415^{th} Wilhelm & Else Heraeus Seminar, 17.-20.March 2008
“Quarks and Hadrons in strong QCD”, Sankt Goar

Spontaneous P-parity breaking in strong QCD at large baryon density

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A.A. & D.E. E-arXiv: **0709.0049** [hep-ph]

A.A., V.A. Andrianov, S.S.Afonin, J.Math.Sci.NY, 143/1 (2007) 2697

S.S.Afonin, A.A., V.A.Andrianov, D.E. E-arXiv: **0710. 0362** [hep-ph]

We guess P- breaking to occur at zero temperature but large baryon number density due to condensation of parity-odd mesons (pions, kaons,… and their heavy twins)

$$\varrho_B \gg \varrho_N \simeq 0.17 fm^{-3} = (1.8 fm)^{-3}$$

How large?

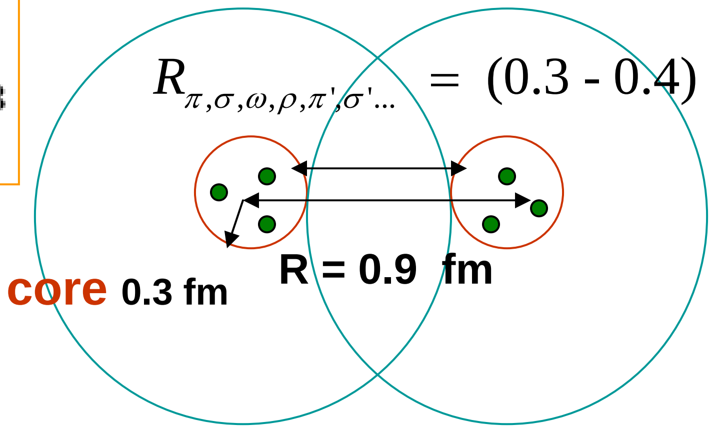
<u>Beyond the range of validity</u> of pion-nucleon effective Lagrangian but <u>not large enough</u> for quark percolation, $\rho_B \sim (3 \div 8)\rho_N$

i.e. **in the hadronic phase with heavy meson excitations playing an essential role in dense nuclear matter where quark-nuclear matter duality can be effectively used.**

Example

$$\varrho_B = 8\varrho_N \simeq (0.9 fm)^{-3}$$

$R_{\pi,\sigma,\omega,\rho,\pi',\sigma'...} = (0.3 - 0.4)\ \text{fm} = (500\text{-}700\ \text{MeV})^{-1}$



Higher-mass mesons + dressed quarks

Effective lagrangian approach

Low energies ⟹ **Chiral lagrangian for pions:**

$$L_\pi = \frac{1}{4} F_\pi^2 \, \mathrm{tr}\left(\mathrm{D}_\mu U \, \mathrm{D}^\mu U^\dagger + m_\pi^2 \left(U + U^\dagger \right) \right)$$

Vector field

$$\mathrm{U} = \exp\left(\mathrm{i} \frac{\pi^a \tau^a}{\mathrm{F}_\pi} \right) \qquad \mathrm{D}_\mu U \equiv \partial_\mu U + \left[\mathrm{V}_\mu , U \right]$$

Density *vs.* chemical potential ***(symmetric nuclear matter)***

$$\left\langle N^\dagger N(x) \right\rangle = \rho_B \quad \Leftrightarrow \quad \int dx \, \mu_\mathrm{B} \left(\bar{N} \gamma_0 N(x) - \rho_B \right)$$

Corresponds to singlet vector current!

$$V_0 = \mu_\mathrm{B}$$

Disappears from pion lagrangian?

No! It is hidden in structural constants and masses. $F_\pi^2(\mu_\mathrm{B}) \qquad m_\pi^2(\mu_\mathrm{B})$

We need a model of formation for these parameters:

QCD ? **At least an extension to QCD motivated Linear sigma models and/or Quark models !**

Heavy meson states are in game!

Extended linear sigma model

with two multiplets of scalar and pseudoscalar mesons
(with matching as close to QCD as possible –spectral sum rules)

$$H_j = \sigma_j \mathbf{I} + i\hat{\pi}_j, \quad j = 1,2; \quad H_j H_j^\dagger = (\sigma_j^2 + (\pi_j^a)^2)\mathbf{I}, \qquad \hat{\pi}_j \equiv \pi_j^a \tau^a$$

Chiral limit ⟶ $SU_L(2) \times SU_R(2)$ **symmetry**

$$\begin{aligned} V_{\text{eff}} \;=\; & \frac{1}{2}\text{tr}\Big\{ -\sum_{j,k=1}^{2} H_j^\dagger \Delta_{jk} H_k \\ & +\lambda_1 (H_1^\dagger H_1)^2 + \lambda_2 (H_2^\dagger H_2)^2 + \lambda_3 H_1^\dagger H_1 H_2^\dagger H_2 \\ & +\frac{1}{2}\lambda_4 (H_1^\dagger H_2 H_1^\dagger H_2 + H_2^\dagger H_1 H_2^\dagger H_1) + \frac{1}{2}\lambda_5 (H_1^\dagger H_2 + H_2^\dagger H_1) H_1^\dagger H_1 \\ & +\frac{1}{2}\lambda_6 (H_1^\dagger H_2 + H_2^\dagger H_1) H_2^\dagger H_2 \Big\} + \mathcal{O}\Big(\frac{|H|^6}{\Lambda^2}\Big), \end{aligned}$$

9 real constants

$$\Delta_{jk} \sim \lambda_A \sim N_c$$

Chiral expansion in $1/\Lambda$
in hadron phase of QCD

$$\Lambda \simeq 4\pi F_\pi \Leftrightarrow M_{\text{dyn}}$$

Chirally symmetric parameterization

$$H_1(x) = \sigma_1(x)U(x) = \sigma_1(x)\xi^2(x); \qquad \langle H_1 \rangle = \langle \sigma_1 \rangle > 0$$

$$H_2(x) = \xi(x)\big(\sigma_2(x) + i\hat{\pi}_2(x)\big)\xi(x) = \sigma_2(x)U(x) + i\xi(x)\hat{\pi}_2(x)\xi(x)$$

