Schwinger-Dyson Studies in Coulomb gauge

P. Watson

Institüt für Theoretische Physik
Universität Tübingen

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Phys. Rev. D75:045021 (2007) [arXiv:hep-th/0612114];
Phys. Rev. D76:125016 (2007) [arXiv:0709.0140];
Phys. Rev. D77:025030 (2008) [arXiv:0709.3963].

Outline

- Why Coulomb gauge?
  (comparison to other gauges/approaches notwithstanding)
  - Gauß' law
  - formal physicality
- DSe's
  - structure
  - some perturbative results
- STid's (realising the formal?)
  - Gauß-BRST invariance
  - 2-point functions
  - 3-point functions...
- Summary and outlook

Gauß’ law

Reminder – electrodynamics ($\sigma \equiv A^0$):

$$\begin{aligned}\mathcal{S} &= \int \left[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}\right] = \int \left[\frac{1}{2}E^2 - \frac{1}{2}B^2\right] \\ \vec{E} &= -\partial_t \vec{A} - \vec{\nabla}\sigma\end{aligned}$$

equation of motion for $\vec{E}$-field: $-\nabla^2\sigma - \partial_t \vec{\nabla}\cdot\vec{A} = -\rho$
Coulomb gauge, $\vec{\nabla}\cdot\vec{A} = 0 \longrightarrow$ constraint equation for σ.

$\Rightarrow$ charges and fields are closely related in Coulomb gauge
Gauß’ law is especially prominent

QCD: color confinement needs charge conservation/Gauß’ law,
we have fields...

1st order formalism

Consider the functional integral

$$Z = \int \mathcal{D}\Phi \exp\{\imath \mathcal{S}_{YM}\}$$

where $\mathcal{S}_{YM}$ is invariant under gauge transforms:

$$\begin{aligned}
\mathcal{S}_{YM} &= \int \left[-\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu}\right] = \int \left[\frac{1}{2}E^2 - \frac{1}{2}B^2\right] \\
\vec{E}^a &= -\partial_t \vec{A}^a - \vec{D}^{ac}\sigma^c \qquad \left(\vec{D}^{ac} = \delta^{ac}\vec{\nabla} - gf^{abc}\vec{A}^b\right)
\end{aligned}$$

Avoid zero modes by fixing the (Coulomb) gauge:

$$Z \to \int \mathcal{D}\Phi \;\mathrm{det}\left[-\vec{\nabla}\cdot\vec{D}\right]\delta(\vec{\nabla}\cdot\vec{A}) \exp\{\imath \mathcal{S}_{YM}\}$$

1st order formalism

Introduce 'momentum conjugate' field $\vec{\pi}^a$:

$$\exp\left\{\imath\int\frac{1}{2}E^2\right\} = \int\mathcal{D}\pi\exp\left\{\imath\int\left[-\frac{1}{2}\pi^2 - \vec{\pi}^a\cdot\vec{E}^a\right]\right\}$$

E^2 has 4-point interactions, $\vec{\pi}\cdot\vec{E}$ has only 3-point interactions.

$\Rightarrow$ 1st order formalism may be 'simpler'.

$\Rightarrow$ action is linear in σ.

Decompose $\vec{\pi}$ into transverse ($\vec{\nabla}\cdot\vec{\pi} = 0$) and longitudinal ($\vec{\nabla}\phi$) parts…

1st order formalism

Implement integration over σ-field as δ-functional:

$$\delta(-\vec{\nabla}\cdot\vec{D}^{ab}\phi^b - gf^{ade}\vec{A}^d\cdot\vec{\pi}^e), \quad \text{(Gauß' law)}$$

define inverse Faddeev-Popov operator: $-\vec{\nabla}\cdot\vec{D}^{ab}M^{bc} = \delta^{ac}$

$$= \det\left[-\vec{\nabla}\cdot\vec{D}\right]^{-1}\delta(\phi^a - M^{ac}gf^{cde}\vec{A}^d\cdot\vec{\pi}^e)$$

Inverse determinant cancels ghost sector!

