Measuring Landau Gauge Gluon and Ghost Infrared Exponents from Lattice QCD

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Outline

- Introduction and Motivation
- Lattice setup
- New method for extracting infrared exponents
  Ratios between consecutive momenta
  - gluon propagator
  - ghost propagator
- Running coupling
- Conclusions

Introduction and motivation

- Study of IR limit of QCD can be useful for the understanding of confinement mechanism.
  - Kugo-Ojima confinement criterion $\frac{1}{G(0)} = 1 + u = 0$
  - Zwanziger horizon condition $D(0) = 0$
- This limit requires nonperturbative methods:
  - Dyson-Schwinger equations;
  - Lattice QCD.

	Good features	Bad features
DSE	analitical solution in the IR	truncation of infinite tower of equations
Lattice	include all non-perturbative physics	finite volume and finite lattice spacing

Infrared exponent κ

- DSE Infrared analysis

$$Z_{gluon}(p^2) \sim (p^2)^{2\kappa}\,,\; Z_{ghost}(p^2) \sim (p^2)^{-\kappa}$$

$$\kappa = 0.595, \alpha(0) = 2.972$$

[Lerche, von Smekal, Phys. Rev. **D65**(2002)125006]

- Flow equation

$$0.52 \leq \kappa \leq 0.595$$

[Pawlowski *et al*, Phys. Rev. Lett. **93**(2004)152002 ; Fischer, Gies, JHEP 0410 (2004) 048]

- Time independent stochastic quantisation

$$\kappa = 0.52145$$

[Zwanziger, Phys. Rev. **D65**(2002)094039,**D67**(2003)105001]

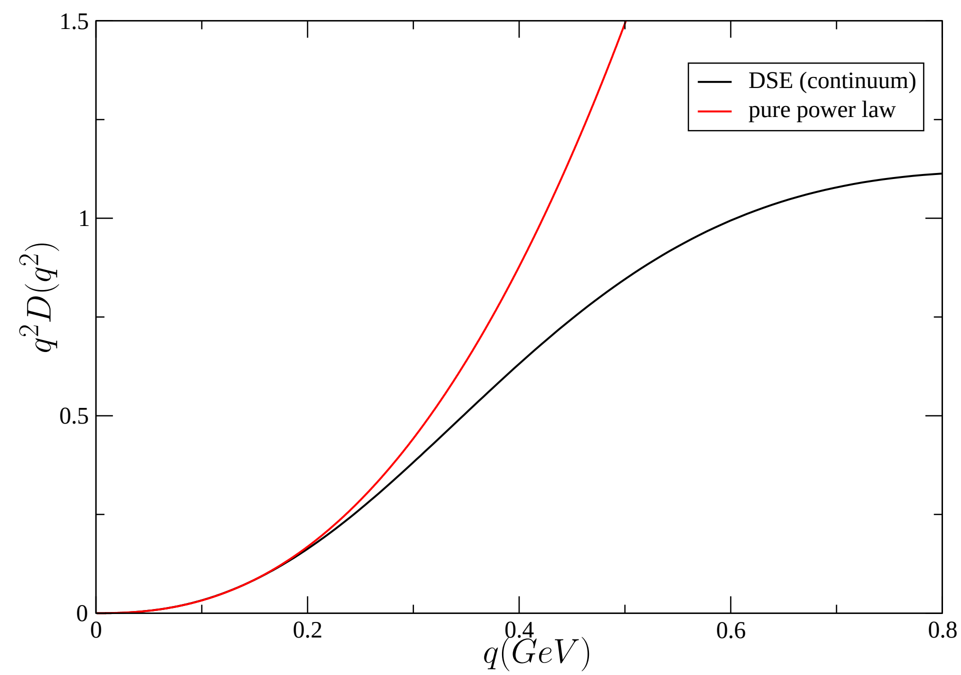
Infrared exponent κ

As an infrared analytical solution of DSE, the pure power law $Z(q^2) \sim (q^2)^{2\kappa}$ is valid only for very low momenta.

[From DSE solution of Alkofer *et al*: $q < 200 MeV$]

On the lattice:
($\beta = 6.0$)

$V = L^4$	$q_{min}(MeV)$
32^4	381
48^4	254
64^4	191
128^4	95
256^4	48



In order to try to measure κ and study IR properties of QCD, we will consider large SU(3) 4D asymmetric lattices ($L_s^3 \times L_t$, with $L_t \gg L_s$)

Lattice setup

- pure gauge, Wilson action SU(3) configurations
- $\beta = 6.0$ ($a^{-1} = 1.943 GeV \Longrightarrow a = 0.106 fm$)
- generated with MILC code [http://physics.indiana.edu/ sg/milc.html]
- combinations of over-relaxation (OVR) and Cabibbo-Mariani (HB) updates

Lattice	Update	therm.	Sep.	Conf
$8^3 \times 256$	7OVR+4HB	1500	1000	80
$10^3 \times 256$	7OVR+4HB	1500	1000	80
$12^3 \times 256$	7OVR+4HB	1500	1000	80
$14^3 \times 256$	7OVR+4HB	3000	1000	128
$16^3 \times 256$	7OVR+4HB	3000	1500	155
$18^3 \times 256$	7OVR+4HB	2000	1000	150
$16^3 \times 128$	7OVR+2HB	3000	3000	164

- L_t large allow access to deep IR region [$L_t = 256 \Rightarrow p_{min} = 48 MeV$]
- Finite volume effects motivated by small L_s

Lattice setup

- Gauge fixing to Landau gauge $\partial_\mu A_\mu = 0$
  - Steepest descent (SD) with Fourier acceleration

    C.T.Davies *et al*, Phys. Rev. **D37**(1988)1581

$$\theta < \ 10^{-15}$$

- Continuum / Lattice momentum

$$q_\mu \ = \ \frac{2}{a} \sin \frac{\hat{q}_\mu a}{2}$$

- Bare gluon propagator

$$D^{ab}_{\mu\nu}(\hat{q}) \ = \ \frac{1}{V} \langle A^a_\mu(\hat{q}) \, A^b_\mu(-\hat{q}) \rangle \ = \ \delta^{ab} \left(\delta_{\mu\nu} \ - \ \frac{q_\mu q_\nu}{q^2} \right) \ D \left(q^2 \right)$$

- Gluon dressing function

$$Z(q^2) = q^2 D(q^2)$$

Ghost propagator

- ghost propagator: inverse of Faddeev-Popov matrix
- FP matrix: second variation $F_U[g]$ in order to infinitesimal gauge transformations