Effective potential

$$\begin{aligned} V_{\rm eff} \;=\; & -\sum_{j,k=1}^{2} \sigma_j \Delta_{jk} \sigma_k - \Delta_{22}(\pi_2^a)^2 \\ & + \lambda_1 \sigma_1^4 + \lambda_2 \sigma_2^4 + (\lambda_3 + \lambda_4)\sigma_1^2\sigma_2^2 + \lambda_5 \sigma_1^3 \sigma_2 + \lambda_6 \sigma_1 \sigma_2^3 \\ & + (\pi_2^a)^2 \Big((\lambda_3 - \lambda_4)\sigma_1^2 + \lambda_6 \sigma_1 \sigma_2 + 2\lambda_2 \sigma_2^2 \Big) + \lambda_2 \Big((\pi_2^a)^2 \Big)^2 \end{aligned}$$

Minimal nontrivial choice admitting SPB

$$\boxed{\lambda_5 = \lambda_6 = 0 \;\Rightarrow\; \Delta_{12} = 0}$$ **from consistency**

Such an effective potential is symmetric under $\boxed{Z_2 \times Z_2}$

$$H_1 \to -H_1 \quad \text{or} \quad H_2 \to -H_2$$

Vacuum states

Neutral pseudoscalar condensate breaking P-parity?

$$\pi_2^a = \delta^{a0}\rho$$

No! in QCD at zero quark density $\rho = 0$ **(Vafa-Witten theorem)**

Eqs. for vacuum states

$$\begin{aligned}
2(\Delta_{11}\sigma_1 + \Delta_{12}\sigma_2) &= 4\lambda_1\sigma_1^3 + 3\lambda_5\sigma_1^2\sigma_2 + 2(\lambda_3+\lambda_4)\sigma_1\sigma_2^2 + \lambda_6\sigma_2^3 \\
&\quad +\rho^2\Big(2(\lambda_3-\lambda_4)\sigma_1 + \lambda_6\sigma_2\Big), \\
2(\Delta_{12}\sigma_1 + \Delta_{22}\sigma_2) &= \lambda_5\sigma_1^3 + 2(\lambda_3+\lambda_4)\sigma_1^2\sigma_2 + 3\lambda_6\sigma_1\sigma_2^2 + 4\lambda_2\sigma_2^3 \\
&\quad +\rho^2\Big(\lambda_6\sigma_1 + 4\lambda_2\sigma_2\Big), \\
0 &= 2\pi_2^a\Big(-\Delta_{22} + (\lambda_3-\lambda_4)\sigma_1^2 + \lambda_6\sigma_1\sigma_2 + 2\lambda_2\sigma_2^2 + 2\lambda_2\rho^2\Big)
\end{aligned}$$

Necessary and sufficient condition to avoid P-parity breaking in normal QCD vacuum (μ **= 0)**

$$(\lambda_3-\lambda_4)\sigma_1^2 + \lambda_6\sigma_1\sigma_2 + 2\lambda_2\sigma_2^2 > \Delta_{22}$$

Second variation for $\rho = 0$

$$\frac{1}{2}V^{(2)}_{11} = -\Delta_{11} + 6\lambda_1\sigma_1^2 + 3\lambda_5\sigma_1\sigma_2 + (\lambda_3+\lambda_4)\sigma_2^2, \quad > 0$$

$$V^{(2)}_{12} = -2\Delta_{12} + 3\lambda_5\sigma_1^2 + 4(\lambda_3+\lambda_4)\sigma_1\sigma_2 + 3\lambda_6\sigma_2^2,$$

$$\frac{1}{2}V^{(2)}_{22} = -\Delta_{22} + (\lambda_3+\lambda_4)\sigma_1^2 + 3\lambda_6\sigma_1\sigma_2 + 6\lambda_2\sigma_2^2 \quad > 0$$

$$\frac{1}{2}V^{(2)\pi}_{ab} = \delta_{ab}\Big(-\Delta_{22} + (\lambda_3-\lambda_4)\sigma_1^2 + \lambda_6\sigma_1\sigma_2 + 2\lambda_2\sigma_2^2\Big) \quad > 0$$

Conditions for CSB minimum

$$\mathrm{tr}\left\{\hat{V}^{(2)}\right\} > 0 \qquad \mathrm{Det}\hat{V}^{(2)} > 0$$

but $\mathrm{Det}\Delta > 0,\ \mathrm{tr}\{\Delta\} > 0$ **or** $\mathrm{Det}\Delta < 0$

for the absence of competitive chirally symmetric minimum

Embedding a chemical potential

$$\int dx\ \mu\ \left(\bar{\mathbf{q}}\gamma_0 \boldsymbol{q}(x) - \rho_B \right)$$

Local coupling to quarks

$$\mathcal{L}_{int} = -(\bar{q}_R H_1 q_L + \bar{q}_L H_1^\dagger q_R)$$

Superposition of physical meson states

From a quark model

$$\Delta V_{\rm eff}(\mu) = \frac{\mathcal{N}}{2}\Theta(\mu - |H_1|)\left[\mu|H_1|^2\sqrt{\mu^2 - |H_1|^2} - \frac{2\mu}{3}(\mu^2 - |H_1|^2)^{3/2} - |H_1|^4 \ln\frac{\mu + \sqrt{\mu^2 - |H_1|^2}}{|H_1|}\right]\left(1 + O\left(\frac{\mu^2}{\Lambda^2}\right)\right)$$

$$\mathcal{N} \equiv \frac{N_c N_f}{4\pi^2}$$

Density and Fermi momentum

$$\varrho_B = -\frac{1}{3}\partial_\mu \Delta V_{\rm eff}(\mu) = \frac{N_c N_f}{9\pi^2} p_F^3 = \frac{N_c N_f}{9\pi^2}(\mu^2 - |\langle H_1 \rangle|^2)^{3/2}$$

Dense matter drivers of minimum

Only the first Eq. for stationary points is modified

$$2(\Delta_{11}\sigma_1 + \Delta_{12}\sigma_2) = 4\lambda_1\sigma_1^3 + 3\lambda_5\sigma_1^2\sigma_2 + 2(\lambda_3+\lambda_4)\sigma_1\sigma_2^2 + \lambda_6\sigma_2^3$$
$$+\rho^2\Big(2(\lambda_3-\lambda_4)\sigma_1 + \lambda_6\sigma_2\Big) + 2\mathcal{N}\Theta(\mu-\sigma_1)\left[\mu\sigma_1\sqrt{\mu^2-\sigma_1^2} - \sigma_1^3\ln\frac{\mu+\sqrt{\mu^2-\sigma_1^2}}{\sigma_1}\right]$$

