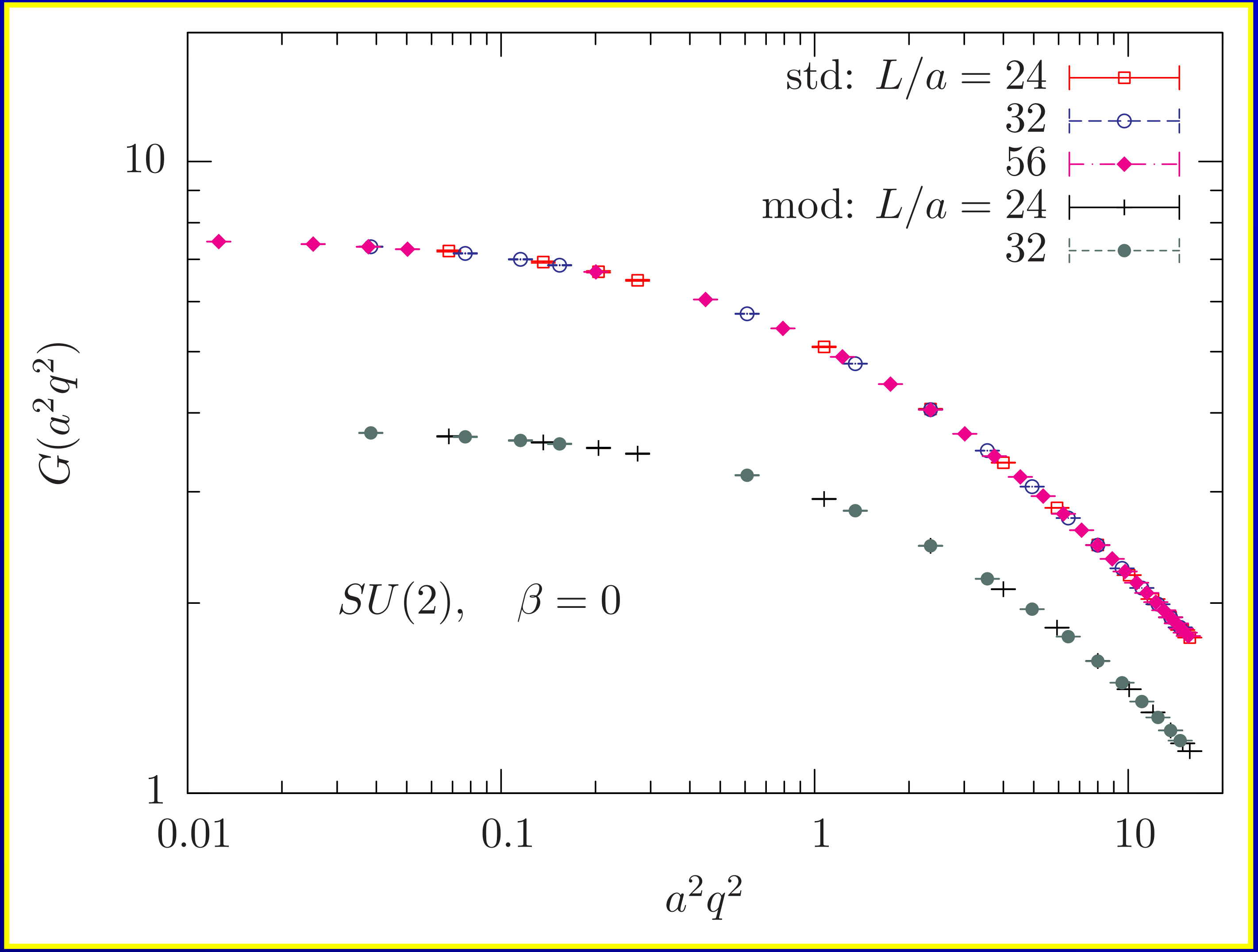
terms

$\propto (a^2p^2)^{2\kappa}$,

$\kappa = 0.6$

$$Z_{\text{fit}}(x) = c\,x + d\,x^{2\kappa}, \quad x = a^2p^2$$