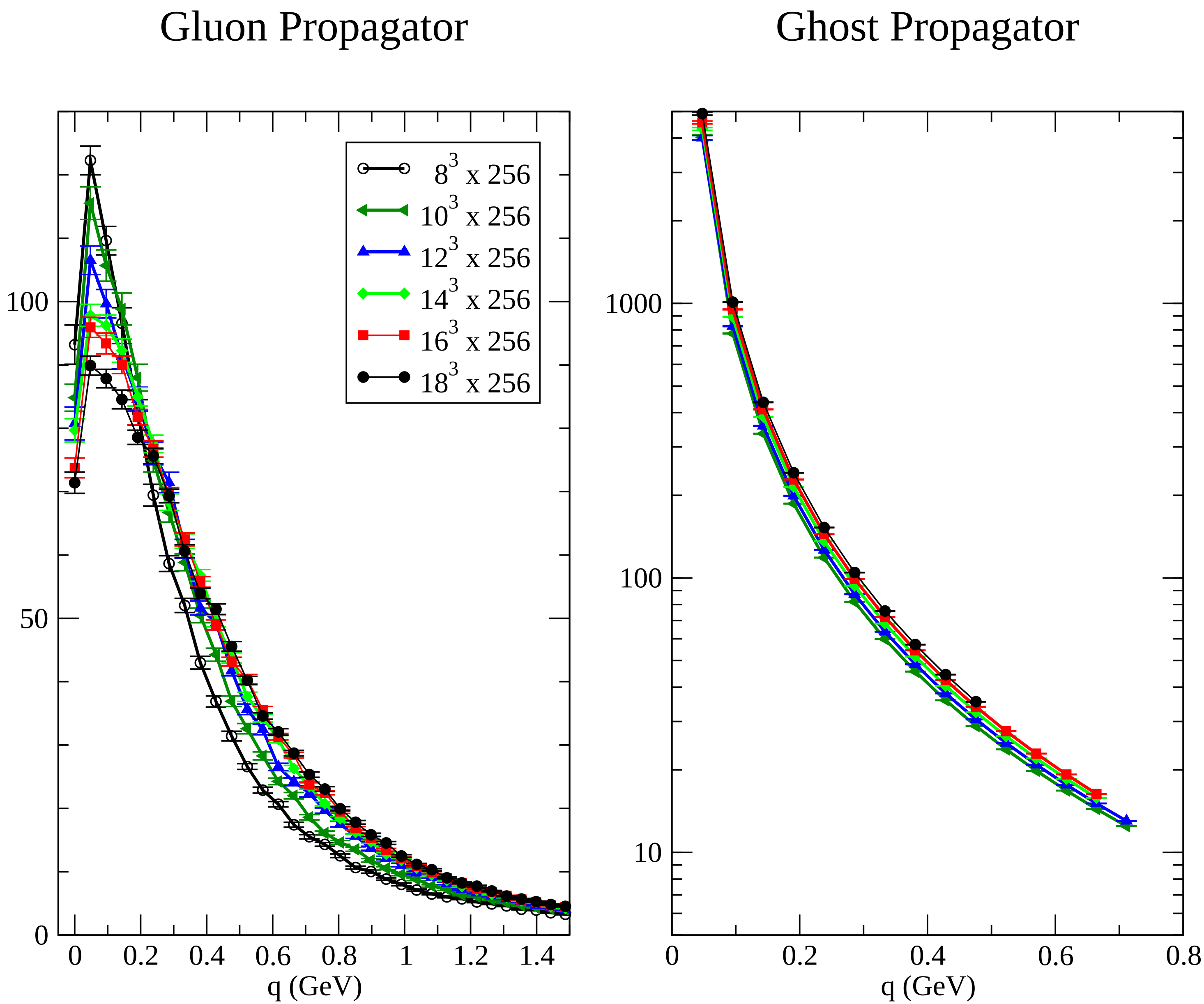
$$\frac{\partial^2 F}{\partial \tau^2} \sim \frac{1}{2}(\omega, M(U)\omega)$$

- propagator in momentum space:

$$G^{ab}(k) = \frac{1}{V} \left\langle \sum_{x,y} (M^{-1})^{ab}_{xy} e^{ik\cdot(x-y)} \right\rangle = \delta^{ab} G(k)$$