Second variation matrix depends on μ **only in the first element**

$$\begin{aligned}\frac{1}{2}V_{11}^{(2)\sigma} \;&=\; -\Delta_{11} + 6\lambda_1\sigma_1^2 + 3\lambda_5\sigma_1\sigma_2 + (\lambda_3+\lambda_4)\sigma_2^2 + (\lambda_3-\lambda_4)\rho^2 \\ &\quad +\mathcal{N}\Theta(\mu-\sigma_1)\left[\mu\sqrt{\mu^2-\sigma_1^2} - 3\sigma_1^2\ln\frac{\mu+\sqrt{\mu^2-\sigma_1^2}}{\sigma_1}\right]\end{aligned}$$

Necessary conditions to approach to P-breaking phase

$$\partial_\mu\Big[(\lambda_3-\lambda_4)\sigma_1^2+\lambda_6\sigma_1\sigma_2+2\lambda_2\sigma_2^2\Big]<0$$

$$\downarrow$$

$$-\Big(2(\lambda_3-\lambda_4)\sigma_1+\lambda_6\sigma_2\Big)V_{22}^{(2)}+\Big(\lambda_6\sigma_1+4\lambda_2\sigma_2\Big)V_{12}^{(2)}<0$$

One condition for 7 parameters:

P-breaking is not exceptional but rather typical!

P-parity breaking phase

Above a critical point $\mu > \mu_{crit}$

$$(\lambda_3 - \lambda_4)\sigma_1^2 + \lambda_6\sigma_1\sigma_2 + 2\lambda_2\left(\sigma_2^2 + \rho^2\right) = \Delta_{22}$$

From Eqs. for extremum

$$\lambda_5\sigma_1^2 + 4\lambda_4\sigma_1\sigma_2 + \lambda_6\left(\sigma_2^2 + \rho^2\right) = 2\Delta_{12}$$

If $\lambda_2 = 0$ **and/or** $\lambda_6 = 0$ **these Eqs. give a relation between** $\sigma_1 \text{ and } \sigma_2$

Otherwise, for $\lambda_2\lambda_6 \neq 0$

$$\sigma_2 = A\sigma_1 + \frac{B}{\sigma_1} \qquad A \equiv \frac{2\lambda_5\lambda_2 + \lambda_6(\lambda_4 - \lambda_3)}{\lambda_6^2 - 8\lambda_2\lambda_4} \qquad B \equiv \frac{\lambda_6\Delta_{22} - 4\lambda_2\Delta_{12}}{\lambda_6^2 - 8\lambda_2\lambda_4}$$

All these relations don't depend on P-breaking v.e.v. ρ
and on chemical potential ! Robust !

Critical points μ_c when $\rho(\mu_c) = 0$

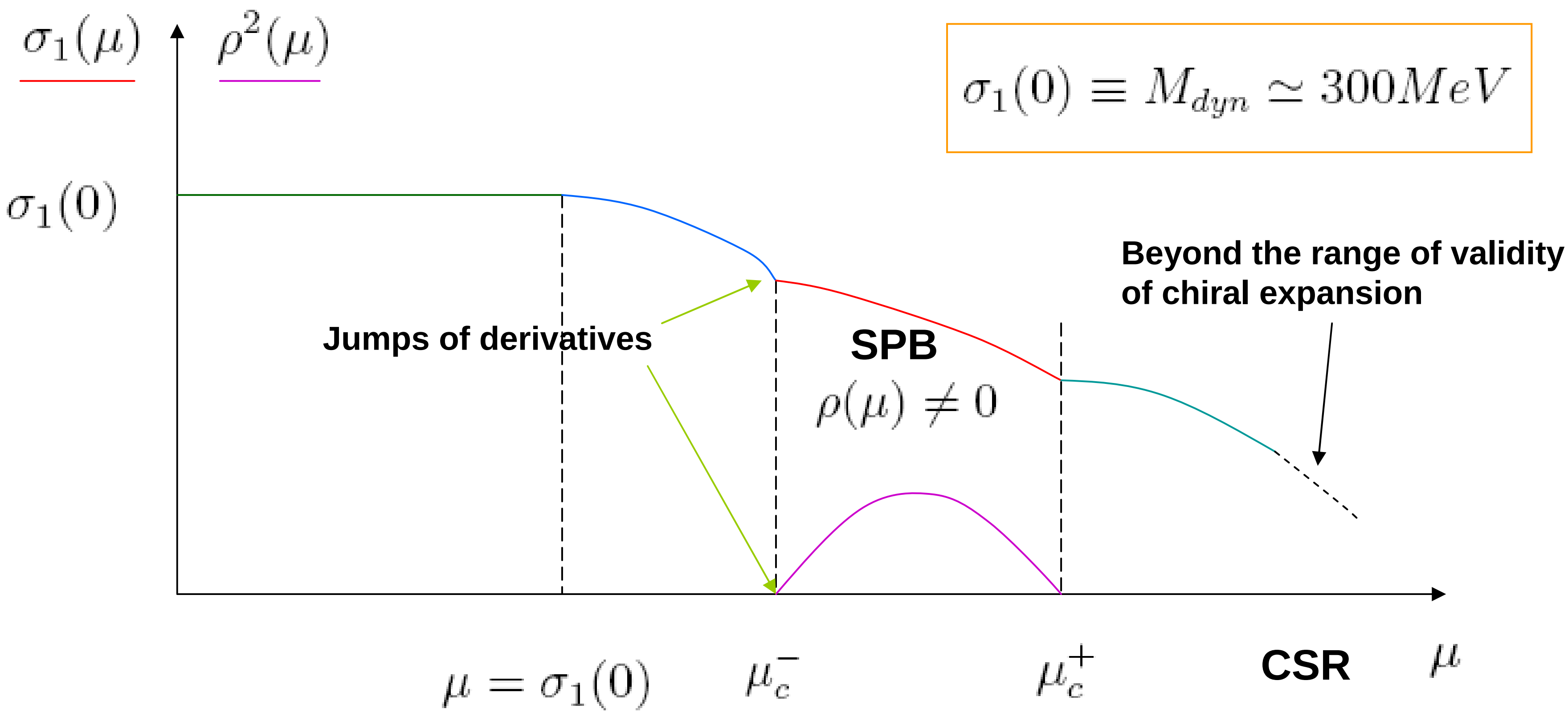
$$(4\lambda_2\Delta_{12} - \lambda_6\Delta_{22})r^2 + (2\lambda_6\Delta_{12} - 4\lambda_4\Delta_{22})r + 2(\lambda_3 - \lambda_4)\Delta_{12} - \lambda_5\Delta_{22} = 0$$