$SU(2)$ Ghost Propagator – $\beta = 0$



SU(2) Coupling – β = 0

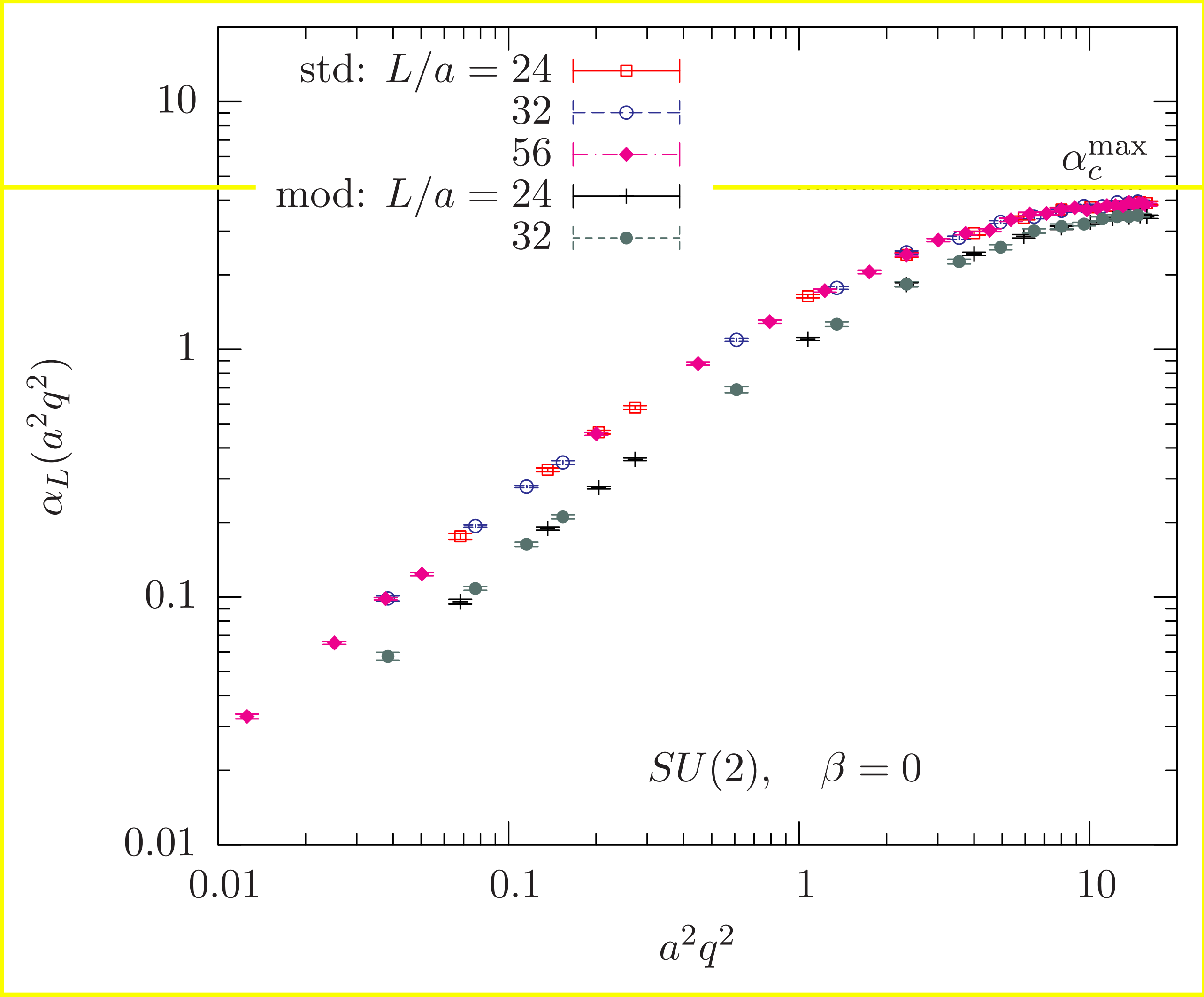
$$\alpha_L = \frac{g^2}{4\pi} Z_L(q^2)\, G_L^2(q^2)$$

Lerche & L.v.S., 2002:

$$\alpha_c = \frac{4\pi}{N_c I(\kappa)}$$

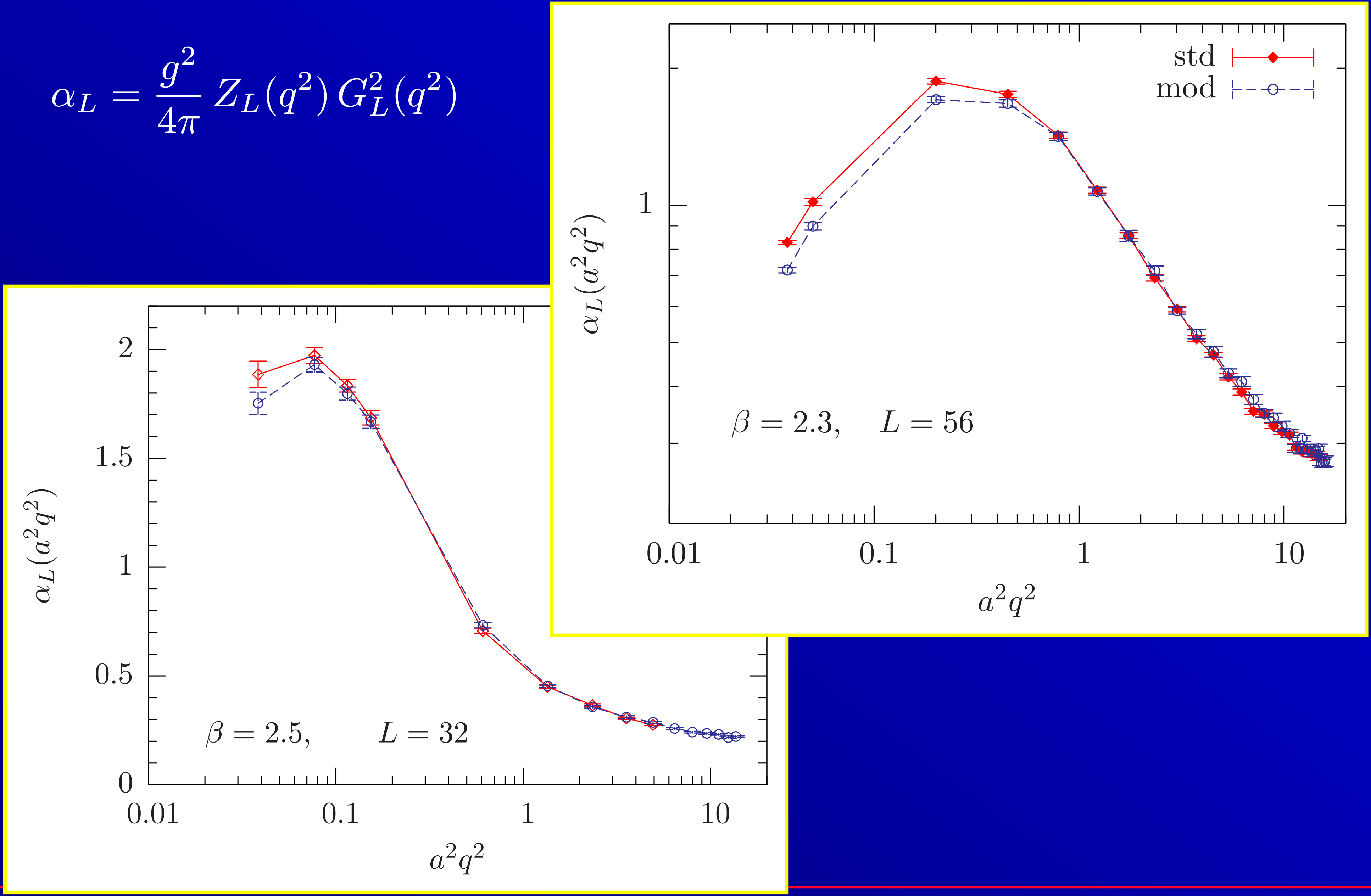
$$\alpha_c = \alpha_c^{\max} = 4.46\ldots, \; N_c = 2,$$

max for $\kappa = 0.595\ldots$



SU(2) Running Coupling – finite β

$$\alpha_L = \frac{g^2}{4\pi} Z_L(q^2)\, G_L^2(q^2)$$



Conclusions

- **Strong Coupling Limit of Lattice Landau Gauge**

facilitate $\pi/L \ll p \ll \Lambda_{\text{QCD}} \to \infty$

observe infrared behaviour of functional methods at large momenta:

$$D_{\text{gluon}} \sim (a^2p^2)^{2\kappa-1}, \quad D_{\text{ghost}} \sim (a^2p^2)^{-\kappa-1}, \qquad \kappa \approx 0.6,$$

in particular, $\kappa_{\text{gluon}} = \kappa_{\text{ghost}}$, and $\alpha_S \to \alpha_c \approx 4 \quad (SU(2))$.

However, this all happens for $1 \ll a^2p^2$

deviations at small momenta are *not* a finite-size effect!

- **Modified Lattice Landau Gauge**

solves Neuberger $0/0$ problem of lattice BRST.

alternative gauge field definition.

will allow to perform (Landau) gauge-fixed MC.