$$G(k) = \frac{1}{N_c^2 - 1} \sum_a G^{aa}(k)$$

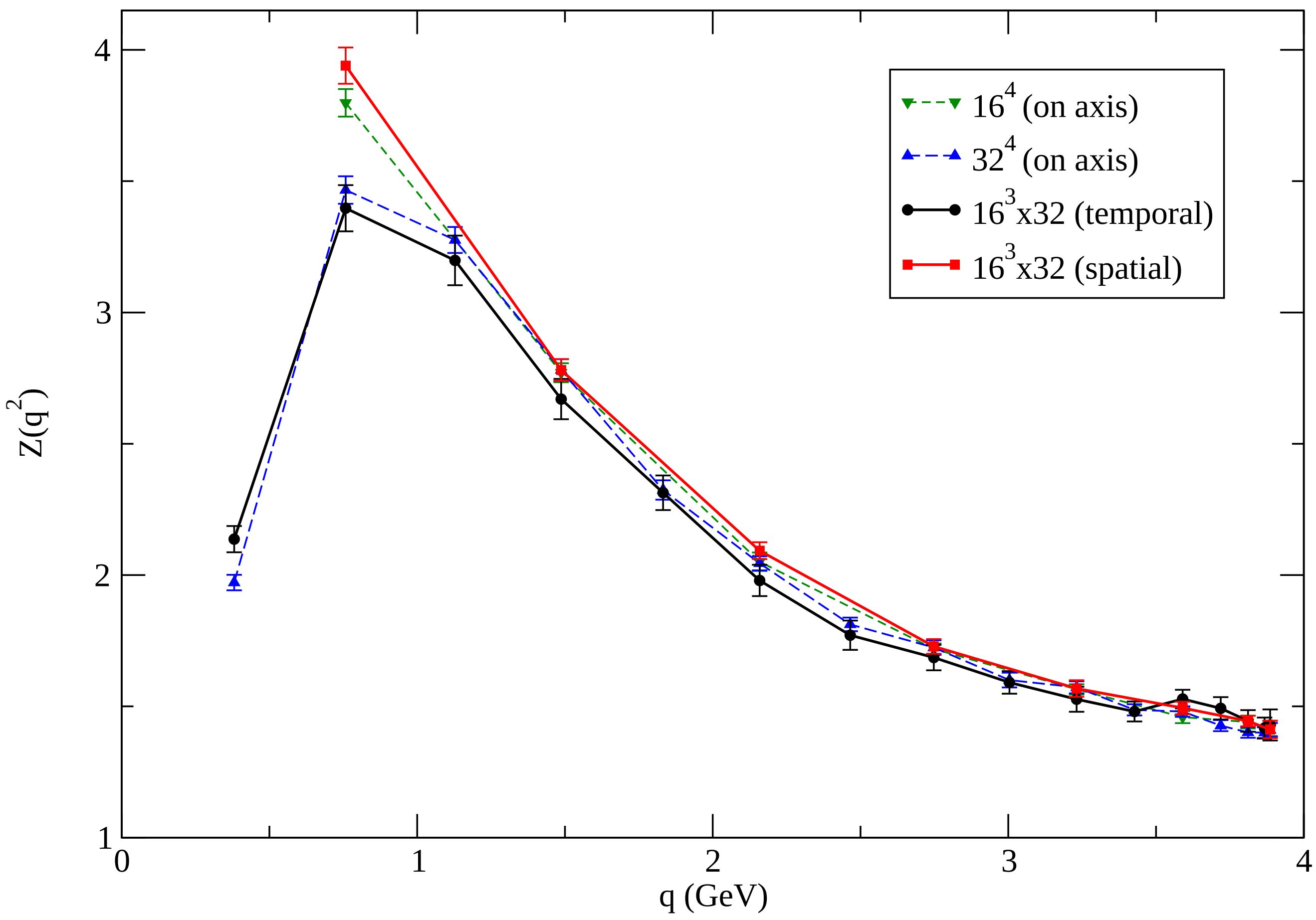
Gluon and ghost propagators



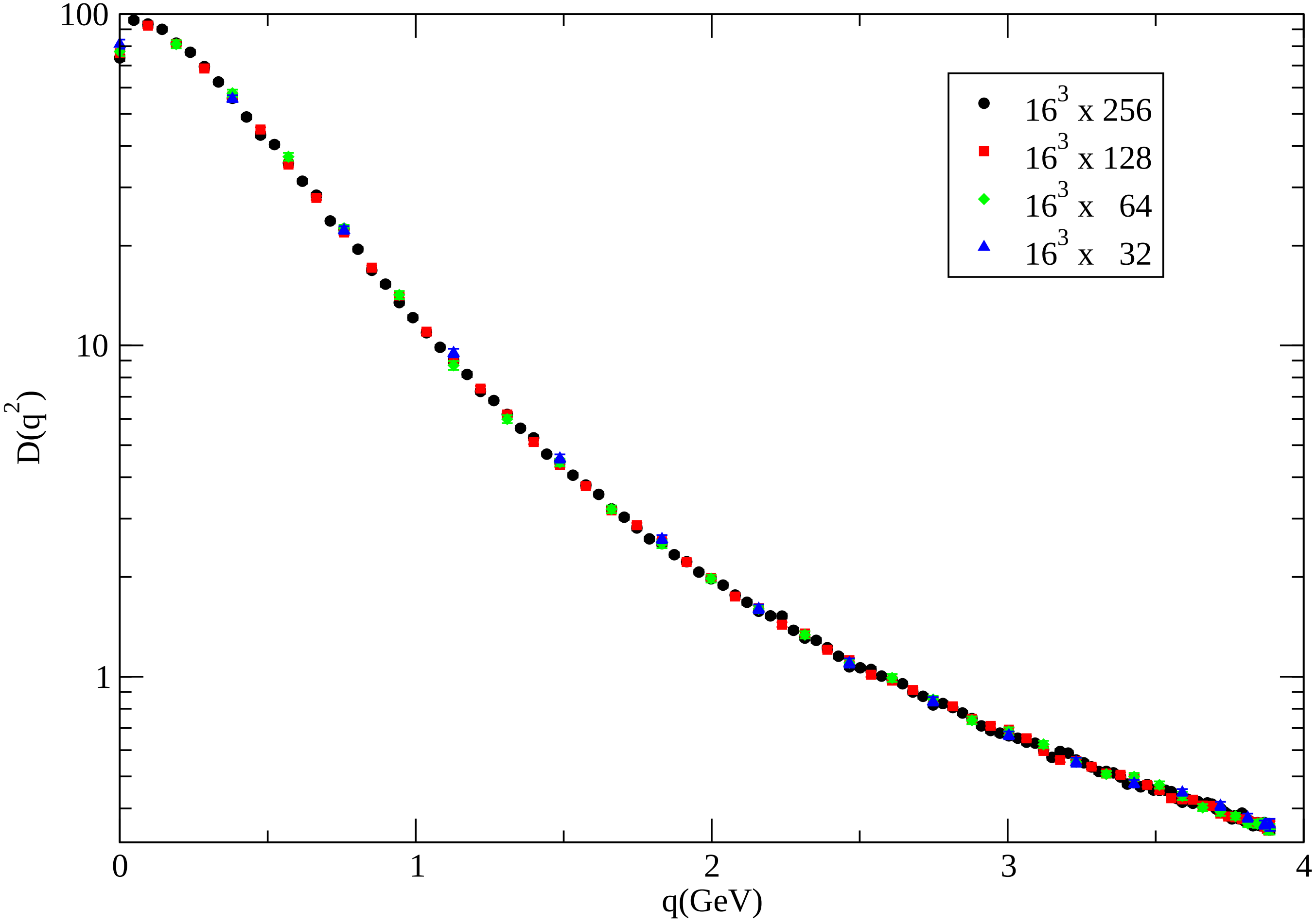
Lattice effects in gluon propagator

Analysis under way: various lattice volumes and shapes

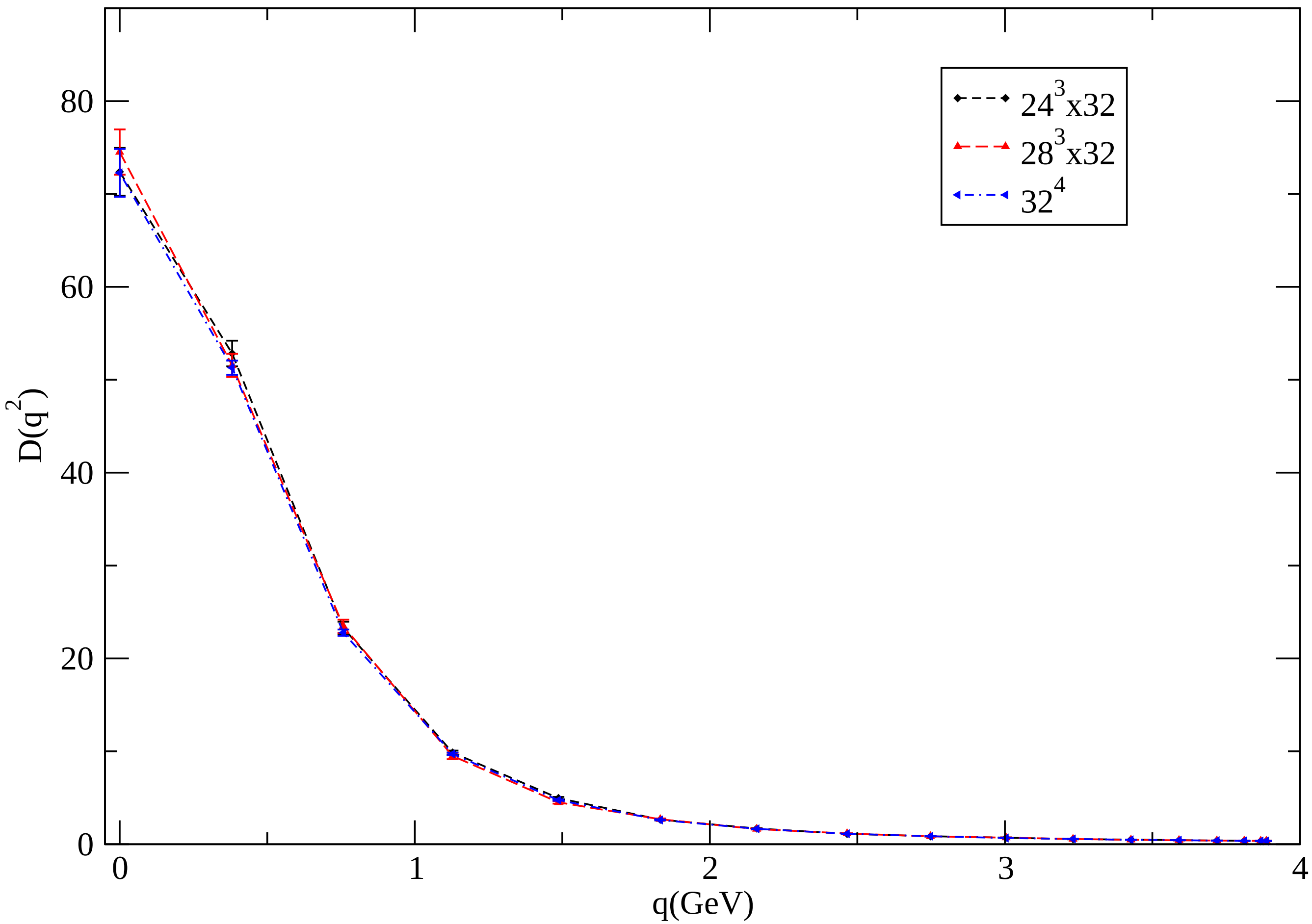
Preliminary results



Gluon propagator computed with $L_s = 16$



Gluon propagator computed with $L_t = 32$



Ratios between consecutive momenta

- On the lattice, ratios help to suppress systematic errors
- Example: a quantity A, dependent of x;
  supposing systematic errors given by $1 + \delta(x)$;
  supposing also $x' \sim x$, and $\delta \ll 1$

$$
\begin{aligned}
\frac{A(x')(1+\delta')}{A(x)(1+\delta)} &\simeq \frac{A(x')}{A(x)}(1+\delta')(1-\delta) \\
&\sim \frac{A(x')}{A(x)}(1-\delta^2)
\end{aligned}
$$

Error of 2^{nd} order on the ratio!!!