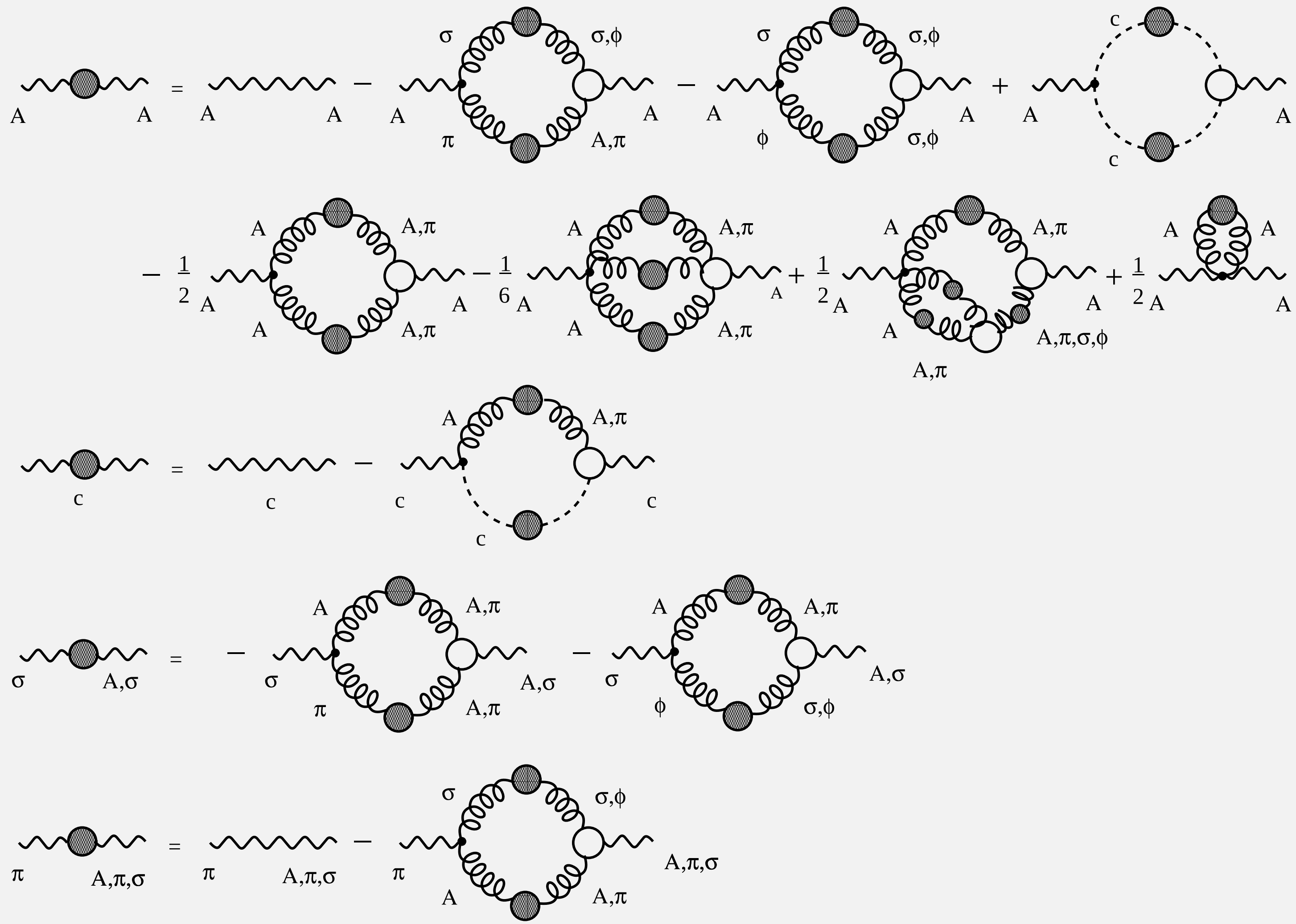
$$\begin{aligned} Z &= \int \mathcal{D}\Phi\delta(\vec{\nabla}\cdot\vec{A})\delta(\vec{\nabla}\cdot\vec{\pi})\exp\left\{\imath\mathcal{S}_B + \imath\mathcal{S}\right\} \\ \mathcal{S} &\sim \int\left[-\frac{1}{2}\pi^2 - \frac{1}{2}\left(\vec{A}\cdot\vec{\pi}\right)M(-\nabla^2)M\left(\vec{A}\cdot\vec{\pi}\right) + \vec{\pi}\cdot\partial_t\vec{A}\right] \end{aligned}$$

D. Zwanziger, Nucl. Phys. B518 (1998) 237.

1st order formalism

- 1st order formalism formally reduces to ‘physical’ (transverse) degrees of freedom
- Gauß’ law gets rid of the ghosts!
- BUT: we need a local theory
  – in practise, the above is still not obvious

DSe's (1st order formalism)



Perturbation theory

In Coulomb gauge,

$$D_{AA}(k) \sim \left[\delta_{ij} - \frac{k_i k_j}{\vec{k}^2}\right] \frac{1}{(k_0^2 - \vec{k}^2)}$$

and this gives rise to (nasty) Euclidean loop integrals of the form

$$I(k_4^2, \vec{k}^2) = \int \frac{\bar{d}\omega\ \omega_4}{\omega^2 (k-\omega)^2 \vec{\omega}^2}.$$

We need a new technique!