For $r \equiv \frac{\sigma_2}{\sigma_1}$

There are in general two solutions for two $\mu_c^- < \mu_c^+$

But for $4\lambda_2\Delta_{12} = \lambda_6\Delta_{22}$ **only one solution and P-breaking phase may be left via Ist order phase transition**

Spontaneous P-parity breaking (IId order phase transition)



With increasing μ one enters SPB phase and leaves it before (?) encountering any new phase (CSR, CFL …)

Meson spectrum in SPB phase

Neutral pi-prime condensate breaks vector SU(2) to U(1) and two charged pi-prime mesons become massless

$$\frac{1}{2}V_{11}^{(2)\sigma} = 4\lambda_1\sigma_1^2 + 2\lambda_5\sigma_1\sigma_2 + 2\lambda_4\sigma_2^2 - 2\mathcal{N}\sigma_1^2 \ln\frac{\mu+\sqrt{\mu^2-\sigma_1^2}}{\sigma_1}$$

$$V_{12}^{(2)\sigma} = 2\lambda_5\sigma_1^2 + 4\lambda_3\sigma_1\sigma_2 + 2\lambda_6\sigma_2^2$$

$$\frac{1}{2}V_{22}^{(2)\sigma} = 2\lambda_4\sigma_1^2 + 2\lambda_6\sigma_1\sigma_2 + 4\lambda_2\sigma_2^2$$

$$V_{10}^{(2)\sigma\pi} = \Big(4(\lambda_3-\lambda_4)\sigma_1 + 2\lambda_6\sigma_2\Big)\rho$$

$$V_{20}^{(2)\sigma\pi} = \Big(2\lambda_6\sigma_1 + 8\lambda_2\sigma_2\Big)\rho$$

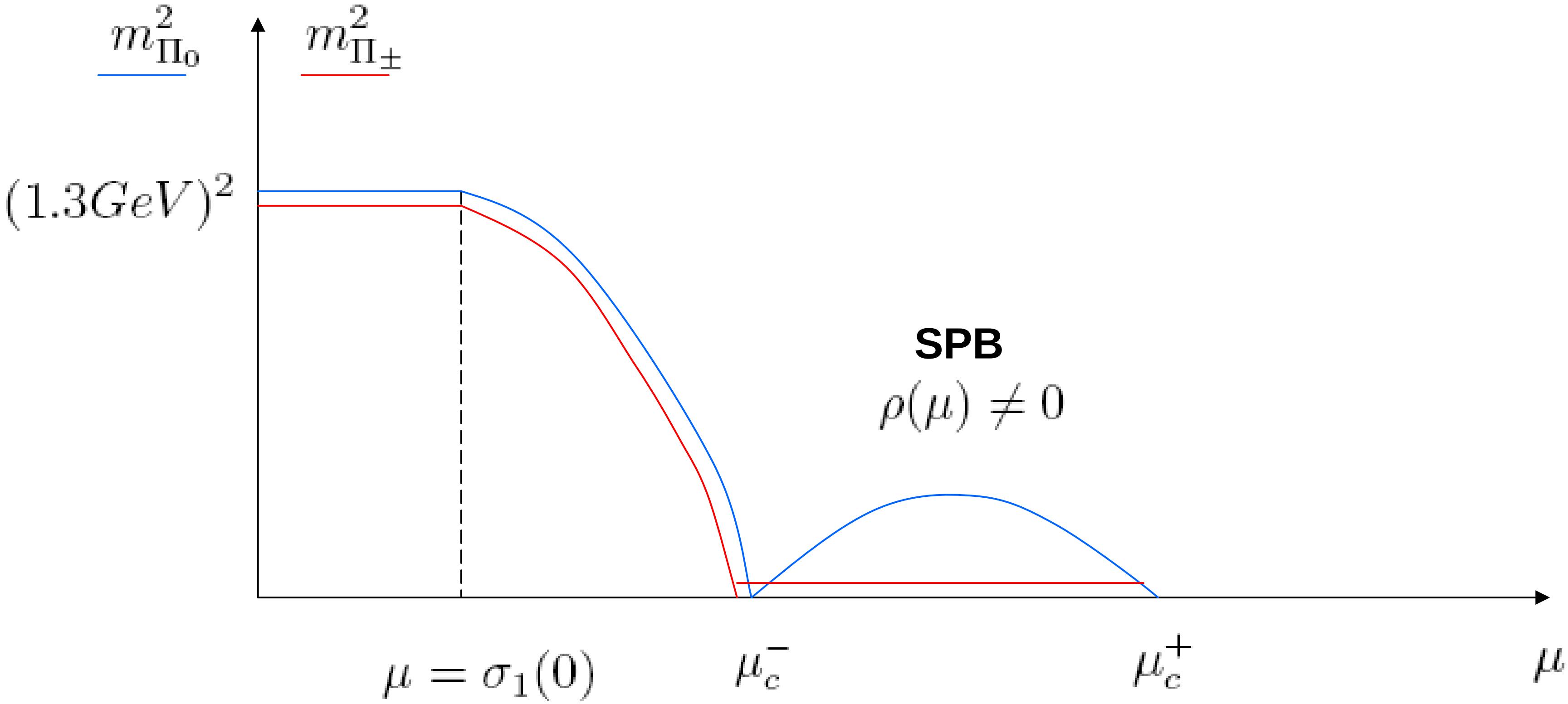
Mixture of massive scalar and neutral pseudoscalar states

$$\frac{1}{2}V_{00}^{(2)\pi} = 4\lambda_2\rho^2$$

$$\boxed{\frac{1}{2}V_{\pm\mp}^{(2)\pi} = 0}$$

Mass spectrum of “pseudoscalar” states

(chiral limit)



In P-breaking phase there are 5 massless pseudoscalar mesons

$\pi_0 \quad \pi_\pm \quad \Pi_\pm$

Kinetic terms

symmetric under $SU_L(2) \times SU_R(2)$

$$\mathcal{L}_{kin} = \frac{1}{4} \sum_{j,k=1}^{2} A_{jk} \mathrm{tr} \left\{ \partial_\mu H_j^\dagger \partial^\mu H_k \right\}$$