Ratios: gluon propagator

O. Oliveira, P.J. Silva, arXiv:0705.0964[hep-lat]

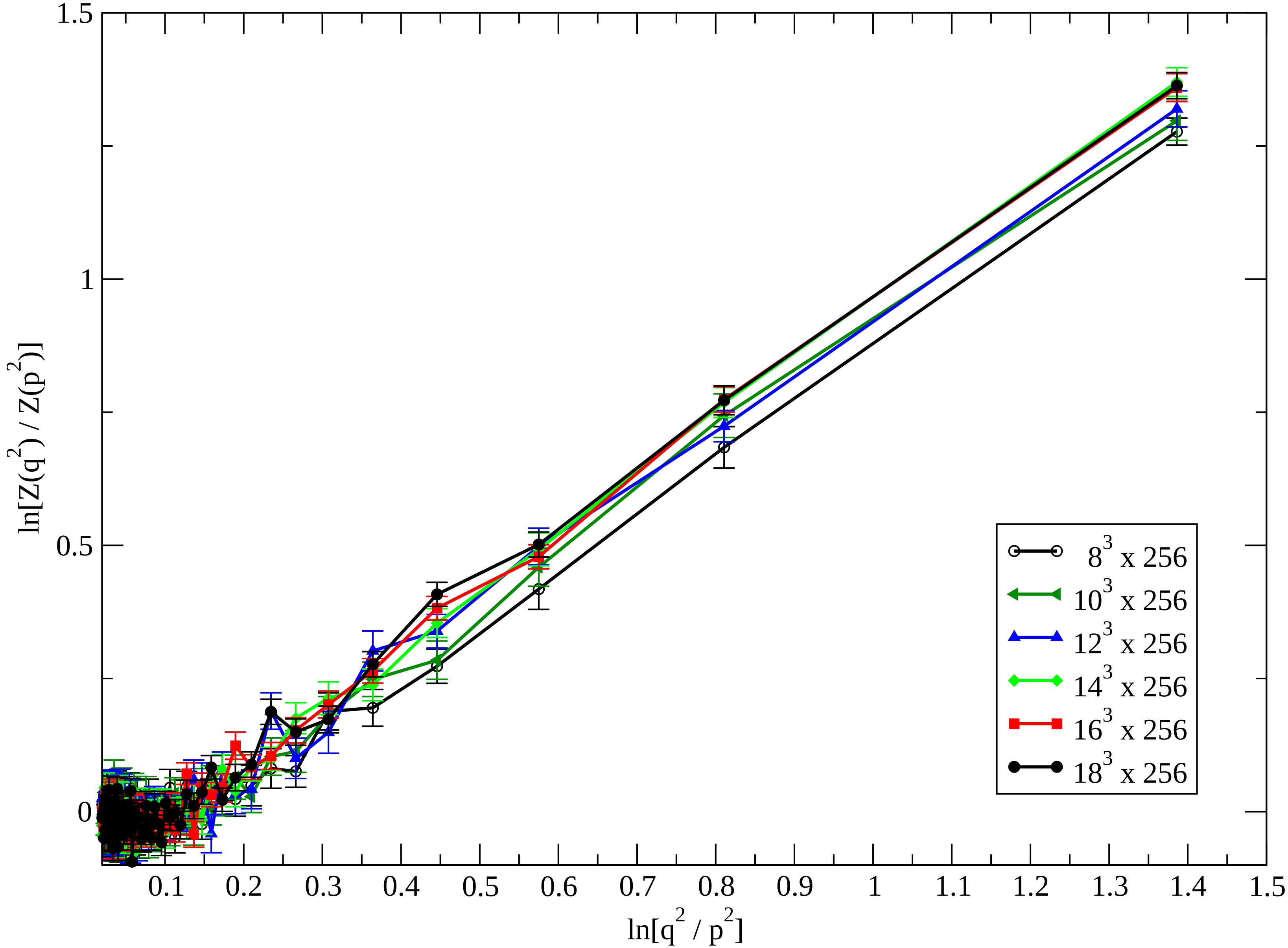
- continuum: $Z(q^2) = (q^2)^{2\kappa}$
  finite lattice: $Z_{latt}(q^2) = (q^2)^{2\kappa}\Delta(q)$
- ratios of dressing function between consecutive temporal momenta:

$$q[n] = q_4[n] = \frac{2}{a}\,\sin\left(\frac{\pi n}{T}\right), \qquad n \;=\; 0,\,1,\,\ldots\,\frac{T}{2}$$

$$\ln\left[\frac{Z_{latt}(q^2[n+1])}{Z_{latt}(q^2[n])}\right] \;=\; 2\kappa\ln\left[\frac{q^2[n+1]}{q^2[n]}\right] \;+\; C(q)$$

- $C(q) = C$ constant!!!

Ratios: gluon propagator



Linear fit to gluon ratios

L		191 MeV	238 MeV	286 MeV	333 MeV	381 MeV
8	κ	$0.526(27)$	$0.531(19)$	$0.531(13)$	$0.522(16)$	$0.527(12)$
	C	$-0.179(54)$	$-0.194(34)$	$-0.193(19)$	$-0.171(28)$	$-0.184(18)$
	$\chi^2/d.o.f.$	0.12	0.11	0.08	0.48	0.54
10	κ	$0.511(35)$	$0.531(25)$	$0.525(21)$	$0.523(17)$	$0.527(16)$
	$\chi^2/d.o.f.$	0.69	0.98	0.74	0.56	0.50
12	κ	$0.509(31)$	$0.517(21)$	$0.508(18)$	$0.521(18)$	$0.530(14)$
	$\chi^2/d.o.f.$	0.11	0.16	0.33	0.84	1.03
14	κ	$0.536(24)$	$0.540(19)$	$0.548(16)$	$0.545(12)$	$0.542(11)$
	C	$-0.114(44)$	$-0.123(30)$	$-0.140(21)$	$-0.134(15)$	$-0.127(12)$
	$\chi^2/d.o.f.$	0.33	0.20	0.39	0.34	0.34
16	κ	$0.539(22)$	$0.528(17)$	$0.534(12)$	$0.536(12)$	$0.539(11)$
	$\chi^2/d.o.f.$	1.77	1.24	0.96	0.78	0.68
18	κ	$0.529(20)$	$0.516(16)$	$0.523(14)$	$0.536(11)$	$0.5398(95)$
	C	$-0.099(36)$	$-0.068(25)$	$-0.085(19)$	$-0.111(14)$	$-0.119(13)$
	$\chi^2/d.o.f.$	0.39	0.77	0.85	1.79	1.58

Understanding finite volume effects

$$\left.\begin{array}{rcr} Z_{latt}(q^2) & = & (q^2)^{2\kappa}\Delta(q) \\ \ln\left[\frac{Z_{latt}(q^2[n+1])}{Z_{latt}(q^2[n])}\right] & = & 2\kappa\ln\left[\frac{q^2[n+1]}{q^2[n]}\right] + C \end{array}\right\} \qquad \Delta(q[n+1]) = \Delta(q[n])e^C$$

$$\frac{d\Delta(q)}{dq} \sim \frac{\Delta(q[n+1]) - \Delta(q[n])}{q[n+1]-q[n]} \sim \Delta(q)\,\frac{e^C - 1}{\frac{2\pi}{aT}} = \Delta(q)\,A$$

$$\Delta(q) = \Delta_0\, e^{Aq}$$

Exponential correction to $Z(q^2)$

$$Z_{latt}(q^2) = \omega\left(q^2\right)^{2\kappa} e^{Aq}$$

Finite volume effects parametrized by A.