Differential equations and integration by parts

Notice that everything is a function of energy *and* momentum, unlike covariant gauges.

Perturbation theory

Solution with $x = k_4^2$, $y = \vec{k}^2$, $z = x/y$:

$$I(x,y) \;=\; k_4 \frac{(x+y)^{-1-\varepsilon}}{(4\pi)^{2-\varepsilon}}(1+z)f(z)$$

$$f(z) \;=\; 4\ln 2\frac{\arctan(\sqrt{z})}{\sqrt{z}} - \int\limits_0^1 \frac{dt\;\ln(1+zt)}{\sqrt{t}(1+zt)}$$

(integral is dilogarithmic)

- first part of integral is standard $\ln(k^2)/k^2$
- rest is some (finite) function of z
- singularities at $z = -1$ (*not* $z = 0$)

Perturbation theory

Results:
with $g(z) = 2\ln 2 - \ln(1+z)$

$$\begin{aligned} D_{AA}(x,y) &\sim& 1 + \frac{g^2 N_c}{(4\pi)^{2-\varepsilon}} \left\{ \left[\frac{1}{\varepsilon} - \gamma - \ln\left(\frac{x+y}{\mu}\right)\right] - \frac{64}{9} + 3z \right. \\ && \left. + \left[-\frac{1}{2z} + \frac{14}{3} - \frac{3}{2}z \right] g(z) + \left[\frac{1}{4z} - \frac{1}{4} + \frac{11}{4}z - \frac{3}{4}z^2 \right] f(z) \right\} \\ D_{\sigma\sigma}(x,y) &\sim& \ldots \end{aligned}$$

Features:

- Explicit non-covariance, sometimes even for $1/\varepsilon$.
- Singularity at $z = -1$ (light-cone) only.
- $g^2 D_{AA} D_c^2$ *and* $g^2 D_{\sigma\sigma}$ renormalisation group invariant.
- 1st and 2nd order formalism results obtained.

Slavnov-Taylor identities*

Action is invariant under a Gauß-BRST transform

– a *time-dependent* BRS transform:

$$\theta_x^a = c_x^a \delta\lambda_t.$$

– timescale t serves to inject energy q_0 into the identities!

NB*: 2nd order formalism here

Slavnov-Taylor identities

After some work, for the 2-point functions...

$$
\begin{aligned}
k_0\Gamma_{\sigma\sigma}(k_0,\vec{k}) &= \imath\frac{k_i}{\vec{k}^2}\Gamma_{\sigma Ai}(k_0,\vec{k})\Gamma_c(q_0+k_0,\vec{k}) \\
k_0\Gamma_{A\sigma k}(k_0,\vec{k}) &= \imath\frac{k_i}{\vec{k}^2}\Gamma_{AAki}(k_0,\vec{k})\Gamma_c(q_0+k_0,\vec{k})
\end{aligned}
$$

- gluon polarization is not transverse (even at tree-level)
- inverse ghost propagator independent of energy
- σ Green's functions known in terms of others
  – (local) elimination of σ-field (Gauß' law)!
- G.I. $\leftrightarrow$ STid $\leftrightarrow$ Gauß' law $\leftrightarrow$ charge cons. $\leftrightarrow$ Kugo-Ojima
  – all the same in Coulomb gauge!