Three more real constants A_{jk}
but one, A_{22} **is redundant**

Chirally symmetric parameterization

$$H_1(x) = \sigma_1(x) U(x) = \sigma_1(x) \xi^2(x)$$

$$H_2(x) = \xi(x)(\sigma_2(x) + i\hat{\pi}_2)(x)\xi(x) = \sigma_2(x)U(x) + i\xi(x)\hat{\pi}_2(x)\xi(x)$$

and expansion around a vacuum configuration

$$U = 1 + i\hat{\pi}/F_0 + \cdots \qquad \xi = 1 + i\hat{\pi}/2F_0 + \cdots$$

$$\sigma_j \equiv \bar{\sigma}_j + \Sigma_j \qquad \hat{\pi} = \tau_3 \rho + \hat{\Pi}$$

Kinetic part quadratic in meson fields

$$\mathcal{L}^{(2)}_{kin} = \frac{1}{2}\sum_{j,k=1}^{2} A_{jk}\partial_\mu\Sigma_j\partial^\mu\Sigma_k + \frac{\rho^2}{2F_0^2}A_{22}\partial_\mu\pi^0\partial^\mu\pi^0 - \frac{\rho}{F_0}\sum_{j=1}^{2} A_{j2}\partial_\mu\Sigma_j\partial^\mu\pi^0$$
$$+\sum_{j,k=1}^{2}\left[\frac{1}{2F_0^2}A_{jk}\bar{\sigma}_j\bar{\sigma}_k\partial_\mu\pi^a\partial^\mu\pi^a + \frac{1}{F_0}A_{j2}\bar{\sigma}_j\partial_\mu\pi^a\partial^\mu\Pi^a\right] + \frac{1}{2}A_{22}\partial_\mu\Pi^a\partial^\mu\Pi^a$$

Normalizations and notations

$$F_0^2 = \sum_{j,k=1}^{2} A_{jk}\bar{\sigma}_j\bar{\sigma}_k \simeq (90\mathrm{MeV})^2, \quad \zeta \equiv \frac{1}{F_0}\sum_{j=1}^{2} A_{j2}\bar{\sigma}_j$$

Pion weak decay constant

P-symmetric phase $\rho = 0$

After shift $\tilde{\pi}^a = \pi^a + \zeta \Pi^a$

$$\mathcal{L}^{(2)}_{kin,\pi} = \frac{1}{2}\partial_\mu \tilde{\pi}^a \partial^\mu \tilde{\pi}^a + \frac{1}{2}(A_{22} - \zeta^2)\partial_\mu \Pi^a \partial^\mu \Pi^a, \qquad A_{22} - \zeta^2 = \frac{\bar{\sigma}_1^2 \det A}{F_0^2} > 0,$$

<u>Mass of pi-prime meson(s)</u>

$$m_\Pi^2 = \frac{-\Delta_{22} + (\lambda_3 - \lambda_4)(\bar{\sigma}_1)^2 + \lambda_6 \bar{\sigma}_1 \bar{\sigma}_2 + 2\lambda_2 (\bar{\sigma}_2)^2}{A_{22} - \zeta^2} \simeq 1300\mathrm{MeV}$$

F_0^2 **and** m_Π^2 **represent the major scale normalizations,**

fit of masses of scalar mesons is more subtle

P-breaking phase

resolution of mixing with massless pions is different for neutral and charged ones because vector isospin symmetry is broken

$$\tilde{\pi}^{\pm} = \pi^{\pm} + \zeta\Pi^{\pm}, \qquad \tilde{\pi}^{0} = \pi^{0} + \frac{F_0^2}{F_0^2 + A_{22}\rho^2}\Big(\zeta\Pi^0 - \frac{\rho}{F_0}\sum_{j=1}^{2} A_{j2}\partial_\mu\Sigma_j\Big).$$

Partially diagonalized kinetic term

$$\begin{aligned}\mathcal{L}_{kin}^{(2)} &= \partial_\mu\tilde{\pi}^{\pm}\partial^\mu\tilde{\pi}^{\mp} + (A_{22} - \zeta^2)\partial_\mu\Pi^{\pm}\partial^\mu\Pi^{\mp} + \frac{1}{2}\Big(1 + \frac{A_{22}\rho^2}{F_0^2}\Big)\partial_\mu\tilde{\pi}^0\partial^\mu\tilde{\pi}^0 \\ &+ \frac{1}{2}(A_{22} - \frac{F_0^2}{F_0^2 + A_{22}\rho^2}\zeta^2)\partial_\mu\Pi^0\partial^\mu\Pi^0 + \frac{1}{2}\sum_{j,k=1}^{2}\frac{A_{jk}F_0^2 + \rho^2 \det A\,\delta_{1j}\delta_{1k}}{F_0^2 + A_{22}\rho^2}\partial_\mu\Sigma_j\partial^\mu\Sigma_k \\ &- \frac{F_0\rho}{F_0^2 + A_{22}\rho^2}\zeta\partial_\mu\Pi^0\sum_{j=1}^{2} A_{j2}\partial^\mu\Sigma_j\end{aligned}$$

Isospin breaking $SU_V(2) \to U(1)$

Further diagonalization $\Pi^0 \quad \Sigma_1 \quad \Sigma_2 \quad \Longrightarrow \quad \tilde{\Pi}^0 \quad \tilde{\Sigma}_1 \quad \tilde{\Sigma}_2$

mixes neutral pseudoscalar and scalar states

Therefore genuine mass states don’t possess a definite parity in decays

$$\tilde{\Pi}^0 \quad \tilde{\Sigma}_1 \quad \tilde{\Sigma}_2 \longrightarrow \pi_0\,\pi_0\,\pi_0 \qquad \pi_0\,\pi_+\,\pi_- \qquad \pi_0\,\Pi_+\,\Pi_- \ldots$$

$$\tilde{\Pi}^0 \quad \tilde{\Sigma}_1 \quad \tilde{\Sigma}_2 \longrightarrow \pi_0\,\pi_0 \qquad \pi_+\,\pi_- \qquad \Pi_+\,\Pi_- \ldots$$

Beyond the chiral limit: $\mathbf{m_q \neq 0}$

Two new lowest-dimensional operators

$$\frac{1}{2} m_q d_1 \mathrm{tr}(H_1 + H_1^\dagger) \qquad \frac{1}{2} m_q d_2 \mathrm{tr}(H_2 + H_2^\dagger)$$