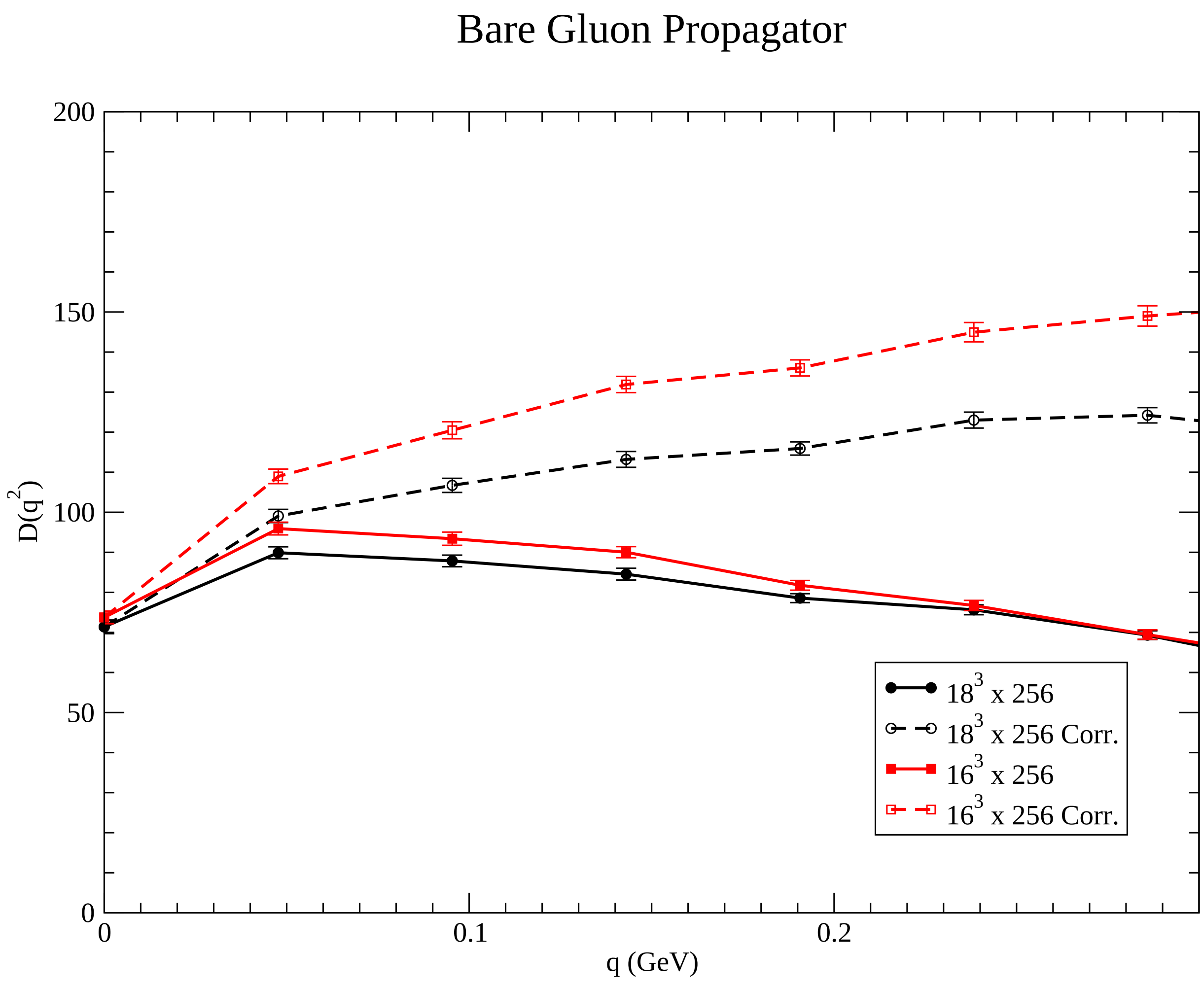
Exponential correction to the pure power law

L		191 MeV	238 MeV	286 MeV	333 MeV	381 MeV
8	κ	$0.526(26)$	$0.533(19)$	$0.534(11)$	$0.523(10)$	$0.524(9)$
	$A(GeV^{-1})$	-3.75 ± 1.1	$-4.06(68)$	$-4.11(34)$	$-3.69(28)$	$-3.73(23)$
	$\chi^2/d.o.f.$	0.09	0.12	0.08	0.62	0.51
10	κ	$0.511(27)$	$0.536(22)$	$0.534(17)$	$0.531(14)$	$0.534(13)$
	$\chi^2/d.o.f.$	0.53	1.08	0.73	0.58	0.49
12	κ	$0.508(31)$	$0.515(22)$	$0.507(15)$	$0.520(12)$	$0.537(9)$
	$\chi^2/d.o.f.$	0.07	0.12	0.24	0.84	1.94
14	κ	$0.538(23)$	$0.542(18)$	$0.552(14)$	$0.551(11)$	$0.546(9)$
	$A(GeV^{-1})$	$-2.42(87)$	$-2.62(59)$	$-3.00(41)$	$-2.96(29)$	$-2.80(21)$
	$\chi^2/d.o.f.$	0.24	0.17	0.47	0.36	0.45
16	κ	$0.541(22)$	$0.532(16)$	$0.535(10)$	$0.539(9)$	$0.543(8)$
	$\chi^2/d.o.f.$	1.15	0.78	0.55	0.50	0.54
18	κ	$0.529(20)$	$0.516(15)$	$0.523(12)$	$0.539(9)$	$0.550(8)$
	$A(GeV^{-1})$	$-2.05(79)$	$-1.50(51)$	$-1.75(33)$	$-2.31(24)$	$-2.66(20)$
	$\chi^2/d.o.f.$	0.28	0.59	0.54	2.14	2.71

Gluon ratios

- **Ratios**
  $\kappa \in [0.508,\ 0.548]$
  $\langle\kappa\rangle\ =\ 0.529(8)$

- **Modelling**
  $\kappa \in [0.507,\ 0.552]$
  $\langle\kappa\rangle\ =\ 0.531(7)$