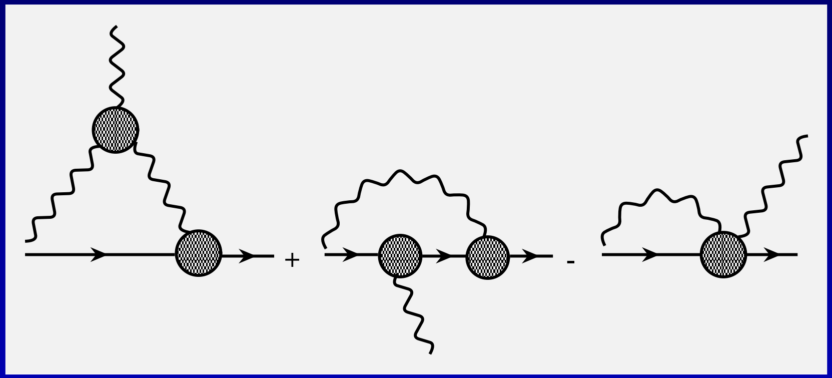
Slavnov-Taylor identities

After some more work, for the 3-point functions...

$$k_0\Gamma_{XY\sigma} + k_i\Gamma_{XYAi} \sim \Gamma_{XY}\tilde{\Gamma}_{\text{nasty}}$$

where $\tilde{\Gamma}_{\text{nasty}}$ is familiar from covariant gauges:

$\tilde{\Gamma}_{\text{nasty}} \sim$



neglecting ghosts, STid's can be solved for $\Gamma_{XY\sigma}$ (IR approx?)

NB. No 'transverse' part because energy is scalar – noncovariance is good for this!

Slavnov-Taylor identities

It turns out though...

$$
\begin{aligned}
\tilde{\Gamma}_{\text{nasty}} &\sim F\left(\Gamma_{\bar{c}c\sigma}, \Gamma_{\bar{c}cA\sigma}\Gamma_{\bar{c}c\sigma\sigma}, \ldots\right) \\
\Gamma_{\bar{c}c\sigma} &\sim F\left(\Gamma_{\bar{c}c\sigma}, \ldots\right) \\
\Gamma_{\bar{c}cX\sigma} &\sim F\left(\Gamma_{\bar{c}c\sigma}, \Gamma_{\bar{c}cA\sigma}, \Gamma_{\bar{c}c\sigma\sigma}, \Gamma_{\bar{c}c\bar{c}c\sigma}, \ldots\right) \\
\Gamma_{\bar{c}c\bar{c}c\sigma} &\sim F\left(\Gamma_{\bar{c}c\bar{c}c\sigma}, \ldots\right)
\end{aligned}
$$

can solve STid's for σ-functions without truncation because the system of equations closes!

but

Slavnov-Taylor identities

- σ Green's functions known in terms of others
  – (local) elimination of σ-field (Gauß' law)!
- G.I. $\leftrightarrow$ STid $\leftrightarrow$ Gauß' law $\leftrightarrow$ charge cons. $\leftrightarrow$ Kugo-Ojima
  – all the same in Coulomb gauge!
- STid's more powerful than Landau gauge
  – given (spatial) A and ghost function ansätze, can 'solve' STid's to get correct charges!
- STid's are the key to showing how the formal aspects of Coulomb gauge are realised in practise!

Summary and conclusions

- Coulomb gauge has great potential in understanding confinement
- there are significant technical issues to be overcome...
- ...and we're beginning to sort them out
- STid's hold the key to releasing the potential (or maybe even confining the potential)

Differential equation technique

First notice that:

$$
\begin{aligned}
k_4 \frac{\partial I}{\partial k_4} &= \int \frac{\bar{d}\omega \; \omega_4}{\omega^2 (k-\omega)^2 \vec{\omega}^2} \left\{ -2 \frac{k_4 (k_4 - \omega_4)}{(k-\omega)^2} \right\} \\
k_i \frac{\partial I}{\partial k_i} &= \int \frac{\bar{d}\omega \; \omega_4}{\omega^2 (k-\omega)^2 \vec{\omega}^2} \left\{ -2 \frac{\vec{k} \cdot (\vec{k} - \vec{\omega})}{(k-\omega)^2} \right\}
\end{aligned}
$$

and use integration by parts [IBP] identities:

$$
\begin{aligned}
0 &= \int \bar{d}\omega \; \frac{\partial}{\partial \omega_4} \frac{\omega_4^2}{\omega^2 (k-\omega)^2 \vec{\omega}^2} \\
0 &= \int \bar{d}\omega \; \frac{\partial}{\partial \omega_i} \frac{\omega_i \omega_4}{\omega^2 (k-\omega)^2 \vec{\omega}^2}
\end{aligned}
$$

Differential equation technique

Eventually…

$$k_4 \frac{\partial I}{\partial k_4} = \left[2(d-3)\frac{\vec{k}^2}{k^2} \right] I + 2\frac{\vec{k}^2}{k^2} \int \frac{\bar{d}\omega \;\omega_4}{(k-\omega)^4 \vec{\omega}^2}$$

$$-2 \int \frac{\bar{d}\omega \;\omega_4}{(k-\omega)^4 \omega^4} \left[(k-\omega)^2 + \omega^2 \right]$$

$$k_i \frac{\partial I}{\partial k_i} = -k_4 \frac{\partial I}{\partial k_4} + (d-4) I$$

- integrals can be done using standard techniques
- partial differential equations reduce with $I = FG$: F solves homogeneous problem, G the rest
- (boundary conditions are trivial)…

but. . .