The spectrum in dense matter

$\mu = 0$ $\quad \mathbf{m}_\pi^2 \sim \mathbf{m_q}$ $\quad \mathbf{m}_\Pi^2 \gg \mathbf{m}_\pi^2$ **heavy**

↓

$\mu = \mu_c$ $\quad \mathbf{m}_\pi^2 = 0$ $\quad \mathbf{m}_\Pi^2 \sim \mathbf{m_q}$ **very light**

↓

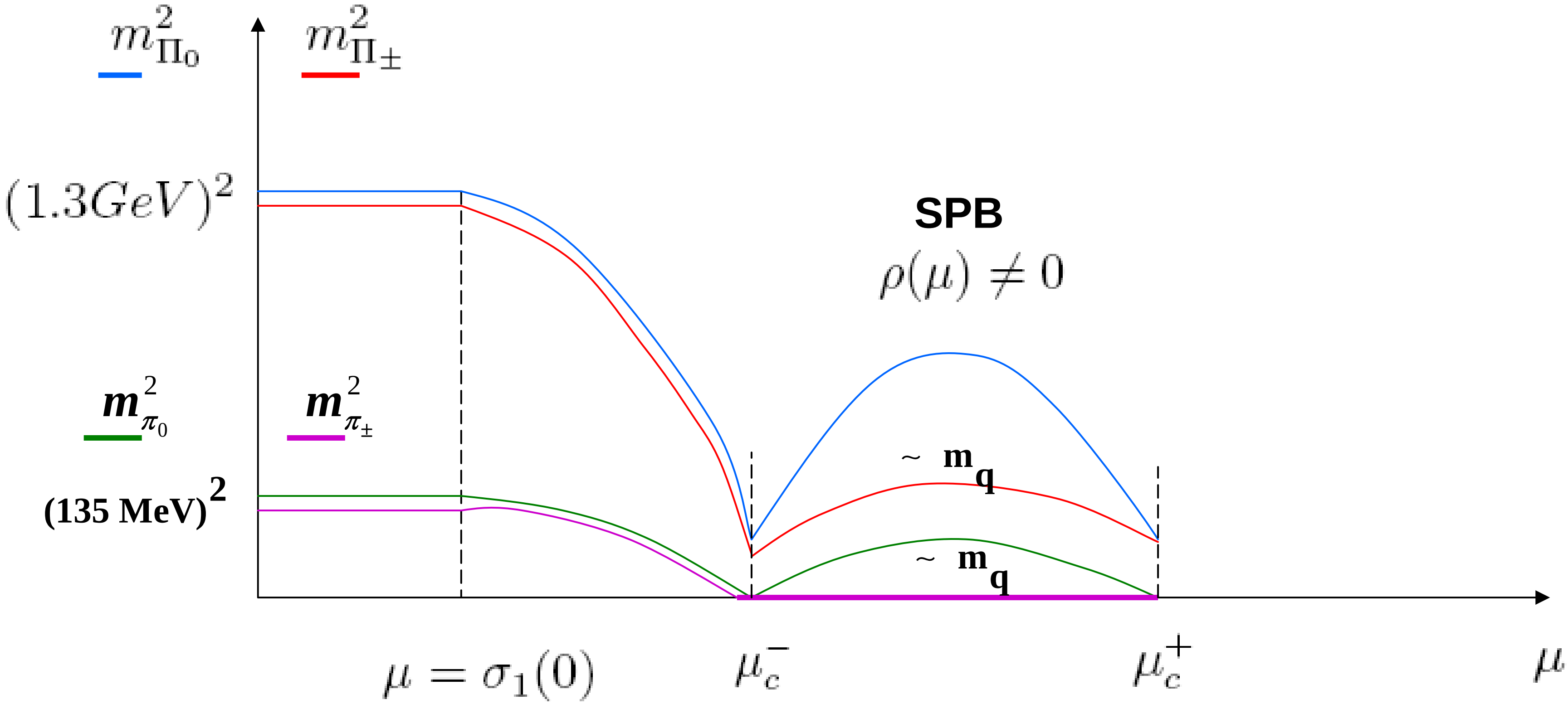
$\mu > \mu_c$ $\quad \mathbf{m}_{\pi_\pm}^2 = 0$ $\quad \mathbf{m}_\Pi^2 > \mathbf{m}_{\pi_0}^2 \sim \mathbf{m_q}$ **light**

Two Goldstone bosons of isospin breaking

Four light massive "pions"

Mass spectrum of “pseudoscalar” states

(massive quarks)



In P-breaking phase there are 2 massless pseudoscalar mesons

$\tilde{\pi}_\pm$

Estimations of coupling constants in Quasilocal Quark Model

$$\mathcal{L}_{\rm QQM} = \bar{q}(i\not{\partial})q + \sum_{k,l=1}^{2} a_{kl}\left[\bar{q} f_k(s) q\, \bar{q} f_l(s) q - \bar{q} f_k(s)\tau^a\gamma_5 q\, \bar{q} f_l(s)\tau^a\gamma_5 q\right]. \qquad (3)$$

Here a_{kl} represents a symmetric matrix of real coupling constants and $f_k(s)$, $s \equiv -\partial^2/\Lambda^2$ are the polynomial form factors specifying the quasilocal (in momentum space) interaction. The form factors are orthogonal on the unit interval and the results of calculations do not depend on a concrete choice of form factors in the large-log approximation. A convenient choice is $f_1(s) = 2 - 3s$, $f_2(s) = -\sqrt{3}s$. The values of couplings λ_i in Eq. (2) are then fixed for $i = 2, \ldots, 6$: $\lambda_2 = \frac{9N_c}{32\pi^2}$, $\lambda_3 = \frac{3N_c}{8\pi^2}$, $\lambda_4 = \frac{3N_c}{16\pi^2}$, $\lambda_5 = -\frac{5\sqrt{3}N_c}{8\pi^2}$, $\lambda_6 = \frac{\sqrt{3}N_c}{8\pi^2}$.

λ_1 is rather arbitrary

P-breaking is possible!