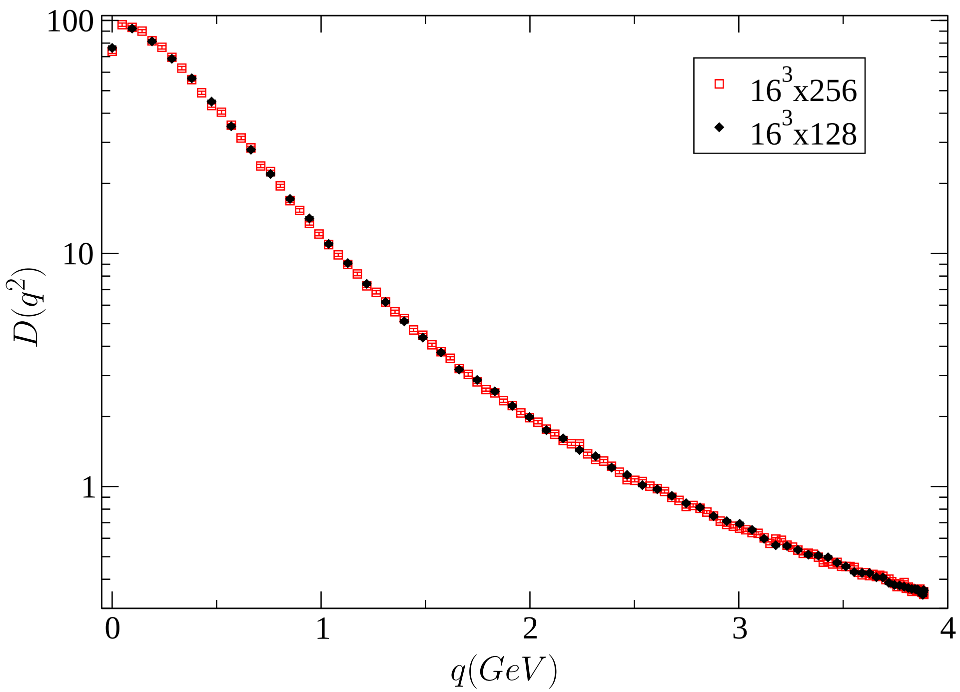


$L_s = 16$ lattices

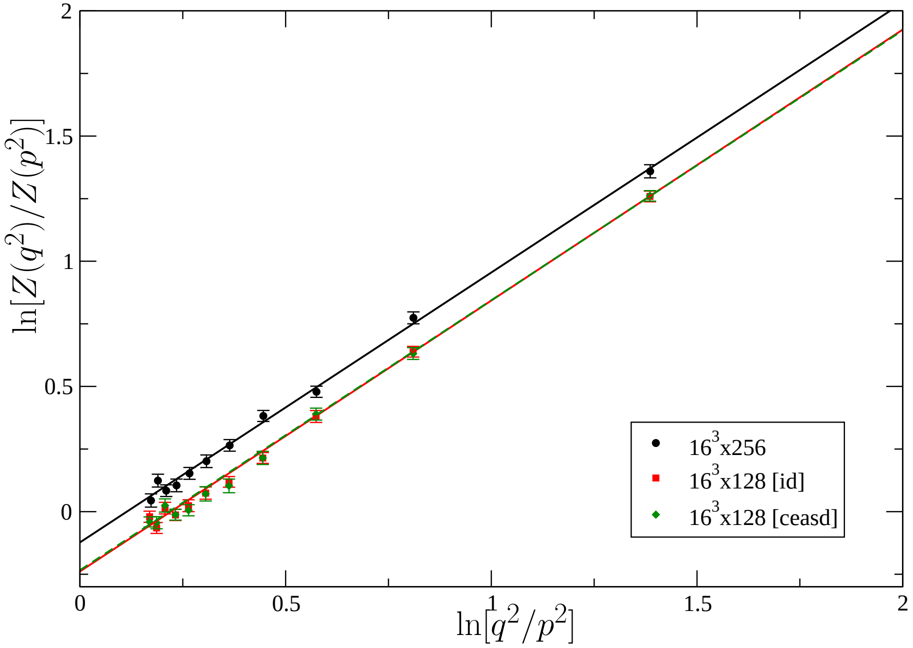
$16^3 \times T$	Ratios			Modelling		
381 MeV	κ	C	χ^2/dof	κ	$A(GeV^{-1})$	χ^2/dof
$T = 256$	$0.539(11)$	$-0.123(12)$	0.68	$0.543(8)$	$-2.66(18)$	0.54
$T = 128$ [ID]	$0.541(19)$	$-0.239(38)$	0.01	$0.542(20)$	$-2.56(39)$	0.01
$T = 128$ [CEASD]	$0.539(19)$	$-0.234(36)$	0.15	$0.539(18)$	$-2.47(36)$	0.10

$$A = \frac{e^C - 1}{\frac{2\pi}{aT}} \sim C\frac{aT}{2\pi}$$

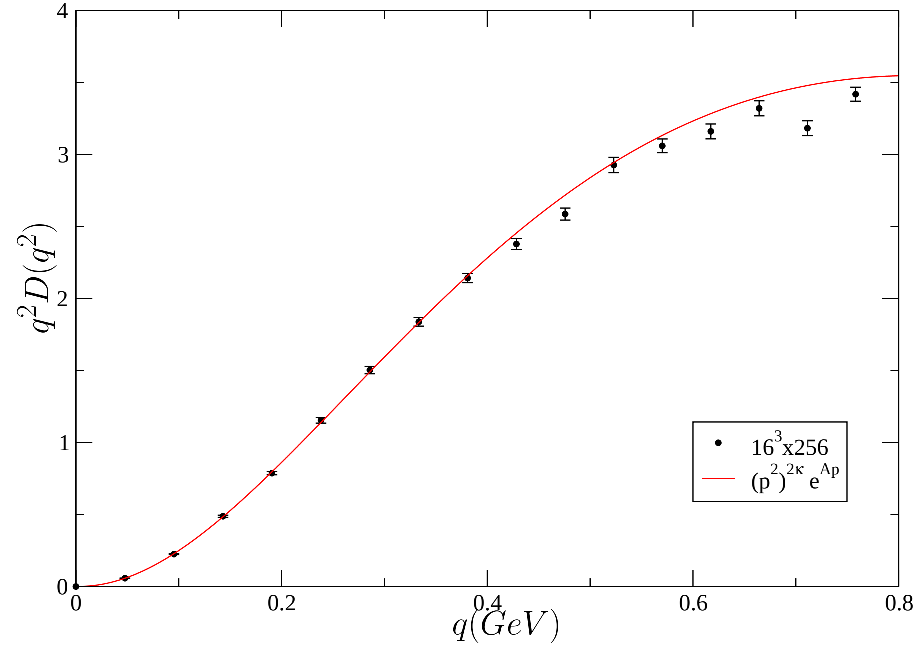
Lattice data for $16^3 \times 256$ and $16^3 \times 128$ compatible within errors $\Rightarrow$ same A $\Rightarrow$ $C_{T=128} \simeq 2 \times C_{T=256}$