14

$$
\begin{aligned}
&\tilde{\Gamma}^{adegf}_{\bar{c};\bar{c}ccc\bar{c}}(p_1,p_2,p_3,p_4,p_5) = gf^{abc}\int đk\, W^{b1}_{\bar{c}c}(k)W^{c2}_{\bar{c}c}(p_1-k)\Big\{-\Gamma^{1gfe2d}_{\bar{c}c\bar{c}c\bar{c}c}(k,p_4,p_5,p_3,p_1-k,p_2)\\
&+2\Gamma^{f42d}_{\bar{c}c\bar{c}c}(p_5,k+p_3+p_4,p_1-k,p_2)W^{43}_{\bar{c}c}(-k-p_3-p_4)\Gamma^{1g3e}_{\bar{c}c\bar{c}c}(k,p_4,-k-p_3-p_4,p_3)\\
&+2\Gamma^{f42e}_{\bar{c}c\bar{c}c}(p_5,k+p_2+p_4,p_1-k,p_3)W^{43}_{\bar{c}c}(-k-p_2-p_4)\Gamma^{1d3g}_{\bar{c}c\bar{c}c}(k,p_2,-k-p_2-p_4,p_4)\\
&+2\Gamma^{f42g}_{\bar{c}c\bar{c}c}(p_5,k+p_2+p_3,p_1-k,p_4)W^{43}_{\bar{c}c}(-k-p_2-p_3)\Gamma^{1e3d}_{\bar{c}c\bar{c}c}(k,p_3,-k-p_2-p_3,p_2)\\
&+2\Gamma^{fd2g\varepsilon}_{\bar{c}c\bar{c}c\varepsilon}(p_5,p_2,p_1-k,p_4,k+p_3)W^{\kappa\varepsilon}_{\kappa\varepsilon}(k+p_3)\Gamma^{1e\kappa}_{\bar{c}c\kappa}(k,p_3,-k-p_3)\\
&+2\Gamma^{fe2d\varepsilon}_{\bar{c}c\bar{c}c\varepsilon}(p_5,p_3,p_1-k,p_2,k+p_4)W^{\kappa\varepsilon}_{\kappa\varepsilon}(k+p_4)\Gamma^{1g\kappa}_{\bar{c}c\kappa}(k,p_4,-k-p_4)\\
&+2\Gamma^{fg2e\varepsilon}_{\bar{c}c\bar{c}c\varepsilon}(p_5,p_4,p_1-k,p_3,k+p_2)W^{\kappa\varepsilon}_{\kappa\varepsilon}(k+p_2)\Gamma^{1d\kappa}_{\bar{c}c\kappa}(k,p_2,-k-p_2)\\
&+2\Gamma^{fd\tau\varepsilon}_{\bar{c}c\tau\varepsilon}(p_5,p_2,p_1+p_3-k,k+p_4)\Gamma^{1g\kappa}_{\bar{c}c\kappa}(k,p_4,-k-p_4)W^{\kappa\varepsilon}_{\kappa\varepsilon}(k+p_4)W^{\tau\lambda}_{\tau\lambda}(k-p_1-p_3)\Gamma^{2e\lambda}_{\bar{c}c\lambda}(p_1-k,p_3,k-p_1-p_3)\\
&+2\Gamma^{fe\tau\varepsilon}_{\bar{c}c\tau\varepsilon}(p_5,p_3,p_1+p_4-k,k+p_2)\Gamma^{1d\kappa}_{\bar{c}c\kappa}(k,p_2,-k-p_2)W^{\kappa\varepsilon}_{\kappa\varepsilon}(k+p_2)W^{\tau\lambda}_{\tau\lambda}(k-p_1-p_4)\Gamma^{2g\lambda}_{\bar{c}c\lambda}(p_1-k,p_4,k-p_1-p_4)\\
&+2\Gamma^{fg\tau\varepsilon}_{\bar{c}c\tau\varepsilon}(p_5,p_4,p_1+p_2-k,k+p_3)\Gamma^{1e\kappa}_{\bar{c}c\kappa}(k,p_3,-k-p_3)W^{\kappa\varepsilon}_{\kappa\varepsilon}(k+p_3)W^{\tau\lambda}_{\tau\lambda}(k-p_1-p_2)\Gamma^{2d\lambda}_{\bar{c}c\lambda}(p_1-k,p_2,k-p_1-p_2)\\