Minimal model admitting SPB

$$\lambda_5 = \lambda_6 = 0 \Rightarrow \Delta_{12} = 0$$ **from consistency**

Such an effective Lagrangian is symmetric under $Z_2 \times Z_2$

$$H_1 \to -H_1 \quad \text{or} \quad H_2 \to -H_2$$

Fit on hadron phenomenology

$$A_{11} = \frac{1}{9} = A_{22}, \quad F_0 = 100 MeV, \quad \sigma_1 = 300 MeV = 3F_0,$$

$$m_{s,1} = 0.7 GeV = 7F_0, \quad m_p = 1.3 GeV = 13 F_0, \quad m_{s,2} = 1.5 GeV = 15 F_0,$$

$$\Delta_{11} \simeq 2.7 F_0^2, \quad \lambda_1 \simeq 0.15, \quad \lambda_4 \simeq 0.35.$$

For $\varrho_{B,crit} \simeq 3\varrho_{B,nuclear}$ → $\mu_c \simeq 4.3 F_0$ ↔ $\sigma_{1,crit} \simeq 1.8 F_0$

$$\lambda_3 \simeq 3.6 \qquad \Delta_{22} \simeq 11 F_0^2$$

$$m_{s,1} \simeq 1.7 F_0, \quad m_{s,2} \simeq 4.5 F_0$$

Where to see and how to check SPB ?

a) **Decays of higher-mass meson resonances (radial excitations) into pions: the same heavy resonance can decay both in two and three pions . Look like the doubling of states of equal masses and different parities!**

b) **At the very point of the P-breaking phase transition one has three massless pion-like state. Below phase transition point one finds an abnormally *light and long-living pseudoscalar resonances*! After phase transition two massless charged pseudoscalars remain as Goldstone bosons enhancing charged pion production .**

c) **One can search for enhancement of long-range correlations in the pseudoscalar channel in lattice simulations or in BSE!! or ERG/FRG!? Hunting for new light pseudoscalars!**

d) F_{Π} **and extended PCAC: it is modified for massless charged pions giving an enhancement of electroweak decays of heavy pions.**

e) **Additional isospin breaking**: $f_{\pi_0} \neq f_{\pi_\pm}$

f) **BSE!!** $\left\langle \bar{\mathbf{q}}\mathbf{q} \ \bar{\mathbf{q}}\gamma_5\tau_0\mathbf{q} \right\rangle \neq 0$

Lattice is "stubborn" (Pauli blocking,complex det) and BSE?? Program for GSI SIS 200 ?

A bit of history:
pion condensation in symmetric nuclear matter $\rho_p = \rho_n$

A. Migdal, 1971

$$\omega^2 = 1 + k^2 - 4\pi n F(k) \quad \longleftarrow \quad \hbar = c = m_\pi = 1$$

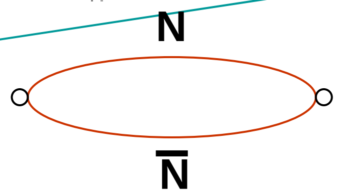
where n is the nucleon density and $F(k)$ is the forward pion-nucleon scattering amplitude *which* for both π^+ and π^- mesons, has the sign corresponding to attraction $(F > 0)$, and therefore at sufficient-density the frequency can vanish, meaning instability of the pion field. However, $F(k)$ is small at small k and instability sets in at $k = k_0$, which corresponds to the minimal value of $k^2 - 4\pi n F(k)$. The instability condition is $\omega^2 = 0$ or

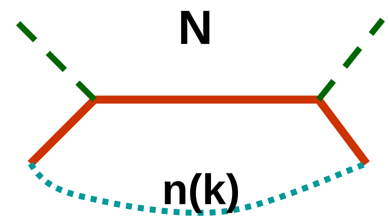
$$1 + k_0^2 = 4\pi n F(k_0)$$

In this approach a pion condensate is spatially inhomogeneous!

A more exact calculation includes the particle-hole excitations of the nuclear medium

$$\omega^2 = 1 + k^2 + \Pi(k, \omega)$$

Polarization operator = 

= 

Off-shell amplitudes!

Polarization operator in the Migdal's approach

$$\omega^2 = \mathbf{k}^2 + \Pi(\omega, \mathbf{k}, \mu)$$

for two pseudoscalar states π, π'

Take masses $m_\pi^2(\mu) = 0$ $\quad m_{\pi'}^2(\mu)\Big|_{\mu \to \mu_{\mathrm{crit}}} \to 0$

and wave function normalizations $Z_\pi \approx Z_{\pi'} \approx 1$

$$\Pi(\omega, \mathbf{k}, \mu) = -\frac{\left(\omega^2 - \mathbf{k}^2\right)^2}{m_{\pi'}^2(\mu) - 2\left(\omega^2 - \mathbf{k}^2\right)}$$

Has a pole in the narrow resonance approach
and changes sign for high energies

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Our inspiration from

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Polarization operator in details

$$\Pi^{(1)\pi^-}(\omega,k) = -2\int\frac{d^3p}{(2\pi)^3}[D_{\pi^-n}(\omega,k)n_n(p) + D_{\pi^-p}(\omega,k)n_p(p)],$$

where D_{π^-n} and D_{π^-p} are the spin-averaged forward scattering amplitudes, $n_n(p)$ and $n_p(p)$ are the occupation functions of the neutrons and the protons respectively,

$$\Pi^{(1)\pi^-}(\omega,k;\rho) = \Pi_N^{(1)\pi^-} + \Pi_\Delta^{(1)\pi^-} + \Pi_D^{(1)\pi^-} + \Pi_\sigma^{(1)\pi^-}$$

Off-shell amplitudes from an effective lagrangian

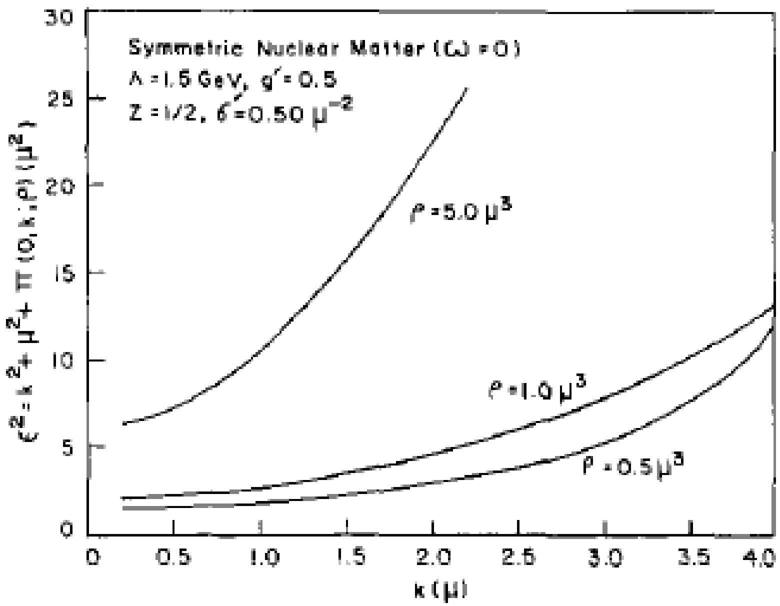
where the subscripts N, Δ, D and σ refer to contributions from nucleon exchange, delta exchange, direct pion–nucleon scattering and the pion–nucleon σ term,

Most pessimistic!