Matching fits with lattice data

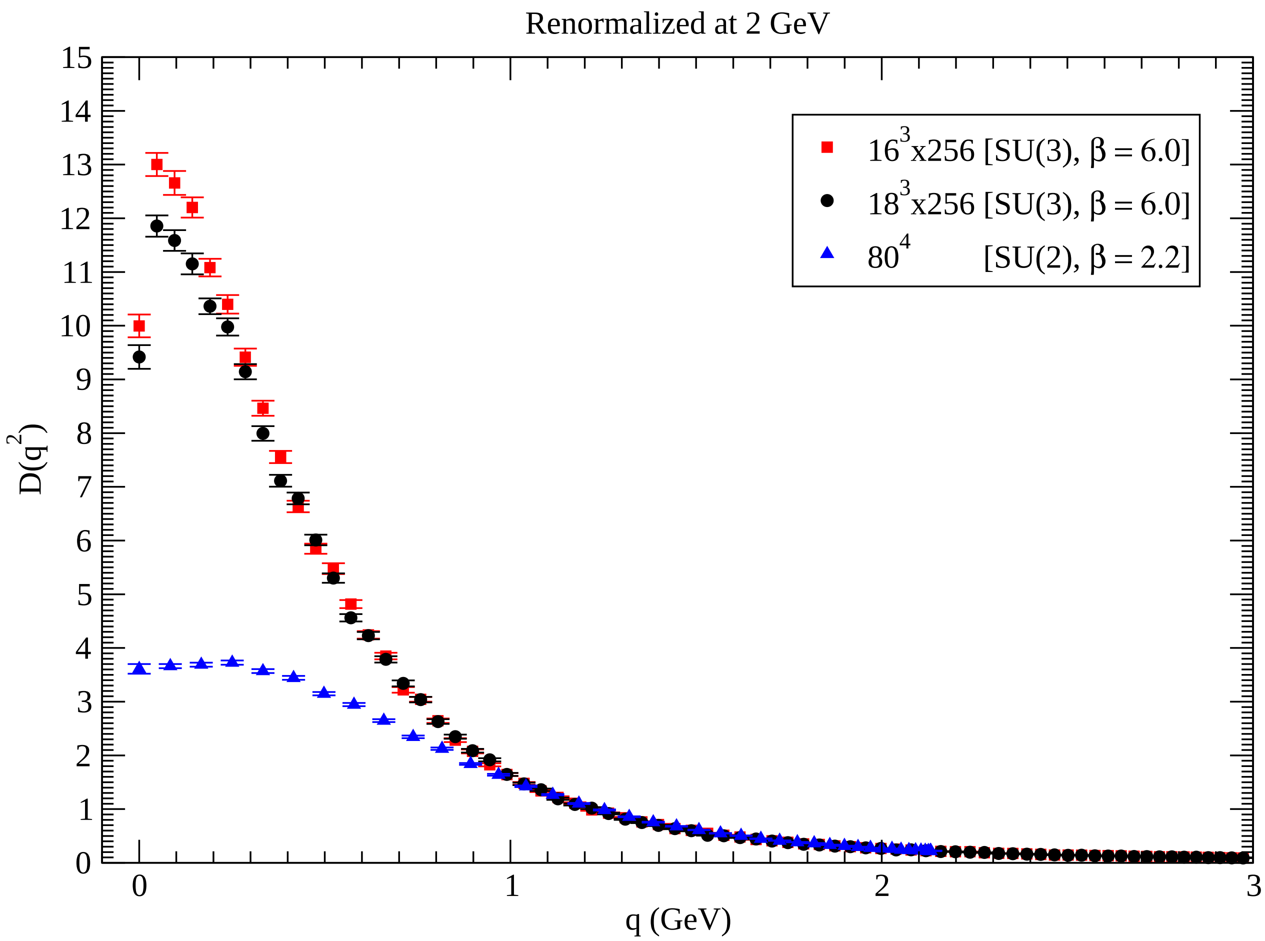


(a) Ratios



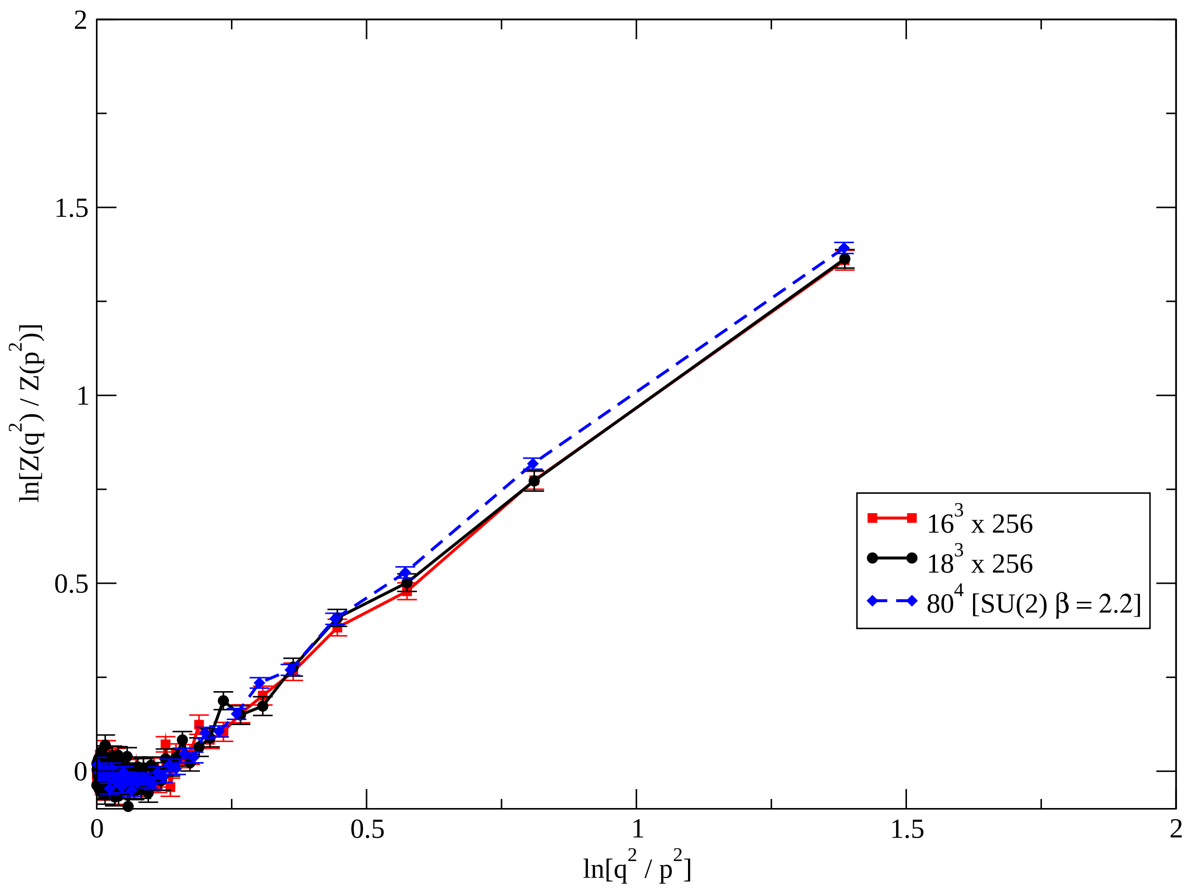
(b) Modelling

Comparing with results from other groups



Results for 80^4, SU(2), $\beta = 2.2$ [Cucchieri and Mendes]

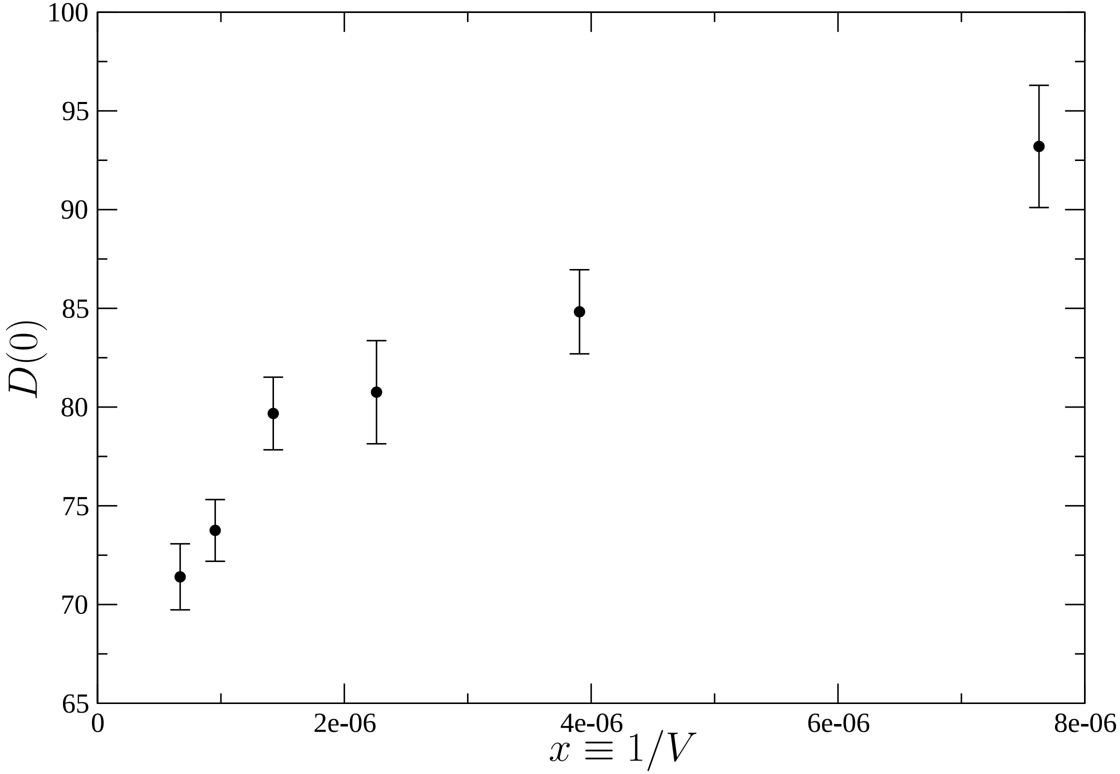
Comparing with results from other groups



- 80^4, SU(2):
  $\kappa \sim 0.53$
  $C \sim -0.06$
- $L_s^3 \times 256$, SU(3):
  $\kappa \sim 0.53$
  $-0.18 < C < -0.09$

Very good agreement

What about D(0)?