&+2\Gamma^{f3\varepsilon}_{\bar{c}c\varepsilon}(p_5,-k-p_2-p_5,k+p_2)\Gamma^{1d\kappa}_{\bar{c}c\kappa}(k,p_2,-k-p_2)\Gamma^{2e\lambda}_{\bar{c}c\lambda}(p_1-k,p_3,k-p_1-p_3)\Gamma^{4g\tau}_{\bar{c}c\tau}(k+p_2+p_5,p_4,p_1+p_3-k)\\
&\times W^{\kappa\varepsilon}_{\kappa\varepsilon}(k+p_2)W^{\tau\lambda}_{\tau\lambda}(k-p_1-p_3)W^{34}_{\bar{c}c}(k+p_2+p_5)\\
&+2\Gamma^{f3\varepsilon}_{\bar{c}c\varepsilon}(p_5,-k-p_3-p_5,k+p_3)\Gamma^{1e\kappa}_{\bar{c}c\kappa}(k,p_3,-k-p_3)\Gamma^{2g\lambda}_{\bar{c}c\lambda}(p_1-k,p_4,k-p_1-p_4)\Gamma^{4d\tau}_{\bar{c}c\tau}(k+p_3+p_5,p_2,p_1+p_4-k)\\
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&\times W^{\kappa\varepsilon}_{\kappa\varepsilon}(k+p_2)W^{\tau\lambda}_{\tau\lambda}(k-p_1-p_3)W^{34}_{\bar{c}c}(-k-p_2-p_4)\\
&+2\Gamma^{f3\tau}_{\bar{c}c\tau}(p_5,k+p_2+p_3,p_1+p_4-k)\Gamma^{1e\kappa}_{\bar{c}c\kappa}(k,p_3,-k-p_3)\Gamma^{2g\lambda}_{\bar{c}c\lambda}(p_1-k,p_4,k-p_1-p_4)\Gamma^{4d\varepsilon}_{\bar{c}c\varepsilon}(-k-p_2-p_3,p_2,k+p_3)\\
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&\times W^{\kappa\varepsilon}_{\kappa\varepsilon}(k+p_4)W^{\tau\lambda}_{\tau\lambda}(k-p_1-p_2)W^{34}_{\bar{c}c}(-k-p_3-p_4)\\
&-2\Gamma^{f3\varepsilon}_{\bar{c}c\varepsilon}(p_5,-k-p_3-p_5,k+p_3)\Gamma^{4d2g}_{\bar{c}c\bar{c}c}(k+p_3+p_5,p_2,p_1-k,p_4)\Gamma^{1e\kappa}_{\bar{c}c\kappa}(k,p_3,-k-p_3)\\
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&+2\Gamma^{f42e}_{\bar{c}c\bar{c}c}(p_5,k+p_2+p_4,p_1-k,p_3)\Gamma^{1g\kappa}_{\bar{c}c\kappa}(k,p_4,-k-p_4)\Gamma^{3d\varepsilon}_{\bar{c}c\varepsilon}(-k-p_2-p_4,p_2,k+p_4)\\
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&-2\Gamma^{f42g}_{\bar{c}c\bar{c}c}(p_5,k+p_2+p_3,p_1-k,p_4)\Gamma^{1e\kappa}_{\bar{c}c\kappa}(k,p_3,-k-p_3)\Gamma^{3d\varepsilon}_{\bar{c}c\varepsilon}(-k-p_2-p_3,p_2,k+p_3)\\
&\times W^{\kappa\varepsilon}_{\kappa\varepsilon}(k+p_3)W^{43}_{\bar{c}c}(-k-p_2-p_3)\Big\}\,.
\end{aligned}
\tag{3.24}
$$

$$\tilde{\Gamma}^{adegf}_{\bar{c};\bar{c}cccc\bar{c}}(p_1,p_2,p_3,p_4,p_5)!!!$$