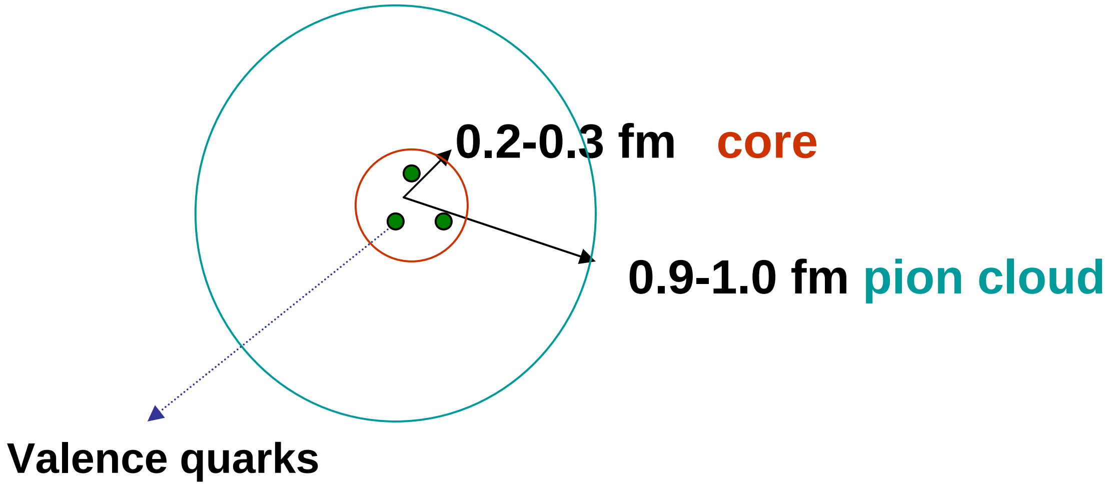
T Shamsunnahar et al.

J. Phys. G: Nucl. Part. Phys. **17** (1991) 887

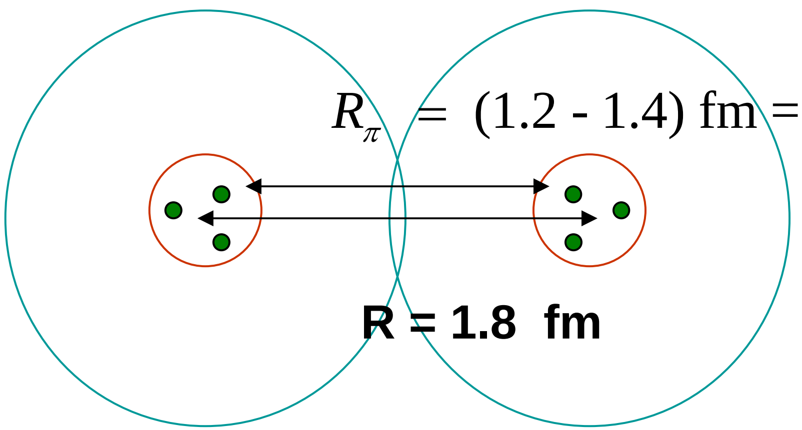
No pion condensate ?



Nucleon in consensus



Normal nuclear matter



$$R_\pi = (1.2 - 1.4)\ \mathrm{fm} = (140\text{-}160\ \mathrm{MeV})^{-1} \approx (m_\pi)^{-1}$$

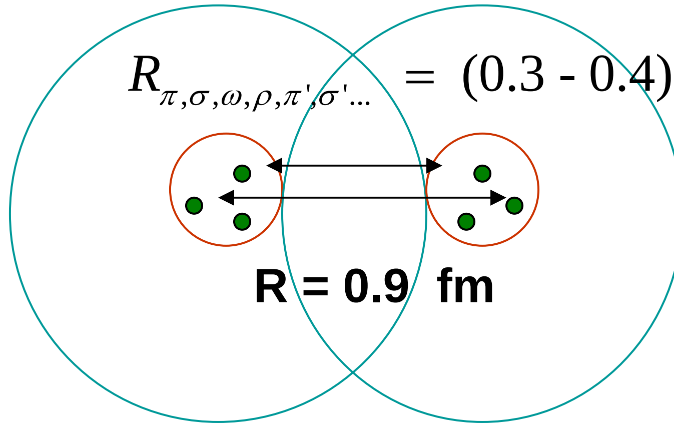
Pion-nucleon realm

Dense nuclear matter

$$R_{\pi,\sigma,\omega,\rho,\pi',\sigma'...} = (0.3 - 0.4)\ \text{fm} = (500\text{-}700\ \text{MeV})^{-1}$$

$$\varrho_B = 8\varrho_N \simeq (0.9fm)^{-3}$$

Neutron stars

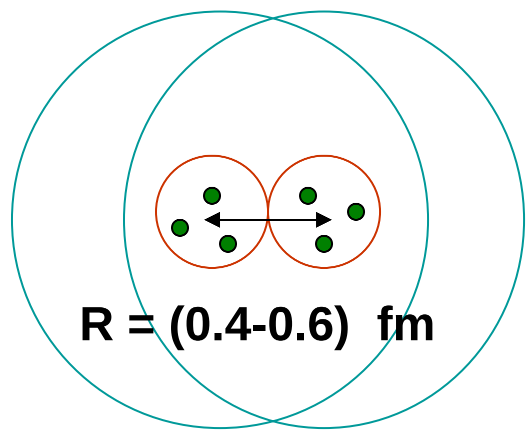


Higher-mass mesons +
undressed nucleons
⟷ dressed quarks

Superdense nuclear matter

$$\varrho_B = (25 \div 100)\varrho_N$$
$$\simeq (0.4 \div 0.6fm)^{-3}$$

Quark stars?



Quark percolation
2SC, CFL …

It represents a consensus of several models:
nuclear potentials, meson-nucleon effective Lagrangians,
extended Skyrme models, chiral bag models …
still the NJL ones give larger core sizes due to lack of confinement