Extrapolations $V \to \infty$:

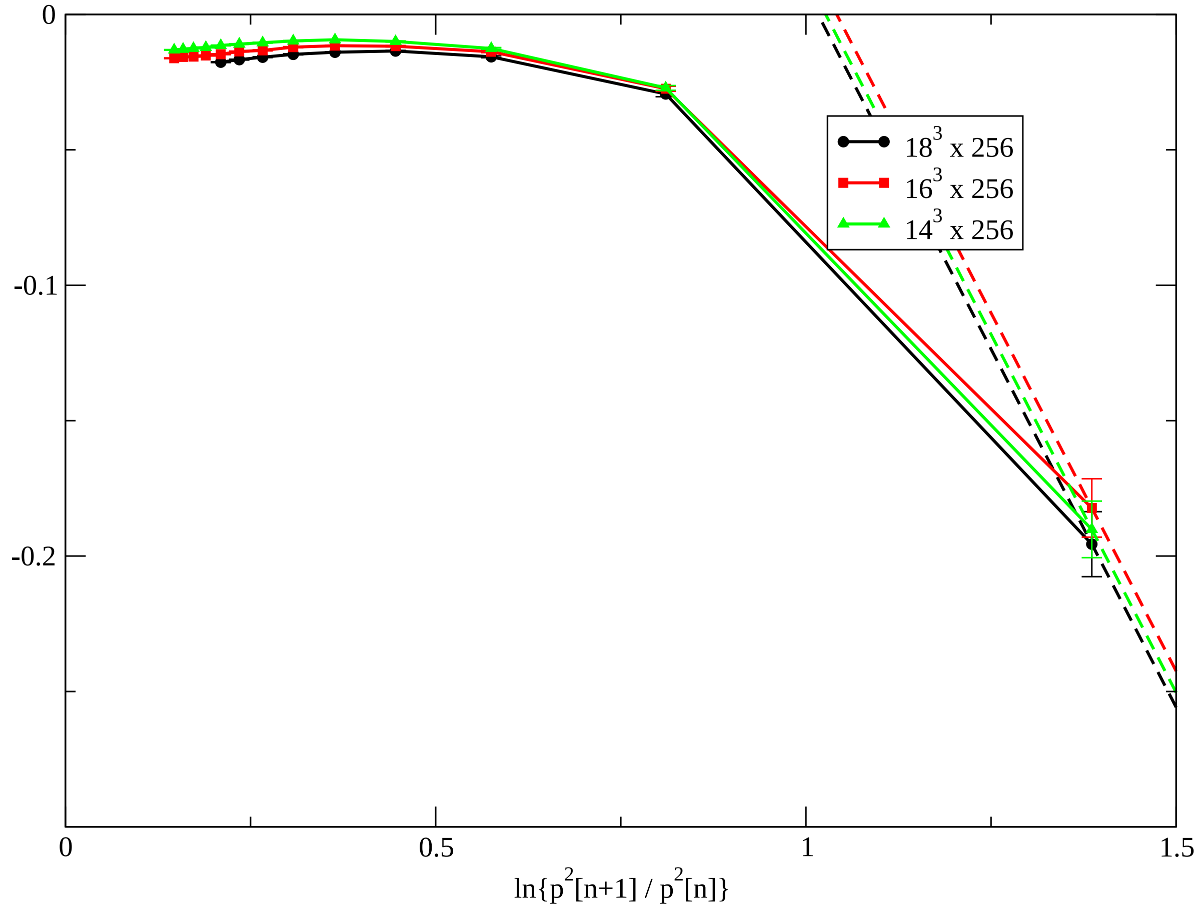
- linear
  $D(0) + bx$
  $D(0) = 71.7$
- quadratic
  $D(0) + bx + cx^2$
  $D(0) = 68.9$
- power law
  ax^b, $D(0) = 0$
  $b = 0.10$

No conclusive answer!!!

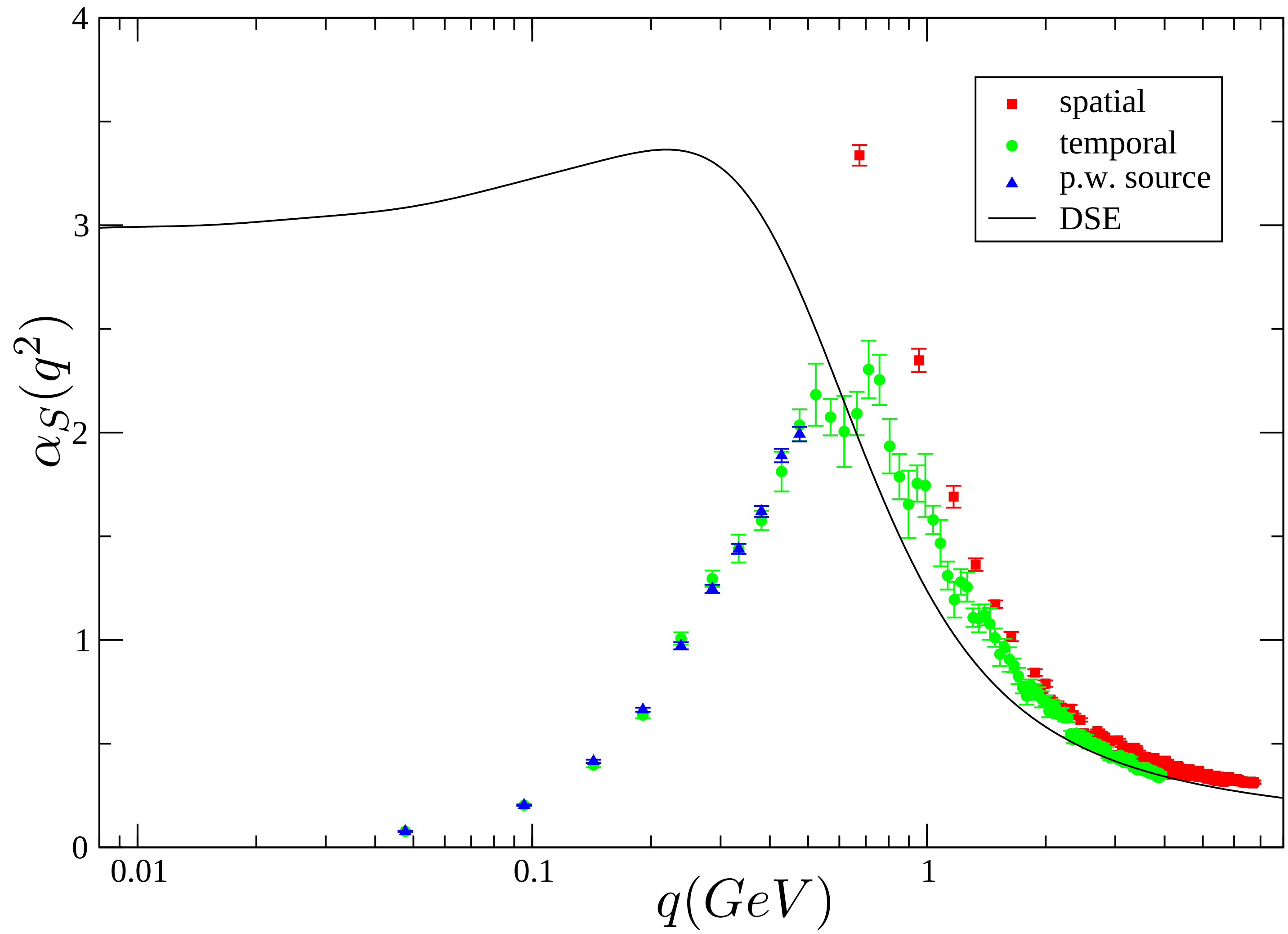
$b = 0.10$ very similar to that obtained in DSE on a torus ($b \sim 0.095$)

Ratios for ghost dressing function

- No linear behaviour!
- Far from $\kappa \sim 0.53$
- Lower bound: $\kappa \sim 0.29$ (two lowest points)



Strong running coupling – $18^3 \times 256$



Conclusions

- New method for extracting infrared exponents
- without relying in extrapolations to infinite volume
- Gluon propagator:
  - stable, volume independent $\kappa \sim 0.53$
  - compares well with results from 80^4 SU(2)
  - modelling the finite volume effects

$$Z_{latt}(q^2) \; = \; Z_{cont}(q^2) \, e^{Aq}$$

    volume dependent A

- Ghost propagator:
  - No pure power law behaviour; lower bound $\kappa > 0.29$
- **FUTURE WORK:**
  - careful analysis of lattice effects under way
  - larger lattices